

Question 1

Problem a

$$P = \binom{n}{k} p^k (1-p)^{n-k}$$

Problem b

Even :

$$S_n = 0$$

$$\Rightarrow k = n - k$$

$$\Rightarrow n = 2k$$

$$\Rightarrow P(S_n = 0) = \binom{2k}{k} p^k (1-p)^k$$

Odd :

With one step at a time, it cannot return to origin after odd steps

$$\Rightarrow P(S_n = 0) = 0$$

Problem c

$$E(S) = 80 \cdot 0.45 = 36$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{80 \cdot 0.45 \cdot (1-0.45)}$$

$$\Rightarrow P(S_{80} = 0) = P(S = 40) = P(39.5 < S < 40.5)$$

$$\Rightarrow P(S_{80} = 0) = P(S < 40.5) - P(S < 39.5)$$

$$\Rightarrow P(S_{80} = 0) = P\left[Z < \frac{40.5 - 36}{\sigma}\right] - P\left[Z < \frac{39.5 - 36}{\sigma}\right]$$

$$\approx P[Z < 1.0113] - P[Z < 0.7866]$$

$$= 0.8438 - 0.7823$$

$$= 0.0615$$

$$\Rightarrow P(S_{80} = 0) \approx 0.0615$$

Question 2

Problem a

$$\begin{aligned}2\Phi(2\varepsilon\sqrt{n}) - 1 &\geq 0.95 \\ \Rightarrow \Phi(2 \times 0.02\sqrt{n}) &\geq 0.975 \\ 0.04\sqrt{n} &\geq 1.96 \\ \sqrt{n} &\geq 49 \\ \Rightarrow n &\geq 2401\end{aligned}$$

Problem b

$$\begin{aligned}P(S_n = 5050) &= \Phi\left(\frac{5000 + \frac{1}{2} - 10000 \times 0.51}{\sqrt{2499}}\right) - \Phi\left(\frac{5000 - \frac{1}{2} - 10000 \times 0.51}{\sqrt{2499}}\right) \\ \Rightarrow P(S_n = 5050) &= \Phi(-0.9902) - \Phi(-1.0102) \\ \Rightarrow P(S_n = 5050) &= 0.1611 - 0.1562 = 0.0049\end{aligned}$$