Extraction of Inclusive Cross Sections

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EXPLORING THE NATURE OF MATTER

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In this talk I will discuss the exraction of inclusive cross sections from Counting experiments.

Although I will focus on the HMS spectrometer in Hall C, these ideas are generally applicable to other spectrometers.

Outline

- Methods for extracting cross sections from yields
    i. Acceptance correction method
    ii. Monte Carlo ratio method
- Eliminating backgrounds
- Acceptance corrections
- θ bin centering
- Radiative corrections
- Application of extraction methods

Cross Section Extraction Methods

For each bin in ΔE, $\Delta\Omega$, the number of detected electrons is:

$$N^{-} = L \cdot (d\sigma/d\Omega dE') \cdot (\Delta E' \, \Delta\Omega) \cdot \varepsilon \cdot A(E',\theta) + BG$$

with L: Integrated Luminosity (*# of beam electrons · targets/area*)

ε: Total efficiency for detection

$A(E',\theta)$: Acceptance for bin

BG: Background events.

The efficiency corrected electron yield is

$$Y = (N^{-} - BG)/\varepsilon \quad = \quad L \cdot \sigma^{data} \cdot (\Delta E \, \Delta\Omega) \cdot A(E',\theta)$$

For known $A(E',\theta)$, $\sigma^{data}(E',\theta) = Y(E',\theta)/[(\Delta E \, \Delta\Omega) \cdot A(E',\theta) \cdot L]$

From previous slide:

$$d\sigma/d\Omega dE' = Y(E',\theta) / [(\Delta E\, \Delta\Omega) \cdot A(E',\theta) \cdot L]$$ (Acceptance correction method)

$A(E',\theta)$ is the probability that a particle will make it through the spectrometer and *must be measured or determined from simulation!*

$\Delta\Omega_{eff} = \Delta\Omega \cdot A(E',\theta)$ is the *effective solid angle* or *solid angle acceptance*.

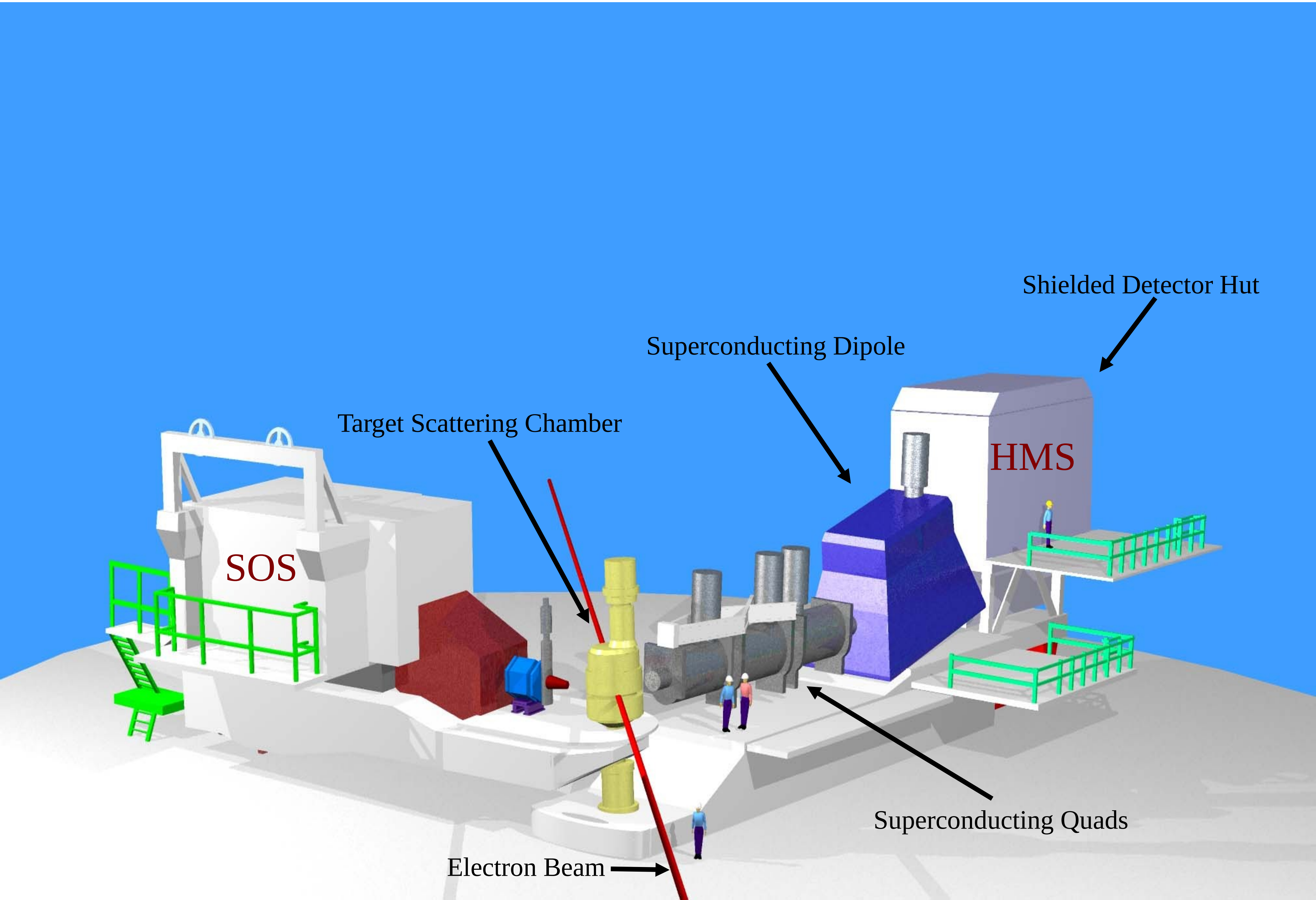
Conversely, we can simulate Monte Carlo data using a cross section model to obtain

$$Y_{MC}(E',\theta) = L \cdot \sigma^{mod} \cdot (\Delta E\, \Delta\Omega) \cdot A_{MC}(E',\theta):$$

Taking ratio to data and assuming that $A_{MC} = A$, yields

$$d\sigma/d\Omega dE' = \sigma^{mod} \cdot [Y(E',\theta)/Y_{MC}(E',\theta)]$$ (MC ratio method)

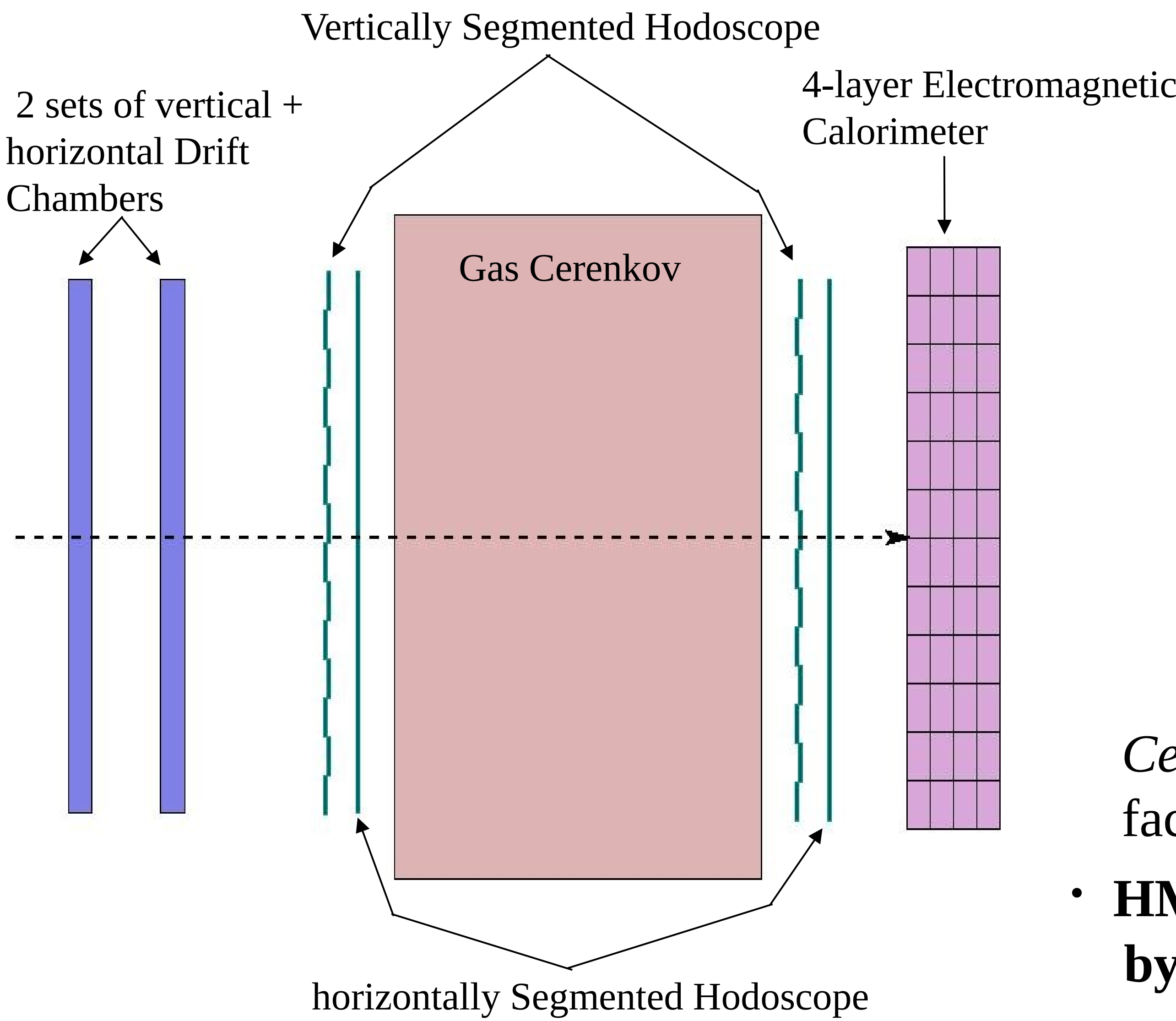
Jlab Hall C



Shielded Detector Hut
Superconducting Dipole
Target Scattering Chamber
HMS
SOS
Superconducting Quads
Electron Beam

