08/30/16

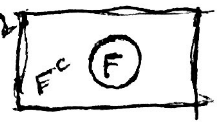
pipes / cardinality

- $\in, \subseteq, \subset, \cup, \cap, \setminus, |\ |$
- sets $A, B, \ldots$
- special sets $\emptyset$ Ω

universe - everything you care about right now

ex:
- $F \cap \Omega = F$
- $F \cup \Omega = \Omega$

Ω



- $\emptyset \cap \Omega = \emptyset$
- $\emptyset \cup \Omega = \Omega$

$F \subseteq \Omega$

$F \in Z^{\Omega}$

- $F \setminus \Omega = \emptyset$
- $\Omega \setminus F = F^c$

set complement

everything in the universe that is not F.

- $(F^c)^c = F$
- $A \cup A^c = \Omega$
- $\{A, A^c\}$ are called **COLLECTIVELY EXHAUSTIVE**.

Defn. collectively exhaustive - put them together and we get everything.

$\{A_1, A_2, A_3, \ldots\}$

- if $\bigcup_{i=1}^{\infty} A_i = \Omega$
- $A \cap A^c = \emptyset$

$\{A, A^c\}$ are mutually exclusive ("disjoint")

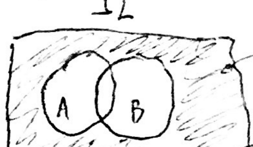
$\{A_1, A_2, \ldots\}$

are mutually exclusive if $A_i \cap A_j = \emptyset \ \forall i \neq$

- $|\Omega| = |A| + |A^c|$ for $|\Omega|$ finite
- $|A| = |\Omega| - |A^c|$

Consider $A, B \subseteq \Omega$

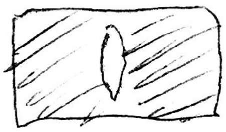
Ω



- $(A \cup B)^c = ?$

can $(A \cup B) \overset{?}{=} \underbrace{(A^c) \cup (B^c)}$?

NO.



- can $(A \cup B) \overset{?}{=} A^c \cap B^c$ ✓
- can $(A \cap B) \overset{?}{=} A^c \cap B^c$ - NO
- can $(A \cap B) \overset{?}{=} A^c \cup B^c$ - ✓

$$\boxed{\begin{array}{l}\underline{\text{DeMorgan's law}} \\ \cdot\ (A \cup B) = A^c \cap B^c \\ \cdot\ (A \cap B) = A^c \cup B^c\end{array}}$$

- $\{x : x \geq 3, x \leq 7\} \subset \mathbb{R}$

$[3,7]$

- $(10,17) := \{x : x > 10, x < 17\} \subset \mathbb{R}$

"rationals" "irrationals"

$\mathbb{R} := \mathbb{Q} \cup \{\text{all holes}\}$

"Reals"

"Continuum

- Consider $(0,1) \subset \mathbb{R}$

$|(0,1)| \leq |\mathbb{R}|$

~~Assum~~ Assume $|(0,1)| \neq \aleph_0$

WRONG because we cannot enumerate them.

UNCOUNTABLE ∞ $|\mathbb{R}| = c > \aleph_0$

- ORDERED PAIR

$\langle a,b \rangle := \{\{a\}, \{a,b\}\}$

element a, element b in that order.

$\langle a,b \rangle \neq \langle b,a \rangle$

$\langle a,b \rangle \neq \{a,b\}$

CARTESIAN PRODUCT

- $A \times B := \{\langle a,b \rangle : a \in A, b \in B\}$

- $|A \times B| = 4$

$|A| = 2$ $|B| = 2$

$|A \times B| = |A||B|$ if finite

Think of GRAPH paper...

$\langle 2,3 \rangle \neq \langle 3,2 \rangle$, etc.

$A = \{1,2\}$

$B = \{3,4\}$

$A \times B = \{\langle 1,3 \rangle, \langle 1,4 \rangle, \langle 2,3 \rangle, \langle 2,4 \rangle\}$

- $A^2 := A \times A$
- $A^3 := A \times A \times A$

$n \in \mathbb{N}$

- $|A^n| = |A|^n$

END OF SET THEORY

Ω : sample space

experimental outcome space

$\Omega = \{\omega_1, \omega_2, \dots\}$

little omega

These elements are called outcomes.

- Experiment... $\omega \in \Omega$ is chosen.

Ex:

COIN FLIP : $\Omega = \{H,T\}$ $|\Omega| = 2$

$A \subseteq \Omega$

"event": set of outcomes

$A \in 2^{\Omega} = \{\emptyset, \{H\}, \{T\}, \{H,T\}\}$

event space - ie., all events

PROBABILITY : Working Definition

- $P(A) = \frac{|A|}{|\Omega|}$ if Ω finite

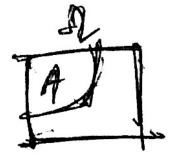
H is not a set... Does not compute.

- $\cancel{P(H) = \frac{|H|}{|\Omega|}}$

- $P(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2}$

$P : 2^{\Omega} \longrightarrow [0,1]$

events



- $P(\emptyset) = \cancel{\emptyset}$ $\longrightarrow$ $P(\underbrace{\{H\} \cap \{T\}}_{\emptyset}) = 0$
- $P(\Omega) = 1$
- $P(\{H,T\}) = 1$
- $P(\{H\} \cup \{T\}) = 1$

- $P(A^c) = \frac{|A^c|}{|\Omega|} = \frac{|\Omega| - |A|}{|\Omega|} = 1 - \frac{|A|}{|\Omega|}$

$= 1 - p(A)$

Complete Rule

$\boxed{\Rightarrow p(A) = 1 - p(A^c)}$

- Two coin flips

$\Omega' = \Omega^2$

$= \{\langle H,H \rangle, \langle H,T \rangle, \langle T,H \rangle, \langle T,T \rangle\}$