Schedule

1. **What are models?**
   Exercise: Python Skills

2. **Energy budget of the Earth**
   Exercise: Simple Energy Balance Model

3. **Nonlinearity, Feedback, and Predictability**
   Exercise: Nonlinearity and Feedbacks
   Exercise: Revised Energy Balance Model

4. **Parametrization and Sensitivity**

5. **Radiative budget**
   Exercise: 1-layer greenhouse model
   Exercise: 2-layer greenhouse model

6. **Introduction to fluid dynamics**
   Exercise: Analytical katabatic flow model

7. **Finite difference method**
   Exercise: Advection-Diffusion Equation
   Exercise: Boundary layer Evolution
   Exercise: Numerical katabatic flow model
   Exercise: Heat Equation

8. **Implicit finite difference methods**
   Exercise: Boundary layer evolution

9. **Optimization problem**
   Exercise: Surface energy balance
   Exercise: Sublimation

10. **COSIPY snow model**
    Exercise: Simulations with COSIPY

11. **Introduction to PALM**
    Exercise: Simulations with PALM

12. **How to write an article**

Nonlinearity, feedback and predictability

Learning objectives

- A basic understanding of nonlinearity and predictability
- What are feedbacks
- Sensitivity of climate models
- Equilibrium climate sensitivity and feedback factors
- What are parametrizations
- Equilibrium climate states
- What is hidden behind the term ‘butterfly-effect’?

Von-May equation

Problem description:

The starting point for our analysis is the 'Von-May-Equation', which is given by

$$y_{t+1} = r \cdot y_t \cdot (1 - y_t),$$

with r an pre-defined parameter and y the function value at time t and $t+1$.

> **Task 1**: Write a function which solves the Von-May-Equation.

> **Task 2**: Run the code for several initial and parameter combination. What is particularly striking about increasing r-values?
>
> ```
> y(0)=0.5 and r=2.80 (alternatively, use y(0)=0.9)
> y(0)=0.5 and r=3.30 (alternatively, use y(0)=0.9)
> y(0)=0.5 and r=3.95 (alternatively, use y(0)=0.495)
> y(0)=0.8 and r=2.80
> ```

> **Task 3**: Extend this Von-May function by generating 20 random r-values and run simulations with them. Sample the values from a normal distribution with mean 3.95 and standard deviation 0.015 (limit the r-values between 0 and 4). Then average over all time series. Plot both the time series, the averaged time series and the histogram of the averaged time series. What do you observe?

Fluctuations and white noise

Climate system: many non-linear processes simultaneously

White noise is a random signal having equal intensity at different frequencies, giving it a constant power spectral density (temporally uncorrelated) and thus no predictability or reconstruction.

Example: instead of an unknown number of different real mechanisms, only 20 non-linear processes (May) with small differences in r

Damped system with disturbances

Revisit the EBM-Model

So far, the greenhouse effect has been parameterised by τ in the energy balance model. However, the transmissivity (clouds etc.) fluctuates with the weather. At this point, the simple model does not account for this dynamic. In order to include dynamics, we slightly modify the energy balance model and generate a new τ at each time step. To do this, we sample the transmission values from a normal distribution with a standard deviation of 10 percent.

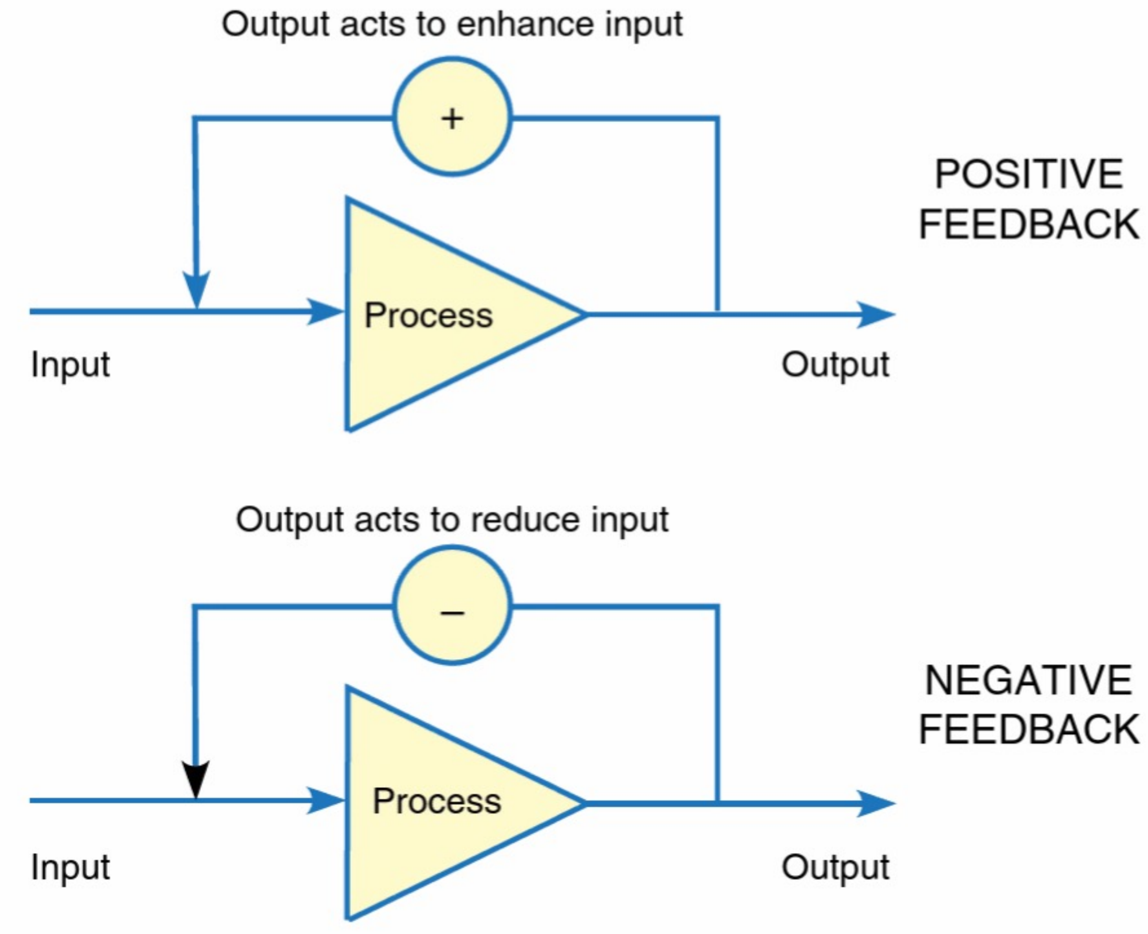
> **Task 4**: Run the energy balance model T(0)=288 K, C_w = 2·10^8 J/(m^2· K), α =0.3, and τ_{mean} =0.608 ($\pm$ 10%). Describe the time series.

