Smart Rail Network Optimization Phase 2

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1 Problem Description

After a successful initial Phase I of the project, our team has been rehired by SmartRail, a transportation company that moves shipping containers within its existing network of train stations and tracks. With various shipments having a series of different arrival and departure windows as well as various origin and destination points, we endeavor to help SmartRail create a transportation plan to deliver these shipments at a minimal cost. This Phase II version of the problem is expanded in its inclusion of time in the model, requiring a balanced flow of the network on each day. With the inclusion of time, it also means that containers are only available to be picked up at or after a certain time and should be delivered by a certain time, otherwise it will be late and incur a penalty as a result. Additionally, this expanded version of the problem now allows for the construction of additional reloading zones and additional tracks. These additional tracks can only be built on routes that already have an existing track. Still, the goal is to minimize the costs. In this Phase II version of the problem, there are now four components of the costs – the cost for building new tracks, the cost for building new reloading zones, the penalty fee for late containers, and the cost to ship containers. We note that there are no longer considerations for reload costs.

There are also constraints within the SmartRail network. Containers can only be picked up at their station of origination on or after their specified pickup date – nothing can be moved prior. Additionally, all containers must arrive at their final destination by the end of the time horizon. On a single day, containers can only be shipped from one station to an adjacent station. While new tracks and new reloading zones are allowed, there is a limit on the number of new tracks and new reloading zones that can be built. Lastly, the entire network must have flow balance of containers – for each shipment at each station on every day in the time horizon.

In order to optimize this process, we are creating a MILP (mixed-integer linear program) to model and minimize SmartRail's cost structure in AMPL. The model will be derived and explained in this report (see section 2). Like before, our model will enable SmartRail to directly load .csv files into the model so they can use our model on different sets of data. The code will output comprehensive log files with all the information that SmartRail needs to implement the transportation plan. For the two cases provided by SmartRail to our team, we will provide a detailed summary of the results and a comprehensive analysis (see section 3). The analysis will help give context to the results – explaining why certain new tracks were built or why certain containers were delivered late. This report will provide SmartRail with all the information needed to implement the optimal transportation plan to accommodate their shipping needs at the minimum possible cost.

2 Model

As mentioned in the Problem Description, this project seeks to find an optimal solution using a custom MILP implemented in AMPL. This section discusses the components of the MILP and then presents the program itself.

2.1 Overview

This optimization problem will be modeled as a MILP, due to the types of decision variables, objective, and constraints. It is expanded upon the previous model derived in Phase I, as the new model now includes time (in these cases, with a bi-weekly shipping schedule). In order to make this model functional as SmartRail changes and grows their network, we model this MILP data-independently. This is done through the creation of sets and parameters described in sections 2.2 and 2.3 below. Then, decision variables are created in 2.4, enabling the entire model to be written in section 2.6. We explain the derivation of the model more in sections 2.5, 2.7, and 2.8. To actually use this model and compute optimal shipping routes and objective values, the specific data is loaded into the data-independent model. In both Case 1 and Case 2, SmartRail is providing data through CSV files. We will present and analyze the results from both cases in section 3.

2.2 Sets

The first step in the creation of a data-independent model is through the creation of sets. A set is used to describe a component of the model that helps define the dimensions of the problem within the decision variables and the constraints. For this problem, four sets are needed: a set for each of the tracks (origin, destination), a set for each of the stations (number), a set for each of the shipments (letter), and a set for the number of days (number from 1 to T - a parameter we will define in 2.3). In this version of the problem, set tracks only include those that are in one direction (from station i to station j). We will define a decision variable in section 2.4 that will handle shipments moving in the other direction (from station j to station i).

Set	Description	Example
stations	The given stations	stations $= \{1, 2, 3, \ldots.\}$
tracks	Listed by origin and destination station	tracks $= \{(1, 3), (1, 4), \ldots.\}$
shipments (abbr. ship)	A list of the shipments	ship $= \{A, B, \ldots.\}$
days	The timespan for container movement	days $= \{1, \ldots, \mathrm{T}\}$

Table 1. Table of Sets

2.3 Parameters

Next, parameters are needed to define the objective and the constraints (they are given data). For both Case 1 and Case 2, a parameter was created for each component of the .csv files that do not correspond to one of the sets in Table 1 above. We also included a parameter for the length of a track, which is calculated from the x_coord and y_coord parameters (using the distance formula) This model includes both parameters that are constant and parameters that depend on the sets defined above. Table 2 below defines each parameter used in the model as well as what it depends on (if anything).

Parameter	Description	Units
T	The number of days in this shipment schedule.	# days
cost_ship	Cost to ship.	$ per container per mile
capa_track	Fixed capacity per track.	Containers per track
capa_reload	Fixed reloading capacity for all stations.	Containers per station
cost_new_reload	Fixed cost of building a new reload zone	$ per reload zone
cost_penalty	The cost penalty for a container being late per day.	$ per day
cost_track	The cost of building new track per mile.	$ per mile of new track
max_reload	The maximum number of reload zones.	# reload zones
max_tracks	The maximum number of tracks allowed.	# tracks
orig{shipments}	Origin station by shipment.	(station)
dest{shipments}	Destination station by shipment.	(station)
volume{shipments}	Volume by shipment.	# containers
avail_day{shipments}	The day that a shipment becomes available.	(day)
deliv_day{shipments}	The day that a shipment is due for delivery.	(day)
num_reload{stations}	Number of reloading zones per station.	# reloading zones
x_coord{stations}	X coordinates indexed by station.	(latitude, longitude)
y_coord{stations}	Y coordinates indexed by station.	(latitude, longitude)
len_track{(i,j) in tracks}	Length between tracks.	Miles

Table 2. Table of Parameters

2.4 Decision Variables

Section 2.4.1 contains the "standard" decision variables, which SmartRail needs to know in order to implement the plan. Section 2.4.2 contains a series of variables, as functions of these decision variables, that calculate the objective function. They are variables in the sense that they vary on the decision variables (rather than parameters, which are constants)!

2.4.1 Standard Decision Variables

In order to find an optimal solution, variables that define the objective function and the constraints are needed. The key for decision variables is that they not only are capable of defining the objective and constraints but also that they can adequately answer the question at hand. The "standard" decision variables are shown in Table 3 below. Firstly, a decision variable is needed to track the number of containers that move along a track (in a certain direction) on each day for each shipment. Thus, we defined a variable x as the number of containers belonging to shipment s traveling along the track from station i to station j. Next, we defined a variable y to be the number of containers belonging to shipment s traveling along the track from station j to station j. By reversing the j and i (but still using the set tracks), the y variables handle the reverse direction of travel. These decision variables encode the primary answer the the question at hand – determining what containers are moved for each shipment on each day along each route.

In this version of the problem, there is also the possibility of building additional reloading stations and additional tracks. We need a decision variable to handle each of these cases. As can be seen in Table 3 below, we define a variable r that is the number of new reloading zones built at each station k. Then, we define a variable t that is the number of new tracks built along each track from station i to station j. We note that this t variable will also implicitly (based on our definition of y) include the expanded routing in the reverse direction.

Finally, with the inclusion of time, it now becomes possible for containers to leave the origin and be delivered at the destination on different days. Thus, we need a decision variable to handle each of these cases. Since for a given shipment, containers can only originate from one node (the parameter orig) or be delivered at one node (the parameter dest), there is no need to make these variables a function of stations, as they only exist at one station. We create ori as the variable to handle the number of containers from shipment s leaving the origination station on day d. We define del to be the number of containers from shipment s that are delivered on day d. (Note: the use of del and not d is to not be confused with d which is the parameter for days. We use ori - for origination - instead of o for consistency).

Combining all the decision variables for a given shipment will give Smart Rail all of the information it needs in order to implement the optimal shipping schedule. It will also enable the objective function to be defined and each of the constraints to be written in terms of these decision variables and the parameters.

Decision Variable	Description	Example	Explanation
$x_{s,d,(i,j)}$	Number of containers belonging to shipment s traveling along the track from station i to j on day d.	$x_{A,1,(1,3)}$	The number of containers in shipment A traveling by track from station 1 to 3 on day 1.
$y_{s,d,(j,i)}$	Number of containers belonging to shipment s traveling along the track from station j to i on day d.	$y_{A,1,(3,1)}$	The number of containers in shipment A traveling by track from station 3 to 1 on day 1.
r_k	Number of new reloading zones constructed at each station k.	r_1	The number of new reloading zones built at station 1.
$t_{(i,j)}$	Number of new tracks built along the track (i, j) bi-directionally.	$t_{(1,3)}$	The number of new tracks built along track (1, 3).
$\text{ori}_{s,d}$	Number of containers leaving the origination station belonging to shipment s on day d.	$\text{ori}_{1,1}$	Number of containers leaving shipment 1's origination station on day 1
$\text{del}_{s,d}$	Number of containers delivered at the destination station belonging to shipment s on day d.	$\text{del}_{1,9}$	Number of containers delivered to shipment 1's destination station on day 9

Table 3. Decision Variable Table

2.4.2 Variables to Determine Objective Costs (Computed from Standard Decision Variables)

$$var \text{ total\_ship\_cost } = \sum_{s \in \text{Ship}} \sum_{d \in \text{Days}} \sum_{(i,j) \in \text{Tracks}} \text{cost\_ship} * \text{len\_track}_{(i,j)} * x_{s,d,(i,j)} * y_{s,d,(j,i)} \tag{1}$$

$$var \text{ new\_track\_cost } = \sum_{(i,j) \in \text{Tracks}} \text{cost\_track} * \text{len\_track}_{(i,j)} * T * t_{(i,j)} \tag{2}$$

$$var \text{ new\_reload\_cost } = \sum_{k \in \text{Stations}} \text{cost\_new\_reload} * T * r_k \tag{3}$$

$$var \text{ delay\_cost } = \sum_{s \in \text{Ship}} \sum_{d \in \text{Days}\{d > \text{deliv\_date}_s\}} \text{cost\_penalty} * (d - \text{deliv\_date}_s) * \text{del}_{(s,d)} \tag{4}$$

Equations 1-4 explain the four different variable values that will later be used to define the objective function (see sections 2.6/2.7). We note that these equations which make up the objective are variable, as they depend on the values of the decision variables from 2.4.1 above!

Firstly, equation 1 represents the total shipping cost. For each container, there is a fixed cost to ship it per mile. By multiplying this by the length of track, it is then possible to compute the total cost to ship each container. As defined in 2.4.1, the x and y decision variables represent the number of containers that move on a track on a given day (x in one direction, y in the other). Thus, summing across every shipment, day, and track for the x and y variables enables the cost of shipping containers along a given track to be computed per shipment per day through eq 1.

Equation 2 represents the total cost for building new tracks. The cost for a new track has been amortized to a fixed daily cost per mile. Thus, to get the cost for the entire period, the fixed cost is multiplied by the length of the time period. Then, the number of miles a track covers (len_track) is multiplied to get the cost for building a specific track for the shipping schedule. This is multiplied by the t decision variable, which contains the number of new tracks built. That gets the cost for building a variable (t) number of tracks between two stations. Finally, we sum up over every track in the set tracks to get the total cost of building new tracks for the whole network. We note that because t is defined simply as being between two stations, it covers both the forward and backward directions. This is handled more specifically in the track limit constraint (eq 15).

Equation 3 represents the total cost for new reloading zones. The cost for a reloading zone has been amortized to a fixed daily cost. Thus, to get the cost for the entire period, the fixed cost of a new reloading zone per day is multiplied by the length of the time period. Then, this is multiplied by the decision variable r, which reflects the number of new reloading zones at a given station k. Finally, this needs to be summed up across all k in stations in order to compute the reloading costs across the entire network.

