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SHORT-RANGE CORRELATIONS IN NUCLEI AND THEIR INVESTIGATION WITH THE REACTION (e, e'd)

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Abstract: A model-independent definition of short-range correlations is given in terms of the two-body density. It is contrasted to other concepts of the short-range correlations, particularly to deviations from a Slater determinant one-body density. It is shown how a two-nucleon knock-out reaction like (e, e'd) may give information about short-range correlations if its cross section is formulated in terms of a two-hole spectral function. This cross section is calculated in a simple model to demonstrate the possibility of extracting detailed information about two-body quantities from a complicated expression for the cross section. The results show that the distortion of the outgoing deuteron does not obscure the dependence on the relative motion of the two nucleons before the reaction.

1. Introduction

The experimental search for and the theoretical description of short-range correlations in nuclei (SRC) has found wide interest in recent years. There were rather controversial discussions in the literature [1–5], as to whether the introduction of SRC leads to a meaningful improvement of the description of various experiments, among which I mention the most important ones:

(i) Elastic scattering of high-energy projectiles (electrons, protons) [6–13].

(ii) One-nucleon knock-out reactions like (e, e'p) and (p, 2p) [refs. [14,15]].

(iii) Absorption processes with either one or two nucleons emitted, (γ, N), (π, N), (γ, 2N) and (π, 2N) [refs. [16–18,47]].

(iv) Deuteron knock-out, (e, e'd) and (p, p'd) [refs. [19–22]].

The question as to whether SRC are a useful concept was not really settled. One reason for this was the occurrence of many different definitions and aspects of the term SRC so that in fact often different kinds of effects were compared, some of which were not correlation effects in the proper sense. A second reason was that some methods of incorporating SRC suffered from theoretical inconsistencies, as was pointed out in ref. [1]). To overcome the second shortcoming, correct expansion methods were worked out for the use of Jastrow-modified wave functions in the

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description of different reactions [23]). But the first problem still seems to exist, and I will concentrate on it in this paper.

Some questions which have to be answered before firm conclusions can be drawn from an analysis of the data, which are said to be relevant to SRC, are the following:

(a) Are the usual [harmonic oscillator, Woods-Saxon, or even (Brueckner-) Hartree-Fock] single-particle wave functions a good basis to define the uncorrelated system?

(b) Are the correlations defined with respect to these s.p. wave functions sufficiently model-independent to give meaningful information about the amount of correlation in the investigated nuclei?

(c) Is there a unique way to distinguish between s.p. properties and two-nucleon correlation effects? Can this distinction be made by looking at quantities which depend only on the s.p. density of the nucleus?

(d) Is there a direct way to measure two-particle quantities (which depend on the two-particle density) in the nucleus?

I want to stress that this work is not concerned with the problem of determining in which way a given s.p. model of the nucleus can be improved by the introduction of SRC, as most of the works on SRC are. Rather my question is, how far are SRC necessary or detectable at all, if one only uses all the information one has about the s.p. properties of a nucleus to construct a "best" s.p. basis, and takes this basis as an uncorrelated system.

One very elegant way to deal with two-nucleon correlations would be to work with two-nucleon overlap functions, as was pointed out in ref. [24]). I will use in this work a two-nucleon overlap function but, as yet, results about properties and relations of two-nucleon overlap functions, as they were worked out for one-nucleon overlap functions in the above mentioned paper, are not available.

During the preparation of the manuscript a completely different approach to correlations in the case of photonuclear reactions appeared [47]). In these papers it is shown that by reasons of gauge invariance, the introduction of NN correlations makes it also necessary to couple explicitly the electromagnetic field to the NN correlations. This coupling term contributed a large amount to the cross section. Whether this term is as important in electron induced reactions has not yet been investigated. Since I will not use correlated wave functions the conclusions of ref. [47]) do not directly apply to the model calculation performed in sect. 5. The reason for using the most simple wave functions is that the aim of this investigation is not to calculate in a realistic way the cross section for the reaction (e, e'd). Rather, it will be investigated how far it is possible to extract with sufficient reliability information concerning the short-range behaviour of relative wave functions even if this quantity is involved in the cross section in a very intricate manner. Once this possibility has been shown, it can be applied also to more realistic electron nucleus interactions and nucleon wave functions.

This paper proposes in sect. 2 some answers to the above-mentioned questions.

The connection of a two-hole spectral function to SRC is pointed out in sect. 3. In sect. 4 the process (e, e'd) is proposed as a possible means to detect SRC, and its cross section is derived. Sect. 5 describes the influence of distortion of the emitted deuteron, sect. 6 discusses the numerical results of a model calculation and sect. 7 gives a summary.

2. The concept of short-range correlations (SRC) and its difficulties

It is rather obvious, and was therefore early mentioned [25]), that the term two-body correlations is only meaningful in connection with the two-body density of the system under study. Nevertheless, many of the discussions about short-range NN correlations in nuclei concentrated on one-particle properties of the nucleus. This means that the nuclear reactions investigated depended only on the single-particle wave functions or, equivalently, on the one-body density or the one-particle momentum distribution of the nucleus. It was often argued [1]) that those particular one-particle phenomena which are sensitive to details of the short-range part of the NN interaction should be understood as an indication of SRC. But this is somewhat misleading because the whole existence of a nucleus is due to the NN interaction. Specifically, the repulsive part (or anything equivalent) produces the saturation without which no mean field could exist in which the nucleons can move relatively freely. It is therefore rather arbitrary to introduce a limit in the one-body density or the one-particle momentum distribution, below which s.p. properties prevail and above which the existence of strong momentum components indicates so-called two-particle effects. *One has rather to distinguish between the effect the NN interaction with its core has on the one-body density and the effect it has on the two-body density. Only the latter can be called a correlation effect.*

Many authors, with varying success, have tried to improve the description of experiments where high momentum transfer plays a role by artificially enhancing the high momentum components in the s.p. orbitals [shell model (SM) or Hartree-Fock (HF)]. This was done either phenomenologically [2,6,8,14,16]) by multiplying two s.p. orbitals by a Jastrow correlation factor with adjustable parameters, or microscopically [7,17,26]), in which case a two-body wave function is calculated via the Bethe-Goldstone equation. In both cases a correction to a Slater determinant (SD) one-body density built from SM or HF orbitals is derived. In the SM case these corrections consist of 1p-1h and 2p-2h corrections, whereas in the HF formalism the 1p-1h corrections are eliminated by the self-consistency condition [1]). This shows that the 1p-1h corrections are a model-dependent feature of the SM description. However, the significance of the much smaller 2p-2h corrections is also not clear. The following reasons make it questionable whether HF (or BHF) orbitals are a "best" choice of orbitals if not only gross features of the nucleus like the g.s. energy are investigated but also the finer details (in our case the high momentum components of the s.p. wave functions).

The HF orbitals arise from a minimization of the energy functional which as an average is certainly not sensitive to details of the s.p. wave functions. This is stressed by the fact that HF orbitals are only the first-order term in an expansion of maximum-overlap orbitals (MOO) in orders of the residual interaction V [refs. [27,28])]. The MOO themselves give the best SD approximation to the nuclear wave function corresponding to a given NN interaction. The BHF orbitals would be equivalent to MOO if they fulfilled the Brillouin-Brueckner condition† $\langle\phi_0|t\phi_i^*\rangle = 0$ with the full many-body reaction operator t [ref. [27])]. But this is not the case in usual calculations, where only the two-body reaction operator is considered and many higher order terms are left out. As long as these orbitals are only tested by some few eigenvalues (like the g.s. energy, the radius and some s.p. energies) where high momentum components do not play a rôle, this approximation is sufficient. One might therefore suspect that the high momentum components are provided just by the neglected higher order terms. Furthermore, even the MOO are only an approximation (of first order in t) to the natural orbitals (NO) [27,28]), a SD which gives the best approximation to the true one-body density and hence to all matrix elements of one-body operators.

The MOO and NO seem to be of theoretical interest only because both are burdened with the difficulty or even impossibility of calculating them. But in any case there is evidence for the necessity to go beyond the HF (BHF) orbitals for an optimal set of s.p. orbitals. This is supported by refs. [5,29]), where it has been shown that a set of A realistic orbitals can be found which are sufficient to describe the one-body density or the elastic form factor as measured by elastic electron scattering. These orbitals give also satisfying results for g.s. and s.p. energies [29]). Whether other available one-particle experiments like (e, e'p) can also be described by these orbitals should be investigated.

