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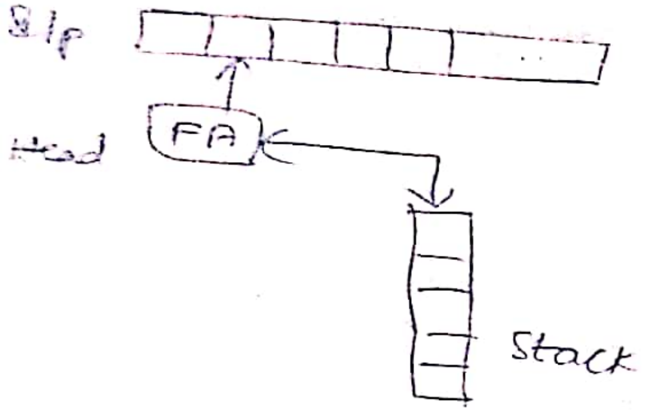
UNIT - III

Pushdown Automata (FA + Stack)



1.) Pushdown Automaton :-

Defⁿ of PDA is similar to that of FA, except for auxiliary mem unit stack.

→ Stack is a mem. unit with stores elements ; PDA may use diff alphabets for its i/p & its stack.

Defⁿ :- A PDA is defined as 7-tuple,

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$$

where ; Q : Finite set of states

Σ : " " " i/p symbols

Γ : " " " pushdown symbols / stack alphabets including Z_0, used to rept empty stack

q_0 : Starting state

($F \subseteq Q$) F : finite set of final states

δ : transitiⁿ fⁿ, $Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma$.

Domain of " " is $Q \times \Sigma \times \Gamma$

range " " " is $Q \times \Gamma$

(14) current state, next i/p alphabet to be processed & topmost symbol of stack determines next move of PDA.

* **Instantaneous Desc^n (ID) :-**

Def^n :- An ID of PDA is a 3-tuple (q, w, α) where

q denotes current state $(q \in Q)$

w " position of alphabet i/p yet to be read & $(w \in \Sigma^*)$

α - contents of pushdown store $(\alpha \in \Gamma^*)$

Ex :- $(q_0, aaabbb, XYZ_0)$ is an ID describing PDA current state of PDA is q_0, i/p string to be processed is aaabbb, & stack contains symbols XYZ_0, where X is topmost symbol on stack.

→ Initial ID of PDA can be defined as (q_0, w, Z_0)

→ If $(q, aa_1a_2 \cdots a_n, zz_1 \cdots z_n)$ is an ID & $\delta(q, a, z)$ contains $(p, B_1, \cdots B_m)$ then next ID is $(p, a_1 \cdots a_m, B_1, \cdots B_m\, z_1, \cdots z_n)$, $a \in \Sigma \cup \{\epsilon\}$.

→ This is denoted by $(q, aa_1 \cdots a_n, zz_1 \cdots z_n) \vdash (p, a_1 \cdots a_n, B_1 \cdots B_m\, z_1, \cdots z_n)$

→ $\vdash^*$ is reflexive transitive closure of $\vdash$.

→ set of strings accepted by PDA M by emptying pushdown store is denoted as Null(M) or N(M).

$N(M) = \{w \mid w \in \Sigma^*, (q_0, w, Z_0) \vdash^* (q, \epsilon, \epsilon)$ for some $q \in K\}$

- This means any string 'w' on i/p tape will be accepted by PDA M by empty store, if

M started in q_0 with its i/p head pointing to the leftmost symbol of 'w' & 'Z_0' on its pushdown store, will read whole of 'w' & go to state 'q' & pushdown store will be emptied. This is cld acceptance by empty store. F is taken as empty set.

– Another way of acceptance cld acceptance by final state. Finally after some moves, reads whole i/p & reaches one of final states. Pushdown store need not be emptied in this case. Lang. accepted by PDA by final state is denoted as T(M)

$$T(M) = \{ w \mid w \in \Sigma^*, (q_0, w, Z_0) \vdash^* (q_f, \epsilon, \gamma) \text{ for some } q_f \in F \text{ \& } \gamma \in \Gamma^* \}$$

Ex:- ① $L = \{ a^i b^j c^k \mid i, j, k \geq 1$ & $i + j = k \}$

a) Find PDA (wch accepts via final state) that recognizes L

b) Find PDA (" " " empty stack) "

Sol:- a) push a's & b's into stack & match them with each c & clear.

PDA, $M = \{ q_0, q_1$

Transitions are :

$\delta(q_0, a, \$)$ contains $(q_0, a\$)$

$\delta(q_0, a, a)$ " (q_0, aa)

$\delta(q_0, b, a)$ " (q_1, ba)

$\delta(q_1, b, b)$ " (q_1, bb)

$\delta(q_1, c, b)$ " (q_2, ϵ)

$\delta(q_2, c, a) \;\therefore\; (q_2, \epsilon)$

$\delta(q_2, c, b) \;\therefore\; (q_2, \epsilon)$

$\delta(q_2, \epsilon, \$) \;\therefore\; (q_3, \epsilon)$

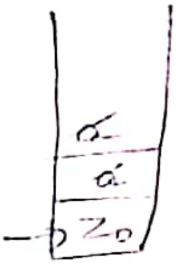
↓
final state

6) All transitn remain same.
This m/c accepts L via both by empty stack & final state. This is a DPDA.

