# Accurate SoC Estimation of Lithium-Ion Batteries Based on Parameter-Dependent State-Space Model

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***Abstract*—This paper is concerned with the State-of-Charge (SoC) estimation of lithium-ion batteries based on an equivalent circuit model and the extended Kalman filtering technique. The physical parameters in the equivalent circuit are dependent on both the temperature and the SoC of the battery, though the previous method assume the parameters to be constant. We propose a new method for improving the estimation accuracy based on a parameter-dependent state-space model. To be more specific,we derive a parameter-dependent state-space model by viewing these physical parameters as time-varying parameters,and then apply the extended Kalman filter to estimate the SoC. The effectiveness of the proposed method is verified by experimental results.**

## I. Introduction

Rechargeable batteries play crucial roles as useful and efficient storage devices in the recent energy management issues. Among such batteries, lithium-ion batteries have been increasingly used in various electronic devices due to their high energy density. On the other hand, it is well known that, some critical situations such as over-charge or over-discharge accelerate the aging of the battery capacity.

For the safe and long lasting operation of a lithium-ion battery, it is required to accurately estimate the State-of-Charge (SoC) of the battery from its terminal voltage and current measurements. Recently, the extended Kalman filtering (EKF) approaches have been reported as effective SoC estimation methods based on an equivalent circuit model (e.g. [1],[2]). In these model-based estimation methods, it is assumed that the physical parameters in the equivalent circuit are constant. However, in reality, these parameters are dependent on both the temperature and the SoC of the battery as shown in the experimental data in Sub-section IV-A. The temperature dependency of the equivalent circuit parameters was also pointed out by Ishizaki *et al*. [3]. To avoid this difficulty, Baba and Adachi [4] proposed an indirect SoC estimation method to firstly estimate the open circuit voltage (OCV), and then to compute the SoC based on the SoC-OCV curve.

In this paper, we present a new direct SoC estimation method based on a *parameter-dependent state-space model*. To be more specific,we derive a parameter-dependent state-space model by viewing the equivalent circuit parameters as time-varying parameters,and then apply the extended Kalman filter. It is the major advantage of the proposed method that the estimation accuracy is improved by taking account of the time variations of the temperature and the SoC. We will also verify the effectiveness of the proposed method through some experiments.

It may also be noted that this kind of approach has been known as *the gain scheduling approach* in the control community, and was applied to $H_\infty$ filtering problem [5]. Hu and Yourkovich [6] also considered a gain scheduling observer design for SoC estimation taking hysteretic nonlinearity as a scheduling parameter. The advantage of our estimation method compared with the previously proposed gain scheduling observers is that a near optimal estimation accuracy is guaranteed by the EKF technique with reasonable computational effort.

## II. Battery model

In this section, we will derive a paramter-dependent state-space model for the SoC estimation via an equivalent circuit of the lithium-ion battery.

### A. Equivalent circuit model

An example of equivalent circuit models of a lithium-ion battery is illustraited in Fig. 1. This model is called Thevenin model. We start with this circuit model under the assumption that the physical paramters are constant.

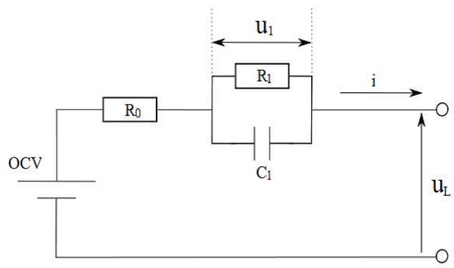


Fig. 1. Equivalent circuit model of a lithium-ion battery

In Fig. 1, $R_0$ is the charge transfer resistance, and $(R_1, C_1)$ represents the interfacial transfer between the electrolyte and the electrodes. Moreover, the open circuit voltage (OCV) is the internal voltage source which is a function of SoC and cannot be directly measured when a load is connected to the battery. The voltage $u_1$ is the potential difference across the $(R_1, C_1)$ circuit. The terminal voltage and current are defined by $u_L$ and $i$. Then, the dynamics of this equivalent circuit is described by

$$C_1 \frac{du_1}{dt} + \frac{u_1}{R_1} = i, \tag{1}$$

$$u_L = \mathrm{OCV}(\mathrm{SoC}) + u_1 + R_0 i. \tag{2}$$

Based on the idea of current integration method, the State-of-Charge (SoC) is defined by

$$\mathrm{SoC}(t) = \frac{1}{\mathrm{FCC}} \int_0^t i(\tau) d\tau + \mathrm{SoC}(0),$$

or equivalently

$$\frac{d}{dt}\mathrm{SoC} = \frac{i}{\mathrm{FCC}}, \tag{3}$$

where FCC is a constant called the full charge capacity.

