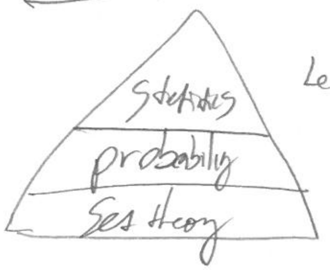
Math 241 Lec 1 8/25/16



Lec 20-23

Lec 2 – 20

Lec 1-2

1870's formalized all of math by 20th century. Everything is built on sets.

assignment / definition   elements

$F := \{ \text{Jane, Mary, Susan, Dawn} \}$

elements have no order

(order of elements in enumeration doesn't matter)

separators

braces denote enumeration begin/end

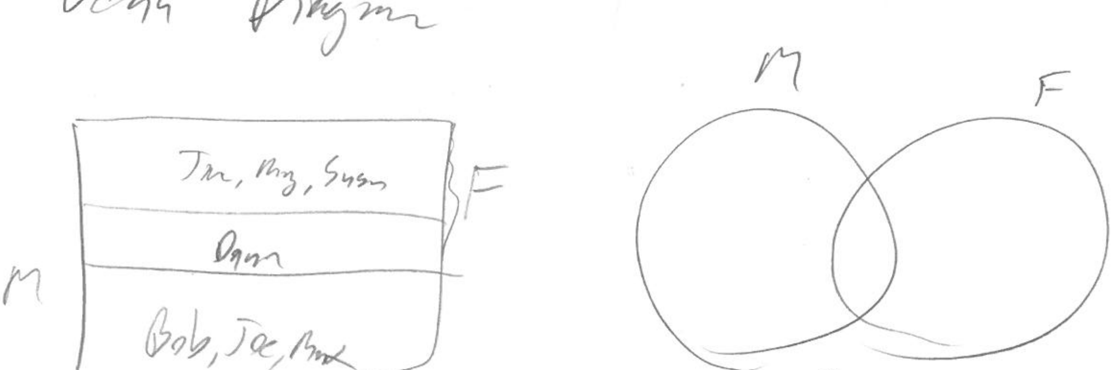
descriptive name

F for "female"

two hard problems in CS...

$M := \{ \text{Bob, Joe, Max, Dawn} \}$

Venn Diagram



logic matters

sets can have infinite elements $N = \{1, 2, 3, \ldots\}$ natural #'s $\mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$ integers (2

ellipses means continues the pattern is

Operations on sets

set inclusion "element of", "in"

$\underbrace{\text{Jane}}_{\text{element}} \in \underbrace{F}_{\text{set}}$ evaluates to T/F

set exclusion "not element of", "in"

$\text{Joe} \notin F$

order?

$\{\text{Jane}, \text{Mary}, \text{Dan}, \text{Susan}\} = F$ set equality all elements the same

$M \neq F$ set inequality

Subsets

$\underbrace{\{\text{Jane}, \text{Mary}\}}_{\text{set}} \subseteq \underbrace{F}_{\text{set}}$

all elements in the set on the l.h.s are in the set on the r.h.s

$\{\text{Jane}, \text{Mary}\} \subset F$

" but the sets are not equal

$\subseteq$ same as "$\subset$" or "$=$"

$\{\text{Jane}\} \subset F$?, $\text{Jane} \subset F$?, $\{\text{Jane}\} \subseteq F$?, $\text{Jane} \subseteq F$?

$\{\text{Jane}\} \in F$, $\text{Jane} \in F$?

Jane ⊂ F does not parse ... it's like $\frac{3=}{7+}$? $f(x)=x^2$

it's a "compiler error" in math   double int 7new $f(☺)$

$\in, \notin, =, \neq, \subset, \subseteq$ were predicates as in they are functions which return T/F

technically... $\neq(M,F) = T$ $\neq : \text{set} \to \{T,F\}$

What about functions from sets to sets?

