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Nucleon-Nucleon Short-Ranged Correlations, β Decay and the Unitarity of the CKM Matrix

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The influence of nucleon-nucleon short-ranged correlations on nuclear super-allowed β decay is examined. A combination of formalism, experimental information and phenomenology leads to the result that the isospin-breaking correction can be reduced by about 20% or more compared with previous calculations. This reduction is consistent with the observed depletion of spectroscopic strength obtained in studies of (e, e') and $(d, {}^3\text{He})$ reactions on a wide variety of nuclei. The ${}^{46}\text{V}$ nucleus is used as an example. The resulting impact on studies of the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element can be substantial.

The dominant contribution to the unitarity test of the Standard Model (SM) CKM matrix comes from the up-down quark matrix element V_{ud} . The value of V_{ud} has been extracted by Hardy and Towner (HT) [1–10] with the highest precision from $0^+ \rightarrow 0^+$ decays from nuclei ranging from ${}^{10}\text{C}$ to ${}^{74}\text{Rb}$. The remarkably consistent nature of the values of V_{ud} obtained from many different decays has lead to a very small uncertainty. Their latest paper [10] states

$$V_{ud} = 0.97373 \pm 0.00031. \quad (1)$$

Despite the considerable success of the HT approach, the crucial importance of the process in testing the Standard Model has long mandated that the theory behind the analysis be continually re-examined, an especially urgent process now because a more recent evaluation [11] of an electro-weak radiative correction claims a 4 standard deviation violation of unitarity. Our focus is on the isospin-breaking correction δ_C . A variation of this quantity, $\Delta\delta_C$ would cause a change in V_{ud} given by

$$\frac{\Delta(V_{ud}^2)}{V_{ud}^2} \approx \Delta\delta_C \quad (2)$$

As a typical example, $\delta_C = 0.960(63)\%$ for the $0f_{7/2}$ orbital of ${}^{42}\text{Ti}$ [10]. A 20 % change in that number is about 0.2% and V_{ud} would be changed by half that, 10^{-3} , a number that is 3.5 times the uncertainty quoted in Eq. (1). The Particle Data Group [12] finds a similar central value of V_{ud} but a smaller uncertainty of ± 0.00014 . In that case the 20% change in δ_C would be almost 8 times the uncertainty.

The purpose of this paper is to argue that the influence of short-ranged correlations between nucleons, unaccounted for by Towner & Hardy [6] (TH), causes a reduction in δ_C that can be reasonably expected to be about 20%. This means that, depending on future theoretical and experimental work, either the uncertainty in the value of V_{ud} is significantly larger than that of Eq. (1), or that the value is itself reduced by a significant amount, enhancing the probability that there is a violation of unitarity of the CKM matrix.

Superaligned β decays are generated by the isospin operator τ that obeys the usual commutation relations. The theoretical formalism of TH is based on using a weak interaction operator different than τ , designed instead to fit into a small shell model space. This procedure was criticized by Miller & Schwenk (MS) [13, 14] who argued that the effects of the TH approximation were substantial. Similar ideas based on the collective isovector monopole state were presented in [15, 16]. Work on the effects of short-ranged correlations appears in [17] that concludes, “we present a new set of isospin-mixing corrections to sd - shell $0^+ \rightarrow 0^+$ β decay rates. All Hamiltonians provide surprisingly similar results, however, different from the values of Towner and Hardy. A more advanced study of these corrections should be performed.”

The TH restriction is motivated by a shell-model picture in which radial excitations of energy $2\hbar\omega$ and higher above the relevant orbitals can be neglected. However their approach specifically eliminates the influence of short-ranged nucleon-nucleon correlations that involve nucleons in orbitals high above the given shell model space. This presence, a strong interaction effect, reduces the probability that a decaying nucleon is in a valence single-particle orbital and suggests that the magnitude of δ_C is smaller than that of previous calculations.

An exact formalism for evaluating δ_C was presented in [13, 14], but this was not taken up by Hardy & Towner. Ref. [7] used the phrase “not a practical proposition”, when referring to the work of Ref. [13, 14]. The present effort presents a more detailed version of the MS formalism that focuses on the influence of short-ranged correlations, now known to be important because of significant experimental and theoretical work.