Ex (2) $L = \{a^n b^n \mid n \geq 1\}$

a a b b ε
↑ ↑ ↑ ↑ ↑

completed string.



$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

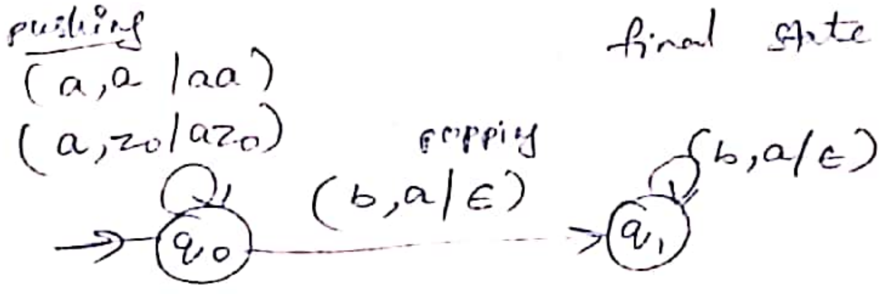
$\delta(q_0, a, a) = (q_0, aa)$

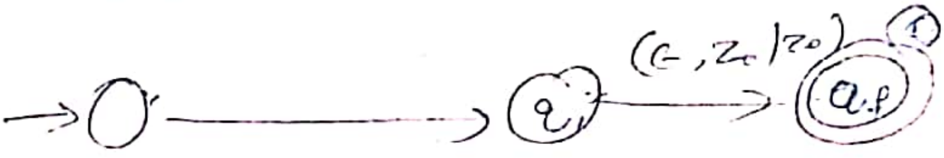
$\delta(q_0, b, a) = (q_1, a)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$ or (q_1, ϵ)

acceptance by final state ∴ empty stack.





acceptance by final state

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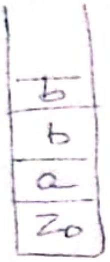
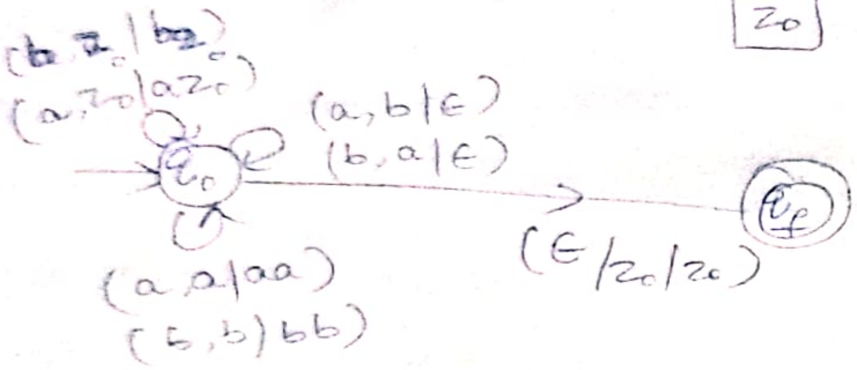
Q) $L = \{ w \mid n_a(w) = n_b(w) \}$ diff frm $\{a^n b^n \mid n \geq 1\}$

after b's, a's shld not come

ab   aab
ba   bbaa
aabb
⋮

abbaaε
↑↑





Q.) Draw Transitⁿ diag for PDA defined as:

$\delta(q_0, x, Z_0) = (q_0, xZ_0)$

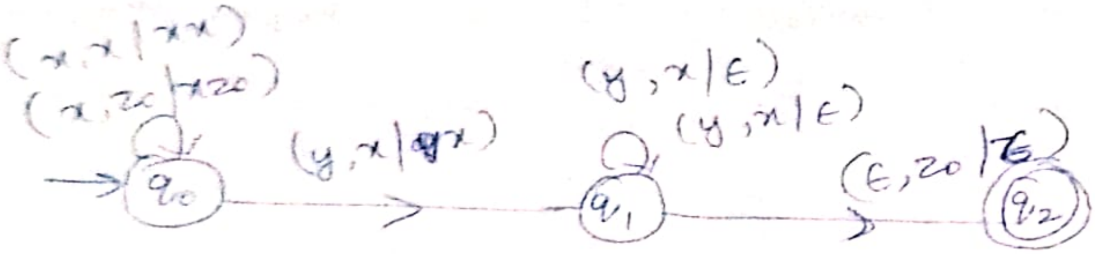
$\delta(q_0, x, x) = (q_0, xx)$

$\delta(q_0, y, x) = (q_1, x)$

$\delta(q_1, y, x) = (q_1 x)$

$\delta(q_1, x, x) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_2, \epsilon)$



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2.) **Equivalence b/w Acceptance by Empty Store**
& " " Final State :-

theorem :- L is accepted by PDA M_1 by empty store, iff L " " " M_2 by final state.