HMS Spectrometer

Detector Stack
(view from above)



HMS Properties (pt-pt tune)

Kinematic Range:

Momentum:	0.5 – 7.5 GeV/c
Angular:	10.5^{o} - 80^{o}

Acceptance:

$\Delta\Omega$:	~6.5 msr
Dp/p:	+/-9%

Resolution:

Dp/p:	< 0.1 %
Θ:	~ 1 mrad

Cer + *Cal* provide p rejection factor ~ 10000/1 At 1 GeV

- **HMS Acceptance is dominated by the octagonal collimator!**

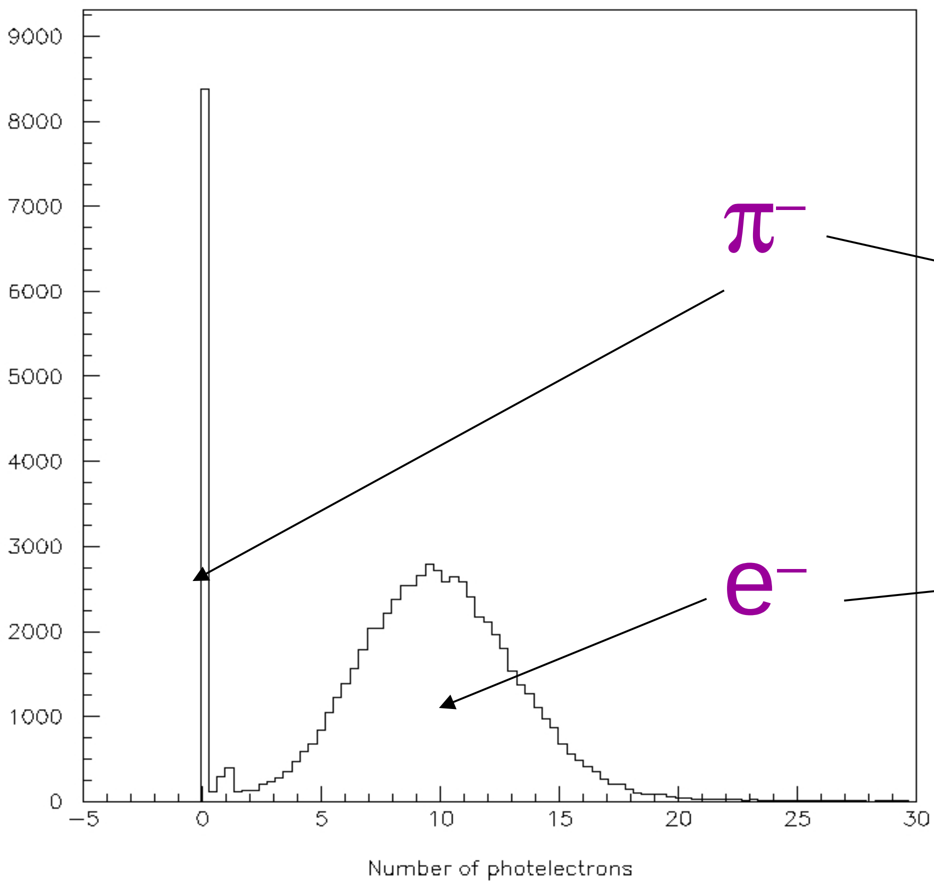
Dominant background for e^- Yields

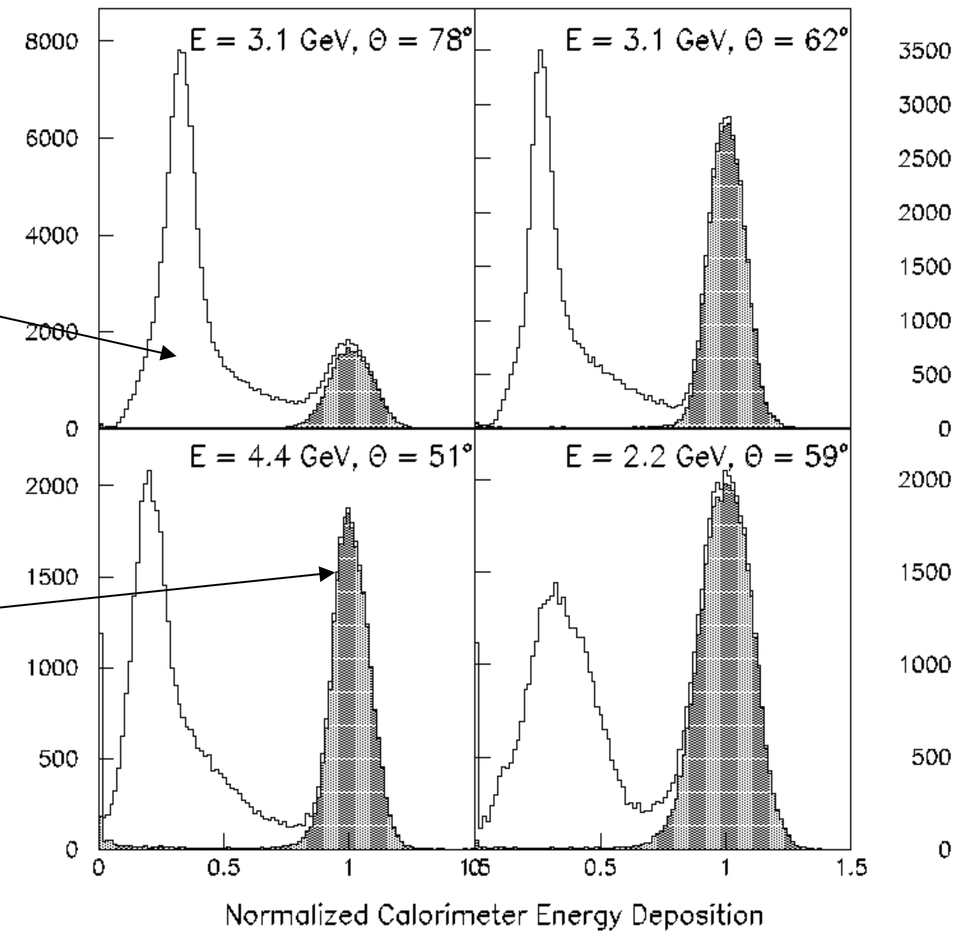
Background	Eliminate by
e^- scattered from Al walls of cryogenic target cell.	Subtracting measured e^- from Al dummy target.
e^- from charge-symmetric p^0, g production and decay.	Subtracting measured e^+ yields.
e^- from radiative events.	Applying radiative corrections.
π^-	Cerenkov and calorimeter cuts

PID Detectors and π^- elimination

Cerenkov # photo-electrons

Calorimeter energy deposition

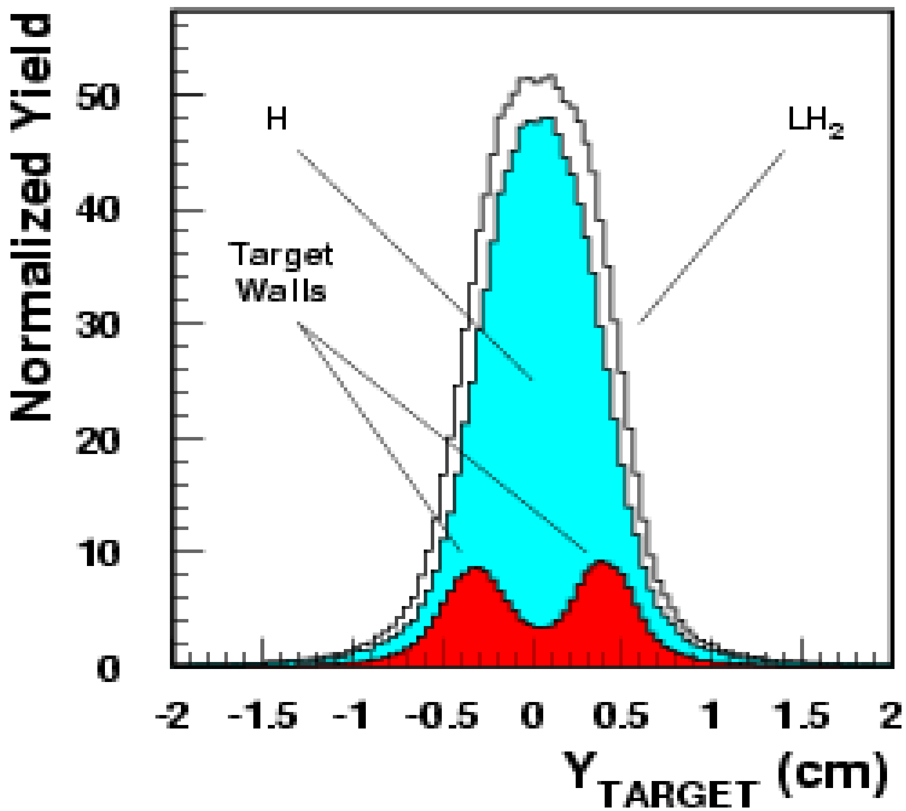




Background from Cryo Cell end caps

Endcaps: $N_{ec} = L_{ec} \cdot \sigma_{Al} / RC_{ec} \sim Q_H (\rho t)_{ec} \cdot \sigma_{Al} / RC_{ec}$

