Ejercicios A

1. Resuelva cada uno de los siguientes ejercicios

1. $\frac{dy}{dx} = 1 + \frac{y}{x}$

— ★ Cambio de variable

$$u = \frac{y}{x}$$

$$\frac{dx}{x} = \frac{du}{1+u-u}$$

— ★ Tenemos

$$\int \frac{dx}{x} = \int du$$

$$\ln x = u + C$$

$$\ln x = \frac{y}{x} + C$$

$$\ln x = \frac{y + xC}{x}$$

Respuesta: $y = x \ln x + xC$

Ejercicios A

Resolver cada uno de los siguientes ejercicios

3. $xy' = 2x + 3y$

Cambio de variable

$$u = \frac{y}{x}$$

$$y' = \frac{2x + 3y}{x}$$

$$y' = 2 + 3\frac{y}{x}$$

$$y' = 2 + 3u$$

$$\frac{dx}{x} = \frac{du}{2+3u-u}$$

★ Integramos

$$\int \frac{dx}{x} = \int \frac{du}{2(1+u)}$$

$v = 1 + u$
$dv = du$

Cambio de variable

$$\ln x = \frac{1}{2}\ln(v) + c$$

$$x^2 = \left(1 + \frac{y}{x}\right) + c$$

$$x^2 - \frac{y}{x} = c$$

$$\frac{x^3 - y}{x} = c$$

$$\frac{x^3 - y - xc}{x} = 0$$

Respuesta: $y = x^3 - xc$

||Ejercicios A

Resolver los siguientes ejercicios

5. $(x+2y)dx+(2x+y)dy=0$

$(1+2\frac{y}{x})dx+(2+\frac{y}{x})dy=0$

$(1+2u)dx+(2+u)dy=0$

$(2+u)dy=-(1+2u)dx$

$\frac{dy}{dx}=-\frac{1+2y}{2+y}$

$\frac{dx}{x}=\frac{du}{-\frac{1+2u}{2+u}-u}=\int\frac{dx}{x}=\int\frac{2+u}{u+3}du$

$\int\frac{2}{u-3}du+\int\frac{u}{u-3}du$

||Cambio de variable

$v=u-3$

$dv=du$

$\int\frac{v+3}{v}dv=\int dv+\int\frac{3dv}{3}=v+3\ln v+C$

$\int\frac{2}{u-3}=2\ln(v)$

$\Rightarrow \ln x=2\ln(v)+v+3\ln v+C$

exponencial

$e^{\ln x}=e^{2\ln v+3\ln v+v+C}$

Ejercicios A

Resuelva cada uno de los siguientes ejercicios:

Fórmula

$$\frac{dx}{x} = \frac{du}{fu} - u$$

7. $xy' = y - \sqrt{x^2+y^2}$

$$y' = \frac{y - \sqrt{x^2+y^2}}{x}$$

Acomodar:

$$y' = \frac{y}{x} - \sqrt{1+\left(\frac{y}{x}\right)^2}$$

//Cambio de variable

$$u = \frac{y}{x}$$

//sustituir

$$y' = \boxed{u - \sqrt{1+(u)^2}} \rightarrow fu$$

//sustituir en la formula

$$\frac{dx}{x} = \frac{du}{\cancel{u} - \sqrt{1+(u)^2}} \cancel{-u}$$

$$\frac{dx}{x} = \frac{du}{-\sqrt{1+(u)^2}}$$

//Integrar

$$\int \frac{dx}{x} = \int \frac{du}{-\sqrt{1+(u)^2}}$$

$$\int \frac{dx}{x} = -\int \frac{du}{\sqrt{1+(u)^2}}$$

$$\ln|x| = -\ln\left|\sqrt{1+u^2} + u\right| + C$$

//Cambiar $u = \frac{y}{x}$

$$\ln|x| = -\ln\left|\sqrt{1+\left(\frac{y}{x}\right)^2} + \left(\frac{y}{x}\right)\right| + C$$

$$\boxed{-\ln|x| = \ln\left|\sqrt{1+\left(\frac{y}{x}\right)^2} + \left(\frac{y}{x}\right)\right| + C}$$

$$-\ln|x| = \ln\left|\sqrt{1+\left(\frac{y}{x}\right)^2} + \left(\frac{y}{x}\right)\right| + C$$

$$-\ln|x| + C = \ln\left|\sqrt{1+\left(\frac{y}{x}\right)^2} + \left(\frac{y}{x}\right)\right|$$

