Schedule

1. **What are models?**
   Exercise: Python Skills

2. **Energy budget of the Earth**
   Exercise: Simple Energy Balance Model

3. **Nonlinearity, Feedback, and Predictability**
   Exercise: Nonlinearity and Feedbacks
   Exercise: Revised Energy Balance Model

4. **Parametrization and Sensitivity**

5. **Radiative budget**
   Exercise: 1-layer greenhouse model
   Exercise: 2-layer greenhouse model

6. **Introduction to fluid dynamics**
   Exercise: Analytical katabatic flow model

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7. **Finite difference method**
   Exercise: Heat Equation
   Exercise: Advection-Diffusion Equation
   Exercise: Boundary layer Evolution
   Exercise: Numerical katabatic flow model

8. **Implicit finite difference methods**
   Exercise: Boundary layer evolution

9. **Optimization problem**
   Exercise: Surface energy balance
   Exercise: Sublimation

10. **COSIPY snow model**
    Exercise: Simulations with COSIPY

11. **Introduction to PALM**
    Exercise: Simulations with PALM

12. **How to write an article**

Exercise: Heat Equation

Ground heat flux: $$Q_G = -k_g \frac{\partial T}{\partial z}$$ (Fourier's law)

Temporal temperature change: $$\frac{\partial T}{\partial t} = -\left(\frac{1}{c_g}\right)\frac{\partial Q_g}{\partial z}$$

Heat conductivity equation: $$\frac{\partial T}{\partial t} = \nu_g \frac{\partial^2 T}{\partial z^2}$$

with k_g *: Thermal molecular conductivity [W m^{-1} K^{-1}]*
c_g *: Soil heat capacity [Wm^{-3} K^{-1}]*
ν_g *: Soil thermal diffusivity ($\nu_g = k_g/c_g$) [m^2 s^{-1}]*

Exercise: Heat Equation

Surface	Thermal molecular conductivity k_g in $[Wm^{-1}K^{-1}]$	Soil heat capacity Cg in 10^6 $[Wm^{-3}K^{-1}]$	Soil thermal diffusivity ν_g in 10^{-6} $[m^2s^{-1}]$
Granit	2.73	2.13	1.28
Wet sand	2.51	2.76	0.91
Dry sand	0.30	1.24	0.24
Sandy clay	0.92	2.42	0.38
Swamp	0.89	3.89	0.23
Old snow	0.34	0.84	0.40
Fresh snow	0.02	0.21	0.10
Pure ice	2.10	2.09	1.09

Exercise: 1D Heat Equation

Heat conductivity equation:

$$\frac{\partial T}{\partial t} = \nu_g \frac{\partial^2 T}{\partial z^2} = \nu_g \nabla^2 T$$

with ν_g *: Soil thermal diffusivity (*$\nu_g = k_g / c_g$*) [*$m^2\ s^{-1}$*]*

Strategy:

- ▷ Forward difference in time
- ▷ Central difference in space

$$\frac{\partial T}{\partial t} = \nu_g \frac{\partial^2 T}{\partial z^2} \simeq \nu_g \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta z^2}$$

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \nu_g \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta z^2}$$

$$T_i^{n+1} = T_i^n + \nu_g \frac{\Delta t}{\Delta z^2} (T_{i+1}^n - 2T_i^n + T_{i-1}^n)$$

