

Pion and proton transparencies in a relativistic multiple-scattering Glauber approach

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FACULTEIT WETENSCHAPPEN

Outline

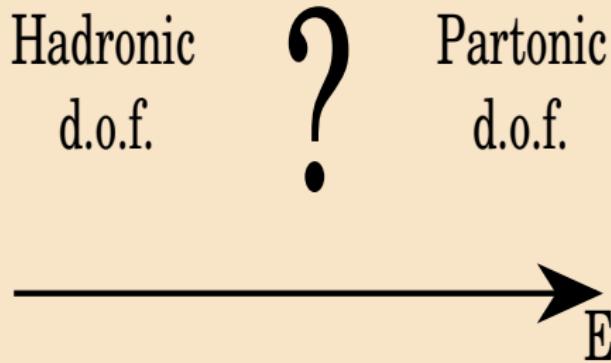
1 Introduction and Motivation

2 Model description

3 Applications and Results

4 Conclusion

Motivation



When and in what way?

- Emergence of the concepts of “nuclear physics” (**baryons and mesons**) out of QCD (**quarks and gluons**) remains elusive
- Explore the limits of the shell model description of nuclei
- Use of removal reactions

Motivation

- Emergence of the concepts of “nuclear physics” (**baryons and mesons**) out of QCD (**quarks and gluons**) remains elusive
- Explore the limits of the shell model description of nuclei
- Use of **removal reactions**

Fig. from Jefferson Lab

- Look for phenomena predicted in QCD that introduce **deviations** from traditional nuclear physics observations
- the nuclear transparency as a function of a tunable scale parameter (t or Q^2) is a good quantity to study the crossover between the two regimes

Nuclear transparency:

effect of nuclear attenuations on escaping hadrons

$$T(A, Q^2) = \frac{\text{cross section on a target nucleus}}{A \times \text{cross section on a free nucleon}}$$

- Interpretation of the transparency experiments requires the availability of reliable and advanced traditional nuclear-physics calculations to compare the data with

Building a Model



- * To interpret the data from experiments, comparison to results from up-to-date **nuclear models** is necessary to identify deviations originating from QCD effects
- * Semi-classical models are available
- * Develop a **relativistic and quantum mechanical** model

Ingredients

- **Relativistic wave functions** for beam, target and residual nucleus, outgoing particles
- **Impulse approximation:** incoming particle (leptonic or hadronic) interacts with one nucleon
- Describe the **final state interactions** of the ejected particles with **Glauber scattering theory**
NPA A728 (2003) 226
- The possibility to use FSI based on an **optical potential** at low ejectile momenta (ROMEA)

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NPA **A728** (2003) 226
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Glauber scattering theory



- Uses the **eikonal approximation**, originating from optics: $\phi_{\text{out}}(\vec{r}) = e^{i\chi(\vec{r})} \phi_{\text{in}}(\vec{r}) = (1 - \Gamma(\vec{r}))\phi_{\text{in}}(\vec{r})$
- Works when the wavelength of the particle is a lot smaller than the range of the scattering potential → **OK** for the performed experiments!
- Particles scatter over small angles and follow a **linear trajectory**
- Second order eikonal corrections have been computed → **small**

Profile function in $N - N$ and $\pi - N$ scattering

$$f(\Delta, E) = \frac{K}{2\pi i} \int d\vec{b} e^{i\vec{\Delta} \cdot \vec{b}} \Gamma_{\pi N}(\vec{b})$$

- Profile function can be related to the scattering amplitude
- Three energy-dependent parameters
 - total cross section
 - slope parameter
 - real to imaginary ratio
- Fit parameters to $N - N$ and $\pi - N$ scattering data
- range $\sqrt{2}\beta$ is of the order 0.75 fm → short range

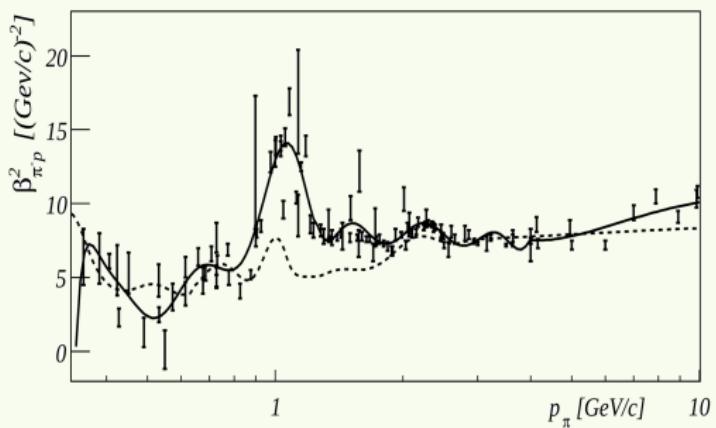
Profile function in $N - N$ and $\pi - N$ scattering

$$\Gamma_{\pi N}(\vec{b}) = \frac{\sigma_{\pi N}^{\text{tot}}(1 - i\epsilon_{\pi N})}{4\pi\beta_{\pi N}^2} \exp\left(-\frac{\vec{b}^2}{2\beta_{\pi N}^2}\right)$$

