Poisson Distributions and Processes Problem Set

1. We know that the mean and variance of a homogeneous Poission process are equal, but what about an inhomogeneous Poisson?

   A. Suppose you have a Poisson spike train that has a mean rate of λ=5 Hz. What is the expected number of spikes in 10 seconds? What is the variance of the number of spikes?

   B. Suppose instead that the process has two rates, λ=5 Hz for $0< t \leq 5$ and $\lambda = 10$ Hz for $5 <t\leq 10$ seconds. What is the expected number of spikes in 10 seconds? What is the variance of the number of spikes? Hint: remember that the Poisson process is history independent.

   C. What is the relationship between the expected number of spikes in the 10 second period and $\int_0^{10} \lambda(t)dt$?

   D. Suppose that the rate of the process is given by 5*sin(t) + 5 over the 10 second interval. What is the mean and variance of the spike count? You can solve this analytically or numerically.

   E. What is the relationship between the mean and variance of an inhomogeneous Poission process?

2. Load the data from ps5_2.mat. spiketimes is a set of simulated spike times from an inhomogeneous Poission process with rate $\lambda(t) = \begin{Bmatrix} 10 & 0 < t < 50 \\ 20 & 50 \leq t < 100 \end{Bmatrix}$.

   A. Plot the interspike interval distributions for
      1. all of the data
      2. t < 50 seconds.
      3. t > 50 seconds.

   B. Use expfit to estimate the mean of the exponential distribution associated with the interspike intervals from 1-3 in part A above. How do these means related to $\lambda(t)$?

3. Suppose you are performing a whole cell recording experiment with low amplitude stimulation of afferent fibers that synapse onto your neuron. Load the data from ps5_3.mat. exp1 contains data from your first experiment where you have identified the size of a quantal event and counted the size of each event relative to the smallest quantal event. Each element of exp1 is the number of quanta released after each stimulating pulse and there were 500 pulses total.

   A. One quick test for the Poisson-ness of a distribution is to determine whether the number of times where an event was not observed is consistent with a Poisson process with a mean given by the data. What is the mean number of quanta (n) released for a single stimulation?

   B. Given that mean, what is the expected number of failures and how does that compare with the actual number of failures?

   C. You can also compare the CDF of the observed distribution to that of the predicted distribution. Do they agree?

4. In your second experiment you record from a different cell and get the data in exp2 from ps5_4.mat. Are the data Poission? If so, what is the mean number of quanta released during a single event? If not, do you have any hypotheses about what might cause the observed distribution?

5. Bonus: show that the probability of observing a given spike train $0 < u_1 < u_2 < \ldots < u_J \leq T$ from an inhomogeneous Poisson process is $\Pr\left(N_0^T\right) = \prod_{i=1}^{N} \lambda\left(u_i\right) \exp\left(-\int_0^T \lambda\left(u_i\right) du\right)$.
Hint: break the spike train down into times without a spike and times with a spike.