State of Charge Estimation Based on Improved $LiFePO_4$ Battery Model and Kalman Filtering

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***Abstract*—The Lithium iron phosphate ($LiFePO_4$) batteries are widely used as power source for electric vehicles. By reason of safety issues, it is significant to estimate the state of charge (SOC) of a battery precisely. On the basis of the characteristics of $LiFePO_4$ battery, the improved Thevenin equivalent circuit model is proposed, and the model parameters are identified by experimental testing. Additionally, a novel algorithm, which combines the Open-circuit voltage method, Ampere-hour integration and Kalman filtering, for SOC online estimation is proposed. The experimental verification of the improved battery model and SOC estimation algorithm is presented. The improved battery model shows better estimation accuracy than traditional Thevenin equivalent circuit model. The proposed SOC estimation algorithm is able to obtain $LiFePO_4$ SOC precisely even with inaccurate initial values and distinguish the performances of different batteries.**

I. INTRODUCTION

Automobile industry has been increasing the penetration of electric vehicles (EVs) in the market because of the global oil resources reduction annually. Lithium iron phosphate ($LiFePO_4$) battery packs are widely used as power sources for EVs. In practice, $LiFePO_4$ battery is usually configured in series or parallel in order to achieve a higher voltage level or larger capacity. The differences of dynamic characteristics among batteries would cause State of Charge (SOC) unbalanced, as well as, inevitably influence the efficiency and lifetime of the battery packs. Therefore, it is important to estimate the SOC of $LiFePO_4$ battery in real-time in order to ultimately improve the battery performance and lengthen the useful lifetime of battery packs.

Owing to SOC is not a direct measurement, it must be estimated by some algorithm based on certain battery physical quantities (e.g., battery voltage and current). To obtain the exact SOC estimation of the $LiFePO_4$ battery is difficult due to several factors such as weather, EV state, traffic conditions and battery nonlinearity. Generally, Ampere-hour integration [1, 2], Open-circuit voltage (OCV) method [1, 3, 4], Neural network [5] and Kalman filtering [6-8] can be used for battery SOC estimation. Ampere-hour integration is the most commonly used method as it is theoretically the most precise method for SOC estimation and easy-to-implement. However, the estimation error of this method could be significant because of incorrect initial SOC and current measurement errors. In real-time system, the Open-circuit voltage method is only used to estimate initial SOC. The Neural network is also inapplicable since it introduces huge computational burden to a microprocessor to estimate multiple SOC values using a large amount of experimental data. Kalman filtering might be a suitable SOC estimation approach applied in EV. However, it still has a disadvantage that the convergence of Kalman filtering highly relies on accurate battery models.

This paper presents an accurate SOC estimation method for battery packs. Firstly, an improved Thevenin equivalent circuit model that more accurately represents $LiFePO_4$ characteristics is proposed. Secondly, Open-circuit voltage method, Ampere-hour integration and Kalman filtering are combined for SOC estimation. Finally, the experimental verification of the proposed battery model and the SOC estimation algorithm are presented.

II. BATTERY MODEL AND SOC ESTIMATION ALGORITHM

A. The Improved Thevenin Equivalent Model

It is well-known that battery characteristics are different when it is under charging or discharging conditions, as well as at different SOC [9]. In order to get the $LiFePO_4$ battery behaviors under charging and discharging conditions respectively, the traditional Thevenin equivalent circuit model [10] of $LiFePO_4$ battery, as shown in Fig.1 (a), is improved. Fig.1 (b) presents the improved battery equivalent circuit, where U_{oc} is an ideal voltage source to represent the open circuit voltage of battery cell, I_o is the load current, U_o is the battery output voltage, I_p is the battery polarization current, C_p is the polarization capacitor. R_i and R_p are the battery internal resistor and polarization resistor respectively, where subscripts c and d mean charging and discharging. With the help of these diodes in the equivalent circuit, different internal and polarization resistors will be adopted during battery charging and discharging. Furthermore, the range of SOC is divided into 10 segments (10% in each segment), and corresponding values of R_i, R_p and τ are identified for each segment.