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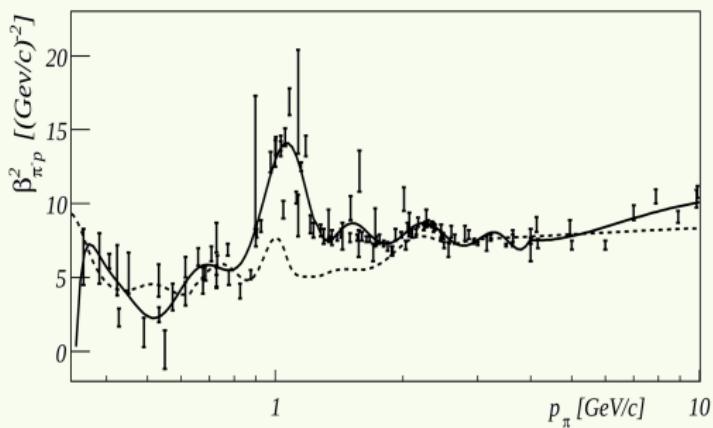
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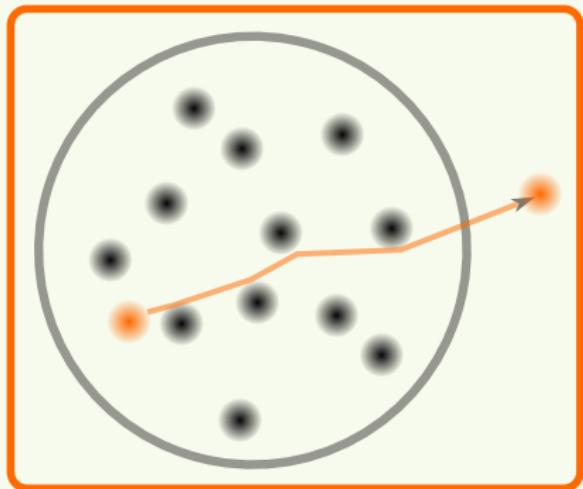
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Relativistic Multiple-Scattering Glauber Approximation

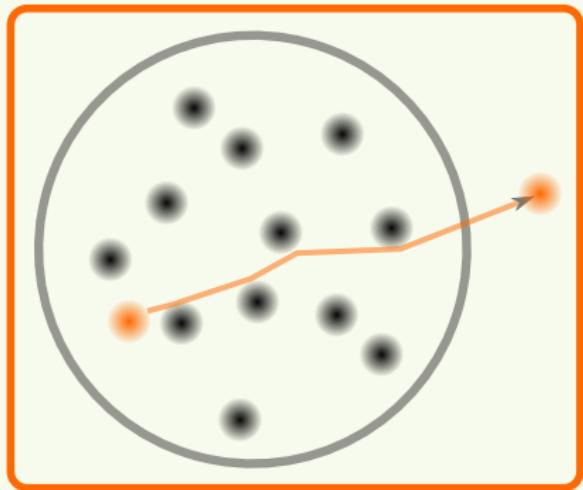


Multiple scattering

- Frozen approximation is adopted
- Phase-shift additivity
$$e^{i\chi_{\text{tot}}} = \prod_i (1 - \Gamma_i(\vec{b}_i))$$
- Profile functions are weighted with the Dirac wave function
- Only nucleons in forward path contribute

$$\mathcal{G}(\vec{b}, z) = \prod_{a_{\text{occ}} \neq a} \left[1 - \int d\vec{r}' |\phi_{a_{\text{occ}}}(\vec{r}')|^2 [\theta(z' - z) \Gamma(\vec{b}' - \vec{b})] \right]$$

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$\vec{b}, z \rightarrow$ point of creation

Implementing Short-Range Correlations

- In standard Glauber: effect of intranuclear attenuations is computed as if the density remains unaffected by the presence of a nucleon at $\vec{r} = (\vec{b}, z)$
- $\sqrt{2}\beta \sim 0.75\text{fm}$ → attenuations will be mainly affected by the **short-range structure** of the transverse density in the residual nucleus
- Mean field does not contain repulsive short-range behavior of the $N - N$ force
- Introduce **correlated** two-body density

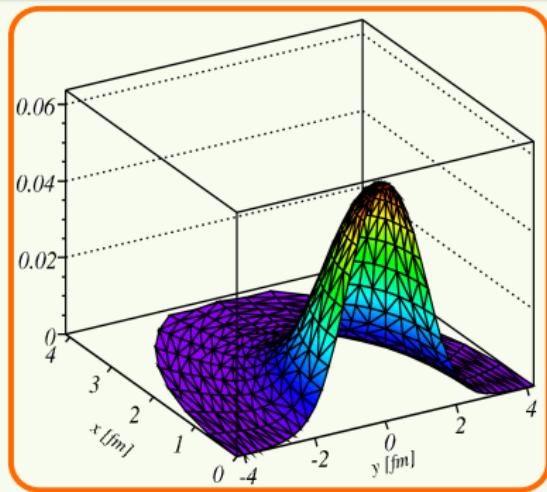
$$\rho_A^{[2]}(\vec{r}', \vec{r}) = \frac{A-1}{A} \gamma(\vec{r}) \rho_A^{[1]}(\vec{r}) \gamma(\vec{r}') \rho_A^{[1]}(\vec{r}') g(|\vec{r} - \vec{r}'|)$$

- $\gamma(\vec{r})$ ensures normalization

Densities in Glauber calculations (${}^4\text{He}$ case)

A nucleon or pion is created in the center of ${}^4\text{He}$: how does the nuclear density looks like for this hadron?

