Computational practicum

Analytical solution (exact solution)

Practicum

$$\begin{cases} y' = 2x(x^2+y) \\ y(0) = 0 \\ x \in (0, 10) \end{cases}$$

The equation has a form: $y' + a(x)y = b(x)$

$y' - 2xy = 2x^3$ Let's solve complementary equation

$y' - 2xy = 0$ complementary equation

$\frac{dy}{dx} = 2xy$ Let's transform it into differential form and integrate it

$\int \frac{dy}{y} = \int 2x\,dx$

$\ln|y| = x^2 + C$

$y = C_1 e^{x^2}$ where $C_1 = C_1(x)$ is a function depends on x

$y' = C_1' e^{x^2} + 2xC_1 e^{x^2}$

To find C_1 let's substitute y and y' into original equation and solve it

$C_1' e^{x^2} + 2xC_1 e^{x^2} = 2x^3 + 2xC_1 e^{x^2}$

$C_1' e^{x^2} = 2x^3$

$C_1' = \frac{2x^3}{e^{x^2}} = \frac{dC_1}{dx}$

$\int dC_1 = \int \frac{2x^3}{e^{x^2}} dx$

$C_1 + C_2 = 2\int x^3 e^{-x^2} dx = \begin{matrix} u = x^2 \\ du = 2x\,dx \end{matrix} = \int u e^{-u} du = -ue^{-u} - \int(-e^{-u})du = -ue^{-u} - e^{-u} = -e^{-x^2}(x^2+1)$

$y = -e^{-x^2}(x^2+1)e^{x^2} + C_2 e^{x^2} = -x^2 - 1 + C_2 e^{x^2}$

$C_2 = (y + x^2 + 1)e^{-x^2}$ formula for calculating the constant

Initial value problem $y(0) = 0$

$C_2 = (0+0+1)\cdot 1 = 1$

Answer: $y = -x^2 - 1 + e^{x^2}$; This function is symmetric (even) without points of discontinuity.

Points of discontinuity: There is no points of discontinuity in the equation.

Exact solution for given IVP: $y = e^{x^2} - x^2 - 1$

Program's part

The program allows user to see the graph of the solution of the equation $y = C_2 e^{x^2} - x^2 - 1$ with opportunity to change initial conditions, range and number of grid steps.

For calculating new exact solution the program use the following formula to calculate the constant C_2: $C_2 = (y + x^2 + 1)e^{-x^2}$

