

Question 1

$$\begin{aligned}
 & P(X = Y) \\
 &= \sum_{1 \leq k < n} P(X = k, Y = n) \\
 &= \sum_{n=1}^{\infty} \sum_{k=1}^{n-1} p(1-p)^{k-1} q(1-q)^{n-1} \\
 &= pq \sum_{n=1}^{\infty} \frac{1 - (1-p)^{n-1}}{1 - (1-p)} (1-q)^{n-1} \\
 &= q \sum_{n=1}^{\infty} (1 - (1-p)^{n-1}) (1-q)^{n-1} \\
 &= q \sum_{n=1}^{\infty} (1-q)^{n-1} - q \sum_{n=1}^{\infty} (1-p)^{n-1} (1-q)^{n-1} \\
 &= 1 - \frac{q}{1 - (1-p)(1-q)} \\
 &= 1 - \frac{q}{p + q - pq} \\
 &= \frac{p - pq}{p + q - pq}
 \end{aligned}$$

Question 2

$$\begin{aligned}& \int_0^1 \int_y^{2-y} 3(2-x)y dx dy \\&= \int_0^1 3y(2-2y) dy \\&= 1\end{aligned}$$

Problem a

$$\begin{aligned}f_X &= \int_{-\infty}^{\infty} 3(2-x)y dy \\& \quad 0 < x \leq 1 \\&= \int_0^x 3(2-x)y dy \\&= \int_0^x 6y - 3xy dy \\&= -\frac{3}{2}x^2 + 3x^2 \\& \quad 1 < x \leq 2 \\&= \int_0^{2-x} 3(2-x)y dy \\&= \int_0^{2-x} 6y - 2xy dy \\&= -\frac{3}{2}(x-2)^2 \\f_Y &= \int_{-\infty}^{\infty} 3(2-x)y dx \\&= \int_y^{2-y} 3(2-x)y dx \\&= \int_y^{2-y} 6y - 6xy dx \\&= 6y - 6y^2\end{aligned}$$

Problem b

$$\begin{aligned} & E[XY] \\ &= \int_0^1 \int_y^{2-y} xy3(2-x)y dx dy \\ &= \int_0^1 \int_y^{2-y} 6xy - 3x^2y^2 dx dy \\ &= \int_0^1 2y^5 - 6y^4 + 4y^2 dy \\ &= \frac{7}{15} \end{aligned}$$

Problem c

$$\begin{aligned} & P(x+y \leq 1) \\ &= \int_0^{\frac{1}{2}} \int_y^{1-y} xy3(2-x)y dx dy \\ &= 3 \int_0^{\frac{1}{2}} \frac{3}{2}(1-y)y dy \\ &= \frac{3}{16} \end{aligned}$$