Damped system with disturbances

Observations

- although the transmissivity is temporally incoherent, the model shows the long-term "climate oscillation"
- Disturbances produce dynamic effect
- dynamic equilibrium (no trend), as system contains "damping mechanisms"

Feedbacks



Source: McGuffie and Henderson-Sellers, 2014.

Feedback

If the energy absorbed by a planet changes and causes an increase in temperature, there will likely be some processes that respond to the increase in temperature and these changes the amount of energy absorbed by the surface.

System without feedback

$$\Delta E = C_w \Delta \mathrm{T}$$

System with feedback

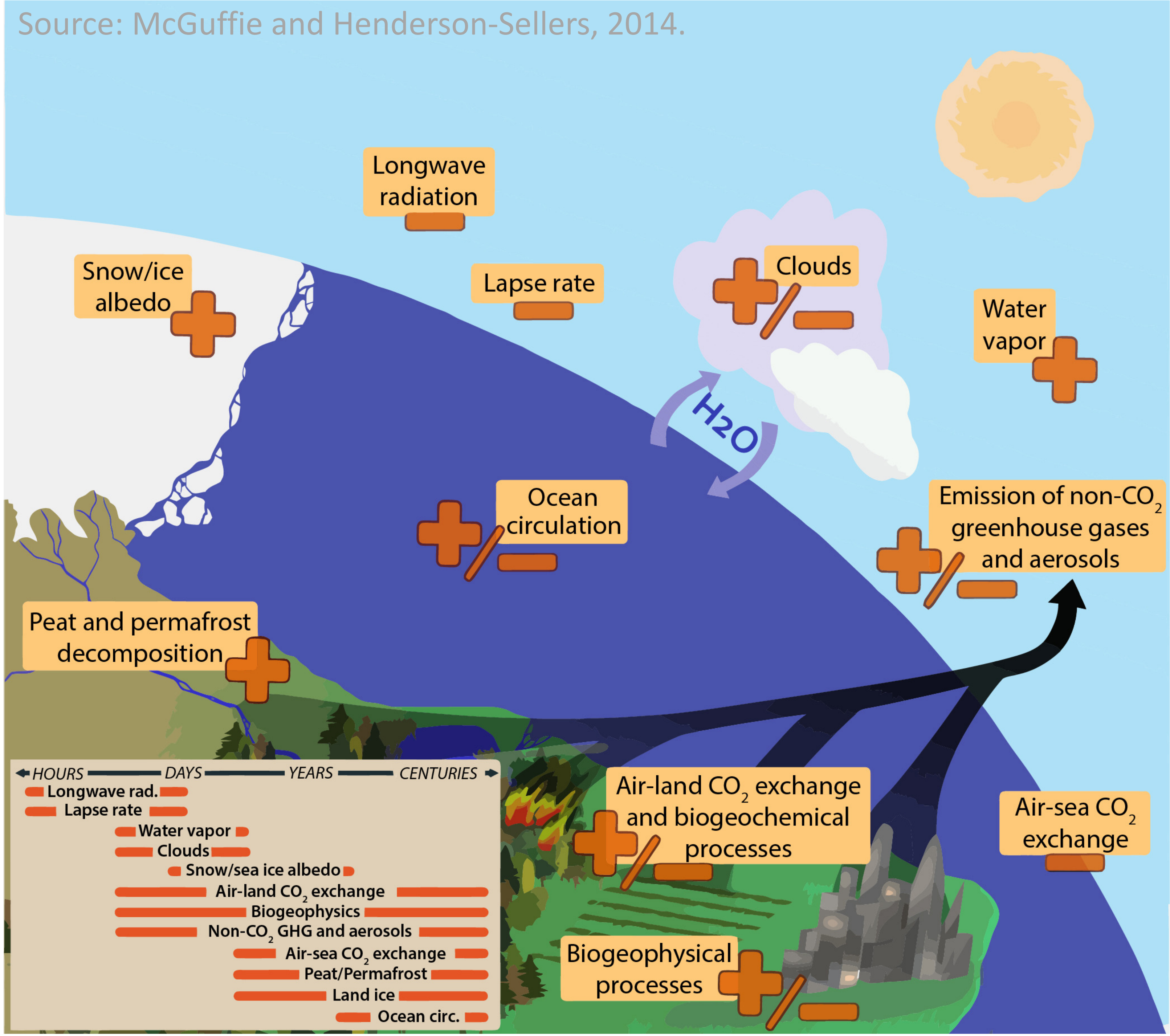
$$\Delta E = (C_w \Delta \mathrm{T} + f C_w \Delta \mathrm{T} + f^2 C_w \Delta \mathrm{T} + \ldots)$$

$$\Delta E = (1 + f + f^2 + \cdots) \cdot C_w \Delta \mathrm{T}$$

$$\Delta E = \frac{1}{1 - f} \cdot C_w \Delta \mathrm{T}$$

Where f is the feedback amount which is between 0 and 1

Feedbacks


Source: McGuffie and Henderson-Sellers, 2014.
Longwave radiation
Snow/ice albedo
Lapse rate
Clouds
Water vapor
H_2O
Ocean circulation
Emission of non-CO_2 greenhouse gases and aerosols
Peat and permafrost decomposition
Air-land CO_2 exchange and biogeochemical processes
Air-sea CO_2 exchange
Biogeophysical processes
HOURS
DAYS
YEARS
CENTURIES
Longwave rad.
Lapse rate
Water vapor
Clouds
Snow/sea ice albedo
Air-land CO_2 exchange
Biogeophysics
Non-CO_2 GHG and aerosols
Air-sea CO_2 exchange
Peat/Permafrost
Land ice
Ocean circ.

