

Question 1

Problem a

$$\begin{aligned} E[Z^3] &= \int_{-\infty}^{\infty} Z^3 f(Z) dz \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Z^3 e^{-\frac{z^2}{2}} dz \\ &= 0 \end{aligned}$$

Problem b

$$\begin{aligned} E[Z^2] &= \int_{-\infty}^{\infty} Z^2 f(Z) dz = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Z^2 e^{-\frac{z^2}{2}} dz = 1 \\ X^3 &= (\sigma Z + \mu)^3 = \sigma^3 Z^3 + 3\sigma Z\mu^2 + 3\sigma^2 Z^2\mu + \mu^3 \\ E[X^3] &= \sigma^3 E[Z^3] + 3\sigma^2 E[Z^2]\mu + 3\sigma E[Z]\mu^2 + \mu^3 \\ \Rightarrow E[X^3] &= 0 + 3\sigma^2\mu + 0 + \mu^3 = 3\sigma^2\mu + \mu^3 \end{aligned}$$

Question 2

$$m = 0$$

$$\sigma = 6$$

$$P(x > 5) = P\left(\frac{x - 0}{6} > \frac{5 - 0}{6}\right) = P\left(z > \frac{5}{6}\right)$$

$$P\left(z > \frac{5}{6}\right) = 1 - P\left(z \leq \frac{5}{6}\right) = 1 - 0.79673 = 0.20327$$