8/30

Review from last class

or and (minus)

$\in, \subseteq, \subset, \cup, \cap, \setminus, |\cdot|$

Set $A, B, \ldots$

Special sets $\emptyset$ (nothin), Ω (Universal) → has 100 elements.

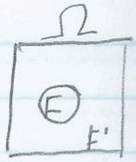
Set function 2^A.

Ex:

- $F \cap \Omega = F$
- $F \cup \Omega = \Omega$
- $\emptyset \cup \Omega = \Omega$
- $\emptyset \cap \Omega = \emptyset$
- $F \setminus \Omega = \emptyset$
- $F^c = \Omega \setminus F$
- $(F^c)^c = F$
- $F \cup F^c = \Omega$

★ $\Omega^c = \Omega \setminus \Omega = \emptyset$

$\emptyset^c = \Omega$



$\{F, F^c\}$ is a collectively exhaustive set.

Def: $\{A_1, A_2, A_3, \ldots\}$ is collectively exhastive.

if $\bigcup_{i=1}^{\infty} A_i = \Omega$

$= A_1 \cup A_2 \cup A_3 \cup A \ldots\ldots$

$\sum_{i=1}^{3} i = 6.$

★By def- share is nothin

$F \cap F^c = \emptyset$.

$\{F, F^c\}$ are mutually exclusive ("disjoint")

Def: $\{A_1, A_2, \ldots\}$ are mutally exclusive.

if $A_i \cap A_j = \emptyset$

$\forall_{i \neq j}$

Any 2 unique sets has no intersection. (nothin in common).

$F \overset{?}{\subseteq} F^c$ → False.

nothin in this will be in F.

- $|\Omega| \overset{?}{=} |A| + |A^c|$ (100) if $|A| = 0$ (nothin) + $|A^c|$ = 100.
- $|A| = |\Omega| - |A^c|$

- $A, B \subseteq \Omega$

$= (A \cup B)^c = A^c \cap B^c$ (or; c ← negation)

Ex: $(3 \cdot 4)^7 = 3^7 4^7$

$A^c \cup B^c$

DeMorgan's Laws.

$= (A \cap B)^c = A^c \cup B^c$

must remember . $\mathbb{R}$ = all real #s. $\mathbb{N}$ = Z=

☆

$\mathbb{N} := \{1, 2, 3, \ldots\}$

$|\mathbb{N}| = \aleph_0$ / ∞ infinity.

↳ we call this ∞ = (countable ∞).

$|\mathbb{Z}| = \aleph_0$ $|\{\ldots, -1, 0, 1, \ldots\}|$

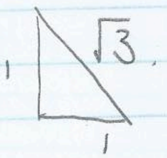
$\{1, 2, 3, 4, 5\}$

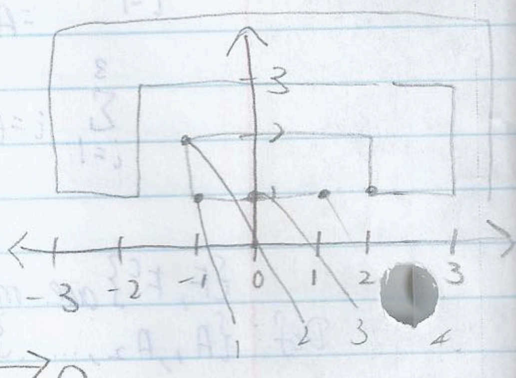
$Q := \{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\} = \{0.157, -0.682, \ldots$

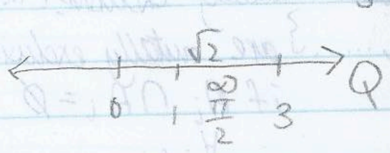
AKA all fractions, decimals.

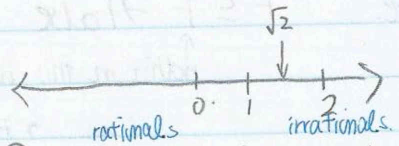
"the rationals"

$|Q| = \aleph_0$









"continuum"

- $\mathbb{R} := \mathbb{Q} \cup \{\text{all holes}\}$
  $= \{\sqrt{2}, \pi, \ldots\}$.

