VISPAD INSTITUTE OF
TECHNOLOGY
Nea onnim no sua a ohu,

Matrix Decomposition

Fall 2025

(October – December Virtual Internship)

At the end of the session the students or candidates should be able to understand and work with:

- Introduction to Matrix Decompositions
- Singular Value Decomposition (SVD)
- Relationship between SVD and Eigen-decomposition.

We have come far. Do not give up!!!

- Matrices represent data and transformations, but their structure can sometimes be complex.
- A **matrix decomposition** is a way of reducing a matrix into its constituent parts.
- It is an approach that can simplify more complex matrix operations that can be performed on the decomposed matrix rather than on the original

matrix

- A common analogy for matrix decomposition is the factoring of numbers, such as the factoring of 10 into 2×5 in to its prime components.
- For this reason, matrix decomposition is also called *matrix factorization*.

- Matrix decomposition enables potentially complex matrix to be expressed as a product of two or more “simpler” matrices.
- These constituent matrices typically possess specific structures, such as being diagonal, triangular, or orthogonal, which make them easier to analyze and work with computationally.

- There are many ways to decompose a matrix, hence there are a range of different matrix decomposition techniques.
- Two simple and widely used matrix decomposition methods includes:
  - LU matrix decomposition
  - QR matrix decomposition

- **LU decomposition** provides efficient factorization specifically tailored for square matrices and solving linear systems.
- It’s a workhorse algorithm often employed behind the scene of numerical software.
- LU decomposition factors a **square matrix** A into the product of two matrices:

- ❖ A lower triangular matrix L.
- ❖ An upper triangular matrix U.

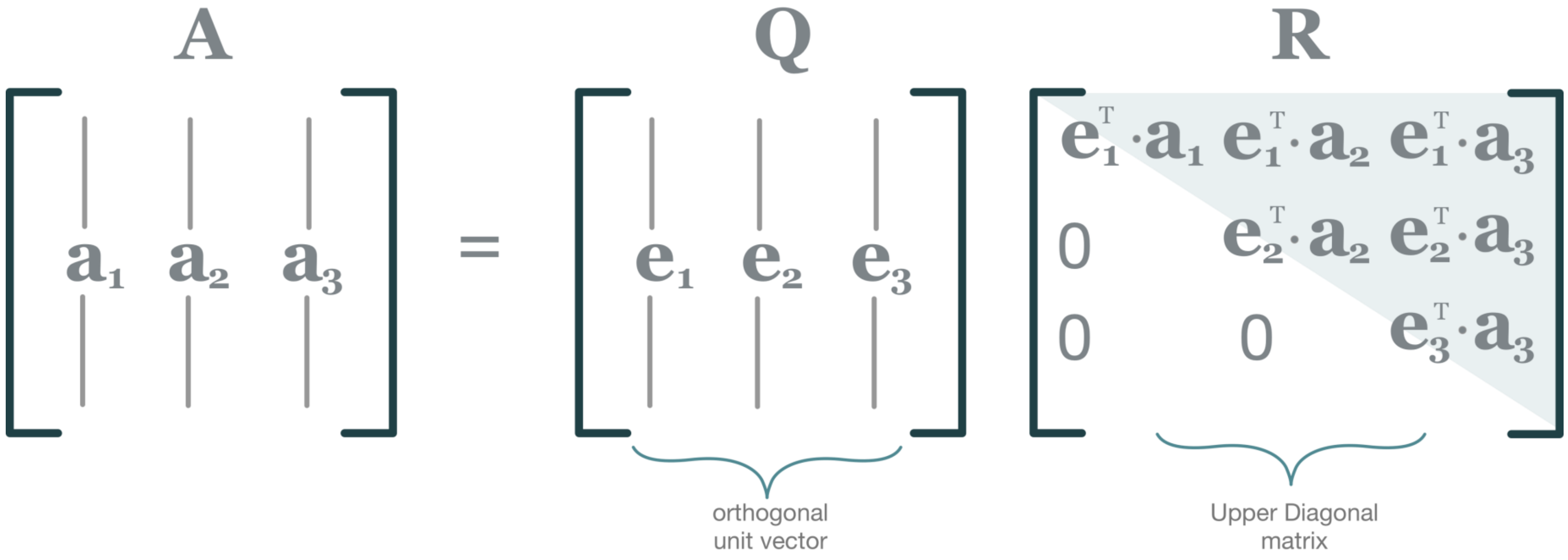
Lower Triangular

$$\begin{bmatrix} A00 & A01 & A02 \\ A10 & A11 & A12 \\ A20 & A21 & A22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L10 & 1 & 0 \\ L20 & L21 & 1 \end{bmatrix} \begin{bmatrix} U00 & U01 & U02 \\ 0 & U11 & U12 \\ 0 & 0 & U22 \end{bmatrix}$$

Upper Triangular

- **QR** decomposition is a highly method for matrix factorization, comparable for SVD and LU decomposition.
- For an $m \times n$ matrix A (often $m \geq n$ and linearly independent columns), QR decomposition finds two specific matrices, Q and R, such that: $A = QR$

- Q is an $m \times n$ matrix with orthonormal columns, and R is an $n \times n$ upper triangular matrix.
- QR decomposition is used in solving least-squares problems.