In the time since TH started their epic sequence of calculations many new experimental and theoretical results have obtained unambiguous evidence that nucleon-nucleon short-ranged correlations do exist in an observable fashion [18–45]. The effects of short-ranged correlations between nucleons, predicted long ago, have finally been measured and are significant. Such correlations in-

volve the excitations of nucleons to intermediate states of high energy. Consequently, radial excitations are now known to be important in nuclear physics.

Spectroscopic factors, essentially the occupation probability of a single-particle, shell-model orbital, play an important role in what follows. As reviewed in Ref. [21], electron scattering experiments typically observe only about 60-70% of the expected number of protons. This depletion of the spectroscopic factor was observed over a wide range of the periodic table at relatively low-momentum transfer for both valence nucleon knockout using the $(e, e'p)$ reaction [46] and stripping using the $(d, {}^3\text{He})$ reaction [47]. The missing strength of 30%-40% implies the existence of collective effects (long-range correlations) and short-range correlations in nuclei. See also the substantial theoretical analyses [48–51] that used detailed many-body evaluations to find that including the effects of both long and short-range correlations must be included to reproduce the results of experiments that measure spectroscopic factors.

Ref. [30] made a quantitative effort to analyze the separate long (LRC) and short range (SRC) contributions to the quenching of the spectroscopic factors. Their result is the SRC contribution amounts to $22\% \pm 8\%$ and the LRC contribution to $\delta = 14\% \pm 10\%$. This is in accordance with expectations [18–21, 29] and with the results of [48–51]. In the following we argue that, in analogy with the $(e, e'p)$ and $(d, {}^3\text{He})$ reactions, the super-allowed beta decay measurements are impacted by the short-ranged correlations that reduce the spectroscopic strength by about 20%.

Therefore we re-examine the calculations of super-allowed beta decay rates with an eye toward including the effects of short-ranged correlations absent in the TH formalism. Our goal is to provide a plausibility argument, rather than a detailed evaluation. Therefore we rely on simple arguments, starting from the basics.

The shell model model is the starting point for nuclear physics. In its simplest form, the nucleons are in single particle orbitals and the β decay matrix element is simply an overlap between neutron and proton wave functions. If the Hamiltonian commutes with all components of the isospin operator, this overlap would be unity. But the non-commuting interactions, such as Coulomb interaction and the nucleon mass difference cause the overlap to be less than unity. This leads to a non-zero value of the isospin correction known as δ_C .

But there is a further modification of the value of the matrix element because there is no fundamental single-nucleon, mean-field potential in the nucleus. The mean field that binds the orbitals is only a first approximation to the interactions that bind the nucleus. The mean-field arises from the average of two- (or more) body interactions, but residual two- (or more) nucleon effects must remain. There are residual interactions that cause long-range correlations, such as particle-vibration coupling and those that cause the short-ranged correlations mentioned above. The MS formalism easily adapts to

including the effects of these residual interactions.

The fundamental theory for the Fermi interaction of proton beta decay involves the isospin operator τ_+ and the Fermi matrix element is then given by $M_F = \langle f | \tau_+ | i \rangle$, $|i\rangle$ and $|f\rangle$ the exact initial and final eigenstates of the full Hamiltonian $H = H_0 + V_C$, with energy E_i and E_f , respectively and V_C denotes the sum of *all* interactions that do not commute with the vector isospin operator. We use round bra and ket states to denote the eigenstates of the isospin-symmetric part of the Hamiltonian, so $H_0 |n\rangle = E_n^{(0)} |n\rangle$. Obtaining the states $|n\rangle$ requires a solution of the A -body problem. Here we follow MS by expanding in the *difference*, ΔV_C , of the charge-dependent interactions between the initial proton-rich and final neutron-rich states. This amounts to a redefinition of $|f\rangle$ as the isospin rotation of the state $|i\rangle$. Hence, ΔV_C includes all charge-dependent interactions of the extra proton with the other nucleons in the initial state. Then $|f\rangle \rightarrow |f\rangle$ and

$$|i\rangle = \sqrt{Z_C} \left[|i\rangle + \frac{1}{E_i - \Lambda_i \tilde{H}_0 \Lambda_i} \Lambda_i \Delta V_C |i\rangle \right], \quad (3)$$

where $\Lambda_i \equiv 1 - |i\rangle\langle i|$, and \tilde{H}_0 includes the effects of V_C common to the initial and final states, for example the Coulomb interactions in the core. The final state is an eigenstate of \tilde{H}_0 and obeys $\langle f | \tau_+ \Lambda_i = 0$, so that

