Passive Motion Paradigm Equations

$$f(x) = k_{ext}(x_d - x) \tag{1}$$

$$\tau = J^T f(x) \tag{2}$$

$$\dot{q} = A_{int}\tau \tag{3}$$

$$\dot{x} = J\dot{q} \tag{4}$$

$$x(t) = \int J\dot{q}dt \tag{5}$$

Equations related to the floating base system
Generalized Cooridinates

$$q = \begin{bmatrix} x_b \\ y_b \\ \theta_b \\ q_1 \\ q_2 \end{bmatrix}$$

Dynamics,

$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$

$h(q,\dot{q}) = C(q,\dot{q})\dot{q} + G(q)$
Using the above equation
$M(q)\ddot{q} + h(q,\dot{q}) = \tau$
Control Laws,

$M(q)\ddot{q} + h(q,\dot{q}) - J_1^T F_1 - J_2^T F_2 = 0$

External Forces,

$$F_1 = \begin{bmatrix} {}^1F_x \\ {}^1F_y \\ {}^1\tau \end{bmatrix}$$

$$F_2 = \begin{bmatrix} {}^2F_x \\ {}^2F_y \\ {}^2\tau \end{bmatrix}$$

Rearranging the control law,

$$B = \begin{bmatrix} J_1^T & J_2^T \end{bmatrix}$$

$$u = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$M(q)\ddot{q} + h(q,\dot{q}) = Bu$

States Variables,

$$x_1 = q \tag{6}$$

$$x_2 = \dot{q} \tag{7}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} x_2 \\ M^{-1}[Bu - h(x_1, x_2)] \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} x_2 \\ -M^{-1}h(x_1, x_2) \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix} u$$

$$\dot{x} = f(x) + g(x)u$$

Output Variables

$$y = \begin{bmatrix} x_b \\ y_B \\ \theta_b \end{bmatrix}$$

$$y = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

Output Equation,

$$\dot{y} = S\dot{q} \tag{8}$$

$$\ddot{y} = S\ddot{q} \Rightarrow SM^{-1}[Bu - h] \tag{9}$$

$$\ddot{y} = -SM^{-1}h + SM^{-1}Bu \tag{10}$$

$$\ddot{y} = h(u) \tag{11}$$

Considering the joint constraints, Control Laws,

$M(q)\ddot{q} + h(q, \dot{q}) - J_1^T F_1 - J_2^T F_2 - J_c^T F_c = 0$

$M(q)\ddot{q} + h(q, \dot{q}) = Bu + J_c^T F_c$

Estimating constraint forces,

$J_c\ddot{q} = 0 \Rightarrow J_c M^{-1}[Bu + J_c^T F_c - h] = 0$

$F_c = (J_c M^{-1} J_c^T)^{-1} J_c M^{-1}[h - Bu]$