Equation 4 is the penalty costs for delayed containers. As described in the problem statement, containers have a fixed penalty fee per day they are late. As such, only days in which the given day is after the delivery day should be evaluated. As defined in 2.4.1, our del decision variable say how many containers were delivered on a certain day d for each shipment. Thus, the difference of the current day d and the scheduled delivery day multiplied by the del decision variable says how late those containers were. Multiplying by the fixed cost per day gives the penalty fee for any late containers. Finally, this is summed across all shipments and days (for days after the delivery date for a given shipment) in order to get the total cost of delays within the network across the entire time horizon.

2.5 Optimization Model Explanation

Before presenting the model and explaining why the MILP models the problem through its objectives and constraints, it is first essential to define what an optimization model does. The purpose of an optimization problem is to try and find an optimal solution for a given problem (assuming such a feasible solution exists and the objective is bounded). In order to solve this mixed-integer linear program, we will use the CPLEX algorithm in AMPL which will iterate until it finds an optimal solution (assuming one exists; there can also be multiple optimal solutions).

Once the decision variables are defined and the data is provided, such an MILP can be crafted. First, an objective function is needed to either maximize or minimize some combination of the decision variables that define the goal of the project. In this case, as described in the Project Description, the goal of this project is to minimize costs. For simplicity, the components of the objective were stored as variables (see section 2.4.2), such that the objective could simply be the sum of variables and is easy to read and interpret (see sections 2.6/2.7).

Then, a series of constraints are created that bound the set of feasible solutions based on either requirements or limitations within the problem. This is explained in section 2.8. In the context of this problem, such limits include the number of containers on a track, the number of containers that can be reloaded at a station, and balanced flow throughout the network (among others). Combining the sets, parameters, and decision variables into a series of objectives and constraints creates the optimization model – in this case a MILP. The complete model is written out in section 2.6 below.

2.6 Model

Equations (5-18) below create the entire MILP. Equation (5) is the objective function, equations (6-17) are the main constraints, and equation (18) are the sign and variable type constraints. An explanation of the MILP and its objectives and constraints is given in sections 2.4.2 above, as well as 2.7 and 2.8 below. Note that the set Shipments is abbreviated as Ship.

Mixed-Integer Linear Program

$$min \quad total\_ship\_cost + new\_track\_cost + new\_reload\_cost + delay\_cost \tag{5}$$

$$s.t \quad \mathrm{ori}_{s,d} = \sum_{(k,j)\in \mathrm{Tracks}} (x_{s,d,(k,j)}) + \sum_{(j,k)\in \mathrm{Tracks}} (y_{s,d,(k,j)}) \tag{6}$$

$$\forall\, s \in \mathrm{Shipments} \quad k \in \mathrm{Stations}\{k = \mathrm{orig}_s\} \quad d = 1$$

$$0 = \sum_{(k,j)\in \mathrm{Tracks}} (x_{s,d,(k,j)}) + \sum_{(j,k)\in \mathrm{Tracks}} (y_{s,d,(k,j)}) \tag{7}$$

$$\forall\, s \in \mathrm{Shipments} \quad k \in \mathrm{Stations}\{k \neq \mathrm{orig}_s\} \quad d = 1$$

$$\sum_{(i,k)\in \mathrm{Tracks}} (x_{s,(d-1),(i,k)}) + \sum_{(k,i)\in \mathrm{Tracks}} (y_{s,(d-1),(i,k)}) + \mathrm{ori}_{s,d} = \sum_{(k,j)\in \mathrm{Tracks}} (x_{s,d,(k,j)}) + \sum_{(j,k)\in \mathrm{Tracks}} (y_{s,d,(k,j)}) \tag{8}$$

$$\forall\, s \in \mathrm{Shipments} \quad k \in \mathrm{Stations}\{k = \mathrm{orig}_s\} \quad d \in \mathrm{Days}\,\{2 \ldots T\}$$

$$\sum_{(i,k)\in \mathrm{Tracks}} (x_{s,(d-1),(i,k)}) + \sum_{(k,i)\in \mathrm{Tracks}} y_{s,(d-1),(i,k)} = \sum_{(k,j)\in \mathrm{Tracks}} (x_{s,d,(k,j)}) + \sum_{(j,k)\in \mathrm{Tracks}} y_{s,d,(k,j)} + \mathrm{del}_{s,d} \tag{9}$$

$$\forall\, s \in \mathrm{Shipments} \quad k \in \mathrm{Stations}\{k = \mathrm{dest}_s\} \quad d \in \mathrm{Days}\,\{2 \ldots T\}$$

$$\sum_{(i,k)\in \mathrm{Tracks}} (x_{s,(d-1),(i,k)}) + \sum_{(k,i)\in \mathrm{Tracks}} (y_{s,(d-1),(i,k)}) = \sum_{(k,j)\in \mathrm{Tracks}} (x_{s,d,(k,j)}) + \sum_{(j,k)\in \mathrm{Tracks}} (y_{s,d,(k,j)}) \tag{10}$$

$$\forall\, s \in \mathrm{Shipments} \quad k \in \mathrm{Stations}\{k \neq \mathrm{orig}_s, \mathrm{dest}_s\} \quad d \in \mathrm{Days}\,\{2 \ldots T\}$$

$$\sum_{d \in \text{Days}\{\text{avail\_day}_s \ldots (T-1)\}} (\text{ori}_{s,d}) = \text{volume}_s \quad \forall\, s \in \text{Shipments} \tag{11}$$

$$\sum_{d \in \text{Days}\{(\text{avail\_day}_s+1) \ldots T\}} (\text{del}_{s,d}) = \text{volume}_s \quad \forall\, s \in \text{Shipments} \tag{12}$$

$$(x_{s,d,(i,j)} + y_{s,d,(j,i)}) = 0 \quad \begin{array}{l} \forall\, s \in \text{Shipments} \\ (i,j) \in \text{Tracks} \\ d \in \text{Days}\,\{d < \text{avail\_day}_s\} \end{array} \tag{13}$$

$$\sum_{s \in \text{Ship}} \sum_{(i,k) \in \text{Tracks}} (x_{s,d,(i,k)}) + \sum_{s \in \text{Ship}} \sum_{(k,i) \in \text{Tracks}} (y_{s,d,(i,k)}) \leq \text{capa\_reload} * (r_k + \text{num\_reload}_k) \quad \begin{array}{l} \forall\, s \in \text{Shipments} \\ k \in \text{Stations} \end{array} \tag{14}$$

$$\sum_{s \in \text{Ship}} (x_{s,d,(i,j)} + y_{s,d,(j,i)}) \leq \text{capa\_track} * (1 + t_{(i,j)}) \quad \begin{array}{l} \forall\, (i,j) \in \text{Tracks} \\ d \in \text{Days} \end{array} \tag{15}$$

$$(1 + t_{(i,j)}) \leq \mathit{max\_tracks} \quad \forall\, (i,j) \in \text{Tracks} \tag{16}$$

$$(\mathit{num\_reload}_s + r_k) \leq \mathit{max\_reload} \quad \begin{array}{l} \forall\, s \in \text{Shipments} \\ k \in \text{Stations} \end{array} \tag{17}$$

$$x_{s,d,(i,j)},\ y_{s,d,(j,i)},\ r_k,\ t_{(i,j)},\ \text{ori}_{s,d},\ \text{del}_{s,d} \geq 0,\ \text{integer} \quad \begin{array}{l} \forall\, s \in \text{Shipments} \\ k \in \text{Stations} \\ (i,j) \in \text{Tracks} \\ d \in \text{Days} \end{array} \tag{18}$$

2.7 Objective Function Explanation

The components of the objective function are the total shipping cost, the cost for building new tracks, the cost for building the new reloading zones, and the cost for delayed containers (those that arrive after their required delivery date). Each of those was defined as a variable in Section 2.4.2 to simplify the objective function within the model. We attribute each component to their respective names (see eq 5) which are defined by equations 1-4 (in section 2.4.2).

SmartRail is required to ensure that all of these shipments are made on a biweekly basis. It wants to do so at the lowest possible cost. The constraints will ensure that the shipments are made in a feasible and timely manner. The objective is to minimize the total costs for Smart Rail. Thus, the objective function (eq 5) is simply minimizing the sum of the four costs – shipping costs, new track costs, new reload costs, and delay costs.

2.8 Constraints Explanation

We first begin by looking at the flow balance constraints (eqs 6-10). The general principle of the flow balance is that anything arriving on the previous day (d-1) plus anything that originates at the current station (on day d - assuming that station is the origination station) must equal anything departing from the current station (on day d) plus anything that is being delivered at the current station (on day d - assuming that station is the destination station). This must hold for every shipment, at every station, on every day. However, there is a unique problem on day 1, as 1-1 = 0, and no day 0 exists.

Thus, for day 1 we modify the flow balance. As can be seen in equations 6 and 7, the flow balance on day one is simply that the number of containers originating at the current station for a given shipment (assuming the current station is the origin station for that shipment) on day 1 must equal the number of containers leaving that node on day 1. The first of the two equations (equation 6) handles the case where the current station is the origin station. In that case the origination term is included in the equation. The second of the two equations (equation 7) handles the case where this is not true and the origination term is not included. We note that because the data provided by SmartRail only contains tracks in one direction, we reverse the (j, k) indices for trains departing from station k, to handle the reverse direction for the y decision variable. Since multiple tracks can leave from a node, we need to sum up the containers for all tracks departing from station k. These constraints must hold for all shipments and stations – with either eq 6 or eq7 used depending on the specific station number. Because of how the del decision variable was defined, no container can ever be delivered on day 1. Thus, we simply exclude it from the equation.

For days 2 until the end time horizon, the flow balance holds as originally described. The number of containers originating at the current station for a given shipment on the current day d (assuming the current station is the origin station for that shipment) plus the number of containers entering into the station on the previous day d-1 (summed up across all tracks - in both directions - coming INTO the station k) must equal the number of containers leaving the station on the current day d (summed up across all tracks - in both directions - leaving OUT OF the station k) plus the number of containers being delivered to station k on the current day d (assuming station k is the destination station k for shipment s). We note that because the data provided by SmartRail only contains tracks in one direction, we reverse the (j, k) indices for trains departing from station k, to handle the reverse direction for the y decision variable. A similar thing is done with the reversing of the (i,k) indices to handle containers that are entering the current node for tracks in both directions - enabling the valid use of the x and y variables. For clarity in the MILP model, we write this flow balance in three separate equations – equations 8, 9, and 10. The first equation (eq 8) handles a case where the current station is the origin station. It includes the origination term but not the delivery term. The second equation (eq 9) handles the case where the current station is the destination station. It includes the delivery term but not the origination term. Finally, equation 10 handles the case where the current station is neither the destination nor the origin case. In this case, there is a simple flow balance of the x and y decision variables handling containers coming into and out of the station. These conditions must hold for every shipment s at every station k on every day d. The specific one of the three constraints used (eq 8,9,10) depends on the specific station k and whether it is the origin or destination station for a given shipment s. Still, one of these type II flow balance constraints – the one that meets the conditions – must hold as per the conditions on the right – per qualifying station for all shipments on all days (from 2 to the end of the time horizon).

These flow balance constraints are the heart of the model. However, they are meaningless on their own, as there is no requirement for containers actually moving through the network, containers only moving after they become available, or containers being delivered on time. The next three constraints solve those issues.

Equation 11 is the constraint that ensures that the total shipment must originate from its origination station. Because of how the decision variable ori is defined and how the flow balance is written, containers can originate at any point from their available date to the second-to-late day in the time horizon (a container cannot originate and be delivered on the same day, so a container originating on day T would never be delivered, violating the project requirements). Thus, for each shipment, the number of containers originating is summed up from the available day to the second-to-last day. This sum must equal the total volume that shipment must move. This ensures that all containers will originate from their correct starting station at a valid time.

Equation 12 is the constraint that ensures that the total shipment must be delivered to its destination station. Because of how the decision variable del is defined and how the flow balance is written, containers can be delivered at any point from the day after they become available to the last day in the time-horizon (a container cannot originate and be delivered on the same day, so a container originating on its first available day could not be delivered on that same day).