Dummy: $N_{dum} = L_{dum} \cdot \sigma_{Al} / RC_{dum} \sim Q_{dum} (\rho t)_{dum} \cdot \sigma_{Al} / RC_{dum}$



Efficiency corrected yield of hydrogen only events:

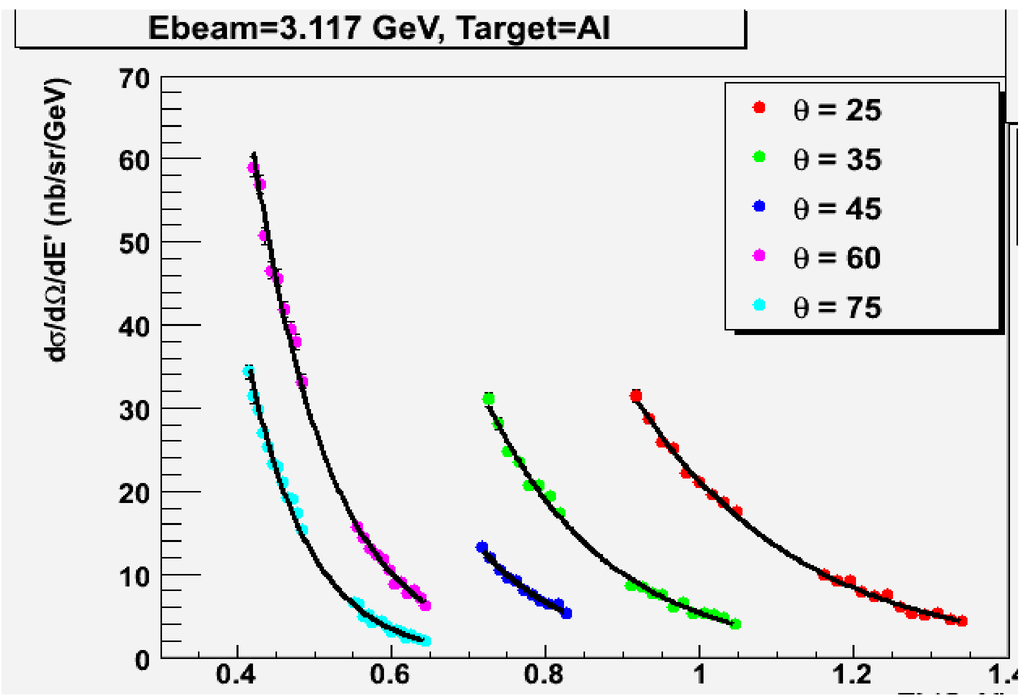
$$Y_H = Y_{cryo} - Y_{dummy} \cdot \frac{L_{ec} \cdot RC_{dummy}}{L_{dummy} \cdot RC_{ec}}$$

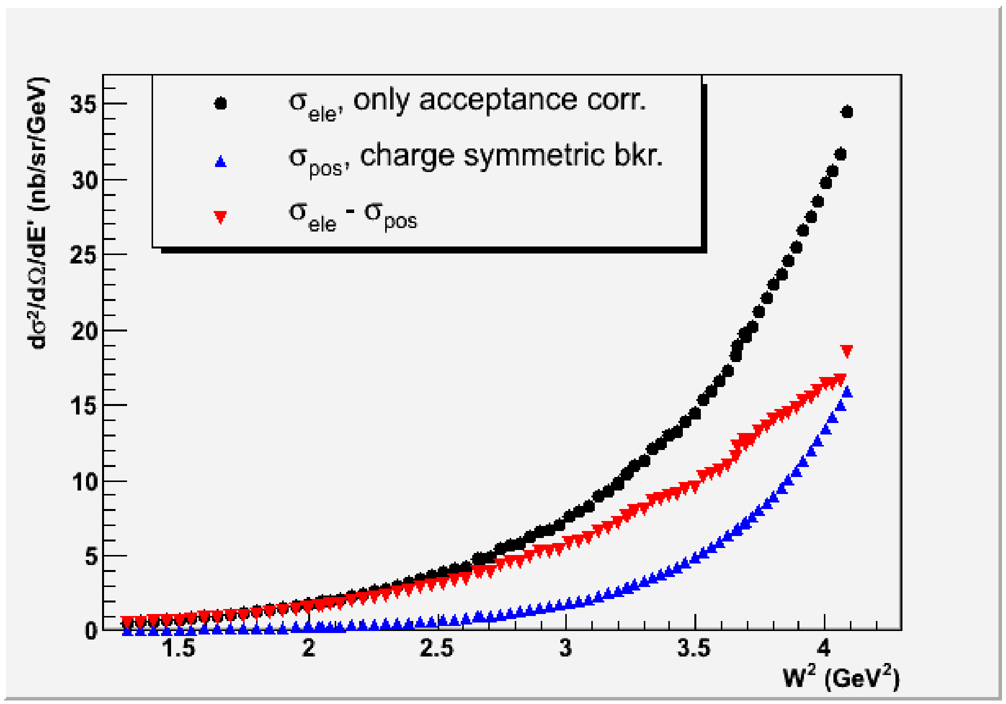
Plot from Vladas Tvaskis' PhD Thesis

Charge Symmetric Background

- Electrons can be produced from charge symmetric processes such As π_0 production and decay to 2γ or direct γ production
  => γ converts in target to e^+ e^- pairs with e^- detected in spectrometer.

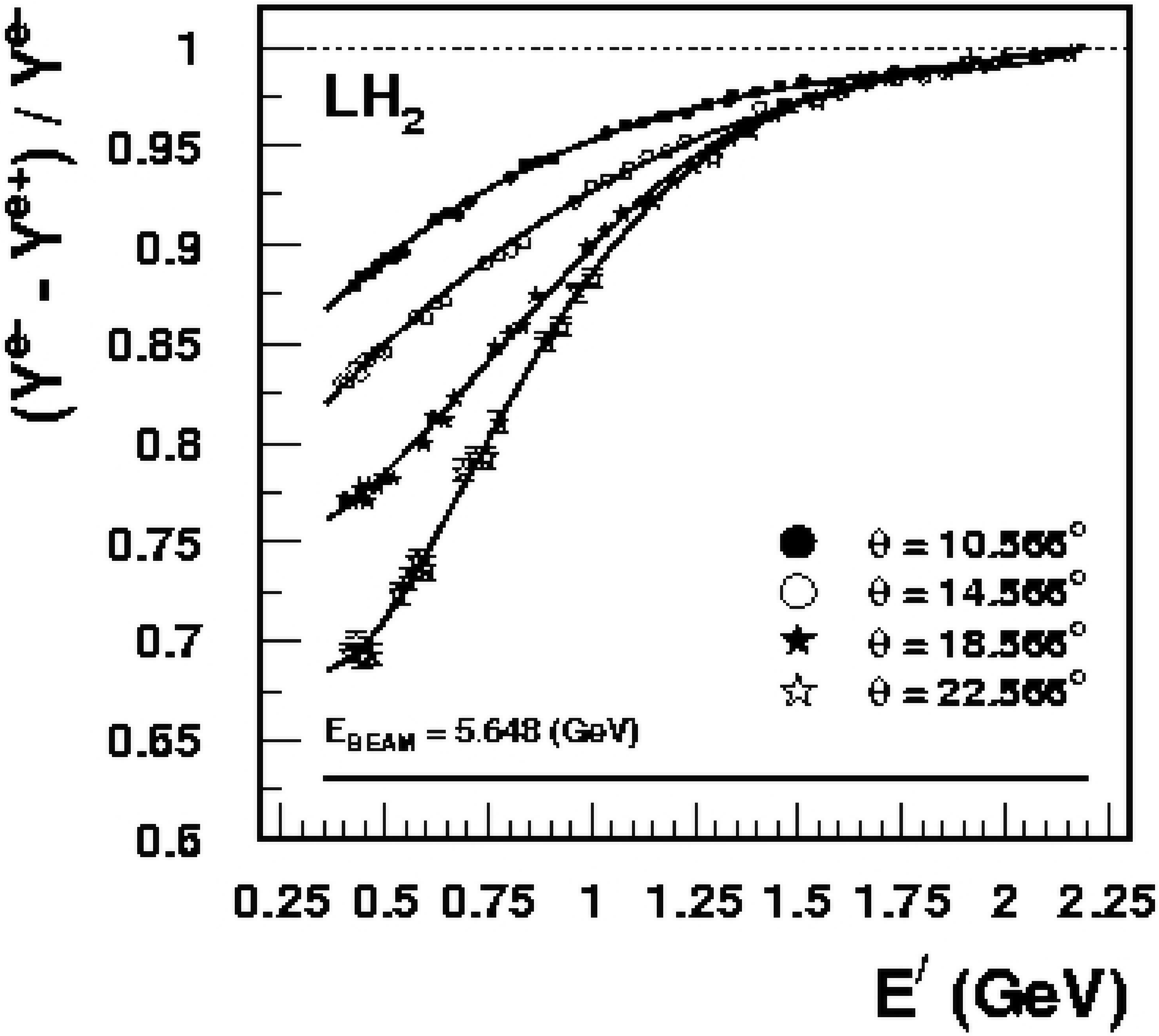
- These electrons look like inclusive scatterers.
- Measure e+ yield or cross section and subtract as background.