proof :- (i.) Let L be accepted by PDA,
$M_2 = (Q, \Sigma, \Gamma, \delta_2, q_0, Z_0, F)$ by final state
→ construct M_1 as : $M_1 = (K \cup \{q_0', q_e\}, \Sigma, \Gamma \cup \{X_0\}, \delta_1, q_0', X_0, \phi)$

δ mappings are :

1.) $\delta_1(q_0', \epsilon, X_0)$ contains (q_0, Z_0X_0)

2.) $\delta_1(q, a, Z)$ includes $\delta_2(q, a, Z)$ for all $q \in Q$, $a \in \Sigma \cup \{\epsilon\}, Z \in \Gamma$

3.) $\delta_1(q_f, \epsilon, Z)$ contains (q_e, ϵ) for $f_f \in F$ & $Z \in \Gamma \ominus \{X_0\}$

4.) $\delta(q_e, \epsilon, Z)$ " (q_e, ϵ) for $Z \in \Gamma \cup \{X_0\}$

→ If w is i/p accepted by M_2, then
$(q_0, w, Z_0) \vdash^{*}_{M_2} (q_f, \epsilon, \gamma)$.

– This can happen in M_1 also.

$(q_0, w, Z_0) \vdash^{*}_{M_1} (q_f, \epsilon, \gamma)$

→ M_1 accepts 'w' as follows:

$(q_0', w, X_0) \vdash (q_0, w, Z_0X_0) \vdash^{*} (q_f, \epsilon, \gamma X_0) \vdash^{*} (q_e, \epsilon, \epsilon)$

(146) Hence, $(q_0, w, Z_0X_0) \overset{*}{\underset{M}{\vdash}} (q_f, \epsilon, \gamma X_0)$ wch means $(q_0, w, Z_0) \overset{*}{\underset{M}{\vdash}} (q_f, \epsilon, \gamma)$ & 'w' will be accepted by M_2

(ii) L is accepted by M_1 by empty store, it will be accepted by M_2 by final state.

→ Let $M_1 = \{Q, \Sigma, \Gamma, \delta, q_0, Z_0, \phi)$. Then M_2 is constructed as:

$$M_2 = (Q \cup \{q_0', q_f\}, \Sigma, \Gamma \cup \{X_0\}, \delta_2, q_0', X_0, \{q_f\})$$

→ δ mappings are:-

1.) $\delta_2(q_0', \epsilon, X_0)$ contains (q_0, Z_0X_0)

2) $\delta_2(q, a, z)$ includes all ele. of $\delta(q, a, z)$ for $q \in Q, a \in \Sigma \cup \{\epsilon\}, Z \in \Gamma$

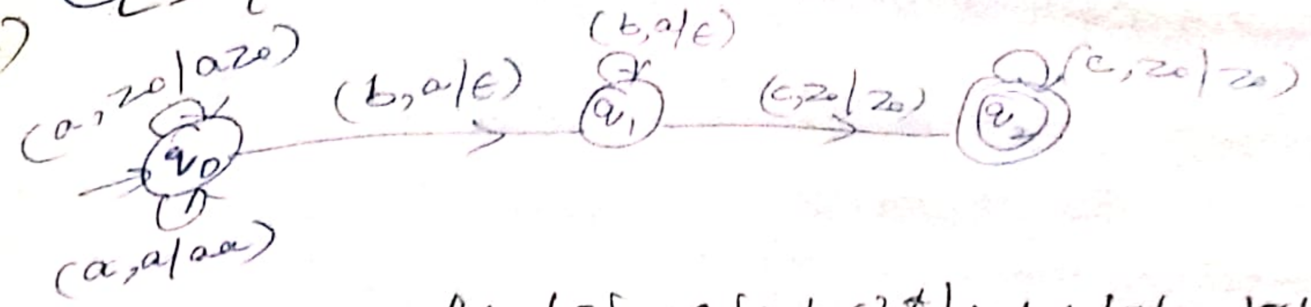
3) $\delta_2(q, \epsilon, X_0)$ contains (q_f, X_0) for each $q \in Q$

→ Moves of M_2 in accepting an i/p 'w' can be described as:

$$(q_0', w, X_0) \vdash (q_0, w, Z_0X_0) \overset{*}{\vdash} (q, \epsilon, X_0) \vdash (q_f, \epsilon, X_0)$$

(147) $L = \{ a^n b^n c^m \mid n, m >, 1 \}$

Q)



Q) Find PDA for $L = \{ x \in \{a, b, c\}^* \mid |x|_a + |x|_b = |x|_c \}$

Transitions are:

$\delta(q_0, a, \$)$ contains $\delta(q_0, a\$)$

$\delta(q_0, b, \$)$ '' $\delta(q_0, b\$)$

$\delta(q_0, c, \$)$ '' $\delta(q_0, c\$)$

$\delta(q_0, a, a)$ '' $\delta(q_0, aa)$

$\delta(q_0, a, b)$ '' $\delta(q_0, ab)$

$\delta(q_0, b, b)$ '' $\delta(q_0, bb)$

$\delta(q_0, c, c)$ '' $\delta(q_0, cc)$

$\delta(q_0, a, c)$ '' $\delta(q_0, \epsilon)$

$\delta(q_0, b, c)$ '' $\delta(q_0, \epsilon)$

$\delta(q_0, c, a)$ '' $\delta(q_0, \epsilon)$

$\delta(q_0, c, b)$ '' $\delta(q_0, \epsilon)$

$\delta(q_0, \epsilon, \$)$ '' $\delta(q_f, \epsilon)$

↓

final state

a) $L = \{a^n b^n c^m d^m \mid n, m \geq 1\}$ Find PDA

$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b, c, d\}, \{a, c, \$\}, \delta, q_0, \$, \{q_4\})$

