Question 1

Problem a

$$E[x] = \sum_x xP(X = x)$$
$$= P(X = 1) + 2P(X = 2) + ...$$
$$= (P(X = 1) + P(X = 2) + ...) + (P(X = 2) + P(X = 3) + ...) + ...$$
$$= P(X \geqslant 1) + P(X \geqslant 2) + ...$$
$$= \sum_{k=1}^{\infty} P(X \geqslant k)$$

Problem b

$$p = \frac{1}{3}$$
$$E[X] = \sum_{k=1}^{\infty} P(X \geqslant k)$$
$$P(X \geqslant k) = 1 - P(x < k) = 1 - \sum_{i=0}^{k-1} P(X = i) = (1 - p)^{k-1}$$
$$E[X] = \sum_{k=1}^{\infty} P(X \geqslant k) = \sum_{k=1}^{\infty} (1 - p)^{k-1}$$
$$= \sum_{k=1}^{\infty} (1 - \frac{1}{3})^{k-1} = \frac{1}{1 - (1 - \frac{1}{3})} = 3$$

Question 2

$$f(x) = \frac{d}{dx}P(X \leqslant x) = \frac{1}{60}$$
$$P = P(X \in [5, 10]) + P(X \in [15, 20]) + P(X \in [25, 30])$$
$$+ P(X \in [35, 40]) + P(X \in [45, 50]) + P(X \in [55, 60])$$
$$= \int_5^{10} \frac{1}{60}dx + \int_{15}^{20} \frac{1}{60}dx + \int_{25}^{30} \frac{1}{60}dx$$
$$+ \int_{35}^{40} \frac{1}{60}dx + \int_{45}^{50} \frac{1}{60}dx + \int_{55}^{60} \frac{1}{60}dx$$
$$= \frac{1}{12} \times 6$$
$$= \frac{1}{2}$$