I will, however, in the following concentrate on proper two-particle effects, i.e. I will deal with the two-body density.

An A-particle system is in a strict sense called uncorrelated if the probability distribution of one particle is independent from that of the others [30]). Mathematically expressed, the two-body density is then a product of the one-body densities. If one wants to incorporate the Pauli correlations in a so-called free part of the two-body density one has to take an antisymmetrized product. The free two-body density is then defined as

$$\rho_2^f(x, x') = \tfrac{1}{2}[\rho(x)\rho(x') - \rho(x, x')\rho(x', x)],$$

where $\rho(x, x')$ is the off-diagonal one-body density. The dynamical two-particle correlations are then given by the difference between the true two-body density

† The g.s. SD is given by ϕ_0 and ϕ_i^* is a 1p-1h state arising from ϕ_0 by pushing one particle above the Fermi sea. The reaction operator t satisfies the Lippmann-Schwinger equation $t = V + VG_0t$, where G_0 is the unperturbed propagator.

$\rho_2(x, x')$ and its free part †

$$C_{\mathrm{dyn}}(x, x') = \rho_2(x, x') - \rho_2^{\mathrm{f}}(x, x').$$

The term "short range" in SRC corresponds to the strong repulsive core of the NN interaction with a range of $r_c = 0.4$ fm. If SRC exist, the probability to find two nucleons simultaneously in a volume with radius r_c must be either zero or very small. Proper SRC manifest themselves therefore in the "short relative distance" properties of the two-body density.

3. The two-hole spectral function (2h-SF)

The direct access to the two-body density is not easy. This is shown by the difficulties which arise in describing data which depend on the two-body density, like total inelastic cross sections [31]) and the elastic proton scattering [12,13,32]) where double scattering must be taken into account. It is therefore more convenient to investigate the relative momentum distribution of two nucleons in the nuclear g.s. This can be done with reactions where two nucleons emitted by the target are measured in the final channel. In sect. 4 it will be shown that for one-step or direct reaction mechanisms all the information about the nuclear g.s. which is relevant for these two-particle knock-out reactions is contained in the two-hole spectral function (2h-SF). This is true also for more complicated reaction mechanisms like double scattering. However, in these cases the dependence on the 2h-SF is not so simple [33]). The definition and properties of the 2h-SF and its connection with SRC will be given in this section.

The 2h-SF has the following general form (without spin and isospin):

$$S_{\mathrm{hh}}(\boldsymbol{k}, \boldsymbol{k}', \boldsymbol{p}, \boldsymbol{p}'; W) = \langle \Psi_0 | a^+(\boldsymbol{p}') a^+(\boldsymbol{p}) \delta(W + H - W_0) a(\boldsymbol{k}) a(\boldsymbol{k}') | \Psi_0 \rangle. \tag{1}$$

The operator $a(\boldsymbol{k})$ destroys a nucleon with momentum $\boldsymbol{k}$, the wave function of the target nucleus is given by Ψ_0 and W_0 is the g.s. binding energy. The Hamiltonian of the system is given by H.

This 2h-SF is most easily interpreted in its diagonal form with $\boldsymbol{k} = \boldsymbol{p}$ and $\boldsymbol{k}' = \boldsymbol{p}'$. In this case it can be written as a product of the diagonal two-particle momentum distribution $\rho_2(\boldsymbol{k}, \boldsymbol{k}')$, which gives the probability to find two nucleons with momenta $\boldsymbol{k}$ and $\boldsymbol{k}'$ in the nucleus, and a function $P_{\boldsymbol{k}\boldsymbol{k}'}(W)$. The latter is defined as

$$P_{\boldsymbol{k}\boldsymbol{k}'}(W) = \langle \Psi_0 | a^+(\boldsymbol{k}') a^+(\boldsymbol{k}) \delta(W + H - W_0) a(\boldsymbol{k}) a(\boldsymbol{k}') | \Psi_0 \rangle / \rho_2(\boldsymbol{k}, \boldsymbol{k}'), \tag{2}$$

and gives the probability that the $(A-2)$-particle residual nucleus after knock-out of two nucleons with momenta $\boldsymbol{k}$ and $\boldsymbol{k}'$ is in a state with energy $W_0 - W$. The energy integral over $P_{\boldsymbol{k}\boldsymbol{k}'}(W)$ gives one, since it gives the probability to find the nucleus with any energy. This leads to the following relation between the 2h-SF and the two-

† Spin and isospin dependences are not given explicitly here.

particle momentum distribution:

$$\int S_{\mathrm{hh}}(k, k', k, k'; W)\mathrm{d}W = \rho_2(k, k'). \tag{3}$$

In analogy to the one-hole spectral function (1h-SF) [34]) the 2h-SF is given by the residues of the poles and the discontinuity at the cut along the real energy axis of the two-hole Green function (2h-GF). The latter is given by [35])

$$K_{\mathrm{hh}}(k, k', p, p'; \omega) = i^{-1}\langle\Psi_0|a^+(p')a^+(p)(\omega+H-W_0)^{-1}a(k)a(k')|\Psi_0\rangle, \tag{4}$$

and the 2h-SF as

$$S_{\mathrm{hh}}(k, k', p, p'; \omega) = \lim_{\eta\to 0}(4\pi)^{-1}(K_{\mathrm{hh}}(k, k', p, p'; \omega-i\eta)-K_{\mathrm{hh}}(k, k', p,p'; \omega+i\eta)). \tag{5}$$

One splits the two-body Green function into a free part, which is given by an anti-symmetrized product of one-body Green functions, and a rest [35]). The discontinuity of this free part corresponds to a free 2h-SF. If one uses the Lehmann representation of one-body Green functions in terms of 1h-SF one ends up with a relation between the free 2h-SF $S^{\mathrm{f}}_{\mathrm{hh}}$ and the 1h-SF S_{h}:

$$S^{\mathrm{f}}_{\mathrm{hh}}(k, k', p, p'; W) = \tfrac{1}{2}\int[S_{\mathrm{h}}(k, p; \omega)S_{\mathrm{h}}(k', p'; W-\omega)-S_{\mathrm{h}}(k, p'; \omega)S_{\mathrm{h}}\ (k', p; W-\omega)]\mathrm{d}\omega. \tag{6}$$

This expression becomes more transparent if one looks at the 1h-SF and the 2h-SF in a pure shell model.

Generally, the SF can be expanded in SM one-particle and two-particle wave functions, respectively:

$$S_{\mathrm{h}}(k, p; W) = \sum_{\alpha,\gamma}\varphi^*_\gamma(p)\varphi_\alpha(k)S_{\alpha\gamma}(W), \tag{7}$$

$$S_{\mathrm{hh}}(k, k', p, p'; W) = \tfrac{1}{4}\sum_{\alpha,\beta,\gamma,\delta}\varphi^*_{\gamma\delta}(p, p')\varphi_{\alpha\beta}(k, k')S_{\alpha\beta\gamma\delta}(W). \tag{8}$$

The sums run over all discrete and continuous quantum numbers. In the shell model the expansion coefficients are given by

$$S_{\alpha\gamma}(W) = n_\alpha\delta_{\alpha\gamma}\delta(W-\varepsilon_\gamma), \tag{9}$$

$$S_{\alpha\beta\gamma\delta}(W) = n_\alpha n_\beta(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma})\delta(W-\varepsilon_{\alpha\beta}). \tag{10}$$

The s.p. and two-particle SM wave functions φ_α and $\varphi_{\alpha\beta}$ have the energy eigenvalues ε_α and $\varepsilon_{\alpha\beta}$. The step function n_α is 1 for occuped and 0 for unoccupied states. The

2h-SF is then given in the SM by

$$S_{\mathrm{hh}}^{\mathrm{SM}}(k, k', p, p'; W) = \tfrac{1}{2} \sum_{\alpha,\beta=1}^{A} \varphi_{\alpha\beta}^{*}(p, p')\varphi_{\alpha\beta}(k, k')\delta(W-\varepsilon_{\alpha\beta}), \tag{11}$$

and of course identical with its free part. As eqs. (7) to (10) show, the 1h-SF is in the SM a function which is defined for all energies ε_α of s.p. states, and the 2h-SF for all energies of two-particle states $\varepsilon_{\alpha\beta}$ which are in the SM, simply the sums of two s.p. energies ε_α and ε_β. This result holds qualitatively even if the energies for which S_{h} is defined are not discrete but have a certain distribution. One sees from eq. (6) that if S_{h} has maxima at the energies ε_1 and ε_2 then the free 2h-SF constructed from this S_{h} has a maximum at the energy $\varepsilon_{12} = \varepsilon_1 + \varepsilon_2$.

