

```

% LIMPIEZA DE PANTALLA
clear all
close all
clc

% =====
% DEFINICIÓN DE VARIABLES SIMBÓLICAS
% =====
syms theta1 theta2 theta3 theta4 theta5 real
syms L real % misma longitud usada entre eslabones

% =====
% MATRICES HOMOGÉNEAS SIMBÓLICAS H1-H5
% =====
Rt = @(R, t) [R, t; 0 0 0 1];

% H1: Rotación Z ( $\theta_1$ ) + Traslación Z (L)
Rz1 = [cos(theta1), -sin(theta1), 0;
       sin(theta1),  cos(theta1), 0;
       0,           0,           1];
H1 = Rt(Rz1, [0; 0; L]);

% H2: Rotación Y ( $\theta_2$ ) + Traslación Z (L)
Ry2 = [cos(theta2), 0, sin(theta2);
       0,           1, 0;
       -sin(theta2), 0, cos(theta2)];
H2 = Rt(Ry2, [0; 0; L]);

% H3: Rotación Z ( $\theta_3$ ) + Traslación Z (L)
Rz3 = [cos(theta3), -sin(theta3), 0;
       sin(theta3),  cos(theta3), 0;
       0,           0,           1];
H3 = Rt(Rz3, [0; 0; L]);

% H4: Rotación Y ( $\theta_4$ ) + Traslación Z (L)
Ry4 = [cos(theta4), 0, sin(theta4);
       0,           1, 0;
       -sin(theta4), 0, cos(theta4)];
H4 = Rt(Ry4, [0; 0; L]);

% H5: Rotación X ( $\theta_5$ ) + Traslación Z (L)
Rx5 = [1, 0, 0;
       0, cos(theta5), -sin(theta5);
       0, sin(theta5),  cos(theta5)];
H5 = Rt(Rx5, [0; 0; L]);

% =====
% MATRIZ DE TRANSFORMACIÓN GLOBAL SIMBÓLICA
% =====
T = simplify(H1 * H2 * H3 * H4 * H5);

```

```
disp('=====')
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```
disp('Matrices homogéneas simbólicas H1, H2, H3, H4, H5:')
```

Matrices homogéneas simbólicas H1, H2, H3, H4, H5:

```
disp('H1 ='), disp(H1)
```

H1 =

$$\begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
disp('H2 ='), disp(H2)
```

H2 =

$$\begin{pmatrix} \cos(\theta_2) & 0 & \sin(\theta_2) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta_2) & 0 & \cos(\theta_2) & L \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
disp('H3 ='), disp(H3)
```

H3 =

$$\begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
disp('H4 ='), disp(H4)
```

H4 =

$$\begin{pmatrix} \cos(\theta_4) & 0 & \sin(\theta_4) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta_4) & 0 & \cos(\theta_4) & L \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
disp('H5 ='), disp(H5)
```

H5 =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_5) & -\sin(\theta_5) & 0 \\ 0 & \sin(\theta_5) & \cos(\theta_5) & L \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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disp('=====')
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```
disp('Matriz de transformación homogénea global T =')
```

Matriz de transformación homogénea global T =

```
disp(T)
```

$$\begin{pmatrix} -\cos(\theta_4) \sigma_8 - \cos(\theta_1) \sin(\theta_2) \sin(\theta_4) & -\sin(\theta_5) \sigma_6 - \cos(\theta_5) \sigma_2 & \sin(\theta_5) \sigma_2 - \\ \cos(\theta_4) \sigma_7 - \sin(\theta_1) \sin(\theta_2) \sin(\theta_4) & \sin(\theta_5) \sigma_5 + \cos(\theta_5) \sigma_1 & \cos(\theta_5) \sigma_5 - \\ -\cos(\theta_2) \sin(\theta_4) - \cos(\theta_3) \cos(\theta_4) \sin(\theta_2) & \sin(\theta_5) (\sigma_4 - \sigma_3) + \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) & \cos(\theta_5) (\sigma_4 - \sigma_3) - \sin(\theta_5) \sin(\theta_2) \sin(\theta_3) \\ 0 & 0 & 0 \end{pmatrix}$$

where

$$\sigma_1 = \cos(\theta_1) \cos(\theta_3) - \cos(\theta_2) \sin(\theta_1) \sin(\theta_3)$$

$$\sigma_2 = \cos(\theta_3) \sin(\theta_1) + \cos(\theta_1) \cos(\theta_2) \sin(\theta_3)$$

$$\sigma_3 = \cos(\theta_3) \sin(\theta_2) \sin(\theta_4)$$

$$\sigma_4 = \cos(\theta_2) \cos(\theta_4)$$

$$\sigma_5 = \sin(\theta_4) \sigma_7 + \cos(\theta_4) \sin(\theta_1) \sin(\theta_2)$$

$$\sigma_6 = \sin(\theta_4) \sigma_8 - \cos(\theta_1) \cos(\theta_4) \sin(\theta_2)$$

$$\sigma_7 = \cos(\theta_1) \sin(\theta_3) + \cos(\theta_2) \cos(\theta_3) \sin(\theta_1)$$

$$\sigma_8 = \sin(\theta_1) \sin(\theta_3) - \cos(\theta_1) \cos(\theta_2) \cos(\theta_3)$$

