Simplify inequality
→ Ignore the lower order terms for larger

$2n^2 \geq 7cn$

→ Divide both sides by n

$2n \geq 7c$

Solve for n

$n = 7c/2$

(i) Choose constants

$c = 1$

$n \geq \frac{7 \cdot 1}{2} = 3.5$

for $n \geq n$ inequality holds

$2n^2+5 \geq 7n$ for all $n \geq n$

$2n^2+5 \geq 7n$

we can conclude

$f(n) = 2n^2+5 = \Omega(7n)$

is Ω notation the dominant term $2n^2$ is Ω
clearly grows faster n. Hence

$f(n) = \Omega(n^2)$

However, the specific comparison asked $f(n) = \Omega(7n)$
is also correct

showing that $f(n)$ grows at least as
fast as $7n$.

2) $T(n) = \begin{cases} 2T(n/2)+1 & \text{if } n>1 \\ 1 & \text{otherwise} \end{cases}$

By applying master's theorem

$T(n) = aT(n/b)+f(n)$ where $a\geq 1$, $b>1$

$T(n) = 2T(n/2)+1$

$a=2$ $b=2$ $f(n)=1$

By comparison of $f(n)$ and $n^{\log_b a}$

If $f(n) = O(n^c)$ $c<\log_b a$ then $T(n) = \Theta(n^{\log_b a})$

If $f(n) = \Theta(n^{\log_b a})$ then $T(n) = \Theta(n^{\log_b a}\log n)$

If $f(n) = \Omega\, n^c$ where $c>\log_b a$ then $T(n) = \Theta(f(n))$

Lets calculate $\log_b a$

$\log_b a = \log_2 2 = 1$

$f(n) = 1$

$n^{\log_b a} = n^1 = n$

$f(n) = O(n^c)$ with $c<\log_b a$

In this case $c=0$ and $\log_b a = 1$

$c<1$ so $T(n) = O(n^{\log_b a}) = O(n^1) = O(n)$

Time complexity is Recurrence relation

$T(n) = 2T(n/2)+1$ is $O(n)$

4)

$T(n) = \begin{cases} 2T(n-1) & \text{if } n>0 \\ 1 & \text{otherwise} \end{cases}$

here when $n = 0$

$T(0) = 1$

Recurrence relation

for $n > 0$:

$T(n) = 2T(n-1)$

$T(n) = 2T(n-1)$

$T(n-1) = 2T(n-2)$

$T(n-2) = 2T(n-3)$

$T(1) = 2T(0)$

$T(n) = 2 \cdot 2 \cdot 2 \cdots 2 T(0) = 2^n T(0)$

Since $T(0) = 1$ we have $T(n) = 2^n$

Recurrence relation $T(n) = 2T(n-1)$ for $n > 0$

$T(0) = 1$ is $T(n) = 2^n$.

5) Big O Notation show that $f(n) = n^2 + 3n + 5$ is $O(n^2)$

$f(n) = O(g(n))$ means $c > 0$ and $n_0 \geq 0$

$f(n) \leq c g(n)$ for all $n \geq n_0$

given is $f(n) = n^2 + 3n + 5$

$c > 0$ $n_0 > 0$ such that $f(n) \leq cn^2$

$f(n) = n^2 + 3n + 5$

Let's choose $c = 2$

$f(n) \leq 2n^2$

$f(n) = n^2 + 3n + 5 \leq n^2 + 3n^2 + 5n^2 = 9n^2$

so $c = 9$ $n_0 = 1$ $f(n) \leq 9n^2$ for all $n \geq 1$

for all

$f(n) = n^2 + 3n + 5$ is $O(n^2)$

6) ~~Rig~~

Lower bound =

$n(n) = 4n^2 + 3n$

$n(n) \geq cn^2$

$4n^2 + 3n \geq 4n^0$

$4 + 3/n \geq 4$

$n(n) \geq 4n^2 + 3n$

Let $f(n) = n^3 - 2n^2 + n$ and $g(n) = n^2$ show whether $f(n) = \Omega(g(n))$ is true or false and justify answer.

$f(n) \geq g(n)$

$f(n)$ and $g(n)$ inequality added

$n^3 - 2n^2 + n \geq (n^2)$

find c and n

$n^3 - 2n^2 + n \geq cn^2$

$n^3 + (-2)n^2 \geq 0$

$n^3 + (-2)n^2 + n \geq 0$

$n^2 + (1-2)n^2 + n = n^2 + n^2 + n$

therefore so that $f(n) = \Omega\, g(n)$

Big Omega Notation prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

given $n^3 + 2n^2 + 4n$

$g(n) = c \cdot n^3$

$g(n) = n^3 + 2n^2 + 4n$

$= n^2(n+2) + 4n$

$g(n) = n^3$

$g(n) = n^2(n+2) + 4n \geq cn^3$

$n^2(n+2) + 4n - cn^3 \geq 0$

$n^2(n+2) + 4n - cn^3 \geq 0$

this is equality is not always true

when n is close to 0. $n^2(n+2) + 4n - cn^3$

can be -ve)

$g(n) \neq \Omega(n^3)$

Big theta Notation determine whether $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

upper bound $(O(n^2))$

$h(n) = 4n^2 + 3n$

$h(n) \leq C_2 n^2$

$4n^2 \; 3n \leq C_2 n$

$4n^2 + 3n \leq 5n^2$

$C_2 \geq 5$

divide both sides by n^2

q.) Determine whether $n(n) = \log n$ is $\Theta(n\log n)$ prove a rigerous proof your answer

<u>sol)</u>

Upper bound :-

$f(n) \leq C_2 n \log n$

$f(n) = n\log n + n$

$n\log n + n \leq C_2 n\log n$

divide on both sides by $n\log n$

$1 + \frac{n}{n\log n} \leq C_2$

$1 + \frac{1}{\log n} \leq C_2$

$1 + \frac{1}{\log n} \leq 2$

$\Theta(n\log n)$ $C_2 = 2$ $n = 2$

lower bound

$f(n) \geq C_1 n\log n$

$n(n) = n\log n + n$

$n\log n + n \geq C_1 n\log n$

divide both sides by $n\log n$

$1 + \frac{1}{\log n} \geq C_1$

$1 + \frac{1}{\log n} \geq 4$

$1 + \frac{1}{\log n} = 1$

$\frac{1}{\log n} \leq n$

$n(n)$ is $\Omega(n\log n)$

$n(n) = n\log n$ is $\Theta(n\log n)$

10) Solve the following recurrence relations and find the order of growth for solutions

Sol

$$T(n) = 4T(n/2) + n^2 \quad T(0) = 1$$

$$a = 4 \quad b = 2 \quad f(n) = n^2$$

applying master theorem

$$T(n) = aT(n/b) + f(n)$$

$$f(n) = a(n^{\log b - 1})$$

$$f(n) = O(n^{\log_b a})$$

Calculating $\log_b a$

$$\log_b a = \log_2 4 = 2$$

$$f(n) = n^2 = \Theta(n^2)$$

$$f(n) = \Theta(n^2) = \Theta(n^{\log_b a})$$

$$f(n) = 4T(n/2) + n^2$$

$$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n \log n)$$

Order of growth.

$$T(n) = 4T(n/2) + n^2 \quad T(1) = 1$$

$$\Theta(n^2 \log n)$$