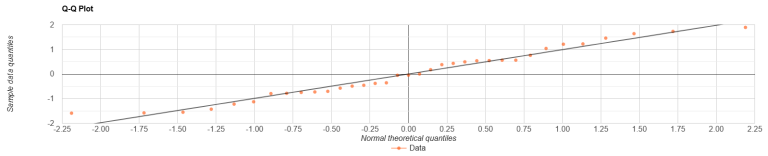
Question 1

a



The plot suggests that assumption of normal is acceptable.

$$H_0 : {\sigma_1}^2 = {\sigma_2}^2 = {\sigma_3}^2 = {\sigma_4}^2 = {\sigma_5}^2$$
$$H_a : \exists i, j \in \{1, 2, 3, 4, 5\} : {\sigma_i}^2 \neq {\sigma_j}^2$$
Use Levene's test :
$$f_{Levene} = \frac{\frac{\sum_{i=1}^{k} N_i (Z_{i.} - Z_{..})^2}{k-1}}{\frac{\sum_{i=1}^{k} \sum_{j=1}^{N_1} (Z_{ij} - Z_{i.})^2}{N-k}}$$
$$f_{Levene} \geqslant F_{a,(k-1,N-k)} : H_0 \text{ fails}$$

b

$$\alpha = 0.01$$
$$Z_{ij} = |Y_{ij} - \overline{Yi.}|$$

								$\overline{Y_{i.}}$
4 inch	309.2	409.5	311.0	326.5	316.8	349.8	309.7	333.21
6 inch	402.1	347.2	361.0	404.5	331.0	348.9	381.7	368.06
8 inch	392.4	366.2	351.0	357.1	409.9	367.3	382.0	375.13
10 inch	346.7	452.9	461.4	433.1	410.6	384.2	362.6	407.36
12 inch	407.4	441.8	419.9	410.7	473.4	441.2	465.8	437.17

								$Z_{i\bullet}$
4 inch	24.01	76.29	22.21	6.71	16.41	16.59	23.51	26.53
6 inch	34.04	20.86	7.06	36.44	37.06	19.16	13.64	24.04
8 inch	17.27	8.93	24.13	18.03	34.77	7.83	6.87	16.83
10 inch	60.66	45.54	54.04	25.74	3.24	23.16	44.76	36.73
12 inch	29.77	4.63	17.27	26.47	36.23	4.03	28.63	21.00

$$\begin{aligned} Z_{\bullet\bullet} &= \frac{1}{N}\sum_{i=1}^{k}\sum_{j=1}^{N_i} Z_{ij} \\ &= \frac{1}{35}\sum_{i=1}^{k}\sum_{j=1}^{N_i} Z_{ij} \\ &= 25.029 \end{aligned}$$

								$\sum_{j=1}^{N_i}(Z_{ij} - Z_{i\bullet})^2$
4 inch	6.35	2475.16	18.67	392.85	102.42	98.98	9.12	3103.56
6 inch	100.12	10.11	288.31	153.91	169.53	23.81	108.03	853.82
8 inch	0.19	62.47	53.23	1.43	321.80	81.07	99.23	619.43
10 inch	572.28	77.58	299.57	120.82	1121.70	184.35	64.36	2440.67
12 inch	76.87	268.16	13.93	29.89	231.79	288.17	58.13	966.93

$$\begin{aligned} f_{Levene} &= \frac{\frac{\sum_{i=1}^{k} N_i(Z_{i\bullet}-Z_{\bullet\bullet})^2}{k-1}}{\frac{\sum_{i=1}^{k}\sum_{j=1}^{N_1}(Z_{ij}-Z_{i\bullet})^2}{N-k}} \\ &= \frac{\frac{7((26.53-25.029)^2+\ldots+(21-25.029)^2)}{5-1}}{\frac{3103.56+\ldots+966.93}{35-5}} \\ &= 1.47 \\ F_{0.01,(4,30)} &= 4.01 \\ f_{Levene} &< F_{0.01,(4,30)} \\ \Rightarrow H_0 &\text{ holds} \end{aligned}$$

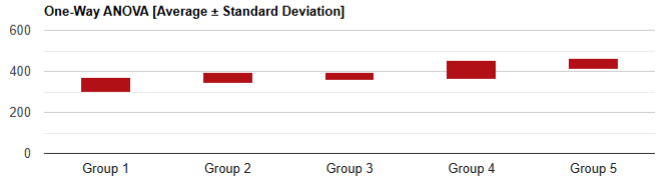
Analysis of Variance Results

F-statistic value = 10.48272

P-value = 0.00002

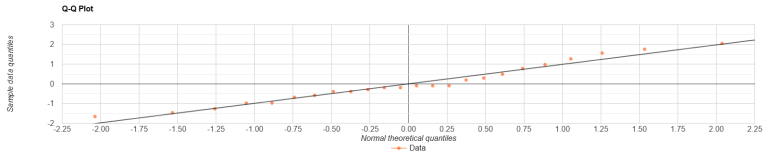
Data Summary				
Groups	N	Mean	Std. Dev.	Std. Error
Group 1	7	333.2143	36.5882	13.829
Group 2	7	368.0571	28.5721	10.7992
Group 3	7	375.1286	20.8278	7.8722
Group 4	7	407.3571	44.5098	16.8231
Group 5	7	437.1714	25.9972	9.826

ANOVA Summary					
Source	Degrees of Freedom DF	Sum of Squares SS	Mean Square MS	F-Stat	P-Value
Between Groups	4	43992.5122	10998.128	10.4827	0
Within Groups	30	31475.0114	1049.167		
Total:	34	75467.5236			



