Question 1

$$P(X_i = 1) = \frac{17}{40}$$

$$P(X_i = 0) = \frac{23}{40}$$

$$E[X_i] = \frac{17}{40} \times 1 + \frac{23}{40} \times 0 = \frac{17}{40}$$

$$\mu = E[X] = 15 \times E[X_i] = \frac{51}{8}$$

$$\sigma^2 = 15P(X_i = 1)P(X_i = 0)$$

$$\Rightarrow \sigma^2 = 15 \times \frac{17}{40} \times \frac{23}{40} = \frac{1173}{320}$$

Question 2

$$x + y = 6$$
$$y = -x + 6$$
$$\Rightarrow \text{Cov}(X, Y)$$
$$= \text{Cov}(X, 6 - X)$$
$$= \text{Cov}(X, X)$$
$$= -E[X^2] + E[X]^2$$

$$E[X] = \sum_{x=1}^{5} x P_X(X = x)$$
$$= 1 \times \frac{\binom{6}{1}\binom{4}{4}}{\binom{10}{5}} + 2 \times \frac{\binom{6}{2}\binom{4}{3}}{\binom{10}{5}} + 3 \times \frac{\binom{6}{3}\binom{4}{1}}{\binom{10}{5}} + 4 \times \frac{\binom{6}{4}\binom{4}{1}}{\binom{10}{5}} + 5 \times \frac{\binom{6}{5}\binom{4}{0}}{\binom{10}{5}}$$
$$= 3$$

$$E[X^2] = x^2 P_X(X = x)$$
$$= \frac{29}{3}$$
$$\Rightarrow (X, Y) = -\frac{29}{3} + 3^2 = -\frac{2}{3}$$