*B. Discrete-time state space model*

By the forward Euler approximation, the equations (1) and (3) are descretized with the sampling period $\Delta t$ as

$$u_{1k+1} = \left(1 - \frac{\Delta t}{R_1 C_1}\right) u_{1k} + \frac{\Delta t}{C_1} i_k, \tag{4}$$

$$\mathrm{SoC}_{k+1} = \mathrm{SoC}_k + \frac{\Delta t}{\mathrm{FCC}} i_k, \tag{5}$$

where $u_{1k} := u_1(k\Delta t)$, $i_k := i(k\Delta t)$, $\mathrm{SoC}_k := \mathrm{SoC}(k\Delta t)$, $u_{Lk} := u_L(k\Delta t)$, and the nonnegative integer $k$ represents a step number.

By taking the state $x_k$ and the output $y_k$ as

$$x_k = \begin{bmatrix} \mathrm{SoC}_k \\ u_{1k} \\ R_{0k} \end{bmatrix}, \qquad y_k = u_{Lk},$$

we obtain the state-space model of the lithium-ion battery as

$$x_{k+1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \left(1 - \frac{\Delta t}{R_1 C_1}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{\Delta t}{\mathrm{FCC}} \\ \frac{\Delta t}{C_1} \\ 0 \end{bmatrix} i_k + w_k, \tag{6}$$

$$y_k = \mathrm{OCV}(\mathrm{SoC}_k) + R_{0k} i_k + u_{1k} + v_k. \tag{7}$$

To apply the extended Kalman filter, we have introduced the system disturbance $w_k$ and the measurement noise $v_k$ into the above state-space model. We assume that $w_k$ and $v_i$ are the zero mean Gaussian white noise processes with covariances $Q$ and $R$, respectively.

Note that the state equation (6) is linear if $R_1$ and $C_1$ are constant, while the measurement equation (7) is nonlinear because of $\mathrm{OCV}(\mathrm{SoC}_k)$. The previous works such as [1], [2], [3] applied the EKF to the above state-space model under the assumption that $R_1$ and $C_1$ are constant.

However, the physical parameters $R_1$, $C_1$ are dependent on both the atomospheric temperature, denoted by $T$, and the SoC of the battery as shown in Sub-section IV-A.Hence, the state equation (6) is highly nonlinear and time-varying in realistic situations. This leads to the degradation of the estimation accuracy of the EKF if we neglect the dependency of $R_1$, $C_1$ on $T$ and SoC.

We employ the gain scheduling technique to cope with the above difficulty. Define

$$\psi_k := \frac{1}{R_1 C_1}, \quad \phi_k := \frac{1}{C_1}$$

By viewing $\theta_k := (\psi_k, \phi_k)$ as the time-varying parameters which can be measured on-line, the state-space model (6),(7) is re-written as

$$x_{k+1} = F(\theta_k) + G(\theta_k) i_k + w_k, \tag{8}$$

$$y_k = h(x_k, i_k) + v_k, \tag{9}$$

where

$$F(\theta_k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 - \psi_k \Delta t & 0 \\ 0 & 0 & 1 \end{bmatrix}, G(\theta_k) = \begin{bmatrix} \frac{\Delta t}{\mathrm{FCC}} \\ \phi_k \Delta t \\ 0 \end{bmatrix}$$

$$h(x_k, i_k) = \mathrm{OCV}(\mathrm{SoC}_k) + u_{1k} + R_{0k} i_k.$$

We refer to the above state-space model as the *parameter-dependent state-space model*. The state equation (8) is apparently linear as long as $\theta_k$ is measured on-line, though the measurement equation (9) is the same as (7).

