10 Discrete-Time Fourier Series

Solutions to Recommended Problems

S10.1

The output of a discrete-time linear, time-invariant system is given by

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k],$$

where $h[n]$ is the impulse response and $x[n]$ is the input. By substitution, we have the following.

(a)
$$y[n] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k e^{j\pi(n-k)} = e^{j\pi n} \sum_{k=0}^{\infty} \left(\frac{e^{-j\pi}}{2}\right)^k$$
$$= \frac{e^{j\pi n}}{1 - \frac{1}{2}e^{-j\pi}} = \frac{2}{3}(-1)^n$$

(b)
$$y[n] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k e^{j[\pi(n-k)/4]} = e^{j(\pi n/4)} \sum_{k=0}^{\infty} \left[\frac{e^{-j(\pi/4)}}{2}\right]^k$$
$$= \frac{e^{j(\pi n/4)}}{1 - \frac{1}{2}e^{-j(\pi/4)}}$$

(c)
$$y[n] = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \left[\frac{1}{2}e^{j(\pi/8)}e^{j[\pi(n-k)/4]} + \frac{1}{2}e^{-j(\pi/8)}e^{-j[\pi(n-k)/4]}\right],$$ where we have used Euler's relation

$$= \frac{1}{2}e^{j(\pi/8)}e^{j(\pi n/4)} \sum_{k=0}^{\infty} \left[\frac{e^{-j(\pi/4)}}{2}\right]^k + \frac{1}{2}e^{-j(\pi/8)}e^{-j(\pi n/4)} \sum_{k=0}^{\infty} \left[\frac{e^{j(\pi/4)}}{2}\right]^k$$
$$= \frac{\frac{1}{2}e^{j[(\pi/8)+(\pi n/4)]}}{1 - \frac{1}{2}e^{-j(\pi/4)}} + \frac{\frac{1}{2}e^{-j[(\pi/8)+(\pi n/4)]}}{1 - \frac{1}{2}e^{j(\pi/4)}}$$
$$= \frac{\cos\left(\frac{\pi}{4}n + \frac{\pi}{8}\right) - \frac{1}{2}\cos\left(\frac{\pi}{4}n + \frac{3\pi}{8}\right)}{\frac{5}{4} - \cos\left(\frac{\pi}{4}\right)}$$

S10.2

(a) $\tilde{x}_1[n] = 1 + \sin\left(\frac{2\pi n}{10}\right)$

To find the period of $\tilde{x}_1[n]$, we set $\tilde{x}_1[n] = \tilde{x}_1[n+N]$ and determine N. Thus

$$1 + \sin\left(\frac{2\pi n}{10}\right) = 1 + \sin\left[\frac{2\pi}{10}(n+N)\right]$$
$$= 1 + \sin\left(\frac{2\pi}{10}n + \frac{2\pi}{10}N\right)$$

Since

$$\sin\left(\frac{2\pi}{10}n + 2\pi\right) = \sin\left(\frac{2\pi}{10}n\right),$$

the period of $\tilde{x}_1[n]$ is 10. Similarly, setting $\tilde{x}_2[n] = \tilde{x}_2[n+N]$, we have

$$1 + \sin\left(\frac{20\pi}{12}n + \frac{\pi}{2}\right) = 1 + \sin\left[\frac{20\pi}{12}\left(n + N\right) + \frac{\pi}{2}\right]$$
$$= 1 + \sin\left(\frac{20\pi}{12}n + \frac{\pi}{2} + \frac{20\pi}{12}N\right)$$

Hence, for $\frac{20}{12}\pi N$ to be an integer multiple of 2π, N must be 6.

(b) $\tilde{x}_1[n] = 1 + \sin\left(\frac{2\pi n}{10}\right)$

Using Euler's relation, we have

$$x_1[n] = 1 + \frac{1}{2j}e^{j(2\pi/10)n} - \frac{1}{2j}e^{-j(2\pi/10)n} \qquad \text{(S10.2-1)}$$

Note that the Fourier synthesis equation is given by

$$\tilde{x}_1[n] = \sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n},$$

where $N = 10$. Hence, by inspection of eq. (S10.2-1), we see that

$$a_0 = 1, \qquad a_{1-1} = \frac{-1}{2j},$$
$$a_{11} = \frac{1}{2j}, \qquad \text{and}$$
$$a_{1k} = 0, \qquad 2 \le k \le 8,$$
$$-8 \le k \le -2$$