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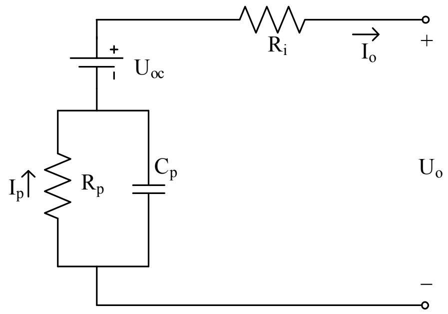
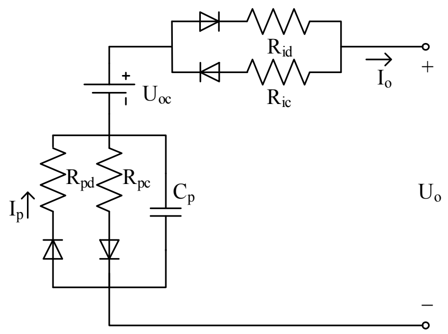


(a) Traditional Thevenin equivalent circuit model



(b) Improved Thevenin equivalent circuit model

Fig. 1: Equivalent circuit models of $LiFePO_4$ battery

For a single $LiFePO_4$ battery, the variable calculation equations are obtained by choosing SOC and I_p to be the system state variables, U_o to be the system output and I_o to be the system input. I_o is positive during discharging and negative during charging.

The discrete expression of SOC based on Ampere-hour integration is:

$$\begin{aligned} SOC(k) &= SOC(k-1) \\ &-\eta\left[I_o(k)+I_o(k-1)\right]\left[T(k)-T(k-1)\right]/2C \end{aligned} \tag{1}$$

where C represents the battery static capacity, η is the coulombic efficiency that is different under charging and discharging conditions, k is the time index, and T is the sampling instance. The initial SOC value is calculated by Open-circuit voltage method [1, 3].

The calculation of state variable I_p is as:

$$\begin{aligned} I_p(k) &= \left[1-\frac{1-e^{-\Delta T/\tau}}{\Delta T/\tau}\right] I_o(k) \\ &+\left[\frac{1-e^{-\Delta T/\tau}}{\Delta T/\tau} - e^{-\Delta T/\tau}\right] I_o(k-1) \\ &+e^{-\Delta T/\tau} \times I_p(k-1) \end{aligned} \tag{2}$$

where τ is the time constant of polarization to represent the battery dynamic response, and $\Delta T = T(k) - T(k\text{-}1)$.

Expression (3) represents the battery output:

$$U_o = U_{oc} - R_i I_o - R_p I_p \tag{3}$$

where U_{oc} is the OCV of battery and has nonlinear relationship with respect to the battery SOC. In this paper, linear interpolation is employed to obtain the segmented linearized expression of OCV related to SOC:

$$\begin{aligned} U_{oc}(k) &= \frac{OCV_{end}-OCV_{start}}{SOC_{end}-SOC_{start}}\left[SOC(k)-SOC_{start}\right] \\ &+OCV_{start} + C_c \text{ or } C_d \end{aligned} \tag{4}$$

where subscripts "start" and "end" represent the start and the end of each segment respectively, C_c and C_d are compensations for the OCV when the battery is being charged or discharged.

The system can be expressed in a discrete-time state-space form from expression (1) and (2), as below:

$$\begin{aligned} \begin{bmatrix} SOC(k) \\ I_p(k) \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & e^{-\Delta T/\tau} \end{bmatrix} \begin{bmatrix} SOC(k-1) \\ I_p(k-1) \end{bmatrix} \\ &+\begin{bmatrix} \eta\Delta T/2C \\ 1-\dfrac{1-e^{-\Delta T/\tau}}{\Delta T/\tau} \end{bmatrix} I_o(k) \\ &+\begin{bmatrix} \eta\Delta T/2C \\ \dfrac{1-e^{-\Delta T/\tau}}{\Delta T/\tau} - e^{-\Delta T/\tau} \end{bmatrix} I_o(k-1) \end{aligned} \tag{5}$$

The system output function can be derived from expression (3) and (4):

$$\begin{aligned} U_o(k) &= \begin{bmatrix} \dfrac{OCV_{end}-OCV_{start}}{SOC_{end}-SOC_{start}} & R_p \end{bmatrix} \begin{bmatrix} SOC(k)-SOC_{start} \\ I_p(k) \end{bmatrix} \\ &+OCV_{start} - I_o R_i + C_c or C_d \end{aligned} \tag{6}$$

B. Kalman Filtering Based SOC Estimation

Kalman filtering [11] is a set of mathematical equations that provides an efficient computational recursive method to estimate the state of a dynamic system, even though the initial state is uncertain or the measurement is incomplete due to noise.