$$M_F = \sqrt{Z_C} M_0, \quad (4)$$

a theorem of MS (see also [52, 53]), that there are no first-order ISB corrections to Fermi matrix elements, and in this case $\delta_C = 1 - Z_C$ has a straightforward perturbative expansion in ΔV_C . In second order:

$$\delta_C \approx \langle i | \Delta V_C \Lambda_i \left(\frac{1}{E_i - \Lambda_i \tilde{H}_0 \Lambda_i} \right)^2 \Lambda_i \Delta V_C | i \rangle \quad (5)$$

which follows from the normalization condition $\langle i | i \rangle = 1$. To second order in ΔV_C , the full energy E_i can be taken as the energy $\tilde{E}_i^{(0)}$ of \tilde{H}_0 . Furthermore, in this first assessment of the influence of the short-range correlations, SRC, we treat ΔV_C as a single-nucleon operator that acts only on a valence proton, in an orbital v . Then we may define the operator appearing in Eq. (5) to be a one-body operator \mathcal{O}_C : $\delta_C \approx \langle i | \mathcal{O}_C | i \rangle$. The presence of the intermediate propagator mandates that \mathcal{O}_C is non-local (off-diagonal) when expressed in the coordinate-space representation.

A deeper level of analysis is needed to display the effects of SRC. The state $|i\rangle$ can be written in term of a mean-field component $|i_0\rangle$ that also includes the effects of long-ranged correlations contained in the model space of TH and a reaction matrix G ¹ that sums ladder diagrams

¹ For two-nucleon interactions G is the two-nucleon T -matrix evaluated at negative energy and modified by Pauli blocking effects.

involving two- and three-nucleon interactions:

$$|i\rangle = \sqrt{Z_S} \left[|i_0\rangle + Q \frac{G}{e} |i_0\rangle \right], \quad (6)$$

in which the projection operator $Q = 1 - |i_0\rangle\langle i_0|$, e is a schematic representation of an energy denominator, all iterations of the potential that correct the state $|i_0\rangle$ are included in the schematic factor $Q \frac{G}{e}$, and Z_S insures the normalization—it is the spectroscopic factor. The operator Q is constructed to exclude the long-ranged correlations so that *only* the short-ranged correlations are included in the correction to the state $|i_0\rangle$.

Defining an operator $\Omega \equiv Q \frac{G}{e}$ allows Eq. (5) to be expressed as

$$\delta_C = Z_S \left[\langle i_0 | \mathcal{O}_C | i_0 \rangle + \langle i_0 | \Omega^\dagger \mathcal{O}_C + \mathcal{O}_C \Omega + \Omega^\dagger \mathcal{O}_C \Omega | i_0 \rangle \right]. \quad (7)$$

The matrix element $\langle i_0 | \mathcal{O}_C | i_0 \rangle \equiv \delta_{C0}$ is a compact manner of expressing the isospin-violating terms of TH, including the correlations within their shell-model space. Observe that the formalism requires a multiplicative factor, $Z_S < 1$ that reduces the size of the isospin-violating effects. The other terms in the bracket, defined as $\Delta\delta_{C0}$, represent β decay from nucleons correlated by short-ranged effects. It will be necessary to determine if those terms counteract the reduction caused by the factor Z_S or further reduce the correction.

The remainder of this analysis is concerned with providing a first estimate of the influence of SRC. We proceed by using the Jastrow correlation [54–63] approximation to the nuclear wave function. The most important and best-measured SRC involve two nucleons, in which the correlated wave function, $\psi^{(2)}$ is related to the mean field approximation $\phi^{(2)}$ with $\psi^{(2)} = (1 + f)\phi^{(2)}$. In the present situation one of the nucleons is the decaying proton in orbital v and the other is any nucleon in orbital α , so $\phi^{(2)}$ represents the product state $|v\alpha\rangle$. Here the correlations are represented by a function $f(r)$, in which r is the separation distance and in *e.g.* Bruckner theory $f(r) = Q \frac{G}{e}$ [62, 63]. The relative density of the state is given by the term $(1 + f)^2$. A schematic notation, in which various quantum numbers of the two-nucleon wave function are not explicitly specified, is used to simplify our presentation. With this notation, the operator Ω and $f(r)$ are related by $f(r) = \langle r | \Omega | \phi^{(2)} \rangle$. The function $f(r)$ represents *only* the short-ranged correlations, as mandated by the proper construction of the operator Q .

