Question 2

$$
\begin{aligned}
&20 - 15 = 5 \\
&30 - 18 = 12 \\
&20 - 16 = 4 \\
&32 - 19 = 13 \\
&35 - 10 = 25 \\
&33 - 12 = 21 \\
\Rightarrow &\overline{d} = \frac{5 + 12 + 4 + 13 + 25 + 21}{6} = 13.33 \\
&Var_d = \frac{\sum (d_i - \overline{d})^2}{6 - 1} \\
&\sigma_d = \sqrt{Var_d} = \sqrt{\frac{\sum (d_i - \overline{d})^2}{6 - 1}} = 8.33 \\
&H_0 : \mu_d = 0 \\
&H_a : \mu_d > 0 \\
&t = \frac{\overline{d}}{\frac{\sigma_d}{\sqrt{n}}} \\
&= \frac{13.33}{\frac{8.33}{\sqrt{6}}} \\
&= 3.96 \\
&\alpha = 0.01, df = 5 \\
\Rightarrow &t_{0.01,5} = 3.365 \\
&t > t_{0.01,5} \\
\Rightarrow &H_0 \text{ is rejected} \\
&CI = (\overline{d} - t_{0.975,5} \frac{\sigma_d}{\sqrt{n}}, \overline{d} - t_{0.975,5} \frac{\sigma_d}{\sqrt{n}}) \\
&= (13.33 - 2.571 \times \frac{8.33}{\sqrt{6}}, 13.33 + 2.571 \times \frac{8.33}{\sqrt{6}}) \\
&= (4.58, 22.08)
\end{aligned}
$$

Question 3

$H_0 : p_1 = p_2$

$H_a : p_1 \neq p_2$

$m_1 = 300$

$x_1 = 63$

$\hat{p_1} = \frac{x_1}{m_1} = 0.21$

$m_2 = 180$

$x_2 = 75$

$\hat{p_2} = \frac{x_2}{m_2} = 0.4167$

$\hat{p} = \frac{x_1 + x_2}{m_1 + m_2} = \frac{63 + 75}{300 + 480} = 0.2875$

$SE = \sqrt{0.2875(1 - 0.2875)(\frac{1}{300} + \frac{1}{180})} = 0.0507$

$z = \frac{\hat{p_1} - \hat{p_2}}{SE} = \frac{0.21 - 0.4167}{0.0507} = -4.07$

$\alpha = 0.05$

$\Rightarrow z_{0.025} = 1.96$

$|z| > z_{0.025}$

$\Rightarrow H_0$ is rejected

$\hat{p_1} - \hat{p_2} = -0.2067$

$CI = (\hat{p_1} - \hat{p_2} - z_{0.025}SE, \hat{p_1} - \hat{p_2} + z_{-.-25}SE)$

$= (-0.2067 - 0.0994, -0.2067 + 0.0994)$

$= (-0.3061, -0.1073)$

Question 4

$$
\begin{aligned}
&\text{width} = 2 \times z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \\
&10 = 2 \times 1.96 \times \frac{25}{\sqrt{n}} \\
&\sqrt{n} = 1.96 \times 5 = 9.8 \\
&n = 9.8^2 = 96.04 \approx 97 \\
\Rightarrow &n = 97
\end{aligned}
$$

Question 5

$$
\begin{aligned}
&\mu_0 = 96 \\
&\mu_1 = 92 \\
&\sigma = 25 \\
&\alpha = 0.05 \\
&\text{Power} = 0.85 \\
&z_{0.05} = 1.645 \\
&z_{0.85} = 1.04 \\
&n = \left(\frac{(z_\alpha + z_{\text{power}})\sigma}{\mu_0 - \mu_1} \right)^2 \\
&= \left(\frac{(1.645 + 1.04)\sigma}{96 - 92} \right)^2 \\
&= \left(\frac{67.125}{4} \right)^2 \\
&= 281.7 \approx 282 \\
\Rightarrow &n = 282
\end{aligned}
$$

Collaborators

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