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examen

Resuelva

1. Hallar el valor principal de

a) $[(e/2)(-1-\sqrt{3}i)]^{3\pi i}$

Tenemos

$$[(e/2)(-1-\sqrt{3}i)]^{3\pi i} = \exp[3\pi i \operatorname{Log}(\tfrac{e}{2}(-1-\sqrt{3}i))]$$

$$[(e/2)(-1-\sqrt{3}i)]^{3\pi i} = \exp[3\pi i(\log e - i\tfrac{2}{3}\pi)]$$

$$[(e/2)(-1-\sqrt{3}i)]^{3\pi i} = \exp(2\pi^2)[\cos(3\pi) + i\operatorname{sen}(3\pi)]$$

$$[(e/2)(-1-\sqrt{3}i)]^{3\pi i} = \exp(2\pi^2)[-1+0]$$

$$[(e/2)(-1-\sqrt{3}i)]^{3\pi i} = -\exp(2\pi^2)$$

b) $(1-i)^{4i}$

Tenemos

$$(1-i)^{4i} = \exp[4i \operatorname{Log}(1-i)]$$

$$(1-i)^{4i} = \exp[4i(\log\sqrt{2} - i\tfrac{\pi}{4})]$$

$$(1-i)^{4i} = \exp[4i\log\sqrt{2} + \pi]$$

$$(1-i)^{4i} = \exp[\pi]\exp[i4\log\sqrt{2}]$$

$$(1-i)^{4i} = e^{\pi}[\cos(4\log\sqrt{2}) + i\operatorname{sen}(4\log\sqrt{2})]$$

$$(1-i)^{4i} = e^{\pi}[\cos(2\log 2) + i\operatorname{sen}(2\log 2)]$$

2. Encuentre la solución a la ecuación $\operatorname{sen} z = 4$, por dos métodos

1. Probando parte real y parte imaginaria tenemos

$$\operatorname{sen} z = 4$$

$$\operatorname{sen} z = \operatorname{sen} x \cosh y + i \cos x \operatorname{senh} y$$

$$\operatorname{sen} x \cosh y + i \cos x \operatorname{senh} y = 4$$

♡ Parte real

$$\operatorname{sen} x \cosh y = 4$$

♡ parte imaginaria

$$\cos x \cdot \operatorname{senh} y = 0$$

$\Rightarrow$ si $\cos x = 0$ entonces $x = \frac{\pi}{2}$ $\Rightarrow$ $\operatorname{sen} \frac{\pi}{2} \cosh y = 4$

por lo tanto $\cosh y = 4$ $\Rightarrow$ $x = \frac{\pi}{2} + 2n\pi$

Donde $n = 0, \pm 1, \pm 2 \ldots$

Entonces

$$\cosh y = \frac{e^4 + e^{-4}}{2}$$

$$\frac{e^4 + e^{-4}}{2} = 4$$

$$e^4 + e^{-4} = 8$$

$$e^{24} + 1 = 8e^4$$

$$e^{24} - 8e^4 + 1 = 0$$

$$e^4 = \frac{8 \pm \sqrt{64 - 4}}{2}$$

Tenemos

$$e^4 = \frac{8 \pm \sqrt{60}}{2}$$

$$e^4 = \frac{8 \pm 2\sqrt{15}}{2}$$

$$e^4 = 4 \pm \sqrt{15}$$

$$\ln e^4 = \ln(4 \pm \sqrt{15})$$

$$y = \pm \ln(4 + \sqrt{15})$$

nos queda

$$z = \frac{\pi}{2} + 2n\pi \pm i \ln(4 + \sqrt{15})$$

donde $n = 0, \pm 1, \pm 2 \ldots$

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2. Calculando el $\operatorname{sen}^{-1} z$

Tenemos

$$\operatorname{sen}^{-1} z = -i \operatorname{Log}\left(iz + (1-z^2)^{1/2}\right)$$

$$\operatorname{sen}^{-1} z = -i \operatorname{Log}\left(i4 + (1-16)^{1/2}\right)$$

$$\operatorname{sen}^{-1} z = -i \operatorname{Log}\left(i4 + \sqrt{-15}\right)$$

$$\operatorname{sen}^{-1} z = -i \operatorname{Log}\left(4i \pm i\sqrt{15}\right)$$

$$\operatorname{sen}^{-1} z = -i \log\left((4 \pm \sqrt{15})i\right)$$

$$\operatorname{sen}^{-1} z = -i\left(\log(4 \pm \sqrt{15}) + i\left(\frac{\pi}{2} + 2n\pi\right)\right)$$