Why Matrix Decomposition?

- ❑ **Revealing Properties**
  - ❖ Decompositions can expose underlying characteristics of the matrix and the transformation it represents. They can help determine the rank of matrix, identify principal directions in data in PCA, or check if a matrix is in

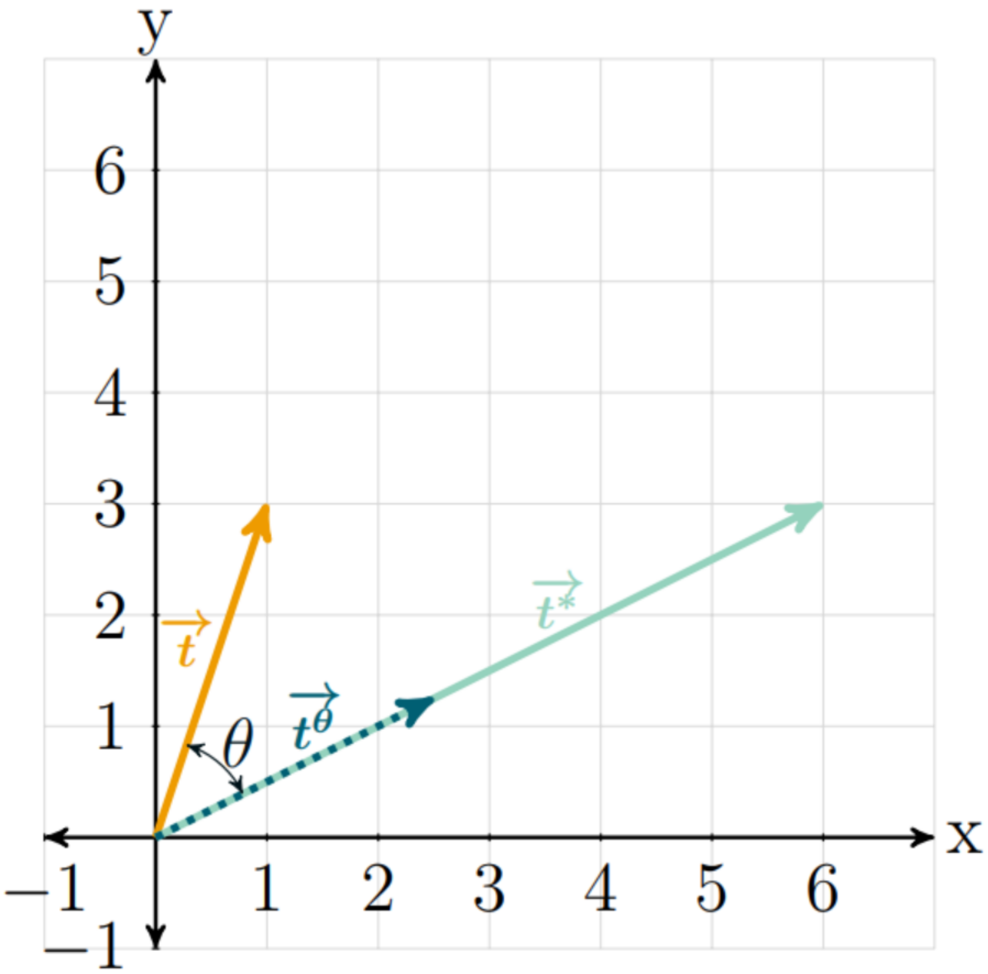
invertible.

- **Simplifying Computations:**
  - Many standard operations become more efficient when performed on the decomposed factors. Solving $Ax = b$ is often faster using LU decomposition.

❑ Data Approximation and Compression:

- ❖ Certain decompositions, like SVD, allow us to construct approximations of the original matrix using less information. This is the basis for techniques like dimensionality reduction and lossy data compression.

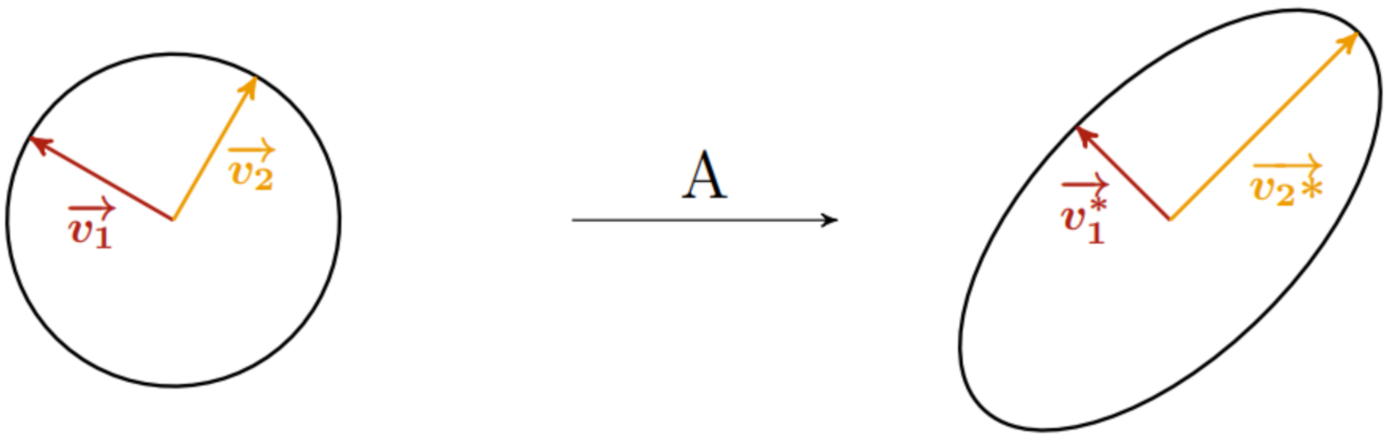
- In **eigendecomposition**, we transformed a matrix into a product of three different matrices, namely two *rotations* and a *scaling* term.