## III. Estimation Algorithm

By applying the EKF to the parameter-dependent state space model (8),(9), we obtain the following SoC estimation algorithm under the assumption that $\theta_k$ is available on-line, where $\hat{x}_{k|l}$ denotes the estimation of $x_k$ based on the measurement and input data up to the time $l$: $\{(i_0, y_0), (i_1, y_1), \ldots, (i_l, y_l)\}$.

Initialization:

$$\hat{x}_{0|-1} = \hat{x}_0, \ P_{0|-1} = \Sigma_0 \tag{10}$$

Prediction step:

$$\hat{x}_{k+1|k} = F(\theta_k)\hat{x}_{k|k} + G(\theta_k) i_k \tag{11}$$

$$P_{k+1|k} = F(\theta_k) P_{k|k} F(\theta_k)^\top + Q \tag{12}$$

Filtering step:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k[y_k - h(\hat{x}_{k|k-1}, i_k)] \tag{13}$$

$$K_k = P_{k|k-1} H_k^\top [R + H_k P_{k|k-1} H_k^\top]^{-1} \tag{14}$$

$$P_{k|k} = P_{k|k-1} - K_k H_k P_{k|k-1} \tag{15}$$

$$\begin{aligned} H_k &= \left. \frac{\partial h(x_k, i_k)}{\partial x_k} \right|_{x = \hat{x}_{k|k-1}} \\ &= \begin{bmatrix} \left. \frac{d\mathrm{OCV}}{d\mathrm{SoC}} \right|_{\mathrm{SoC} = \widehat{\mathrm{SoC}}_{k|k-1}} & 1 & i_k \end{bmatrix} \end{aligned}$$

To implement the above algorithm, we need to know the value of $\theta_k$ at each time instant $k$. Since $\theta$ is a function of $T$ and SoC, we identify the static functional relation $\theta = \theta(T, \mathrm{SoC})$ in advance from the experimental data of the AC impedance test (see Section IV.A). Then, at each time instant $k$, we compute the parameter value as $\theta_k = \theta(T_k, \widehat{\mathrm{SoC}}_{k|k-1})$ by using the measured temperature $T_k$ and the one-step prediction $\widehat{\mathrm{SoC}}_{k|k-1}$.

## IV. Experimental Results

We performed experiments on a 18650 type lithium-ion battery, whose specifications are summarized in TABLE I.

TABLE I. BATTERY SPECIFICATIONS

| CGR-18650CH Li-ion battery | |
|---|---|
| Nominal capacity (FCC) | 2250[mAh] (8250[Asec]) |
| Nominal voltage | 3. 6[V] |
| The highest voltage | 4. 2[V] |
| Cut-off voltage | 3[V] |

### A. *Identification of equivalent circuit parameters*

We first need to identify the temperature and SoC dependencies of the physical parameters $R_1$ and $C_1$ in the equivalent circuit of Fig. 1.

To identify these dependencies of the parameters, we measure the AC impedances over the frequency range from 0.08[Hz] to 3[kHz] for various values of the atomospheric temperatures and SoC's. Experimental conditions of the AC impedance measurements are described in TABLE II. For each combination of the temperature and the SoC, we draw the corresponding Nyquist plot, and read the values of $R_1$ and $C_1$ from the plot. The characteristics of $1/(R_1C_1)$ and $1/C_1$ calculated from the above measurements are indicated by the dashed lines in Figs. 2–5.

