Book

Crest of the Peacock

Question 1

1. split 142884 into pairs of 2: 14, 28, 84
2. $3 \times 3 = 9 \leq 14$ is the largest number whose square is less than 14
3. $14 - 9 = 5$, so 28 becomes 528
4. find greatest $x : x(3 \times 20 + x) \leq 528 \implies x = 7$
5. $528 - 7 \times 67 = 59$, so 84 becomes 5984
6. current root is 37
7. find greatest $x : x(37 \times 20 + x) \leq 5984 \implies x = 8$
8. $5984 - 8 \times 748 = 0$, no remainder
9. $\sqrt{142884} = 378$

Question 2

1. $x \coloneqq 300 + h$
2. $p(x) = (300+h)^4 - 1212(x+h)^3 + 181579(x+h)^2 + 37287672x - 190826960$
3. (a) $(300 + h)^4 = 1 \cdot 300^4 + 4 \cdot 300^3 h + 6 \cdot 300^2 h^2 + 4 \cdot 300 h^3 + 1 \cdot h^4$
   (b) $(300 + h)^3 = 1 \cdot 300^3 + 3 \cdot 300^2 h + 3 \cdot 300 h^2 + 1 \cdot h^3$
   (c) $(300 + h)^2 = 1 \cdot 300^2 + 2 \cdot 300 h + 1 \cdot h^2$
4. substituting and simplified: $p(x) = h^4 - 12h^3 - 369221h^2 - 73004928h + 2713584640$
5. Take the guess of the second digit 3
6. $h \coloneqq 30 + p$
7. $p(x) = 191623900 - 95082588p - 364901p^2 + 108p^3 + p^4$
8. This leaves to $p = 2$
9. So the number is 332.

Question 3

1. Use CRT:
2. $M = 11 \times 3 \times 5 = 165$
3. $M_1 = 165/11 = 15, M_2 = 165/5 = 33, M_3 = 165/3 = 55$
4. $t_i \coloneqq {M_i}^{-1} \mod m_i$
5. (a) $15t_1 \equiv 1 \mod 11 \implies t_1 = 3$
   (b) $33t_2 \equiv 1 \mod 5 \implies t_2 = 2$
   (c) $55t_3 \equiv 1 \mod 3 \implies t_3 = 1$
6. So $N = 3 \times 3 \times 15 + 2 \times 2 \times 33 + 1 \times 1 \times 55 + 165k = 322 + 165k$
7. for $N \leq 165, k = -1, N = 157$