Ejercicios A

1. Resuelva cada uno de los siguientes ejercicios, sujetos a las condiciones donde se den.

a. $\frac{dy}{dx} = -\frac{x}{y}$; $y = 2$ donde $x = 1$

→ Integrar
Lado derecho x
Lado izquierdo y

$$dy\,(y) = (-x)\,dx$$

$$\int y\,dy = \int -x\,dx$$ → más una constante

$$\cancel{(2)}\ \frac{y^2}{\cancel{2}} = -\frac{x^2}{\cancel{2}}\ \cancel{(2)} + C$$

Despejar a la constante

$$C = y^2 + x^2$$

Sustituimos $y = 2$, $x = 1$

$$C = (2)^2 + (1)^2$$

$$C = 4 + 1$$

$$\underline{C = 5}$$ valor de la constante.

$$y^2 + x^2 = 5$$ Solución general de la ecuación diferencial.

1. Resuelva cada uno de los siguientes ejercicios, sujetos a las condiciones donde se den.

b. $\frac{dy}{dx} = -\frac{y}{x}$; $y(1) = 3$

$$\frac{dy}{dx} = -\frac{y}{x}$$

→ Lado izquierdo y
→ Lado derecho x

$$\left(-\frac{1}{y}\right) dy = \left(\frac{1}{x}\right) dx$$

→ integrar

$$\int \left(\frac{1}{y}\right) dy = \int \left(\frac{-1}{x}\right) dx$$

$$\ln y = -\ln x + C$$

→ Multiplicar por exponencial

$$\Rightarrow e^{\ln y} = e^{-\ln x} \cdot e^{e}$$

$$\Rightarrow e^{\ln y} = e^{-\ln x} \cdot e^{e}$$ || cuando se esta restando un ln pasa dividiendo

$$\Rightarrow y = \frac{K}{x}$$ || Despejamos a K

$$\Rightarrow K = xy$$ || cuando $y(1) = 3$

$K = (1)(3)$

$K = 3$

$$\Rightarrow \boxed{xy = 3}$$

Solución general

1. Resuelve cada uno de los siguientes ejercicios, sujetos a las condiciones donde se den.

C $3x(y^2+1)dx + y(x^2+2)dy = 0$

$\rightarrow y(x^2+2)dy = -3x(y^2+1)dx$

$\Rightarrow y\left(\frac{1}{y^2+1}\right)dy = -3x\left(\frac{1}{x^2+2}\right)dx$

$\int\left(\frac{y}{y^2+1}\right)dy = \int\left(\frac{-3x}{x^2+2}\right)dx$

$\Rightarrow$ Integrar por cambio de variable

$\rightarrow \int\left(\frac{y}{y^2+1}\right)dy \quad \Big| \quad u = y^2+1 \quad du = 2y\,dy \qquad dy = \frac{du}{2y}$

Sustituimos

$\rightarrow \int\left(\frac{\cancel{y}}{u}\right)\left(\frac{du}{2\cancel{y}}\right)$