Recall that the nuclear force is repulsive at small separations and attractive at large separations. This means that $f \approx -1$ for small values of s , rises to 0 or slightly above as values of s increase towards the region of attraction and then falls to within 1 fm or so [57, 61, 64]. The details of $f(s)$ are model dependent, but the previous sentence holds in all approaches to the nuclear many-body problem.

Three quantities are needed to evaluate δ_C of Eq. (7). The first is $\delta_{C0} \equiv \langle i_0 | \mathcal{O}_C | i_0 \rangle$, given by

$$\delta_{C0} = \int d^3r d^3r' \phi_v^*(\mathbf{r}) \mathcal{O}_C(\mathbf{r}, \mathbf{r}') \phi_v(\mathbf{r}'). \quad (8)$$

Note that the ranges of \mathbf{r} and \mathbf{r}' vary over the entire size of the nucleus. This expression was shown to account for the overlap correction, δ_{C2} of TH in Ref. [13]. The second term in the bracket of Eq. (7), $\Delta\delta_{C0}$, depends on the operator Ω and gives the contributions of β decay of a proton in a correlated pair. The third quantity (to be discussed first) is Z_S , given by

$$Z_S^{-1} - 1 = \sum_{\alpha} \langle v\alpha | f^2 | v\alpha \rangle. \quad (9)$$

The sum over α leads to the density ρ of the $A - 1$ inert core. Then

$$Z_S^{-1} - 1 = \int d^3r d^3r_2 \rho(r_2) |\phi_v(\mathbf{r})|^2 f^2(|\mathbf{r} - \mathbf{r}_2|). \quad (10)$$

The nucleus is assumed to be large and f , representing the SRC has a range of the order of 1 fm or less. Thus we may approximate

$$Z_S^{-1} - 1 \approx \kappa \int d^3r |\phi_v(\mathbf{r})|^2 \rho(r), \quad (11)$$

with $\kappa \equiv \int d^3s f^2(s)$. Eq. (11) schematically represents a way to compute the SRC contribution to the spectroscopic factor. An order of magnitude estimate establishes that Eq. (11) is reasonable. First take the integral to be given by about 1/2 of the nuclear matter density, ρ_{nm} . This is because the decaying orbitals are located on the edge of the nucleus, but not so far outside as to become unbound. Then the right-hand-side of Eq. (10) is given approximately by $\frac{4\pi}{3} r_1^3 \frac{1}{2} \rho_{\text{nm}}$, with r_1 being the range of the short-ranged correlation. Using $r_1 = 0.8$ fm and $\rho = 0.17 \text{ fm}^{-3}$ leads to $Z_S = 0.82$, in rough agreement with that expected from experimental measurements. and theoretical calculations [18–21, 29, 30, 48–51]. Computing the spectroscopic factor using detailed many-body theory is beyond the scope the present paper, but the stated references display ample evidence that it is reasonable to expect that $Z_S \approx 0.8$, although with significant dependence on the specific state and nucleus.

Next consider the factor $\Delta\delta_{C0}$. Explicitly:

$$\Delta\delta_{C0} = \int d^3r d^3r' \phi_v(\mathbf{r}) I(\mathbf{r}, \mathbf{r}') \mathcal{O}_C(\mathbf{r}, \mathbf{r}') \phi_v(\mathbf{r}'). \quad (12)$$

The function $I(\mathbf{r}, \mathbf{r}')$ is the sum of terms of first-order in Ω , I_1 , and of second order, I_2 . We find

$$I_1(\mathbf{r}, \mathbf{r}') = \int d^3r_2 \rho(r_2) (f(|\mathbf{r} - \mathbf{r}_2|) + f(|\mathbf{r}' - \mathbf{r}_2|)) \approx (\rho(\mathbf{r}) + \rho(\mathbf{r}')) \lambda. \quad (13)$$

with $\lambda = \int d^3s f(s)$, using again the idea the $f(s)$ is non-zero only for values of s that are less than 1 fm or so, and this is very small compared with the radius of the nucleus.

It is important to note that, following our earlier discussion, $\lambda < 0$, because the short-range correlations reduce the two-nucleon wave function at small separations. An estimated comparison between $\Delta\delta_{C_0}$ and δ_{C_0} is given below, but for now the key result is that $\Delta\delta_{C_0}$ takes on a negative value.