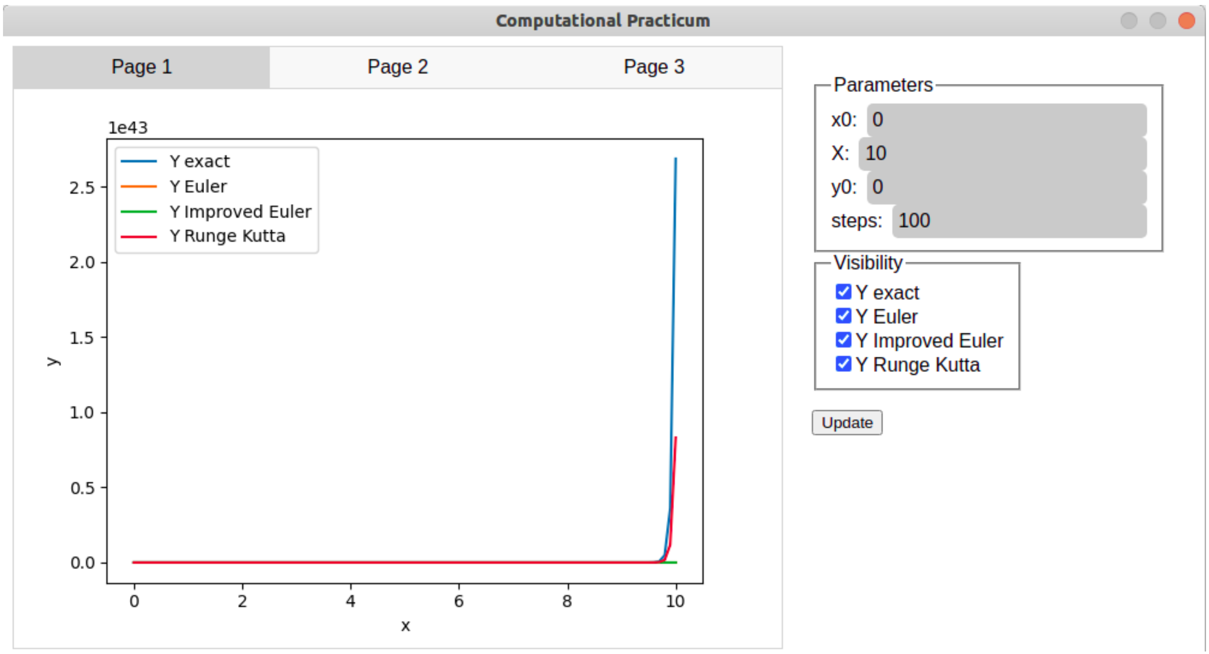
Graphs

Graph of solutions

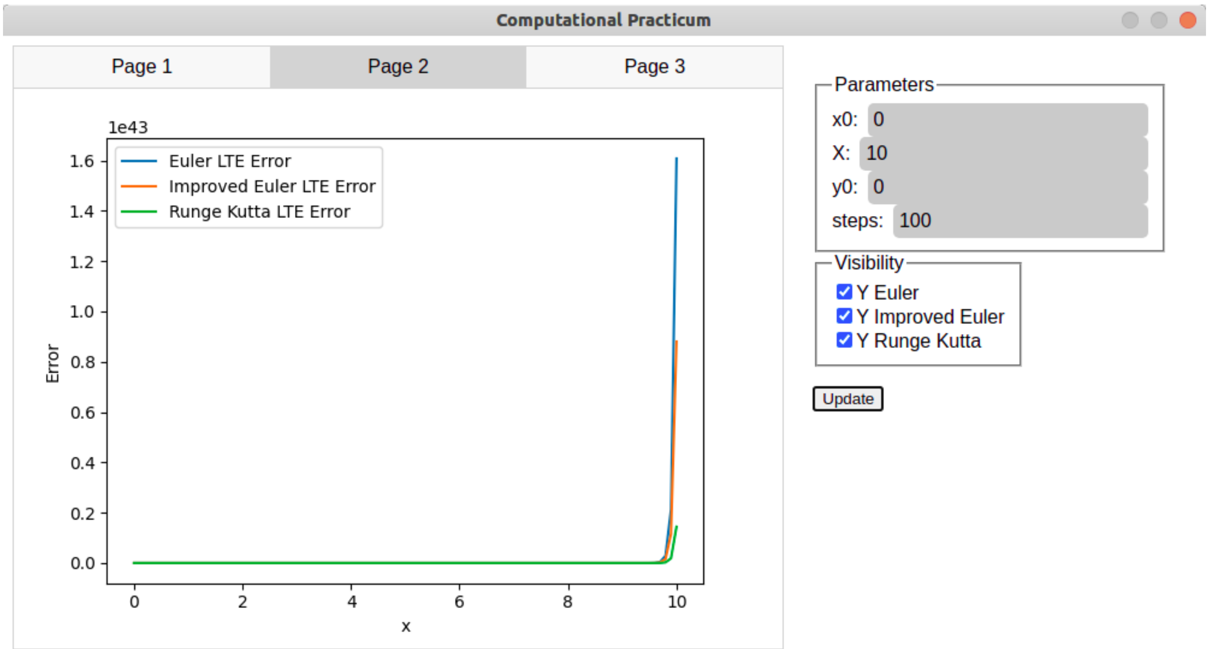
There are 4 lines represented different types of the solution:

- Exact solution;
- Approximate solution using Euler's method;
- Approximate solution using Improved Euler's method;
- Approximate solution using Runge Kutta method.