Analogously to eq. (3), there is a relation between the free 2h-SF and the free part of the two-particle momentum distribution ρ_2^{f}:

$$\int S_{\mathrm{hh}}^{\mathrm{f}}(k, k', p, p'; W)\mathrm{d}W = \tfrac{1}{2}(\rho(k, p)\rho(k', p') - \rho(k, p')\rho(k', p)) \equiv \rho_2^{\mathrm{f}}(k, k', p, p'), \tag{12}$$

where $\rho(k, p)$ is the non-diagonal one-particle momentum distribution. The deviation between a measured 2h-SF and a free 2h-SF, which one might construct from 1h-SF, leads directly to the two-particle correlation function C_{dyn}. For the construction of the free 2h-SF from 1h-SF via eq. (6) it is of course necessary to know the latter in sufficiently large energy and momentum intervals and also in its non-diagonal form. If one looks for SRC one has to investigate the amount of high relative momenta in C_{dyn}.

4. The cross section for (e, e'd)

If one wants to extract information about nuclear structure from any reaction one must be able to distinguish properly those features in the cross section which arise from the reaction mechanism from others which are due to nuclear structure. The best would be of course to have a clean factorization, where one factor depends only on the reaction mechanism, whereas the other depends only on the nuclear structure. Such a case is found in quasi-free one-particle knock-out reactions if no account is given of distortion effects [36-38]). If one wants to investigate two-particle effects the equivalent to this kind of reaction would be a two-particle knock-out, with the projectile and the two emitted particles measured in coincidence. Since this would involve a triple coincidence experiment, which does not seem to be feasable at present, I consider the simpler case of a deuteron knock-out reaction. In this case one integrates over one degree of freedom, namely the relative coordinate of the two knocked-out nucleons. The question arises as to whether the cross section has a structure which makes it possible to identify the influence of nuclear structure. For SRC studies it is particularly the dependence on the relative distance which matters. This question is answered in subsect. 5.2.

In this section, I derive the cross section for the (e, e'd) reaction with and without distortion under the following assumptions:

(i) The projectile interacts only once with one nucleon in the target. Although there are certainly higher order effects present, which may add two-body parts to the interaction, I start with the simplest possible operator in order to show most clearly how to disentangle different contributions which enter in a complicated way the cross section.

(ii) The electron is not distorted, which is a safe assumption at sufficiently high energy [39].

(iii) The interaction of the outgoing nucleons with the residual nucleus is either ignored (plane waves) or described by a distorting potential.

The effects of small violations of the antisymmetry and orthogonality of initial and final state wave functions will be estimated. Numbers are given in the next section, where a model calculation is described.

The differential coincidence cross section for an (e, e'd) reaction is given by ($\hbar = c = 1$):

$$\frac{\mathrm{d}^6\sigma}{\mathrm{d}E'_0\mathrm{d}E_\mathrm{d}\mathrm{d}\Omega_0\mathrm{d}\Omega_\mathrm{d}} = \frac{2\pi}{4E_0E'_0}\int_\mathrm{f}\sum_{\text{spins}}|t_{\mathrm{if}}|^2\rho_\mathrm{f}\delta(W-W_A+W_\mathrm{R}^\mathrm{f}). \tag{13}$$

The symbol $\sum_{\text{spins}}$ indicates the summation and averaging over final and initial electron and deuteron spins, respectively, ρ_f is the three-particle density in the final state given by

$$\rho_\mathrm{f} = (2\pi)^{-6}E_\mathrm{d}K_\mathrm{d}E'_0k'_0(1-\Delta)^{-1},$$

with the recoil correction

$$\Delta = \frac{E_\mathrm{d}}{E_\mathrm{R}}(\boldsymbol{K}_\mathrm{d}\cdot\boldsymbol{q}/K_\mathrm{d}^2-1).$$

Here $W = E'_0+E_\mathrm{d}-E_0-m_\mathrm{d}+E_\mathrm{R}$ is the energy transferred from the target nucleus, $\boldsymbol{q} = \boldsymbol{k}_0-\boldsymbol{k}'_0$ the transfered momentum, and $\boldsymbol{K}_\mathrm{d}$ and $\boldsymbol{K}_\mathrm{R}$ are the momenta of deuteron and recoil nucleus with masses m_d and m_R; E'_0, E_d and E_R are the total energies of the electron, deuteron, and recoil nucleus in the final states and E_0 is the total energy of the electron in the initial state. The binding energies of the target nucleus in the ground state and the residual nucleus in state f are given by W_A and W_R^f. The summation is carried out over the discrete and continuous states f.

The reaction matrix t_{if} is given by

$$t_{\mathrm{if}}\delta(\boldsymbol{K}-\boldsymbol{K}') = \langle\Psi_\mathrm{f}|M|\Psi_\mathrm{i}\rangle \equiv T_{\mathrm{if}}, \tag{14}$$

where $\boldsymbol{K}$ and $\boldsymbol{K}'$ are the total momenta of all A nucleons in the initial and final states with wave functions Ψ_i and Ψ_f. The interaction operator $M = \sum_{j=1}^{A}M(j)$ contains besides the electron spinors, operators which act on the nucleonic coordinates. In

the approximation of McVoy and Van Hove [40]) these are the Coulomb term, the Darwin-Foldy term, the spin term and the convection current term. The first three depend on the nucleonic coordinates only in the from $\exp(i\boldsymbol{q}\cdot\boldsymbol{r}_j)$. The last term contains also the momentum operator $\boldsymbol{p}_j = -i\nabla_j$. I omit it in the following derivation, and I will indicate which change it would effect in the final cross section. The final state wave function Ψ_f consists of a deuteron and a residual nucleus with $A-2$ nucleons. Explicit antisymmetrization between the deuteron and the residual $A-2$ nucleons gives the following two terms for the T-matrix:

$$T_{if} = \sqrt{2A(A-1)}[\langle\Phi_d(1,2)|M(1)|\Omega_{fi}(1,2)\rangle + \tfrac{1}{2}(A-2)\langle\Psi^f_{A-2}(1,4,\ldots,A)|M(1)|G_{dA}(1,4,\ldots,A)\rangle]. \quad (15)$$

An approximate normalization factor of $\sqrt{2/A(A-1)}$ has been used, Ω_{fi} is the two-nucleon overlap integral of the residual and the target nucleus wave functions

$$\Omega_{fi}(1,2) = \int \Psi^{f*}_{A-2}(3,\ldots,A)\Psi_A(1,2,3,\ldots,A)\mathrm{d}3\ldots\mathrm{d}A,$$

and G_{dA} is the overlap integral of deuteron and target wave function:

$$G_{dA}(1,4,\ldots,A) = \int \Phi^*_d(2,3)\Psi_A(1,2,3,\ldots,A)\mathrm{d}2\mathrm{d}3.$$

Space, spin and isospin coordinates of the A nucleons are symbolized by $i = 1,\ldots,A$.

For a sufficiently fast deuteron (I consider deuterons with $E_{kin} > 80$ MeV) the second term of the T-matrix will be smaller by several orders of magnitude than the first one. For numbers see subsect. 5.1.

Retaining only the first part, I use now coordinates relative to the c.m. of the residual nucleus, namely, $\boldsymbol{X}$ for the c.m. of the deuteron and $\boldsymbol{\xi}$ for the relative distance of the two emitted nucleons, with the conjugate momenta $\boldsymbol{Q}$, $\boldsymbol{Q}'$ and $\boldsymbol{\kappa}$, $\boldsymbol{\kappa}'$ before and after the reaction, respectively. After all possible integrations have been carried out, the T-matrix can be written in the following form:

$$T_{if} = \sqrt{2A(A-1)} \sum_{\substack{m'_1 m_1 m_2 \\ \mu_1\mu_2}} \int g_d^{m'_1 m_2\mu_1\mu_2 *}\left(\boldsymbol{Q} + \frac{A-2}{A}\,\boldsymbol{q}, \boldsymbol{\kappa} + \tfrac{1}{2}\boldsymbol{q}\right) \times \langle m'_1|M(q_\lambda)|m_1\rangle g_{fi}^{m_1 m_2\mu_1\mu_2}(\boldsymbol{Q},\boldsymbol{\kappa})\mathrm{d}^3Q\mathrm{d}^3\kappa\,\delta(\boldsymbol{K}'-\boldsymbol{q}). \quad (16)$$

The functions g_d and g_{fi} are the Fourier transforms of the deuteron wave function Φ_d and the overlap integral Ω_{fi}, and g_{fi} is given by

$$g_{fi}^{m_1 m_2\mu_1\mu_2}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \langle\Psi^f_{A-2}|a_{m_1\mu_1}(\boldsymbol{k}_1)a_{m_2\mu_2}(\boldsymbol{k}_2)|\Psi_A\rangle, \quad (17)$$

where $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ are here the lab system momenta of the individual nucleons. The operator $a_{m\mu}(\boldsymbol{k})$ destroys a nucleon with momentum $\boldsymbol{k}$ and spin and isospin projections m and μ.