Usamos exponencial

$$e^{-\ln|x|+C} = e^{\ln\left|\sqrt{1+\left(\frac{y}{x}\right)^2} + \left(\frac{y}{x}\right)\right|}$$

$$\Rightarrow \frac{1}{x}\cdot C = \sqrt{1+\left(\frac{y}{x}\right)^2} + \left(\frac{y}{x}\right)$$

$$\Rightarrow y = \sqrt{x^2+y^2} = \frac{C}{x}$$

$$\Rightarrow y + \sqrt{x^2+y^2} = C$$

Despejando C nos queda:

$$C = y + \sqrt{x^2+y^2}$$

Respuesta: $C = y + \sqrt{x^2+y^2}$

Resuelva cada uno de los siguientes ejercicios:

9. $(x^3+y^3)dx - xy^2 dy = 0 \quad y(1)=0$

$$\frac{dy}{dx} = \frac{(x^3+y^3)}{xy^2} \quad \ldots (1)$$

$$y = ux$$

$$\frac{dy}{dx} = u + \frac{du}{dx}$$

$$u + x\frac{du}{dx} = \frac{x^3+u^3x^3}{x^3u^2}$$

$$u + x\frac{du}{dx} = \frac{1+u^3}{u^2} - u$$

$$x\frac{du}{dx} = \frac{1+u^3}{u^2} - u^3$$

$$\frac{x\,du}{dx} = \frac{1}{u^2}$$

$$u^2 du = \frac{dx}{dx}$$

$$\frac{u^3}{3} = \ln x + \ln c$$

$$u^3 = 3\ln x + \ln c$$

$$\frac{y^3}{x^3} = \ln x^3 + \ln c$$

// sustituir a $u = \frac{y}{x}$

// $\left(\frac{y}{x}\right)^3 = 3\ln|x| + C$

// Despejar a y^3

$$y^3 = 3(x^3)\ln|x| + C$$

Respuesta: $y^3 = 3(x^3)\ln|x| + C$!

Ejercicios A

Resuelva cada uno de los siguientes ejercicios:

11. $y' = \frac{y}{x} + \sec^2 \frac{y}{x}$

Formula

// $\frac{dx}{x} = \frac{du}{fu} - u$

// sustituimos

$u = \frac{y}{x}$

$\Rightarrow y' = \boxed{u + \sec^2 u} \} fu$

//sustituir en la formula

$\frac{dx}{x} = \frac{du}{\cancel{u} + \sec^2 u} \cancel{-u}$

$\frac{dx}{x} = \frac{du}{\sec^2 u}$

//Integrar de ambos lados

$\int \frac{dx}{x} = \int \frac{du}{\sec^2 u}$

Integral directa

$\ln|x| = \boxed{\int \frac{1 + \cos(2u)}{2} du} \}$ identidad trigonometrica

Sacamos constante

$\ln|x| = \frac{1}{2} \int \frac{\cos(2u)}{2} du$

$\Rightarrow \ln|x| = \frac{1}{2}\left(u + \frac{1}{2}\operatorname{sen}(2u)\right) + C$

$\Rightarrow \ln|x| = \frac{1}{4} u + \operatorname{sen}(2u) + C$

Cambiar $u = \frac{y}{x}$

$\Rightarrow \ln|x| = \frac{1}{4}\left(\frac{y}{x}\right) + \operatorname{sen}\left(2\left(\frac{y}{x}\right)\right) + C$

$2\left(\frac{y}{x}\right) + \operatorname{sen} 2\left(\frac{y}{x}\right) = 4 \ln|x|$

Respuesta: $2\left(\frac{y}{x}\right) + \operatorname{sen} 2\left(\frac{y}{x}\right) = 4 \ln|x|$

Ejercicios B

Resuelva los siguientes ejercicios

3. $\frac{dy}{dx} = \frac{6x^2 - 5xy - 2y^2}{6x^2 - 8xy + y^2}$ ★

$\frac{dy}{dx} = \frac{6x^2 - 5xy - 2y^2}{6x^2 - 8xy + y^2}$ || Simplificar

$= \left[\frac{6x^2 - 5xy - 2y^2}{6x^2 - 8xy + y^2}\right] \cdot \frac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)}$ | $f\left(\frac{y}{x}\right) = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{6 - 5\left(\frac{y}{x}\right) - 2\left(\frac{y^2}{x^2}\right)}{6 - 8\left(\frac{y}{x}\right) - \left(\frac{y}{x}\right)^2}$ $\quad u = \frac{y}{x}$