${}^4\text{He}$ in IPM

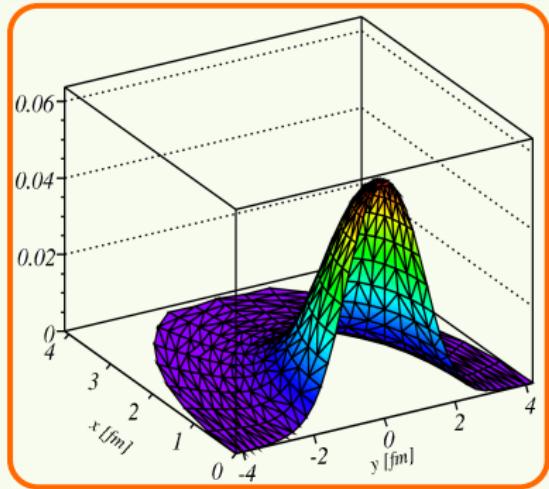


${}^4\text{He}$ with SRC

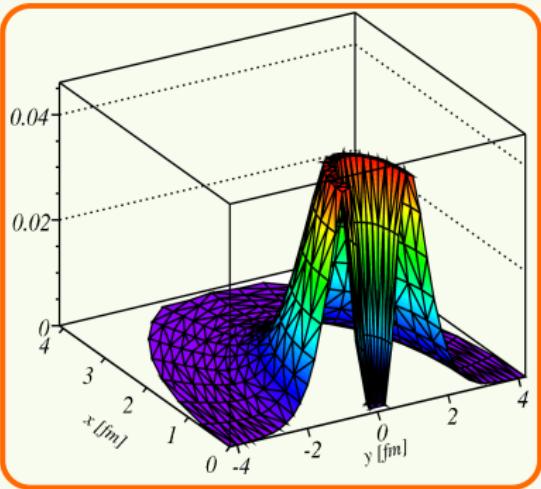
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Color Transparency: Quantum diffusion parametrization

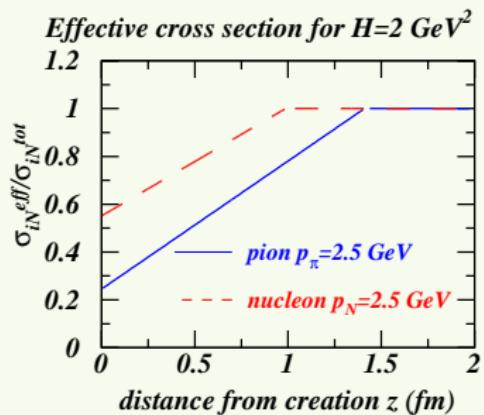
$$\sigma_{iN}^{\text{eff}}(z) = \sigma_{iN}^{\text{tot}} \left\{ \left[\frac{z}{l_h} + \frac{\langle n^2 k_t^2 \rangle}{\mathcal{H}} \left(1 - \frac{z}{l_h} \right) \theta(l_h - z) \right] + \theta(z - l_h) \right\} i = \pi \text{ or } N.$$

- Replace the total cross section with an **effective** one
- Parameters are based on theoretical grounds but values are **educated guesses**
- Pion cross section is more strongly reduced and formation length is **longer**

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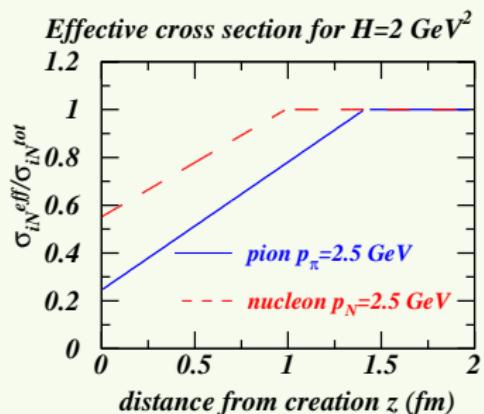
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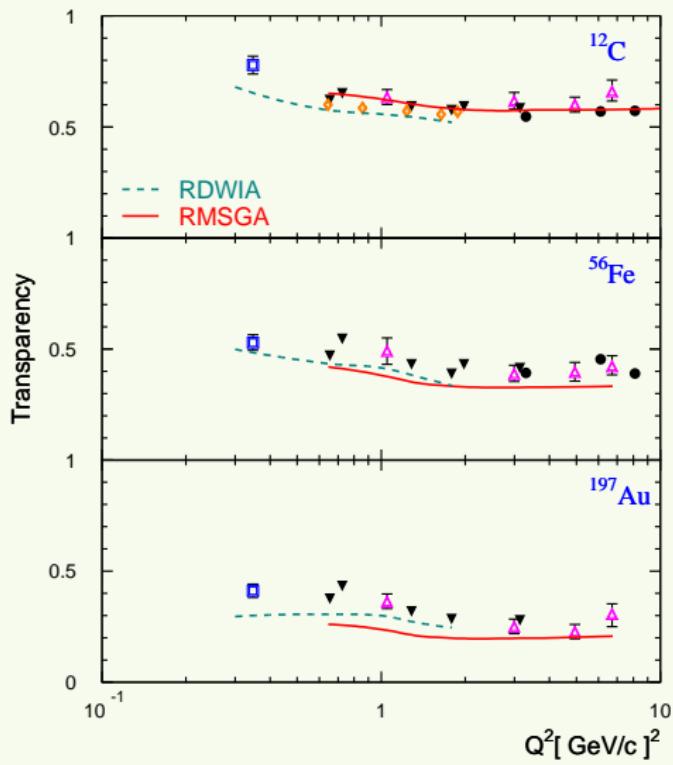
Applications



RMSGA can be applied to a variety of reactions, with incoming leptons or hadrons, outgoing nucleons or mesons, ...

- $A(e, e'\pi)$
- $A(p, 2p)$
- $A(e, e'p)$
- $A(\nu, \nu'p)$
- $A(e, e'NN)$
- ...