Transitions are:-

$\delta(q_0, a, \$)$ contains $\delta(q_0, a\$)$

$\delta(q_0, a, a)$ " $\delta(q_0, aa)$

$\delta(q_0, b, a)$ " (q_1, ϵ)

$\delta(q_1, b, a)$ " (q_1, ϵ)

$\delta(q_1, c, \$)$ " $(q_2, c\$)$

$\delta(q_2, c, c)$ " (q_2, cc)

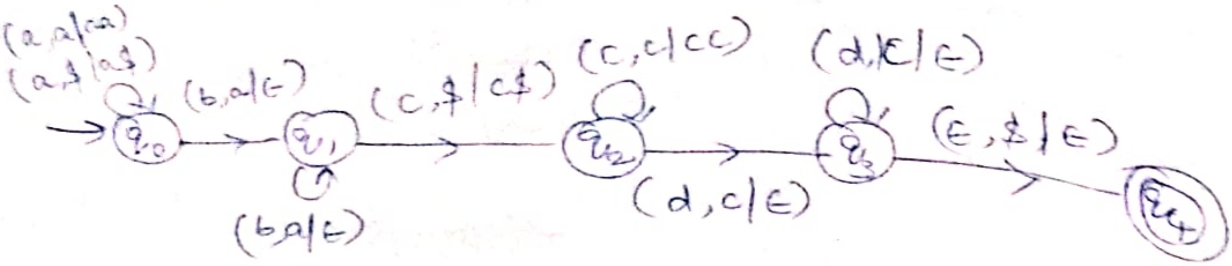
$\delta(q_2, d, c)$ " (q_3, ϵ)

$\delta(q_3, d, c)$ " (q_3, ϵ)

$\delta(q_3, \epsilon, \$)$ " (q_4, ϵ)

→ M/c accepts L by empty stack

→ Taking q_4 as final state L is accepted by final state also



(Q.) ⑭⑨ Find PDA for $L = \{ x0y1 \mid x, y \in \{0,1\}^{+}, |x| = |y| \}$

$M = (\{q_0, q_1, q_2\}, \{0,1\}, (X, \$), \delta, q_0, \$, \phi)$

$\delta(q_0, 0, \$) = \delta(q_0, X\$)$

$\delta(q_0, 0, X) = \delta(q_0, XX)$

$\delta(q_0, 1, \$) = \delta(q_0, X\$)$

$\delta(q_0, 1, X) = \delta(q_0, XX)$

$\delta(q_0, 0, \$) = \delta(q_1, \$)$

$\delta(q_0, 0, X) = \delta(q_1, X)$

$\delta(q_1, 0, X) = \delta(q_1, \epsilon)$

$\delta(q_1, 1, X) = \delta(q_1, \epsilon)$

$\delta(q_1, 1, \$) = \delta(q_2, \epsilon)$

→ Acceptance by empty stack & by taking q_2 as final state

Q.) $L = \{ 0^n 1^n / n > 0 \}$

→ Accepts all 0's followed by equal no. of 1's

→ push 0's & pop 0's when seeing 1's.

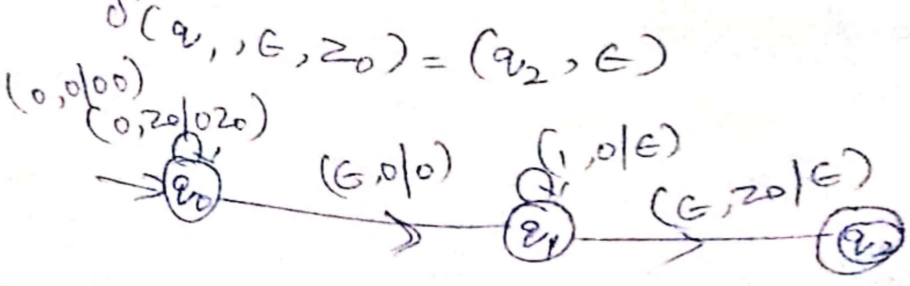
$\delta(q_0, 0, z_0) = (q_0, 0z_0)$

$\delta(q_0, 0, 0) = (q_0, 00)$

$\delta(q_0, \epsilon, 0) = (q_1, 0)$

$\delta(q_1, 1, 0) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$

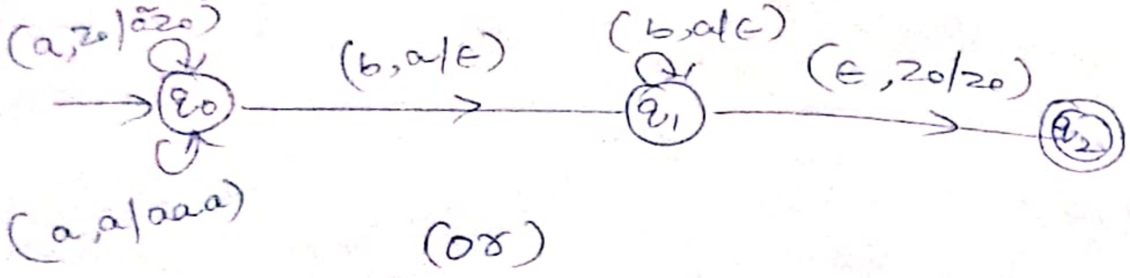


(150)