Charge Symmetric Background

Vladas Tvaskis' PhD Thesis

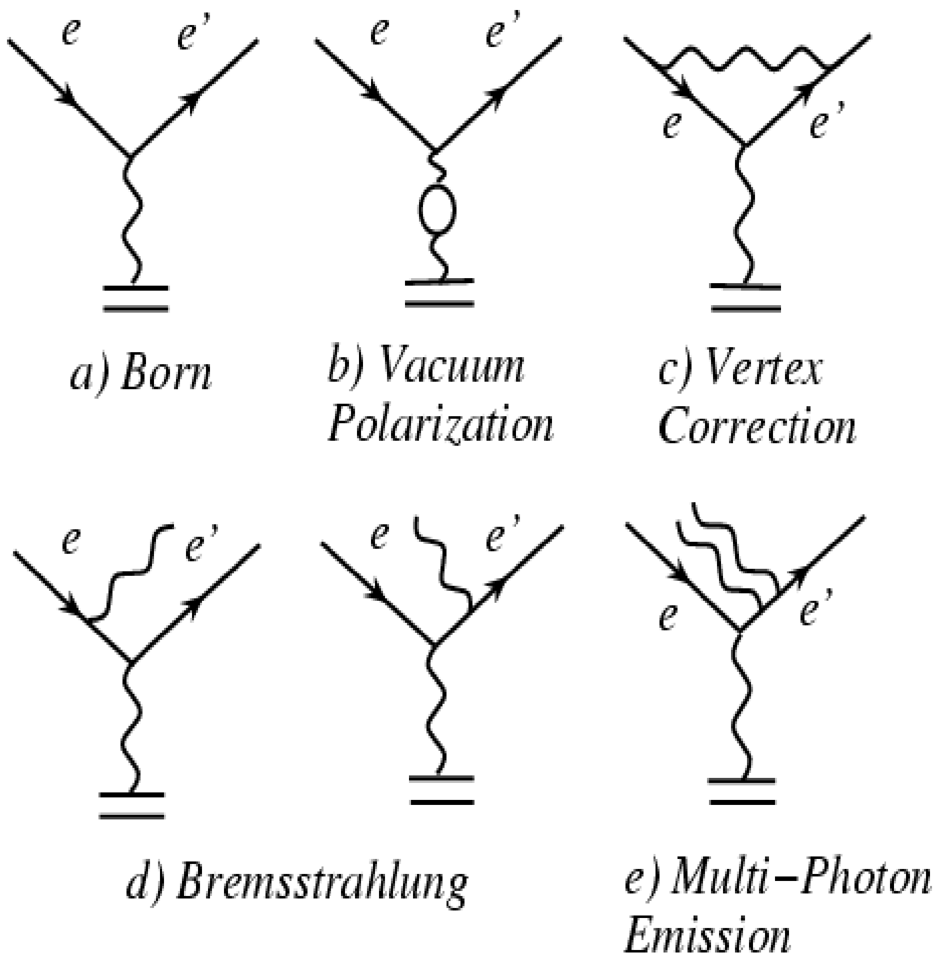


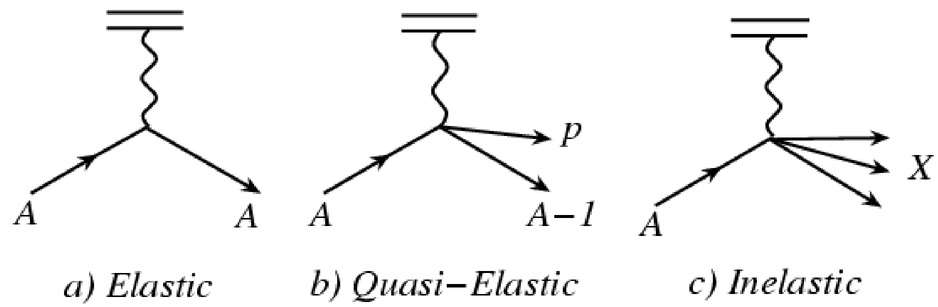
If the CS background is not too large then a multiplicative correction factor can be applied to the electron yield, as

CScor = $(Y^{e-} - Y^{e+})/Y^{e-}$

This can also be added as
A background weighting
Factor in the MonteCarlo

Radiative Corrections





$$\sigma_{\text{Meas}} = \sigma_{\text{Born}} + \sigma_{\text{Elastic}} + \sigma_{\text{Q-Elastic}} + \sigma^{\text{Rad}}_{\text{Inelastic}}$$

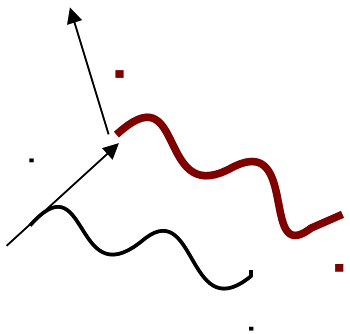
$$\sigma^{Born} = (\sigma^{rad}_{\text{exp}} - \sigma_{el} - \sigma_{qe\,l}) \times \frac{\sigma^{Born}_{Inel}}{\sigma^{rad}_{Inel}}$$

RC

Slide from Vladas Tvaskis

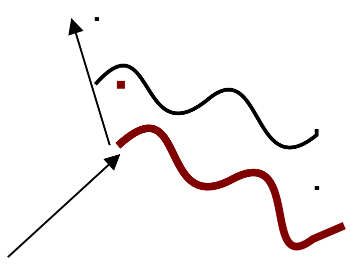
Radiative Corrections II

Bremsstrahlung from beam electron



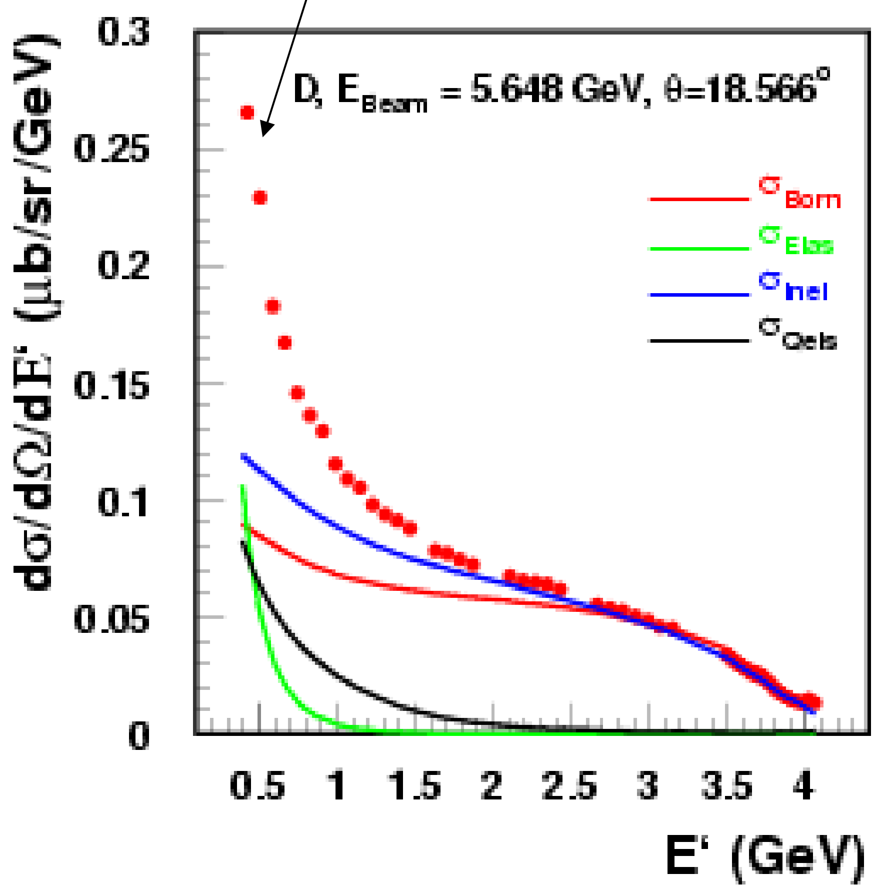
- E_{vertex} is *smaller* than E_{beam}
- Q^2_{vertex} is *smaller* than calculated
- W^2_{vertex} *typically smaller* than calculated.