The spin space matrix $\langle m_1'|M(q_\lambda)|m_1\rangle$ is only dependent on the transfered four-momentum $q_\lambda^2 = -E_0E_0'\sin^2\frac{1}{2}\vartheta_e$, where ϑ_e is the angle of the outgoing electron. With the convection current term taken into account there would be also a dependence on the momenta κ and Q and the interaction could not be taken out of the integral over Q and κ in eq. (16).

Summation over the final states with proper attention paid to the energy conservation results in

$$\sum_f |t_{if}|^2 = 2A(A-1)\sum_{\substack{m_1'm_1m_2\\ n_1'n_1n_2}}\sum_{\substack{\mu_1\mu_2\\ \nu_1\nu_2}}\langle n_1|M^+(q_\lambda)|n_1'\rangle\langle m_1'|M(q_\lambda)|m_1\rangle$$

$$\times\int g_d^{n_1'n_2\nu_1\nu_2}\left(Q'+\frac{A-2}{A}q,\kappa'+\tfrac{1}{2}q\right)g_d^{m_1'm_2\mu_1\mu_2*}\left(Q+\frac{A-2}{A}q,\kappa+\tfrac{1}{2}q\right)$$

$$\times S_{\mu_1\mu_2\nu_1\nu_2}^{m_1m_2n_1n_2}(\tfrac{1}{2}Q+\kappa,\tfrac{1}{2}Q-\kappa,\tfrac{1}{2}Q'+\kappa',\tfrac{1}{2}Q'-\kappa';W)\mathrm{d}^3Q\mathrm{d}^3Q'\mathrm{d}^3\kappa\mathrm{d}^3\kappa'. \qquad (18)$$

The function S is the two-hole spectral function described in sect. 3 and given here with explicit spin and isospin dependence as

$$S_{\mu_1\mu_2\nu_1\nu_2}^{m_1m_2n_1n_2}(k_1,k_2,k_1',k_2';W)$$

$$=\langle\Psi_A|a_{n_2\nu_2}^+(k_2')a_{n_1\nu_1}^+(k_1')\delta(W+H-W_A)a_{m_1\mu_1}(k_1)a_{m_2\mu_2}(k_2)|\psi_A\rangle. \qquad (19)$$

The expression (18) is rather complicated and does not factorize, although only those parts of the interaction were taken into account which depend only on the momentum transfer. It is therefore necessary to investigate the influence of the distortion, whether it can be handled reliably and to what extent it obscures the effect of the relative motion of the two knocked-out nucleons on the cross section. This will be done for a numerical example in sect. 5. With plane waves for the c.m. motion of the deuteron a substantial simplification of the formula (18) is achieved. The integral in eq. (18) reduces to

$$\int\tilde\varphi_d^{m_1'm_2\mu_1\mu_2*}(\kappa+\tfrac{1}{2}q)\tilde\varphi_d^{n_1'n_2\nu_1\nu_2}(\kappa'+\tfrac{1}{2}q)$$

$$\times S_{\mu_1\mu_2\nu_1\nu_2}^{m_1m_2n_1n_2}(P+\kappa,P-\kappa,P+\kappa',P-\kappa';W)\mathrm{d}^3\kappa\mathrm{d}^3\kappa', \qquad (20)$$

where $\tilde\varphi_d$ is the Fourier transform of the internal deuteron wave function and $P=\frac{1}{2}\times(K_d-q)$.

This expression still does not factorize but it is a kind of double folding. In subsect. 5.2 a case is discussed where this folding can be inverted if one knows the dependence of the cross section on the momentum transfer. One might hope therefore to investigate relative momenta in the nucleus even with a deuteron knock-out by varying the momentum transfer.

5. The influence of the distortion on the cross section in a simple model

In this section, a model calculation is described concerning the importance of the distortion of the outgoing deuteron. I will neither try to calculate a realistic two-hole spectral function nor will the derived cross section be directly comparable with experiments. It seems therefore sufficient to use the following simple model:

(i) The c.m. motion of the deuteron is described by a distorted wave and the result compared with a plane wave calculation. The distorted wave is calculated by partial wave expansion from a complex potential. A Woods-Saxon form was used for the real part and a derivative of a WS form for the imaginary part. The spin-orbit potential and the Coulomb potential were neglected.

(ii) The initial and residual nuclei are described by harmonic oscillator (HO) wave functions, the overlap integral $\Omega_{\rm fi}$ thereby becoming a simple HO two-particle wave function. To disentangle c.m. and relative motion the Moshinsky transformation is used [41]).

(iii) In the interaction of electron and nucleus only the Coulomb term is retained. For the purpose of estimating the distortion effects the neglect of the higher order terms is not important and simplifies the calculation.

(iv) For the internal deuteron wave function an analytic function [42]) is taken. Only the S-state is used.

As a starting point for the calculation, I take eq. (16) rather than eq. (18), because the summation over the possible final states is in our model simply a sum over all possible combinations of neutron and proton shell-model states from which the deuteron can be knocked out. It is therefore easier to use the overlap integral directly instead of going via the 2h-SF.

The Fourier transform of the overlap integral $\Omega_{\rm fi}$ is then given by

$$g^{m_1 m_2 \mu_1 \mu_2}_{n_\alpha l_\alpha n_\beta l_\beta j_\alpha j_\beta J M_J}(Q,\kappa) = \langle \tfrac{1}{2}\mu_1 \tfrac{1}{2}\mu_2 | 00\rangle \sum_{\substack{LS \\ M_L M_S}} b^{l_\alpha l_\beta LS}_{j_\alpha j_\beta J} \times \langle LM_L SM_S | JM_J\rangle \langle \tfrac{1}{2}m_1 \tfrac{1}{2}m_2 | SM_S\rangle \sum_{nlNA} d^{n_\alpha l_\alpha n_\beta l_\beta L}_{nlNA} \times \sum_{m_l M_A} \langle l m_l \Lambda M_\Lambda | LM_L\rangle \tilde{\phi}^{\rm HO}_{N\Lambda M_\Lambda}(Q) \tilde{\varphi}^{\rm HO}_{nlm_l}(\kappa). \quad (21)$$

The coefficients $b^{l_\alpha l_\beta LS}_{j_\alpha j_\beta J}$ effect the change from the jj to the LS coupling scheme, $(n_\alpha l_\alpha j_\alpha)$ are the usual s.p. quantum numbers, S, L, J are the two-particle spin, orbital, and total angular momentum. The tilde on the HO c.m. and relative wave functions $\tilde{\phi}^{\rm HO}_{N\Lambda M_\Lambda}$ and $\tilde{\varphi}^{\rm HO}_{nlm_l}$ indicates Fourier transformation. The Moshinsky brackets are given by $d^{n_\alpha l_\alpha n_\beta l_\beta L}_{nlN\Lambda}$ and the Clebsch-Gordan coefficients are used in the convention of Edmonds [43]). The Fourier transform of the deuteron wave function is given by

$$g_{M'_S}{}^{m'_1 m'_2 \mu'_1 \mu'_2}(Q',\kappa') = \tilde{\chi}^{(-)}_{\rm d}(Q') \tilde{\varphi}_{\rm d}(\kappa') \langle \tfrac{1}{2}m'_1 \tfrac{1}{2}m'_2 | 1 M'_S\rangle \langle \tfrac{1}{2}\mu'_1 \tfrac{1}{2}\mu'_2 | 00\rangle, \quad (22)$$

where M'_S is the deuteron spin, $\tilde{\varphi}_{\rm d}$ the Fourier transform of the internal S-state

deuteron wave function and $\tilde{\chi}_d^{(-)}$ the Fourier transform of the distorted (or plane) wave for the c.m. motion of the deuteron, given in coordinate space in partial wave expansion:

$$\chi_d^{(-)}(X) = 4\pi \sum_{L_d M_{L_d}} i^{L_d} U_{L_d}(X, Q_d) Y_{L_d}^{M_{L_d}*}(\hat{Q}_d) Y_{L_d}^{M_{L_d}}(\hat{X}). \tag{23}$$