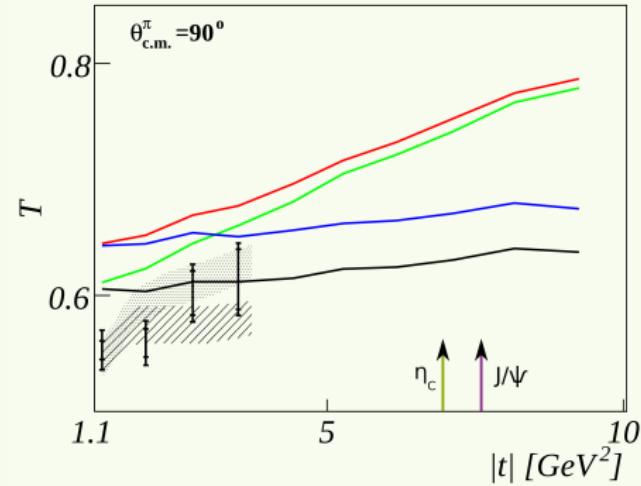
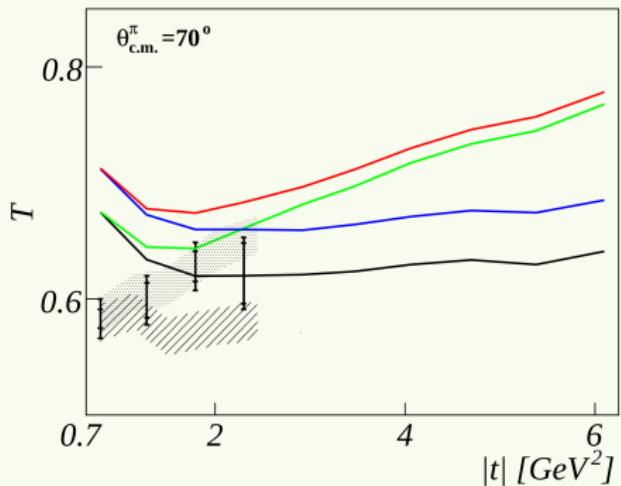
The nuclear transparency from $A(e, e' p)$



- Calculations tend to underestimate the measured proton transparencies
- In the region of overlap: RMSGA and RDWIA predictions are **not** dramatically different !!
- Data from **MIT**, **JLAB** and **SLAC**
- CT effects are very small for $Q^2 \leq 10 \text{ GeV}^2$

P. Lava et al. PLB595 (2004), 177–186

$^4\text{He}(\gamma, p\pi^-)$ transparencies



Glauber

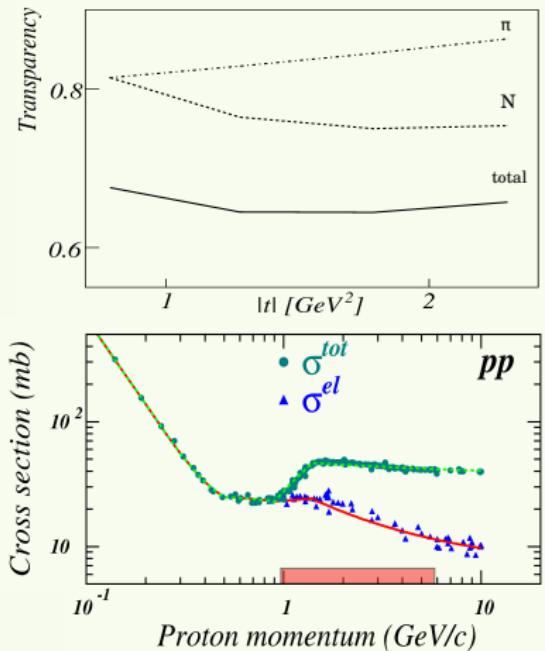
Glauber + SRC

Glauber + CT

Glauber + SRC + CT

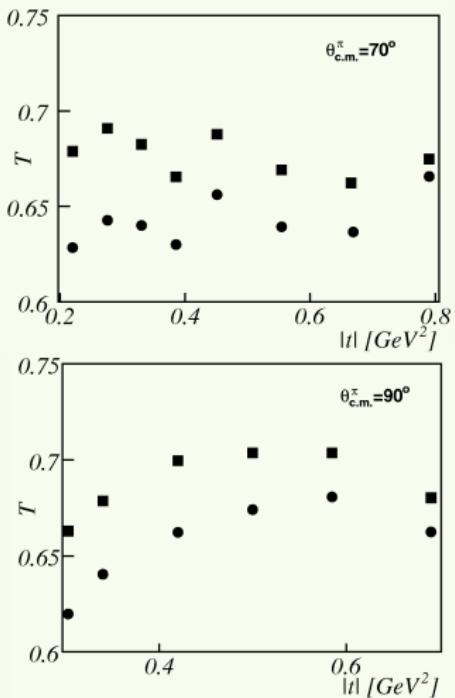
- **Theory:** W. Cosyn et al., PRC74 (2006) 062201
- **Data:** D. Dutta et al., PRC68 (2003) 021001
- **Semiclassical theory:** H. Gao et al., PRC54 (1996) 2779
[normalized to first data point]

${}^4\text{He}(\gamma, p\pi^-)$ transparencies (II)



- Pion transparency is larger than nucleon one
- Low energy behavior can be attributed to nucleon → related to local minimum in σ_{NN}^{tot}
- How reliable are the transparency calculations? [robustness]
- Comparison with ROMEA model (based on nucleon-nucleus scattering) at very low momenta
- Difference about 5% and becomes smaller with rising energy