Bremsstrahlung from scattered electron

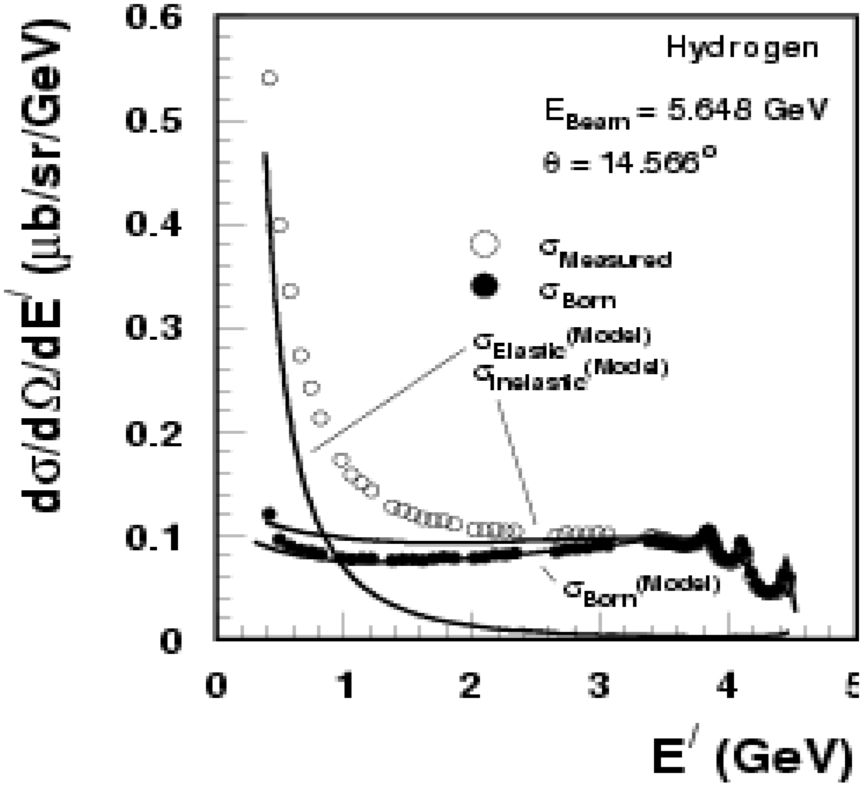


- E'_{vertex} is *larger* than E'_{HMS}
- Q^2_{vertex} is *larger* than calculated
- W^2_{vertex} is smaller than calculated.

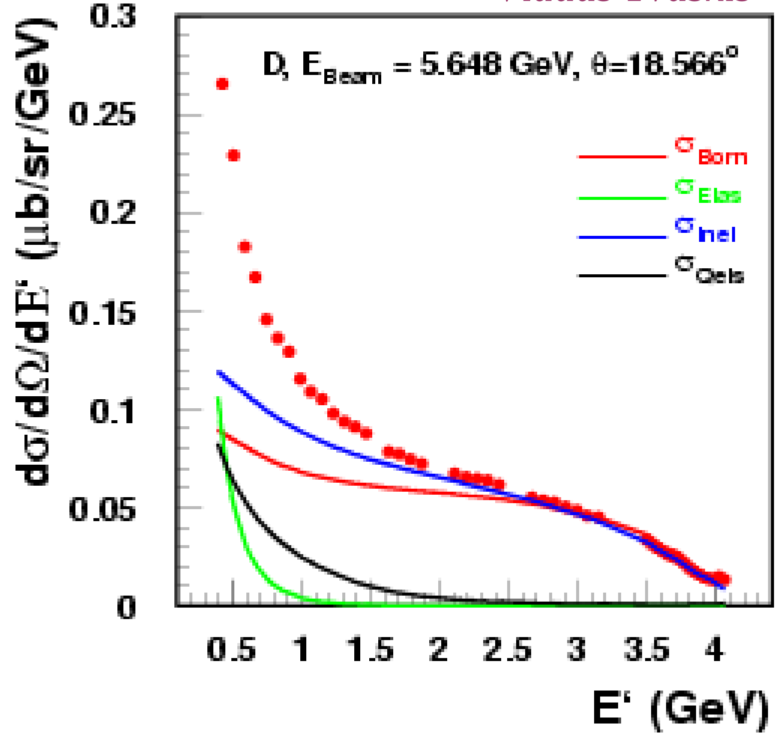
Elastic events at lower Q^2, W^2 radiate to higher Q^2, W^2.



Radiative Corrections III



Vladas Tvaskis



Q: Include QE or nuclear elastic in RC?

A: It depends on the relative size. Safest to subtract large tails from isolated states and correct for remaining.

$$\sigma^{Born} = (\sigma^{rad}_{\exp} - \sigma_{el} - \sigma_{qe\,l}) \times \frac{\sigma^{Born}_{Inel}}{\sigma^{rad}_{Inel}}$$

RC

Notes on Acceptance

- **In general, whether a particle makes it to the detectors without hitting the collimator, beam pipe, or other stopping aperture depends on the full vertex coordinate and the momentum vector of the particle at the target, and *implicitly* on the spectrometer optics) i.e. A = A(E',x,y,z,X',Y').**

- **The acceptance above is purely deterministic. The trajectory through the spectrometer will either intersect with an aperture or not.**

- **However, if we know the fraction of particles at each vertex location then we can properly average over x, y, and z and write A = A(E',X',Y')**

- **The physics only depends on θ (combination of X', Y') => A = A(E',θ)**
  (This has been checked!)

- **Multiple scattering and energy straggling enter only though bin migration and are accounted for in an approximate way.**

Calculating Acceptance Corrections

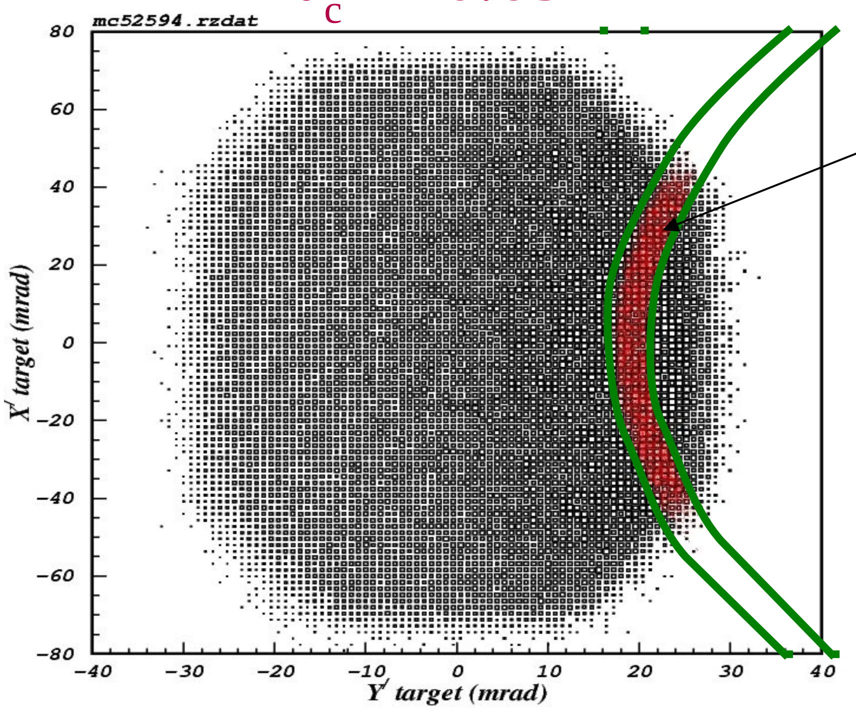
1[st] we generate MC with the same E', θ_c, target, raster, etc... as the data.

Then, for each bin: $\Delta\Omega_{eff}(\delta,\theta) \equiv \mathbf{A(\delta,\theta) * \Delta\Omega_{gen}(\delta,\theta)}$,

where $\mathbf{A}(\delta,\theta) = \mathbf{N_{rec}(\delta,\theta) / N_{gen}(\delta,\theta)}$

Note that $\mathbf{\Delta\Omega_{gen}}(\delta,\theta)$ is the solid angle generated into for that bin and *depends on generation limits in* **Y'** (in-plane angle) and **X'** (out-of-plane angle).

$\theta_c = 10.65^o$



$\Delta\theta$ = -20 +/- 1.75 mrad

If the only aperture were the collimator then the solid angle of this slice would be

$\Delta\Omega_{gen}$ (Green band for $|X'_{gen}| < 80$ mrad)

$\Delta\Omega_{eff}$ (Red area)

However, $\Delta\Omega_{eff}$ does **not** depend on generation limits.