Thus, for each shipment, the number of containers delivered is summed up from the day-after it becomes available to the last day. This sum must equal the total volume that shipment must move. This ensures that all containers will be delivered to their correct destination station within a feasible time and that all will be delivered within the total time horizon.

Even with these origination and destination requirements, there is nothing preventing a container from moving before it becomes available. Thus, a constraint (eq 13) is added to solve this issue. Each shipment has a specific day in which the containers become available. The goal is to ensure that the x and y variables before this date are all zero (which will also ensure that nothing originates or is delivered before the available date). To do this, we say that for each shipment, track, and day (prior to the available day), the sum of the x variable (containers belonging to shipment s moving from station i to j on day d) and y variable (containers belonging to shipment s moving from station j to station i on day d) equal 0. This is shorthand notation (rather than splitting into two constraints) – since both x and y are non-negative, if their sum is zero then they both must be zero. NOTE: There is an alternative formulation involving the ori and del variables described in APPENDIX I. We also explain an important detail regarding solver accuracy with these different formulations in that appendix.

Next, we describe the two constraints related to a limit for the number of containers that can be reloaded and the number of containers that can move on a given track. We note that each of these constraints is per day.

Equation 14 describes the reload capacity limit. For all containers coming INTO station k on a given day d, the total number of containers cannot exceed the reload capacity per reloading zone times the number of reloading zones. As the problem describes, a container is only reloaded when it comes into a station (i.e. not when it originates). Since the set of tracks only includes tracks in one direction, we flip the (i, k) indices to be (k, i) for the y decision variable to handle the reverse direction. Each station begins with a certain number of reloads but then can add a variable number of reloading zones at a certain cost. Thus, the right side of this equation can change depending on the value of the decision variable r. Lastly, we add that this must hold for all stations on all days.

Equation 15 describes the track limits. For all containers moving along a valid track – either from station i to station j or station j to station i – that sum on a given day d cannot exceed the capacity of the track. As seen on the left hand side, we only need to go through tracks (i, j) but can reverse the indices for the y variable to handle containers moving in the reverse direction along a track. Adding the x and y decision variables enables the counting of containers moving in both directions along a track. As seen on the right hand side of the equation, that capacity is a fixed amount per track, multiplied by the number of tracks. The number of tracks in this version of the problem is variable – equal to the starting number of tracks (always 1, no matter the track) plus the number of new tracks built, represented by our decision variable t. Since this condition is dependent on the track not the shipment, we need to sum up the number of containers moving along a given track on a given day of every shipment. Then, we add the condition that this must hold for all tracks on all days.

The final two constraints relate to the limit on the number of new reloading zones and new tracks that are allowed to be built. Those maximum values are given parameters into the problem.

Equation 16 describes the maximum number of tracks allowed for every track in the set of tracks between station i and station j. The starting number of tracks (1) plus the number of new tracks – a decision variable t – must be less than or equal to the maximum number of tracks between two stations. This condition must hold for all tracks. We note that we only need to worry about a single direction in this constraint, as the bidirectional capacity is handled in equation 15 above.

Equation 17 describes the maximum number of reloading zones at a given station k. In this case, the starting number of reloading zones at station k (a parameter) plus the number of new reloading zones built at station k (decision variable r) must be less than or equal to the maximum number of reloading zones allowed (a parameter). This condition must hold for all stations.

Finally, all of the decision variables must be non-negative, as it would not make any sense for there to be a negative number of containers shipped, originated, or delivered, nor would it make sense to have a negative number of tracks or reloading zones constructed. Additionally, because of the nature of all of these decision variables, it makes the most sense for them to be integers. These sign and variable type requirements are defined in the final constraint (eq 18).

Now that the model has been fully developed and the specific cases can be run through the model.

3 Solutions

Our model is designed to take in .CSV files in the format SmartRail has described to us and then output a shipment plan based on that data. The primary result is a shipment schedule that tells SmartRail exactly when to ship a container on a specific route. In addition to that, we tell SmartRail where they should build new tracks and new reloading zones. The solutions and analysis also contain information relating to the cost breakdown, delivery and origination days, and containers. In the analysis, we will add some context to the meaning of these solutions.

The AMPL model can be seen in the files SmartRailSchedulePhase2.run and SmartRailScheduleingPhase2.mod. In addition to all of the solutions presented below, the AMPL program outputs a CSV with the entire shipping schedule (SmartRailCompletePlan.csv) and has a log file that neatly presents all relevant information (SmartRail_Phase2_Case1/2.log). In addition to displaying the values of every decision variable, the log file also contains information about tracks and reloading zones that were expanded and what days/shipments led to those expansions being necessary. All of the information used in the analysis below is contained within the .log files. Thus, if SmartRail wanted to use our software to analyze a new shipment schedule (of the same format), they can use the output of the log file. We explain in more detail in APPENDIX II.

In both Case 1 and Case 2 given to us by Smart Rail, our model successfully produced optimal solutions to minimize the total costs for SmartRail as they successfully moved shipments throughout the network. We will present the results from both cases first and then analyze the results in the following sections.

3.1 Case 1 Optimal Solution

For Case 1, we found the following optimal solution.

Total Minimum Cost = $38,513,829.76	
Total Shipping Cost	$14,931,385.70
New Track Cost	$21,854,444.06
New Reloading Zone Cost	$168,000.00
Delayed Shipment Cost	$1,560,000

Table 4. Case 1 Cost Breakdown

Table 4 above shows the breakdown of the costs SmartRail will incur. These costs come from shipping the containers from their station of origination to their destination, building new tracks between stations, building new reloading zones at stations, and containers arriving past their specified delivery date to their final destination.

In regards to the shipping costs, the plan for how containers will move from their origination station to destination station for all shipments is detailed below in Tables 5-14. The first three columns in the table contain the day (from 1-14), the track that containers from the shipment are passing along, and the number of containers or volume passing along the corresponding track in column two. The fourth column shows the number of containers that are picked up or moved from the station of origination on a given day. The fifth and final column shows how many containers were delivered to their final destination on each day presented in the table.

For Company A's shipment from station 4 to station 7 in Table 5 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
1	4 → 6 4 → 1	10 10	20	–
2	1 → 3 4 → 6 6 → 7	10 10 10	10	–
3	3 → 7 6 → 7	10 10	–	10
4	–	–	–	20

Table 5. Case 1: Company_A_4_7 Transportation Plan

For Company A's shipment from station 3 to station 2 in Table 6 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
3	3 → 5	5	5	–
4	5 → 2	5	–	–
5	–	–	–	5

Table 6. Case 1: Company_A_3_2 Transportation Plan

For Company B's shipment from station 1 to station 11 in Table 7 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
8	1 → 4 1 → 8	10 8	18	–
9	1 → 4 1 → 8 4 → 6 4 → 8 8 → 9	2 8 9 1 8	10	–
10	1 → 8 4 → 8 6 → 9 8 → 9 9 → 10	8 2 9 9 8	8	–
11	8 → 9 9 → 10 10 →11	10 18 8	–	–
12	9 → 10 10 →11	10 18	–	8
13	10 →11	10	–	18
14	–	–	–	10

Table 7. Case 1: Company_B_1_11 Transportation Plan

For Company C's shipment from station 13 to station 8 in Table 8 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
5	13 → 14 13 → 7	3 10	13	–
6	13 → 14 14 → 15 14 → 3 7 → 6 13 → 7	10 1 2 10 8	18	–
7	13 → 14 3 → 1 14 → 3 6 → 4 7 → 6 15 → 12	2 2 10 10 8 1	2	–
8	1 → 8 4 → 8 3 → 1 14 → 3 6 → 4 12 → 4 13 → 7	2 10 10 2 8 1 9	9	–
9	1 → 4 1 → 8 4 → 8 3 → 1 7 → 6	8 2 9 2 9	–	12
10	1 → 8 4 → 8 6 → 4	2 8 9	–	11
11	4 → 8	9	–	10
12	–	–	–	9

Table 8. Case 1: Company_C_13_8 Transportation Plan

For Company C's shipment from station 13 to station 1 in Table 9 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
4	13 → 14	10	10	–
5	13 → 14 14 → 3	7 10	7	–
6	3 → 1 14 → 3	10 7	–	–
7	3 → 1	7	–	10
8	–	–	–	7

Table 9. Case 1: Company_C_13_1 Transportation Plan

For Company C's shipment from station 13 to station 7 in Table 10 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
2	13 → 7	10	10	–
3	13 → 7	10	10	10
4	13 → 7	10	10	10
5	–	–	–	10
6	13 → 7	2	2	–
7	13 → 7	10	10	2
8	–	–	–	10

Table 10. Case 1: Company_C_13_7 Transportation Plan

For Company D's shipment from station 5 to station 6 in Table 11 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
11	5 → 7	6	6	–
12	7 → 6	6	–	–
13	–	–	–	6

Table 11. Case 1: Company_D_5_6 Transportation Plan

For Company D's shipment from station 5 to station 10 in Table 12 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
5	5 → 14	1	1	–
6	5 → 7 14 → 15	2 1	2	–
7	5 → 7 7 → 6 15 → 6	10 2 1	10	–
8	5 → 7 7 → 6 15 → 6	1 3 10	1	
9	5 → 7 6 → 9 9 → 10 7 → 6	10 10 3 1	10	
10	5 → 7 6 → 9 9 → 10 7 → 6	10 1 10 10	10	3
11	6 → 9 9 → 10 7 → 6	10 1 10	–	10
12	6 → 9 9 → 10	10 10	–	1
13	9 → 10	10	–	10
14	–	–	–	10

Table 12. Case 1: Company_D_5_10 Transportation Plan

For Company E's shipment from station 3 to station 13 in Table 13 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
1	3 → 14	10	10	–
2	3 → 14 14 → 13	10 10	10	–
3	14 → 13	10	–	10
4	–	–	–	10
5	–	–	–	–
6	3 → 14	1	1	–
7	3 → 7 14 → 13	1 1	1	
8	3 → 7 3 → 14 7 → 13	10 8 1	18	1
9	7 → 13 14 → 13	10 8	–	1
10	–	–	–	18

Table 13. Case 1: Company_E_3_13 Transportation Plan

For Company E's shipment from station 3 to station 15 in Table 14 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
9	3 → 14	10	10	–
10	14 → 15	10	–	–
11	–	–	–	10

Table 14. Case 1: Company_E_3_15 Transportation Plan

As we have stated above, the total number of containers that can pass along a track is 10 each day. In order to best accommodate all shipments as laid out in the above tables, additional tracks were constructed. Table 15 below shows the newly constructed tracks, where the first and second columns are the two stations that are connected by the track, the third column is the number of tracks SmartRail currently has built between the two stations, and the fourth column is the number of new tracks that should be constructed between the stations.

Station 1	Station 2	# of Original Tracks	# of New Tracks
9	10	1	1
10	11	1	1

Table 15. Case 1: Newly Constructed Tracks

In addition to new tracks having to be constructed, a new reloading zone was also constructed due to the constraint that limits the total number of containers that can arrive at a station on a given day to 12 per reloading zone. Table 16 below shows the additional reloading zones that should be constructed.

Station	# of Original Reloading Zones	# of New Reloading Zones
10	1	1

Table 16. Case 1: Newly Constructed Reloading Zones

3.2 Case 2 Optimal Solution

For Case 2, we found the following optimal solution.

Total Minimum Cost = $187,107,870.50	
Total Shipping Cost	$50,325,824.09
New Track Cost	$129,802,046.38
New Reloading Zone Cost	$1,680,000.00
Delayed Shipment Cost	$5,300,000

Table 17. Case 2 Cost Breakdown

Table 17 above shows the breakdown of the costs SmartRail will incur. These costs come from shipping the containers from their station of origination to their destination, building new tracks between stations, building new reloading zones at stations, and containers arriving past their specified delivery date to their final destination.

In regards to the shipping costs, the plan for how containers will move from their origination station to destination station for all shipments is detailed below in Tables 18-37. The first three columns in the table contain the day (from 1-14), the track that containers from the shipment are passing along, and the number of containers or volume passing along the corresponding track in column two. The fourth column shows the number of containers that are picked up or moved from the station of origination on a given day. The fifth and final column shows how many containers were delivered to their final destination on each day presented in the table.

