Problem 1

Question a

$$\begin{split} B &\coloneqq \text{blue balls} \\ R &\coloneqq \text{red balls} \\ \Rightarrow &\Omega = \{(B,R), (B,B), (R,B), (R,R)\} \end{split}$$

Question b

$$P = P(B,R) + P(R,B)$$

$$= \frac{n}{2n} \times \frac{n}{2n-1} + \frac{n}{2n} \times \frac{n}{2n-1}$$

$$= \frac{2n^2}{2n(2n-1)}$$

$$= \frac{n}{2n-1}$$

${\bf Question}~{\bf c}$

$$p_n = 1 - P$$

$$\Rightarrow p_n = 1 - \frac{n}{2n - 1} = \frac{n - 1}{2n - 1}$$

$$\lim_{n \to \infty} p_n = \frac{1}{2}$$

Problem 2

$$A_1 := person 1 lost all$$

$$A_2 := person 2 lost all$$

$$A_3 := person 3 lost all$$

$$P(A_1) = P(A_2) = P(A_3) = (\frac{2}{3})^4$$

the lost of two people is the win of the third:

$$P(A_1 \cap A_2) = P(A_1 \cap A_3) = P(A_2 \cap A_3) = (\frac{1}{3})^4$$

$$P(A_1 \cap A_2 \cap A_3) = 0$$
 since there must be a winner

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$

$$\Rightarrow P(A_1 \cup A_2 \cup A_3) = 3 \cdot (\frac{2}{3})^4 - 3 \cdot (\frac{1}{3})^4 = \frac{16}{27} - \frac{1}{27} = \frac{5}{9}$$