

2. Suppose we collect data for a group of bank customers with variables
- default A variable with levels No and Yes indicating whether the customer defaulted on their debt
 - student A variable with levels No and Yes indicating whether the customer is a student
 - balance The average balance that the customer has remaining on their credit card after making their monthly payment
 - income of a customer

We fit a logistic regression and produce estimated coefficient, $\hat{\beta}_0 = -15.05$, $\hat{\beta}_{\text{studentYes}} = -0.5149$, $\hat{\beta}_{\text{balance}} = 0.003738$, $\hat{\beta}_{\text{income}} = -0.00000791$.

(a) Estimate the probability that a student with a balance of \$3,000 and income \$70,000 does default on a loan.

$$P(x) = \frac{1}{e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3)} + 1}$$

$$\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = -15.05 - 0.5149 + 0.003738 \times 3000 - 0.00000791 \times 70,000$$

$$= -4.9046$$

$$P(x) = 0.00741$$

(b) Estimate the probability that borrower a balance of \$3,000 and income \$70,000 who is not a student does default on a loan.

$$P(x) = \frac{1}{e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3)} + 1}$$

$$\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 = -15.05 + 0.003738 \times 3000 - 0.00000791 \times 70,000$$

$$= -4.3897$$

$$P(x) = 0.0124$$

(c) How much income would the student in part (a) need to make to have a 90% chance of getting approved for a loan?

$$a = -15.05 - 0.5149 + 0.003738 * 3000 - 0.00000791 * x$$

$$0.9 = \frac{1}{e^{-a} + 1}$$

$$e^{-a} = \frac{1}{9}$$

$$a = 0.9542$$

$$x = \frac{-5.3051}{0.00000791}$$

$$x = -670682.680$$

(d) How much income would the borrower in part (b) need to make to have a 90% chance of getting approved for a loan?

$$a = -15.05 + 0.003738 * 3000 - 0.00000791 * x$$

$$e^{-a} = \frac{1}{9}$$

$$a = 0.9542$$

$$x = \frac{-4.7902}{0.00000791}$$

$$x = 605587.863$$