For Company A's shipment from station 4 to station 7 in Table 18 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
1	$4 \rightarrow 6$ $4 \rightarrow 1$	9 6	15	–
2	$1 \rightarrow 3$ $4 \rightarrow 6$ $6 \rightarrow 7$	6 10 9	10	–
3	$3 \rightarrow 7$ $4 \rightarrow 6$ $6 \rightarrow 7$	6 5 10	5	9
4	$6 \rightarrow 7$	5	–	16
5	–	–	–	5

Table 18. Case 2: Company_A_4_7 Transportation Plan

For Company A's shipment from station 3 to station 2 in Table 19 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
3	$3 \to 5$	5	5	–
4	$5 \to 2$	5	–	–
5	–	–	–	5

Table 19. Case 2: Company_A_3_2 Transportation Plan

For Company B's shipment from station 1 to station 11 in Table 20 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
8	$1 \to 4$ $1 \to 8$	10 10	20	–
9	$1 \to 4$ $1 \to 8$ $4 \to 6$ $4 \to 8$ $8 \to 9$	6 9 9 1 10	15	–
10	$1 \to 8$ $4 \to 6$ $6 \to 9$ $8 \to 9$ $9 \to 10$	1 6 9 10 10	1	–
11	$6 \to 9$ $8 \to 9$ $9 \to 10$ $10 \to 11$	6 1 19 10	–	–
12	$9 \to 10$ $10 \to 11$	9 19	–	10
13	$10 \to 11$	7	–	19
14	–	–	–	7

Table 20. Case 2: Company_B_1_11 Transportation Plan

For Company C's shipment from station 13 to station 8 in Table 21 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
5	13 → 7	10	10	–
6	7 → 6 13 → 7	10 10	10	–
7	13 → 14 6 → 4 7 → 6 13 → 7	1 10 10 2	3	–
8	4 → 8 7 → 3 14 → 3 6 → 4 7 → 6 13 → 7	10 1 1 10 1 3	3	–
9	4 → 8 13 → 14 3 → 1 6 → 4 7 → 6 13 → 7	10 3 2 1 3 4	7	10
10	1 → 8 4 → 8 13 → 14 14 → 3 6 → 4 7 → 6 13 → 7	2 1 4 3 3 4 5	9	10
11	4 → 8 6 → 9 3 → 1 7 → 3 14 → 3 6 → 4 7 → 6	3 3 3 1 4 1 4	–	3

Table. Case 2: Company_C_13_8 Transportation Plan (21A)

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
12	$1 \rightarrow 8$	3	–	3
	$4 \rightarrow 8$	1		
	$6 \rightarrow 9$	1		
	$3 \rightarrow 1$	5		
	$6 \rightarrow 4$	3		
	$9 \rightarrow 8$	3		
13	$1 \rightarrow 8$	5	–	7
	$4 \rightarrow 8$	3		
	$9 \rightarrow 8$	1		
14	–	–	–	9

Table 21. Case 2: Company_C_13_8 Transportation Plan (21B)

For Company C's shipment from station 13 to station 1 in Table 22 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
4	13 → 14 13 → 7	1 3	4	–
5	13 → 14 7 → 3 14 → 3	5 3 1	5	–
6	13 → 14 3 → 1 14 → 3	8 4 5	8	–
7	3 → 1 14 → 3	5 8	–	4
8	3 → 1	8	–	5
9	–	–	–	8

Table 22. Case 2: Company_C_13_1 Transportation Plan

For Company C's shipment from station 13 to station 7 in Table 23 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
2	13 → 7	10	10	–
3	13 → 7	10	10	10
4	13 → 7	7	7	10
5	–	–	–	7
6	–	–	–	–
7	13 → 7	8	8	–
8	13 → 7	7	7	8
9	–	–	–	7

Table 23. Case 2: Company_C_13_7 Transportation Plan

For Company D's shipment from station 5 to station 6 in Table 24 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
9	5 → 14	2	2	–
10	14 → 13	2	–	–
11	5 → 14 13 → 7	4 2	4	–
12	14 → 15 7 → 6	4 2	–	–
13	15 → 6	4	–	2
13	–	–	–	4

Table 24. Case 2: Company_D_5_6 Transportation Plan

For Company D's shipment from station 5 to station 10 in Table 25 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
5	$5 \rightarrow 3$	7	7	–
6	$5 \rightarrow 14$	1	4	–
	$3 \rightarrow 1$	7		
	$5 \rightarrow 3$	3		
7	$1 \rightarrow 8$	7	4	–
	$3 \rightarrow 7$	3		
	$5 \rightarrow 14$	4		
	$14 \rightarrow 3$	1		
8	$3 \rightarrow 7$	1	12	
	$5 \rightarrow 7$	1		
	$5 \rightarrow 14$	9		
	$8 \rightarrow 9$	7		
	$5 \rightarrow 3$	2		
	$14 \rightarrow 3$	4		
	$7 \rightarrow 6$	3		
9	$3 \rightarrow 7$	6	6	–
	$5 \rightarrow 14$	3		
	$6 \rightarrow 15$	3		
	$9 \rightarrow 10$	7		
	$5 \rightarrow 3$	3		
	$14 \rightarrow 3$	4		
	$7 \rightarrow 6$	2		
	$14 \rightarrow 13$	5		
10	$5 \rightarrow 7$	1	1	7
	$6 \rightarrow 9$	1		
	$6 \rightarrow 15$	1		
	$14 \rightarrow 15$	3		
	$3 \rightarrow 1$	7		
	$7 \rightarrow 6$	6		
	$15 \rightarrow 6$	3		
	$13 \rightarrow 7$	5		
11	$1 \rightarrow 8$	7	–	–
	$6 \rightarrow 9$	9		
	$9 \rightarrow 10$	1		
	$7 \rightarrow 6$	6		
	$15 \rightarrow 6$	4		

Table. Case 2: Company_D_5_10 Transportation Plan (25A)

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
12	6 → 9 8 → 9 9 → 10	10 7 9	–	1
13	9 → 10	17	–	9
14	–	–	–	17

Table 25. Case 2: Company_D_5_10 Transportation Plan (25B)

For Company E's shipment from station 3 to station 13 in Table 26 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
1	3 → 14	10	10	–
2	3 → 14 14 → 13	10 10	10	–
3	3 → 5 3 → 14 14 → 13	3 9 10	12	10
4	5 → 14 14 → 13	3 9	–	10
5	14 → 13	3	–	9
6	–	–	–	3
7	–	–	–	–
8	3 → 7	4	4	–
9	7 → 13	4	–	–
10	3 → 7	4	4	4
11	7 → 13	4	–	–
12	–	–	–	4

Table 26. Case 2: Company_E_3_13 Transportation Plan

For Company E's shipment from station 3 to station 15 in Table 27 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
4	3 → 7 3 → 1	1 8	9	–
5	1 → 4 7 → 6	8 1	–	–
6	4 → 12 6 → 15	8 1	–	–
7	3 → 7 12 → 15	1 8	1	1
8	7 → 6	1	–	8
9	6 → 15	1	–	–
10	–	–	–	1

Table 27. Case 2: Company_E_3_15 Transportation Plan

For part a of Company F's shipment from station 9 to station 2 in Table 28 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
1	$9 \rightarrow 6$ $9 \rightarrow 8$	11 9	20	–
2	$6 \rightarrow 7$ $8 \rightarrow 1$ $9 \rightarrow 6$ $9 \rightarrow 8$	11 9 10 2	12	–
3	$1 \rightarrow 3$ $6 \rightarrow 7$ $8 \rightarrow 1$ $7 \rightarrow 3$ $7 \rightarrow 5$ $9 \rightarrow 6$ $9 \rightarrow 8$	9 10 2 1 10 10 3	13	–
4	$1 \rightarrow 3$ $3 \rightarrow 5$ $6 \rightarrow 7$ $8 \rightarrow 1$ $5 \rightarrow 2$ $7 \rightarrow 5$	2 10 10 3 10 10	–	–
5	$1 \rightarrow 3$ $3 \rightarrow 5$ $5 \rightarrow 2$ $7 \rightarrow 5$	3 2 20 10	–	10
6	$3 \rightarrow 5$ $5 \rightarrow 2$	3 12	–	20
7	$5 \rightarrow 2$	3	–	12
8	–	–	–	3

Table 28. Case 2: Company_F_9_2_a Transportation Plan

For part b of Company F's shipment from station 9 to station 2 in Table 29 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
4	9 → 6	29	31	–
	9 → 8	2		
5	6 → 7	19	4	–
	6 → 15	10		
	8 → 1	2		
	9 → 6	2		
	9 → 8	2		
6	1 → 3	2	–	–
	7 → 3	9		
	6 → 4	2		
	8 → 4	2		
	7 → 5	10		
	15 → 14	10		
7	3 → 5	10	–	–
	3 → 14	1		
	4 → 1	4		
	5 → 2	10		
	14 → 5	10		
8	1 → 3	4	–	10
	5 → 2	20		
	14 → 5	1		
9	3 → 5	4	–	20
	5 → 2	1		
10	5 → 2	4	–	1
11	–	–	–	4

Table 29. Case 2: Company_F_9_2_b Transportation Plan

For Company G's shipment from station 11 to station 7 in Table 30 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
9	11 → 10	12	12	–
10	10 → 9	12	–	–
11	9 → 6	12	–	–
12	6 → 7	12	–	–
13	–	–	–	12

Table 30. Case 2: Company_G_11_7 Transportation Plan

For Company G's shipment from station 11 to station 2 in Table 31 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
3	11 → 10	20	20	–
4	10 → 9	20	20	–
	11 → 10	20		
5	9 → 6	17	16	–
	9 → 8	3		
	10 → 9	20		
	11 → 10	16		
6	6 → 7	10	19	–
	6 → 15	3		
	8 → 1	3		
	6 → 4	4		
	9 → 6	18		
	9 → 8	2		
	10 → 9	16		
	11 → 10	19		
7	1 → 3	3	12	–
	6 → 7	10		
	6 → 15	8		
	4 → 1	4		
	8 → 1	2		
	7 → 5	10		
	9 → 6	15		
	9 → 8	1		
	10 → 9	19		
	11 → 10	12		
	15 → 14	3		

Table 31. Case 2: Company_G_11_2 Transportation Plan

For Company H's shipment from station 12 to station 5 in Table 32 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
6	12 → 15	3	3	–
7	12 → 5 15 → 14	2 3	2	–
8	14 → 5 15 → 14	3 2	–	–
9	14 → 5	2	–	3
10	12 → 15 12 → 4	2 10	12	2
11	12 → 15 4 → 1 15 → 6	6 10 2	6	–
12	1 → 3 6 → 7 15 → 14	10 2 6	–	–
13	3 → 5 7 → 5 14 → 5	10 2 6	–	–
14	–	–	–	18

Table 32. Case 2: Company_H_12_5 Transportation Plan

For Company H's shipment from station 12 to station 2 in Table 33 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
1	12 → 15	10	20	–
	12 → 4	10		
2	12 → 15	10	20	–
	4 → 1	10		
	12 → 4	10		
	15 → 14	10		
3	1 → 3	10	19	–
	12 → 15	10		
	4 → 1	10		
	12 → 4	9		
	14 → 5	10		
	15 → 14	10		
4	1 → 3	10	10	–
	3 → 14	10		
	12 → 15	10		
	4 → 1	9		
	5 → 2	10		
	14 → 5	10		
	15 → 14	10		
5	1 → 3	9	10	10
	3 → 5	1		
	3 → 14	9		
	5 → 2	10		
	12 → 4	10		
	14 → 5	20		
	15 → 14	10		
6	3 → 5	4	6	10
	3 → 14	5		
	12 → 15	4		
	4 → 1	10		
	5 → 2	21		
	12 → 4	2		
	14 → 5	19		