Sensitivity of climate models

Assuming a system in which a change of surface temperatur of magnitude ΔT is introduced. If feedback occur, the there will be additional temperature change:

$$\mathbf{\Delta T_{final}} = \boldsymbol{\Delta T} + \boldsymbol{\Delta T_{feedbacks}}\,,$$

where $\Delta T_{feedbacks}$ can be positive or negative. ΔT_{final} is usually related to the perturbation ΔE:

$$\boldsymbol{\Delta T_{final}} = \frac{\mathbf{1}}{\mathbf{1} - \boldsymbol{f}}\boldsymbol{\Delta E}$$

Sensitivity of climate models

A more useful feedback descriptor measures the **perturbation in the global surface temperature** ΔT to the externally pescribed changes in the **net radiative flux crossing the trropospause**, ΔQ:

$$\boldsymbol{C_w \left[\frac{\delta(\Delta T)}{\delta t}\right] + \lambda \Delta T = \Delta Q}$$

where $\lambda\Delta\mathrm{T}$ is the net radiation change at the tropopause. A convenient reference value for λ is the value λ_B which λ would have if the Earth were a simple **black body** with its present-day albedo

$$\boldsymbol{\lambda_B = 4\sigma T_e^3} = 3.75\ \mathrm{Wm^{-2}\,K^{-1}}$$

The overall radiative feedback parameter λ_{total} is composed of **all contributing feedback**:

$$\boldsymbol{\lambda_{total} = \lambda_B + \lambda_{wv} + \lambda_{ice}}$$

Sensitivity of climate models

Thus, for a given system heat capacity:

A **positive** value of **feedback factors** (λ_i) implies a **negative feedback**
A **negative** value of **feedback factors** (λ_i) implies a **positive feedback**

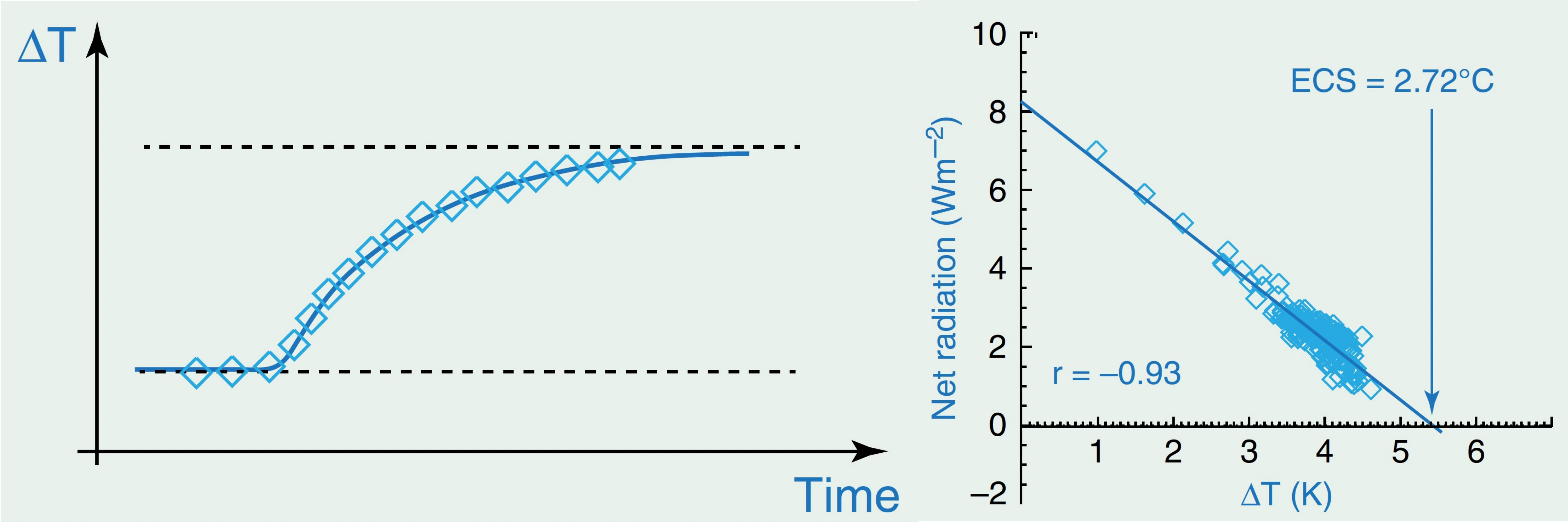
Remember: $C_w \left[\frac{\delta(\Delta T)}{\delta t}\right] + \lambda \Delta T = \Delta Q$

For zero heat capacity it follows $$\Delta T = \frac{\Delta Q}{\lambda}$$

For convenience another definition of a feedback factor has been introduced, the so-called **equlibrium climate sensitivity (ECS)**:

$$\mathbf{ECS} = \frac{\mathbf{1}}{\boldsymbol{\lambda}} = \boldsymbol{\lambda}'$$

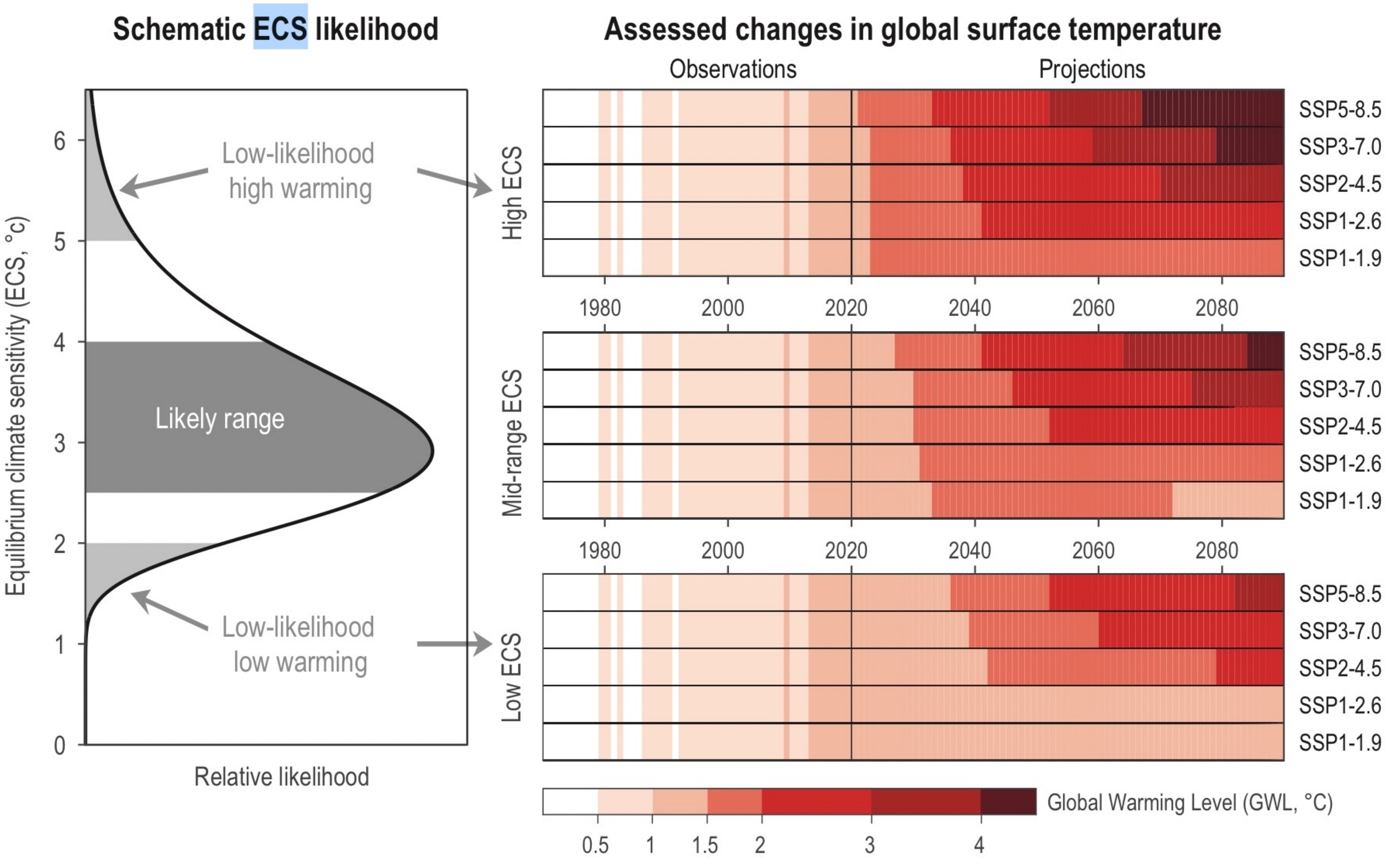
How to estimate the equlibrium climate sensitivity?



