Question 1

a

$$\begin{aligned}
\text{CI} &= \overline{x} \pm Z_{0.95}(\frac{\sigma}{\sqrt{n}}) \\
&= 4.85 \pm 1.96(\frac{0.75}{20}) \\
&= 4.85 \pm 0.33 \\
&= (4.52, 5.18)
\end{aligned}$$

b

$$\begin{aligned}
\text{CI} &= \overline{x} \pm Z_{0.98}(\frac{\sigma}{\sqrt{n}}) \\
&= 4.56 \pm 2.33(\frac{0.75}{\sqrt{16}}) \\
&= 4.56 \pm 0.44 \\
&= (4.12, 5.00)
\end{aligned}$$

c

$$\begin{aligned}
n &= (2Z_{0.95}(\frac{\sigma}{w}))^2 \\
&= (2 \cdot 1.96(\frac{0.75}{0.40}))^2 \\
&= (7.35)^2 \\
&= 54.0225 \\
&\approx 55
\end{aligned}$$

d

$$
\begin{aligned}
w &= 2E \\
&= 0.40 \\
n &= (2Z_{0.99}(\frac{\sigma}{w}))^2 \\
&= (2 \cdot 2.58(\frac{0.75}{0.40}))^2 \\
&= (9.675)^2 \\
&= 93.605625 \\
&\approx 94
\end{aligned}
$$

Question 2

$$
\begin{aligned}
\hat{p} &= \frac{133}{539} \\
&= 0.2468 \\
\text{CI} &= \hat{p} \pm Z_{0.95}(\sqrt{\frac{(\hat{p})(1-\hat{p})}{n}}) \\
&= 0.2468 \pm 1.96(\sqrt{\frac{(0.2468(1-0.2468))}{539}}) \\
&= 0.2468 \pm 0.036399 \\
&= (0.210401, 0.283199) \\
\min(\text{CI}) &= 0.210401
\end{aligned}
$$

Question 3

a

$$
\begin{aligned}
&\hat{p} \coloneqq 0.5 \\
&\hat{q} \coloneqq 0.5 \\
&w^2 = 0.01 \\
&Z_{0.95} = 1.96 \\
&n = \frac{2{Z_{0.95}}^2\hat{p}\hat{q} - {Z_{0.95}}^2 w^2 \pm \sqrt{4{Z_{0.95}}^2\hat{p}\hat{q}(\hat{p}\hat{q} - w^2) + w^2{Z_{0.95}}^4}}{w^2} \\
&= \frac{2 \cdot 1.96^2 \cdot 0.5 \cdot 0.5 - 1.96^2 \cdot 0.01 \pm \sqrt{4 \cdot 1.96^4 \cdot 0.5 \cdot 0.5(0.5 \cdot 0.5 - 0.01) + 0.01 \cdot 1.96^4}}{0.01} \\
&= \frac{1.9208 - 0.038416 \pm \sqrt{3.68947264}}{0.01} \\
&= (\frac{1.88238 - 1.9208}{0.01}, \frac{1.88238 + 1.9208}{0.01}) \\
&= (-4.321, 380.318) \\
\Rightarrow &n = 380.318 \approx 381
\end{aligned}
$$

b

$$\hat{p} \coloneqq \frac{2}{3}$$
$$\hat{q} \coloneqq \frac{1}{3}$$
$$w^2 = 0.01$$
$$Z_{0.95} = 1.96$$
$$n = \frac{2{Z_{0.95}}^2 \hat{p}\hat{q} - {Z_{0.95}}^2 w^2 \pm \sqrt{4{Z_{0.95}}^2 \hat{p}\hat{q}(\hat{p}\hat{q} - w^2) + w^2 {Z_{0.95}}^4}}{w^2}$$
$$= \frac{2 \cdot 1.96^2 \cdot \frac{2}{3} \cdot \frac{1}{3} - 1.96^2 \cdot 0.01 \pm \sqrt{4 \cdot 1.96^4 \cdot \frac{2}{3} \cdot \frac{1}{3}(\frac{2}{3} \cdot \frac{1}{3} - 0.01) + 0.01 \cdot 1.96^4}}{0.01}$$
$$= \frac{1.70738 - 0.038416 \pm \sqrt{2.93154}}{0.01}$$
$$= (\frac{1.66896 - 1.71217}{0.01}, \frac{1.88238 + 1.71217}{0.01})$$
$$= (-4.321, 338.113)$$
$$\Rightarrow n = 338.113 \approx 339$$

Question 4

a

$$
\begin{aligned}
&n = 14 \\
&\overline{x} = 8.48 \\
&s = 0.79 \\
&df = n - 1 = 13 \\
&t_{0.95, df=13} = 2.160 \\
&\text{CI} = \overline{x} \pm t_{0.95, df=13}(\frac{s}{\sqrt{n}}) \\
&= 8.48 \pm 2.160(\frac{0.79}{\sqrt{n}}) \\
&= 8.48 \pm 0.456 \\
&= (8.023, 8.937) \\
&= \min(\text{CI}) = 8.023
\end{aligned}
$$

Collaborators

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