

Question 1

$$\begin{aligned}P(4 \text{ of a kind}) &= \frac{13 \times 48}{\binom{52}{4}} \\ \Rightarrow P(4 \text{ of a kind}) &\approx 0.00024 \\ p &= 0.00024 \\ \Rightarrow \lambda = np &= 10000 \times 0.00024 = 2.4 \\ P(X = x) &= \frac{e^{-\lambda} \lambda^x}{x!} \\ \Rightarrow P(X \leq 2) &= P(X = 0) + P(X = 1) + P(X = 2) \\ P(X \leq 2) &= \frac{e^{-2.4} 2.4^0}{0!} + \frac{e^{-2.4} 2.4^1}{1!} + \frac{e^{-2.4} 2.4^2}{2!} \\ \Rightarrow P(X \leq 2) &= 0.569709\end{aligned}$$

Question 2

$$p_X = 0.1$$

$$n = 400$$

$$\Rightarrow \mu_X = np = 40$$

$$\sigma_X = \sqrt{np_X(1 - p_X)} = 6$$

Use normal distribution

$$P(X \geq 48)$$

$$= P(X \geq 48 - 0.5)$$

$$= P\left(\frac{X - \mu_X}{\sigma_X} \geq \frac{47.5 - 40}{6}\right)$$

$$= P(z \geq 1.25)$$

$$= 1 - P(z < 1.25)$$

$$= 1 - 0.8944$$

$$= 0.1056$$

$$p_Y = 0.0025$$

$$\Rightarrow \mu_Y = 1$$

Use Poisson distribution

$$P(Y \geq 2)$$

$$= 1 - (P(Y = 0) + P(Y = 1))$$

$$= 1 - \left(\frac{e^{-1} \times 1^0}{0!} + \frac{e^{-1} \times 1^1}{1!}\right)$$

$$= 1 - (0.36788 + 0.36788)$$

$$= 0.2642$$