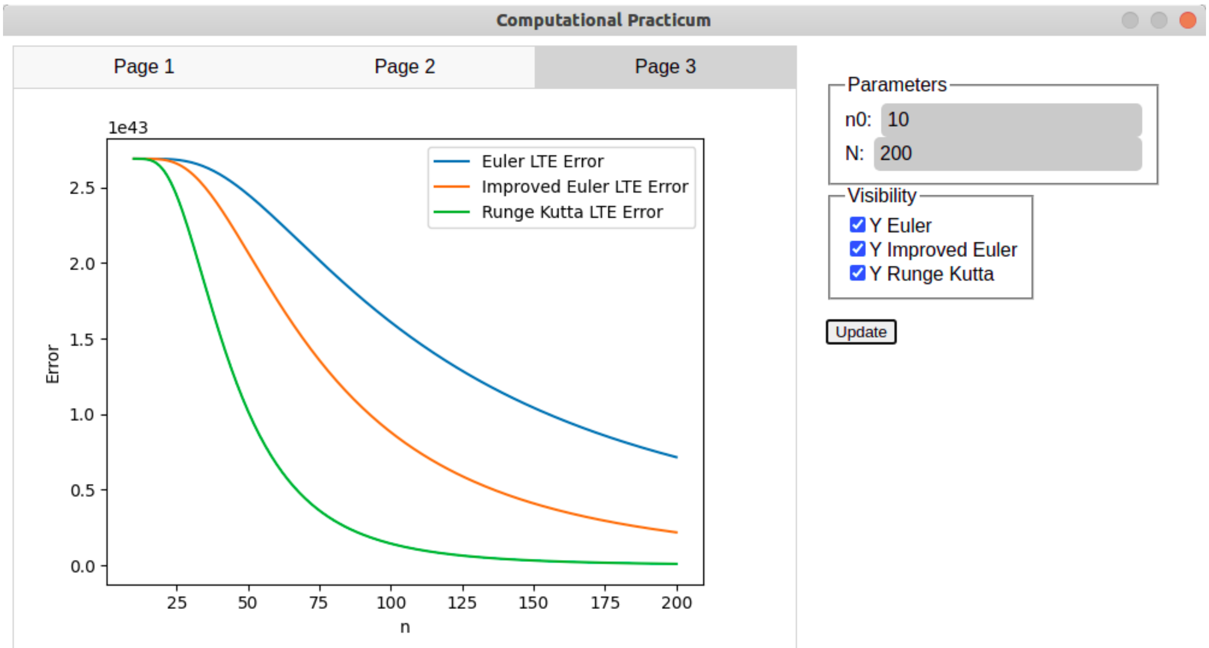
$y - axis$ represents solution for given x with values $\in [0, 2.7 * 10^{43}]$.