Table. Case 2: Company_H_12_2 Transportation Plan (33A)

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
7	$1 \to 3$	10	–	21
	$4 \to 1$	2		
	$5 \to 2$	23		
	$14 \to 5$	5		
	$15 \to 14$	4		
8	$1 \to 3$	2	13	23
	$3 \to 5$	8		
	$3 \to 14$	2		
	$12 \to 15$	10		
	$5 \to 2$	5		
	$12 \to 4$	3		
	$14 \to 5$	4		
9	$3 \to 14$	2	14	5
	$12 \to 15$	4		
	$4 \to 1$	3		
	$5 \to 2$	12		
	$12 \to 4$	10		
	$14 \to 5$	2		
	$15 \to 14$	10		
10	$1 \to 3$	3	8	12
	$12 \to 15$	8		
	$4 \to 1$	10		
	$5 \to 2$	2		
	$14 \to 5$	12		
	$15 \to 14$	4		
11	$1 \to 3$	10	–	2
	$3 \to 14$	3		
	$5 \to 2$	12		
	$14 \to 5$	4		
	$15 \to 14$	8		
12	$3 \to 5$	10	–	12
	$5 \to 2$	4		
	$14 \to 5$	11		
13	$5 \to 2$	21	–	4
14	–	–	–	21

Table 33. Case 2: Company_H_12_2 Transportation Plan (33B)

For Company I's shipment from station 1 to station 2 in Table 34 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
9	1 → 3	14	14	–
10	1 → 3 3 → 5 3 → 14	5 10 4	5	–
11	3 → 5 5 → 2 14 → 5	5 10 4	–	–
12	5 → 2	9	–	10
13	–	–	–	9

Table 34. Case 2: Company_I_1_2 Transportation Plan

For Company I's shipment from station 1 to station 10 in Table 35 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
2	1 → 8	1	1	–
3	1 → 8 8 → 9	8 1	8	–
4	1 → 4 1 → 8 8 → 9 9 → 10	1 5 8 1	6	–
5	1 → 4 1 → 8 4 → 6 8 → 9 9 → 10	2 8 1 5 8	10	1
6	1 → 8 4 → 8 6 → 9 8 → 9 9 → 10	7 2 1 8 5	7	8
7	1 → 8 8 → 9 9 → 10	1 9 9	1	5
8	8 → 9 9 → 10	1 9	–	9
9	9 → 10	1	–	9
10	–	–	–	1

Table 35. Case 2: Company_I_1_10 Transportation Plan

For Company J's shipment from station 15 to station 9 in Table 36 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
6	15 → 6	15	15	–
7	6 → 9 15 → 6	15 12	12	–
8	6 → 9 15 → 6	12 18	18	15
9	6 → 9 15 → 6	18 10	10	12
10	6 → 9	10	–	18
11	–	–	–	10

Table 36. Case 2: Company_J_15_9 Transportation Plan

For Company K's shipment from station 8 to station 15 in Table 37 below:

Day	Track and Direction	Volume	Volume Picked Up	Volume Delivered
10	8 → 1 8 → 4	7 19	26	–
11	1 → 3 4 → 6 4 → 12 8 → 9 8 → 4	7 9 10 9 17	26	–
12	3 → 14 4 → 6 4 → 12 6 → 15 12 → 15 9 → 6	7 7 10 9 10 9	–	–
13	6 → 15 12 → 15 14 → 15	16 10 7	–	19
14	–	–	–	33

Table 37. Case 2: Company_K_8_15 Transportation Plan

As stated previously, the total number of containers that can pass along a track is 10 each day. In order to best accommodate all shipments as laid out in the above tables, additional tracks were constructed. Table 38 below shows the newly constructed tracks, where the first and second columns are the two stations that are connected by the track, the third column is the number of tracks SmartRail currently has built between the two stations, and the fourth column is the number of new tracks that should be constructed between the stations.

Station 1	Station 2	# of Original Tracks	# of New Tracks
1	3	1	1
2	5	1	3
4	8	1	1
5	14	1	1
6	7	1	1
6	9	1	2
6	15	1	1
9	10	1	2
10	11	1	1

Table 38. Case 2: Newly Constructed Tracks

In addition to new tracks having to be constructed, a new reloading zone was also constructed due to the constraint that limits the total number of containers that can arrive at a station on a given day to 12 per reloading zone. Table 39 below shows the additional reloading zones that should be constructed for Case 2.

Station	# of Original Reloading Zones	# of New Reloading Zones
1	1	1
2	1	2
5	2	1
6	2	2
9	2	2
10	1	1
15	2	1

Table 39. Case 2: Newly Constructed Reloading Zones

3.3 Analysis

There are significant similarities between the two cases. For that reason, prior to analyzing each case individually, we will first analyze the structure of the problem overall. Specifically, we will look at how the specific tracks that are available leads to specific areas where expansion might be necessary. Then, we will look at the cost tradeoffs between building new tracks and reloading zones, having to take a longer shipping route, and having containers be late. Following the general analysis, we will explain how these results are applied directly to the answers from our optimization program in Case 1 and Case 2 individually.

3.3.1 General Analysis

We begin the general analysis by looking at the SmartRail track network map that applies to Case 1 and Case 2 (see Figure 1 below). Based on this map, a few things are interesting, but what really stands out are the routes in which there is only a single option for travel. For example, there is only one track connection station 2 to the rest of the map – station 2 to station 5. Similarly, there is only one route to get to stations 10 and 11, which is station 9 - station 10 - station 11. For any container that needs to go to station 10 it must go on the track from 9-10. For any container that needs to go to station 11, it must go on the track from 9-10 and then 10-11. Similarly for the reverse direction – sending containers into the network from stations 11, 10, or 2. While the cost of building new tracks is expensive (see table 40 below), all containers are required to be delivered within the biweekly period. Thus, there may be no choice in some cases but to build a new track on these specific routes to ensure that the problem is feasible. To that end, SmartRail would very likely benefit from preventing these "one-option" routes to a station. Adding a track from 14-2 or from 13-11-7 are examples that in the future could end up being good choices for SmartRail, assuming the cost of building these new routes is less than the cost savings would be.

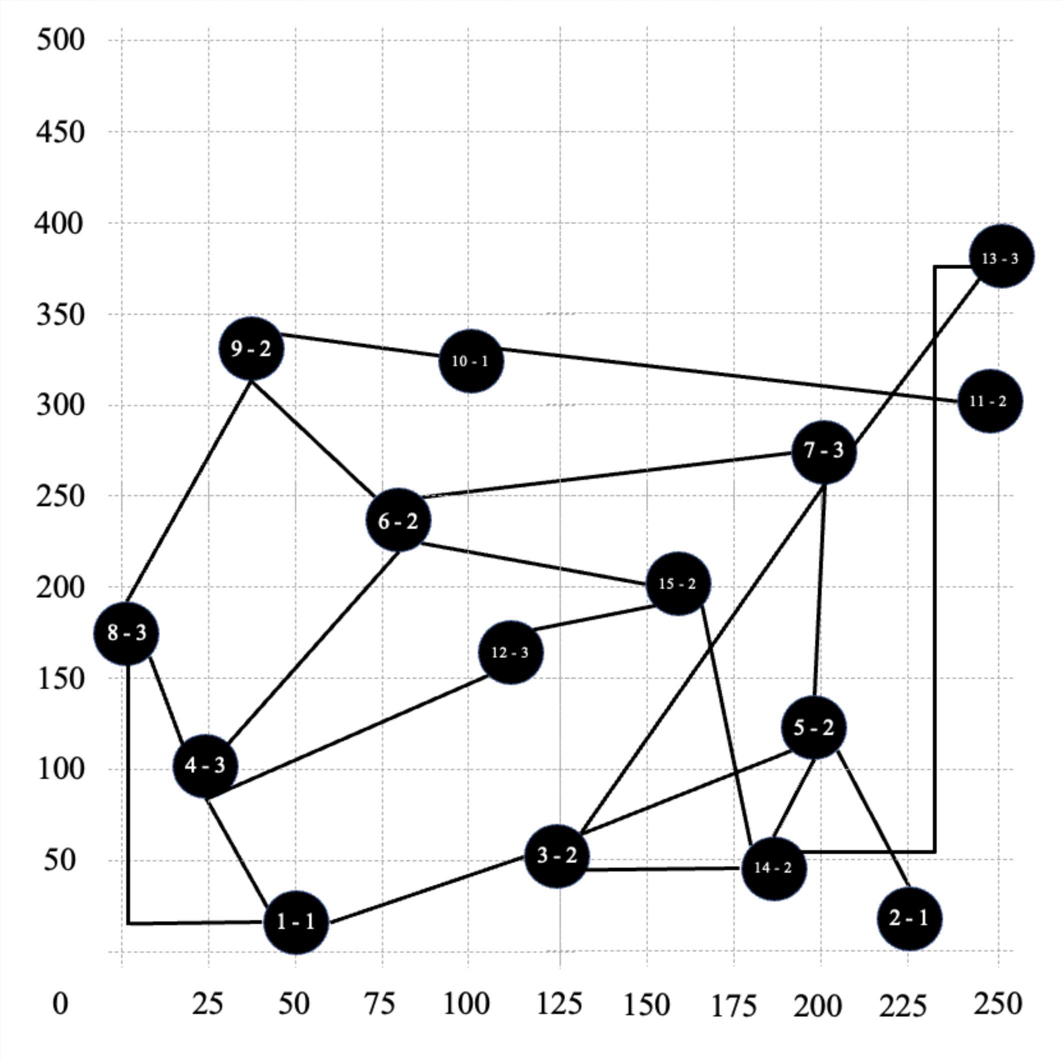

Figure 1.

The next area to investigate is the cost tradeoffs. While we will look at specific cases in the following sections, the general costs are the same across a case. As can be seen in the Cost of New Track Over Biweekly Period, the cheapest new track to build exceeds $6,500,000. The highest cost to ship a container along any track is less than $51,000. Thus, depending on the case, it may be more efficient to ship a container in a more round-a-bout manner and have higher shipping costs than it would be to build a new track. Additionally, the cost of a new reloading zone and cost for delayed containers is relatively low compared to other costs (these costs are shown in Table 41). For example, eight containers can be one day late or one reloading zone could be built. There are only a few cases where rerouting eight containers might be cheaper and no cases where building a new track would be cheaper. Therefore, we expect to see that – especially on congested routes – there may be willingness to re-route containers (on a longer path) or allow some containers to be delayed.

Station 1	Station 2	Track Length in Miles	Cost of New Track Over Biweekly Period	Cost to Ship a Single Container Along Track
1	3	82.41	$8,652,672.87	$12,360.96
1	4	91.05	$9,560,648.08	$13,658.07
1	8	161.39	$16,945,902.67	$24,208.43
2	5	105.25	$11,050,973.11	$15,787.10
3	5	95.57	$10,035,025.57	$14,335.75
3	7	230.78	$24,231,863.92	$34,616.95
3	14	63.73	$6,691,442.48	$9,559.20
4	6	147.23	$15,458,759.57	$22,083.94
4	8	70.37	$7,389,319.40	$10,556.17
4	12	105.72	$11,100,732.54	$15,858.19
5	7	151.02	$15,857,505.51	$22,653.58
5	14	74.71	$7,844,659.62	$11,206.66
6	7	124.44	$13,065,833.54	$18,665.48
6	9	103.57	$10,874,875.55	$15,535.54
6	15	88.89	$9,333.057.11	$13,332.94
7	13	116.82	$12,266,595.10	$17,523.71
8	9	162.12	$17,023,035.75	$24,318.62
9	10	64.38	$6,759,388.35	$9,656.27
10	11	143.76	$15,095,055.71	$21,564.37
12	15	67.34	$7,070,576.35	$10,100.82
13	14	338.06	$35,496,260.45	$50,708.94
14	15	156.80	$16,464,834.50	$23,519.76

Table 40. Shipping and New Track Costs

Type of Cost	Cost	Unit
Penalty for Delayed Containers	$20,000	Per day per container
Cost of New Reloading Zone over Biweekly Period	$168,000	Per new reloading zone

Table 41. Delay and New Reload Costs

3.3.2 Case 1 Analysis

Knowing the information from the general analysis above, we can now investigate the results from Case 1 directly. The first step in our analysis will be to look at when the containers originate and are delivered and investigate those containers that are late.