TABLE II. EXPERIMENTAL CONDITION

| | |
|---|---|
| Atomospheric temperature[$^\circ$C] | 0, 5, 10, 15, 20, 25, 30, 35, 40, 45 |
| SoC[%] | 20, 30, 40, 50, 60, 70, 80, 90, 100 |
| Set voltage[V] | 3. 004. 2 |
| Discharge current[A] | 1. 125(0. 5C) |
| Alternating current | 8 portion of the discharge current |
| Set frequency[Hz] | 30000. 08 |

TABLE III. EXPERIMENTAL EQUIPMENTS

| | |
|---|---|
| Applications software | FC Tester |
| AC impedance measuring instrument (Kikusui) | KFM2150 |
| Electronic Load (Kikusui) | PLZ664WA |
| Low temperature incubator (ESPEC) | LU-113 |

In view of the temperature characteristics in Figs. 2 and 4, we model the static relation between ($\psi$, $\phi$) and ($T$, SoC) by

$$\psi = \frac{1}{R_1C_1} = A_1 \exp(\alpha_1 T), \tag{16}$$

$$\phi = \frac{1}{C_1} = A_2 \exp(\alpha_2 T), \tag{17}$$

where $A_1$, $A_2$, $\alpha_1$, and $\alpha_2$ are functions of SoC.

We first find the values of $A_1$, $A_2$, $\alpha_1$ and $\alpha_2$ for various SoC's by applying the least-squares method to fit (16),(17) to the dashed lines in Figs. 2 and 4. Then, we interpolate the obtained data of $A_1$, $A_2$, $\alpha_1$ and $\alpha_2$ by the least-squares method under the assumption that these parameters are 8th-order polynomials of SoC. The results of this identification are indicated by the solid curves in Figs. 2–5.

### B. *Identification of Open Circuit Voltage*

We characterize the OCV-SoC relation in terms of the following nonlinear function which was proposed by Plett [1].

$$\mathrm{OCV} = K_0 - \frac{K_1}{\mathrm{SoC}} - K_2\mathrm{SoC} + K_3\ln(\mathrm{SoC}) + K_4\ln(1-\mathrm{SoC}).$$

The coefficients $K_0$,$K_1$,...,$K_4$ are determined by the least squares method. The SOC-OCV curve to be fitted is obtained

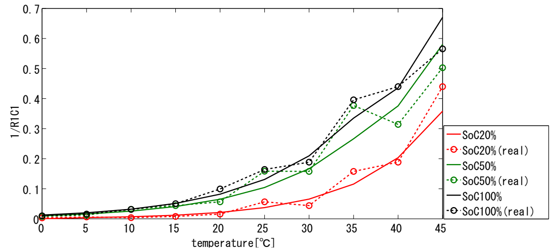


Fig. 2. Temperature dependency of $1/(R_1C_1)$ (dashed: experiment, solid: identified)

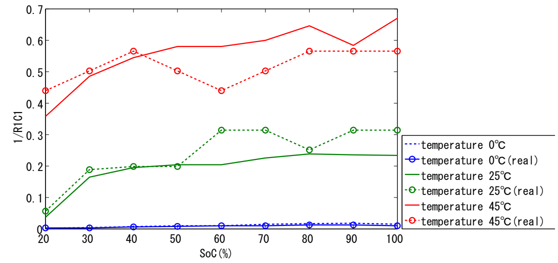


Fig. 3. SoC dependency of $1/(R_1C_1)$ (dashed: experiment, solid: identified)

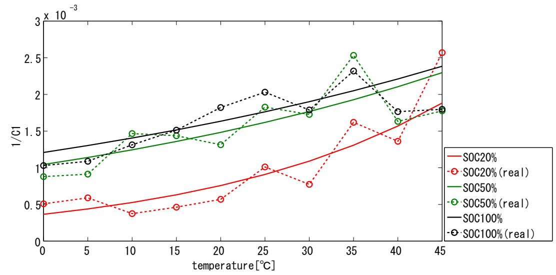


Fig. 4. Temperature dependency of $1/C_1$ (dashed: experiment, solid: identified)

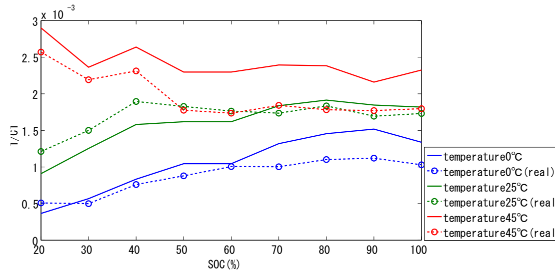


Fig. 5. SoC dependency of $1/C_1$ (dashed: experiment, solid: identified)

by averaging the results for the discharge and charge experiments with small current 0.02C. The identification result is shown in Fig. 6, where the red dotted curve and the blue solid curve denote the measured data and the identified curve, respectively.