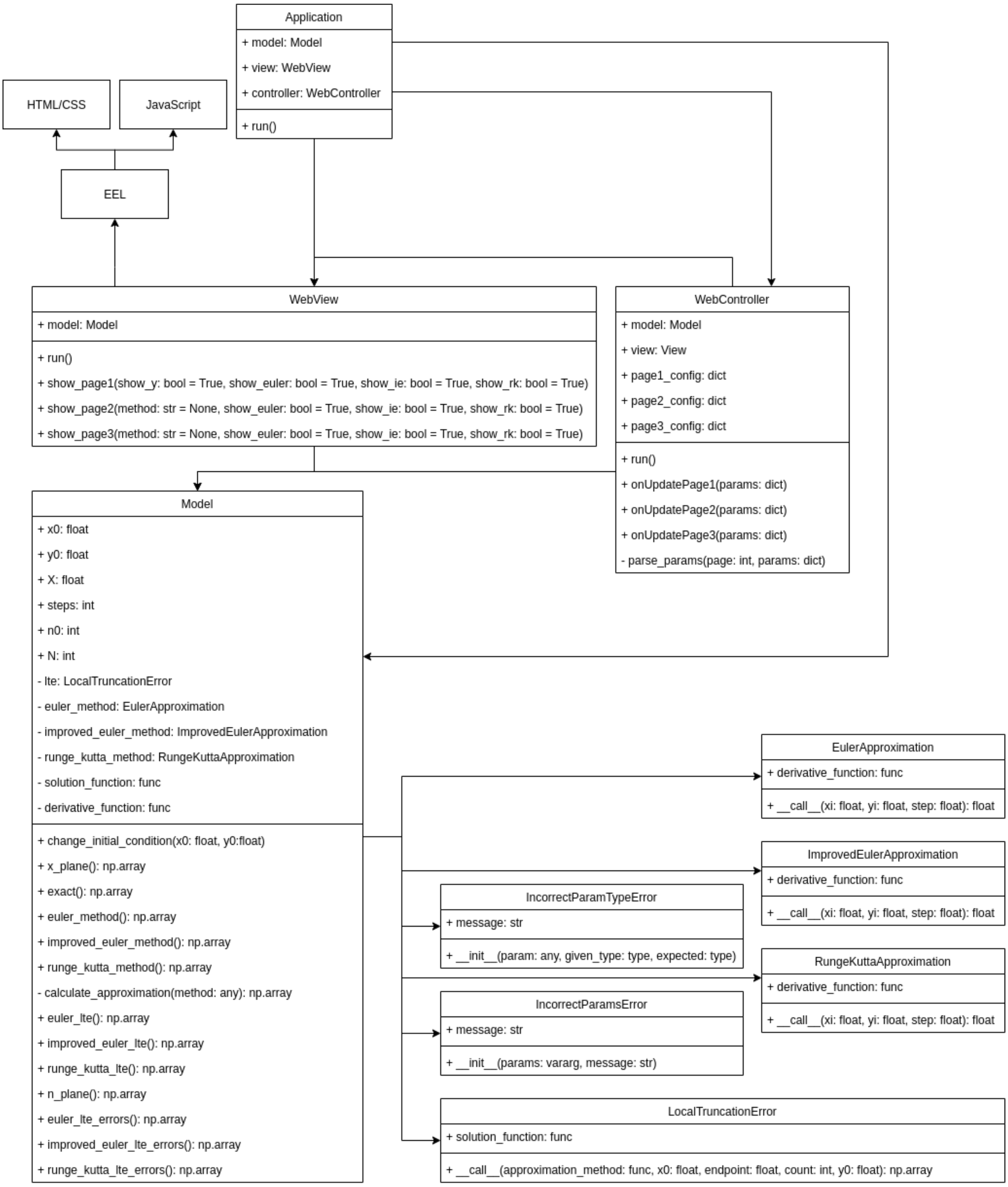
Graph of local errors



Graph of total approximation error



UML class diagram



Parts of the code

Run application (__main__.py)

```
if __name__ == '__main__':
    app = Application(model, view, controller)
    app.run()
```

Run graphical user interface (web_view.py)

```
def run(self) -> None:
    self._change_image({}, 1, callback_needed=False)
    eel.init('view/static')
    eel.start('index.html', size=(1000, 600))
```

Calculation of LTE (lte.py)

```
arr = np.zeros(shape=steps, dtype=np.float64)
xi = x0
y_real = y0
for i, x in enumerate(np.linspace(x0, endpoint, steps)):
    if i == 0:
        continue
    y_approximate = approximation_method(xi, y_real, step)
    y_real = self.solution_function(x)
    arr[i] = abs(y_real - y_approximate)
    xi = x
return arr
```

Plotting and saving a graph (web_view.py)

```
def _change_image(table: dict, page_number: int, callback_needed=True) -> None:
    for key in table.keys():
        if key == 'X':
            continue
        plt.plot(table['X'], table[key], label=key)
    if page_number == 1:
        plt.xlabel('x')
        plt.ylabel('y')
    elif page_number == 2:
        plt.xlabel('x')
        plt.ylabel('Error')
    elif page_number == 3:
        plt.xlabel('n')
        plt.ylabel('Error')
    if len(table) > 1:
        plt.legend()
    plt.savefig('view/static/img/graph.png', bbox_inches='tight', transparent=True)
    if callback_needed:
        eel.updateImage()()
    plt.close()
```