The deuteron momentum Q_d is taken relative to the recoil nucleus. The Y_L^M are spherical harmonics. Using the expressions (21) and (22) in eq. (16) and writing the integrals in coordinate space leads to

$$t_{if} = \sqrt{2A(A-1)} M_e(q_\lambda^2) \sum_{LM_L} b_{j_\alpha' j_\beta' J}^{l_\alpha' l_\beta' L 1} \langle LM_L 1 M_S' | J M_J \rangle$$

$$\times \sum_{nlNA} a_{nlNA}^{n_\alpha l_\alpha n_\beta l_\beta L} \sum_{m_l M_A} \langle lm_l \Lambda M_A | LM_L \rangle \tag{24}$$

$$\times \int \exp\left(i \frac{A-2}{A} q \cdot X\right) \chi_d^{(-)*}(X) \phi_{N\Lambda M_\Lambda}^{HO}(X) d^3X \int \exp(i \tfrac{1}{2} q \cdot \xi) \varphi_d^*(\xi) \varphi_{nlm_l}^{HO}(\xi) d^3\xi.$$

The remaining Coulomb term of the electron-nucleus interaction is given by [40])

$$M_e(q_\lambda^2) = \frac{4\pi e^2}{q_\lambda^2} (4E_0 E_0' + q_\lambda^2) F(q_\lambda^2), \tag{25}$$

where $F(q_\lambda^2)$ is the form factor of a proton. The results of the calculation are discussed in sect. 6.

5.1. ORTHOGONALITY AND ANTISYMMETRIZATION

Because model wave functions, which are not eigenfunctions of the same Hamiltonian, are used for the initial and final state, they are not automatically orthogonal. A comparison between the c.m. integrals in eq. (24) with the corresponding overlap between the deuteron scattering wave and the c.m. part of the bound state two-nucleon wave function,

$$O_{N\Lambda M_\Lambda} = \int \chi_d^{(-)*}(X) \phi_{N\Lambda M_\Lambda}^{HO}(X) d^3X, \tag{26}$$

gives a measure for the importance of the non-orthogonality. Both quantities are compared for knock-out from the $s_{\frac{1}{2}}$ shells in fig. 1.

According to the remark in sect. 4, the smallness of expression (26) secures also that the second term which arises in the antisymmetrization (eq. (15)) is neglegible compared to the first one.

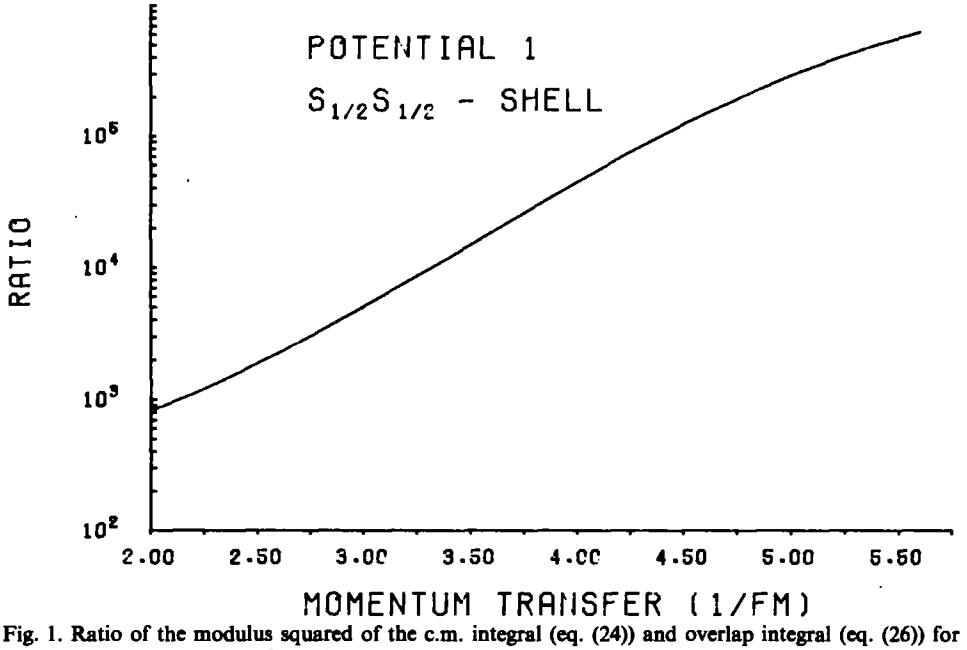

Fig. 1. Ratio of the modulus squared of the c.m. integral (eq. (24)) and overlap integral (eq. (26)) for knock-out from the $s_{1/2}$ shells calculated with distortion potential 1.

5.2. UNFOLDING OF THE 2h-SF IN A SPECIAL CASE

In the case when both nucleons are knocked out from the $s_{\frac{1}{2}}$-shell the expression (24) for the reaction matrix greatly simplifies to

$$t_{if} = \sqrt{2A(A-1)}M_e(q_\lambda^2)\int \exp\left(i\frac{A-2}{A}\boldsymbol{q}\cdot\boldsymbol{X}\right)\chi_d^{(-)*}(\boldsymbol{X})\varphi_{000}^{HO}(X)\mathrm{d}^3X$$
$$\times \int \exp\left(i\tfrac{1}{2}\boldsymbol{q}\cdot\boldsymbol{\xi}\right)\varphi_d^*(\xi)\varphi_{000}^{HO}(\xi)\mathrm{d}^3\xi. \qquad (27)$$

The reaction matrix factors into a centre-of-mass part $\mathcal{F}^{c.m.}(q)$ and a relative motion part $\mathcal{F}^{rel}(q)$. If one is able to calculate the c.m. integral one can extract the relative motion integral from the experimental cross section. Since in momentum space the relative motion integral is a simple folding it can be inverted to give the wave function of the relative motion of the two knocked-out nucleons before the reaction

$$\varphi_{000}(\xi) = \frac{\mathcal{F}^{rel}(\xi)}{8(2\pi)^{\frac{3}{2}}\varphi_d^*(\xi)}, \qquad (28)$$

where $\mathcal{F}^{rel}(\xi)$ is the Fourier transform of the relative motion integral $\mathcal{F}^{rel}(q)$. To calculate this Fourier transform one has of course to know $\mathcal{F}^{rel}(q)$ in a sufficiently large interval of the transfered momentum q.

Fig. 3b shows; however, that hard-core effects can clearly be seen in the relative

motion integral. The relative motion integral for a harmonic oscillator wave function and a Jastrow-modified wave function [2]) is compared. A Jastrow factor $f(\xi) = 1 - j_0(q_c\xi)$ with two different values for the exchanged momentum q_c was used. The introduction of a hard core greatly changes the dependence of the relative motion integral on the momentum transfer. Hard-core effects will therefore show up as well in the unfolding of the relative motion integral. More details about this unfolding and an extension to knock-out from other shells will be published later.

6. Discussion of the numerical results

The calculations were performed for the reaction ^{16}O(e, e'd)^{14}N. The s.p. energies used for the different shells were the same as in ref. [29]). The oscillator parameter for the harmonic oscillator wave functions was taken from ref. [44]). The calculation was mainly intended to investigate the influence of the distortion on the cross section. If the cross section is strongly dependent on the distorting potential, this would make it difficult to extract unique information about the relative motion integral in eqs. (27) or (24), which one is interested in. This is because for deuterons of such high energies distortion potentials are not well known. I take as a starting point a potential which was recently fitted to deuteron elastic scattering on ^{12}C up to energies of 90 MeV [ref. [45])]. The change in the potential depth if one goes from ^{12}C to ^{14}N, which is the residual nucleus in the calculated reaction, is of minor importance. The depths of real and imaginary wells were then varied to show their effect on the cross section. Other parameters of the potential were held fixed. The potential parameters used are given explicitly in table 1. The energy of the deuteron relative to the residual nucleus

TABLE 1

Parameters of the used optical potentials for an emitted deuteron with energy E_d

No.	V (MeV)	W_s (MeV)		
1	66.0	18.3	E_d (MeV)	91
2	60.0	18.3	r_V (fm)	1.25
3	72.0	18.3	a_V (fm)	0.67
4	66.0	9.3	r_W (fm)	1.12
5	66.0	27.3	a_W (fm)	0.70

The potential is of Woods-Saxon form with a surface imaginary part.

is always 91 MeV in the calculated example. This was chosen in order to simplify the lengthy calculations of the c.m. integrals.