Sensitivity of climate models

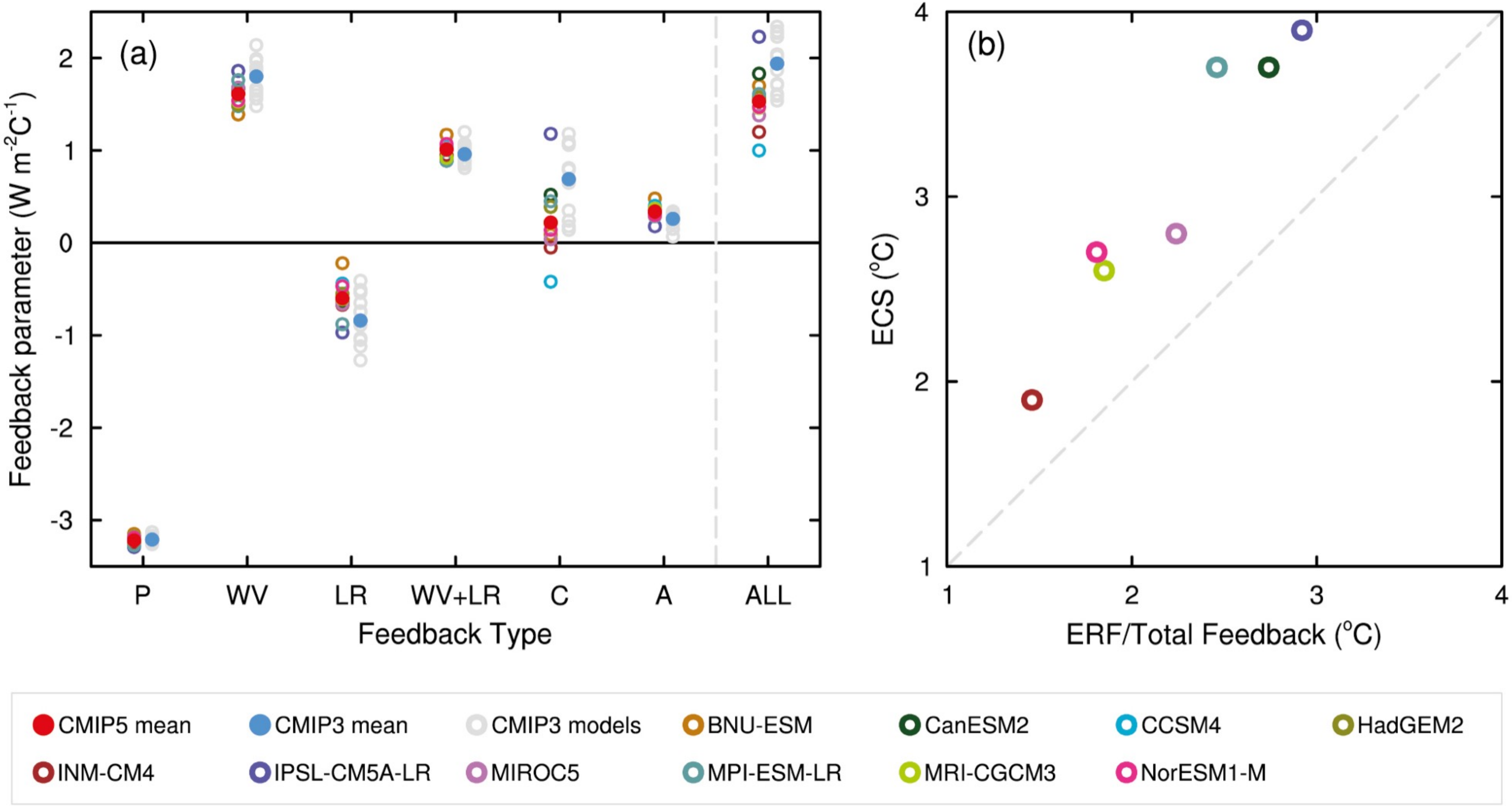
Task: What are the ECS values in the latest IPCC report (AR6)?
What are the feedback factors?

ECS



Spurce: IPCC, AR6, TR, 2021

Feedback parameter and ECS



Source: Rose, 2020.

Exercise: Ice-Albedo Feedback

Task 5: Yet, the model does not take into account changes in albedo that result from changes in glaciation and land use as a consequence of a changing climate. Therefore, we are now extending the model with a simple ice/land use albedo parameterisation. In this parameterisation, the albedo is solely a function of the mean temperature. To add some nonlinearity we assume a sigmoid function with

$$\alpha(T_i) = 0.3 \cdot (1 - 0.2 \cdot \tanh(0.5 \cdot (T_i - 288))).$$

(i) Plot the albedo function.

(ii) Carry out the following simulations:

- Run the energy balance model with four different initial conditions for T(0)=286.0, 287.9, 288.0, and 293.0 K, while fixing the other parameters to C_w = 2·10^8 J/(m^2· K) and τ_{mean} = 0.64%)

What can be said about equilibrium climatic states?

What is meant by parameterisation

Problem: The climate system possesses infinite degrees of freedom → need to represent parts of the system by imprecise or semi-empirical mathematical expressions which are called **parametrizations**.