Below in Table 42 is a color-coded schedule of when containers leave from their origination station. Gray boxes represent days in which no containers are allowed to leave. Green boxes indicate the first day a package could leave and purple boxes indicate the day containers are due. A purple box with a * is a special case – the containers are due on that day but can NOT originate on that day or else they will not be delivered by day 14.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Company_A_3_2	0	0	5	0	0	0	0	0	0	0	0	0	0	0
Company_A_4_7	20	10	0	0	0	0	0	0	0	0	0	0	0	0
Company_B_1_11	0	0	0	0	0	0	0	18	10	8	0	0	0	0
Company_C_13_1	0	0	0	10	7	0	0	0	0	0	0	0	0	0
Company_C_13_7	0	10	10	10	0	2	0	0	0	0	0	0	0	0
Company_C_13_8	0	0	0	0	13	18	2	9	0	0	0	0	0	0
Company_D_5_10	0	0	0	0	1	2	10	1	10	10	0	0	0	0*
Company_D_5_6	0	0	0	0	0	0	0	0	0	0	6	0	0	0
Company_E_3_13	10	10	0	0	0	1	1	18	0	0	0	0	0	0
Company_E_3_15	0	0	0	0	0	0	0	0	10	0	0	0	0	0

Legend: No Container is allowed to originate
Container becomes available for origination
Containers are due for delivery
Containers due for delivery, cannot originate

Table 42. Case 1 Origination Schedule

A similar table was constructed for the delivery of the containers in Case 1 (Table 43 below). Gray boxes again indicate a day in which no packages can be delivered. The dark-green boxes indicate when the containers are eligible to enter into the network (we note that no container can be delivered by making the box dark green, compared to lime green above). Purple again indicates the day that the containers are due for delivery. Finally, red boxes indicate that if a container was delivered on that day, it would incur a penalty fee. Late containers are bolded.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Company_A_3_2	0	0	0	0	5	0	0	0	0	0	0	0	0	0
Company_A_4_7	0	0	10	20	0	0	0	0	0	0	0	0	0	0
Company_B_1_11	0	0	0	0	0	0	0	0	0	0	0	8	18	**10**
Company_C_13_1	0	0	0	0	0	0	10	7	0	0	0	0	0	0
Company_C_13_7	0	0	10	10	10	0	2	**10**	0	0	0	0	0	0
Company_C_13_8	0	0	0	0	0	0	0	0	12	**11**	**10**	**9**	0	0
Company_D_5_10	0	0	0	0	0	0	0	0	0	3	10	1	10	10
Company_D_5_6	0	0	0	0	0	0	0	0	0	0	0	0	6	0
Company_E_3_13	0	0	10	10	0	0	0	1	1	18	0	0	0	0
Company_E_3_15	0	0	0	0	0	0	0	0	0	0	10	0	0	0

Legend: No Container is allowed to be delivered
Container becomes available for shipping (cannot be delivered)
Containers are due for delivery
Containers are delivered late

Table 43. Case 1 Delivery Schedule

Firstly, we investigate the origination of the containers. In this case, only companies D_5_10, D_5_6, and E_3_15 have a significant number of their containers originating after the origination date (see Table 42). In the case of D_5_6, E_3_15, and D_5_10, all of the containers were able to be delivered on time (see Table 43). Thus, these shipments had enough extra room within their shipment windows that they could wait to originate until the network was clear and still arrive on time – without requiring any expansion. Since origination appears fairly normal, we will now investigate the delivery of late containers.

As can be seen in Table 43, there are three shipments that contain late containers. Company B_1_11 has 10 late containers (one-day-late), Company_C_13_7 has 10 late containers (one-day-late), and Company C_13_8 has 30 late containers (11 one day late, 10 two days late, 9 three days late). As a result the total late fee for the entire network is $1,560,000. We note that this late fee is CHEAPER than adding any additional track to the network. In the case of Company B's shipment from station 1 to station 11, 18 of the containers are shipped on the day 8 (first available day), 10 are shipped on day 9 (second available day), and 8 are shipped on day 10 (third available day) (see Table 7 in Section 3.1). They're split in how they are shipped – some go along the route 1-8-9, some go along the route 1-4-8-9, and the rest go along the route 1-4-6-9. As can be seen in the figure of SmartRail's track network presented above, the 1-8-9 route is the most direct (costs the same as 1-4-8-9) and takes fewer days as a result. However, due to the track limit of ten and the high cost of $16,945,902.67 to add another track, it is cheaper to have these containers take longer routes (such as 1-4-6-9) with higher shipping costs and resulting in eight containers being delayed than it would be to add a track between 1-8 or 1-4 or 4-8.

In the case of Company C's shipment from 13-7, the routing map below shows that there is really only one option to go from station 13 to station 7 and that is the direct route from 13-7. The only other option would be 13-14-5-7 which would make a large loop and cost $86,000+ per container compared with 13−7 which costs \$86,000+ per container compared with 13-7 which costs \$17,523.71 per container. As can be seen in the results for C_13_8, it needs to ship some containers along the route from 13-7 on day six in order to reach its destination. Thus, the cheapest option for Company C's shipment is to have 10 containers be one day late and pay the \$200,000 delay fee. That's cheaper than expanding the 13−7 route(would cost \$200,000 delay fee. That's cheaper than expanding the 13-7 route (would cost 12,266,595.10) or rerouting the containers (would cost \$860,000+).

Finally, we look at the case of Company C's shipment from station 13 to station 8 which has 30 containers that are late – 11 one day, 10 two days, and 9 three days. That means there is a late fee equal to \$1,116,000. As shown in the figure of SmartRail's track network presented above, the shortest possible routes to station 8 are 13-7-6-4-8, 13-7-6-9-8, and 13-14-3-1-8. It takes a MINIMUM of four days for a container to go from station 13 to station 8. In the case of this shipment, it is available on day 5 and due on day 9 – a gap of 4 days. Thus, without any track expansions, a maximum of 20 containers could possibly be on time (and shipment C from 13-8 has a volume of 42 containers). The late fee is \$1,116,000 while the cheapest track expansion exceeds \$1,116,000 while the cheapest track expansion exceeds \$5,000,000, so there is no optimal solution that expands tracks. Due to the shipments from 13-7 (see paragraph above) and the shipments that travel along the track from 1-8 (see two paragraphs above), not even 20 containers can be on time, as there are limits on the 13-7 and 1-8 routes. Thus, the cheapest possible option as described in Table 43 is to allow these containers to be late.

We have discussed the reasons that these specific containers are late and why it is cheaper for these containers to be delayed than it would be to reroute them (if possible) or expand a track. While track and reload expansion in Case 1 is cannot create a more optimal solution with respect to delayed containers – it is necessary to create a feasible solution. Specifically, all containers are required to be delivered on or before day 14 – the end of the time horizon – so they must arrive on or before day 13 to be delivered the next day.

As described in the general analysis, there are three specific stations that only have one route to them – and thus for feasibility concerns – may require track expansion. In this problem, those belong to Company A's shipment from 3-2 (must go along 5-2), Company B's shipment from 1 to 11 (must go along 9-10 and 10-11) and Company D's shipment from 5 to 10 (must go along 9-10). We note that for the Days eligible for moment column, the time periods were derived assuming every container could move along the shortest route (going through the fewest number of stations). This is the widest possible range – the true range could be significantly more narrow. That is summarized in Table 44 below.

Station 1	Station 2	Shipment	Track Volume by Shipment	Total Volume on Track	Days Eligible For Movement on Track
10	11	Company_B_1_11	36	36	10-13
9	10	Company_ B_1_11	36	70	9-12
		Company D_5_19	34		8-13
2	5	Company A_3_2	5	5	4-13

Table 44. Case 1 Single Option Tracks

First we look at the track from station 10 to station 11, which only is used by shipment B_1_11. The results of the optimization tell us that we will add 1 additional track on this route, bringing the total number of tracks to 2 with a total capacity of 20 containers per day. Looking more specifically at the shipment for Company_B_1_11, there would need to be ten containers per day moving along the most optimal route from 1-8-9-10. However, this cannot happen due to capacity constraints on 1-8 and 8-9 as can be seen for the results for Company B_1_11. In this case, there ends up being 36 containers that need to move on days 11 (8 containers), 12 (18 containers), and 13 (10 containers). However, as given in the problem, the track from station 10 to station 11 only has a capacity of 10. Because all containers must arrive in the time horizon, the only way to make this problem feasible is to expand the capacity of the track from (10, 11) to twenty containers, which is exactly what happens. This must happen for feasibility purposes, despite the very high cost of $15,095,055.71 for expansion.

With respect to the track from station 9 to station 10, we note that 70 containers are required to move within a six day period (36 of which must move within a four day subset). The results of the optimization tell us that we will add 1 additional track on this route, bringing the total number of tracks to 2 with a total capacity of 20 containers per day. The reason for this expansion is as follows. To begin with, the track capacity is ten containers. It is immediately clear that in order to have a feasible solution, the track capacity must be expanded. Specifically, in a six day period there are 70 containers that are being moved. This can only happen if the track capacity is expanded beyond ten. Without the expansion, there would not be a feasible solution as the containers would not arrive on time. Thus, while it costs $6,759,388.35 to add an additional track, this is a required cost order to have a feasible, optimal solution.

This leads to one additional problem. The number of reloading zones at station 10 is just one to begin with. That means only 12 containers can pass through station ten on a given day. However, on days 10, 11, and 12, in order to reduce the number of late containers, 18+ are shipped through station ten. It would theoretically be possible to spare the 168,000 for a new reload zone and have a feasible solution, but that would result in 10+late containers, which costs 168,000 for a new reload zone and have a feasible solution, but that would result in 10+ late containers, which costs 200,000+. Thus, the cheapest option to ensure feasibility and optimality is to add an extra reloading zone to station 10! That is exactly what the results of the optimization outputs.

Lastly, we note that the track from 2-5 only has one shipment with five containers, so there are no additional concerns there. As such, we do not build any additional tracks. In the future, assuming they could get permits to do so, SmartRail should do a cost-benefit analysis on adding an extra track connecting stations 10 and 11 to the network. It may be cheaper connecting stations 11-7 than adding an extra track from 10-11, and therefore could create a more optimal solution. Finally, we note that as a result of the way these shipments are oriented in the map, there is no need for additional reloading zones throughout the network. In the rest of the cases, there are either already enough reloading zones for the optimal shipment or there is enough of a gap between the time it takes to ship and the due date to still have an on time arrival of containers without exceeding any reloading zones. In fact, the .log file for Case_1 demonstrates that none of the reloading zones are even at their capacity (including zone 10 – which must expand but does not fill to capacity upon expansion). Thus, there is no other reloading zone that even has pressure on it to possibly expand as they all have extra capacity available on every day.

3.3.3 Case 2 Analysis

For Case 2, there are twenty shipments, resulting in a problem of a much larger scope. Thus, the following analysis will primarily focus on the big picture of costs – the tradeoffs between new tracks, new reloading zones, shipping costs, and the delay penalties – rather than looking at every late container or every new track individually.

The first step in our analysis will be looking at when containers originate and are delivered and investigating what the general patterns were that led to containers being late.