As explained in subsect. 5.2 for SRC studies it is interesting to vary the momentum transfer q.

In a plane wave calculation the c.m. integral in eq. (24) becomes

$$\mathcal{I}^{\text{c.m.}}_{N\Lambda M_\Lambda}(Q_R) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int \exp\left(-i(K_d - q)\cdot X\right)\Phi^{HO}_{N\Lambda M_\Lambda}(X)\mathrm{d}^3X$$

$$= \frac{4\pi}{(2\pi)^{\frac{3}{2}}}(-i)^\Lambda Y^{M_\Lambda}_\Lambda(\hat{Q}_R)\int j_\Lambda(Q_R X)R_{N\Lambda}(X)X^2\mathrm{d}X, \qquad (29)$$

where $R_{N\Lambda}(X)$ is the radial part of a harmonic oscillator wave function and j_Λ a spherical Bessel function. Keeping the modulus of the recoil momentum $|Q_R| = |K_d - q|$ fixed, the c.m. integral depends via a spherical harmonic only on the angular orientation of the recoil momentum. For knock-out from the $s_{\frac{1}{2}}$ shell the only quantum numbers are $N = \Lambda = M_\Lambda = 0$ and therefore the corresponding c.m. integral is constant if q is varied. This is not the case for distorted waves because the c.m. integral depends then separately on the momentum transfer q and the deuteron momentum K_d.

How much different distorting potentials change the constancy of the c.m. integral is shown in fig. 2 for knock-out from the $s_{\frac{1}{2}}$ shell. One sees clearly that there is not much difference between the results of distorting potentials with different depths of the wells. The important fact is that the dependence on q is very smooth. How the rise of the c.m. integral with increasing q in the distorted case is reflected by the

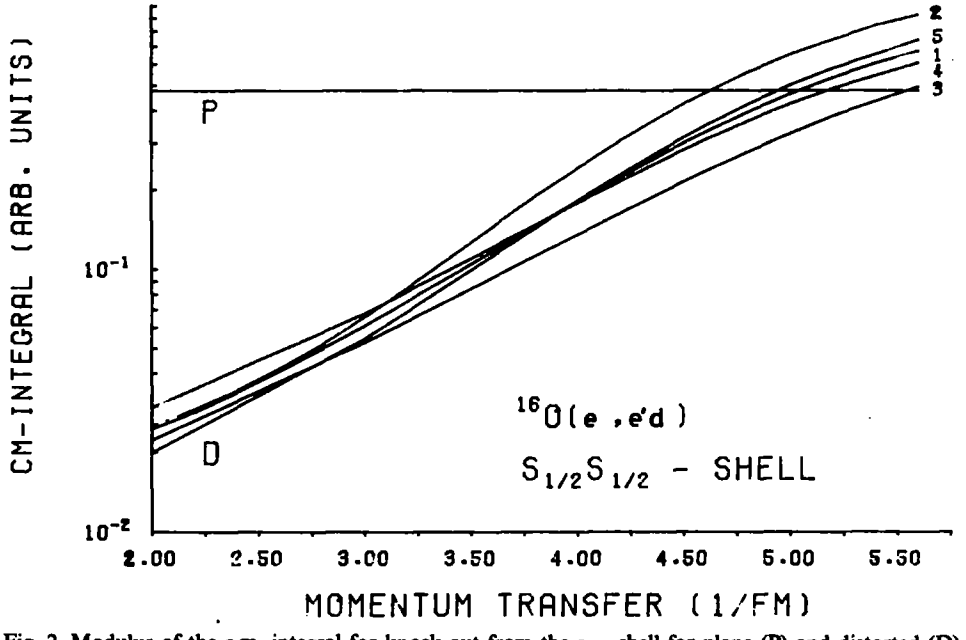

Fig. 2. Modulus of the c.m. integral for knock-out from the $s_{1/2}$ shell for plane (P) and distorted (D) waves. The numbers at the lines indicate the used distorting potential. The recoil momentum is fixed at 2.0 fm^{-1}.

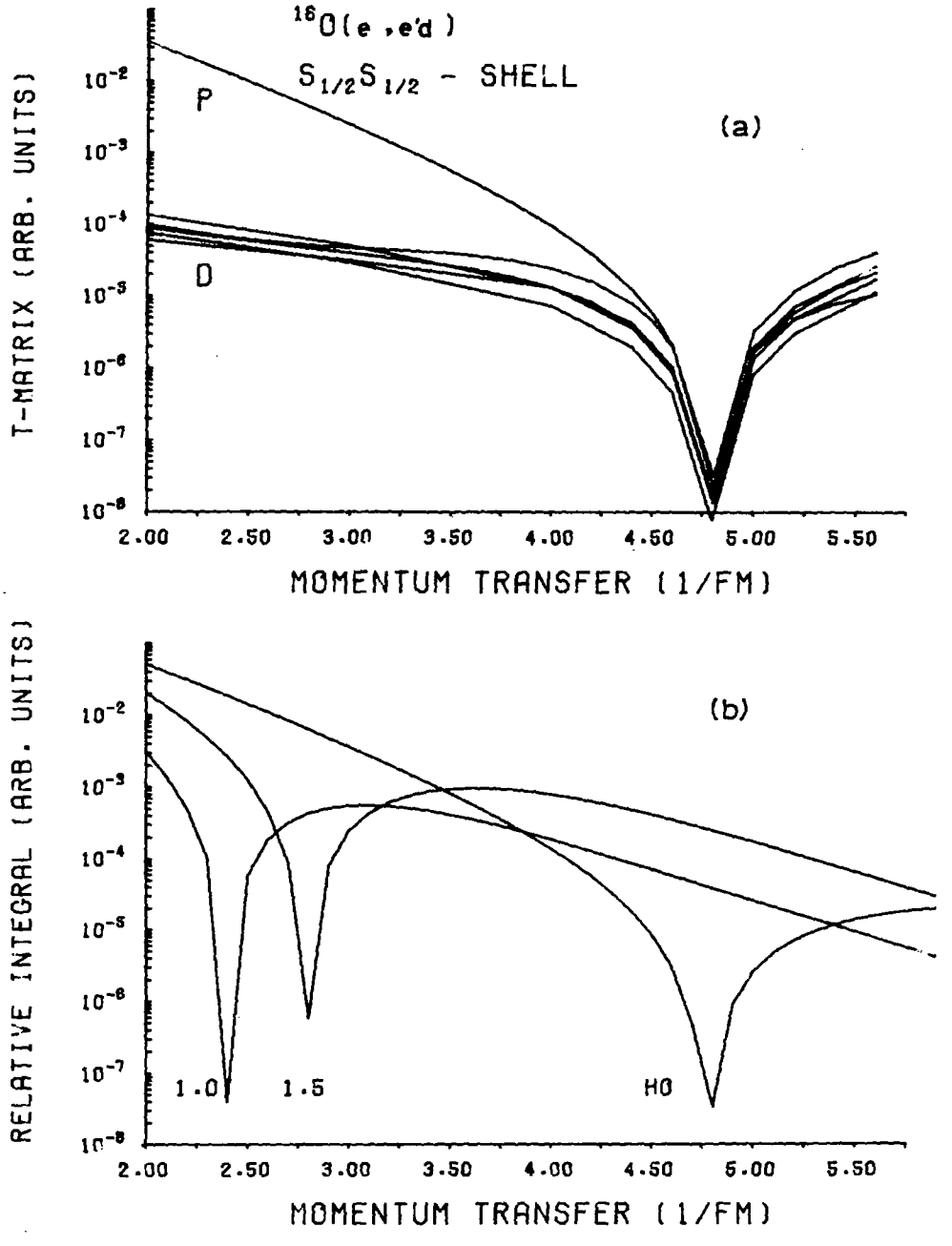

Fig. 3. (a) Nuclear part of the modulus squared of the reaction matrix for knock-out from the $s_{1/2}$ shell for plane (P) and distorted (D) waves. The recoil momentum is $Q_R = 2.0$ fm^{-1}. (b) Modulus squared of the relative motion integral for quantum numbers $n = l = 0$. Pure harmonic oscillator (HO) and Jastrow modified ($q_c = 1.0$ and 1.5 fm^{-1}) relative wave functions are used.

