Question 1

$$
\begin{aligned}
&P(X=Y)\\
=&\sum_{1\leqslant k<n} P(X=k, Y=n)\\
=&\sum_{n=1}^{\infty}\sum_{k=1}^{n-1} p(1-p)^{k-1}q(1-q)^{n-1}\\
=&pq\sum_{n=1}^{\infty}\frac{1-(1-p)^{n-1}}{1-(1-p)}(1-q)^{n-1}\\
=&q\sum_{n=1}^{\infty}(1-(1-p)^{n-1})(1-q)^{n-1}\\
=&q\sum_{n=1}^{\infty}(1-q)^{n-1}-q\sum_{n=1}^{\infty}(1-p)^{n-1}(1-q)^{n-1}\\
=&1-\frac{q}{1-(1-p)(1-q)}\\
=&1-\frac{q}{p+q-pq}\\
=&\frac{p-pq}{p+q-pq}
\end{aligned}
$$

Question 2

$$
\begin{aligned}
&\int_0^1 \int_y^{2-y} 3(2-x)ydxdy \\
=&\int_0^1 3y(2-2y)dy \\
=&1
\end{aligned}
$$

Problem a

$$
\begin{aligned}
f_X =& \int_{-\infty}^{\infty} 3(2-x)ydy \\
& 0 < x \leqslant 1 \\
=& \int_0^x 3(2-x)ydy \\
=& \int_0^x 6y - 3xydy \\
=& -\frac{3}{2}x^3 + 3x^2 \\
& 1 < x \leqslant 2 \\
=& \int_0^{2-x} 3(2-x)ydy \\
=& \int_0^{2-x} 6y - 2xydy \\
=& -\frac{3}{2}(x-2)^3 \\
f_Y =& \int_{-\infty}^{\infty} 3(2-x)ydx \\
=& \int_y^{2-y} 3(2-x)ydx \\
=& \int_y^{2-y} 6y - 6xydx \\
=&6y - 6y^2
\end{aligned}
$$

Problem b

$$
\begin{aligned}
&E[XY]\\
=&\int_0^1\int_y^{2-y} xy3(2-x)ydxdy\\
=&\int_0^1\int_y^{2-y} 6xy-3x^2y^2dxdy\\
=&\int_0^1 2y^5-6y^4+4y^2dy\\
=&\frac{7}{15}
\end{aligned}
$$

Problem c

$$
\begin{aligned}
&P(x+y\leqslant 1)\\
=&\int_0^{\frac{1}{2}}\int_y^{1-y} xy3(2-x)ydxdy\\
=&3\int_0^{\frac{1}{2}}\frac{3}{2}(1-y)ydy\\
=&\frac{3}{16}
\end{aligned}
$$