Assuming a discrete system in expression (7):

$$x_k = A_k x_{k\text{-}1} + B_k u_k + \omega_k \qquad z_k = H_k x_k + \upsilon_k \tag{7}$$

where x_k is the state vector, u_k is the input vector, and z_k is the measurement vector. A_k is the state matrix, B_k is the input matrix, and H_k is the measurement matrix. The random variables ω_k and υ_k represent the process and measurement noises respectively, which are assumed to be independent and with normal probability distributions.

At time step k, the prior estimation of system states and their estimated error covariance matrix are:

$$\hat{x}_k^- = A_k \hat{x}_{k-1} + B_k u_{k-1} \qquad P_k^- = AP_{k-1}A^T + Q \tag{8}$$

where Q is the process noise covariance matrix. Then, the Kalman gain or blending factor matrix K_k that minimizes the posterior error covariance can be updated accordingly:

$$K_k = \frac{P_k^- H^T}{HP_k^- H^T + R} \tag{9}$$

where R is the measurement noise covariance matrix.

Therefore, the posterior estimation of system states and their estimated error covariance matrix can be acquired:

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-) \qquad P_k = (I - K_k H)P_k^- \tag{10}$$

where I is identity matrix.

The system state matrix A and the measurement matrix H can be written according to the system state functions as:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & e^{-\Delta T/\tau} \end{bmatrix} \quad H = \begin{bmatrix} \dfrac{OCV_{end} - OCV_{start}}{SOC_{end} - SOC_{start}} & R_p \end{bmatrix} \tag{11}$$

Substituting matrix A and H in (11) into the expressions (8) – (10), the equation for $LiFePO_4$ battery SOC estimation can be derived.

III. Parameters Identification

In order to identify the parameters of the improved Thevenin equivalent circuit model in Fig.1 (b), experiments are conducted on a 50 Ah $LiFePO_4$ battery.

A. Battery Static Capacity and Coulombic Efficiency

Battery static capacity and coulombic efficiency are used for SOC calculation based on Ampere-hour integration. In order to identify the static capacity, the battery is fully charged and discharged at C/3 rate constant current. The battery static capacity can be calculated by integrating the charging and discharging currents. One hour rest for the testing battery is required between the charging and discharging processes. The average static capacity of multiple tests is 48.5 Ah.

After static capacity identification, the battery is fully charged and discharged at different current rates in order to

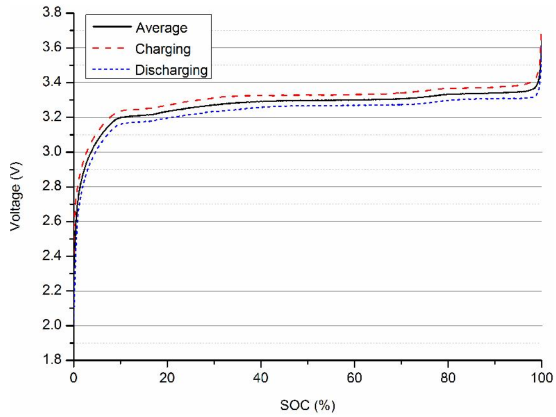


Fig. 2: OCV vs SOC

TABLE I. Ten sets of parameters R_i, R_p and τ

SOC(%)	τ (s)	R_i(mohm)		R_p(mohm)	
		Charging	Discharging	Charging	Discharging
100%-90%	45	1.710	1.918	0.615	0.599
90%-80%	25	1.449	1.474	0.984	0.786
80%-70%	20	1.340	1.458	0.754	0.695
70%-60%	20	1.350	1.282	0.998	0.841
60%-50%	20	1.206	1.297	1.235	0.958
50%-40%	28	1.259	1.315	1.435	0.786
40%-30%	20	1.350	1.427	0.998	0.865
30%-20%	20	1.340	1.562	0.754	0.822
20%-10%	25	1.449	1.392	0.984	1.259
10%-0%	45	1.710	1.004	0.615	1.707