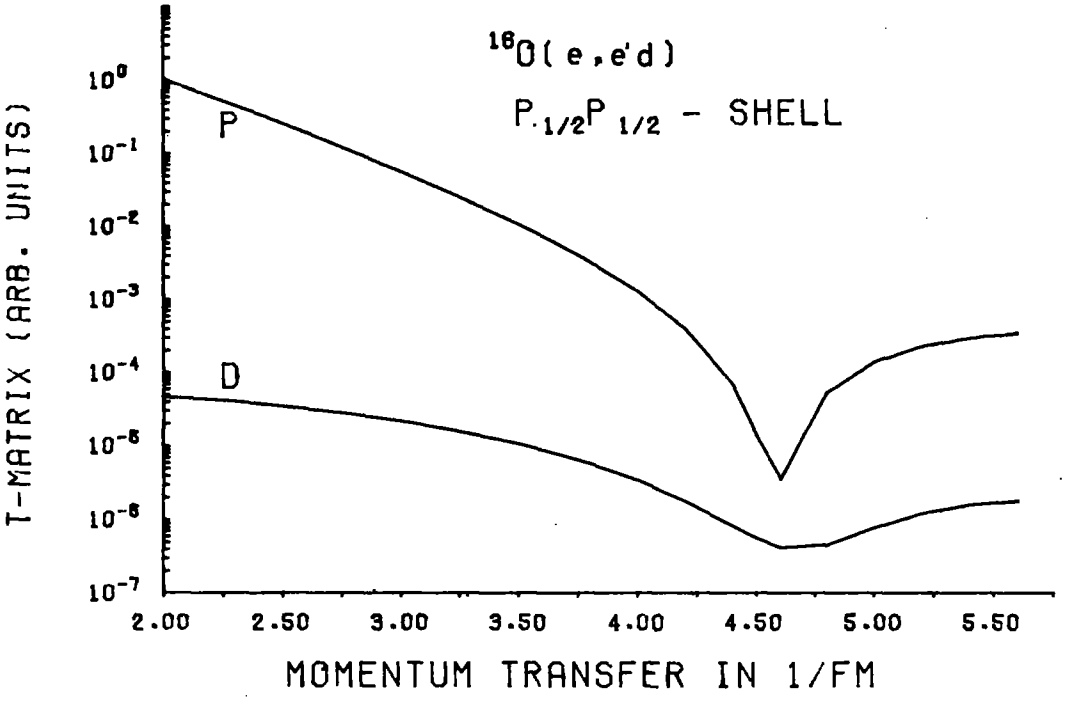

Fig. 4. The same as fig. 3a for knock-out from the $p_{1/2}$ shell. Only distorting potential 1 is used.

reaction matrix is shown in fig. 3a. There the modulus squared of the nuclear part of the reaction matrix is shown as function of q; the kinematical factors present in the cross section and the q-dependent electron-proton interaction (eq. (25)) are left out in order to demonstrate the properties of the nuclear system. For comparison the modulus squared of the relative motion integral is shown in fig. 3b. Its dependence on q is clearly reflected by the reaction matrix. Distortion does not obscure this picture. The most prominent feature of the relative motion integral is the sharp minimum at $q = 4.8$ fm^{-1} for the simple HO wave function. For the Jastrow-modified wave functions this minimum is shifted.

The following conclusions can now be drawn: The c.m. integral can be calculated fairly reliably. If one therefore measures the cross section of the reaction (e,e'd) as a function of the momentum transfer and keeps the recoil momentum constant one can extract the nuclear part of the reaction matrix. Having calculated the c.m. integral one obtains the relative motion integral. It contains the information about two-nucleon correlations in a very clear fashion, namely, in the position of the minimum.

The case when one or both of the knocked-out nucleons originate from the $p_{\frac{1}{2}}$ shell or the $p_{\frac{3}{2}}$ shell is more complicated to analyze and will be discussed in a different paper. In fig. 4, the reaction matrix for knock-out from the $p_{\frac{1}{2}}$ shell is given for

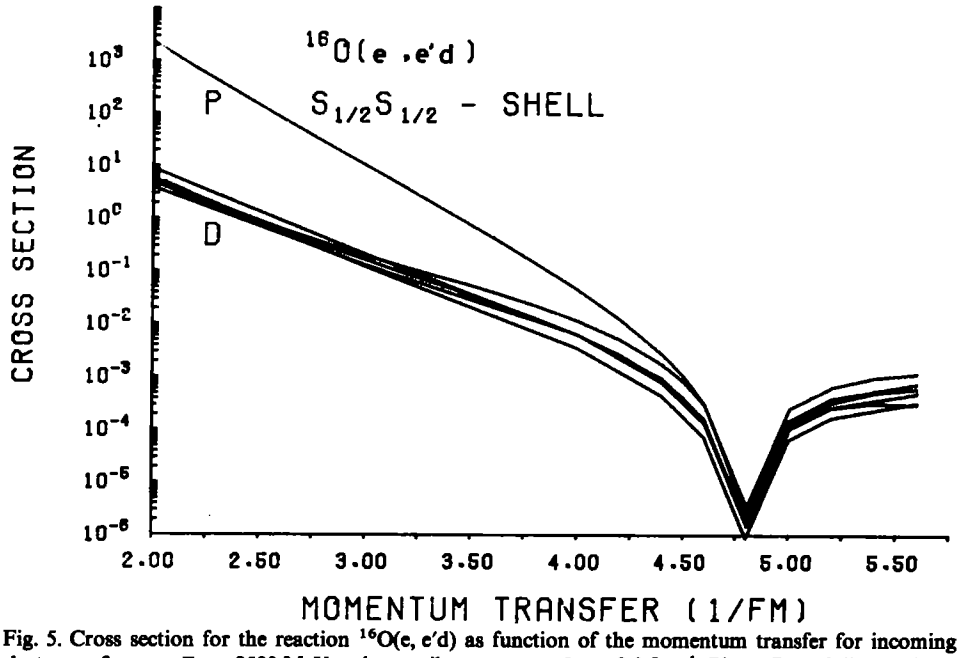

Fig. 5. Cross section for the reaction ^{16}O(e, e'd) as function of the momentum transfer for incoming electrons of energy $E_0 = 2500$ MeV and a recoil momentum $Q_R = 2.0$ fm^{-1}. Plane (P) and distorted (D) waves with potential 1 to 5 were used. The deuteron is knocked-out from the $s_{1/2}$- shell. The cross sections are given in units of pb/MeV^2 · sr^2.

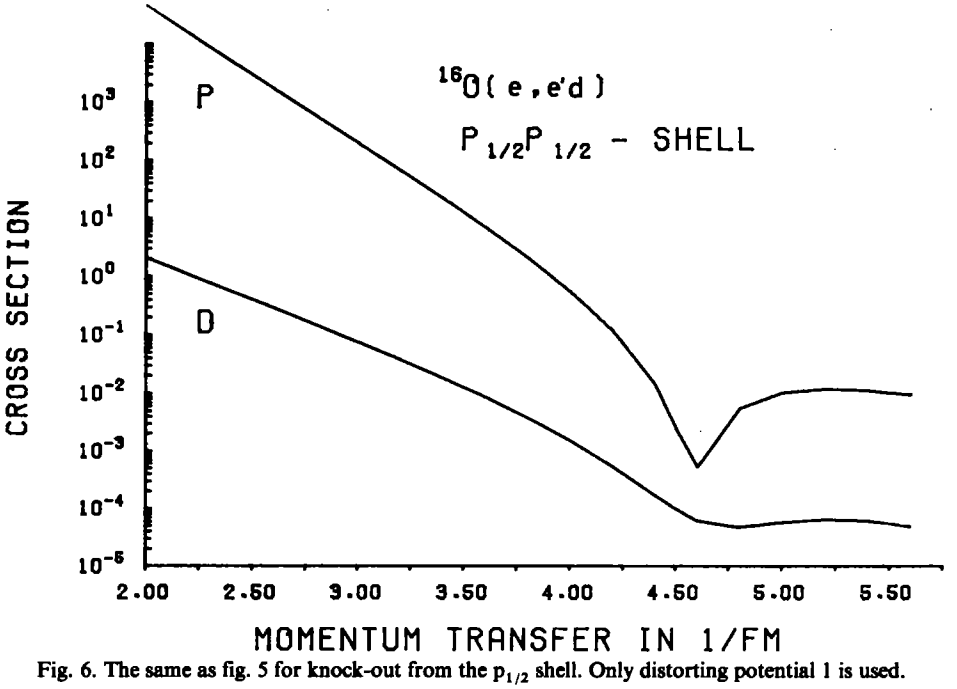

Fig. 6. The same as fig. 5 for knock-out from the $p_{1/2}$ shell. Only distorting potential 1 is used.