$\rightarrow \int\left(\frac{1}{2u}\right)du$ integrar

$\rightarrow \frac{1}{2}\ln|u|+c$

$\int\left(\frac{-3x}{x^2+2}\right)dx$

cambio de variable

$u = x^2+2 \qquad dx = \frac{du}{2x}$

$du = 2x\,dx$

Sustituimos

$\int\left(\frac{-3\cancel{x}}{u}\right)\left(\frac{du}{2\cancel{x}}\right)$

$\int\left(-\frac{3}{2u}\right)du$

Integramos

$\rightarrow -\frac{3}{2}\ln|u|+c$

$\rightarrow \frac{1}{2}\ln|u| = -\frac{3}{2}\ln|u|+c$

$\rightarrow \frac{1}{2}\ln|y^2+1| = -\frac{3}{2}\ln|x^2+2|+c$

$\rightarrow (\ln|y^2+1|)^{1/2} = (\ln|x^2+2|)^{-3/2}+c$

$(\ln|y^2+1|)^{1/2} = (\ln|x^2+2|)^{-3/2}\cdot c$

$\rightarrow e^{(\ln|y^2+1|)^{1/2}} = e^{(\ln|x^2+2|)^{-3/2}}\cdot c$

$((|y^2+1|)^{1/2})^2 = ((|x^2+2|^{-3/2}))^2\cdot c$

$|y^2+1| = |x^2+2|^{\frac{9}{4}}\cdot c$

$$|y^2+1| = |x^2+2|^{-3} \cdot C$$

→ Despejar la constante

$$|y^2+1| = \frac{C}{(|x^2+2|)^3}$$

$$C = \frac{y^2+1}{(x^2+2)^3}$$

$$\boxed{C = \frac{y^2+1}{(x^2+2)^3}}$$ Solución general

→ $(x^2+2)^3(y^2+1) = C$

1. Resuelve cada uno de los siguientes ejercicios, sujetos a las condiciones donde se den.

d) $2y\,dx + e^{-3x}\,dy = 0$

→ Acomodar las variables

$$\left(\frac{1}{2y}\right)dy = -dx\left(\frac{1}{e^{-3x}}\right)$$

$$\rightarrow \left(\frac{1}{2y}\right)dy = \underbrace{\left(\frac{1}{e^{-3x}}\right)}_{\text{cambiar por el signo}}dx$$

Integramos

$$\int\left(\frac{1}{2y}\right)dy = \int\left(e^{3x}\right)dx$$

$$\rightarrow \frac{1}{2}\ln|y| =$$

Cambio de variable

$u = 3x \qquad dx = \frac{du}{3}$

$du = 3\,dx$

$$\int\left(\frac{e^{u}}{3}\right)du$$

$$= \frac{1}{3}\int e^{u}\,du \Rightarrow \frac{1}{3}e^{u}$$

$$\rightarrow \frac{1}{2}\ln|y| = \frac{1}{3}e^{3x} + C$$

$$\ln|y| = -\frac{2}{3}e^{3x} + C$$

$$3\ln|y| = -2e^{3x} + C$$

Despejamos C

$$\rightarrow C = 3\ln|y| + 2e^{3x}$$

$$C = 3\ln|y| + 2e^{3x}$$ Solución general.

1. Resuelva cada uno de los siguientes ejercicios, sujetos a las condiciones donde se den.

e $y' = \frac{x + xy^2}{4y}$; $y(1) = 0$

→ ordenar

$$\frac{dy}{dx} = \frac{x + xy^2}{4y}$$

$$\frac{dy}{dx} = \frac{x(1 + y^2)}{4y}$$

$$\frac{4y\,dy}{(1+y^2)} = x\,dx$$

→ integramos

$$\int \frac{4y\,dy}{(1+y^2)} = \int x\,dx$$

→ Cambio de variable

$u = 1 + y^2$ $\quad dy = \frac{du}{2y}$

$du = 2y\,dy$

$$= \int \frac{4y}{u}\frac{du}{2y} = 2\int \frac{du}{u}$$

$2\ln|u| + c$

→ $2\ln|1+y^2| = \frac{x^2}{2} + C$ | Multiplicar por (2)

→ $(2)(2)\ln|1+y^2| = \left(\frac{x^2}{2}\right)(2) + C$

→ $4\ln|1+y^2| = x^2 + C$

→ $C = 4\ln|1+y^2| - x^2$

sustituimos valores

$C = 4\ln|1 + 0| - x^2$

$C = 4\ln|1| - x^2$

$C = -x^2$

$C = -1$

Sustituimos el valor de C

$C = 1$

$$1 = 4 \ln|1 - y^2| - x^2$$

$$(-1)(-1) = (-1)(4 \ln|1 - y^2| - x^2)$$

$$1 = -4 \ln|1 - y^2| + x^2$$

$$1 = -4 \ln|1 - y^2| + x^2$$ Solución general

$\therefore x^2 - 4 \ln(1 + y^2) = 1$

1. Resuelva cada uno de los siguientes ejercicios, sujetos a las condiciones donde se den.

$\boxed{F}$ $r\dfrac{d\phi}{dr} = \phi^2 + 1$

⇒ Acomodar

$$d\phi\left(\frac{1}{\phi^2+1}\right) = \left(\frac{1}{r}\right)dr$$

⇒ Integramos

$$\int\left(\frac{1}{\phi^2+1}\right)d\phi = \int\left(\frac{1}{r}\right)dr$$

$$\Rightarrow \arctan(\phi) = \ln|r| + C$$

Despejamos la constante

$$\Rightarrow C = \arctan(\phi) - \ln|r|$$

$$C = \arctan(\phi) - \ln|r|$$

1. Resuelva cada uno de los siguientes ejercicios, sujetos a las condiciones donde se den.

9 $\operatorname{sen}^2 y\, dx + \cos^2 x\, dy = 0;\quad y\frac{\pi}{0^4} = \frac{\pi}{4}$