Similarly,

$$\tilde{x}_2[n] = 1 + \frac{1}{2j}e^{j(\pi/2)}e^{j(20\pi/12)n} - \frac{1}{2j}e^{-j(\pi/2)}e^{-j(20\pi/12)n}$$

Therefore, $N = 12$.

$$a_{20} = 1, \quad a_{2-1} = -\frac{e^{-j(\pi/2)}}{2j} = \frac{1}{2}, \quad a_{21} = \frac{1}{2j}e^{j(\pi/2)} = \frac{1}{2}, \quad \text{and}$$
$$a_{2\pm 2}, \ldots, a_{2\pm 11} = 0$$

(c) The sequence a_{1k} is periodic with period 10 and a_{2k} is periodic with period 12.

S10.3

The Fourier series coefficients can be expressed as the samples of the envelope

$$a_k = \frac{1}{N}\cdot\frac{\sin[(2N_1+1)\Omega/2]}{\sin(\Omega/2)}\bigg|_{\Omega=2\pi k/N} \qquad \text{where } N_1 = 1 \text{ (see Example 5.3 on page 302 of the text)}$$
$$= \frac{1}{N}\cdot\frac{\sin(3\Omega/2)}{\sin(\Omega/2)}\bigg|_{\Omega=2\pi k/N}$$

(a) For $N = 6$,

$$a_k = \frac{1}{6}\,\frac{\sin\left[\frac{3}{2}\left(\frac{2\pi k}{6}\right)\right]}{\sin\left[\frac{1}{2}\left(\frac{2\pi k}{6}\right)\right]} = \frac{1}{6}\,\frac{\sin\left(\frac{\pi k}{2}\right)}{\sin\left(\frac{\pi k}{6}\right)}$$

(b) For $N = 12$,

$$a_k = \frac{1}{12}\frac{\sin\left[\frac{3}{2}\left(\frac{2\pi k}{12}\right)\right]}{\sin\left[\frac{1}{2}\left(\frac{2\pi k}{12}\right)\right]} = \frac{1}{12}\frac{\sin\left(\frac{\pi k}{4}\right)}{\sin\left(\frac{\pi k}{12}\right)}$$

(c) For $N = 60$,

$$a_k = \frac{1}{60}\frac{\sin\left[\frac{3}{2}\left(\frac{2\pi k}{60}\right)\right]}{\sin\left[\frac{1}{2}\left(\frac{2\pi k}{60}\right)\right]} = \frac{1}{60}\frac{\sin\left(\frac{\pi k}{20}\right)}{\sin\left(\frac{\pi k}{60}\right)}$$

S10.4

(a) The discrete-time Fourier transform of the given sequence is

$$\begin{aligned} X(\Omega) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} \\ &= \tfrac{1}{2}e^{j\Omega} + 1 + \tfrac{1}{2}e^{-j\Omega} \\ &= 1 + \cos\Omega \end{aligned}$$

$X(\Omega)$ is sketched in Figure S10.4.

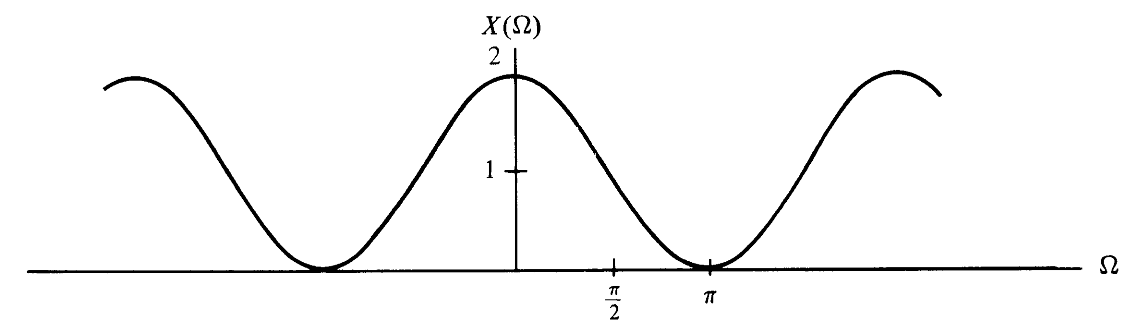

Figure S10.4

(b) The first sequence can be thought of as

$$\tilde{y}_1[n] = x[n] * \left[\sum_{k=-\infty}^{\infty} \delta[n - 3k]\right]$$