Q) $L = \{a^n b^{2n} \mid n \geq 1\}$

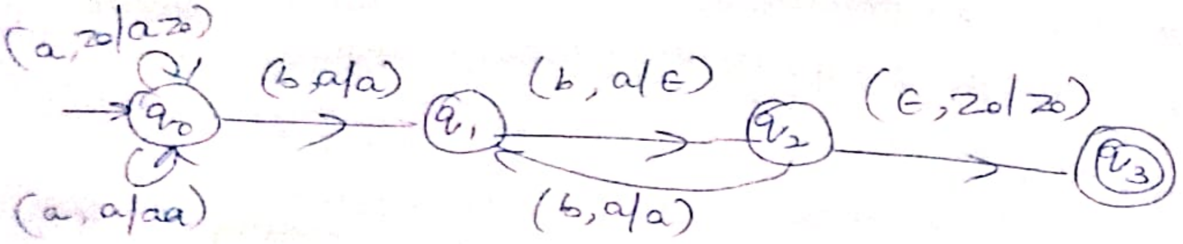
abb, aabbbb, ...

For every 'a' push 2a's pop 'a' for 'b'



(or)

For 'a' push 'a' ; } pop 1a (for 2nd b, pop a)
'b' push 2b's ;



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* **Deterministic PDA :-**

→ For given i/p symbol & any stack top, only one move is possible from one state.

DPDA ⇒ $M = \{Q, \Sigma, \Gamma, \delta, q_0, F)$ if the cond^n's are satisfied, for every $q \in Q$, $x \in \{\Sigma \cup \{\epsilon\}\}$, & $z \in \Gamma$

i) $\delta(q, \epsilon, z)$ contains atmost one ele

ii) $\delta(q, \epsilon, z)$ is not empty, DPDA can have only one move.

NDPA $\xrightarrow{to}$ DPDA

Q.) Cons DPDA for $L = \{wcw^R \mid w \in \{0,1\}^*\}$

$\delta(q,0,Z_0) = \delta(q_0, 0Z_0)$

$\delta(q,1,Z_0) = (q_0, 1Z_0)$

$\delta(q,0,0) = (q_0, 00)$

$\delta(q,0,1) = (q_0, 01)$

$\delta(q,1,0) = (q_0, 10)$

$\delta(q,1,1) = (q_0, 11)$

$\delta(q,c,1) = (q_1, 1)$

$\delta(q,c,0) = (q_1, 0)$

$\delta(q_1,0,0) = (q_1, \epsilon)$

$\delta(q_1,1,1) = (q_1, \epsilon)$

$\delta(q_1,\epsilon,Z_0) = (q_2, Z_0)$

$\delta(q,\epsilon,Z_0) = (q_2, \epsilon)$

Q.) Lang wch is accepted by PDA but not by DPDA

$L = \{0^i 1^i \mid i \geq 0\} \cup \{0^j 1^{2j} \mid j \geq 0)$

$\delta(q_1,0,Z_0) = (q_1, 0Z_0)$

$\delta(q_1,0,0) = (q_1, 00)$

$\delta(q_1,1,0) = (q_1, \epsilon)$

$\delta(q_1,1,0) = (q_2, 0)$

$\delta(q_2,1,0) = (q_1, \epsilon)$

$\delta(q_1,\epsilon,Z_0) = (q_2, \epsilon) \rightarrow$ Accept

⑮

3) Equivalence of CFG & PDA :-

Theorem:- There exists an NPDA 'M' for every CFL L, with implies that $L = L(M)$

Proof :- $CFG = (N, T, P, S)$

$M = \{\{q_0, q_1, q_f\}, T, N \cup Z_0, q_0, q_f, \delta\}$

δ can be defined as:

Rule 1 : Convert all starting productions to $\delta(q_0, \epsilon, Z_0) = (q_1, SZ_0)$, where Z_0 is starting symbol on stack to mark empty stack & S is starting production

Rule 2 : Convert all productions of the form $A \to a\alpha$ to $\delta(q_1, a, A) = (q_1, \alpha)$

Rule 3 : $A \to a$

to $\delta(q_1, a, A) = (q_1, \epsilon)$

Rule 4 : For all elements 'a' of T, $\delta(q_1, a\, a) = (q_1, \epsilon)$

Rule 5 : Add $\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$ to reach to final state