TABLE II. Statistical analysis list of the absolute values of battery terminal voltage

Model	Max error (V)	Mean error (V)	Std. Deviation (V)	Error rate (%)
The traditional model	0.1039	0.0083	0.01507	0.413
The improved model	0.0887	0.0014	0.00708	0.194

find out the relationship between the coulombic efficiency and current [12]. The coulombic efficiency as a function of the battery current is as equation (12).

$$\eta = \begin{cases} -0.0001I_o + 0.99 & (I_o > 0) \\ \dfrac{0.9896}{-0.00026I_o + 0.99} & (I_o < 0) \end{cases} \tag{12}$$

B. The OCV vs SOC Curve

In order to find the relationship between the OCV and SOC, the battery is fully charged and discharged at the C/25 rate [6]. The extremely low current rate is used to minimize the influence of the internal resistor on the battery output voltage. However, there still exists a gap between the charging and discharging terminal voltages, known as hysteresis as illustrated in Fig. 2. In order to eliminate such hysteresis, the average battery terminal voltage is calculated based on the charging and discharging voltage obtained in this set of experiments. It can be assumed that the average voltage is approximately the same as OCV. In real applications, a compensation value is added or subtracted from the averaged OCV during charging or discharging accordingly.

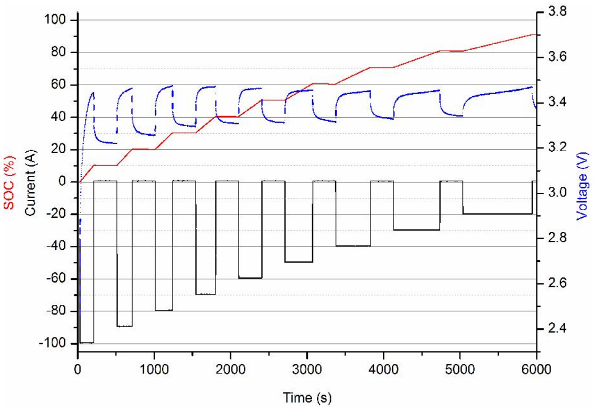
C. Identification of Ri, Rp and τ

The internal resistors, polarization resistors and time constants of the improved Thevenin equivalent circuit model could be identified by the pulse charging and discharging test. The data recorded in this set of experiments include current, voltage, time and SOC, as shown in Fig. 3.

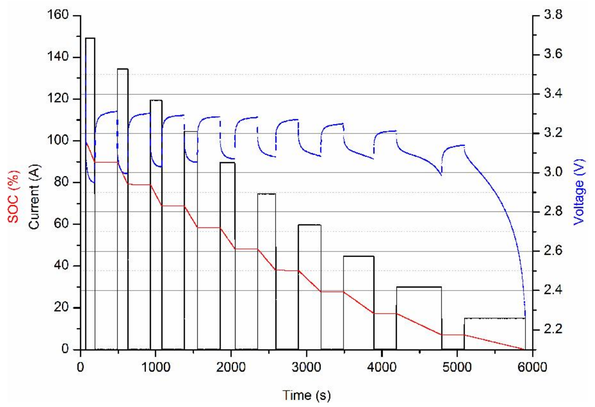
Ten sets parameters of *Ri, Rp* and **τ** under charging and discharging conditions are calculated by Linear Regression and Least Square methods based on the experimental data aforementioned. The identified parameters are shown in TABLE I.