Hence

$$Y_1(\Omega) = X(\Omega)\frac{2\pi}{3}\sum_{k=-\infty}^{\infty} \delta\left(\Omega - \frac{2\pi k}{3}\right)$$

Therefore, the Fourier series of $y_1[n]$ is given by

$$a_k = \frac{1}{2\pi}Y_1\left(\frac{2\pi}{3}k\right) = \frac{1}{3}\left(1 + \cos\frac{2\pi k}{3}\right), \qquad \text{for all } k$$

The second sequence is given by

$$y_2[n] = x[n] * \left[\sum_{k=-\infty}^{\infty} \delta[n - 5k] \right]$$

Similarly, the Fourier series of this sequence is given by

$$a_k = \frac{1}{5}\left[1 + \cos\left(\frac{2\pi k}{5}\right)\right], \qquad \text{for all } k$$

This result can also be obtained by using the fact that the Fourier series coefficients are proportional to equally spaced samples of the discrete-time Fourier transform of one period (see Section 5.4.1 of the text, page 314).

S10.5

(a) The given relation

$$x[n] = \sum_{k=0}^{3} a_k e^{jk(2\pi/4)n}$$

results in the following set of equations

$$\begin{aligned} a_0 + a_1 + a_2 + a_3 &= x[0] = 1, \\ a_0 + a_1 e^{j(\pi/2)} + a_2 e^{j\pi} + a_3 e^{j(3/2)\pi} &= x[1] = 0, \\ a_0 + a_1 e^{j\pi} + a_2 e^{j2\pi} + a_3 e^{j3\pi} &= x[2] = 2, \\ a_0 + a_1 e^{j(3/2)\pi} + a_2 e^{j3\pi} + a_3 e^{j(9/2)\pi} &= x[3] = -1 \end{aligned}$$

The preceding set of linear equations can be reduced to the form

$$\begin{aligned} a_0 + a_1 + a_2 + a_3 &= 1, \\ a_0 + ja_1 - a_2 - ja_3 &= 0, \\ a_0 - a_1 + a_2 - a_3 &= 2, \\ a_0 - ja_1 - a_2 + ja_3 &= -1 \end{aligned}$$

Solving the resulting equations, we get

$$a_0 = \frac{1}{2}, \qquad a_1 = -\frac{1+j}{4}, \qquad a_2 = +1, \qquad a_3 = -\frac{1-j}{4} \qquad \text{(S10.5-1)}$$

By the discrete-time Fourier series analysis equation, we obtain

$$a_k = \tfrac{1}{4}[1 + 2e^{-j\pi k} - e^{-j(3\pi k/2)}],$$

which is the same as eq. (S10.5-1) for $0 \le k \le 3$.

S10.6

(a) $a_k = a_{k+10}$ for all k is true since $\tilde{x}[n]$ is periodic with period 10.

(b) $a_k = a_{-k}$ for all k is false since $\tilde{x}[n]$ is not even.

(c) $a_k e^{jk(2\pi/5)}$ is real. This statement is true because it would correspond to the Fourier series of $\tilde{x}[n + 2]$, which is a purely real and even sequence.

(d) $a_0 = 0$ is true since the sum of the values of $\tilde{x}[n]$ over one period is zero.

Solutions to Optional Problems

S10.7

The Fourier series coefficients of $x[n]$, which is periodic with period N, are given by

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk(2\pi/N)n}$$

For $N = 8$,

$$a_k = \frac{1}{8}\sum_{n=0}^{7} x[n]e^{-jk(\pi/4)n} \tag{S10.7-1}$$

(a) We are given that

$$a_k = \cos\left(\frac{\pi k}{4}\right) + \sin\left(\frac{3\pi k}{4}\right), \tag{S10.7-2}$$

$$a_k = \frac{1}{2}e^{j(\pi k/4)} + \frac{1}{2}e^{-j(\pi k/4)} + \frac{1}{2j}e^{j(3\pi k/4)} - \frac{1}{2j}e^{-j(3\pi k/4)}$$

Hence, by comparing eqs. (S10.7-1) and (S10.7-2) we can immediately write

$$x[n] = 4\delta[n-1] + 4\delta[n-7] - 4j\delta[n-3] + 4j\delta[n-5], \quad 0 \le n \le 7$$