Q) Find PDA to the grammar: $S \to 0X1 / 1$
$X \to X0 / \epsilon$

→ Conversion of grammar into GNF form

Step 1 : Remove ε-productions

$S \to 0X1 / 1 / 01$
$X \to X0 / 0$

Step 2 : Conversion into GNF :

$S \to 0XY / 1 / 0Y$
$X \to 0X / 0$
$Y \to 1$

(152) $M = \{Q, \Sigma, \Gamma, \delta, \overset{\to q_0}{q_0}, \overset{\to q_f}{F}\}$

where ; $Q = \{q_0, q_1, q_f\}$

$\Sigma = T = \{0, 1\}$

$\Gamma = N \cup Z_0 = \{S, X, Y, Z_0\}$

→ Transitions of PDA as:

1 $\delta(q_0, \epsilon, Z_0) = (q_1, SZ_0)$
2 $\delta(q_1, 0, S) = (q_1, XY)$
3 $\delta(q_1, 0, S) = (q_1, Y)$
4 $\delta(q_1, 1, S) = (q_1, \epsilon)$
5 $\delta(q_1, 0, X) = (q_1, X)$
6 $\delta(q_1, 0, X) = (q_1, \epsilon)$
7 $\delta(q_1, 1, Y) = (q_1, \epsilon)$
8 $\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

→ check whether NPDA accepts same string as CFG

→ consider $\underline{0001}$.

$\delta(q_0, \epsilon, Z_0) = (q_1, SZ_0)$ (using 1)

$\Rightarrow \delta(q_1, 0001, S)$

$= \delta(q_1, 001, XYZ_0)$ (using 2)

$= (q_1, 01, XYZ_0)$ (" 3)

$= (q_1, 1, YZ_0)$ (" 6)

$= (q_1, \epsilon, Z_0)$ (" 7)

$= (q_f, Z_0)$ (" 8)

There is more than one choice for making transitions on same i/p symbol such as:

$\delta(q_1, 0, S) = (q_1, XY)$

$\delta(q_1, 0, S) = (q_1, Y)$

- This gives NPDA

Q.) Cons NPDA equivalent to the CFG:

$S \rightarrow 0XX$

$X \rightarrow 0S / 1S / 0$

$M = \{Q, \Sigma, \Gamma, \delta, \overset{q_0 \rightarrow q_f}{q_0}, F\}$

- $Q = \{q_0, q_1, q_f\}$
- $\Sigma = T = \{0, 1\}$
- $\Gamma = N \cup Z_0 = \{S, X, Z_0\}$

→ Transitions for NPDA are:

1. $\delta(q_0, \epsilon, Z_0) = (q_1, SZ_0)$
2. $\delta(q_1, 0, S) = (q_1, XX)$
3. $\delta(q_1, 0, X) = (q_1, S)$
4. $\delta(q_1, 1, X) = (q_1, S)$
5. $\delta(q_1, 0, X) = (q_1, \epsilon)$
6. $\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

→ Test the string 001000 of CFL given by above CFG:

$\delta(q_0, \overset{001000}{\cancel{\epsilon}}, Z_0) = (q_1, SZ_0)$ (Using 1)

$\delta(q_1, 001000, SZ_0)$ (∵ 2)

$= \delta(q_1, 001000, XXZ_0)$ (Using 3)

$= \delta(q_1, 1000, XZ_0)$ (" 4)

$= \delta(q_1, 000, SZ_0)$ (" 5)

$= \delta(q_1, 00, X\cancel{X}Z_0)$ (" 6)

$= \delta(q_1, 0, XZ_0)$ (" 7)

$= \delta(q_1, \epsilon, Z_0)$ (" 8)

$= \delta(q_f, \epsilon, Z_0)$ = Accept

Q) Cons PDA for grammar: $X \rightarrow 0\cancel{X}3$

$X \rightarrow 0A3$

$A \rightarrow 1A2$

$A \rightarrow 12$

2 methods:

I:- convert grammar into GNF

Method I

$X \rightarrow 0XB$

$X \rightarrow 0AB$

$A \rightarrow 1AC$

$A \rightarrow 1C$

$B \rightarrow 3$

$C \rightarrow 2$

→ Convert grammar into PDA :-

$\delta(q_0, \epsilon, Z_0) = \delta(q_1, XZ_0)$

$\delta(q_1, 0, X) = (q_1, AB)$

$\delta(q_1, 1, A) = (q_1, AC)$

$\delta(q_1, 1, A) = (q_1, C)$

$\delta(q_1, 3, B) = (q_1, \epsilon)$

$\delta(q_1, 2, C) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_f, \epsilon)$

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II method w/o Converting grammar into GNF

$\delta(q, \epsilon, X) = (q, 0X3)$

$\delta(q, \epsilon, X) = (q, 0A3)$

$\delta(q, \epsilon, A) = (q, 1A2)$

$\delta(q, \epsilon, A) = (q, 12)$

$\delta(q, 0, 0) = (q, \epsilon)$

$\delta(q, 1, 1) = (q, \epsilon)$

$\delta(q, 2, 2) = (q, \epsilon)$

$\delta(q, 3, 3) = (q, \epsilon)$

Q) Convert PDA for grammar:

$X \rightarrow 0 / 0X / 1XX / XX1 / X1X$

$\delta(q, \epsilon, X) = (q, 0)$

$\delta(q, \epsilon, X) = (q, 0X)$

$\delta(q, \epsilon, A) = (q, 1XX)$

$\delta(q, \epsilon, A) = (q, XX1)$

$\delta(q, \epsilon, A) = (q, X1X)$

$\delta(q, 0, 0) = q, \epsilon)$

$\delta(q, 1, 1) = (q, \epsilon)$

Q) Const. PDA for lag., $L = \{0^n 1^m 2^m 3^n / m, n \geq 1\}$

$\delta(q_0, 0, z_0) = (q_0, 0z_0)$

$\delta(q_0, 0, 0) = (q_0, 00)$

$\delta(q_0, 1, 0) = (q_1, 10)$

$\delta(q_1, 1, 1) = (q_1, 11)$

$\delta(q_1, 2, 1) = (q_2, \epsilon)$

$(q_2, 2, 1) = (q_2, \epsilon)$

$(q_2, 3, 0) = (q_4, \epsilon)$

$\delta(q_4, 3, 0) = (q_4, \epsilon)$

$\delta(q_4, \epsilon, Z_0) = (q_f, Z_0)$

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d) Cons PDA for $L = \{0^n 1^m 2^{n+m} / n, m > 0\}$

$L = \{0122, 00112222, \ldots\}$

smlst string.

→ Cons CFG for this grammar as:

$X \rightarrow AB$

$A \rightarrow 0A1 / 01$

$B \rightarrow 22 / 2B2$

→ Convert into GNF ;

$A \rightarrow 0AC / 0C$

$B \rightarrow 2D / 2BD$

$C \rightarrow 1$

$D \rightarrow 2$

$X \rightarrow 0ACB / 0CB$

→ PDA for this grammar as:

$\delta(q_0, \epsilon, Z_0) = (q_1, XZ_0)$

$\delta(q_1, 0, X) = (q_1, ACBX)$

$\delta(q_1, 0, X) = (q_1, CBX)$

$\delta(q_1, 0, A) = (q_1, AC)$

$\delta(q_1, 0, A) = (q_1, C)$

$\delta(q_1, 2, B) = (q_1, D)$

$\delta(q_1, 0, Z_0) = (q_1, Z_0)$

$\delta(q_1, 1, C) = (q_1, \epsilon)$

$\delta(q_1, 2, D) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

Q) Cons PDA for $L = \{0^{n+2} 1^{2n+1} / n \geq 0\}$

$\delta(q_0, 0, Z_0) = (q, Z_0)$

$\delta(q_1, 0, Z_0) = (q_2, Z_0)$

$\delta(q_2, 0, Z_0) = (q_2, 11Z_0)$

$\delta(q_2, 0, Z_0) = (q_1, Z_0)$

$\delta(q_0, 0, Z_0) = (q, Z_0)$

Q.) Convert CFG into NPDA:

1.) $A \to 0B$

$B \to 0B1 / 1B / \epsilon$

2.) $X \to X_1 / X_2$

$X_1 \to AX_1 / 0X_11 / \epsilon$

$A \to 0A / 0$

$X_2 \to X_2B / 0X_21 / \epsilon$

$B \to 1B / 1$

3) $A \to BC$

$B \to 0B1 / 01$

$C \to 2C3 / 23$

4.) $L = \{ab^n c d^n / n \geq 1\}$

5.) $S \to aB / bA$

$A \to a / aS / bAA$

$B \to b / bS / aBB$

6) $L = \{a^n b^m / n \geq 0 \ \& \ n \neq m\}$

Q.) (159) const. grammar for $L = \{ w \in \{0,1\}^* \mid n_0(w) > n_1(w) \}$

$L = \{ 0, 00, 001, 010, 100, \ldots \}$

$X \to 0$
$X \to 0X$
$X \to XX1$
$X \to X1X$
$X \to 1XX$

Theorem :- There exists a CFG 'G' for every NPDA M, wch implies $L(G) = L(M)$.

Proof :- $PDA, M = \{ Q, \Gamma, \Sigma, \delta, q_0, F, Z_0 \}$

→ Cons. CFG, $G = \{N, T, P, S\}$ to this NPDA by using following rules:

1.) $T = \Sigma$

2.) $N = S \cup [q Z q']$ where q & $q' \in Q$ & $Z \in \Gamma$

3.) Starting products for grammar are written as:

a.) $S \to [q_0 Z_0 q]$, where q_0 is starting state & Z_0 is starting stack alphabet used to rept. empty stack & $q \in Q$.

b.) If $\delta(q, a, Z) = (q', \epsilon)$ then for such transitions from product as $[q, Z, q'] \to a$

c.) If $\delta(q, a, Z) = (q', Z_1 Z_2 Z_3 \cdots Z_n)$ then form product as $[q, Z, q_1] \to a [q', Z_1, q_1][q_1, Z_2, q_2] \ldots [q_n, Z_n, q']$ where $q_1, q_2 \ldots q_n$ are ele. of Q & $Z_1, Z_2 \ldots Z_n$ are stack alphabets i.e, ele. of Γ.