The second-order term is given by

$$I_2(\mathbf{r}, \mathbf{r}') \equiv \int d^3r_2 \rho(\mathbf{r}_2) f(|\mathbf{r} - \mathbf{r}_2|) f(|\mathbf{r}' - \mathbf{r}_2|). \quad (14)$$

The contributions of I_2 occur only when both \mathbf{r} and \mathbf{r}' are close to \mathbf{r}_2 , and this restriction severely reduces this contribution to $\Delta\delta_{C_0}$ compared to λ . One therefore expects the contribution of this term to be of order $(r_0/R_A)^3$, where R_A is the nuclear radius relative to I_1 . This simple estimate is borne out using a momentum-space analysis, as follows. First, define the Fourier transform ρ to be F and that of f to be \tilde{f} . Then

$$I_2(\mathbf{r}, \mathbf{r}') = \int \frac{d^3P}{(2\pi)^3} \frac{d^3Q}{(2\pi)^3} F(Q) e^{i\mathbf{P} \cdot (\mathbf{r} - \mathbf{r}')} e^{i\mathbf{Q} \cdot (\mathbf{r} + \mathbf{r}')/2} \times \tilde{f}(\mathbf{P} + \mathbf{Q}/2) \tilde{f}(\mathbf{P} - \mathbf{Q}/2), \quad (15)$$

which is simplified by realizing the \tilde{f} contains high momentum components of the order of fm^{-1} , which depending on the size of the large nucleus, is much larger than that of $F(Q)$. Thus we ignore the factors $\pm\mathbf{Q}/2$ that appear in the arguments of \tilde{f} to find

$$I_2(\mathbf{r}, \mathbf{r}') \approx \int \frac{d^3P}{(2\pi)^3} \tilde{f}^2(P) e^{i\mathbf{P} \cdot (\mathbf{r} - \mathbf{r}')} \rho\left(\frac{1}{2}(\mathbf{r} + \mathbf{r}')\right). \quad (16)$$

The Fourier transform of the square of \tilde{f} is again a short-ranged operator and we may approximate

$$\int \frac{d^3P}{(2\pi)^3} \tilde{f}^2(P) e^{i\mathbf{P} \cdot (\mathbf{r} - \mathbf{r}')} \approx \lambda^2 \delta(\mathbf{r} - \mathbf{r}'). \quad (17)$$

Finally

$$I(\mathbf{r}, \mathbf{r}') \approx (\rho(\mathbf{r}) + \rho(\mathbf{r}'))\lambda + \lambda^2 \delta(\mathbf{r} - \mathbf{r}')\rho(\mathbf{r}). \quad (18)$$

Using the second term in Eq. (12) leads to a contribution: $\lambda^2 \int d^3r |\phi_v(\mathbf{r})|^2 \rho(r) \mathcal{O}_C(\mathbf{r}, \mathbf{r})$. This term is negligible compared with the first term because it is reduced by a ratio of order $(r_0/R_A)^3$, where r_0 is the range of the interaction and R_A is the nuclear radius. Therefore we neglect this term to obtain

$$\Delta\delta_{C_0} \approx 2\lambda \int d^3r d^3r' \phi_V^*(\mathbf{r}) \phi_V(\mathbf{r}') \rho(\mathbf{r}) \mathcal{O}_C(\mathbf{r}, \mathbf{r}'). \quad (19)$$

At this point, we see that using $\Delta\delta_{C_0}$ in Eq. (7) leads to the result that

$$\delta_C \approx Z_S(\delta_{C_0} + \Delta\delta_{C_0}). \quad (20)$$

More detailed evaluations of δ_C ultimately be necessary to establish the necessary precision, but for now observe that $\lambda < 0$ and the operator \mathcal{O}_C , being an absolute square, is positive definite so that all quantities in

the integral that yields $\Delta\delta_{C_0}$ are positive, implying that $\Delta\delta_{C_0} < 0$. Thus we may state an inequality:

$$\delta_C < Z_S \delta_{C_0}. \quad (21)$$

This means that the dominant isospin correction is reduced, compared with the result of TH, by a typically reasonable factor of about $Z_S \approx 0.8$ or smaller. This seemingly innocuous 20 % effect on a small correction, δ_C , has major implications for the extraction of V_{ud} because of the great precision required.