```
% =====
% SIMULACIÓN NUMÉRICA DE LAS TRAMAS
% =====
theta1_val = pi/6;
theta2_val = pi/4;
theta3_val = -pi/6;
theta4_val = pi/3;
theta5_val = pi/8;
L_val = 2;

H1n = double(subs(H1, [theta1, L], [theta1_val, L_val]));
```

```

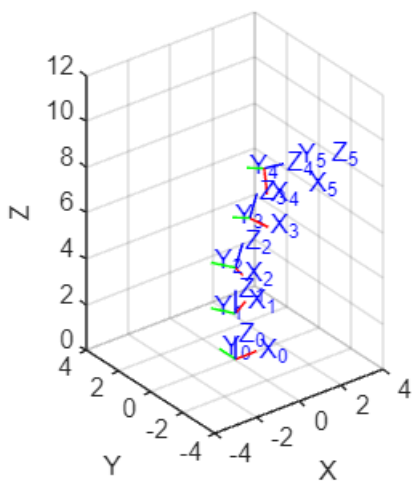
H2n = double(subs(H2, [theta2, L], [theta2_val, L_val]));
H3n = double(subs(H3, [theta3, L], [theta3_val, L_val]));
H4n = double(subs(H4, [theta4, L], [theta4_val, L_val]));
H5n = double(subs(H5, [theta5, L], [theta5_val, L_val]));

H0 = eye(4);
H0_1 = H1n;
H1_2 = H0_1 * H2n;
H2_3 = H1_2 * H3n;
H3_4 = H2_3 * H4n;
H4_5 = H3_4 * H5n;

% =====
% VISUALIZACIÓN DE LAS TRAMAS
% =====
figure
axis equal
view(3)
grid on
xlabel('X')
ylabel('Y')
zlabel('Z')
axis([-4 4 -4 4 0 12])
hold on

trplot(H0, 'frame', '0', 'rgb', 'length', 1)
trplot(H0_1, 'frame', '1', 'rgb', 'length', 1)
trplot(H1_2, 'frame', '2', 'rgb', 'length', 1)
trplot(H2_3, 'frame', '3', 'rgb', 'length', 1)
trplot(H3_4, 'frame', '4', 'rgb', 'length', 1)
trplot(H4_5, 'frame', '5', 'rgb', 'length', 1)

```



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% =====
% CINEMÁTICA DIFERENCIAL SIMBÓLICA

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```

% =====
syms t
syms theta1(t) theta2(t) theta3(t) theta4(t) theta5(t)

Rz1t = [cos(theta1(t)), -sin(theta1(t)), 0;
        sin(theta1(t)),  cos(theta1(t)), 0;
        0,               0,              1];
H1t = Rt(Rz1t, [0;0;L]);

Ry2t = [cos(theta2(t)), 0, sin(theta2(t));
        0,              1, 0;
        -sin(theta2(t)), 0, cos(theta2(t))];
H2t = Rt(Ry2t, [0;0;L]);

Rz3t = [cos(theta3(t)), -sin(theta3(t)), 0;
        sin(theta3(t)),  cos(theta3(t)), 0;
        0,               0,              1];
H3t = Rt(Rz3t, [0;0;L]);

Ry4t = [cos(theta4(t)), 0, sin(theta4(t));
        0,              1, 0;
        -sin(theta4(t)), 0, cos(theta4(t))];
H4t = Rt(Ry4t, [0;0;L]);

Rx5t = [1, 0, 0;
        0, cos(theta5(t)), -sin(theta5(t));
        0, sin(theta5(t)),  cos(theta5(t))];
H5t = Rt(Rx5t, [0;0;L]);

Tt = simplify(H1t * H2t * H3t * H4t * H5t);

% Velocidad lineal
pt = Tt(1:3,4);
v = simplify(diff(pt,t));

% Velocidad angular
Rt3 = Tt(1:3,1:3);
R_dot = simplify(diff(Rt3, t));
w_skew = simplify(R_dot * Rt3. ');

wx = (w_skew(3,2) - w_skew(2,3))/2;
wy = (w_skew(1,3) - w_skew(3,1))/2;
wz = (w_skew(2,1) - w_skew(1,2))/2;
w = simplify([wx; wy; wz]);

% =====
% RESULTADOS
% =====
disp('=====')

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disp('=====')
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```