Q.) (160) Find CFG that generates same lang. as generated by NPDA : $\{M = \{q_1, q_2\}, \{0, 1\}, \{X, Z_0\}, q_1, \delta, q_2\}$ with transitions given below:

$$\delta(q_1, \epsilon, Z_0) = (q_1, XZ_0)$$

$$\delta(q_1, 1, X) = (q_1, XX)$$

$$\delta(q_1, 0, X) = (q_2, \epsilon)$$

→ Grammar, G is defined by $\{N, T, P, S\}$, where

& $N = \{[q_1 Z_0 q_1], [q_1 Z_0, q_2], [q_2 Z_0 q_1], [q_2 Z_0 q_2], [q_1 X q_1], [q_1 X q_2], [q_2 X q_1], [q_2 X q_2]\}$

& $T = \{0, 1\}$

& starting productions are written as:

1.) $S \rightarrow [q_1 Z_0 q_1]$

2.) $S \rightarrow [q_0 Z_0 q_2]$

3.) $S \rightarrow [q_1 X q_1]$

4.) $S \rightarrow [q_1 X q_2]$

& Productions to $\delta(q_1, 0, Z_0) = (q_1, XZ_0)$ are written as:

5) $[q_1, Z_0, q_1] \rightarrow 0[q_1, X, q_1][q_1, Z_0, q_1]$

6) $[q_1, Z_0, q_1] \rightarrow 0[q_1, X, q_2][q_2, Z_0, q_1]$

7) $[q_1, Z_0, q_2] \rightarrow 0[q_1, X, q_1][q_1, Z_0, q_2]$

8) $[q_1, Z_0, q_2] \rightarrow 0[q_1, X, q_2][q_2, Z_0, q_2]$

* Productions to $\delta(q_1, 1, X) = (q_1, XX)$ are written as:

9) $[q_1, X, q_1] = 1[q_1, X, q_1][q_1, X, q_1)$

10) $[q_1, X, q_1] = 1[q_1, X, q_2][q_2, X, q_1]$

11) $[q_1, X, q_2] = 1[q_1, X, q_1)[q_1, X, q_2]$

12) $[q_1, X, q_2] = 1[q_1, X, q_2][q_2, X, q_2]$

* Productions to $\delta(q_1, 0, X) = (q_2, \epsilon)$ are written as:

13) $[q_1, X, q_2] \rightarrow 0$

NOTE:- Also Convert CFG into PDA by using following rules & w/o converting grammar into GNF form:

1) $\delta(q, \epsilon, X) = (q, w)$ for all productions of the form $X \rightarrow w$

2) $\delta(q, a, a) = (q, \epsilon)$ for all every terminal symbol of the grammar

---

Q) convert PDA $P = (\{p, q\}, \{0, 1\} \{x, Z_0\}, \delta, q_0, Z_0)$ to a CFG if δ is given by

$\delta(q, 1, Z_0) = \{(q, xZ_0)\}$

$\delta(q, 1, x) = (q, xx)$

$\delta(q, 0, x) = (p, x)$

$\delta(q, \epsilon, x) = (q, \epsilon)$

$\delta(p, 1, x) = (p, \epsilon)$

$\delta(p, 0, Z_0) = \{(q, Z_0)\}$

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Q) Convert PDA, $P = (\{q_0, q_1\}, \{0, 1\}, \{x, z_0\}, \delta, q, z_0)$ to a CFG if δ is given by:

$\delta(q_0, 0, z_0) = \{(q_0, xz_0)\}$

$\delta(q_1, \epsilon, x) = (q_1, \epsilon)$

$\delta(q_0, 1, x) = (q_1, \epsilon)$

$\delta(q_1, 1, x) = (q_1, \epsilon)$

$\delta(q_0, 0, x) = (q_0, xx)$

$\delta(q_1, \epsilon, z_0) = (q, \epsilon)$

Q) Cons. CFG wch accepts N(A) & Simplify same, where $A = (\{q_0, q_1\}, \{a, b\}, \{z_0, z\}, -\delta, q_0, z_0 - \}$ & δ is given by,

$-(q_0, b, z_0) = \{(q_0, zz_0)\}$

$-(q_0, \epsilon, z_0) = (q_0, \epsilon)$

$-(q_0, b, z) = (q_0, zz)$

$-(q_0, a, z) = (q_1, z)$

$-(q_1, b, z) = (q_1, \epsilon)$

$-(q_1, a, z_0) = (q_0, z_0)$

Q) Const. CFG wch accepts N(A) where A is $(\{q_0, q_1, q_2\}, \{a, b, c\}, \{a, b, z_0\}, \delta, q_0, z_0, q_2)$ & δ is given by:

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$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, b, z_0) = (q_0, bz_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, a, b) = (q_0, ab)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, \epsilon, z_0) = (q_1, z_0)$

$\delta(q_0, c, a) = (q_1, a)$

$\delta(q_0, c, b) = (q_1, b)$

$\delta(q_1, a, a) = (q_1, 2)$

$\delta(q_1, b b) = (q_1, 2)$

$\delta(q_1, 2, z_0) = (q_2, z_0)$