Below in Table 45 is a color-coded schedule of when containers leave from their origination station for the twenty shipments in Case 2. Gray boxes represent days in which no containers are allowed to leave. Green boxes indicate the first day a package could leave and purple boxes indicate the day a package is due. A purple box with a * is a special case – the containers are due on that day but can NOT originate on that day or else they will not be delivered by day 14.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Company_A_3_2	0	0	5	0	0	0	0	0	0	0	0	0	0	0
Company_A_4_7	15	10	5	0	0	0	0	0	0	0	0	0	0	0
Company_B_1_11	0	0	0	0	0	0	0	20	15	1	0	0	0	0
Company_C_13_1	0	0	0	4	5	8	0	0	0	0	0	0	0	0
Company_C_13_7	0	10	10	7	0	0	8	7	0	0	0	0	0	0
Company_C_13_8	0	0	0	0	10	10	3	3	7	9	0	0	0	0
Company_D_5_10	0	0	0	0	7	4	4	12	6	1	0	0	0	0*
Company_D_5_6	0	0	0	0	0	0	0	0	2	0	4	0	0	0
Company_E_3_13	10	10	12	0	0	0	0	4	0	4	0	0	0	0
Company_E_3_15	0	0	0	9	0	0	1	0	0	0	0	0	0	0
Company_F_9_2_a	20	12	13	0	0	0	0	0	0	0	0	0	0	0
Company_F_9_2_b	0	0	0	31	4	0	0	0	0	0	0	0	0	0*
Company_G_11_2	0	0	20	20	16	19	12	10	0	0	0	0	0	0
Company_G_11_7	0	0	0	0	0	0	0	0	12	0	0	0	0	0
Company_H_12_2	20	20	19	10	10	6	0	13	14	8	0	0	0	0
Company_H_12_5	0	0	0	0	0	3	2	0	0	12	0	0	0	0
Company_I_1_10	0	0	8	6	10	7	1	0	0	0	0	0	0	0
Company_I_1_2	0	0	0	0	0	0	0	0	14	5	0	0	0	0
Company_J_15_9	0	0	0	0	0	15	12	18	10	0	0	0	0	0
Company_K_8_15	0	0	0	0	0	0	0	0	0	26	26	0	0	0*

Legend: No Container is allowed to originate
Container becomes available for origination
Containers are due for delivery
Containers due for delivery, cannot originate

Table 45. Case 2 Origination Schedule

Table 46 below displays the delivery of containers. Gray boxes again indicate a day in which no packages can be delivered. The dark-green boxes indicate when the containers are eligible to enter into the network (we note that no container can be delivered by making the box dark green, compared to lime green above). Purple again indicates the day that the containers are due for delivery. Finally, red boxes indicate that if a container was delivered on that day, it would incur a penalty fee. Late containers are bolded.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Company_A_3_2	0	0	0	0	5	0	0	0	0	0	0	0	0	0
Company_A_4_7	0	0	9	16	**5**	0	0	0	0	0	0	0	0	0
Company_B_1_11	0	0	0	0	0	0	0	0	0	0	0	10	19	**7**
Company_C_13_1	0	0	0	0	0	0	4	5	**8**	0	0	0	0	0
Company_C_13_7	0	0	10	10	7	0	0	**8**	**7**	0	0	0	0	0
Company_C_13_8	0	0	0	0	0	0	0	0	10	**10**	**3**	**3**	**7**	**9**
Company_D_5_10	0	0	0	0	0	0	0	0	0	7	0	1	9	17
Company_D_5_6	0	0	0	0	0	0	0	0	0	0	0	0	2	**4**
Company_E_3_13	0	0	10	10	9	3	0	0	0	4	0	**4**	0	0
Company_E_3_15	0	0	0	0	0	0	1	8	0	1	0	0	0	0
Company_F_9_2_a	0	0	0	0	10	20	12	**3**	0	0	0	0	0	0
Company_F_9_2_b	0	0	0	0	0	0	0	10	20	1	4	0	0	0
Company_G_11_2	0	0	0	0	0	0	0	0	10	12	24	13	23	**15**
Company_G_11_7	0	0	0	0	0	0	0	0	0	0	0	0	**12**	0
Company_H_12_2	0	0	0	0	10	10	21	23	5	12	2	12	4	21
Company_H_12_5	0	0	0	0	0	0	0	0	3	2	0	0	0	**18**
Company_I_1_10	0	0	0	0	1	8	5	9	9	**1**	0	0	0	0
Company_I_1_2	0	0	0	0	0	0	0	0	0	0	0	10	9	0
Company_J_15_9	0	0	0	0	0	0	0	15	12	18	10	0	0	0
Company_K_8_15	0	0	0	0	0	0	0	0	0	0	0	0	19	33

Legend: No Container is allowed to be delivered
Container becomes available for shipping (cannot be delivered)
Containers are due for delivery
Containers are delivered late

Table 46. Case 2 Delivery Schedule

The first portion of our analysis will be with respect to delayed containers. To begin, we will investigate the origination schedule to see if there were any shipments that were prevented from originating on their availability day – as doing so could be the cause for delays (Table 45). The shipments belonging to Company D_5_6, Company E_13_15, Company_I_1_10, and Company_I_1_2 had no containers left on their available day. Company D_5_6 ended up having four containers be one day late while Company I_1_10 had one container late. Looking through their shipment schedules, congestion along these routes meant that it was cheaper to have them originate late than find an alternative route or expand a track.

Of concern in Table 45 is the fact that there are some containers that originate on or after the day they are due for delivery. Shipment C_13_7 has 15 containers originate on or after the delivery date, shipment C_13_8 has 16 that originate on or after the delivery date, Shipment E_13_3 has 4 that originate on or after the delivery date and company H_12_5 has four that originate on or after the delivery date. With three of these shipments there is a pattern – they all originate from station 13. As can be seen in the SmartRain network image we previously displayed, station 13 is far from the other stations and is only connected to station 7 and station 14. A new track from 13-14 costs \$35,496,260.45 while a new track from 13−7 costs \$35,496,260.45 while a new track from 13-7 costs \$12,266,595.10. Combining the late shipments from all these shipments originating from station 13 is \$2,380,000. Thus, it is cheaper to originate these containers late than expand track capacity along these routes.

One interesting note about the origination is that for Shipment C_13_8, it originates on day 5 and is due for delivery on day 9. The shortest possible routes between these two stations are 13-14-3-1-8, 13-7-6-9-8 and 13-7-6-4-8. In all of these cases, everything would need to be shipped on days five and six to be feasible. However, this doesn't account for the fact that there are other shipments such as C_13_7 that must use these routes. As described above, the cost penalty for all of the delays is far lower than any track expansions would be. Thus, it is more optimal for these containers to be late. This is a similar problem that leads to C_13_1 having late containers, as the routing leaving station 13 is too congested, but it is too expensive to build new tracks. Since they can be delivered within the allotted time horizon without expansion, the best option is for them to be late.

Now, we look more closely at the delivery table to investigate other containers that are late. Besides the ones involving station 13, the next biggest area of concern involves stations 2 and 11. Specifically, Company B_1_11 has 7 late containers, Company G_11_2 has 15 late containers, Company G_11_7 has 12 late containers, and Company F_9_2_a has 3 late containers. For feasibility reasons (as described in general analysis and as will be investigated more below), there may be required track expansion between stations 2-5, 9-10 and 10-11. However, it is clear that even with track expansions, there still may be late containers. In all of these cases, these containers are only a single day late. That brings the total late cost for these specific containers to \$740,000. As can be seen in Table 40, that is far cheaper than any track expansion along any of these routes (2-5 is \$11,050,973.11; 9-10 is \$6,759,388.35; 10−11 is \$6,759,388.35; 10-11 is \$15,095,055.71). In these specific cases, there is no option for re-routing, as there is just a single path to reach stations 2, 10, and 11.

Finally, there are a few outstanding late containers that do not fall into the categories above – such as A_4_7 with 5 containers late, I_1_10 with 1 container late, and J_15_9 with 10 containers late. Looking at the specific shipment schedules for each of these containers, there is not anything that stands out as extraordinary concerning. For A_4_7, there is some limitations on the track capacity from 4-6 and 4-1 on the first day, so five containers are late without track expansions. For I_1_10, there is a problem with congestion along the tracks. Because only one container is one day late (\$20,000),there is no possible expansion that would be worthwhile. Finally, for J159, there is a limit on the tracks from 6−9. While there is a new track built, that still is not enough for the containers to arrive on time as they have a very tight window(available day 6 and due day 10). It would cost \$20,000), there is no possible expansion that would be worthwhile. Finally, for J_15_9, there is a limit on the tracks from 6-9. While there is a new track built, that still is not enough for the containers to arrive on time as they have a very tight window (available day 6 and due day 10). It would cost \$10,874,875.55 compared to the \$200,000 for these ten containers to be one day late.

Now that we have investigated why certain containers are late in the optimal solution, it is now time to investigate the reasons in which new tracks were built and reloading zones were constructed. As was described in the general analysis, the cost of new tracks is so high that the cheapest track is greater than $6,500,000 while the most expensive is just over $6,500,000 while the most expensive is just over $35,000,000. On the contrary, shipping one container falls within the range of ∼($9500−$9500) - ∼($51,000). Thus, it would require the rerouting of a large number of containers on a long out-of-the-way route in order that would make them very late in order for it to be cost effective to build new tracks (that could happen in some version of this problem, but would be tough to make improvements from).

Instead, we can use the feasibility requirement to easily understand why certain new tracks must be built. In this problem, it is a requirement that every container must arrive to its destination on day 13 so that it can be delivered by day 14. In Case 2, we are moving 753 containers (compared with 262 in Case 1) within the same time period. Thus, it is likely feasibility concerns will cause the need for additional tracks to be produced.

First, we will analyze the three tracks highlighted as concerning in the general analysis – 9-10, 10-11, and 5-2. To reiterate, these are influential as there is only one route to stations 10, 11, and 2 – so expansion might be necessary to ensure feasibility. In the Table 47 below, we write out the shipments going to these stations, their volume, as well as the days eligible for their moment. We note that the days eligible for movement was calculated assuming every container can arrive on the shortest possible route (which may not be true due to other track limits). This is the widest possible time range – so if the analysis holds in this case, it holds in any case.

Station 1	Station 2	Shipment	Track Volume by Shipment	Total Volume on Track	Days Eligible For Movement on Track
10	11	Company_B_1_11	36	145	11-13
		Company_G_11_7	12		9-10
		Company_G_11_2	97		3-8
9	10	Company_B_1_11	36	212	10-12
		Company_G_11_7	12		10-11
		Company_G_11_2	97		4-9
		Company_D_5_10	34		8-13
		Company_I_1_10	33		4-13
2	5	Company A_3_2	5	321	4-13
		Company_F_9_2_a	45		4-13
		Company_F_9_2_b	35		7-13
		Company_H_12_2	120		3-13
		Company_I_1_2	19		10-13

Table 47. Case 2 Single Option Tracks

For the track from station 10 to station 11, the results say that we build 1 additional track – such that there are 2 tracks with a total capacity of 20 containers. As can be seen in Table 47 above, there are three different shipments that move along this track. Based on the days in which these containers are able to move along the track, the immediate area of concern is with shipment G_11_2. In order to get to the final destination by day 13 (assuming everything went on the most optimal route), all 97 containers need to leave station 11 by day 8 (but only become available on day 3). This is exactly what we see happen in the results for G_11_2. Everything goes on track 11-10 on days 3-8. This can only happen if the track 10-11 is expanded to double its capacity up to 20 containers per day. Raising it to 30 is not necessary for feasibility purposes, and as described above, feasibility is the reason for expansion of this route. This expansion also enables B_1_11 to be delivered without feasibility issues.

For the track from station 9 to station 10, the results say that we build 2 additional tracks – such that there are 3 total tracks with a total capacity of 30 containers. As can be seen in Table AA above, there are five different shipments that move along this track. The time ranges vary, but there is an overall need to move 212 containers between days 4-13 – a ten day period. Even with a track capacity of 20, it is clear that this would leave 12 containers unable to reach their final destination. The largest number of containers are moved on days 5 and 7, when 28 are shipped along the track from 9-10 between shipments G_11_2 and I_1_10. While there are many ways the shipments could be varied – all of them result in the necessity for 2 extra tracks. The solution provided utilizes the capacity of 30 in order to deliver containers in the most optimal way possible. Thus, two new tracks are needed on 9-10 to have a capacity of 30 containers for feasibility purposes!