```
disp('Matriz antisimétrica de velocidad angular w_skew =')
```

```
Matriz antisimétrica de velocidad angular w_skew =
```

```
disp(w_skew)
```

$$\begin{pmatrix} \sigma_{18} \sigma_{10} + \sigma_{14} \sigma_3 - \sigma_{13} \sigma_4 & \sigma_{11} \sigma_4 - \sigma_{12} \sigma_3 - \sigma_{17} \sigma_{10} & \sigma_2 \sigma_{10} - \sigma_{15} \sigma_4 - \sigma_{16} \sigma_3 \\ \sigma_{18} \sigma_9 - \sigma_{14} \sigma_5 + \sigma_{13} \sigma_6 & \sigma_{12} \sigma_5 - \sigma_{17} \sigma_9 - \sigma_{11} \sigma_6 & \sigma_{16} \sigma_5 + \sigma_{15} \sigma_6 + \sigma_2 \sigma_9 \\ -\sigma_{18} \sigma_1 - \sigma_{14} \sigma_8 - \sigma_{13} \sigma_7 & \sigma_{11} \sigma_7 + \sigma_{17} \sigma_1 + \sigma_{12} \sigma_8 & \sigma_{16} \sigma_8 - \sigma_2 \sigma_1 - \sigma_{15} \sigma_7 \end{pmatrix}$$

where

$$\sigma_1 = -\cos(\theta_2(t)) \cos(\theta_4(t)) \frac{\partial}{\partial t} \theta_4(t) + \sin(\theta_2(t)) \sin(\theta_4(t)) \frac{\partial}{\partial t} \theta_2(t) - \cos(\theta_2(t)) \cos(\theta_3(t)) \cos(\theta_4(t)) \frac{\partial}{\partial t} \theta_2(t).$$

$$\sigma_2 = \cos(\theta_2(t)) \sin(\theta_4(t)) + \cos(\theta_3(t)) \cos(\theta_4(t)) \sin(\theta_2(t))$$

$$\sigma_3 = \sin(\theta_5(t)) \sigma_{19} - \cos(\theta_5(t)) \sigma_{20} - \sin(\theta_5(t)) \sigma_{27} \frac{\partial}{\partial t} \theta_5(t) + \cos(\theta_5(t)) \sigma_{26} \frac{\partial}{\partial t} \theta_5(t)$$

$$\sigma_4 = \cos(\theta_5(t)) \sigma_{19} + \sin(\theta_5(t)) \sigma_{20} - \cos(\theta_5(t)) \sigma_{27} \frac{\partial}{\partial t} \theta_5(t) - \sin(\theta_5(t)) \sigma_{26} \frac{\partial}{\partial t} \theta_5(t)$$

$$\sigma_5 = \sin(\theta_5(t)) \sigma_{21} - \cos(\theta_5(t)) \sigma_{22} - \sin(\theta_5(t)) \sigma_{25} \frac{\partial}{\partial t} \theta_5(t) + \cos(\theta_5(t)) \sigma_{24} \frac{\partial}{\partial t} \theta_5(t)$$

$$\sigma_6 = \sin(\theta_5(t)) \sigma_{22} + \cos(\theta_5(t)) \sigma_{21} - \cos(\theta_5(t)) \sigma_{25} \frac{\partial}{\partial t} \theta_5(t) - \sin(\theta_5(t)) \sigma_{24} \frac{\partial}{\partial t} \theta_5(t)$$

$$\sigma_7 = \cos(\theta_5(t)) \sigma_{23} + \sin(\theta_5(t)) \sigma_{28} \frac{\partial}{\partial t} \theta_5(t) + \cos(\theta_2(t)) \sin(\theta_3(t)) \sin(\theta_5(t)) \frac{\partial}{\partial t} \theta_2(t) + \cos(\theta_3(t)) \sin(\theta_2(t)) \sin(\theta_5(t))$$

$$\sigma_8 = -\sin(\theta_5(t)) \sigma_{23} + \cos(\theta_5(t)) \sigma_{28} \frac{\partial}{\partial t} \theta_5(t) + \cos(\theta_2(t)) \cos(\theta_5(t)) \sin(\theta_3(t)) \frac{\partial}{\partial t} \theta_2(t) + \cos(\theta_3(t)) \cos(\theta_5(t)) \sin(\theta_2(t))$$

$$\sigma_9 = \cos(\theta_4(t)) \sigma_{30} + \sin(\theta_4(t)) \sigma_{31} \frac{\partial}{\partial t} \theta_4(t) + \cos(\theta_1(t)) \sin(\theta_2(t)) \sin(\theta_4(t)) \frac{\partial}{\partial t} \theta_1(t) + \cos(\theta_2(t)) \sin(\theta_1(t)) \sin(\theta_4(t))$$

$$\sigma_{10} = \cos(\theta_4(t)) \sigma_{29} - \sin(\theta_4(t)) \sigma_{32} \frac{\partial}{\partial t} \theta_4(t) + \cos(\theta_1(t)) \cos(\theta_2(t)) \sin(\theta_4(t)) \frac{\partial}{\partial t} \theta_2(t) + \cos(\theta_1(t)) \cos(\theta_4(t)) \sin(\theta_2(t))$$

$$\sigma_{11} = \sin(\theta_5(t)) \sigma_{25} - \cos(\theta_5(t)) \sigma_{24}$$

$$\sigma_{12} = \cos(\theta_5(t)) \sigma_{25} + \sin(\theta_5(t)) \sigma_{24}$$

$$\sigma_{13} = \sin(\theta_5(t)) \sigma_{27} - \cos(\theta_5(t)) \sigma_{26}$$

$$\sigma_{14} = \cos(\theta_5(t)) \sigma_{27} + \sin(\theta_5(t)) \sigma_{26}$$

$$\sigma_{15} = \cos(\theta_5(t)) \sigma_{28} - \sin(\theta_2(t)) \sin(\theta_3(t)) \sin(\theta_5(t))$$

$$\sigma_{16} = \sin(\theta_5(t)) \sigma_{28} + \cos(\theta_2(t)) \sin(\theta_3(t)) \sin(\theta_5(t))$$


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disp('=====')
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```
=====
```

```
disp('Vector de velocidad angular w =')
```

```
Vector de velocidad angular w =
```

```
disp(w)
```

