

Question 1

Problem a

$$\begin{aligned}P(FB|Y) &= \frac{P(FB \cap Y)}{P(Y)} \\P(FB \cap Y) &= \frac{4}{10} \times \frac{1}{2} \\&= \frac{1}{5} \\P(Y) &= \frac{4}{10} \times \frac{1}{2} + \frac{4}{7} \times \frac{1}{2} \\&= \frac{17}{35} \\P(FB|Y) &= \frac{\frac{1}{5}}{\frac{17}{35}} \\&= \frac{7}{17}\end{aligned}$$

Problem b

$$\begin{aligned}P &= \frac{\frac{4}{10} \times \frac{1}{2} \times \frac{4}{10}}{\frac{4}{7} \times \frac{1}{2} \times \frac{4}{7}} \\&= \frac{\frac{2}{25}}{\frac{2}{25} + \frac{8}{49}} \\&= \frac{49}{149} \\&\quad \frac{49}{149} < \frac{7}{17} \\&\Rightarrow \text{less likely}\end{aligned}$$

Question 2

Problem a

$$\begin{aligned}
 \{A\} &= \text{DNA matches} \\
 \{B\} &= \text{Guilty} \\
 P(B|A) &= \frac{P(A \cap B)}{P(A)} \\
 &= \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)} \\
 &= \frac{\frac{9999}{10000} \times \frac{1}{100000}}{\frac{9999}{10000} \times \frac{1}{100000} + \frac{1}{10000} \times \frac{99999}{100000}} \\
 &= \frac{1111}{12222}
 \end{aligned}$$

Problem b

$$\begin{aligned}
 P(B|A) &= \frac{P(A \cap B)}{P(A)} \\
 &= \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)} \\
 &= \frac{\frac{49999}{50000} \times \frac{1}{100000}}{\frac{49999}{50000} \times \frac{1}{100000} + \frac{1}{50000} \times \frac{99999}{100000}} \\
 &= \frac{49999}{149998}
 \end{aligned}$$

Problem c

Old :

$$\begin{aligned}
 P(B|A) &= \frac{P(A \cap B)}{P(A)} \\
 &= \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)} \\
 &= \frac{\frac{9999}{10000} \times \frac{1}{1000}}{\frac{9999}{10000} \times \frac{1}{1000} + \frac{1}{10000} \times \frac{999}{1000}} \\
 &= \frac{1111}{1222}
 \end{aligned}$$

New :

$$\begin{aligned}
 P(B|A) &= \frac{P(A \cap B)}{P(A)} \\
 &= \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)} \\
 &= \frac{\frac{49999}{50000} \times \frac{1}{1000}}{\frac{49999}{50000} \times \frac{1}{1000} + \frac{1}{50000} \times \frac{999}{1000}} \\
 &= \frac{49999}{50998}
 \end{aligned}$$