Finally, for the track from station 2 to station 5, the results say that we build 3 additional tracks – such that there are a total of 4 tracks with a total capacity of 40 containers. As can be seen in Table 47, there are five shipments that must move along this track. The time ranges vary, but there is an overall need to move 321 containers between days 4-13 – a ten day period. Even with a track capacity of 30, this would leave 21 containers that would not reach their final destination – an infeasible result. As such, the track capacity needs to be 40 to enable all of these shipments to reach their final destinations within the time horizon. The largest number of containers are moved on days 6, 7, and 8 with 33, 36, and 35 containers moved among all the shipments (except I_1_2). While 31+ are not moved on all days, at least some days need more than 30 containers to be moved to get a feasible solution. The solution provided utilizes the capacity of 30 in order to deliver containers in the most optimal way possible. Thus, three new tracks are needed on 2-5 to have a capacity of 40 containers for feasibility purposes!

As described above, we are moving 753 containers in Case 2 (this case) compared to 262 in Case 1. With the same number of tracks, stations, and days, moving triple the number of containers was bound to lead to some additional tracks needing to be expanded in order to have feasible and optimal solutions. Tracks (1, 3), (4,8), (6, 7), (6,15) all had one additional track while (6,9) had two additional tracks. We discussed a little above about the movements from 1-8 and how some containers can move 1-4-8 while there is also a direct path. Based on the expansion here, it is likely the case that adding capacity to this short route was cheaper than adding to (1,8) – including the effects of rerouting containers. With respect to (1,3), it is a good connector between the left and right sides of the map (especially for the routes from 9/12 going to station 2) without having to interfere with all the traffic in the middle of the map. Speaking of that traffic, the remainder of the new tracks built are related to station 6. Station 6 is the most central station in the map and the shortest path to connect stations 11/10 to the rest of the map. Therefore, it makes a lot of sense that the routes for stations 6-7, 6-9, and 6-15 need to be expanded to a capacity of 20, 30, and 20 respectively. Looking through the shipment schedule, there are a LOT of different shipments that move across these different shipments. As such, it is likely the case that these shipments would NOT be able to reach their final destination without having the capacity expanded on these tracks.

Expanding these tracks has led to an additional issue – the need for more reloading zones. Additionally, there are some routes even without expanded tracks that have so many containers coming into them from different tracks – more reloading zones are necessary. The cost for adding a reloading zone is a fixed \$168,000 per extra reloading zone and enables an additional 12 containers to move through the node on a day (see Table 41). For reference, delaying 12 containers for one day costs

\$168,000 per extra reloading zone and enables an additional 12 containers to move through the node on a day. For reference, delaying 12 containers for one day costs \$240,000 and rerouting 12 containers would cost at least \$80,000(3−14−5 vs 3−5; one exception is 1−8 vs 1−4−8 which costs the same but can have issues with delay costs). For most rerouting, the costs are \$80,000 (3-14-5 vs 3-5; one exception is 1-8 vs 1-4-8 which costs the same but can have issues with delay costs). For most rerouting, the costs are \$200,000+ not including possible delay costs due to the extra time. While there may be a few small exceptions, the vast majority of the costs for adding a reloading zone are far lower than that of rerouting the containers or delaying the shipment.

For the feasibility reasons with respect to on time delivery, it makes sense that we would see expanded reloading zones at stations 2,5, 9, 10, and/or 11. Station 11 already has two reloading zones, and since at most 20 containers could enter station 11 on a day, there is no need to expand it. However, as can be seen in the results, stations 2, 5, 9, and 10 all expand their reloading zones. Station 2 adds two reloading zones to have a total of 3 reloading zones (to handle the maximum of 36 containers; Note: this is one reason a maximum of 36 containers are moved even though there is a capacity of 40 containers on track 2-5). Station 5 begins with two reloading zones, so it only needs to add 1 in order to handle the 36 containers coming through it at maximum. The same logic will follow for stations 9 and 10. Station 9 starts with two reload zones and adds 2 to reach a capacity of 48. This happens because in addition to the 30 containers that are moving from station 9-10, there are other containers that need to pass through container 9 (such as those going from 6-8).

Finally, we investigate the additional reloading zones created at stations 1, 6, and 15. For station 1, it began with only one reloading zone at a capacity of 12. Since track 1-3 was expanded to a capacity of 20, it makes sense that an extra reloading zone is needed at 1 to make the track expansion actually have an impact. The same story is true for station 15. While station 15 begins with two reloading zones, it has an additional track coming into it from 6-15. In addition to that, station 15 is central in the map, so many containers pass through it (especially to go from 14/15 – 12/6/9/10/11). As described above, the cost of a new reloading zone is so low relative to rerouting costs, it makes sense that an extra reload station is built at station 15 to bring its capacity to 36 containers. And lastly, station 6 had a LOT of new tracks built that go through it (4 new tracks in total). In theory, there could be 80 containers going through station 6 at any time based on track capacity. However, in reality, that many containers never go through the station as it would be suboptimal for network flow. These results add two reloading zones to the station such that it can handle up to 48 containers (with 4 reloading zones). This follows from the addition of the tracks, so it makes sense that the station would need to handle a total number of containers above 36 - especially with 8 tracks going to it.

In summary, Case 2 is a really complex network moving 753 containers in just a 14 day time horizon. Because of the cost of building new tracks, it is often preferable to reroute containers and accept the relatively low delay penalties as seen in the first portion of the analysis. However, the feasibility condition that all containers must be delivered means that certain tracks must be built. Then, reloading zones can be added at a really low relative cost to ensure that the addition of the tracks becomes useful and a minimal amount of re-routing is required due to reload zone capacity. In the future, SmartRail should investigate the ability to build more tracks connecting the isolated stations on the edges. These stations in particular that have a large number of containers going into them require the building of new tracks on long routes (such as 10-11) purely for feasibility purposes. A cost-benefit analysis would be needed to see if new tracks connecting unconnected stations is actually cheaper than adding these additional tracks. Overall, the output of our team's scheduling model provides an optimal solution that is feasible and carefully balances the costs of adding reloading zones, rerouting containers, and having containers arrive late. In this careful balance of the different trade-offs, our recommended shipping schedule and construction plan provides the optimal routing of the 20 shipments in Case 2 – at a minimum total cost to SmartRail.

4 Appendix I

4.1 Alternative Formulation

As described in section 2.8, there is an alternative form of the solution. That alternative form uses equations (19 and 20) below instead of equation 13 that is used in the formulation in the model in section 2.6.

$$s.t \qquad \sum_{d \in \text{Days}} \text{ori}_{s,d} = \text{volume}_s \qquad \forall\, s \in \text{Shipments} \tag{19}$$

$$\sum_{d \in \text{Days}} \text{del}_{s,d} = \text{volume}_s \qquad \forall\, s \in \text{Shipments} \tag{20}$$

In the model, we stated that no container can move for any shipment on a day prior to its available day. This, in the view of our team, was the more intuitive way of explaining this constraint. The alternative way is to write it above. We have already stated that during the time period from when a container becomes available to the second-to-last day, the sum of the ori decision variables must equal the volume of the shipment. So, if we sum up across every day, that means that all other ori variables must be zero (since they are all $>= 0$).

We employ the same strategy with the del decision variable. We had already summed it up from the day after it became available to the final day. Then, by summing up all del variables, we implicitly set the rest of them to be zero (since they are all $>= 0$).

By setting the ori and del decision variables to be zero in all cases in which they cannot contain a value, we implicitly prevent any travel prior to the available date. Additionally, by ensuring that the ori value on the final day is zero, we ensure that everything must originate during the correct time period. If that is the case, then nothing can move prior to the available date and we receive the exact same feasible optimal solution. However, this formulation is a little more implicit in how it prevents movement than the one described in model 2.6. It is easier to think that if no containers are allowed to move, then nothing could have possibly originated. Including both forms of this would be redundant.

4.2 A Note About AMPL

When testing the solution with both formulations, the team received different results in each case. Because the formulations are equivalent, this did not make sense. Upon discussion with the instructional team, we learned that AMPL has an error tolerance built into CPLEX. If the solution is within a certain threshold of the optimal solution, CPLEX stops. This results in a solution that is close to the optimal solution, but not exact. In case 1, the numbers were small enough the error tolerance did not play a factor. In case 2, this tolerance led to one solution that was suboptimal.

Thus, the team has included the following line in the run file to REMOVE any allowed tolerance (i.e. it must be the exact solution).

```
option cplex_options 'mipgap 0';
```

Despite taking longer for the code to run, the team strongly recommends to SmartRail that they continue using this version of the code with no error tolerance. While a solution within \$10,000 might be "good enough" on the order of 100 million, the cost of computation in this case is low enough that SmartRail should happily accept the most optimal possible solution. With this line of code implemented, both formulations get identical optimal outputs!!

5 Appendix II

Appendix 2 Log File Explanation

The goal of our log file was to provide SmartRail with all information necessary to inform their decision making. Some duplicate data is presented throughout the file, however each it is done so in different manners to allow SmartRail to understand the results from all angles.

The log file being outputted by our AMPL model can be split into 14 different sections. We note that throughout the log file, if there is any information not present, it has a value of 0

- Section 1: Breakdown of the costs SmartRail will incur at the optimal solution
  - The optimal, minimum total cost of the entire plan
  - Four cost components: total_ship_cost, new_reload_cost, new_track_cost, delay_cost
  - This is the objective value of the MILP
- Section 2: Shipping Schedule by shipment
  - This section goes through each shipment one-by-one and says how many containers are moved along what route on what day
  - Ex. Container load per track for shipment Company_A_4_7 on day 1 :
    - 4 –> 6: 10
      - For company A from 4 to 7, 10 containers go from station 4 to station 6 on day 1
    - 4 –> 1: 10
  - Note: This section is made up of the x[s, d, i, j] and y[s, d, j, i] decision variables
- Section 3: Shipping Schedule by Day (1-14)
  - This section has the same information as section 2, but instead does every shipment on day 1 followed by every shipment on day 2
- Section 4: Total load per track (per day)
  - This section details the total number of containers that move along a track on a given day
- Section 5: Track at Maximum (per day)
  - This section displays the tracks in which they are at capacity for a given day. It uses the final capacity (i.e. if a track was expanded to have a capacity of 20 but only ships 18, it would not be included).

- Section 6: Expansion Necessary for Tracks (per day)
    - This section details the days in which the number of containers moving along a track exceeded its original capacity. These are the specific days that led to the need for the construction of an additional track/s.
- Section 7: Total Reloaded Containers (per day)
    - This section shows the number of containers that are reloaded at each station on each day.
    - It also displays the maximum number of containers that could be reloaded at that station.
- Section 8: Total Reloaded Containers at Maximum (by day)
    - This section highlights any instances in which the reloading zones are used at capacity.
- Section 9: Expansion Necessary for Reloads (per day)
    - This section shows the day(s) in which the number of containers exceeds the original capacity, leading to the building of additional reloading zones.
- Section 10: Delayed Containers (per day)
    - This section shows the number of containers that arrived late for each shipment per day
    - The output tells the user when the containers were supposed to arrive, making it easy to see how many days late the containers were
- Section 11: New Track Construction Decision Variable
    - This section explicitly states the tracks in which there was an expansion and the number of new tracks that were constructed
    - It is the result of the t[i,j] decision variable
- Section 12: New Reload Zones Built Decision Variable
    - This section shows the stations at which new reloading zones were built and the number of new reloading zones that were constructed
    - It is the result of the r[k] decision variable
- Section 13: Origination Decision Variable
    - This section displays the number of containers that originate on each day for each shipment.
    - This is the result of the ori[s, d] decision variable
- Section 14: Delivery Decision Variable
    - This section displays the number of containers that are delivered on each day for each shipment.
    - This is the result of the del[s, d] decision variable

6 References

<table>
<tr><td>Description</td><td>Author</td></tr>
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