Exercise: 1D Heat Equation

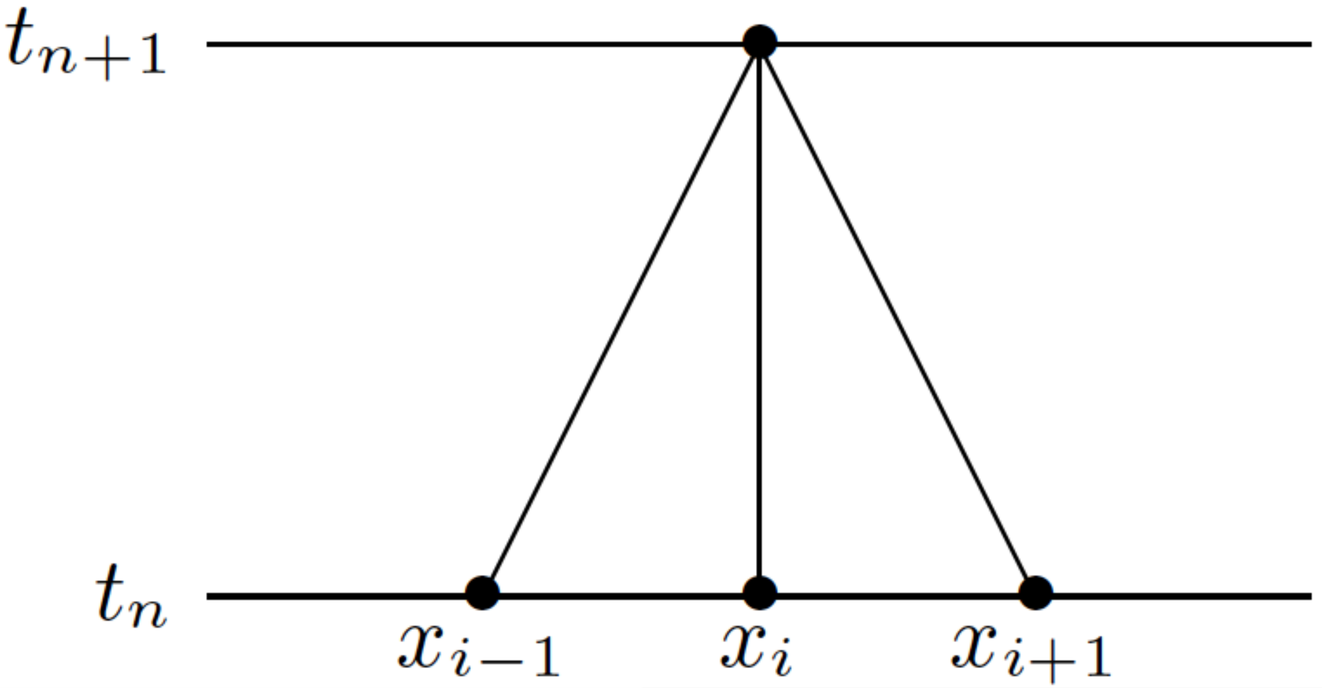
$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = v_g \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta z^2}$$