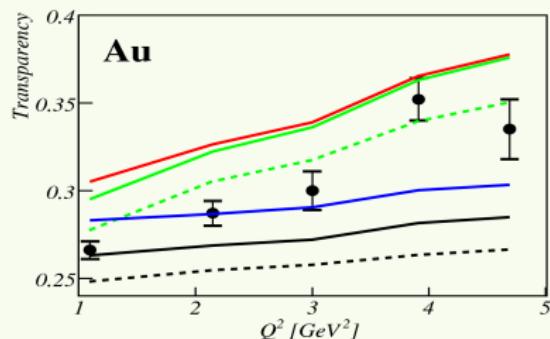
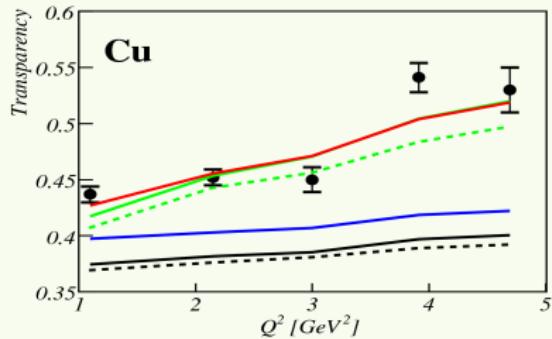
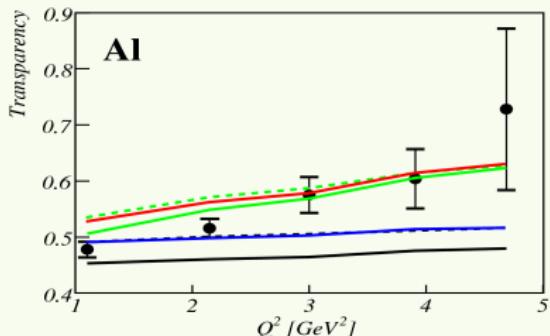
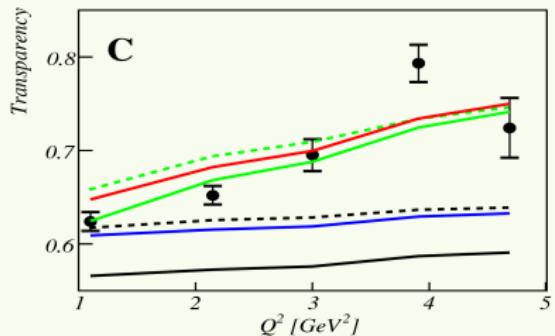
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RMSGA ■
ROMEA ●

$A(e, e' \pi^+)$ transparencies: Q^2 dependence



Glauber

Glauber + SRC

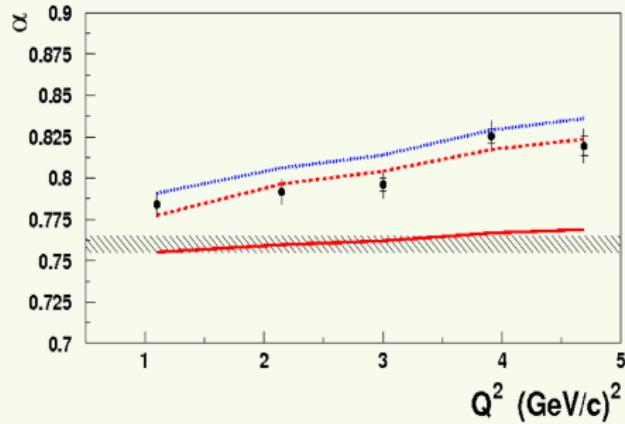
Glauber + CT

Glauber + SRC + CT

$A(e, e' \pi^+)$ data from JLab, B. Clasie *et al.*, PRL99 (2007) 242502

Dashed lines from semi-classical calc. by A. Larson *et al.*, PRC79 (2006) 018201

$A(e, e' \pi^+)$ transparencies: A dependence



GI.+SRC+CT

Semi-classical Larson

Hatched area: value from $\pi - A$ scatt.

- Parametrize $T = A^{\alpha-1}$
- Clear Q^2 dependence, deviates from expected value
- Models in good agreement

$A(e, e' \pi^+)$ transparencies: JLab 12 GeV

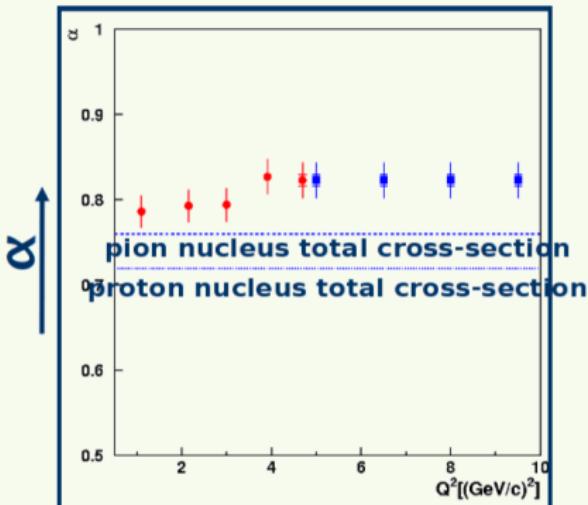
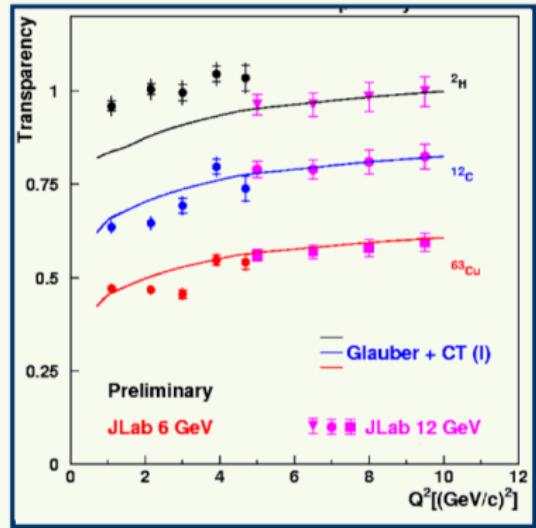
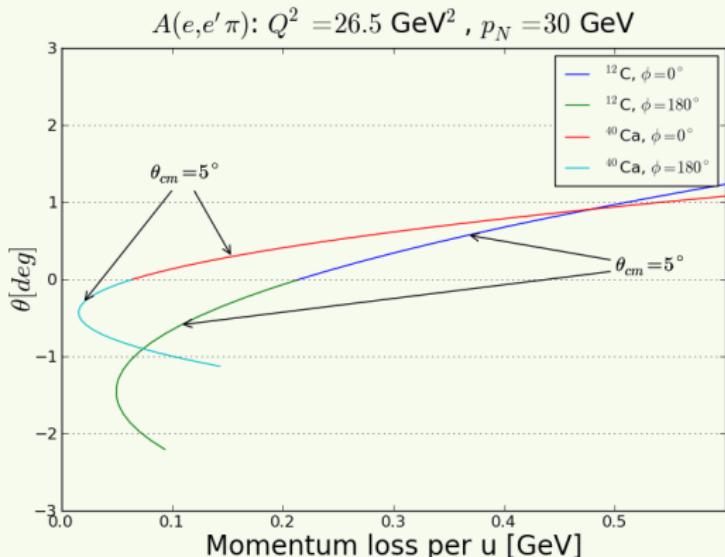