⇒ ordenar

$$\operatorname{sen}^2 y\, dx + \cos^2 x\, dy = 0$$

$$dx(\operatorname{sen}^2 y\, dx + \cos^2 x\, dy) = dx\,(0)$$

$$\operatorname{sen}^2 y + \cos^2 x \frac{dy}{dx} = 0$$

$$\operatorname{sen}^2 y = -\cos^2 x \frac{dy}{dx}$$

$$\operatorname{sen}^2 y\, dx = -\cos^2 x\, dy$$

$$\Rightarrow \left(\frac{-1}{\cos^2 x}\right) dx = \left(\frac{1}{\operatorname{sen}^2 y}\right) dy$$

→ integrar

$$\int \left(\frac{-1}{\cos^2 x}\right) dx = \int \left(\frac{1}{\operatorname{sen}^2 y}\right) dy$$

$$\Rightarrow \int (-\sec^2 x)\, dx = \int (\csc^2 y)\, dy$$

$$\Rightarrow -\tan(x) = -\cot(x) + C$$

→ Despejar Constante

$$-\tan(x) = \cot(y) + C$$

$$C = -\tan(x) - \cot(y)$$

Sustituimos

$$C = -\tan(0) - \cot\left(\frac{\pi}{4}\right)$$

$$C = 1 - 0$$

$$C = 1$$

$$\boxed{C = 1}$$

$$-\tan(x) = \cot(y) + 1$$

$$-\tan(x) - \cot(y) = 1$$

$$\therefore \tan x \tan y = 1$$

$$\therefore \sqrt{1+x^2} - \sqrt{1+y^2} = C$$

solución general

1. Resuelva cada uno de los siguientes ejercicios, sujetos a las condiciones donde se den.

h $x\sqrt{1+y^2}\,dx = y\sqrt{1+x^2}\,dy$

⇒ ordenar

$$\left(\frac{1}{\sqrt{1+x^2}}\right)x\,dx = \left(\frac{1}{\sqrt{1+y^2}}\right)y\,dy$$

⇒ integrar

$$\int\left(\frac{1}{\sqrt{1+x^2}}\right)x\,dx = \int\left(\frac{1}{\sqrt{1+y^2}}\right)y\,dy$$

Cambio de variable

$u = 1+x^2 \quad dx = \frac{du}{2x}$

$du = 2x\,dx$

$$\int\left(\frac{1}{\sqrt{u}}\right)\cancel{x}\,\frac{du}{2\cancel{x}}$$

$$\frac{1}{2}\int\frac{1}{\sqrt{u}}\,du \;\Big\|\; \frac{1}{2}\int u^{-\frac{1}{2}}\,du \;\Big\|\; \frac{1}{2}\,\frac{u^{1/2}}{\frac{1}{2}} \;\Big\|\; u^{1/2} \Rightarrow (1+x^2)^{\frac{1}{2}}$$

Cambio de variable

$u = 1+y^2 \quad dy = \frac{du}{2y}$

$du = 2y\,dy$

$$\int\left(\frac{1}{\sqrt{u}}\right)\cancel{y}\,\frac{du}{2\cancel{y}} \;\Big\|\; \frac{1}{2}\int\left(\frac{1}{\sqrt{u}}\right)du \;\Big\|\; u^{1/2}$$

$$(1+x^2)^{\frac{1}{2}} = (1+y^2)^{\frac{1}{2}} + C$$

$$\sqrt{1+x^2} = \sqrt{1+y^2} + C$$

⇒ despejar C

$$\sqrt{1+x^2} = \sqrt{1+y^2} + C$$

$$C = \sqrt{1+x^2} - \sqrt{1+y^2}$$

$$C = \sqrt{1+x^2} - \sqrt{1+y^2}$$ solución general

1. Resuelva cada uno de los problemas, sujetos a las condiciones donde se den

i) $2y\cos x\,dx + 3\operatorname{sen} x\,dy = 0; \quad y\left(\frac{\pi}{2}\right) = 2$