### C. *State-of-Charge estimation*

Discharge experiment was performed under the situation where the change of the atomospheric temperature is very

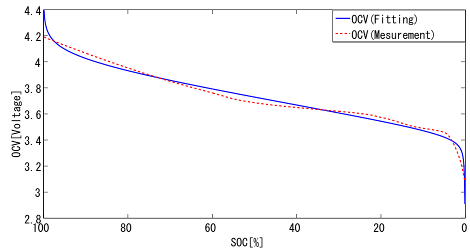


Fig. 6. Identification result of OCV

large. The input current and the atomospheric temperature in this experiment are shown in Figs. 7 and 8, respectively.

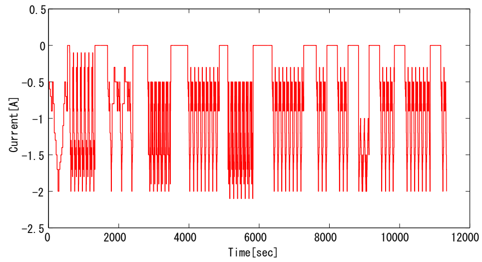


Fig. 7. Input current profile

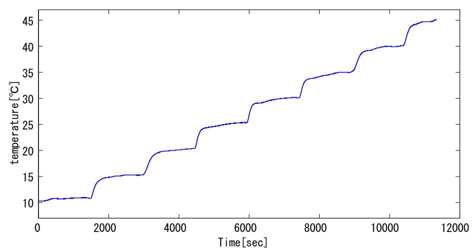


Fig. 8. Atomospheric temperature profile

To carry out the EKF algorithm, we set the noise covariances $Q = 10^{-5}I_3$, $R = 0.1$ and the sampling perod $\Delta t = 1$ [sec]. We also set the initial values to $\hat{x}_{0|-1} = [1.0 \quad 0.0092 \quad 0.0226]^\top$ and $\Sigma_0 = I_3$ (identity matrix).

For comparison, we perform the estimation by using the conventional EKF algorithm with $(R_1, C_1)$ fixed to the values measured by the AC impedance method for $T = 30$[deg] and $\mathrm{SoC} = 50[\%]$.

The estimation results are illustrated in Fig. 9. For the reference of the estimation accuracy, we consider the current accumulation values from the fully charged state measured by the commercial battery test system Kikusui PFX2021S to be the true SoC values. It is seen from the figure that the estimation accuracy of the conventional EKF with the fixed parameters largely degrades after 7000 seconds. On the other hand, the proposed parameter-dependent EKF algorithm keeps the estimation error small until the battery gets almost empty. As a result, we conclude that the proposed method improves the estimation accuracy by taking account of the temperature and SoC dependencies of the equivalent circuit parameters.

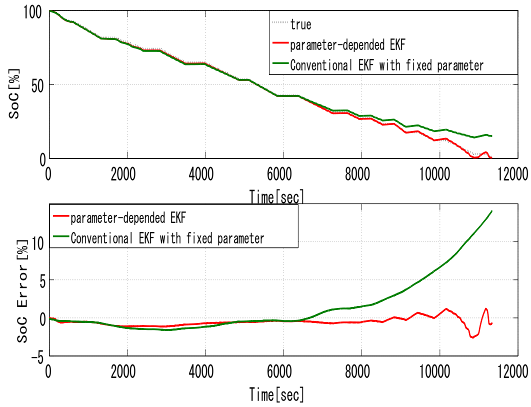


Fig. 9. Error comparison of SoC

## V. Conclusion

We have proposed a new EKF-based method for estimating the SoC of a lithium-ion battery based on the parameter-dependent state-space model. The proposed method improves the estimation accuracy by taking account of the variation of the equivalent circuit parameters due to their dependencies on the temperature and the SoC. The effectiveness of the proposed method has been verified by the experimental results. It remains as a future challenge to extend the proposed method to a more detailed battery model which can explain various physical phenomena inside the battery.

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