- Let's rotate, stretch and rotate a vector again.
- If A is a linear transformation, we can write A as product of rotations and stretches.

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \qquad \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

- A linear transformation will take any vector from the vector space, rotate it by the name angle, and then stretch it by some amount.



- If we stretch and rotate a circle, we end up with an ellipse, with $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ now constituting the minor and major axes of this newly mapped elliptical shape.

- How do we preserve the angle between the vectors and also their norms after the linear transformation?
- An orthonormal matrix can be used to achieved this.
- An orthonormal matrix is a matrix whose columns are orthonormal vectors, meaning that the angle between them is $90°$ and each of the vectors have a norm or length equal to one.

- Consider $Q = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
- For the matrix to be orthonormal, each column vector has to be of length one and they need to be at $90°$ to one another, which is the same as having a dot product equal to zero.

$$\overrightarrow{q_1} = (0, -1) \text{ and } \overrightarrow{q_2} = (1, 0)$$

$$\|\overrightarrow{q_1}\| = \sqrt{0^2 + (-1)^2} = 1$$

$$\|\overrightarrow{q_2}\| = \sqrt{1^2 + (0)^2} = 1$$

The dot product it:

$$\overrightarrow{q_1} \cdot \overrightarrow{q_2} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0 \cdot 1 + (-1) \cdot 0 = 0$$

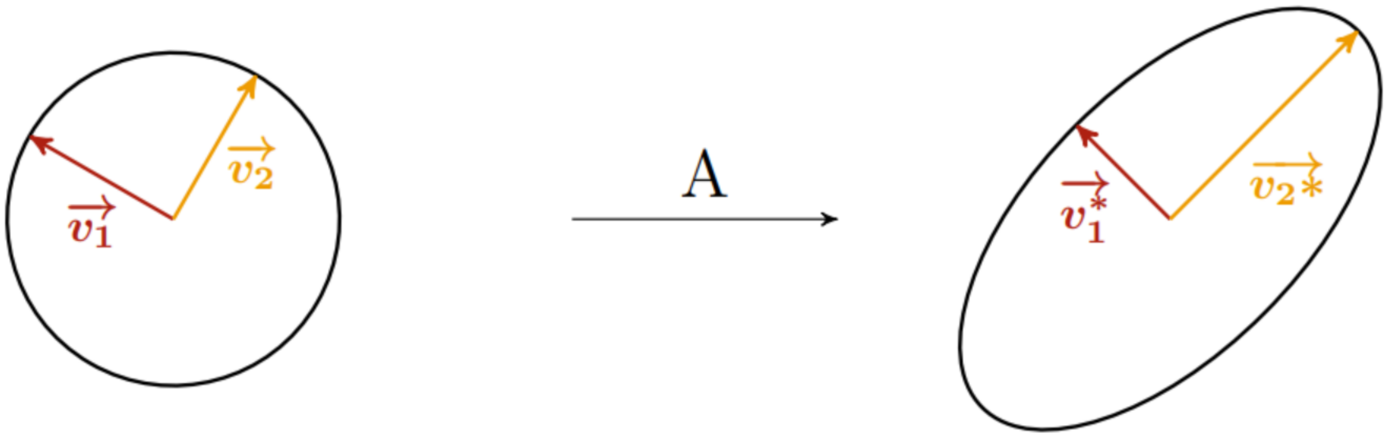
- Multiplying Q^T and Q is something we can leverage on.

$$Q = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, Q^T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$Q^T Q = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$Q = \begin{pmatrix} | & | & & | \\ \overrightarrow{q_1} & \overrightarrow{q_2} & \dots & \overrightarrow{q_n} \\ | & | & & | \end{pmatrix} \quad Q^T = \begin{pmatrix} — & \overrightarrow{q_1}^T & — \\ — & \overrightarrow{q_2}^T & — \\ & \vdots & \\ — & \overrightarrow{q_n}^T & — \end{pmatrix}$$

$$Q = \begin{pmatrix} | & | & & | \\ \overrightarrow{q_1} & \overrightarrow{q_2} & \dots & \overrightarrow{q_n} \\ | & | & & | \end{pmatrix} \cdot \begin{pmatrix} — & \overrightarrow{q_1}^T & — \\ — & \overrightarrow{q_2}^T & — \\ & \vdots & \\ — & \overrightarrow{q_n}^T & — \end{pmatrix} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$



- Let $\overrightarrow{v_1^*}$ and $\overrightarrow{v_2^*}$ be the how much they are scaled by A.
- The vectors $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ are scaled and rotated by A.
- Orthonormal vectors are used to keep the angle from changing.