⇒ ordenar

$$2y\cos x\,dx + 3\operatorname{sen}x\,dy = 0$$

$$3\operatorname{sen}x\,dy = -2y\cos x\,dx$$

$$\left(\frac{1}{\cos x}\right)3\,dy = -2y\left(\frac{1}{\operatorname{sen}x}\right)dx$$

$$\Rightarrow \left(-\frac{3}{2y}\right)dy = \left(\frac{1}{\operatorname{sen}x}\right)(\cos x)\,dx$$

⇒ integrar

$$\int\left(-\frac{3}{2y}\right)dy = \int\frac{1}{\operatorname{sen}x}(\cos x)\,dx$$

$$\| -\frac{3}{2}\int\frac{1}{y}\,dy = \int\frac{1}{\operatorname{sen}x}(\cos x)\,dx \Big\}\ \text{identidad}$$

$$\| -\frac{3}{2}\ln|y| = \int\cot g(x)\,dx$$

$$\| -\frac{3}{2}\ln|y| = \ln|\operatorname{sen}(x)| + C$$

$\Rightarrow -\frac{3}{2} \ln|y| = \ln|\operatorname{sen}(x)| + C$

$(\ln|y|)^{-3/2} = \ln|\operatorname{sen}(x)| + C$

$e^{(\ln|y|)^{-3/2})} = e^{\ln|\operatorname{sen}(x)|} \cdot C$

$(y)^{-3/2} = \operatorname{Sen}(x)$

Elevar al cuadrado

$((y)^{-3/2})^2 = (\operatorname{sen}(x))^2 + C$

$(y)^{-3} = \operatorname{sen}(x)2 \cdot C$

$\frac{1}{y^3} = \operatorname{sen}(x)^2 + C$

$\Rightarrow$ despejar C

$C = \operatorname{sen}(x)^2 \cdot (y)^3$

$\Rightarrow$ Evaluamos $y(\frac{\pi}{2}) = 2$

$C = \operatorname{sen}(\frac{\pi}{2})^2 \cdot (2)^3$

$C = \operatorname{sen}(\frac{\pi}{2})^2 (8)$

$C = (1)(8)$

$C = 8$

Sustituimos

$8 = \operatorname{sen}(x)^2 \cdot (y)^3$

$8 = \operatorname{sen}(x)^2 \ (y)^3$

Solución general

1. Resuelva cada uno de los siguientes ejercicios, sujetos a las condiciones donde se den.

j $y' = 8xy + 3y$

$$\frac{dy}{dx} = 8xy + 3y$$

$$\frac{dy}{dx} = y(8x + 3)$$

$$dy\left(\frac{1}{y}\right) = (8x + 3)\,dx$$

⇒ integrar

$$\int \left(\frac{1}{y}\right) dy = \int (8x + 3)\,dx$$

$$\ln|y| = \int 8x\,dx + 3\int dx$$

$$\ln|y| = 4x^2 + 3x + C$$

$$e^{\ln|y|} = e^{4x^2+3x} \cdot C$$

$$y = e^{4x^2+3x} \cdot C$$

⇒ despejar C

$$C = \frac{y}{e^{4x^2+3x}} \quad \Big\| \quad y = C \cdot e^{4x^2+3x}$$

$$\boxed{y = C \cdot e^{4x^2+3x}}$$ solución general.

1. Resuelva cada uno de los siguientes ejercicios, sujetos a las condiciones donde se den.

$\boxed{1}$ $y\,dx + (x^3y^2 + x^3)dy = 0$

⇒ Acomodar

$$y\,dx + x^3(y^2+1)dy = 0$$

$$x^3(y^2+1)dy = -y\,dx$$

$$\frac{(y^2+1)\,dy}{-y} = \left(\frac{1}{x^3}\right)dx$$

⇒ Integrar

$$\left(\frac{y^2}{-y} - \frac{1}{y}\right)dy = \left(\frac{1}{x^3}\right)dx$$

$$\int \frac{-y^2}{y}\,dy - \int \frac{1}{y}\,dy = \int \frac{1}{x^3}\,dx$$

$$\int -y\,dy - \int \frac{1}{y}\,dy = \int \frac{1}{x^3}\,dx$$

$$-\frac{y^2}{2} - \ln|y| = \int x^{-3}\,dx$$

$$-\frac{y^2}{2} - \ln|y| = \frac{x^{-2}}{2} + C$$

$$\cancel{(2)}\left(\frac{-y^2}{\cancel{2}}\right) - \ln|y| = \left(\frac{1}{\cancel{2}x^2}\right)\cancel{(2)} + C \quad \| \text{ Multiplicar } (-2)$$

