Question 1

$$
\begin{aligned}
&P(t_5 \geqslant 24) = P(t_4 < 24)\\
&X = \#\text{ of students arriving in 24 minutes}\\
&\lambda = 24 \div 4 = 6\\
\Rightarrow &X \sim \text{Poisson}(\lambda = 6)\\
&P(t_5 \geqslant 24) = P(t_4 < 24) = \text{ppois}(4, \lambda = 6) = 0.285
\end{aligned}
$$

Question 2

$p =$ passengers showing up

$P(p \leqslant 95) = 1 - P(95 < p \leqslant 100)$

$$= 1 - \sum_{x=0}^{4} \binom{100}{x} 0.05^x (1-0.05)^{100-x}$$

$X = \#$ of passengers that do not show up

$n = 100$

$p = 0.05$

$X \sim \text{Binomial}(n = 100, p = 0.05)$

$P(95 < p \leqslant 100) = P(X < 5)$

$$\Rightarrow P(X \geqslant 5) = 1 - P(X < 5) = 1 - \sum_{x=0}^{4} \binom{100}{x} 0.05^x (1-0.05)^{100-x}$$

$$= 1 - \text{pbinom}(4, \text{size} = 100, \text{prob} = 0.05) = 0.564$$

Question 3

$$\begin{aligned}
&X = \#\text{ of calls from the 30 minutes}\\
&\lambda = 4 \times \frac{30}{60} = 2\\
&X \sim \text{Poisson}(\lambda = 2)\\
\Rightarrow &P(X = 0) = \text{dpois}(0, 2) = 0.135
\end{aligned}$$

Question 4

$X =$ runtime of the batteries
$\mu = 120$
$\sigma = 20$
$X \sim N(\mu = 120, \sigma = 20)$
$P(X > 147) = 1 - P(x \leqslant 147) = 1 - P(\frac{x - 120}{20} \leqslant \frac{147 - 120}{20})$
$= 1 - P(z \leqslant 1.35) = 1 - 0.9115 = 0.0885$

Question 5

$X =$ the time for the first plane to land
$\lambda = 8 \div 60 = \frac{2}{15}$
$X \sim \text{Exp}(\lambda = \frac{2}{15})$
$P(X < 7) = \text{pexp}(q = 7, \text{rate} = 0.133) = 0.606$

Page 127 # 49 a

$$E[X] = M_X{}'(0) = (4t+5)e^{5t+2t^2} = 5$$
$$V[X] = E[X^2] - E[X]^2 = M_X{}''(0) - (M_X{}'(0))^2$$
$$E[X^2] = M_X{}''(0) = 4e^{5t+2t^2} + e^{5t+2t^2}(5+4t)^2 = 4 + 25 = 29$$
$$V[X] = E[X^2] - E[X]^2 = 4$$

Page 127 # 44

$$M_X(t) = 0.04e^{0t} + 0.2e^{1t} + 0.34e^{2t} + 0.2e^{3t} + 0.15e^{4t} + 0.04e^{5t} + 0.03e^{6t}$$
$$E[X] = M_X{}'(0) = 0.2e^{t} + 0.68e^{2t} + 0.6e^{3t} + 0.6e^{4t} + 0.2e^{5t} + 0.18e^{6t}$$
$$= 0.2 + 0.68 + 0.6 + 0.6 + 0.2 + 0.18 = 2.46$$
$$E[X^2] = M_X{}''(0) = 0.2e^{t} + 1.36e^{2t} + 1.8e^{3t} + 2.4e^{4t} + e^{5t} + 1.08e^{6t}$$
$$= 0.2 + 1.36 + 1.8 + 2.4 + 1 + 1.08 = 7.84$$
$$V[X] = E[X^2] - E[X]^2 = 7.84 - 2.46^2 = 1.7884$$

Page 136 # 64 c

$$n = 10$$
$$p = 0.6$$
$X = \#$ of people want the oversize ball
$$X \sim \text{Binomial}(m = 10, p = 0.6)$$
$$P(X \leqslant 7) = \sum_{x=0}^{7} \binom{10}{x} 0.6^x (1-0.6)^{10-x}$$
$$= 0.833$$

Page 151 # 97 b

$$p = \frac{1}{200} = 0.005$$
$$n = 1000$$
$$\lambda = np = 5$$
$$X = \#\text{ of people carrying the gene}$$
$$X \sim \exp(5)$$
$$P(X \geqslant 8) = 1 - P(x < 7) = 1 - \sum_{x=0}^{7} \frac{5^x e^{-5}}{x!}$$
$$= 1 - 0.867 = 0.133$$