Fig from D. Dutta

Solid lines: Kundu et al., PRD 62,
113009 (2000)
Cu curve scaled to match data

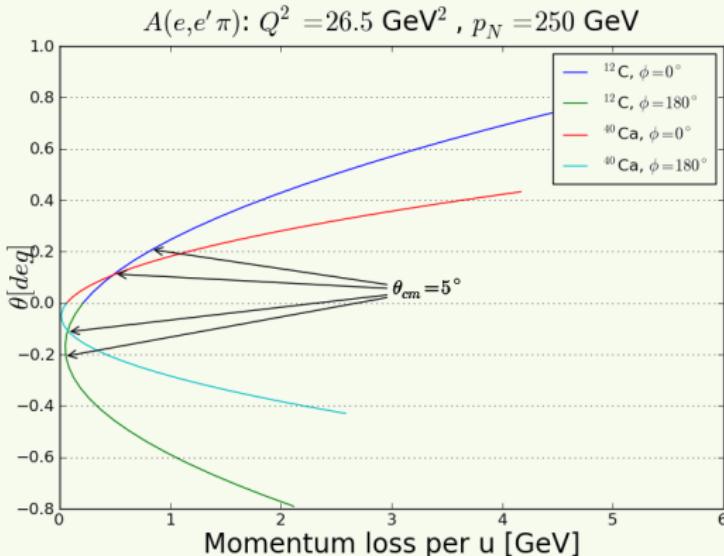
$A(e, e' \pi)$ results will verify the strict applicability of factorization theorems for meson electroproduction

$A(e, e' \pi^+)$ @ EIC: Recoil nucleus detection



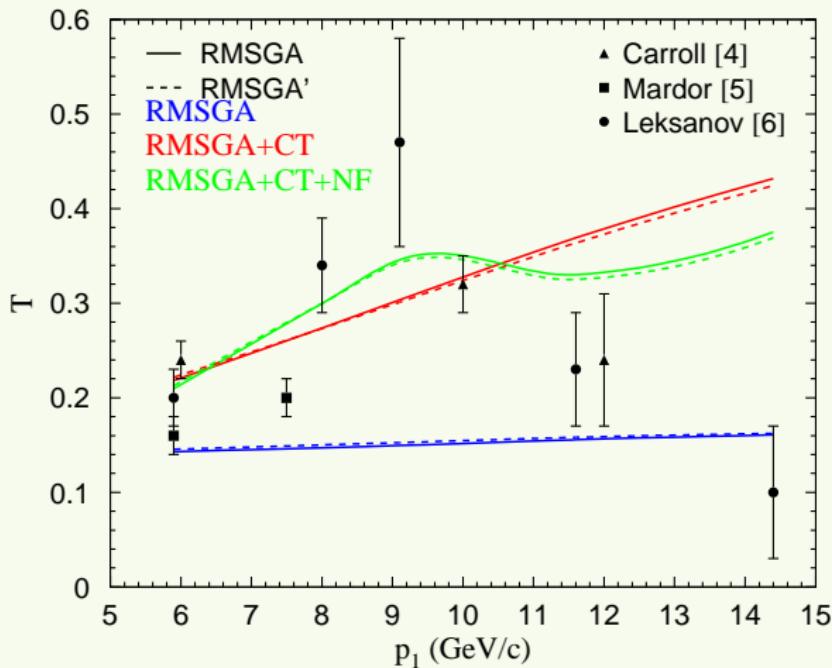
- very forward angles
- small energy loss
- same observations for $A(e, e' p)$ and keep $x \approx 1$

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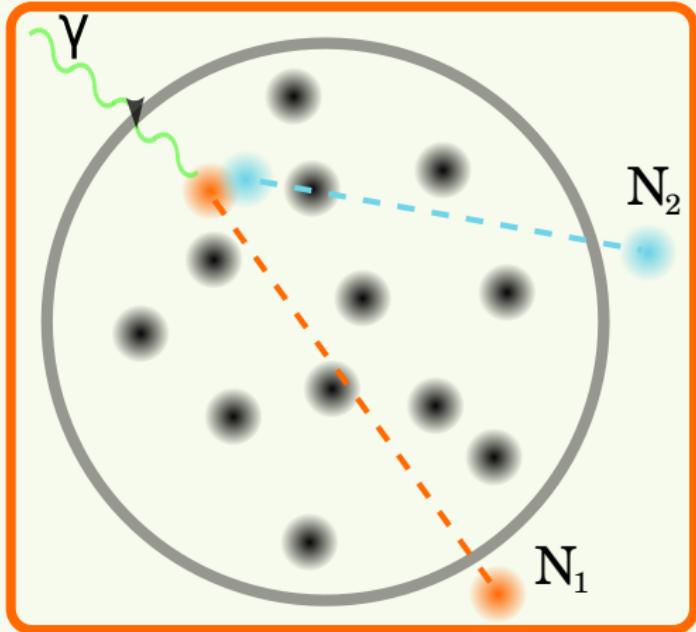
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The nuclear transparency from $^{12}\text{C}(p, 2p)$



Parameterization of the CT effects compatible with pion production results!