- How much the vectors are scaled remain unknown.

$$\overrightarrow{v_1^*} = \alpha_1 \overrightarrow{u_1} \text{ and } \overrightarrow{v_2^*} = \alpha_2 u_2$$

- $\overrightarrow{u_1}$ and $\overrightarrow{u_2}$ tells us the direction of the new set of axes formed by $\overrightarrow{v_1^*}$ and $\overrightarrow{v_2^*}$
- The change in length can be quantified by applying A.

$$\overrightarrow{v^*} = \underbrace{\sigma}_{\text{single values}} \cdot \underbrace{\overrightarrow{u}}_{\text{principal axes}}$$

- A transformation of $\overrightarrow{v_1}$ by the matrix A can then be represented as: $A \cdot \overrightarrow{v_1} = \alpha_1 \cdot \overrightarrow{u_1}$
- In simple terms, a transformation of $\overrightarrow{v_1}$ enabled by A is equal to a scaled rotation of an orthonormal vector $\overrightarrow{u_1}$

- For all of the vectors, we can have an equation like: $A \cdot \overrightarrow{v_i} = \alpha_i \cdot u_i$ with $i = 1, 2, \dots, r$ with $\alpha_1 \geq \alpha_2 \geq \cdots \geq a_r > 0$

$$\begin{pmatrix} \\ \\ & A & \\ \\ \\ \end{pmatrix} \cdot \begin{pmatrix} | & | & & | \\ \overrightarrow{v_1} & \overrightarrow{v_2} & \dots & \overrightarrow{v_r} \\ | & | & & | \end{pmatrix} = \begin{pmatrix} | & | & & | \\ \overrightarrow{u_1} & \overrightarrow{u_2} & \dots & \overrightarrow{u_r} \\ | & | & & | \end{pmatrix} \cdot \begin{pmatrix} \sigma_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_r \end{pmatrix}$$

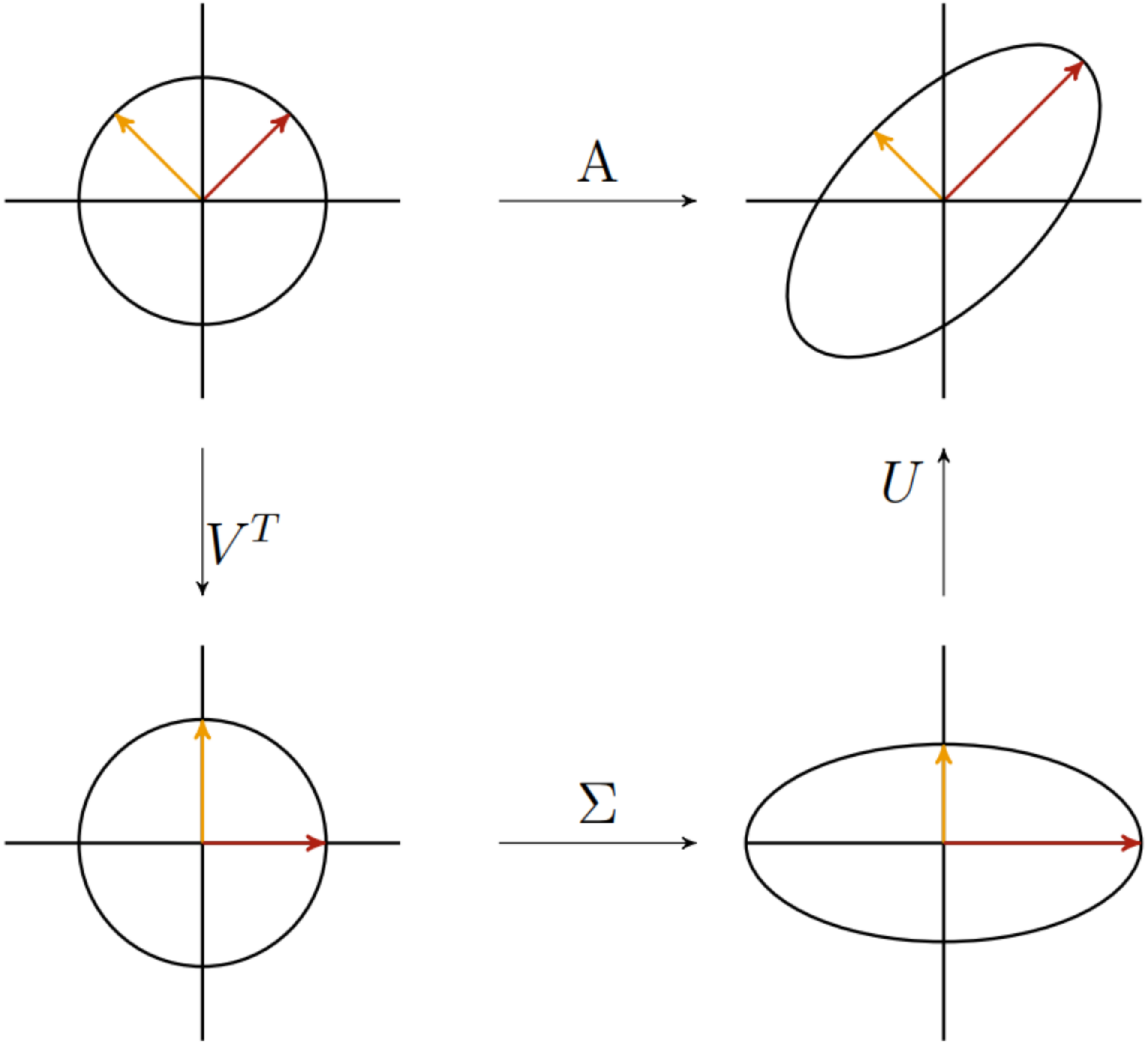
- The following notation can be used to represent the equation: $A \cdot V = U \cdot \Sigma$.
- We need A to be isolated, which can be achieved by multiplying each of the sides of the equation by the inverse of V.

