Non-Graded Questions

Question 1

$$
\begin{aligned}
&A \cap B \cap C = \{5, 11, 17\} \\
&(A \cap B) \setminus C = \{2, 8, 14, 20\} \\
&(A \cap C) \setminus B = \{1, 3, 7, 9, 13, 15, 19\}
\end{aligned}
$$

Question 4

Graded Questions

Question 1

Proof.

$$
\begin{aligned}
&S \coloneqq \{1, 4, 7, 10, 13...\}\\
&f : \mathbb{N} \to S\\
&f(x) \coloneqq 3x - 2\\
&\\
&\text{Injection :}\\
&\forall y_1, y_2 \in S :\\
&\exists x_1, x_2 \in \mathbb{N} : f(x_1) = y_1, f(x_2) = y_2\\
&\text{Suppose : } y_1 = y_2\\
\Rightarrow &f(x_1) = f(x_2)\\
&3x_1 - 2 = 3x_2 - 2\\
\Rightarrow &x_1 = x_2\\
&y_1 = y_2 \implies x_1 = x_2\\
\Rightarrow &f : \text{Injective}\\
&\\
&\text{Surjection :}\\
&\forall y_k \in S, \exists x_k \in \mathbb{N} :\\
&y_k = 3x_k - 2\\
\Rightarrow &f : \text{Surjective}\\
&\\
\Rightarrow &f : \text{Bijective}
\end{aligned}
$$

□

Quesiton 2

Proof.

$$
\begin{aligned}
& f : (0,1) \to \mathbb{R}_{>0} \\
& f(x) = \tan(\frac{\pi}{2}x) \\
& \\
& \text{Injection :} \\
& \forall y_1, y_2 \in \mathbb{R}_{>0} : \\
& \exists x_1, x_2 : y_1 = \tan(\frac{\pi}{2}x_1), y_2 = \tan(\frac{\pi}{2}x_2) \\
& \text{Suppose : } y_1 = y_2 \\
\Rightarrow & \tan(\frac{\pi}{2}x_1) = \tan(\frac{\pi}{2}x_2) \\
& \tan \text{ is bijective in } (0,1) \\
\Rightarrow & \frac{\pi}{2}x_1 = \frac{\pi}{2}x_2 \\
\Rightarrow & x_1 = x_2 \\
& y_1 = y_2 \implies x_1 = x_2 \\
\Rightarrow & f : \text{Injective} \\
& \\
& \text{Surjective :} \\
& \forall y_k \in \mathbb{R}_{>0} : \\
& \exists \frac{\pi}{2}x_k : y_k = \frac{\pi}{2}x_k, x_k \in (0,1) \\
\Rightarrow & \forall y_k \in \mathbb{R}_{>0}, \exists x_k \in (0,1) : f(x_k) = y_k \\
\Rightarrow & f : \text{Surjective} \\
& \\
\Rightarrow & f : \text{Bijective}
\end{aligned}
$$

□

Question 3

Proof.

$$
\begin{aligned}
& f : [0,1) \to \mathbb{R}_{>0} \\
& f = \begin{cases} \tan(\frac{\pi}{2}x), x \in (0,1) \\ \infty, x = 0 \end{cases} \\
& \\
& \text{Injection :} \\
& f(0) \text{ cannot be reached by any other value in } (0,1) \\
& \forall y_1, y_2 \in \mathbb{R}_{>0} : \\
& \exists x_1, x_2 : y_1 = \tan(\frac{\pi}{2}x_1), y_2 = \tan(\frac{\pi}{2}x_2) \\
& \text{Suppose : } y_1 = y_2 \\
\Rightarrow & \tan(\frac{\pi}{2}x_1) = \tan(\frac{\pi}{2}x_2) \\
& \tan \text{ is bijective in } (0,1) \\
\Rightarrow & \frac{\pi}{2}x_1 = \frac{\pi}{2}x_2 \\
\Rightarrow & x_1 = x_2 \\
& y_1 = y_2 \implies x_1 = x_2 \\
\Rightarrow & f : \text{Injective} \\
& \\
& \text{Surjective :} \\
& \infty = f(0) \in \mathbb{R}_{>0} \\
& \forall y_k \in \mathbb{R}_{>0} : \\
& \exists \frac{\pi}{2}x_k : y_k = \frac{\pi}{2}x_k, x_k \in (0,1) \\
\Rightarrow & \forall y_k \in \mathbb{R}_{>0}, \exists x_k \in (0,1) : f(x_k) = y_k \\
\Rightarrow & f : \text{Surjective} \\
& \\
\Rightarrow & f : \text{Bijective}
\end{aligned}
$$

□

Question 4

Proof.

$g \circ f : A \to C$

Surjection :

g is surjective

$\Rightarrow \forall z_k \in C, \exists y_k \in B : g(y_k) = z_k$

f is surjective

$\Rightarrow \forall y_k \in B, \exists x_k \in A : f(x_k) = y_k$

$\Rightarrow \forall z_k \in C, \exists x_k \in A : g(f(x_k)) = z_k$

$\forall z_k \in C, \exists x_k \in A : g \circ f(x_k) = z_k$

$\Rightarrow g \circ f$: surjective

Injection :

g is injective

$\Rightarrow \forall z_1 = g(y_1) = z_2 = g(y_2) \in C : y_1 = y_2 \in B$

f is injective

$\Rightarrow \forall y_1 = f(x_1) = y_2 = f(x_2) \in B, x_1 = x_2 \in A$

$z_1 = z_2 \implies g(y_1) = g(y_2) \implies y_1 = y_2 \implies f(x_1) = f(x_2) \implies x_1 = x_2$

$\Rightarrow z_1 = z_2 \implies x_1 = x_2$

$z_1 = g \circ f(x_1) = z_2 = \circ f(x_2)$

$\Rightarrow g \circ f$: injective

$\square$