Derivatives

For any function y = f(x), the derivative can be written as

$f(x)'$ or $\frac{d}{dx}f(x)$ or $\frac{dy}{dx}$

$(x^n)' = nx^{n-1}$

$(\ln x)' = \frac{1}{x}$

$(\log_k x)' = \frac{1}{x\ln k}$

$(e^x)' = e^x$

$(k^x)' = k^x \ln_k$

Derivatives with Chain Rule

Say $y = (x^2 + 3x)^3$

Then derivative is $\frac{dy}{dx}$

$[(x^2 + 3x)^3]' = 3(x^2 + 3x)^2(2x + 3)$

Think of $x^2 + 3x = u$

Then derivative is $(u^3)' = 3u^2$ (which is really $3(x^2 + 3x)^2$

So we really computed $\frac{dy}{du}$

To get $\frac{dy}{dx}$ need to complete $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Since $u = x^2 + 3x$, $\frac{du}{dx} = 2x + 3$

Derivatives with Chain Rule

Another example:

$y = x^2 \ln^2 x$

Then $\frac{dy}{dx} = x^2(\ln^2 x)' + \ln^2 x(x^2)'$

$X^2(2\ln x * \frac{1}{x}) + 2x\ln^2 x = 2x\ln x + 2x\ln^2 x$

Product Derivatives

$(f(x) * g(x))' = f(x)g'(x) + g(x)f'(x)$

Example:

$(x\ln x)' = x(\ln x)' + \ln x(x)' = 1 + \ln x$

$(x^3\ln^2 x)' = x^3(\ln^2 x)' + \ln^2 x(x^3)'$

$$= x^3\left(\frac{2\ln x}{x}\right) + 3x^2 \ln^2 x = 2x^2\ln x + 3x^2\ln^2 x$$

Integrals

$$\int_a^b x^k dx = \frac{x^{k+1}}{k+1}\Big]_a^b = \left(\frac{b^{k+1}}{k+1} - \frac{a^{k+1}}{k+1}\right)$$

$$\int_a^b \frac{1}{x} dx = \ln|x|\,\Big]_a^b = \ln b - \ln a$$

$$\int_a^b lnx dx = [xlnx - x]_a^b = blnb - b - alna + a$$

$$\int e^x dx = e^x + C$$

$$\int 2^x dx = \frac{2^x}{ln2} + C$$