$$A \cdot V \cdot V^{-1} = U \cdot \Sigma \cdot V^{-1}$$

- Because V is orthonormal, the matrix $V^{-1} = V^T$:

$$A_{n\times m} = U_{m\times r} \cdot \Sigma_{r\times r} \cdot V_{r\times n}^T$$

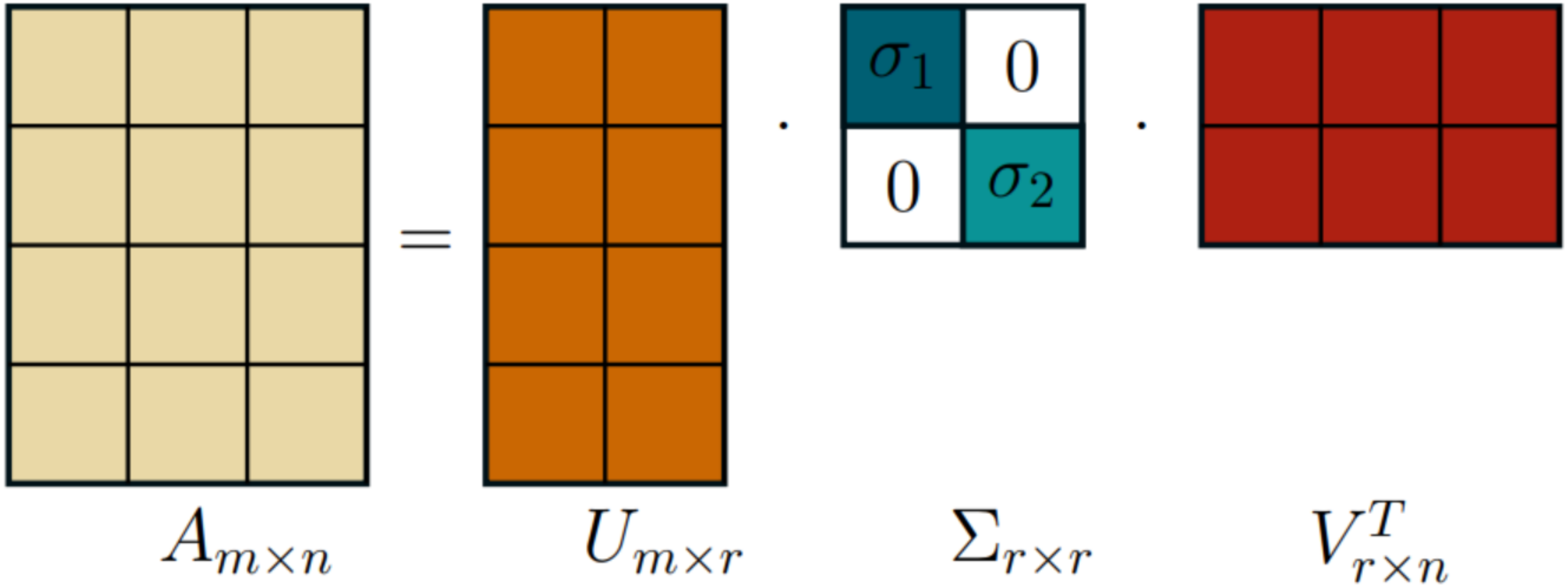
- Where:
  - U represent a rotation
  - Σ is the scaling matrix
  - V^T is another rotation



