Question 1

a

$$
\begin{aligned}
&\mu = \sum[X \cdot P(X)] \\
&= 25 \times 0.2 + 40 \times 0.5 + 65 \times 0.3 = 44.5 \\
&E[\overline{X}] = \sum(\overline{X} \times P[\overline{X}]) \\
&= (25 \times 0.04 + 32.5 \times 0.2 + 40 \times 0.25 + 45 \times 0.12 + 52.5 \times 0.3 + 65 \times 0.09) \\
&= 44.5 \\
\Rightarrow &E[\overline{X}] = \mu
\end{aligned}
$$

b

$$
\begin{aligned}
&\sigma^2 = E[X^2] - E[X]^2 \\
&= \sum(x^2 P(X = x)) - (\sum(xP(X = x)))^2 \\
&= 2192.5 - 1980.25 \\
&= 212.25 \\
&E[S^2] = \sum(S^2 P(S^2)) \\
&= 0 \times (0.04 + 0.25 + 0.09) + 2 \times 7.5^2 \times 0.2 + 2 \times 12.5^2 \times 0.3 + 2 \times 20^2 \times 0.12 \\
&= 212.25 \\
\Rightarrow &E[S^2] = \sigma^2
\end{aligned}
$$

Question 3

a

It is not plausible that the distribution is normal. The median is smaller than the mean and the distribution is not symmetric. The distribution is right skewed. So it is not normal.

b

$$
\begin{aligned}
& P(\overline{X} \geqslant 86.3) \\
=& 1 - P(\overline{X} < 86.3) \\
=& 1 - P(\frac{\overline{X} - 85}{\frac{15}{\sqrt{277}}} < \frac{86.3 - 85}{\frac{15}{\sqrt{277}}}) \\
=& 1 - P(z < 1.4424) \\
=& 1 - 0.9254 \\
=& 0.0746
\end{aligned}
$$

c

$$
\begin{aligned}
& P(\overline{X} \geqslant 86.3) \\
=& 1 - P(\overline{X} < 86.3) \\
=& 1 - P(\frac{\overline{X} - 82}{\frac{15}{\sqrt{277}}} < \frac{86.3 - 82}{\frac{15}{\sqrt{277}}}) \\
=& 1 - P(z < 4.7711) \\
=& 1 - 0.9999 \\
=& 0
\end{aligned}
$$

The probability is almost 0, which is not the case of the sample, so it is not a reasonable value for μ.

Question 4

$$
\begin{aligned}
&\mu = 18\% \\
&\sigma = 6\% \\
&X \sim N(\mu = 18, \sigma = 6) \\
&n = 40 \\
&P(16 \leqslant \overline{X} \leqslant 19) \\
=&P(\frac{16-18}{\frac{6}{\sqrt{40}}} \leqslant \frac{\overline{X}-18}{\frac{6}{\sqrt{40}}} \leqslant \frac{19-18}{\frac{6}{\sqrt{40}}}) \\
=&P(-2.11 \leqslant z \leqslant 1.05) \\
=&P(z \leqslant 1.05) - p(z \leqslant -2.11) \\
=&0.8531 - 0.0174 \\
=&0.8357
\end{aligned}
$$

Question 5

$\sigma = 1$
$\mu = 10$
$X \sim N(\mu = 10, \sigma = 1)$
$n = 4$
$4 \times 10 = 40$
$P(z > \frac{x - 40}{\frac{1}{2}}) = 0.05$
$P(z > 2(x - 40)) = 0.05$
$P(z \leqslant 2(x - 40)) = 0.95$
$\Phi(2(x - 40)) = 0.95$
$2(x - 40) = 1.645$
$x = 40.8225$

Question 7

$\mu_{X_1} = 2$

$\sigma_{X_1} = 1.5$

${\sigma_{X_1}}^2 = 2.25$

$\mu_{X_2} = 9{:}10 - 9{:}00 = 10$

$\sigma_{X_2} = 1$

${\sigma_{X_2}}^2 = 1$

$\mu_{X_3} = 6$

$\sigma_{X_3} = 1$

${\sigma_{X_3}}^2 = 1$

T : time need to make to the second class after first class ends

$\mu_T = \mu_{X_1} + \mu_{X_3} = 2 + 6 = 8$

$\}sigma{a_T}^2 = {\sigma_{X_1}}^2 + {\sigma_{X_3}}^2 = 3.25$

arrive before lecture starts :

$T < X_2$

$\Rightarrow T - X_2 < 0$

$\mu_{T-X_2} = \mu_T - \mu_{X_2} = 8 - 10 = -2$

${\sigma_{T-X_2}}^2 = {\sigma_T}^2 + {\sigma_{X_2}}^2 = 3.25 + 1 = 4.25$

$Y = T - X_2$

$\Rightarrow \mu_Y = -2$

$\sigma_Y = \sqrt{4.25} = 2.0616$

$P(Y < 0) = P(\frac{Y - (-2)}{2.0616} < \frac{0 - (-2)}{2.0616}) = P(z < 0.9701) = 0.8340$

$\rightarrow P = 0.8340$

Collaborators

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