forward

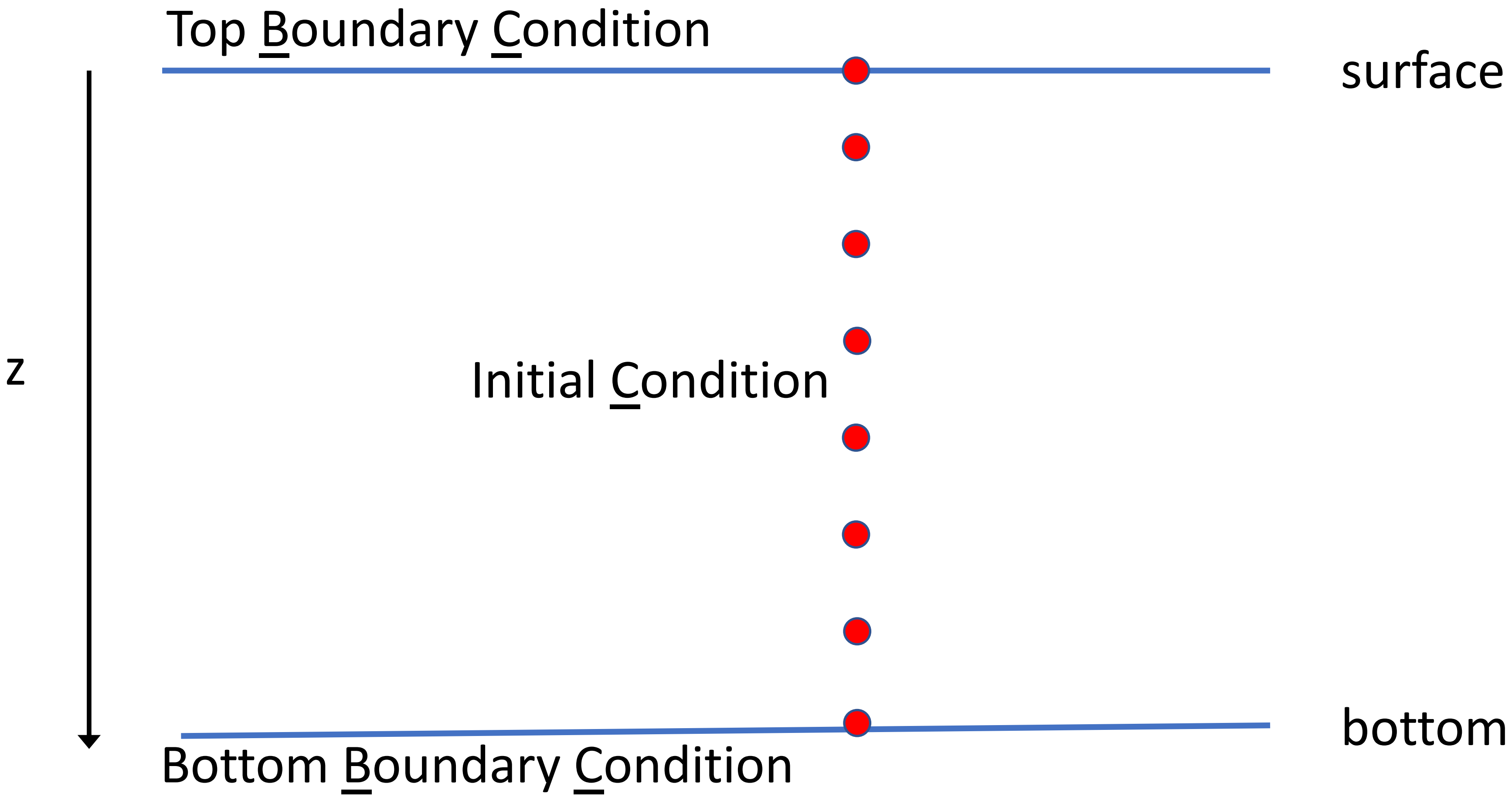
central

Reordered:

$$T_i^{n+1} = T_i^n + v_g \frac{\Delta t}{\Delta z^2} (T_{i+1}^n - 2T_i^n + T_{i-1}^n)$$



Exercise: 1D Heat Equation



Exercise: 1D Heat Equation

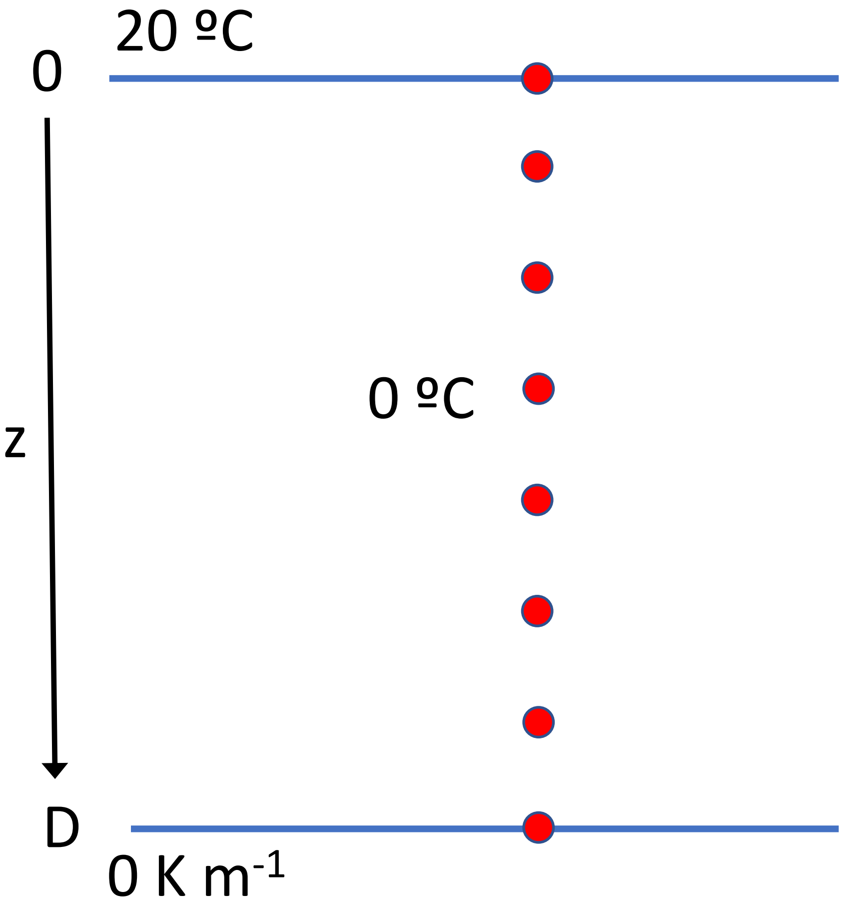
Task 1: Integrate the heat equation for several days using a time step of 1 hour and a heat conductivity of ν_g = 1.2e^{-6} [m^2 s^{-1}]. Plot the result. Once the code works, change the integration time. What happens if you integrate over a very long time?