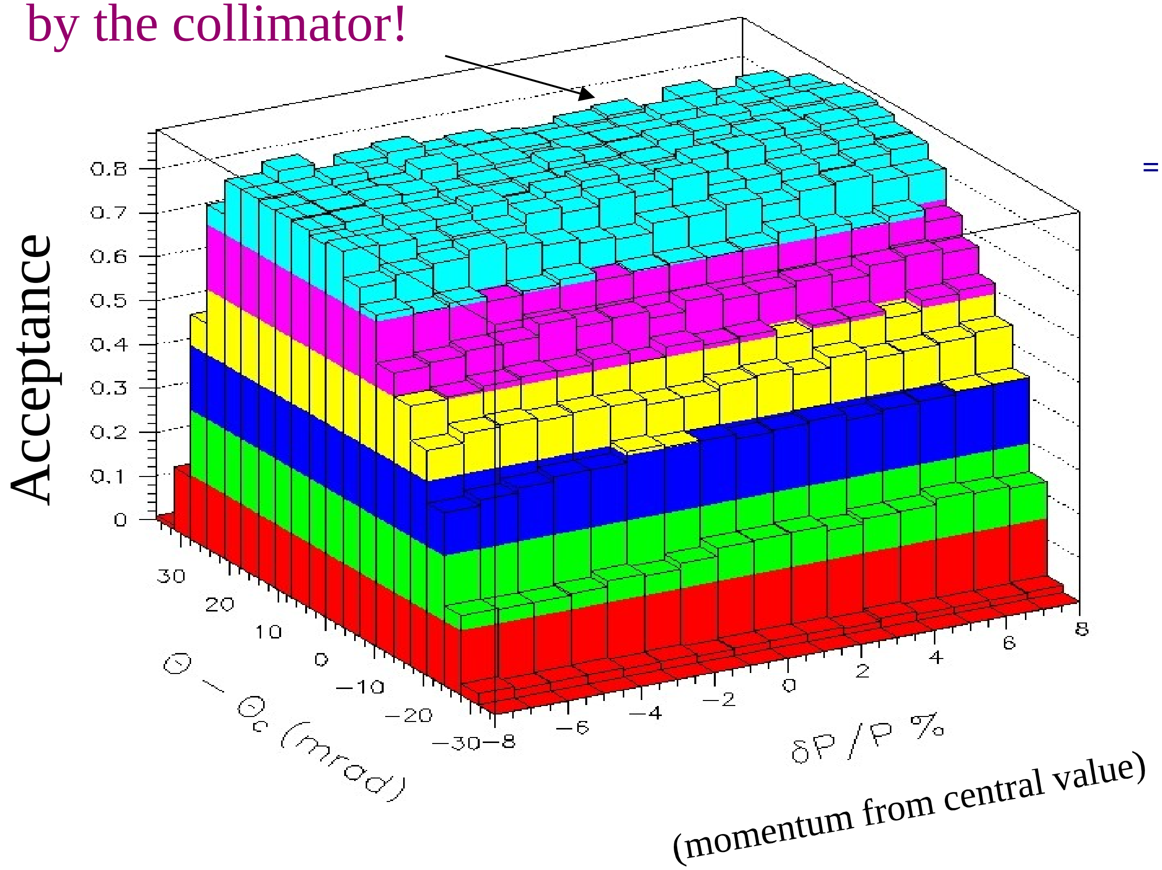
For uniform generation,

$$\Delta\Omega_{eff}(\delta,\theta) \equiv N_{rec}(\delta,\theta) * \frac{\Delta\Omega^{tot}_{gen}(\delta)}{N^{tot}_{gen}}$$

$$= N_{rec}(\delta,\theta) * \frac{N_{gen}(\delta,\theta)}{N_{gen}(\delta,\theta)} \frac{\Delta\Omega^{tot}_{gen}(\delta)}{N^{tot}_{gen}(\delta)}$$

$$= A(\delta,\theta) * \Delta\Omega_{gen}(\delta,\theta)$$

Shape in θ is dominated by the collimator!

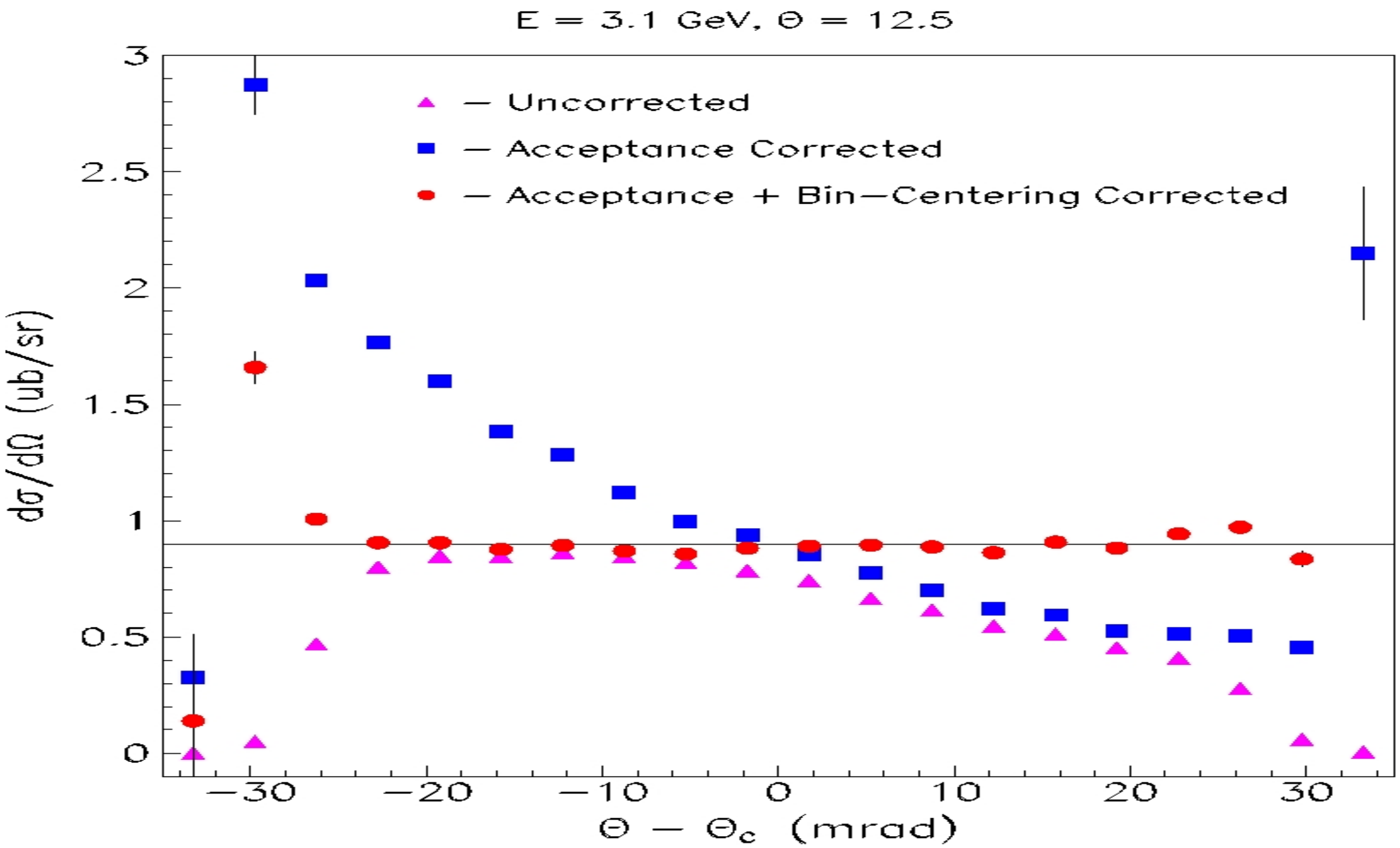


Example application: e-p elastic

Applying Acceptance and θ - BC Corrections

- ***For Each E' bin, we want the cross section at fixed θ (or Q^2).***
- ***Use model to correct dσ to central angle,***

$$d\sigma(\theta_c) = d\sigma(\theta) * \sigma^{mod}(\theta_c)/\sigma^{mod}(\theta)$$

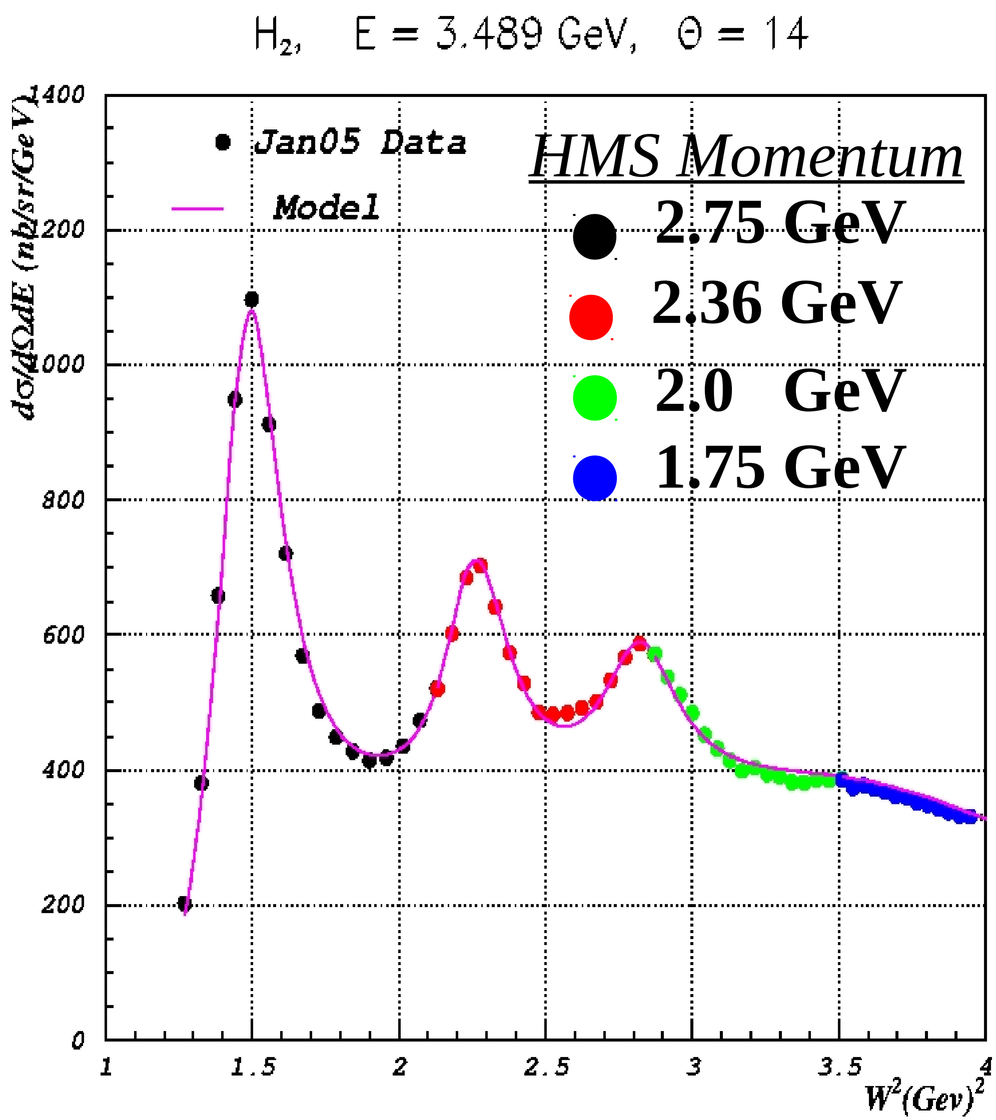


=> Can now average over θ

Notes on Bin-Centering

- A bin-centering correction must be applied to any binned variable for which the yield varies significantly across a bin width.
- The bin-centering correction for a linear function = 1.
- For θ bin-centering, the full radiated cross section model must be used.