$$\left(\begin{array}{l} \frac{(\sin(\theta_5(t)) \sigma_4 - \cos(\theta_5(t)) \sigma_{11}) \sigma_7}{2} - \frac{\sigma_{14} \left(\sin(\theta_5(t)) \sigma_9 - \cos(\theta_5(t)) \sigma_1 - \sin(\theta_5(t)) \sigma_4 \frac{\partial}{\partial t} \theta_5(t) + \cos(\theta_5(t)) \sigma_1 \right)}{2} \\ \frac{(\cos(\theta_4(t)) \sigma_{19} + \cos(\theta_1(t)) \sin(\theta_2(t)) \sin(\theta_4(t))) \sigma_2}{2} - \frac{\sigma_{13} \left(\cos(\theta_5(t)) \sigma_{10} + \sin(\theta_5(t)) \sigma_3 - \cos(\theta_5(t)) \sigma_5 \frac{\partial}{\partial t} \theta_5 \right)}{2} \end{array} \right)$$

where

$$\sigma_1 = \cos(\theta_3(t)) \sin(\theta_1(t)) \frac{\partial}{\partial t} \theta_1(t) + \cos(\theta_1(t)) \sin(\theta_3(t)) \frac{\partial}{\partial t} \theta_3(t) + \cos(\theta_1(t)) \cos(\theta_2(t)) \sin(\theta_3(t)) \frac{\partial}{\partial t} \theta_1(t) +$$

$$\sigma_2 = -\cos(\theta_2(t)) \cos(\theta_4(t)) \frac{\partial}{\partial t} \theta_4(t) + \sin(\theta_2(t)) \sin(\theta_4(t)) \frac{\partial}{\partial t} \theta_2(t) - \cos(\theta_2(t)) \cos(\theta_3(t)) \cos(\theta_4(t)) \frac{\partial}{\partial t} \theta_2(t) \cdot$$

$$\sigma_3 = -\cos(\theta_1(t)) \cos(\theta_3(t)) \frac{\partial}{\partial t} \theta_1(t) + \sin(\theta_1(t)) \sin(\theta_3(t)) \frac{\partial}{\partial t} \theta_3(t) - \cos(\theta_1(t)) \cos(\theta_2(t)) \cos(\theta_3(t)) \frac{\partial}{\partial t} \theta_3(t) \cdot$$

$$\sigma_4 = \cos(\theta_1(t)) \cos(\theta_3(t)) - \cos(\theta_2(t)) \sin(\theta_1(t)) \sin(\theta_3(t))$$