A
V^T
U
Σ

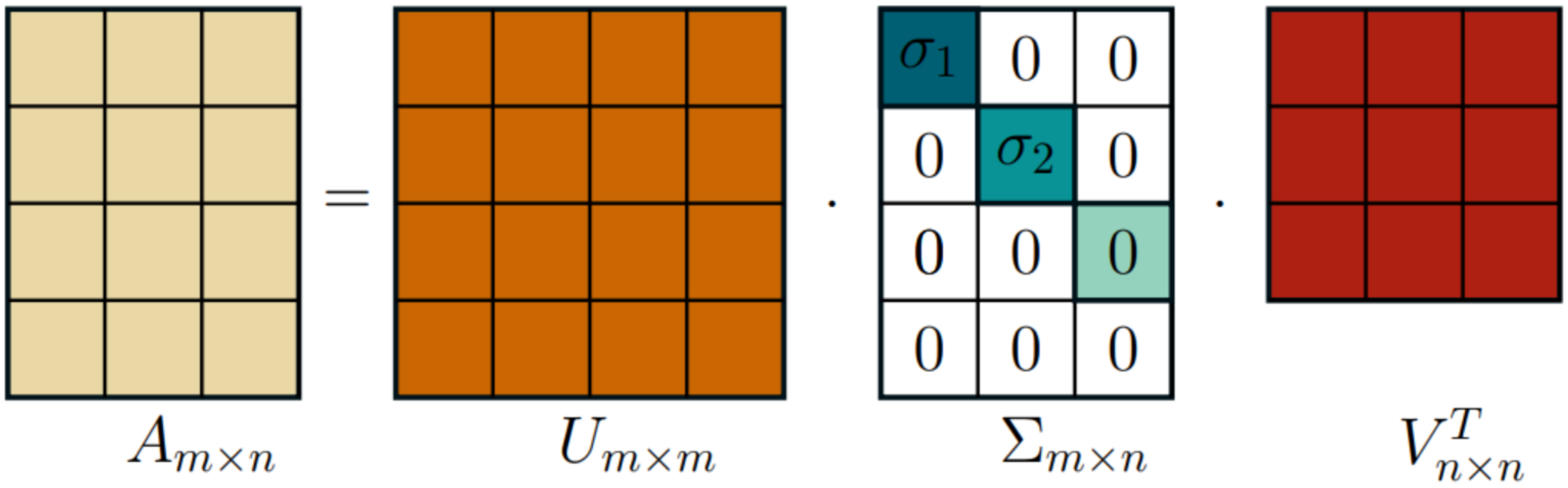
- The vector's basis, or the axes, are rotated into new set of orthonormal vectors via V^T.
- The resulting set of vectors is then scaled by the matrix Σ before rotated by U to produce the desired version of the transformed vectors.

- $A = U_r \Sigma_r V_r^T$ is the reduced form of the decomposition.
- Where r is the rank of matrix A



❑The equation of the full single value decomposition is:

$$A_{m\times n} = U_{m\times m}\Sigma_{m\times n}V_{n\times n}^{T}$$



- There is no information loss between the complete SVD and the reduced version of the same algorithm.
- The reduced version is cheaper to compute.

- The SVD is probably the most potent decomposition from linear algebra that you can apply in machine learning.
- The algorithm has many applications, including **recommendation systems, dimensionality reduction, creating latent variables, and computer vision.**

- Given a matrix A, how do we compute V, U and Σ.
- Single value decomposition is defined in such a way that the rotation matrices must be orthonormal.
- This is a constraint that will mean that the matrices from which we want to obtain a decomposition have to have some kind of properties that ensure orthogonality after some transformation.

- If a matrix is ***symmetrical***, not only are the eigenvalues real but also the eigenvectors are orthogonal.
- A symmetric matrix is such that $A = A^T$

$$A = \begin{pmatrix} 2 & 3 & 6 \\ 3 & 4 & 5 \\ 6 & 5 & 9 \end{pmatrix}$$

- In the SVD equation, A does not have to be a square matrix, let alone symmetrical.
- We have to find a way to transform A into a square symmetrical matrix.
- If A is of size $m \times n$, then only way to have a square is if we multiply A by a matrix of size $n \times m$

- Multiplying matrix A by its transpose A^T will result in a square matrix.
- How can we verify the resultant matrix will be symmetrical?

$$A = \begin{pmatrix} 2 & 1 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 5 & -1 & 4 & 3 \end{pmatrix}_{3\times 4} \quad \text{and} \quad A^T = \begin{pmatrix} 2 & 3 & 5 \\ 1 & 5 & -1 \\ 7 & 2 & 4 \\ 4 & 1 & 3 \end{pmatrix}_{4\times 3}$$

- What will be dimension of the $B = A \cdot A^T$

- Given $A = U \cdot \Sigma \cdot V^T$
- We multiply both side by A^T at the left of A.

$$A^T \cdot A = (U \cdot \Sigma \cdot V^T)^T \cdot U \cdot \Sigma \cdot V^T$$

- From $(A \cdot B)^T = B^T \cdot A^T$. Simplifying will give you

$$A^T \cdot A = V \cdot \Sigma^T \cdot U^T \cdot U \cdot \Sigma \cdot V^T$$

- Σ is a scaling matrix and therefore a diagonal matrix. The transpose is equal to the original matrix, i.e $\Sigma = \Sigma^T$.
- The matrix U is orthonormal. The transpose of is equal to the inverse: $U^{-1} = U^T$.

- By simplifying gives: $A^T \cdot A = V \cdot \Sigma^T \cdot U^T \cdot U \cdot \Sigma \cdot V^T$
- The multiplication of U^T by U will result in the identify matrix such that:

$$A^T \cdot A = V \cdot \Sigma \cdot \Sigma \cdot V^T$$

$$A^T \cdot A = V \cdot \Sigma^2 \cdot V^T$$

- How do we find the value of V?