$$-y^2 - \ln|y| = \frac{1}{x^2} + C$$

⇒ despejar

$$-y^2 - 2\ln|y| = \frac{1}{x^2} + C$$

$$C = -y^2 - 2\ln|y| - \frac{1}{x^2}$$

$$C = -y^2 - 2\ln|y| - \frac{1}{x^2}$$ Solución general.

Ejercicios B

Resuelva cada uno de los siguientes ejercicios.

1. $$\frac{dy}{dx} = \frac{(y-1)(x-2)(y+3)}{(x-1)(y-2)(x+3)}$$

⇒ Agruparlos

$$\int \left(\frac{y-2}{(y-1)(y+3)}\right) dy = \frac{(x-2)}{(x-1)(x+3)}\, dx$$

⇒ Fracciones Parciales

$$\int \left(\frac{y-2}{(y-1)(y+3)}\right) dy$$

$$\frac{A}{(y-1)} + \frac{B}{(y+3)} = \frac{A(y+3) - B(y-1)}{(y-1)(y+3)}$$

$$Ay + 3A - By + B = y - 2$$

$$Ay - By = 1$$

$$3A + B = -2$$

$$\begin{aligned} A - B &= 1 \\ 3A + B &= -2 \\ 4A &= -1 \\ A &= -\frac{1}{4} \end{aligned}$$

$$\begin{aligned} -\frac{1}{4} - B &= 1 \\ -B &= 1 + \frac{1}{4} \\ -B &= \frac{5}{4} \\ B &= -\frac{5}{4} \end{aligned}$$

$$\Rightarrow -\frac{1}{4} \ln|y-1| + \frac{5}{4} \ln|y+3|$$

Fracciones Parciales

$$-\frac{1}{4}\ln|x-1| + \frac{5}{4}\ln|x+3| + C$$

$$-\frac{1}{4}\ln|y-1| + \frac{5}{4}\ln|y+3| = -\frac{1}{4}\ln|x-1| + \frac{5}{4}\ln|x+3| + C$$

exponencial

$$(\ln|y-1|)^{-\frac{1}{4}} + (\ln|y+3|)^{5/4} = (\ln|x-1|)^{1/4} + (\ln|x+3|)^{5/4} \cdot C$$

$$e^{(\ln|y-1|)^{-\frac{1}{4}}} + e^{(\ln|y+3|)^{5/4}}$$

$$(y-1)^{-\frac{1}{4}} + (y+3)^{5/4} = (x-1)^{-\frac{1}{4}} + (x+3)^{5/4}$$

$$((y-1)^{-\frac{1}{\cancel{4}}})^{\cancel{4}} + ((y+3)^{5/\cancel{4}})^{\cancel{4}} = ((x-1)^{-\frac{1}{\cancel{4}}})^{\cancel{4}} + ((x+3)^{5/\cancel{4}})^{\cancel{4}}$$

$$(y-1)^{-1} + (y+3)5 = (x-1)^{-1} + (x+3)^5$$

$$\frac{(y+3)^5}{(y-1)} = \frac{(x+3)^5}{(x-1)} \cdot C$$

$$\boxed{\frac{(y+3)^5}{(y-1)} = \frac{(x+3)^5}{(x-1)} \cdot C}$$ Solución general.

2. Resuelva cada uno de los siguientes ejercicios.

$$\frac{\partial r}{d\phi} = \frac{\operatorname{sen}\phi + e^{2r}\operatorname{sen}\phi}{3e^{r} + e\cos 2\phi} \quad re = 0 \text{ donde } \phi = \frac{\pi}{2}$$

* Acomodar

$$\rightarrow \frac{dr}{d\phi} = \frac{\operatorname{sen}\phi + e^{2r}\operatorname{sen}\phi}{3e^{r} + e^{r}\cos 2\phi}$$

$$\frac{dr}{d\phi} = \frac{\operatorname{sen}\phi\,(1 + e^{2r})}{e^{r}\,(3 + \cos 2\phi)}$$

$$\frac{e^{r}}{(1+e^{2r})}\,dr = \frac{\operatorname{sen}\phi}{(3+\cos 2\phi)}\,d\phi$$