(b)
$$\begin{aligned}
x[n] &= \sum_{k=0}^{7} a_k e^{jk(2\pi/8)} = \sum_{k=0}^{7} a_k e^{jk(\pi/4)n} \\
&= \sum_{k=0}^{6}\left[\frac{1}{2j}e^{j(k\pi/3)} - \frac{1}{2j}e^{-j(k\pi/3)}\right]e^{jk(\pi/4)n} \\
&= \frac{1}{2j}\sum_{k=0}^{6} e^{jk\pi[(1/3)+(n/4)]} - \frac{1}{2j}\sum_{k=0}^{6} e^{-jk\pi[(1/3)-(n/4)]} \\
&= \frac{1}{2j}\,\frac{1-e^{j[(7\pi n/4)+(7\pi/3)]}}{1-e^{j[(\pi n/4)+(\pi/3)]}} - \frac{1}{2j}\,\frac{1-e^{j[(7\pi n/4)-(7\pi/3)]}}{1-e^{j[(\pi n/4)-(\pi/3)]}} \\
&= \frac{1}{2j}\left[\frac{1-e^{j[(7\pi n/4)+(7\pi/3)]}}{1-e^{j[(\pi n/4)+(\pi/3)]}} - \frac{1-e^{j[(7\pi n/4)-(7\pi/3)]}}{1-e^{j[(\pi n/4)-(\pi/3)]}}\right]
\end{aligned}$$

(c)
$$\begin{aligned}
x[n] &= \sum_{k=0}^{7} a_k e^{jk(2\pi/8)n} = \sum_{k=0}^{7} a_k e^{jk(\pi/4)n} \\
&= 1 + e^{j(\pi/4)n} + e^{j(3\pi/4)n} + e^{j\pi n} + e^{j(5\pi/4)n} + e^{j(7\pi/4)n} \\
&= 1 + (-1)^n + 2\cos\left(\frac{\pi}{4}n\right) + 2\cos\left(\frac{3\pi}{4}n\right), \qquad 0 \le n \le 7
\end{aligned}$$

(d) Using an analysis similar to that in part (c), we find

$$x[n] = 2 + 2\cos\left(\frac{\pi}{4}n\right) + \cancelto{0}{\cos\left(\frac{\pi}{2}n\right)} + \frac{1}{2}\cos\left(\frac{3\pi}{4}n\right), \qquad 0 \le n \le 7$$

S10.8

The impulse response of the LTI system is

$$h[n] = \left(\tfrac{1}{2}\right)^{|n|}$$

The discrete-time Fourier transform of $h[n]$ is

$$H(\Omega) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\Omega n} + \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-n} e^{-j\Omega n} - 1$$
$$= \frac{1}{1 - \frac{1}{2}e^{-j\Omega}} + \frac{1}{1 - \frac{1}{2}e^{j\Omega}} - 1$$
$$= \frac{3}{5 - 4\cos\Omega}$$

(a) (i) $x[n] = \sin\left(\frac{3\pi}{4}n\right) = \frac{1}{2j}e^{j(3\pi/4)n} - \frac{1}{2j}e^{-j(3\pi/4)n}$

The period of $x[n]$ is

$$\sin\left(\frac{3\pi}{4}n\right) = \sin\left[\frac{3\pi}{4}(n + N)\right]$$

Thus

$$\sin\left(\frac{3\pi}{4}n\right) = \sin\left(\frac{3\pi}{4}n + \frac{3\pi}{4}N\right)$$

We set $3\pi N/4 = 2\pi m$ to get $N = 8$ ($m = 3$). Hence, the period is 8.

$$x[n] = \sum_{k=0}^{7} a_k e^{jk(2\pi/8)n}$$

Therefore,

$$a_3 = \frac{1}{2j} = a_5^*$$

All other coefficients a_k are zero. By the convolution property, the Fourier series representation of $y[n]$ is given by b_k, where

$$b_k = a_k H(\Omega)\Big|_{\Omega=(2\pi k)/8}$$

Thus

$$b_3 = \frac{1}{2j}\frac{3}{5 - 4\cos(3\pi/4)}$$
$$= b_5^*$$

All other b_k are zero in the range $0 \le k \le 7$.