- Given $A \cdot \vec{v} = \lambda \cdot \vec{v}$ we can rewrite the SVD equation in the form of eigenvalue and eigenvector equation.
- We know that $V^T = V^{-1}$, we will multiply both sides by V

$$A^T \cdot A \cdot V = V \cdot \Sigma^2 \cdot V^{-1} \cdot V$$

$$\boldsymbol{A}^{\boldsymbol{T}} \cdot \boldsymbol{A} \cdot V = \boldsymbol{V} \cdot \Sigma^2$$

$$A = A^T \cdot A, \qquad V = \vec{v}$$

- We have found a way to compute V and Σ, but we still need U.
- We multiply both sides by A^T but at the right of A.

$$A \cdot A^T = U \cdot \Sigma \cdot V^T \, (U \cdot \Sigma \cdot V^T)^T$$

$$A \cdot A^T = U \cdot \Sigma \cdot V^{-1} \cdot V \cdot \Sigma \cdot U^T$$

$$A \cdot A^T = U \cdot \Sigma^2 \cdot U^T$$

Multiplying both sides by U, it follows that:

$$A \cdot A^T \cdot U = U \cdot \Sigma^2$$

- We have ended up with similar eigenvalue problem, and Σ happens to be the same for the two equations.

$$\boldsymbol{A}^T \cdot \boldsymbol{A} \cdot V = \boldsymbol{V} \cdot \Sigma^2$$

$$\boldsymbol{A} \cdot \boldsymbol{A}^T \cdot U = \boldsymbol{U} \cdot \Sigma^2$$

- ❑ Solving these eigenvalue equations will give us U and V
- ❑ $A \cdot A^T$ is a square symmetric matrix, its eigenvalues will all be distinct and real; when squared rooted, the will give us the single values.

- The eigenvectors that form U and V will be orthogonal, allowing any vector transformed by these matrices to preserve the angles between them.
- The matrix Σ which contains the single values will be sorted such that, $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n$

- Consider the matrix A define as: $A = \begin{pmatrix} 1 & -2 & 0 \\ 0 & -2 & 1 \end{pmatrix}$

  and $A^T = \begin{pmatrix} 1 & 0 \\ -2 & -2 \\ 0 & 1 \end{pmatrix}$

- Compute $A \cdot A^T$ and $A^T \cdot A$:

$$A \cdot A^T = \begin{pmatrix} 1 & -2 & 0 \\ 0 & -2 & 1 \end{pmatrix} \quad \cdot \quad \begin{pmatrix} 1 & 0 \\ -2 & -2 \\ 0 & 1 \end{pmatrix} \quad = \quad \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$$

$$A^T \cdot A = \begin{pmatrix} 1 & 0 \\ -2 & -2 \\ 0 & 1 \end{pmatrix} \quad \cdot \quad \begin{pmatrix} 1 & -2 & 0 \\ 0 & -2 & 1 \end{pmatrix} \quad = \quad \begin{pmatrix} 1 & -2 & 0 \\ -2 & 8 & -2 \\ 0 & -2 & 1 \end{pmatrix}$$

- To compute the matrix U, we need to solve the eigen problem: $\boldsymbol{A} \cdot \boldsymbol{A}^T \cdot U = \boldsymbol{U} \cdot \Sigma^2$

$$\det(AA^T - \lambda \cdot I) = \begin{vmatrix} 5-\lambda & 4 \\ 4 & 5-\lambda \end{vmatrix} = \lambda^2 - 10\lambda + 9$$

$$= (\lambda - 9)(\lambda - 1)$$

$\lambda_1 = 9$ and $\lambda_2 = 1$

❑ For the value of $\lambda_1 = 9$ it follows that:

$$\begin{pmatrix} -4 & 4 \\ 4 & -4 \end{pmatrix} \cdot \begin{pmatrix} u_{11} \\ u_{12} \end{pmatrix} = 0$$

$$\begin{cases} -4u_{11} = -4u_{12} \\ 4u_{11} = 4u_{12} \end{cases} \Leftrightarrow \{u_{11} = u_{12}$$

$$u_{11} = 1 \text{ and } u_{12} = 1$$

$$\overrightarrow{u_1} = (1, 1)^T$$

- U is an orthonormal basis, its vector must have length 1.
- We need to normalize them.

$$\|u_1\| = \sqrt{1+1} = \sqrt{2}$$

$$\overrightarrow{u_1} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

❑ For the value of $\lambda = 1$:

$$\begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix} \cdot \begin{pmatrix} u_{11} \\ u_{12} \end{pmatrix} = 0$$

$$\{4u_{21} = -4u_{22} \Leftrightarrow \{u_{21} = -u_{22}\}$$

$$u_{21} = -1 \text{ and } u_{22} = 1$$

❑ The norm of $\overrightarrow{u_2}$ is $\sqrt{2}$, therefore we have to normalize

$$\overrightarrow{u_2} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

❑Let's compute the eigen values of $A^T \cdot A$:

$$\det(A^TA - \lambda I) = \begin{vmatrix} 1-\lambda & -2 & 0 \\ -2 & 8-\lambda & -2 \\ 0 & -2 & 1-\lambda \end{vmatrix}$$

