Starting from equation (13) in reference (18), *Size effects in thermal conduction by phonons*:(p=2)

$$
\begin{aligned}
\kappa(q) &= \frac{1}{\Omega}\sum_{Q=(\vec{Q},s)} \frac{\hbar\omega_Q v_{Q_x}^2(\partial n_Q/\partial T)}{1/\tau_Q + iqv_{Q_x}} \\
&= \frac{1}{\Omega}\sum_{Q=(\vec{Q},s)} \frac{k_B v_{Q_x}^2}{1/\tau_Q + iqv_{Q_x}} \\
&= \frac{3}{\Omega}\cdot k_B \sum_{\vec{Q}} \frac{v_{Q_x}^2}{1/\tau_Q + iqv_{Q_x}} \\
&= \frac{3k_B}{\Omega}\sum_{\vec{Q}} \frac{v_{Q_x}^2 \tau_Q}{1 + iqv_{Q_x}\tau_Q} \\
&= \frac{3k_B v^2 \tau_D}{\Omega}\sum_{\vec{Q}} \frac{(\frac{Q_x}{Q})^2(\frac{Q_D}{Q})^p}{1 + iqv(\frac{Q_x}{Q})\tau_D(\frac{Q_D}{Q})^p} \\
&= \frac{3k_B v^2 \tau_D}{\Omega}\int_0^{\omega_D} \frac{(\frac{\omega_D}{\omega})^p \cos^2\theta}{1 + iqv\tau_D(\frac{\omega_D}{\omega})^p\cos\theta}\cdot c(\omega)d\omega\cdot[\frac{1}{2}\int_{-1}^{1} d(\cos\theta)] \\
&= \frac{3k_B v^2 \tau_D}{\Omega}\int_0^{\omega_D} (\frac{\omega_D}{\omega})^p\cdot c(\omega)d\omega\cdot[\frac{1}{2}\int_{-1}^{1} \frac{\cos^2\theta}{1 + iqv\tau_D(\frac{\omega_D}{\omega})^p\cos\theta} d(\cos\theta)]
\end{aligned}
$$

Let q=0:

$$
\begin{aligned}
\kappa_0 = \kappa(0) &= \frac{3k_B v^2 \tau_D}{\Omega}\int_0^{\omega_D} (\frac{\omega_D}{\omega})^p\cdot c(\omega)d\omega\cdot[\frac{1}{2}\int_{-1}^{1}\cos^2\theta d(\cos\theta)] \\
&= \frac{k_B v^2 \tau_D}{\Omega}\int_0^{\omega_D} (\frac{\omega_D}{\omega})^p\cdot c(\omega)d\omega \\
&= \frac{k_B v^2 \tau_D}{\cancel{\Omega}}\int_0^{\omega_D} (\frac{\omega_D}{\omega})^p\cdot \frac{\cancel{\Omega}\omega^2}{2\pi^2 v^3} d\omega \\
&\overset{p=2}{=} \frac{k_B v^2 \tau_D}{2\pi^2 v^3}\cdot\omega_D^2\int_0^{\omega_D} d\omega \\
&= \frac{k_B v^2 \tau_D}{2\pi^2 v^3}\cdot\omega_D^3 \\
&= \frac{k_B v^2 \tau_D}{2\cancel{\pi^2}\cancel{v^3}}\cdot\frac{6\cancel{\pi^2}N\cancel{v^3}}{\Omega} = \frac{3k_B v^2 \tau_D}{\Omega}
\end{aligned}
$$

Back to equation (9) in *2NL.pdf*:

$$\begin{aligned}
\kappa(q) &= \frac{1}{\Omega} \sum_{Q=(\vec{Q},s)} k_B v_{Q_x}^2 \tau_Q \cos^2(qd/2) F(q, \Lambda_{Q_x}) \\
&= \frac{1}{\Omega} k_B v^2 \tau_D \sum_{Q=(\vec{Q},s)} (\frac{Q_x}{Q})^2 (\frac{Q_D}{Q})^p \cos^2(qd/2) F(q, \Lambda_{Q_x}) \\
&= \frac{N}{\Omega} k_B v^2 \tau_D \cdot \frac{1}{N} \sum_{Q=(\vec{Q},s)} (\frac{Q_x}{Q})^2 (\frac{Q_D}{Q})^p \cos^2(qd/2) F(q, \Lambda_{Q_x}) \\
&= \underline{\frac{3N}{\Omega} k_B v^2 \tau_D} \times \frac{1}{N} \sum_{\vec{Q}} (\frac{Q_x}{Q})^2 (\frac{Q_D}{Q})^p \cos^2(qd/2) F(q, \Lambda_{Q_x}) \\
&= \underline{\kappa_0} \times \frac{1}{N} \sum_{\vec{Q}} (\frac{Q_x}{Q})^2 (\frac{Q_D}{Q})^p \cos^2(qd/2) F(q, \Lambda_{Q_x})
\end{aligned}$$

$$\begin{aligned}
\kappa(q) &= \kappa_0 \times \frac{1}{N} \sum_{\vec{Q}} (\frac{Q_x}{Q})^2 (\frac{Q_D}{Q})^p \cos^2(qd/2) F(q, \Lambda_{Q_x}) \\
\kappa_0 &= \frac{3N}{\Omega} k_B v^2 \tau_D
\end{aligned}$$

$Q_D = (6\pi^2(N/V))^{1/3}$ in the case of fcc $Q_D = (3/\pi)^{1/3}\frac{2\pi}{a} = 0.985\frac{2\pi}{a}$
Q_x is the projection of $\vec{Q}$ in (1,1,$\bar{1}$) direction.