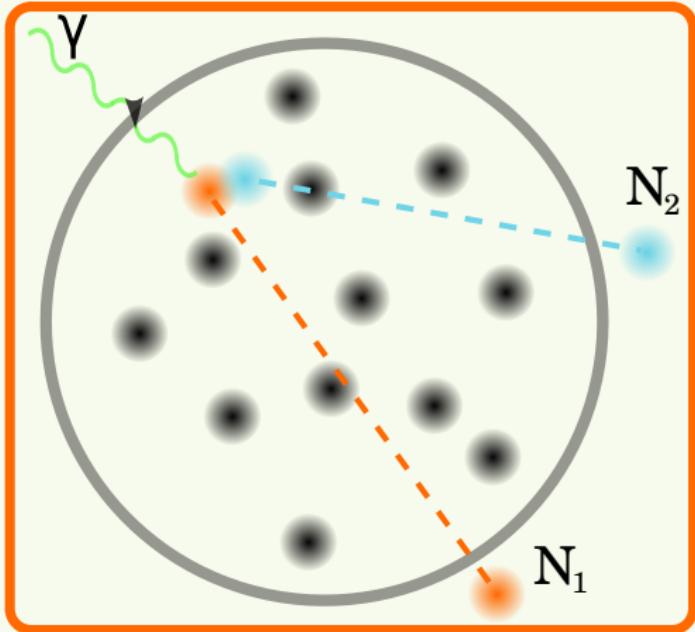
$A(\gamma, pp)$: two competing mechanisms



Knockout of a correlated pair

- **One-step:** beam interacts with one nucleon of the pair, the other nucleon is also ejected
- Two nucleons are assumed to reside in a **relative S-state** ($r_{12} \approx 0$)
- Cross section is unfactorized
- Calculations were done factorized to save computing time

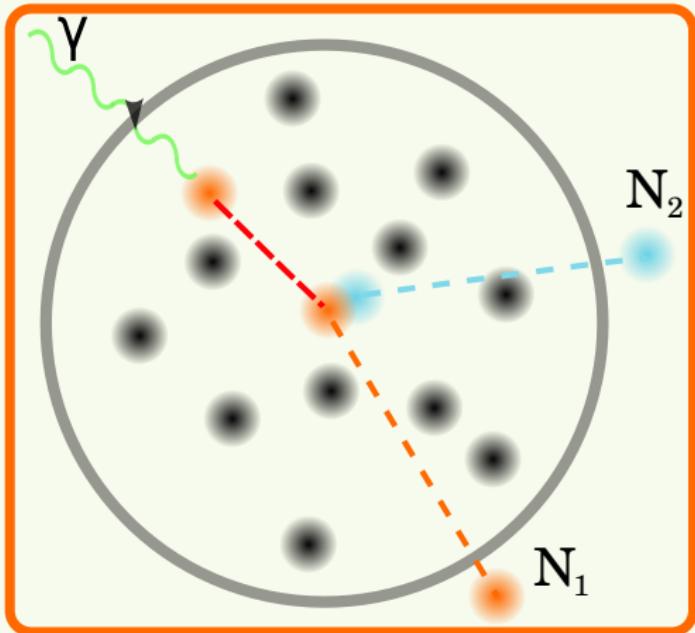
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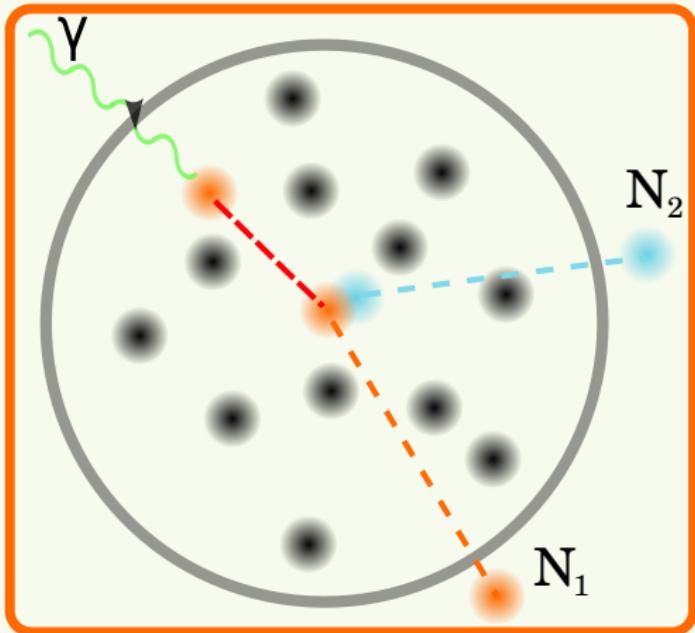
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Hard rescattering

- **Two-step:** beam interacts with a nucleon, nucleon then hits a second one
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- Propagator introduces extra degrees of freedom, a lot of nested integrations

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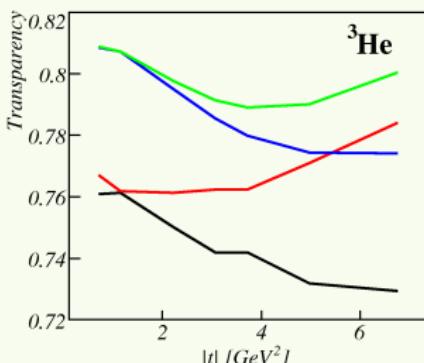
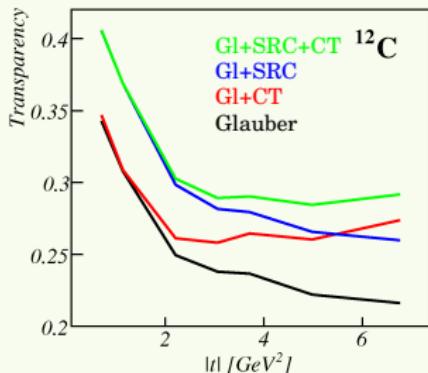