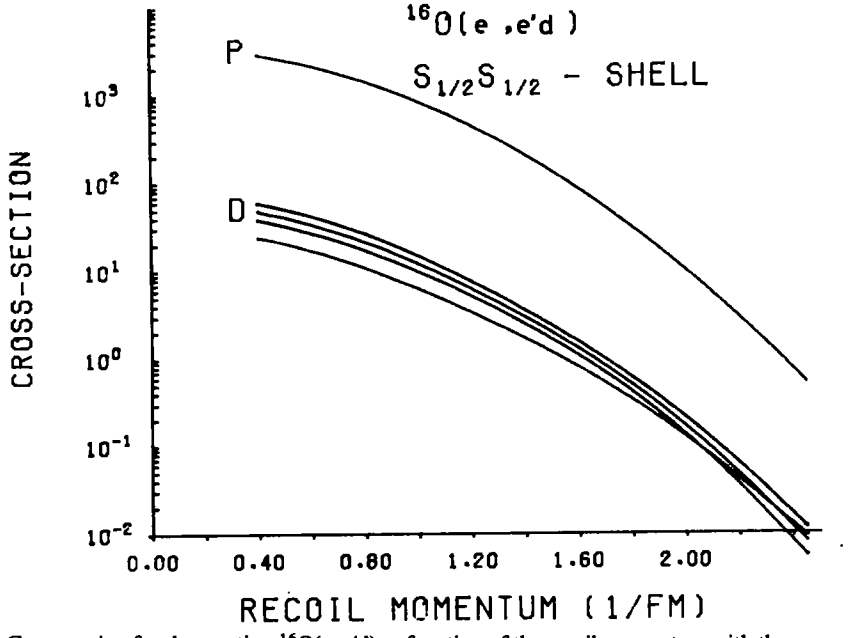

Fig. 7. Cross section for the reaction ^{16}O(e, e'd) as function of the recoil momentum with the momentum transfer fixed at $q = 4.0$ fm^{-1}. Knock-out from the $s_{1/2}$ shell.

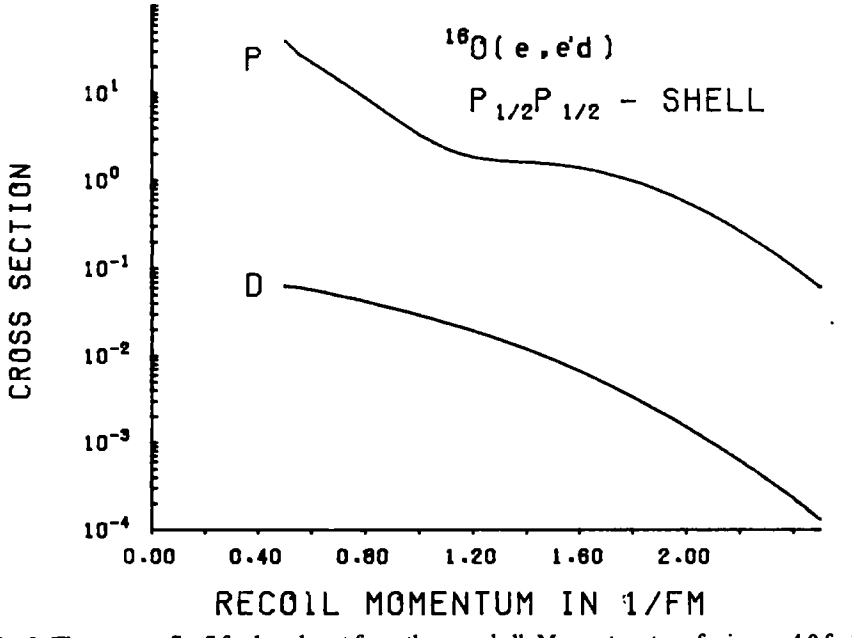

Fig. 8. The same as fig. 7 for knock-out from the $p_{1/2}$ shell. Momentum transfer is $q = 4.0$ fm^{-1}.

comparison. It is similar to the $s_{\frac{1}{2}}$ case, only the minimum is not so pronounced. This arises from the summation of contributions with different quantum numbers N, Λ and M_Λ. The cross sections for $s_{\frac{1}{2}}$ and $p_{\frac{1}{2}}$ knock-out are given as functions of the recoil momentum and the momentum transfer in figs. 5–8.

The calculation described is in several ways not realistic and I will mention some necessary improvements and some further investigations which should be performed.

The use of Woods-Saxon wave functions rather than HO wave functions seems straightforward, but there are two kinds of difficulties. To disentangle c.m. and relative motion one has to expand the Woods-Saxon wave functions in harmonic oscillator wave functions up to a principal quantum number of about six. The calculation needs then a very great amount of computer time. Furthermore, the simple structure of the expression for the reaction matrix particularly in the case of knock-out from the $s_{\frac{1}{2}}$ shell gets lost because of summations in the expansion and the Moshinsky transformation.

The effect of two-step processes like double excitation and the inclusion of two-body operators in the interaction as they appear if a residual interaction (which describes explicitly correlations) [47]) is incorporated in the description of the nucleus, should be investigated.

No allowance was made for break-up of the outgoing deuteron in the distorting potential. This effect might lead to a further reduction of the cross section. Furthermore, it is not clear whether the used g.s. optical potential is adequate for a deuteron in a highly excited nucleus like the residual nucleus. One has to notice, however, that the energy of the emitted deuteron relative to the residual nucleus is always the same in the chosen kinematics. Therefore changes in the energy dependence of the optical potential are not of importance.

7. Summary

In this paper, it was argued that only a clean distinction between one- and two-nucleon quantities can lead to empirical knowledge about SRC and that the usual s.p. wave functions are not suited to define unambiguously an uncorrelated system. By introducing the two-hole spectral function it was shown how the cross section for a two-nucleon knock-out experiment is related to a two-nucleon correlation function.

The results of the calculation with distorted waves indicate that the dependence of the cross section for the (e, e'd) reaction on the relative motion wave function of the nucleons in the target nucleus is not obscured by distortion effects. One might hope that this holds even if some of the proposed improvements of the formalism are made.

A scheme is proposed to extract the relative motion wave function from experiment although it appears in the cross section in an integral. With some modifications this scheme is applicable also if the cross section is not only a mathematical folding of the

relative wave functions before and after the reaction. Work in this direction is in progress.

Since the dependence of the cross section on the relative wave function is very distinct (figs. 3 and 5) the results of this work encourage one to try work along these lines also in more complicated situations.

The precise results for the relative motion wave function does of course depend on the interaction operator used. But even in the presence of two-body operators the measured quantity in two-nucleon knock-out is always the two-hole spectral function [33]), which is directly connected with two-nucleon correlations.

The results of one-particle experiments like elastic electron scattering or one-nucleon knock-out must be compared for compatibility with the two-particle quantities gained in this way. To do this one has to calculate the one-nucleon wave function from the two-nucleon wave function. Only those s.p. model wave functions which are compatible with the two-nucleon wave function gained from a two-particle experiment are good model wave functions. To answer the question whether SRC are detectable, one has then to calculate with these s.p. and two-particle wave functions the dynamical correlation function C_{dyn} defined in sect. 2.

High momentum components in the s.p. wave functions which were necessary to describe one-particle experiments are therefore not a correlation effect but a simple property of the s.p. wave function. All properties of this s.p. wave function are of course determined by the nucleon-nucleon interaction.

From the results of the calculation presented above it seems worthwhile to study experimentally and theoretically more thoroughly experiments of the kind described, in order to learn more about the two-particle properties of the nucleus. This in turn could lead to improvements in the knowledge of the s.p. wave functions, making it necessary to go beyond the approximations of the self-consistent procedures which are used up to now to calculate s.p. wave functions microscopically.

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