- $|\mathbb{R}|$ - $[3,7] = \{x : x \geq 3, x \leq 7\} \subseteq \mathbb{R}$

  - $(10,13) = \{x : x > 10, x < 13\} \subset \mathbb{R}$

  - $|(0,1)| \leq |\mathbb{R}|$

every $x \in (0,1)$ in binary.

Not on the exam.

Assume $|(0,1)| = \aleph_0$ i.e (countable ∞)

0.0101110… → 0.1101

0.1011000… ∉

0.1101001 ↳ {…}

0.1010101…

if we flipped it then our assumption is not true.

Must know this $\boxed{|\mathbb{R}| = c > \aleph_0}$

(uncountable ∞)

rational # is >/< than real #.

T/F $|\Omega| \overset{?}{=} |A| + |A^c|$ if $|\Omega| < \infty$

set theory continue...

Ordered Pairs

Def: $\langle a, b \rangle := \{\{a\}, \{a, b\}\}$

ordered set of elements.

$\langle a, a \rangle$

★ $\langle a,b \rangle \neq \langle b,a \rangle$

$\langle a,b \rangle \neq \{a,b\} \to \{\dots\}$

Cartesian Products

- $A \times B := \{\langle a,b \rangle : a \in A, b \in B\}$

$\underline{A = \{1,2\}, B \in \{3,4\}}$

Ex: $A \times B := \{\langle 1,3 \rangle, \langle 1,4 \rangle, \langle 2,3 \rangle, \langle 2,4 \rangle\}$

$|A \times B| = 4 = |A|\,|B|$ b/c $|A| = 2, |B| = 2.$

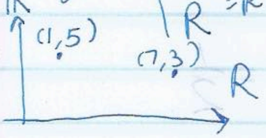
- $A^2 = A \times A$, $|A^2| = |A|^2$

$A^n = \underbrace{A \times A \times \dots}_{n}$, $|A^n| = |A|^n$

$$\left| \mathop{\LARGE\times}_{i=1}^{n} A_i \right| = \prod_{i=1}^{n} |A_i|$$

- $\mathbb{R} \leftarrow \mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$

Nice Fact. graph paper...



"outcomes"

$\Omega = \{\omega_1, \omega_2, \omega_3, \ldots\}$ lower case omega.

sample space

experimental outcome space.

Experiment: one $\omega \in \Omega$ is chosen, / Trials.

Coin Flip experiment

→ only 4 questions you could ask. $\Omega = \{H, T\}$

$|\Omega|$ How many outcomes?. Answer = 2.

$|\Omega| = 2.$

An "event" is: $A \subseteq \Omega$

Is H an event? No.... it's an outcome.

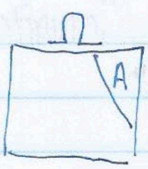
$H \in \Omega$

$\{H\} \subseteq \Omega$ → this is an event.

☆ $A \in 2^{\Omega}$ = event space $= \{\emptyset, \{H\}, \{T\}, \{H, T\}\}$

all possible answers.

events.



☆ • Working Definition: $P(A) = \frac{|A|}{|\Omega|}$ proportion of A working A is.

For all events A

sit still outcome equally like

$\forall \omega_i$

$P(\omega_i) = \frac{1}{|\Omega|}$

Is $P(H) = \frac{|H|}{|\Omega|}$ undefined (not an event).

$P(\{T\}) = \frac{|\{T\}|}{|\{H, T\}|} = \frac{1}{2}.$

Def. • $P: 2^{\Omega} \longrightarrow [0, 1]$

(for the time being)

$P(\emptyset) = \frac{|\emptyset|}{|\Omega|} = \frac{0}{2} = 0$

Complements Rule

Assume Ω finite .... $P(A^c) = \frac{|A^c|}{|\Omega|} = \frac{|\Omega| - |A|}{|\Omega|}$ $P(A) = 1 - P(A^c)$

$P(A^c) = 1 - P(A)$

$$|2^{\Omega}| = 4 = 2^{|\Omega|}$$

$$P(\{H\}) = \frac{1}{2}$$

$$\not\perp\!\!\!\perp$$

$$P(\{T\}) = \frac{1}{2}$$