Question 2

a



The plot suggests that assumption of normal is acceptable.

$$H_0 : {\sigma_1}^2 = {\sigma_2}^2 = {\sigma_3}^2 = {\sigma_4}^2$$
$$H_a : \exists i, j \in \{1, 2, 3, 4\} : {\sigma_i}^2 \neq {\sigma_j}^2$$
Use P test :
$p \leqslant \alpha$: reject H_0

b

$T_1 = 34.3, T_2 = 39.6, T_3 = 33, T_4 = 41.9$

$G = T_1 + T_2 + T_3 + T_4 = 148.8$

$N = 24$

$\sum x^2 = 5.2^2 + ... + 7.2^2 = 946.68$

$SS = \sum x^2 - \frac{G^2}{N} = 946.68 - 24 \times 148.8^2 = 24.12$

$SSB = \sum (\frac{T_1}{n} - \frac{G}{N})^2 = 8.98$

$SSW = SS - SSB = 15.14$

$dfB = 4 - 1 = 3$

$dfW = 24 - 4 = 20$

$p = F_{0.05,(3,20)} = 0.0229$

$0.0229 < 0.05$

$\Rightarrow$reject H_0

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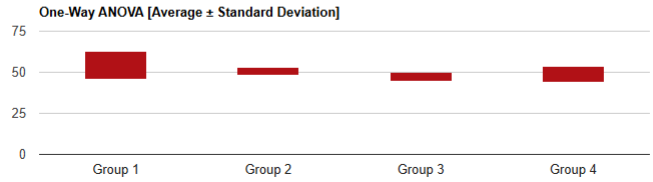
Question 3

a

Data Summary				
Groups	N	Mean	Std. Dev.	Std. Error
Group 1	3	54.3333	8.3865	4.8419
Group 2	3	50.6667	2.5166	1.453
Group 3	3	47.3333	2.5166	1.453
Group 4	3	48.6667	4.9329	2.848

ANOVA Summary					
Source	Degrees of Freedom DF	Sum of Squares SS	Mean Square MS	F-Stat	P-Value
Between Groups	3	83.5829	27.861	1.0383	0.4264
Within Groups	8	214.6669	26.8334		
Total:	11	298.2497			



$H_0 : {\sigma_1}^2 = {\sigma_2}^2 = {\sigma_3}^2 = {\sigma_4}^2$

$H_a : \exists i, j \in \{1, 2, 3, 4\} : {\sigma_i}^2 \neq {\sigma_j}^2$

$\overline{x_{1\bullet}} = 54.33, \overline{x_{2\bullet}} = 50.67, \overline{x_{3\bullet}} = 47.33, \overline{x_{4\bullet}} = 48.67, \overline{x_{\bullet 1}} = 53.75, \overline{x_{\bullet 2}} = 47, \overline{x_{\bullet 3}} = 50$

$X_{\bullet\bullet} = 603$

$\sum\sum {X_{ij}}^2 = (64^2 + ... + 52^2) = 30599$

$CF = \frac{{X_{\bullet\bullet}}^2}{n} = \frac{603^2}{12} = 30300.75$

$SS = \sum\sum {X_{ij}}^2 = (64^2 + ... + 52^2) - CF = 30599 - 30300.75 = 298.25$

$SSB = \frac{1}{3}\sum {X_{i\bullet}}^2 - CF$

$= \frac{1}{3}(163^2 + ... + 146^2) - 30300.75$

$= 83.58$

$SSW = \frac{1}{4}\sum {X_{\bullet j}}^2 - CF$

$= \frac{1}{4}(215^2 + 188^2 + 200^2) - 30300.75$

$= 91.5$

$SSE = SS - (SSB + SSW) = 298.25 - (83.58 + 91.5) = 123.17$

$dfB = 4 - 1 = 3$

$dfW = 11 - 3 = 8$

$dfE = 12 - (3 + 2) = 6$

$df = 11$

$MSB = \frac{SSB}{dfB} = 27.86$

$MSW = \frac{SSW}{dfW} = 45.75$

$MSE = \frac{SSE}{dfE} = 20.53$

$F_B = \frac{MSB}{MSE} = 1.36$

$F_W = \frac{MSW}{MSE} = 2.23$

$F_{0.05,(3,6)} = 4.757$

$\Rightarrow F_B < F_{0.05,(3,6)}$

$\Rightarrow H_0$ holds

b

$$\hat{\mu} = \overline{x_{\bullet\bullet}} = \frac{64 + ... + 52}{12} = 50.25$$
$$\hat{\alpha_1} = \overline{x_{1\bullet}} - \overline{x_{\bullet\bullet}} = 54.33 - 50.25 = 4.08$$
$$\hat{\alpha_2} = \overline{x_{2\bullet}} - \overline{x_{\bullet\bullet}} = 50.67 - 50.25 = 0.42$$
$$\hat{\alpha_3} = \overline{x_{3\bullet}} - \overline{x_{\bullet\bullet}} = 47.33 - 50.25 = -2.92$$
$$\hat{\alpha_4} = \overline{x_{4\bullet}} - \overline{x_{\bullet\bullet}} = 48.67 - 50.25 = -1.58$$
$$\hat{\beta_1} = \overline{x_{\bullet 1}} - \overline{x_{\bullet\bullet}} = 53.75 - 50.25 = 3.5$$
$$\hat{\beta_2} = \overline{x_{\bullet 2}} - \overline{x_{\bullet\bullet}} = 47 - 50.25 = -3.25$$
$$\hat{\beta_3} = \overline{x_{\bullet 3}} - \overline{x_{\bullet\bullet}} = 50 - 50.25 = -0.25$$

Collaborators

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