Next we provide a calculation that represents a proof of principle of our formalism. Calculations that go beyond the scope of the present effort will be ultimately be necessary to precisely determine the value of δ_C . The state of current progress is summarized in [65].

Consider the case of a single proton in a $0f_{7/2}$ state outside an inert core of charge $Z = 22$, schematically representing the calculation for ^{46}V . Then ΔV_C takes the form of a one-body operator, $v_C(r)$. Our numerical results and Hartree-Fock calculations [66] show that the valence radial wave function is very well approximated by that of a three-dimensional harmonic oscillator. We therefore use harmonic-oscillator single-particle wave functions.

The nuclear Coulomb potential arises from the convolution of $Z\alpha/(|\mathbf{r} - \mathbf{r}'|)$ with the charge density $\rho_C(r')$. If we take the latter to be a constant within $r \leq R_C$, the one-body Coulomb potential takes the form used by TH. The value of R_C is chosen to match the Coulomb potential obtained with a Fermi shape using $R_A = 1.1A^{1/3}\text{fm}$ and $a = 0.54\text{ fm}$. Our estimate takes the state $|i\rangle$ to be in the single-particle orbit with radial quantum number $n = 0$ and angular momentum $l = 3$ appropriate for the state appearing in the first line of Table I for ^{46}V of Ref. [6]. The matrix element of v_C between the valence state and the state with n, l is $\langle 0l | v_C | nl \rangle$ so that using Eq. (5), we find

$$\delta_{C_0}(l) = \sum_{n>0} \frac{|\langle 0l | v_C | nl \rangle|^2}{4n^2\omega^2}. \quad (22)$$

Using this yields $\delta_{C_0} = 0.267\%$ in agreement with the result in Table I for ^{46}V in Ref. [6]. This is a second demonstration of the utility of the formalism of MS.

The next step is to handle $\Delta\delta_{C_0}$ of Eq. (19), here given by

$$\Delta\delta_{C_0} = 2\lambda \sum_{n>0} \frac{\langle 0l | \rho v_C | nl \rangle \langle nl | v_C | 0l \rangle}{4n^2\omega^2}. \quad (23)$$

Numerical evaluation using the above-mentioned Fermi form for $\rho(r)$ give $\Delta\delta_{C_0} = 0.69 \delta_{C_0} \lambda \text{fm}^{-3}$. The relatively large value of 0.69 is specific to the use of harmonic oscillator wave functions that falloff as a Gaussian instead of an exponential and is probably an overestimate. Computing the value of λ requires using the best available nucleon-nucleon interactions and many-body theory (because of the presence of the operator Q), and so is

beyond the scope of the present effort. However, an estimate can be obtained by taking the repulsive core to have a radius of about 0.7 fm. Using this value gives $\lambda = -1.4 \text{ fm}^3$, so that the term $\Delta\delta_{C0}$ causes a -99 % reduction in the value of δ_{C0} . If this strong reduction were borne out in a more realistic calculation one would need to also include the effects of the term I_2 that would increase the correction. The net result is that the SRC suppression term of Eq. (19) is potentially a very large effect and its proper evaluation requires serious future work beyond the present estimate.

Turn now to a summary of the implications of the present analysis. The key result is that the isospin correction is decreased by a factor of about 80% or less for a nucleus of the size of ^{46}V . This depletion value is typical of that observed for spectroscopic factors of many nuclei over the periodic table. Although the present analysis is clearly a schematic treatment requiring improvement by more detailed nuclear structure calculations, it is worthwhile to consider the implications for tests of the unitarity of the CKM matrix. A 20% reduction δ_C corresponds,

via Eq. (2) and the ^{42}Ti value of δ_C , to a change in the value of $|V_{ud}|$ of about 10^{-3} , a number that is about three or eight times the uncertainties discussed above. This large increase in the error points to the importance of precisely determining the influence of SRC on super allowed β decay. There is also an intriguing possibility. Suppose that the influence of SRC can eventually be determined with a sufficiently small uncertainty. Then the value of $|V_{ud}|$ would be decreased by about 0.001, a reduction that would vastly increase the potential violation of the unitarity of the CKM Matrix. Thus our first message is that it is reasonable to expect that the constraints on models beyond the Standard Model one need not be as strongly constrained by the result displayed in Eq. (1). As a result, our second message is that doing more detailed state-of-art nuclear calculations of superallowed β decay is a high priority for nuclear theorists.

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