IV. Experimental Rustles

An experimental platform is built to verify the validity of the battery model and parameter identification method, as well as the effectiveness of SOC estimation algorithm. The



(a) Charging



(b) Discharging

Fig. 3: Battery current, voltage and SOC in pulse charging and discharging tests

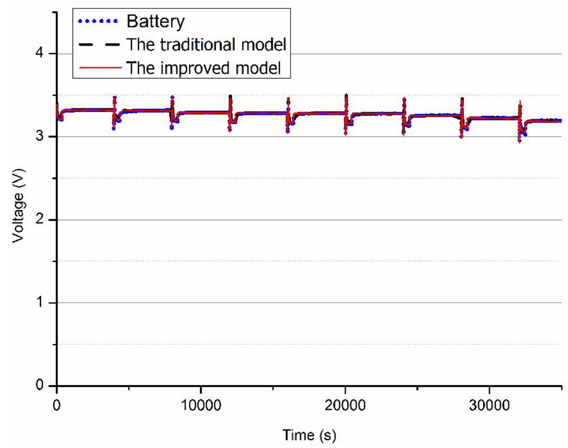


Fig. 4: Battery output voltages

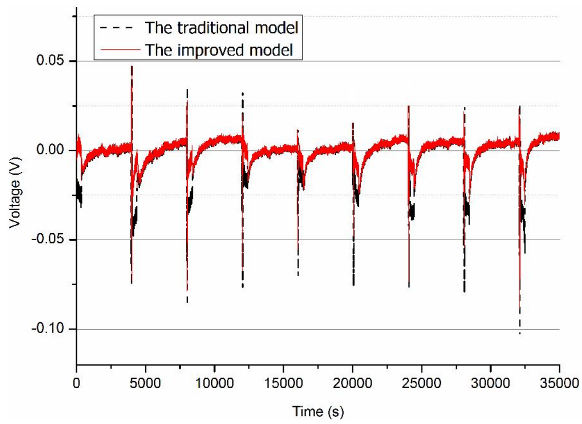


Fig. 5: Voltage estimation errors

experimental platform consists of 1kw electronic load KIKUSUI PLZ1004W, 1kw DC power supply Chroma 6260-60 series, NI data acquisition module USB6009 and Hall effect current sensor HAIS 50-P.

A. *Verification of Battery Model*

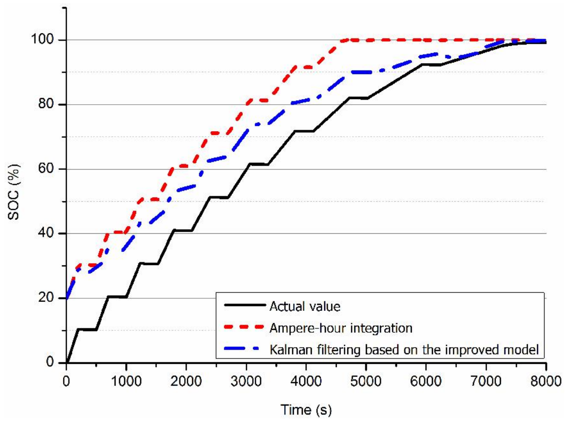
Since the voltage is the only output of the battery model, the estimation error of the output voltage is used to represent the accuracy of the model. The Hybrid Pulse Power Characterization (HPPC) Test [9] is adopted as load profile in this set of experiments. The estimated output voltages and the estimated errors are shown in Fig. 4 and Fig. 5. A statistical analysis and comparison on the absolute values of the terminal voltage errors are conducted and the results are shown in TABLE II. The data of the last discharging segment (SOC between 10% - 0%) is excluded in analysis because on board power batteries are rarely discharged to empty in practical applications.

From Fig. 4, Fig. 5 and TABLE II, it can be seen that the voltage estimation error based on the improved model is within a range of 0.0887 volts, while that of the traditional model fluctuates within a range of 0.1039 volts. The estimation error rate is calculated based on the standard deviation of the estimation error and the max battery voltage, which is indicated in the TABLE II. According to the experiment results, it is clearly proved that the improved model has higher accuracy and characterizes the dynamic responses of the $LiFePO_4$ battery better.