depth timestep

Top BC $T(0,t) = 20\ ºC$

Bottom BC $T(D,t) = \dfrac{\partial T(t,z)}{\partial z} = 0\ K\ m^{-1}$

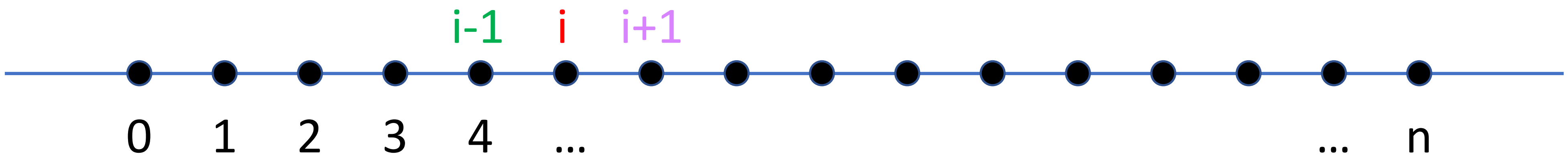
Initial condition $T(n,0) = 0\ ºC$



Hint: 1D Heat Equation using index arrays

A = numpy.arange(0,n+1)

A is an array from 0 to n with Nx = n+1 nodes.



for i in range(0, n, 1):
A[i]

Hint: 1D Heat Equation using index arrays

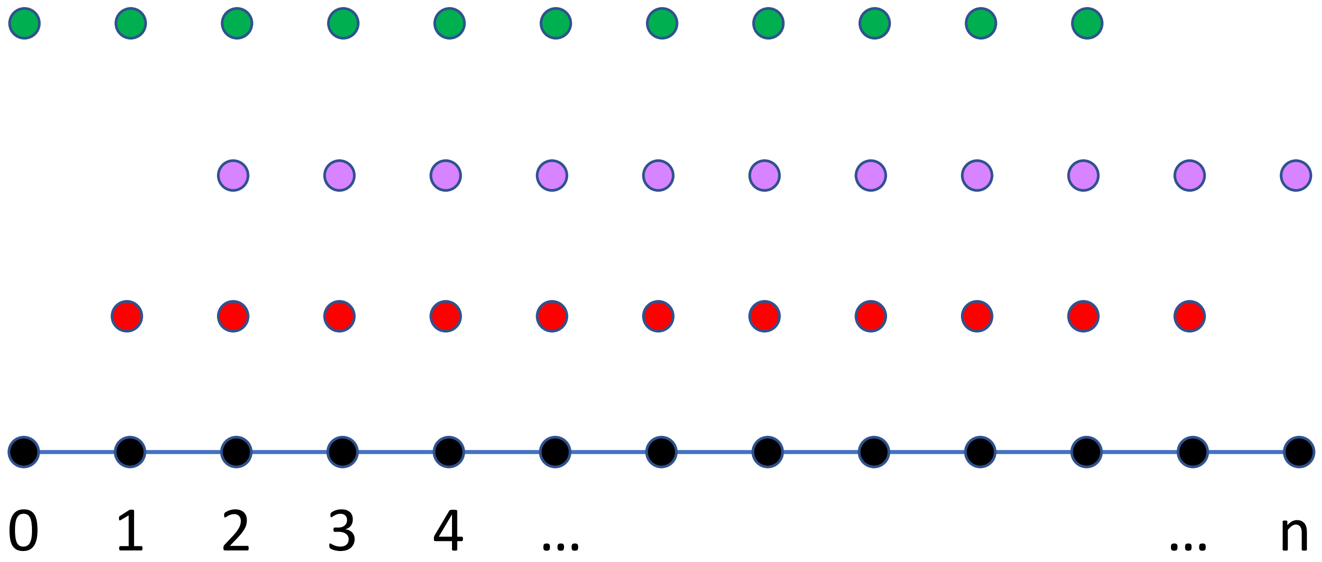
A = numpy.arange(0,n+1)

A is an array from 0 to n with Nx = n+1 nodes.

$$idx_{i-1} = np.arange(0, Nx-2)$$

$$idx_{i+1} = np.arange(2, Nx)$$

$$idx_{i} = np.arange(1, Nx-1)$$



Exercise: 1D Heat Equation

Task 2: Rewrite the 1D heat equation using index arrays

Exercise: Time-dependent Heat Equation

Task 3: Using the previous code, solve the Heat Equation using a temporal varying surface boundary condition. Use the following discretization: I = [0; 20 m], N = 40 grid points, ν_g = 1.2e^{-6} [m^2 s^{-1}], and a daily time step. Integrate the equation for several years, e.g. 5 years. Plot the result as a contour plot. Also plot temperature time series in several depths. Discuss the plot!