* Integrar

$$\int \frac{e^{r}}{(1+e^{2r})}\,dr \quad \Bigg| \quad u = e^{r} \quad e^{2r} \quad dr = \frac{du}{e^{r}} \quad du = e^{r}\,dr$$

$$\int \frac{u e^{r}}{(1+e^{r})}\frac{du}{e^{r}} = \int \frac{1}{(1+e^{r})}\,du = \int \frac{1}{(1+u^{2})}\,du$$

$$\Rightarrow \arctan(u) + C$$

$$= \arctan(e^{r}) + C$$

Propiedad trigonometrica

$$\int \frac{\operatorname{sen}\phi}{(3+\cos 2\phi)}\,d\phi \quad \Bigg\| \quad \int \frac{\operatorname{sen}(\phi)}{3 - 1 + 2\cos^{2}(\phi)}\,d\phi$$

$$\Rightarrow \cos 2\phi = -1 + 2\cos^{2}(\phi)$$

Propiedad trigonometrica.

→ Cambio de variable

$$\int \frac{\operatorname{sen}(\phi)}{3-1+2\cos^2(\phi)}\, d\phi$$

Cambio de variable

$u = \cos\phi \qquad d\phi = \frac{du}{-\operatorname{sen}\phi}$

$du = -\operatorname{sen}\phi\, d\phi$

$$-\int \frac{-1}{2u^2+2}\, du \quad \int \frac{1}{2(u^2+1)} = \frac{1}{2}\int \frac{1}{(u^2+1)} \Big\}\text{ directa}$$

$$\frac{1}{2}\arctan(\cos(\phi)) + C$$

Igualamos las integrales

$$\arctan(u) = \frac{1}{2}\arctan(\cos\phi) + C$$

Despejamos a C

$$C = \arctan(e^r) - \frac{1}{2}\arctan(\cos\phi)$$

valores

$$C = \left(\arctan(e^r) - \frac{1}{2}\arctan\left(\cos\frac{\pi}{2}\right)\right)(2)$$

$$C = 2\arctan(e^1) - \arctan(\cos\pi)$$

$$C = 2\arctan(1) - \arctan(\cos\pi)$$

$$C = \frac{\pi}{2}$$

$$\boxed{C = \frac{\pi}{2}}$$

$$\Rightarrow 2\arctan(e^1) + \arctan(\cos\phi) = \frac{\pi}{2}$$ Solución general

Resuelva cada uno de los siguientes ejercicios.

$\boxed{3.}$ $x^3 e^{2x^2+3y^2}dx - y^3 e^{-x^2-2y^2}dy = 0$

⇒ Acomodar

$$x^3 e^{2x^2} \cdot e^{3y^2} dy - y^3 e^{-x^2} \cdot e^{-2y^2} dy = 0$$

$$y^3 e^{-x^2} \cdot e^{-2y^2} dy = x^3 e^{2x^2} \cdot e^{3y^2} dx$$

⇒ Integrar

$$\int \frac{y^3 \cdot e^{-2y^2} dy}{e^{2y^2}} = \int \frac{x^3 e^{2x^2}}{e^{-x^2}} dx$$

$$\int y^3 \cdot e^{-5y^2} dy = \int x^3 \cdot e^{3x^2} dx$$

* Para y: $u = y^2$, $du = 2y\,dy$ | ¦ $\frac{1}{2}\int u e^{-5u} du$ ★ | ★ Por partes: $p = u$; $dr = e^{-5u}$; $dp = du$; $r = \frac{-e^{-5u}}{5}$

$$\frac{-ue^{-5u}}{5} + \int \frac{e^{-5u}}{5} du = \frac{-ue^{-5u}}{5} - \frac{e^{-5u}}{25} + C$$

¦★ $\frac{1}{2}\int u e^{-5u} du \Rightarrow \frac{ue^{-5u}}{10} - \frac{e^{-5u}}{50} + C$