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Transparency calculations for $A(\gamma, pp)$

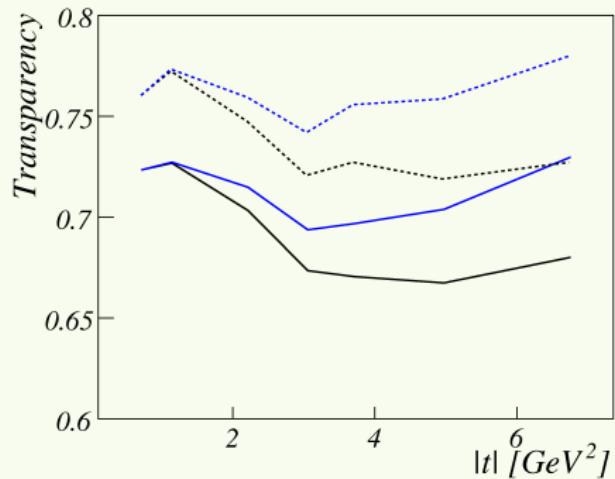
Corr. pair knockout



- Same dependence on the hard scale as the pion transparencies
- Low absolute values! → probes **high density regions** of the nucleus
- HRM transparencies are a little bit lower
- FSI of the propagator lowers the transparency by 5%

Transparency calculations for $A(\gamma, pp)$

Hard rescatt. on ${}^3\text{He}$

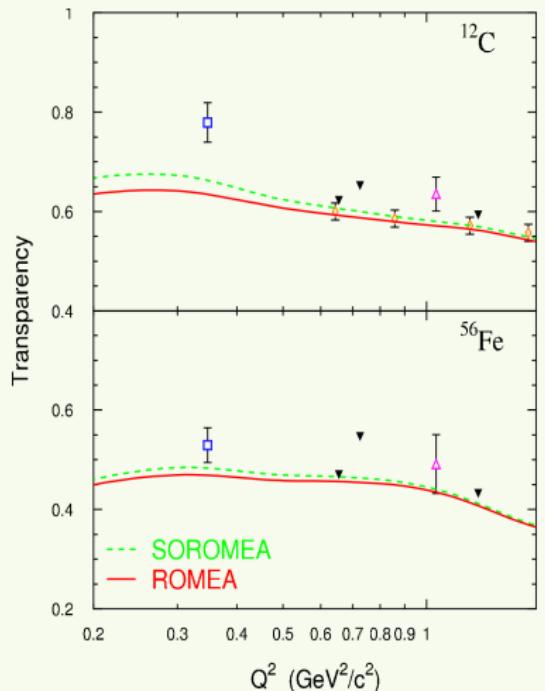


Glauber+CT Glauber

Dashed: without FSI for propagator

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Second-order eikonal corrections for $A(e, e' p)$

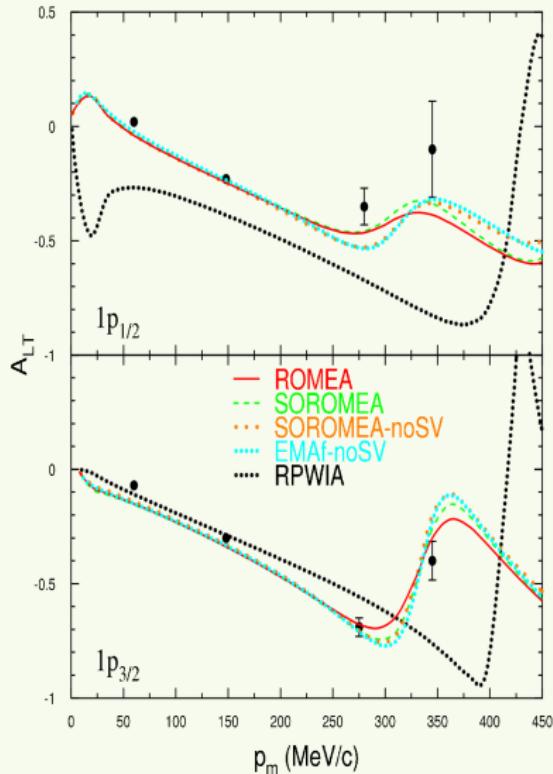


Second-order eikonal corrections to transparencies are very small !!

- One can compute so-called second-order eikonal corrections
- SOROMEA:** Second Order Relativistic Optical Model Eikonal Approximation
- Unfactorized: not an issue in transparency calculations!
- Unfactorized: observables like "left-right" asymmetries can be computed

B. Van Overmeire, J. Ryckebusch, PLB644, 304–310 (2007)

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Conclusions



A “flexible” eikonal framework to model the propagation of fast nucleons and pions through the nuclear medium

- Mean-field approach: can be applied to $A \geq 4$
- Can accommodate relativity (dynamics and kinematics).
- Can be used in combination with both optical potentials (pA) and Glauber Approach (pN).
- Glauber approach computes full $(A - 1)$ multiple-scattering series and has no free parameters
- Provides common framework to describe a variety of nuclear reactions with electroweak and hadronic probes.
- Effect of central short-range correlations and color transparency can be implemented

Conclusions (II)



- Good agreement with non-relativistic and optical potential calculations
- CT and SRC can be clearly separated, due to different hard scale dependence
- Pion electroproduction data in agreement with CT calculations
- EIC: recoil nucleus detection feasible?
- Fair results for $A(p, 2p)$
- Double nucleon knockout probes high density regions
- Second-order eikonal corrections are small