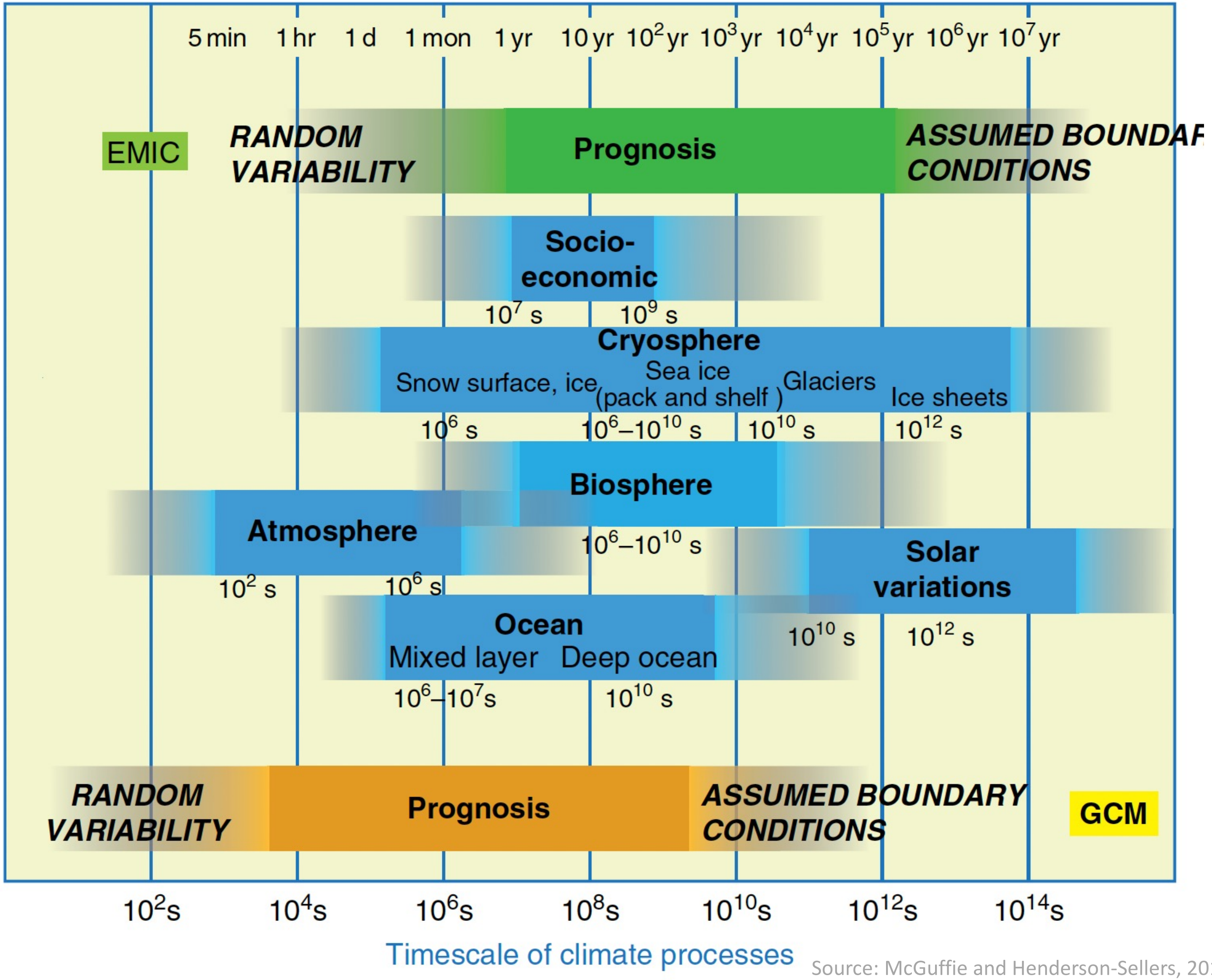
Null parametrization: process or a group of processes are ignored

Climatological specification: Values prescribed by observed climatological averages

Semi-empirical: Processes are related to present-day observations (constants of functions)

→ Parametrizations used for particular processes often depends upon the response time of that feature

Timescales of climate processes



Source: McGuffie and Henderson-Sellers, 2014.

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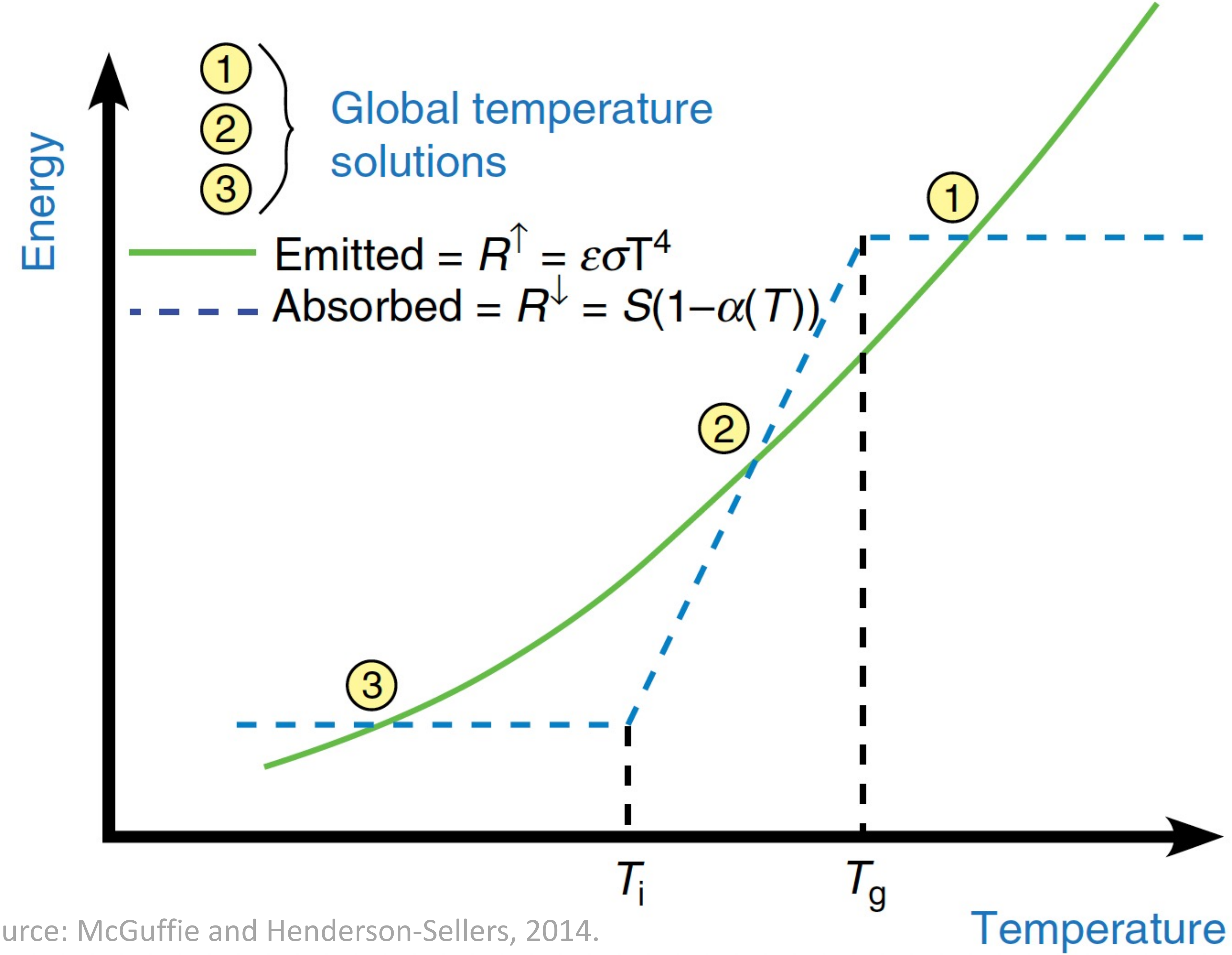
Equilibrium climatic states



