Question 1

$$P(t_5 \geqslant 24) = P(t_4 < 24)$$
$$X = \# \text{ of students arriving in 24 minutes}$$
$$\lambda = 24 \div 4 = 6$$
$$\Rightarrow X \sim \text{Poisson}(\lambda = 6)$$
$$P(t_5 \geqslant 24) = P(t_4 < 24) = \text{ppois}(4, \lambda = 6) = 0.285$$

Question 2

$p =$ passengers showing up

$P(p \leqslant 95) = 1 - P(95 < p \leqslant 100)$

$$= 1 - \sum_{x=0}^{4} \binom{100}{x} 0.05^x (1 - 0.05)^{100-x}$$

$X = \#$ of passengers that do not show up

$n = 100$

$p = 0.05$

$X \sim \text{Binomial}(n = 100, p = 0.05)$

$P(95 < p \leqslant 100) = P(X < 5)$

$$\Rightarrow P(X \geqslant 5) = 1 - P(X < 5) = 1 - \sum_{x=0}^{4} \binom{100}{x} 0.05^x (1 - 0.05)^{100-x}$$

$$= 1 - \text{pbinom}(4, \text{size} = 100, \text{prob} = 0.05) = 0.564$$

Question 3

$$X = \# \text{ of calls from the 30 minutes}$$
$$\lambda = 4 \times \frac{30}{60} = 2$$
$$X \sim \text{Poisson}(\lambda = 2)$$
$$\Rightarrow P(X = 0) = \text{dpois}(0, 2) = 0.135$$

Question 4

X = runtime of the batteries

$\mu = 120$

$\sigma = 20$

$X \sim N(\mu = 120, \sigma = 20)$

$P(X > 147) = 1 - P(x \leqslant 147) = 1 - P(\dfrac{x - 120}{20} \leqslant \dfrac{147 - 120}{20})$

$= 1 - P(z \leqslant 1.35) = 1 - 0.9115 = 0.0885$

Question 5

$X =$ the time for the first plane to land

$\lambda = 8 \div 60 = \dfrac{2}{15}$

$X \sim \text{Exp}(\lambda = \dfrac{2}{15})$

$P(X < 7) = \text{pexp}(q = 7, \text{rate} = 0.133) = 0.606$

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$$E[X] = M_X{}'(0) = (4t+5)e^{5t+2t^2} = 5$$
$$V[X] = E[X^2] - E[X]^2 = M_X{}''(0) - (M_X{}'(0))^2$$
$$E[X^2] = M_X{}''(0) = 4e^{5t+2t^2} + e^{5t+2t^2}(5+4t)^2 = 4 + 25 = 29$$
$$V[X] = E[X^2] - E[X]^2 = 4$$

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$$M_X(t) = 0.04e^{0t} + 0.2e^{1t} + 0.34e^{2t} + 0.2e^{3t} + 0.15e^{4t} + 0.04e^{5t} + 0.03e^{6t}$$
$$E[X] = M_X{}'(0) = 0.2e^t + 0.68e^{2t} + 0.6e^{3t} + 0.6e^{4t} + 0.2e^{5t} + 0.18e^{6t}$$
$$= 0.2 + 0.68 + 0.6 + 0.6 + 0.2 + 0.18 = 2.46$$
$$E[X^2] = M_X{}''(0) = 0.2e^t + 1.36e^{2t} + 1.8e^{3t} + 2.4e^{4t} + e^{5t} + 1.08e^{6t}$$
$$= 0.2 + 1.36 + 1.8 + 2.4 + 1 + 1.08 = 7.84$$
$$V[X] = E[X^2] - E[X]^2 = 7.84 - 2.46^2 = 1.7884$$

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$$n = 10$$
$$p = 0.6$$
$$X = \# \text{ of people want the oversize ball}$$
$$X \sim \text{Binomial}(m = 10, p = 0.6)$$
$$P(X \leqslant 7) = \sum_{x=0}^{7} \binom{10}{x} 0.6^x (1 - 0.6)^{10-x}$$
$$= 0.833$$

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$$p = \frac{1}{200} = 0.005$$

$$n = 1000$$

$$\lambda = np = 5$$

$X = \#$ of people carrying the gene

$X \sim \exp(5)$

$$P(X \geqslant 8) = 1 - P(x < 7) = 1 - \sum_{x=0}^{7} \frac{5^x e^{-5}}{x!}$$

$$= 1 - 0.867 = 0.133$$

Collaborators

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