(ii) $\tilde{x}[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k]$

The Fourier series of $\tilde{x}[n]$ is

$$a_k = \tfrac{1}{4}\sum_{n=0}^{3} \tilde{x}[n]e^{-jk(2\pi/4)n} = \tfrac{1}{4}, \qquad \text{for all } k$$

And the Fourier series of $\tilde{y}[n]$ is

$$b_k = a_k H(\Omega)\Big|_{\Omega=\pi k/2}$$
$$= \frac{1}{4}\frac{3}{5 - 4\cos[(\pi/2)k]} = \frac{3}{20} \qquad \text{for all } k$$

(iii) The Fourier series of $\tilde{x}[n]$ is

$$a_k = \frac{1}{6}\left[1 + 2\cos\left(\frac{\pi}{3}k\right)\right], \qquad 0 \le k \le 5$$

and the Fourier series of $\tilde{y}[n]$ is

$$b_k = a_k H(\Omega)\Big|_{\Omega=(\pi/3)k}$$
$$= \frac{1}{6}\left[1 + 2\cos\left(\frac{\pi}{3}k\right)\right]\frac{3}{5 - 4\cos[(\pi/3)k]}$$

(iv) $x[n] = j^n + (-1)^n$
The period of $\tilde{x}[n]$ is 4. $x[n]$ can be rewritten as

$$x[n] = [e^{j(\pi/2)}]^n + (e^{j\pi})^n$$
$$= \sum_{k=0}^{3} a_k e^{jk(2\pi/4)n}$$

Hence,

$$a_0 = 0, \qquad a_1 = 1,$$
$$a_2 = 1, \qquad a_3 = 0$$

Therefore, $b_0 = b_3 = 0$ and

$$b_1 = \frac{3}{5 - 4\cos(\pi/2)} = \frac{3}{5},$$
$$b_2 = \frac{3}{5 - 4\cos\pi} = \frac{3}{9}$$

(b) $h[n]$ is sketched in Figure S10.8.

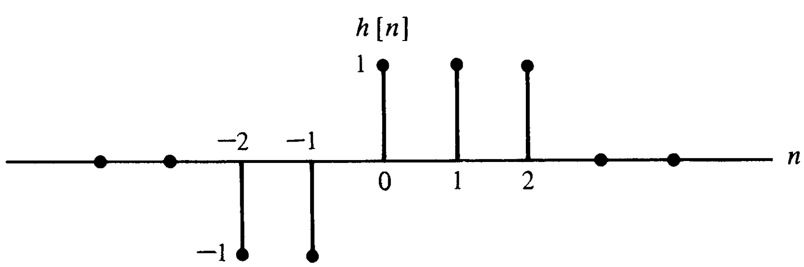

Figure S10.8

$$H(\Omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\Omega n} = -e^{j2\Omega} - e^{j\Omega} + 1 + e^{-j\Omega} + e^{-j2\Omega},$$
$$H(\Omega) = 1 - 2j\sin\Omega - 2j\sin 2\Omega$$

It follows from part (a):

(i) $$b_3 = \frac{1}{2j}H(\Omega)\Big|_{\Omega=3\pi/4} = \frac{1}{2j} - \sin\frac{3\pi}{4} - \sin\frac{3\pi}{2} = b_5^*$$

All other coefficients b_k are zero, in the range $0 \le k \le 7$.

(ii) $b_k = \frac{1}{4}H(\Omega)\Big|_{\Omega=\pi k/2}$

$= \frac{1}{4} - \frac{j}{2}\sin\frac{\pi k}{2} - \frac{j}{2}\sin \pi k = \frac{1}{4} - \frac{j}{2}\sin\frac{\pi k}{2}$

(iii) $b_k = \frac{1}{6}\left[1 + 2\cos\left(\frac{\pi}{3}k\right)\right] H(\Omega)\Big|_{\Omega=\pi k/3}$

(iv) $b_0 = 0,$

$b_1 = H(\Omega)\Big|_{\Omega=\pi/2} = 1 - 2j,$

$b_2 = H(\Omega)\Big|_{\Omega=\pi} = 1,$

$b_3 = 0$

S10.9

$x[n] \overset{\mathcal{F}}{\longleftrightarrow} a_k$

(a) $x[n - n_0] \overset{\mathcal{F}}{\longleftrightarrow} a_k e^{-jk(2\pi/N)n_0}$

(b) $x[n] - x[n-1] \overset{\mathcal{F}}{\longleftrightarrow} a_k[1 - e^{-j(2\pi k/N)}]$