Source: McGuffie and Henderson-Sellers, 2014.

Ice-Albedo Feedback

Task 6: Repeat the previous exercise with a linear parameterisation:

$$f(x) = \begin{cases} \alpha_i, & \text{if } T \leq T_i \\ \alpha_g, & \text{if } T \geq T_g \\ \alpha_g + b(T_g - T) & \text{if } T_i < T < T_g \end{cases}$$

ECS

> **Task 7**: Determine the equilibrium climate sensitivity (ECS) and the feedback factor for the simulation from Task 5.

Spurce: IPCC, AR6, TR, 2021

Equilibrium states

Task 8: Repeat the simulation from Task 5 (sigmoid function for albedo) with $T(0)=289$ K, but again sample the transmissivity on a normal distribution with a standard deviation of 10%. What special feature can now be observed? What conclusions can be inferred regarding the prediction of weather and climate?

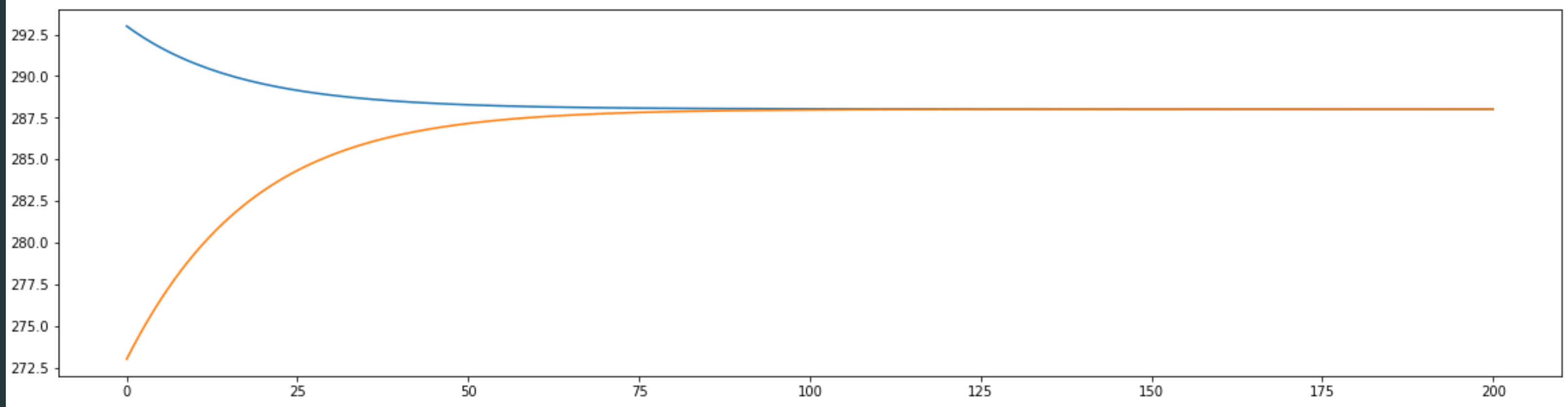
Equilibrium climatic states

Are equilibrium states stable?

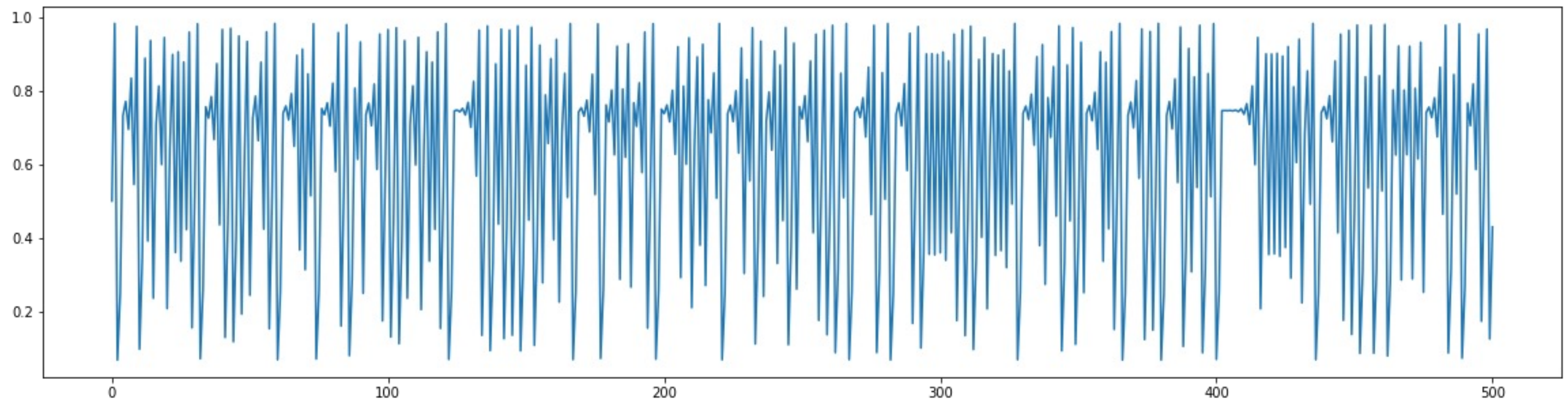
Transivity: if two different initial states of a system evolve to a single resultant state
Intransivity: Has at least two acceptalbe solution states, depending on the initial state

