Question 1

a

$$
\begin{aligned}
H_0 &: \mu = 6 \\
H_a &: \mu > 6
\end{aligned}
$$

b

Suppose that the null hypothesis is true, that the average height is 6 inches

c

$$
\begin{aligned}
z &= \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}} \\
&= \frac{\overline{x} - 6}{\sqrt{72}}
\end{aligned}
$$

d

Significance level is 0.03

e

$$
\begin{aligned}
\overline{x} &= \mu + z_{0.03}(\frac{s}{\sqrt{n}}) \\
&= 6 + 1.88(\frac{1}{\sqrt{72}}) \\
&= 6.22
\end{aligned}
$$

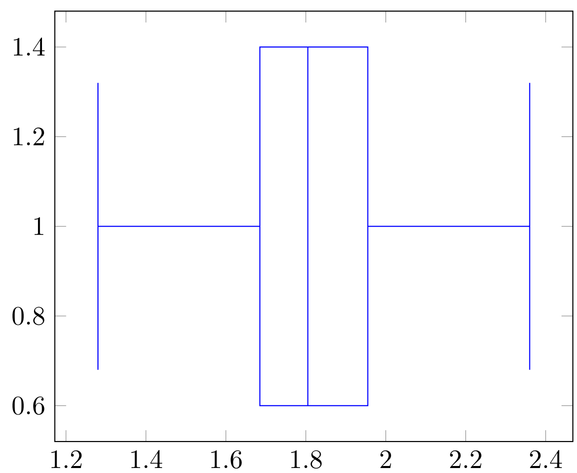
f

$$
\begin{aligned}
&\mu = 6.5 \\
&\text{type 2 error} \\
&\beta = P(\overline{x} < 6.22) \\
&= P(\frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}} < \frac{6.22 - 6.5}{\frac{1}{\sqrt{72}}}) \\
&= P(z < -2.38) \\
&= 0.0086 \\
&\text{Power of test1} : -\beta = 1 - 0.0086 = 0.9914
\end{aligned}
$$

Question 3

a

$$\text{Median} = \frac{1.78 + 1.83}{2} = 1.805$$
$$Q_1 = \frac{1.67 + 1.70}{2} = 1.685$$
$$Q_3 = \frac{1.91 + 2.00}{2} = 1.955$$
$$IQR = Q_3 - Q_1 = 0.27$$
$$\text{upper bound} = Q_3 + 1.5IQR = 2.36$$
$$\text{lower bound} = Q_1 - 1.5IQR = 1.28$$



The boxplot shows there are no outliers, and there the samples are symmetry, suggesting that the distribution is normal and centered around 1.8.

b

$$
\begin{aligned}
&H_0 : \mu = 1.5 \\
&H_a : \mu \neq 1.5 \\
&t = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} \\
&= \frac{1.81625 - 1.5}{0.2105/\sqrt{8}} \\
&= 4.249 \\
&t_{a/2,n-1} = 2.365 \\
\Rightarrow &t > t_{a/2,n-1} \\
&\text{The null hypothesis is rejected}
\end{aligned}
$$

c

Q-Q plot of the eight samples shows that the distribution is roughly normal, so the normality assumption is justified and t test is applicable.

Question 4

a

$$\hat{p} = \frac{14}{100} = 0.14$$
$$p_0 = 0.1$$
$$H_0 : p = 0.1$$
$$H_a : p > 0.1$$
$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{100}}}$$
$$= \frac{0.14 - 0.1}{\sqrt{\frac{0.1\times0.9}{100}}}$$
$$= \frac{0.04}{0.03}$$
$$= 1.33$$
$$P = P[Z > z]$$
$$= 1 - P[Z < 1.33]$$
$$= 1 - 0.9082$$
$$= 0.0918 > 0.05$$

fail,type 2 error: fail to reject the null hypothesis

b

$$p' = 0.15$$

$$n = 100:$$

$$\beta(p') = \Phi\left[\frac{p_0 - p' + z_{0.05}\sqrt{\frac{p_0(1-p_0)}{n}}}{\sqrt{\frac{p'(1-p')}{n}}}\right]$$

$$= \Phi\left[\frac{0.1 - 0.15 + 1.645\sqrt{\frac{0.1(1-0.1)}{100}}}{\sqrt{\frac{0.15(1-0.15)}{100}}}\right]$$

$$= \Phi[-0.0182]$$

$$= 0.4927$$

$$n = 200:$$

$$\beta(p') = \Phi\left[\frac{p_0 - p' + z_{0.05}\sqrt{\frac{p_0(1-p_0)}{n}}}{\sqrt{\frac{p'(1-p')}{n}}}\right]$$

$$= \Phi\left[\frac{0.1 - 0.15 + 1.645\sqrt{\frac{0.1(1-0.1)}{200}}}{\sqrt{\frac{0.15(1-0.15)}{200}}}\right]$$

$$= \Phi[-0.5982]$$

$$= 0.2749$$

c

$$n = (\frac{z_{0.05}\sqrt{0.1(1-0.1)} + z_{0.1}\sqrt{0.15(1-0.15)}}{0.15 - 0.1})^2$$

$$= (\frac{1.645\sqrt{0.1(1-0.1)} + 1.28\sqrt{0.15(1-0.15)}}{0.15 - 0.1})^2$$

$$= 361.419$$

$$\approx 362$$

Collaborators

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