Acceptance Correction Method



- Bin efficiency corrected e- yield in δp/p - θ. (δp/p = +/- 8%, Δθ = +/- 35 mrad)
- Subtract scaled dummy yield bin-by-bin to remove e- Al background.
- Subtract charge symmetric e- yield bin-by-bin.
- Apply acceptance correction for each δ-θ bin.
- Apply radiative corrections bin-by-bin.
- Apply θ bin-centering correction and average over θ => for each δ (or W^2) bin.

Overview of Monte Carlo Ratio Method

- Generate MC events with σ model weighting and radiative contributions included.

- Scale the MC yield by L_{data}/L_{MC}, where L_{MC} is that needed to produce N_{gen} for the given σ_{mod} and phase space generated into.

- Add background contributions to MC or subtract from data.

- $d\sigma(\delta, \theta_c) = d\sigma^{mod}(\delta,\theta_c) * Y(\delta)/Y_{MC}(\delta)$

Where $Y(\delta)$ is the yield for events with any value of θ, i.e. this integrates over θ.

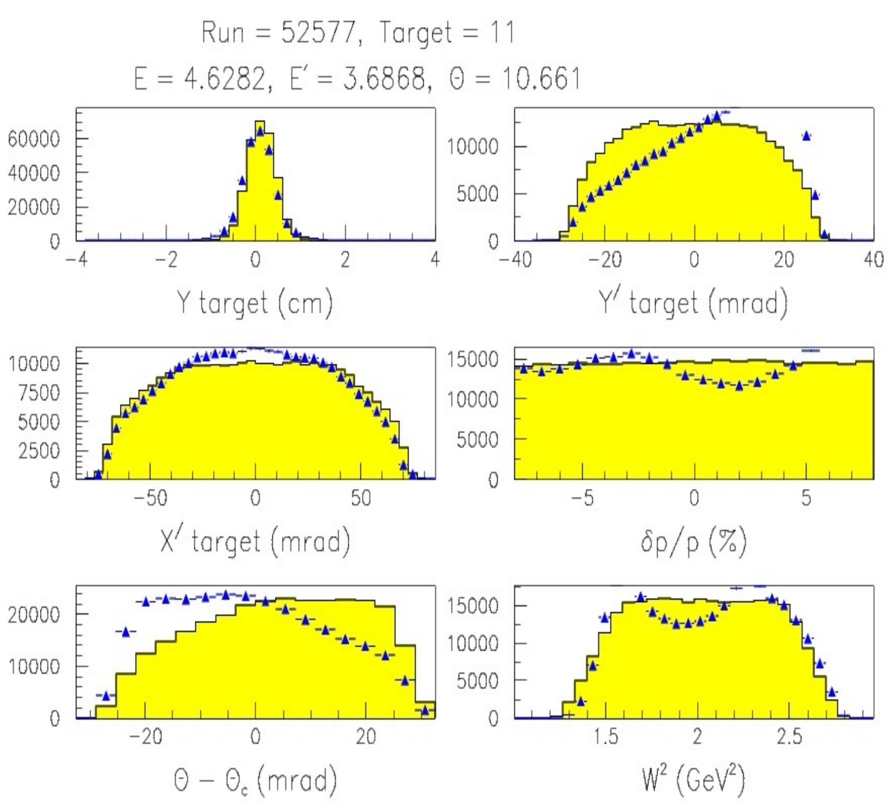
Warning: For inclusive data, radiative events can come from kinematically far away.

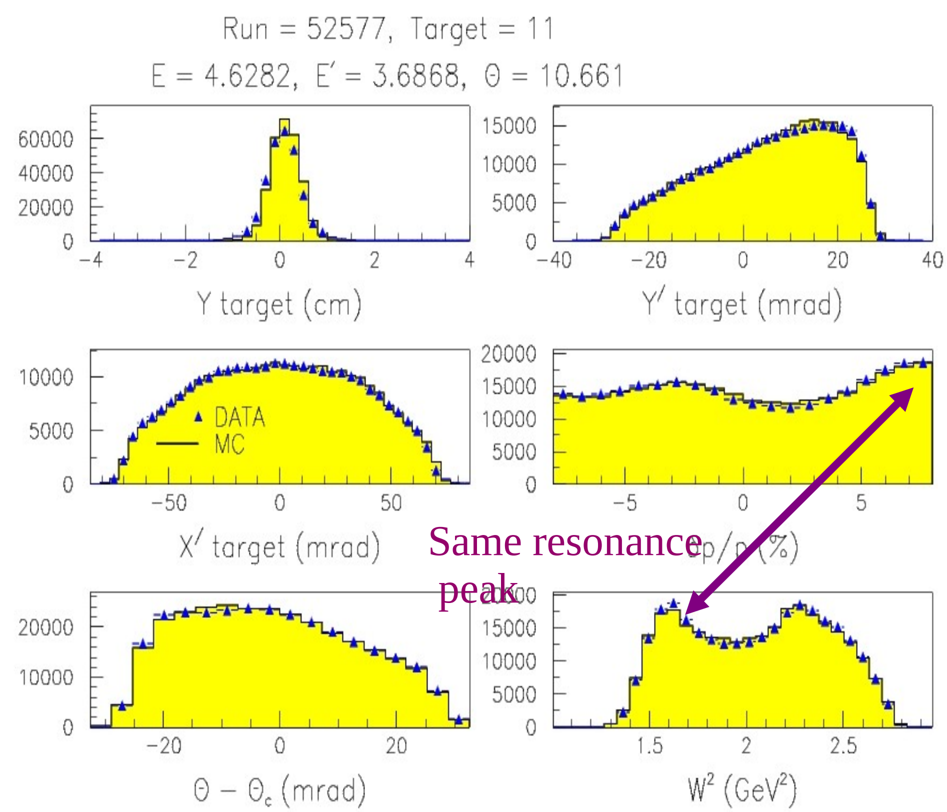
=> Safer to reweight

- Generate e^- scattering *uniformly* in E', Y', X'
- Apply physics weighting for each event based on depending on (E, E', θ)
- Transport events through magnetic fields of optical elements (using transport matrix) and check whether trajectory intercepted an aperture (such as the collimator).
- Compare MC yield to data yield for same normalized luminosity.

Without physics weighting

With physics weighting





Often we will want to generate a fixed # of good events to ensure good statistical precision.

How do I calculate L_{MC}?

If we know the number of generated events, N^{gen}, then

$$N_{gen}(E',\theta) = L \cdot \frac{d\sigma}{d\Omega dE'}^{mod} \cdot \Delta E\, \Delta\Omega \quad (1)$$

In principle we can integrate the model cross section over phase space and solve.

However, if we generated uniformly with $d\sigma = 1$ then

$$\boxed{L_{MC} = N_{gen}(E',\theta) \,/\, (\Delta E\, \Delta\Omega)}$$

Note that if we mutiplied each event in EQ(1) by $d\sigma^{mod}$ then N_{gen} would scale by same amount

Intro to reweighting code (mc_reweight)

- Code takes uniform event generation MC files and reweights utilizing the fully radiated model.

  → For my experiments I utilize the *Externals* program written at SLAC and updated by P. Bosted and others, updating the inclusive models with fits to inclusive cross sections from myself, P. Bosted, et. al.