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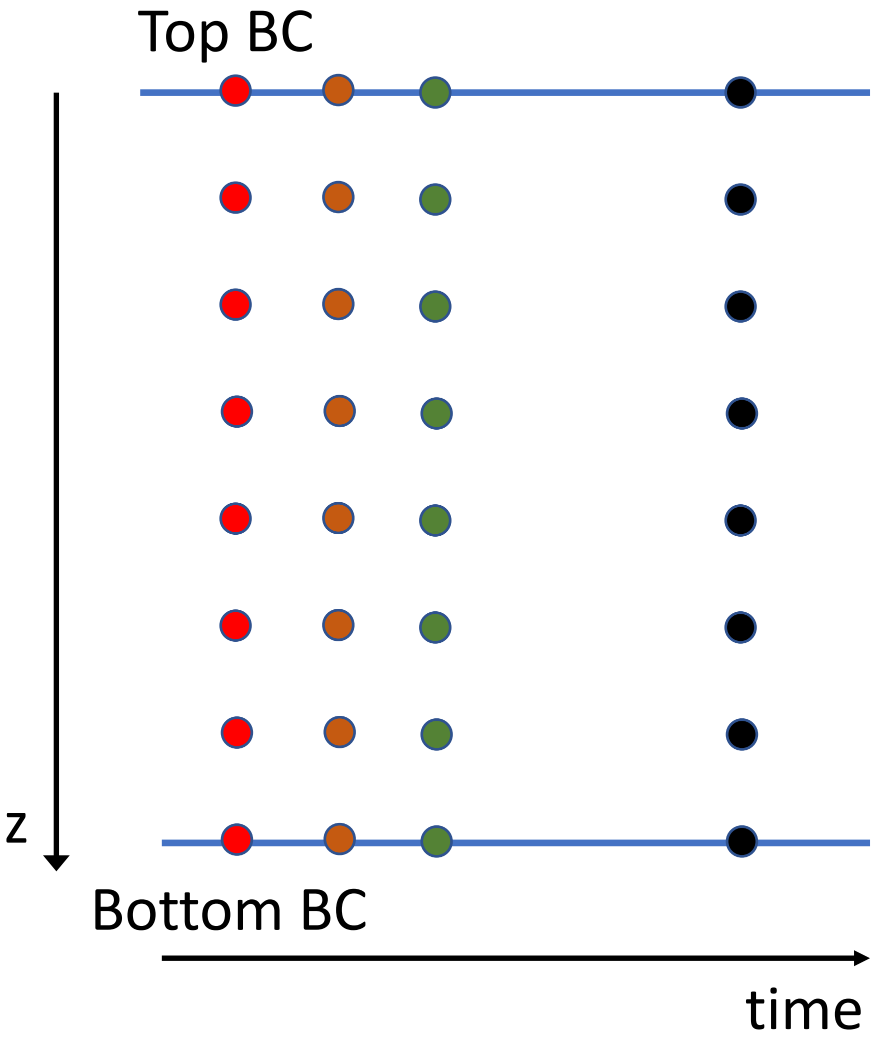
Top BC $$T(t,0) = 0 - 10\sin\left(\frac{2\pi t}{365}\right) ºC$$

Bottom BC $$T(t,D) = \frac{\partial T(t,z)}{\partial z} = 0\ K\ m^{-1}$$

Initial condition $$T(0,n) = 0\ ºC$$

Exercise: Time-dependent Heat Equation



T(1,1)	T(1,2)	T(1,3)	...	T(1,n)
T(2,1)	T(2,2)	T(2,3)		T(2,n)
T(3,1)	T(3,2)	T(3,3)		T(3,n)
T(4,1)	T(4,2)	T(4,3)		T(4,n)
T(..,1)	T(..,2)	T(..,3)		T(..,n)
T(..,1)	T(..,2)	T(..,3)		T(..,n)
T(..,1)	T(..,2)	T(..,3)		T(..,n)
T(m,1)	T(m,2)	T(m,3)	...	T(m,n)