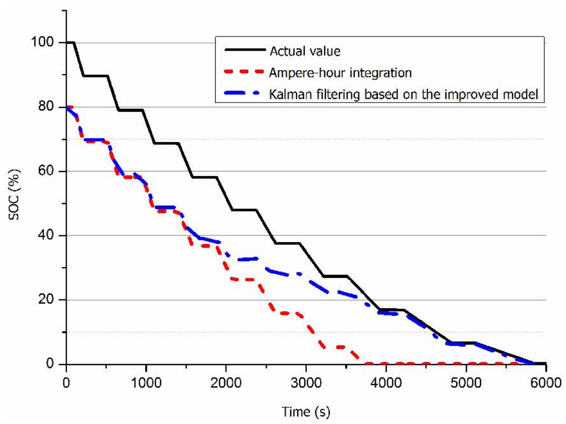
B. *Verification of SOC Estimation Algorithm*

The improved Thevenin equivalent circuit model, which represents the dynamic characteristics of $LiFePO_4$ battery better, is adopted in this set of experiments to verify the SOC estimation algorithm.

In Fig. 6, the battery is charged and discharged under the load profile shown in Fig. 3. The initial SOC is intentionally set to be 20% differ from the actual SOC. It can be seen that the SOC estimation based on Kalman filtering method eventually converges to the actual SOC value. Meanwhile, the 20% gap always exists between the actual SOC and the

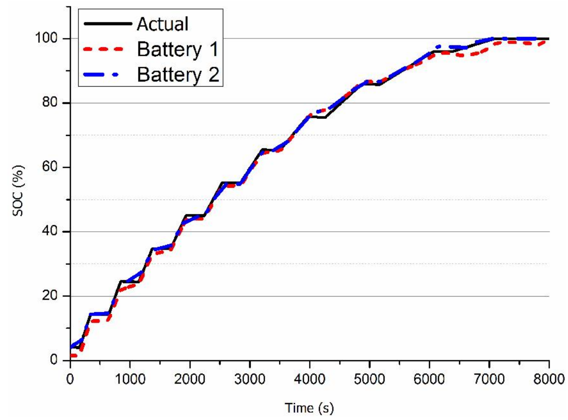


(a) Charging

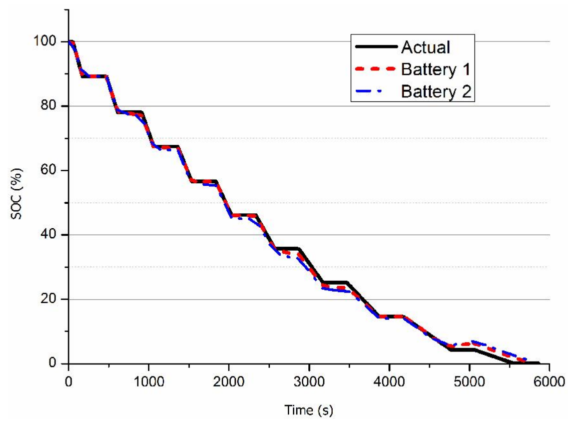


(b)Discharging

Fig. 6: SOC estimation of single battery



(a) Charging



(b)Discharging

Fig. 7: SOC estimation of series batteries

SOC estimated by Ampere-hour integration, which would lead to a misjudgment of battery state and result in a tremendous waste of battery capacity or battery damage. It clearly shows that the Kalman filtering is effective to adjust the SOC with initial error to the actual SOC.

The same experimental process was conducted on two batteries connected in series and the results are presented in Fig. 7. The initial SOCs of these two batteries are calculated by Open-circuit voltage method. It apparently shows, under the same circumstance, even with the same initial SOC value, the SOC estimations are different for different batteries. According to the results, the difference in performance between batteries connected in series can be distinguished by the proposed SOC estimation method. It provides an effective way to protect the batteries with poor performance from damage by over-charging or over-discharging.

V. Conclusion

The improved Thevenin equivalent circuit model is established based on the characteristics of $LiFePO_4$ battery in this paper. The detailed parameter identification process for the improved battery model is introduced accordingly. The proposed SOC estimation algorithm is achieved by combining the Open-circuit voltage method, Ampere-hour integration and Kalman filtering based on the improved model. The conclusions obtained from experiments are as follows:

a) The improved Thevenin equivalent circuit model with identified parameters accurately reflects the $LiFePO_4$ battery static and transient process under charging and discharging conditions.

b) The effectiveness of the proposed SOC estimation algorithm, even with incorrect initial SOC value, has been proved in experiments.

c) The difference of the battery performances can be distinguished by SOC estimation based on the proposed algorithm, which provides a reference for battery protection strategy.

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