$\frac{dy}{dx} = \frac{6 - 5u - 2u^2}{6 - 8u + u^2} = f(u) \rightarrow \frac{dx}{x} = \frac{du}{f(u) - u}$

$\frac{du}{f(u) - u} = \frac{du}{\frac{6 - 5u - 2u^2}{6 - 8u + u^2} - u} \Longrightarrow \frac{dx}{x} = \frac{6 - 8u + u^2}{6 - 5u - 2u^2 - 6u + 8u^2 - u^3}$

$\Rightarrow \frac{dx}{x} = \frac{u^2 - 8u + 6}{-u^3 + 6u^2 - 11u + 6} du \Rightarrow \int \frac{dx}{x} = \int \frac{u^2 - 8u + 6}{(u-1)(u-2)(u-3)} du$

★ fracciones parciales

$\Rightarrow \int \frac{A}{(u-1)} + \int \frac{B}{(u-2)} + \int \frac{C}{(u-3)} = -\int \frac{du}{2(u-1)} + \int \frac{6}{(u-2)} du - \int \frac{9}{2(u-3)} du$

$= \frac{1}{2} \ln|u-1| - 6 \ln|u-2| + \frac{9}{2} \ln|u-3| + C$

Resolvemos el siguiente sistema de ecuaciones

$$A + B + C = 1$$
$$-5A - 4B - C = -B$$
$$6A + 3B + 2C = 6$$

★ Resolvemos el sistema por medio de Gauss Jordan

$$\begin{bmatrix} 1 & 1 & 1 \\ -5 & -4 & -1 \\ 6 & 3 & 2 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ -5 & -4 & -1 & -1 \\ 6 & 3 & 2 & 6 \end{array}\right]$$

★ Matriz de cofactores

Matriz aumentada

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 2 & -9 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & -\frac{9}{2} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & -\frac{9}{2} \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & -4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -\frac{9}{2} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -\frac{9}{2} \end{array}\right]$$

$A = -\frac{1}{2}$

$B = 6$

$C = -9/2$

$$\Rightarrow \int \frac{A}{(u-1)} + \int \frac{B}{(u-2)} + \int \frac{C}{(u-3)}$$

$$\Rightarrow \int -\frac{1}{2} \frac{du}{(u-1)} + \int \frac{6\,du}{(u-2)} - \int \frac{9}{2} \frac{du}{(u-3)}$$

$$\Rightarrow \ln x = \frac{1}{2} \ln|u-1| - \ln|u-2) + \frac{9}{2} \ln(u-3) + C$$

Ejercicios B

Resuelva:

5. $y' = \sqrt{2x+3y}$

$$\frac{y'}{x} = \sqrt{\frac{\cancel{2x}}{\cancel{x}} + \frac{3y}{x}}$$

$$\frac{y'}{x} = \sqrt{2 + \frac{3y}{x}} \quad || \quad \frac{dy}{dx} = \sqrt{2x+3y}$$

//Cambio de variable

$$u = \sqrt{2x+3y}$$

$$u^2 = 2x+3y$$ //Derivar de ambos lados

$$2u\frac{du}{dx} = 2 + 3\frac{dy}{dx}$$

//Despejamos

$$2u\frac{du}{dx} = 2+3u$$

//Cambiar las u al lado izquierdo.

$$2u\frac{du}{dx} - 2 = 3u$$

$$\frac{2}{3}u\frac{du}{dx} - \frac{2}{3} = u$$

$$\frac{2}{3}u\frac{du}{dx} = u + \frac{2}{3}$$

$$\frac{2}{3}u\frac{du}{dx} = \left(u + \frac{2}{3}\right)dx$$

$$\left(\frac{2}{3}\left(\frac{u}{u+\frac{2}{3}}\right)\right) du = dx$$

$\left(\frac{2}{3}\left(\frac{u}{u+\frac{2}{3}}\right)\right)du = dx$

variables separadas

Integrar de amboo lados

$\int \frac{2}{3}\left(\frac{u}{u+\frac{2}{3}}\right)du = \int dx$

Cambio de variable

$\frac{2}{3}\int\left(\frac{u}{u+\frac{2}{3}}\right)du$

$w = u + \frac{2}{3}$

$w - \frac{2}{3} = u$

$\frac{2}{3}\int \frac{w-\frac{2}{3}}{w}\, dw$

$= \frac{2}{3}\left(\int dw - \frac{2}{3}\int \frac{dw}{w}\right)$

$= \frac{2}{3}\left(w - \frac{2}{3}\ln(w)\right)$

$\frac{2}{3}\left(\int dw - \frac{2}{3}\int \frac{dw}{w}\right)$

$\boxed{\int dx = x + C_0}$

$\frac{2}{3}\left(w - \frac{2}{3}\ln(w)\right) = \boxed{x + C_0}$ → integral directa