$$= (1-\lambda)\left[(8-\lambda)(1-\lambda) - 4\right] - 4(1-\lambda))$$

$$= (1-\lambda)^2(8-\lambda) - 8(1-\lambda)$$

$$= (1-\lambda)(8 - 9\lambda + \lambda^2 - 8)$$

$$= \lambda(1-\lambda)(\lambda - 9)$$

$\lambda_1 = 9, \lambda_2 = 1$ and $\lambda_3 = 0$

For $\lambda_1 = 9$:

$$\begin{pmatrix} -8 & -2 & 0 \\ -2 & -1 & -2 \\ 0 & -2 & -8 \end{pmatrix} \cdot \begin{pmatrix} v_{11} \\ v_{12} \\ v_{13} \end{pmatrix} = 0$$

$$\begin{cases} -8v_{11} - 2v_{12} = 0 \\ -2v_{11} - v_{12} - 2v_{13} = 0 \\ -2v_{12} - 8v_{13} = 0 \end{cases} \Leftrightarrow \begin{cases} -8v_{11} = 2v_{12} \\ -2v_{11} - v_{12} - 2v_{13} = 0 \\ -2v_{12} = 8v_{13} \end{cases}$$

$$\begin{cases} -8v_{11} = -8v_{13} \\ -2v_{11} - v_{12} - 2v_{13} = 0 \end{cases} \Leftrightarrow \begin{cases} v_{11} = v_{13} \\ -2v_{11} - v_{12} - 2v_{11} = 0 \end{cases}$$

$$\begin{cases} v_{11} = v_{13} \\ -v_{12} - 4v_{11} = 0 \end{cases} \Leftrightarrow \begin{cases} v_{11} = v_{13} \\ v_{12} = 4v_{11} \end{cases}$$

- For $\overrightarrow{v_1}$ we have the vector $(1, -4, 1)^T$, and its norm is given by:

$$\|v_1\| = \sqrt{1^2 + (-4)^2 + 1^2} = \sqrt{18}$$

- The normalized version of is

$$\overrightarrow{v_1} = \begin{pmatrix} \frac{1}{\sqrt{18}} \\ \frac{-4}{\sqrt{18}} \\ \frac{1}{\sqrt{18}} \end{pmatrix}$$

❑ For $\lambda_2 = 1$, we have:

$$\begin{pmatrix} 0 & -2 & 0 \\ -2 & 7 & -2 \\ 0 & -2 & 0 \end{pmatrix} \cdot \begin{pmatrix} v_{21} \\ v_{22} \\ v_{23} \end{pmatrix} = 0$$

$$\begin{cases} -2.v_{22} = 0 \\ -2v_{21} + 7v_{22} - 2v_{23} = 0 \\ -2v_{22} = 0 \end{cases} \Leftrightarrow \begin{cases} v_{22} = 0 \\ -2v_{21} - 2v_{23} = 0 \end{cases}$$

$$\begin{cases} v_{22} = 0 \\ v_{21} = -v_{23} \end{cases}$$

- For $\overrightarrow{v_2}$ we have the vector $\overrightarrow{v_2} = (-1, 0, 1)^T$ and the norm of $\overrightarrow{v_2}$:

$$\|v_2\| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

- The normalized version of $\overrightarrow{v_2} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

- For $\lambda_3 = 0$, we have:

$$\begin{pmatrix} 1 & -2 & 0 \\ -2 & 8 & -2 \\ 0 & -2 & 1 \end{pmatrix} \cdot \begin{pmatrix} v_{31} \\ v_{32} \\ v_{33} \end{pmatrix} = 0$$

$$\begin{cases} v_{31} - 2v_{32} = 0 \\ -2v_{31} + 8v_{32} - 2v_{33} = 0 \\ -2v_{32} + v_{33} = 0 \end{cases} \Leftrightarrow \begin{cases} v_{31} = 2v_{32} \\ 2v_{32} = 2v_{33} \end{cases}$$

$$\begin{cases} v_{31} = 2v_{32} \\ v_{31} = v_{33} \end{cases}$$

- For $\overrightarrow{v_3}$ we have the vector $(2, 1, 2)^T$, and its norm is equal to:

- The normalized version of $\overrightarrow{v_3} = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$

- Finally, we calculate Σ, which is $\alpha_i = \sqrt{\lambda_i}$

$$\Sigma = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

- Finally, A can be represented as:

$$U\Sigma V^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}_{2\times 2} \cdot \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}_{2\times 3} \cdot \begin{pmatrix} \frac{1}{\sqrt{18}} & -\frac{4}{\sqrt{18}} & \frac{1}{\sqrt{18}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}_{3\times 3}$$

❑The value of A is:

$$= \begin{pmatrix} \frac{3}{\sqrt{2}} & -\frac{1}{2} & 0 \\ \frac{3}{\sqrt{2}} & \frac{1}{2} & 0 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{18}} & -\frac{4}{\sqrt{18}} & \frac{1}{\sqrt{18}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2 & 0 \\ 0 & -2 & 1 \end{pmatrix} = A$$


K YOU
PENGU