$$= \frac{\pi}{2} + 2n\pi - i \operatorname{Log}(4 \pm \sqrt{15})$$

$$\operatorname{sen}^{-1} z = \frac{\pi}{2} + 2n\pi \pm i \log(4 + \sqrt{15})$$

Donde $n = 0, \pm 1, \pm 2 \ldots$

8. Probar que la siguiente función no es analitica en ningun punto.

$f(z) = e^{y} \cdot e^{ix}$

Donde la ecuación de Cauchy-Riemann

Tenemos de la forma

$f(z) = \underbrace{xy}_{u} + i \underbrace{y}_{v}$

Entonces se puede escribir de la forma polar

$f(z) = e^{y}(\cos x + i \operatorname{sen} x)$

$= \underbrace{e^{y}\cos x}_{u} + i \underbrace{e^{y}\operatorname{sen} x}_{v}$

Derivamos

$u_x = -e^{y}\operatorname{sen} x \; ; \; v_x = e^{y}\cos x$

$u_y = e^{y}\cos x \; ; \; v_y = e^{y}\operatorname{sen} x$

$u_x = v_y \Rightarrow -e^{y}\operatorname{sen} x = e^{y}\operatorname{sen} x$

$u_y = -v_x \Rightarrow e^{y}\cos = -e^{y}\cos x$

$2e^{y}\cos x = 0$

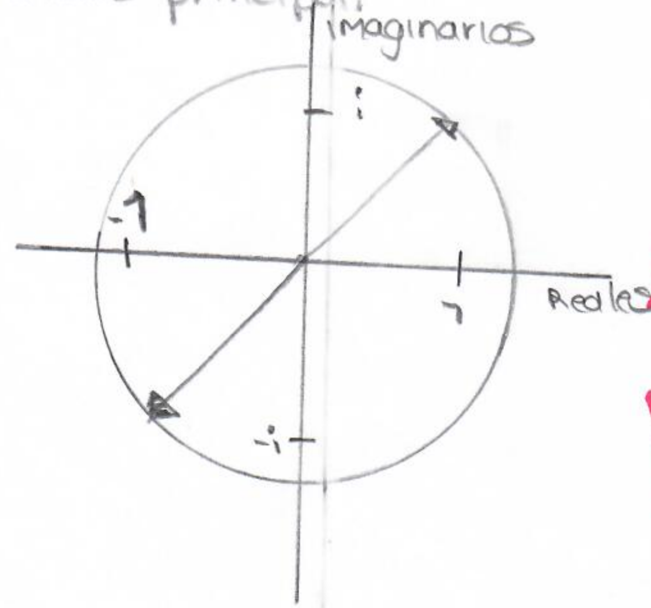
∴ La ecuación de cauchy-Riemann no se satisface en ningun punto, ya que las raices de senx son diferentes de cero al igual que cosx.

Por lo tanto esta función no es analitica.

Bocanegra Hezicpulo Yeotlanczi 4

4. Calcular todas las raices en forma carteciana, representarlas geometricamente e indicar cual es la raiz principal.

a) $(2i)^{1/2}$



$z = z_0^{1/K}$ recordando

$$z = \exp\left[\frac{\ln|z|}{k} + i\left(\frac{\Theta}{k} + \frac{2n\pi}{k}\right)\right]$$

♡ $n = 0$

$$z_0 = \exp\left[\frac{\ln 2}{2} + i\left(\frac{\frac{\pi}{2}}{2} + 0\right)\right]$$

$$z_0 = \exp\left[\ln 2^{1/2} + \frac{i\pi}{4}\right]$$

$$z_0 = e^{\ln 2 \frac{1}{2}} \cdot e^{\frac{i\pi}{4}}$$

$$z_0 = 2^{1/2}\left[\cos\left(\frac{\pi}{4}\right) + i\,sen\left(\frac{\pi}{4}\right)\right]$$

$$z_0 = \sqrt{2}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right]$$

$z_0 = 1 + i$ Raiz Principal

♡ $n = 1$

$$z_1 = \exp\left[\frac{\ln 2}{2} + i\left(\frac{\pi}{2} + \frac{2\pi}{2}\right)\right]$$

$$z_1 = e^{\ln 2 \frac{1}{2}} \cdot e^{i\frac{5}{4}\pi}$$

$$z_1 = 2^{\frac{1}{2}} \cdot e^{i\frac{5}{4}\pi}$$

$$z_1 = \sqrt{2}\left[-\frac{1}{2} - \frac{1}{2}i\right]$$

$z_1 = -[1 + i]$

5. Probar

a) $\overline{iz} = -i\overline{z}$

Podemos separar los términos

$$\overline{i}\,\overline{z} = -i\overline{z}$$

Entonces

$$-i\cdot\overline{z} = -i\overline{z}$$

Por lo que se cumple que

$$-i\overline{z} = -i\overline{z}$$

5. Probar

b) $|(2\overline{z}+5)(\sqrt{2}-i) = \sqrt{3}\,|2z+5|$

Tenemos que

$$|(2\overline{z}+5)(\sqrt{2}-i)| = |2\overline{z}+5||\sqrt{2}-i| \dots (1)$$

Donde

$$|2\overline{z}+5|^2 = (2\sqrt{\overline{z}}+5)\overline{(2\overline{z}+5)}$$

Recordando $|z|^2 = z\overline{z}$

$$|2\overline{z}+5|^2 = (2\overline{z}+5)(2\overline{\overline{z}}+5)$$

$$|2\overline{z}+5|^2 = (2z+5)(2\overline{z}+5)$$

$$|\overline{2z+5}|^2 = |2z+5|^2$$

Entonces

$$|2\overline{z}+5| = |2z+5| \dots (2)$$

nos queda

$$|\sqrt{2}-i| = \sqrt{(\sqrt{2})^2+(-1)^2}$$

$$= \sqrt{2+1}$$

$$= \sqrt{3} \dots (3)$$

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Donde

$$|z| = \sqrt{x^2+y^2}$$
$$= \sqrt{2}$$
$$y = i = -1$$

Sustituyendo (2) y (3) en la ecuación (1)

Tenemos

$$|(2\bar{z}+5)(\sqrt{2}-i)| = |2z+5|\sqrt{3}$$

Por lo tanto se cumple que

$$|(2\bar{z}+5)(\sqrt{2}-i)| = \sqrt{3}|2z+5|$$