$\frac{2}{3}w - \frac{4}{9}\ln(w) = x + C$ | multiplicar por 9 toda la expreoión

$6w - 4\ln(w) = 9x + C$

$6\left(u + \frac{2}{3}\right) - 4\ln\left(u + \frac{2}{3}\right) = 9x + C$ || cambio de variable

$6\left(\sqrt{2x+3y} + \frac{2}{3}\right) - 4\ln\left(\sqrt{2x+3y} + \frac{2}{3}\right) = 9x + C$

||sumar fraccioneo

$6(3\sqrt{2x+3y} + 2 - 4\ln(3\sqrt{2x+3y} + 2 - 9x = C$

Reopucota: $6(3\sqrt{2x+3y} + 2 - 4\ln(3\sqrt{2x+3y} + 2 - 9x = C$

Ejercicios B

Resuelva:

9. $(2x+2y+1)dx+(x+y-1)dy=0$

Acomodar $\frac{dy}{dx}$

Lado izquierdo dy

$$(x+y-1)\,dy = -(2x+2y+1)dx$$

$$\frac{dy}{dx} = -\frac{(2x+2y+1)}{(x+y-1)}$$

Cambio de variable

$v = x+y$

$dv = 1 + \frac{dy}{dx}$

Sustituir

$$\frac{dy}{dx} = -\frac{2v+1}{v-1}$$

Despejamos dy

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

★ sustituimos

$$\frac{dv}{dx} - 1 = \frac{-2v+1}{v-1} \Rightarrow \text{Quitar el } (-1)$$

$$\frac{dv}{dx} = \frac{-1-2v+v-1}{v-1}$$

★ Simplificamos

$$\frac{dv}{dx} = \frac{-v-2}{v-1}$$

★ Integramos

$$-\int \frac{v-1}{v-2} = \int dx$$

$$-\int \frac{v}{v-2} + \int \frac{1}{v-2} = x$$

★ Cambio de variable.

$w = v-2$; $w+2 = v$

$dw = dv$

$$-\int \frac{w+2}{w}\,dw = \int dw + 2\int \frac{dw}{w}$$

$$= w + 2\ln(w)$$

$w + 2\ln(w) = x$

$v - 2 + 2\ln(v-2) = x$

$x + y - 2 + 2\ln(x+y-2) = x$

★ **Respuesta: $x+y-2+2\ln(x+y-2) = x$**

Ejersicios C

Resuelva: (a) 1

(a) $\dfrac{dy}{dx} = \dfrac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}}$

$= x + y$

Cambio de variable

$u^2 = x - y$

$u u' = 1 + y' \quad 2vv' = 1 - y'$

$y' = 2uu' - 1$

$y' = 1 - 2vv' \qquad 2y' = 2(uu' - vv')$

$y' = \dfrac{u+v}{u-v} \qquad y' = uu' - vv'$

$1 - 2vv' = \dfrac{u+v}{u-v} \qquad 2uu' - 1 = \dfrac{u+v}{u-v}$

$2uu' = 1 + \dfrac{u+v}{u-v}$

$\dfrac{u - v - u - v}{u-v} = 2vv'$

$2uu' = \dfrac{u - v + u + v}{u-u}$

$\dfrac{-2v}{u-v} = 2vv'$

$(2u)u' = \dfrac{(2u)}{4-v}$

$-\dfrac{1}{v'} = \dfrac{1}{u} \qquad -v' = u'$

$-v = u + c$

$-\sqrt{x-y} = -\sqrt{x+y} + c$

$\sqrt{x+y} + \sqrt{x-y} = c \Rightarrow \left[\sqrt{x+y} - \sqrt{x-y}\right]^2 = c^2$

$x + y + x - y + 2\sqrt{x+y} \cdot \sqrt{x-y} = c^2 = c$

$2x + 2\sqrt{x^2 - y^2} = c$

$x + \sqrt{x^2 - y^2} = c$