(c) $x[n] - x\left[n - \frac{N}{2}\right] \overset{\mathcal{F}}{\longleftrightarrow} a_k(1 - e^{-jk\pi}), \qquad N \text{ even}$

$$= \begin{cases} 0, & k \text{ even}, \\ 2a_k, & k \text{ odd} \end{cases}$$

(d) $x[n] + x\left[n + \frac{N}{2}\right], \qquad \text{period } \frac{N}{2}$

$$\hat{a}_k = \frac{2}{N}\sum_{n=0}^{(N/2)-1}\left[x[n] + x\left[n + \frac{N}{2}\right]\right] e^{-jk(4\pi/N)n}$$
$$= 2a_{2k}$$

(e) $\hat{a}_k = \frac{1}{N}\sum_{n=0}^{N-1} x^*[-n]e^{-jk(2\pi/N)n},$

$$\hat{a}_k^* = \frac{1}{N}\sum_{n=0}^{N-1} x[-n]e^{jk(2\pi/N)n}$$
$$= \frac{1}{N}\sum_{n=0}^{-N+1} x[n]e^{-jk(2\pi/N)n} = a_k$$

Therefore, $\hat{a}_k = a_k^*$.

S10.10

(a)
$$\tilde{w}[n] = \tilde{x}[n] + \tilde{y}[n],$$
$$\tilde{w}[n + NM] = \tilde{x}[n + NM] + \tilde{y}[n + NM]$$
$$= \tilde{x}[n] + \tilde{y}[n]$$
$$= \tilde{w}[n]$$

Hence, $\tilde{w}[n]$ is periodic with period NM.

(b)
$$c_k = \frac{1}{NM}\sum_{n=0}^{NM-1} \tilde{w}[n]e^{-jk(2\pi/NM)n} = \frac{1}{NM}\sum_{n=0}^{NM-1}[\tilde{x}[n] + \tilde{y}[n]]e^{-jk(2\pi/NM)n}$$
$$= \frac{1}{NM}\sum_{n=0}^{NM-1}\tilde{x}[n]e^{-jk(2\pi/NM)n} + \frac{1}{NM}\sum_{n=0}^{NM=1}\tilde{y}[n]e^{-jk(2\pi/NM)n}$$
$$= \frac{1}{NM}\sum_{n=0}^{N-1}\tilde{x}[n]\sum_{l=0}^{M-1} e^{-jk(2\pi/NM)(n+lN)} + \frac{1}{NM}\sum_{n=0}^{M-1}\tilde{y}[n]\sum_{l=0}^{N-1} e^{jk(2\pi/NM)(n+lM)}$$
$$= \begin{cases} \frac{1}{N}a_{k/M} + \frac{1}{M}b_{k/N}, & \text{for } k \text{ a multiple of } M \text{ and } N, \\ \frac{1}{N}a_{k/M}, & \text{for } k \text{ a multiple of } M, \\ \frac{1}{M}b_{k/N}, & \text{for } k \text{ a multiple of } N, \\ 0, & \text{otherwise} \end{cases}$$

S10.11

(a) $\tilde{x}[n] = \sin\left[\frac{\pi(n-1)}{4}\right]$

To find the period, we set $\tilde{x}[n] = \tilde{x}[n+N]$. Thus,
$$\sin\left[\frac{\pi(n-1)}{4}\right] = \sin\left[\frac{\pi(n+N-1)}{4}\right] = \sin\left[\frac{\pi(n-1)}{4} + \frac{\pi N}{4}\right]$$
Let $(\pi N)/4 = 2\pi i$, when i is an integer. Then $N = 8$ and
$$\tilde{x}[n] = \frac{1}{2j}e^{j[\pi(n-1)/4]} - \frac{1}{2j}e^{-j[\pi(n-1)/4]}$$
$$= \frac{1}{2j}e^{-j(\pi/4)}e^{j(\pi n/4)} - \frac{1}{2j}e^{j(\pi/4)}e^{-j(\pi n/4)}$$
Therefore,
$$a_1 = \frac{e^{-j(\pi/4)}}{2j}, \qquad a_7 = -\frac{e^{j(\pi/4)}}{2j}$$
All other coefficients a_k are zero, in the range $0 \le k \le 7$. The magnitude and phase of a_k are plotted in Figure S10.11-1.