$$\sigma_5 = \cos(\theta_3(t)) \sin(\theta_1(t)) + \cos(\theta_1(t)) \cos(\theta_2(t)) \sin(\theta_3(t))$$

$$\sigma_6 = \cos(\theta_2(t)) \sin(\theta_4(t)) + \cos(\theta_3(t)) \cos(\theta_4(t)) \sin(\theta_2(t))$$

$$\sigma_7 = \cos(\theta_5(t)) \sigma_{15} + \sin(\theta_5(t)) \sigma_{20} \frac{\partial}{\partial t} \theta_5(t) + \cos(\theta_2(t)) \sin(\theta_3(t)) \sin(\theta_5(t)) \frac{\partial}{\partial t} \theta_2(t) + \cos(\theta_3(t)) \sin(\theta_2(t)) \sin(\theta_5(t))$$

$$\sigma_8 = -\sin(\theta_5(t)) \sigma_{15} + \cos(\theta_5(t)) \sigma_{20} \frac{\partial}{\partial t} \theta_5(t) + \cos(\theta_2(t)) \cos(\theta_5(t)) \sin(\theta_3(t)) \frac{\partial}{\partial t} \theta_2(t) + \cos(\theta_3(t)) \cos(\theta_5(t)) \sin(\theta_2(t))$$

$$\sigma_9 = -\sin(\theta_4(t)) \sigma_{16} + \cos(\theta_4(t)) \sigma_{18} \frac{\partial}{\partial t} \theta_4(t) + \cos(\theta_1(t)) \cos(\theta_4(t)) \sin(\theta_2(t)) \frac{\partial}{\partial t} \theta_1(t) + \cos(\theta_2(t)) \cos(\theta_4(t)) \sin(\theta_1(t))$$

$$\sigma_{10} = \sin(\theta_4(t)) \sigma_{17} + \cos(\theta_4(t)) \sigma_{19} \frac{\partial}{\partial t} \theta_4(t) - \cos(\theta_1(t)) \cos(\theta_2(t)) \cos(\theta_4(t)) \frac{\partial}{\partial t} \theta_2(t) + \cos(\theta_4(t)) \sin(\theta_1(t)) \sin(\theta_2(t))$$

$$\sigma_{11} = \sin(\theta_4(t)) \sigma_{18} + \cos(\theta_4(t)) \sin(\theta_1(t)) \sin(\theta_2(t))$$

$$\sigma_{12} = \sin(\theta_4(t)) \sigma_{19} - \cos(\theta_1(t)) \cos(\theta_4(t)) \sin(\theta_2(t))$$

$$\sigma_{13} = \cos(\theta_5(t)) \sigma_{20} - \sin(\theta_2(t)) \sin(\theta_3(t)) \sin(\theta_5(t))$$

$$\sigma_{14} = \sin(\theta_5(t)) \sigma_{20} + \cos(\theta_5(t)) \sin(\theta_2(t)) \sin(\theta_3(t))$$

$$\sigma_{15} = \cos(\theta_4(t)) \sin(\theta_2(t)) \frac{\partial}{\partial t} \theta_2(t) + \cos(\theta_2(t)) \sin(\theta_4(t)) \frac{\partial}{\partial t} \theta_4(t) + \cos(\theta_2(t)) \cos(\theta_3(t)) \sin(\theta_4(t)) \frac{\partial}{\partial t} \theta_2(t) +$$

```

%{
=====
DESCRIPCIÓN DEL PROCEDIMIENTO Y RESULTADOS
=====

Cinemática Directa:
El sistema modela una cadena de 5 transformaciones homogéneas (H1 a H5), cada una
representando una rotación (alrededor de ejes Z, Y o X) seguida de una traslación a
lo largo del eje Z con una misma longitud L. Estas transformaciones se multiplican
secuencialmente para obtener la matriz de transformación global T, la cual describe
la orientación y posición final del extremo del manipulador respecto al sistema
base.

Cinemática Diferencial:
Se reformulan las matrices H1 a H5 como funciones del tiempo. A partir de la matriz
simbólica T(t), se calcula:
- La velocidad lineal `v` como la derivada temporal de la posición del efector
(columna 4 de T).
- La velocidad angular `w` se obtiene al derivar la matriz de rotación y
multiplicarla por su transpuesta, extrayendo luego sus componentes a partir de la
matriz antisimétrica `w_skew`.

Resultados:
El código imprime la matriz de transformación T dependiente del tiempo, junto con
los vectores de velocidad lineal y angular. Estos resultados permiten analizar el
movimiento del efector y son fundamentales para control cinemático o planificación
de trayectorias.
%}

```