Difficulty: When a system exhibits behaviour that mimics transivity for some time, then flips to the alternative state --> **Chaotic behaviour**

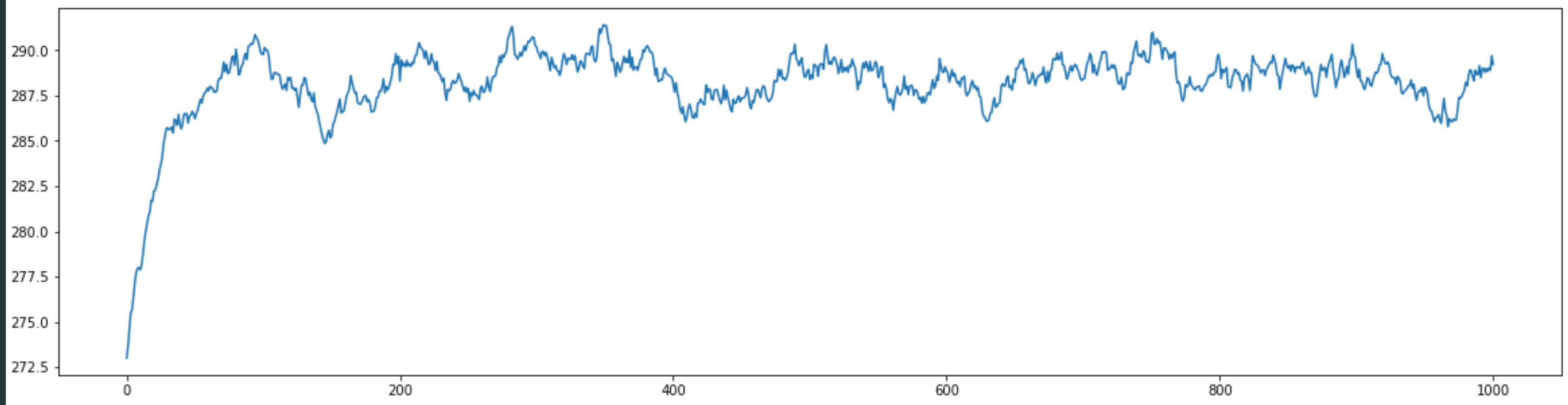
Summary



Damped System

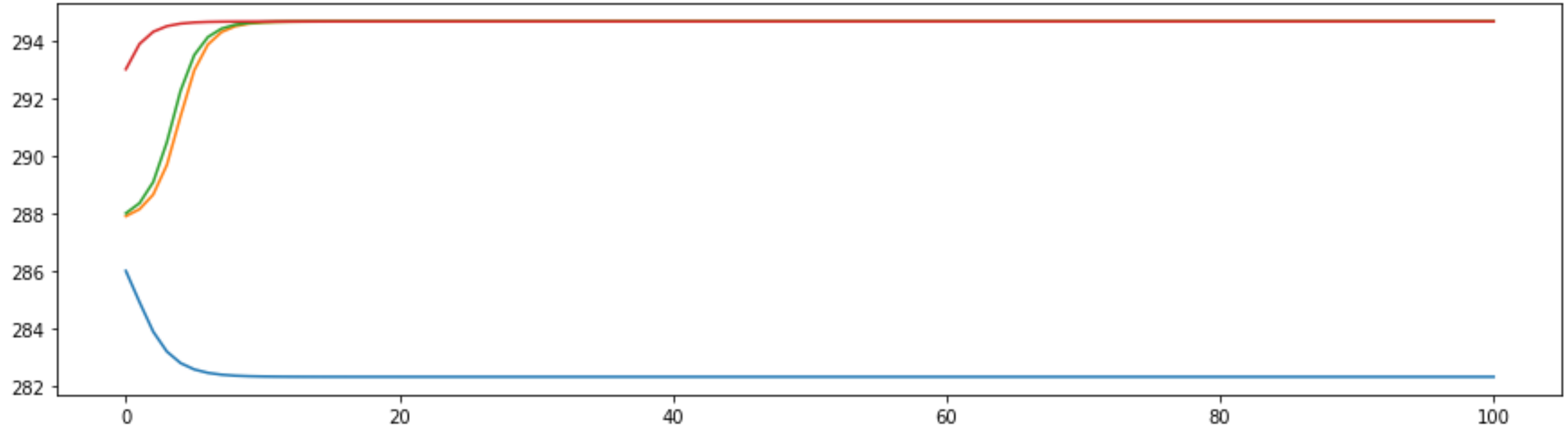


Fluctuation



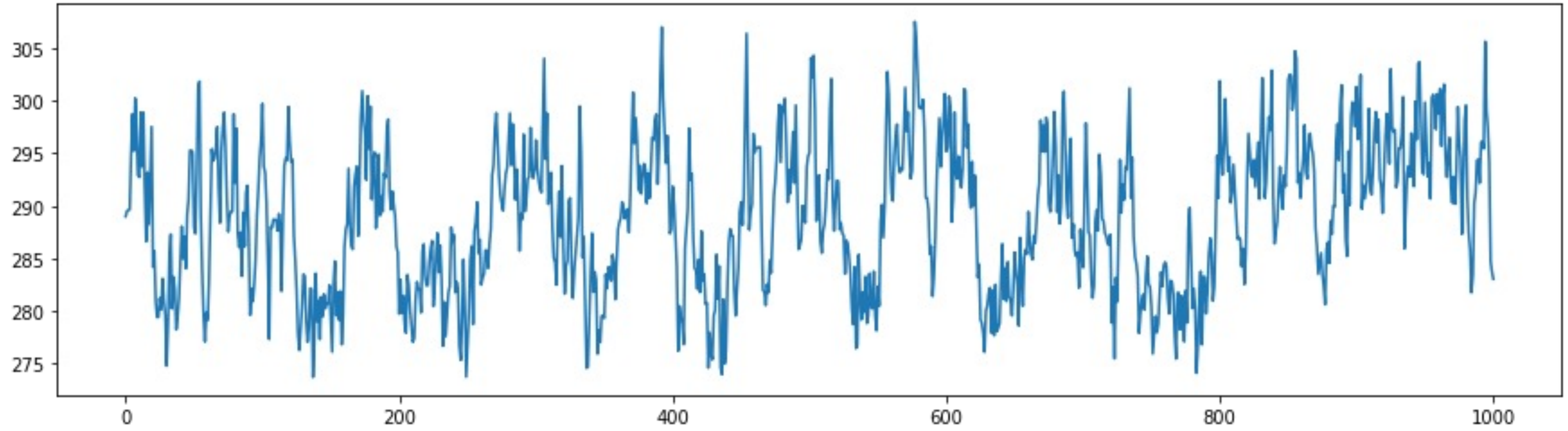
Damped System with fluctuation

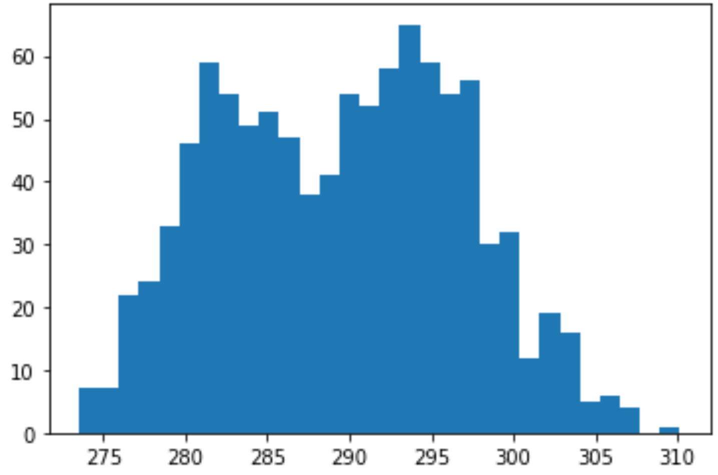
Summary



Feedback

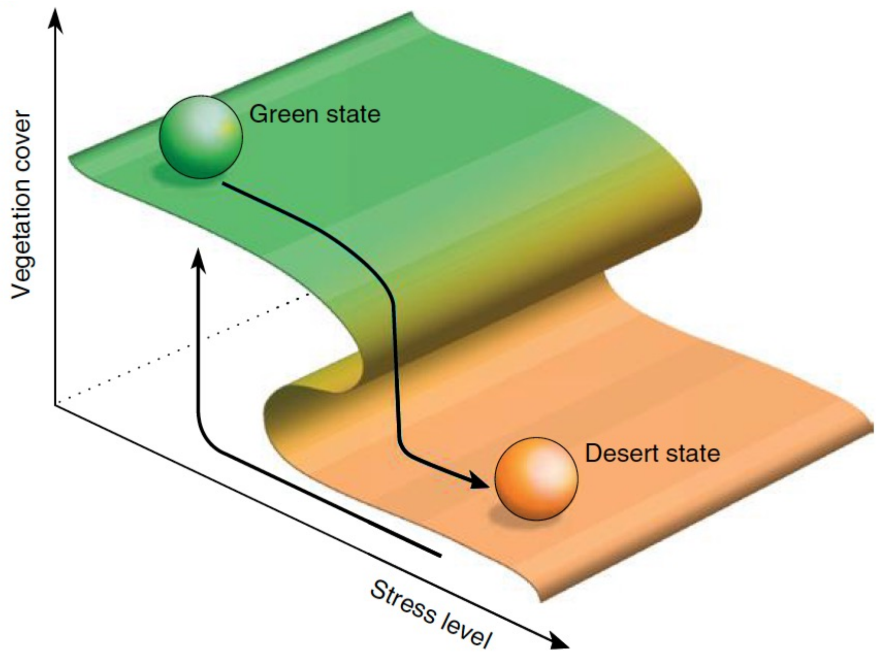
Damped System with disturbances and feedbacks





Tipping points

Idea of tipping points: “**little things can make a big difference**”



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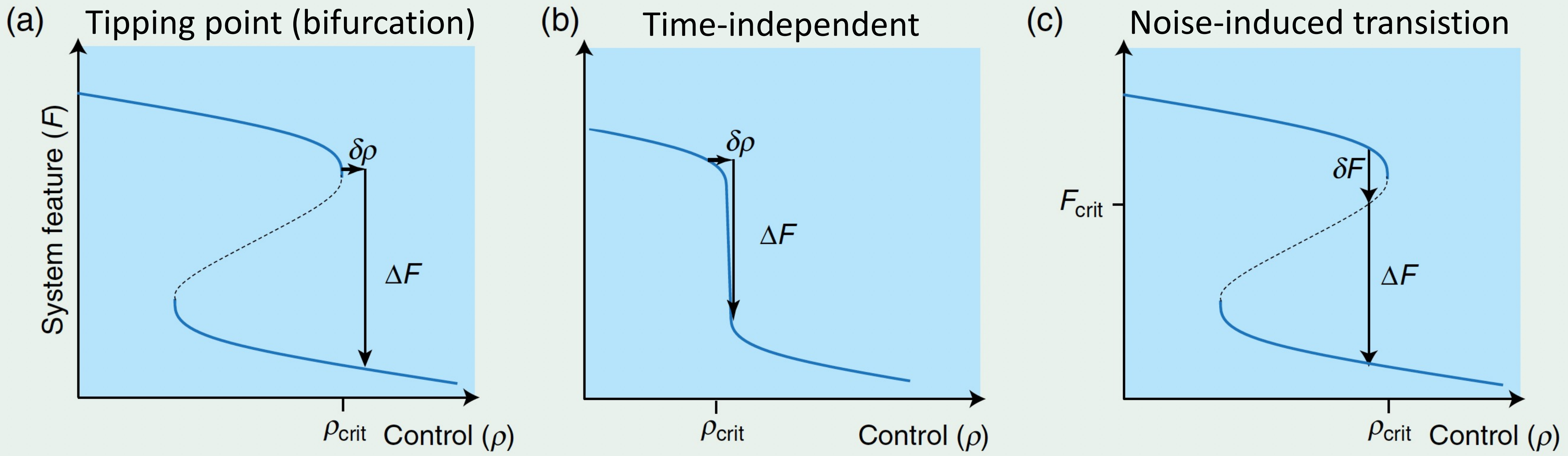

Nonlinearity, Feedbacks and Predictability

Summary

- Damped systems, nonlinearity, feedback, and predictability
- Learned how to quantify the sensitivity of climate models
- Talked about equilibrium climate sensitivity (ECS) and feedback factors
- Have checked current ECS estimates from the IPCC
- Talked about parametrization
- Character of tipping points
- Learned why it is possible to predict the long-term climate even though we it is not possible to predict the weather for more than a couple of days

Nonlinearity, Feedbacks and Predictability

Summary

- Models all agree strongly on the Planck feedback
- The Planck feedback is about 3.3 W m^{-2} K^{-1}
- Water vapor feedback is strongly positive =
- The ice-albedo feedback is positive
- By far the largest spread across the models occurs in the cloud feedback.
- Therefore, most of the spread in the ECS across the models is due to the spread in the cloud feedback.