- Radiated cross section model is taken from a file (called *rc94.dat*) and interpolated from a grid in E' and θ using the format:

$$E,\ E',\ \text{theta},\ \sigma_{\text{radiated}}\ ,\ RC = \sigma_{\text{born}}\ /\ \sigma_{\text{radiated}}$$

mc_reweight input files

1. input.dat

- input MC ntuple filename (example c1_30_1.1.rzdat)
- number of events to reweight (this can be less than the number in the original rzdat)
- the generation range in X' (this is given as the 1/2 range and assumed to be symmetric, example 100. mrad)
- the generation range in Y'
- maximum generation range in dp/p in %
- minimum generation range in dp/p in %
- option to calculate charge-symmetric background weight factor (0 = no, 1 = yes)
- Option to calculate born cross section from interpolated file (0 = no, 1 = yes)

* Note 1: charge-symmetric parametrization must be provided to the code for each

** Note 2: generation ranges **must** be consistent with MC production

2. reconmc.in

contains the scaler information (in 6 GeV grabbed from run scalar file with perl script) :

Target #, E, E'_c, θ_c, prescale, cur_{BCM1}, cur_{BCM2}, Q_{BCM1}, Q_{BCM2}, LT_{comp}, LT_{elec},ε_{track}, ε_{TOF}, $\varepsilon_{3/4}$, ε_{PRLO}, Rate

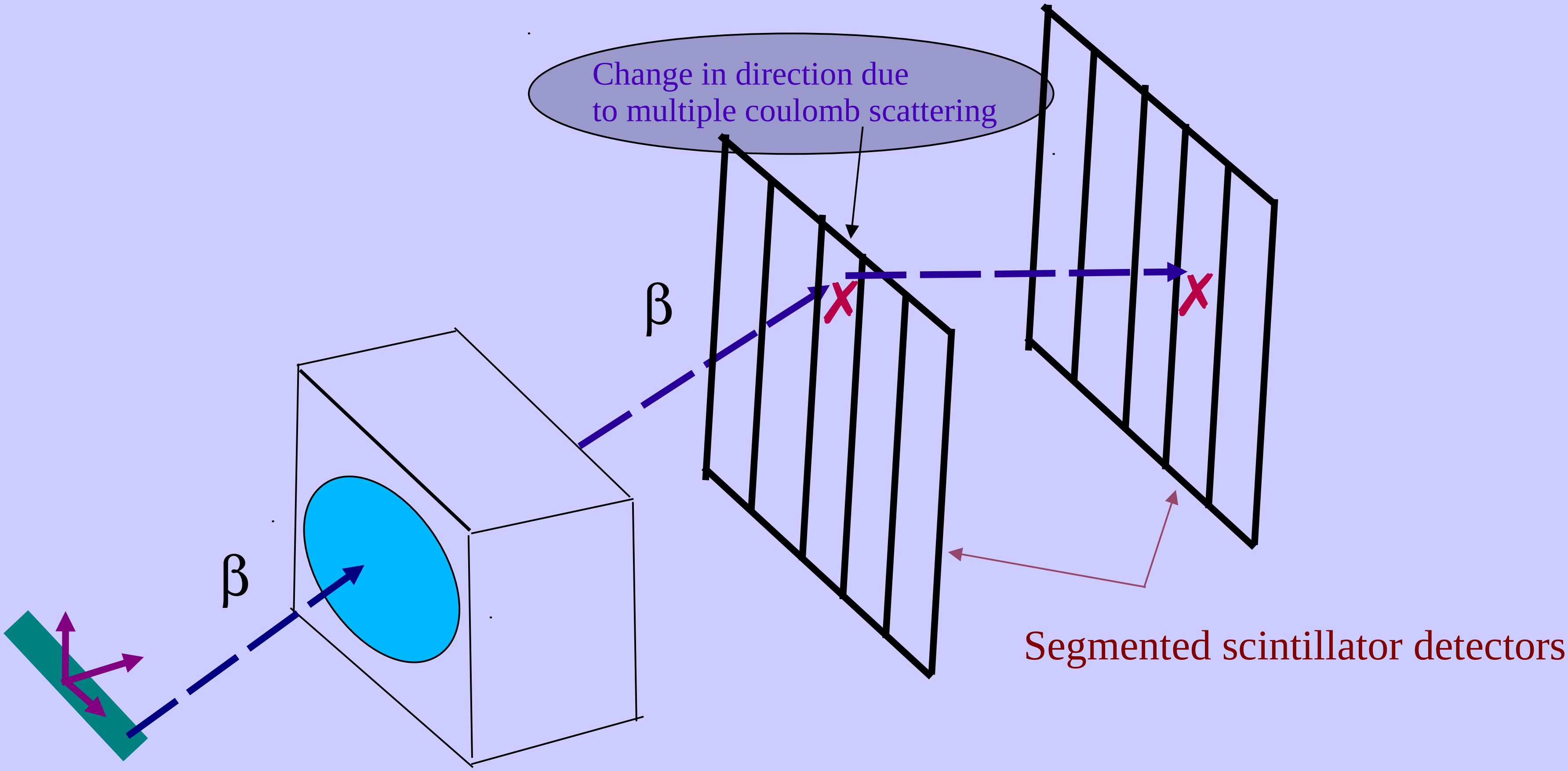
3. targetdata.dat

Contains sequential list of targets

Target #, ρ (g/cm^3), L(cm), Atomic wt., A, ρt (g/cm^2)

Thanks

Multiple scattering effects:



Multiple Coulomb scattering (multiple scattering) causes changes in trajectory and position at detectors after the scattering.

Ignoring the small # of large angle single scatterings, the angular distribution for multiple scattering is approximately Gaussian,

$P(\theta) = 2\theta \exp(-\theta^2 / \langle\theta^2\rangle)d\theta$, with **$\langle\theta^2\rangle$** the mean squared scattering angle

– see previous lecture notes (W.R. Leo pg 46).

Multiple Scattering can:

- affect the acceptance if it occurs before the apertures.

- affect the angle resolution or apparent efficiency of detectors if it happens after the apertures.

Events that and a vertex angle in a particular bin can *migrate* to another bin (typically an adjacent bin)

This is called *bin-to-bin migration* and should be included in the full response function for the detector system (acceptance + bin migration due to resolution, MS, energy loss, etc.)

Energy loss effects

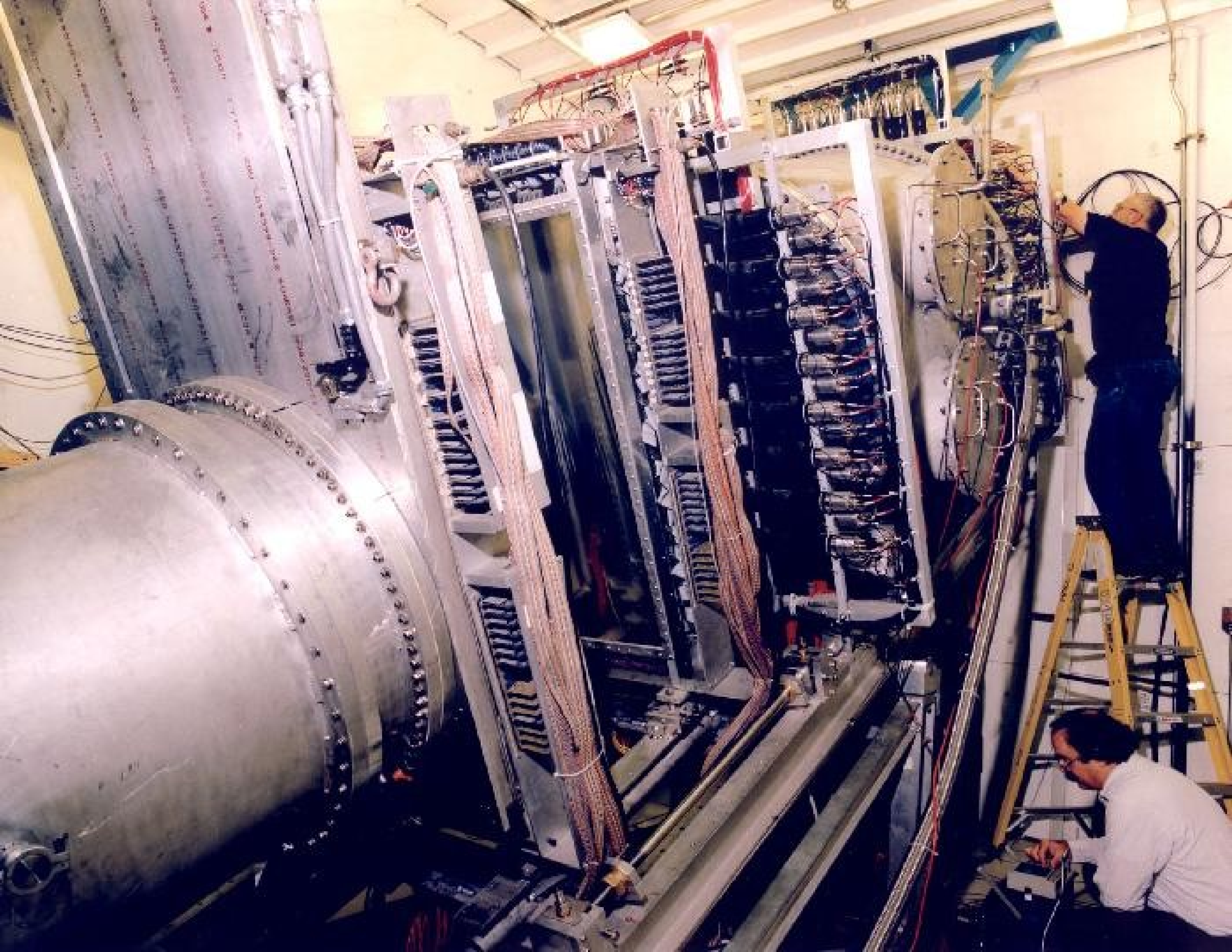
- ➢ affect the acceptance if it occurs before the apertures.

- ➢ affect the angle resolution or apparent efficiency of detectors if it happens after the apertures.

Events that have a vertex energy in a particular bin can *migrate* to another bin (typically an adjacent bin)

This is called *bin-to-bin migration* and should be included in the full response function for the detector system (acceptance + bin migration due to resolution, MS, energy loss, etc.)

** Important: Must generate in kinematics from which events can migrate into kinematic range of interest.



HMS Electron 'Event'