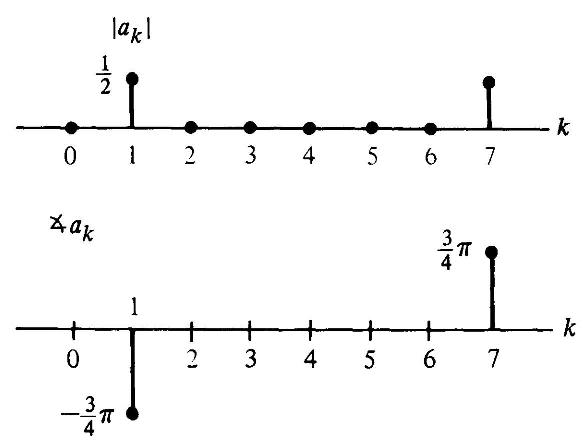

Figure S10.11-1

(b) The period $N = 21$ and the Fourier series coefficients are

$$a_7 = a_{14} = \frac{1}{2}, \qquad a_3 = a_{18}^* = \frac{1}{2j}$$

The rest of the coefficients a_k are zero. The magnitude and phase of a_k are given in Figure S10.11-2.

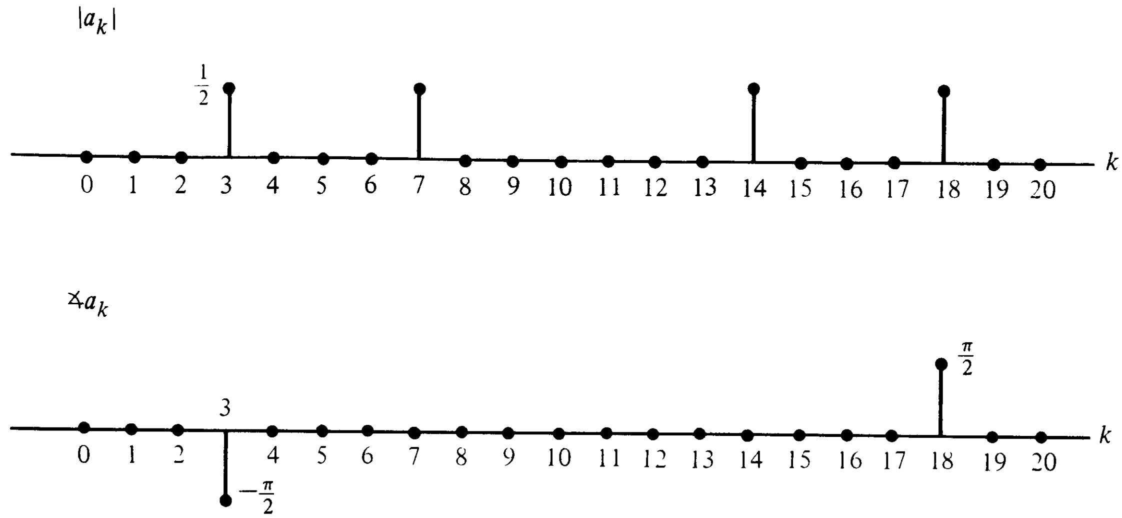

Figure S10.11-2

(c) The period $N = 8$.

$$a_3 = a_5^* = \tfrac{1}{2}e^{-j(\pi/3)}$$

The rest of the coefficients a_k are zero. The magnitude and phase of a_k are given in Figure S10.11-3.

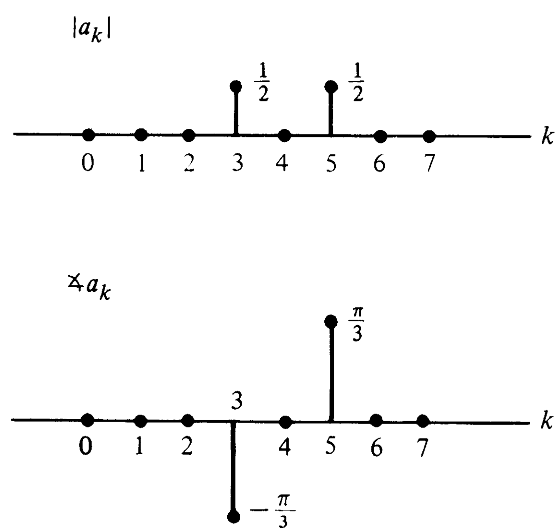

Figure S10.11-3