¦★ $\frac{-y^2 e^{-5y^2}}{10} - \frac{e^{-5y^2}}{50} + C = \frac{-50y^2 e^{-5y^2} - 10e^{-5y^2}}{500}$

$$\Rightarrow \frac{-5y^2 e^{-5y^2} - e^{-5y^2}}{50} + C$$

★ Para x

$u = x^2$
$du = 2x\,dx$

$$\frac{1}{2}\int u e^{3u}\,du$$

★ Por partes

$p = u;\ dr = e^{3u}$
$dp = du;\ r = \frac{e^{3u}}{3}$

$$\frac{ue^{3u}}{3} - \int \frac{e^{3u}}{3}\,du \Rightarrow \frac{ue^{3u}}{3} - \frac{e^{3u}}{9} + C$$

$$\frac{1}{2}\int u e^{3u}\,dx \Rightarrow \frac{ue^{3u}}{6} - \frac{e^{3u}}{18} + C$$

$$\frac{x^2 e^{3x^2}}{6} - \frac{e^{3x^2}}{18} + C = \frac{(3x^2-1)e^{3x^2}}{18} + C$$

$$\Rightarrow \frac{(-5y^2-1)e^{-5y^2}}{50} = \frac{(3x^2-1)e^{3x^2}}{18} + C$$

$$18(-5y^2-1)e^{-5y^2} = 50(3x^2-1)e^{-3x^2} = C$$

★ Dividimos entre -2

$$9(5y^2+1)e^{-5y^2} + 25(3x^2-1)e^{-3x^2} = C$$

$$9(5y^2+1)e^{-5y^2} + 25(3x^2-1)e^{-3x^2} = C$$

4. Resuelva cada uno de los siguientes ejercicios

$$\frac{du}{ds} = \frac{u+1}{\sqrt{s}+\sqrt{su}} \star$$

→ Sacar el factor común

$$\frac{du}{ds} = \frac{u+1}{\sqrt{s}\,(1+\sqrt{u})}$$

$$\frac{1+\sqrt{u}}{u+1}\,du = \frac{1}{\sqrt{s}}\,ds$$

$\star$

→ Integrar

$$\int \frac{1+\sqrt{u}}{u+1}\,du = \int \frac{1}{\sqrt{s}}\,ds$$

⇒ Separar en suma de fracciones

$$\int \frac{1}{u+1} + \int \frac{\sqrt{u}}{u+1} = \int \frac{1}{\sqrt{s}}\,ds$$

Cambio de variable

$$\int \frac{\sqrt{u}}{u+1}du \;\Big\|\; y=\sqrt{u} \quad dy = \frac{1}{2\sqrt{u}}du \qquad du = 2\sqrt{u}\,dy \quad y^2 = u \;\Big\|\; \int \frac{\sqrt{u}\;2\sqrt{u}\;dy}{1+y^2}$$

$$2\int \frac{y^2}{1+y^2}dy \Longrightarrow 2\int \frac{-1}{1+y^2}dy + \int 1\,dy$$

$$2(-\operatorname{arctn}(y) + (y)) + C$$

Primer resultado de integral izquierda

$$\Rightarrow 2((-\operatorname{arctn}(\sqrt{u}) + \sqrt{u}))_{[1]}$$

$$\ln|u+1| + 2((-\operatorname{arctn}(\sqrt{u}) + \sqrt{u}))_{[3]}$$

Integral de $\int \frac{1}{u+1}$ = integral directa

$$= \ln|u+1|$$

$$= \ln|u+1|_{[2]}$$

Para la integral de la derecha

$$\int \frac{1}{\sqrt{S}}\, ds$$

$$\Rightarrow \int (S)^{-1/2}\, ds$$

$$\Rightarrow \int \frac{(S)^{-1/2+1}}{-\frac{1}{2}+1}\, ds$$

$$\Rightarrow \frac{(S)^{1/2}}{\frac{1}{2}}$$

Acomodando

$$\Rightarrow 2\,(S)^{1/2} + C$$

$$2\sqrt{S} + C$$

Igualamos ambos resultados

$$\ln|u+1| + 2((-\arctan(\sqrt{u}) + \sqrt{u})) = 2\sqrt{S} + C$$

$$C = \ln|u+1| + 2((-\arctan(\sqrt{u}) + (\sqrt{u})) - 2\sqrt{S}$$

$$C = \ln|u+1| + 2((-\arctan(\sqrt{u}) + (\sqrt{u})) - 2\sqrt{S}$$ Solución general.