$F \cup M$ "union": combine both sets' elements

$\{\{\text{Jane, Susan, Mary, Dann}\}, \{\text{Bob, Joe, Max, Dann}\}\}$ → $= \{\text{Jane, Mary, Susan, Dann, Bob, Joe, M}$

"collapse", "flatten"
(ruby)

Why is Dann not in twice? Addition without double counting

$\{\text{Jane}\} \cup \{\text{Jane}\} = \{\text{Jane}\}$ "singleton set"

"non-exclusive or"

if $\cup$ was or   Dann $\in F \cup M$?
like in English

$= \mathbb{N} \cup \{0\}$

Jane $\cup$ {Jane} ?   HW... $\mathbb{N}_0 := \{0,1,2,\ldots\}$

$F \cap M$ set intersection .. both returns a set whose elements are both

$= \{\text{Dann}\} \neq \text{Dann}$ "And" in English empty set / null set

$F \cap \{Bob, Joe\} = \{\}$, $\phi := \{\}$

special notation

odd #'s $\cap$ even #'s $= \phi$

both sets infinite

Def: A, B are mutually exclusive if $A \cap B = \phi$

$\phi \subset F$? $\phi \in F$?

We can subtract sets

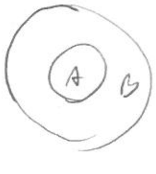
set minus

$F \setminus M :=$ all elements in F that are not in M

$= \{Joe, Mary, Susan\}$

if $A \cap B = \phi$ what is $A \setminus B = A$

if $A \setminus B = \phi$ what is $A \cap B = A$



if $A = B$ $A \cap B = A$ or B

$A \subseteq B$ $A \setminus B = \phi$

Set builder notation

$$E := \{2n : n \in \mathbb{Z}\} = \{\dots,-4,-2,0,2,4,\dots\}$$

"all evens" $2n$ "such that" n is an element of the integers

$$A := \{1,2,3\}$$

$$2^A := \{B : B \subseteq A\} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \underbrace{\{1,2,3\}}_{A}\}$$

"powerset" of A set of all subsets of a set

$$\{3,2\} \in 2^A \ ?$$

Size of set / cardinality

abs val signs / pipes / "overloaded" notation

$|A| = 3$ just count...

$$f : \text{set} \to \mathbb{N}_0 \quad (\text{for now})$$

$$|F \cup M| \stackrel{?}{=} |F| + |M|$$

$$7 \neq 4 + 4$$

$$|F| \,?\, |M|$$

$$\stackrel{?}{=} |F \cap M| = |\{\text{Dan}\}|$$

7 nope... 1

$$|F \setminus M| \stackrel{?}{=} |F| - |M|$$

$$1 \neq 4 - 4$$

ϕ = , $\{1\}$ = , $\{2\}$ = , ..., A = ; $\angle 8$

$$|2^A| = 8 \quad \{ \underset{1}{F}\,\underset{2}{F}\,\underset{3}{F},\ \underset{1}{T}\,\underset{2}{F}\,\underset{3}{F},\ \underset{1}{F}\,\underset{2}{T}\,\underset{3}{F},\ \dots,\ \underset{1}{T}\,\underset{2}{T}\,\underset{3}{T} \}$$

$$|2^F| = \underset{16}{?}$$

What are all possible configurations?

$$\underset{1}{|\{T,F\}|} \cdot \underset{2}{|\{T,F\}|} \cdot \underset{3}{|\{T,F\}|} = 2 \cdot 2 \cdot 2 = 8$$

→ heuristic method of counting

heuristic: an approach to problem solving, practical method, not guaranteed to be optimal but sufficient to get the job done...

for any set S...

$$|2^S| = 2^{|S|}$$

rigorous proof requires ZF set axioms

Special set: Ω "universe", "sample space", "space of discourse", CS: "scope". All elements we are limited to. You define it.

$$\Omega := F \cup M = \{ \ \_\_\_\_ \ \}$$

Note: $F \subseteq \Omega$, $M \subseteq \Omega$, $2^F \subseteq \Omega$?

Coin Flip $\Omega := \{H, T\}$, Die Roll: $\Omega := \{1,2,3,4,5,6\}$

What is the probability a "random" name is a female?

$$P(F) = \frac{|F|}{|\Omega|} = \frac{4}{7}$$

→ definition → definition

always assume scope!

name

$\boxed{\text{working def: } P(A) = \frac{|A|}{|\Omega|}}$