

1. Two dice are rolled at once. Find out the probability for sum of numbers being even and one of the die shows 6.
  - $[(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 3), (3, 1), (3, 5), (4, 4), (4, 2), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)] = 18$
  - The probability of sum of number being even is  $18/36$  i.e.  $1/2$
  - The probability of sum of numbers being even and one of the die shows 6 is  $5/36$
2. Two dice are rolled at once. Find out the probability for sum of numbers being less than 7
  - $[(1,1), (1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (3,3)]$
  - Therefore the probability for sum of numbers being less than 7 is  $9/36$  i.e.  $1/4$
3. You toss a fair coin three times: Given that you have observed *at least* one heads, what is the probability that you observe at least two heads?
  - Possible outcomes when tossed 3 times - hhh, hht, hth, htt, thh, tht, tth, ttt
  - assuming A1 is where 2 heads then (hht, hth, thh, hhh) then probability is  $4/8$
  - and we know that atleast one head then assuming A2 is (hhh, hht, hth, htt, thh, tht, tth) leading to  $7/8$
  - now applying  $P(A2|A1)$  i.e.  $4/8 \mid 7/8$
  - the answer is  $4/7$
4. In my town, it's rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability  $1/2$ , and given that it is not rainy, there will be heavy traffic with probability  $1/4$ . If it's rainy and there is heavy traffic, I arrive late for work with probability  $1/2$ . On the other hand, the probability of being late is  $1/8$  if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25, 0.25. You pick a random day. What is the probability that it's not raining and there is heavy traffic and I am not late?
  - (a) What is the probability that it's not raining and there is heavy traffic and I am not late?
  - (b) What is the probability that I am late?
  - (c) Given that I arrived late at work, what is the probability that it rained that day?
    - Given that we are provided with conditional probability, hence it is better to use Tree diagram here where R is Rainy Event, L is Late event and T is Traffic event.
    - $L = 1/3$  and  $2/3$
    - $R = 1/2$  and  $1/2$
    - $Rc = 1/4$  and  $3/4$
    - $RT = 1/2$  and  $1/2$
    - $RTc = 1/4$  and  $3/4$
    - $RcT = 1/4$  and  $3/4$
    - $RcTc = 1/8$  and  $7/8$ 
      - $A : 2/3 * 1/4 * 3/4 = 1/8$
      - $B : 1/12 + 1/24 + 1/24 + 1/16 = 11/48$
      - $C = 1/12 + 1/24 = 1/8$
5. A box contains three coins: two regular coins and one fake two-headed coin ( $P(\text{Heads})=1$ ), you pick a coin at random and toss it.
  - (a) What is the probability that it lands heads up?
  - (b) You pick a coin at random and toss it and get heads. What is the probability that it is the two-headed coin?
    - What we know -
      - $C1 = \text{regular coin}$
      - $C2 = \text{two-headed coin}$
      - $P(H|C1) = 0.5$
      - $P(H|C2) = 1$
      - $A = P(H) = P(H/C1)P(C1)+P(H/C2)P(C2)$  i.e.  $1/2 \cdot 2/3 + 1 \cdot 1/3 = 2/3$
      - **B = Need to work**

6. Suppose that, of all the customers at a coffee shop,
- (a) 70% purchase a cup of coffee
  - (b) 40% purchase a piece of cake
  - (c) 20% purchase both a cup of coffee and a piece of cake.
- Given that a randomly chosen customer has purchased a piece of cake, what is the probability that he/she has also purchased a cup of coffee?
- What we know -
    - $P(A) = 0.7$
    - $P(B) = 0.4$
    - $P(A \cap B) = 0.2$
  - Thus,  $P(A|B) = P(A \cap B) / P(B)$  i.e.  $0.2/0.4 = \frac{1}{2}$
7. A population has a mean of 50 and a standard deviation of 6.
- (a) What are the mean and standard deviation of the sampling distribution of the mean for  $N = 16$ ?
  - (b) What are the mean and standard deviation of the sampling distribution of the mean for  $N = 20$ ?
- A = Mean is 50 and SD is 1.5
  - B = Mean is 50 and SD is 1.34
8. Given a test that is normally distributed with a mean of 100 and a standard deviation of 12, find:
- (a) The probability that a single score drawn at random will be greater than 110
  - (b) The probability that a sample of 25 scores will have a mean greater than 105
  - (c) The probability that a sample of 64 scores will have a mean greater than 105
  - (d) The probability that the mean of a sample of 16 scores will be either less than 95 or greater than 105
- Need to work
9. In the population, the mean SAT score is 1000. Would you be more likely (or equally likely) to get a sample mean of 1200 if you randomly sampled 10 students or if you randomly sampled 30 students? Explain. Write a python code and try.
- Need to work
10. A population is known to be normally distributed with a standard deviation of 2.8.
- (a) Compute the 95% confidence interval on the mean based on the following sample of nine: 8, 9, 10, 13, 14, 16, 17, 20, 21.
  - (b) Now compute the 99% confidence interval using the same data
- Need to work
11. A is known to tell the truth in 5 cases out of 6 and he states that a white ball was drawn from a bag containing 8 blacks and 1 white ball. Find the probability that the white ball was drawn.
- Truth =  $5/6$
  - Ball =  $1/9$
12. A speaks the truth 4 out of 5 times. A die is tossed. A reports that it is a 6. What are the chances that there actually was a 6?
- Probability
    - i. of getting 6 =  $1/6$  [ $P(S)$ ]
    - ii. of not getting 6 =  $5/6$  [ $P(S\#)$ ]
  - Probability
    - i. Of not cheating =  $4/5$  [ $P(T)$ ]
    - ii. Of cheating =  $1/5$  [ $P(T\#)$ ]
  - $P1 = P(S) \cdot P(T)$  i.e.  $1/6 \cdot 4/5 = 2/15$
  - $P2 = P(S\#) \cdot P(T\#)$  i.e.  $5/6 \cdot 1/5 = 1/6$
  - Probability of actually 6 =  $P1 / (P1 + P2) = 4/9$

