Math 2?? Lec 2 8/30/16

Previously: $\in, \subseteq, \subset, \cup, \cap, \setminus, |A|$, special sets $\phi, \Omega, 2^A$

Ω: "universe" i.e. contains all elements under consideration ... defined by you!

$F \cap \Omega$

$F \cup \Omega$

$\phi \cup \Omega$

$\phi \cap \Omega$ $\quad F \in ?$

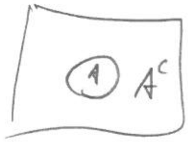
$F \setminus \Omega$

$\Omega \setminus F \Rightarrow$ everything that's not a female name

operation is done a lot $\quad F^c = \{Bob, Joe, Max\} \neq M$

"c" for complement

$A^c := \Omega \setminus A$



$f(A)$ Rules

$(A^c)^c = A$

$A \cup A^c =$ $\quad$ thus $\{A, A^c\}$ are "collectively exhaustive" $\quad \{A_1, A_2, \ldots, A_n\}$, $n \leq \infty$ are collectively exhaustive if $\bigcup_{i=1}^{n} A_i = \Omega$

$A \cap A^c =$ $\quad$ thus, $\{A, A^c\}$ are "mutually exclusive"

$A \subseteq A^c$?

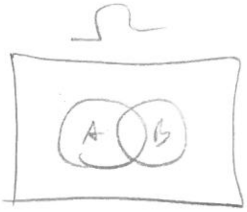
$\{A_1, A_2, \ldots, A_n\}$, $n \leq \infty$ are mutually exclusive if $A_i \cap A_j = \phi$ for all $i \neq j$

$|A| + |A^c| = |\Omega|$ (for finite sets)

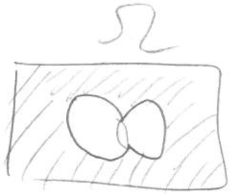
$|A| = |\Omega| - |A^c|$

very helpful fact we will use soon!

Consider A, B. Immediately, you know $A, B \subseteq \Omega$ ← Shorthand



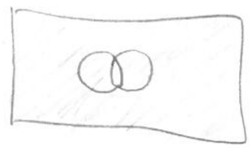
Consider $(A \cup B)^c$



Can I express this a different way?

$A^c \cap B^c$ Proof uses ZF axioms

Consider $(A \cap B)^c$



$= A^c \cup B^c$

These two are deMorgan's Laws

Not covered on exams:

$\mathbb{N} = \{1, 2, \dots\}$

$|\mathbb{N}| = ?\ \infty?$ $\aleph_0 := |\mathbb{N}|$, countable ∞ "Aleph-naught"

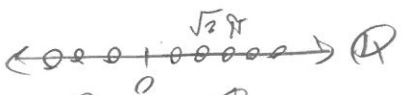
$|\mathbb{Z}| =$ 2 ctbl ∞? No... $\{1, 2, 3, 4, 5, \dots\}$ $\{\dots, -2, -1, 0, 1, 2, \dots\}$ 1:1 function

$\mathbb{Q} = \{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\}$ all fractions (decimals) AKA the rationals

$|\mathbb{Q}| > \aleph_0$? No...

... etc... $|\mathbb{Q}| = \aleph_0$

are all #'s $\in \mathbb{Q}$?? No... eg. $\sqrt{2} \notin \mathbb{Q}$, $\pi \notin \mathbb{Q}$, etc...



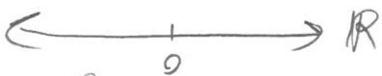
lots of holes!!

$\sqrt{2}$: infinite not repeating decimal, root of polynl (algebraic number)

π: infinite not repeating decimal, not root of polynl (transcendental number)

$\mathbb{R} = \mathbb{Q} \cup \{\sqrt{2}, \pi, \ldots\}$ — all holes / all the other #'s

(late 1800's)



no holes!
AKA the "continuum"

$[3,5] := \{x : x \geq 3, x \leq 5\}$, $(7,10) := \{x : x > 7, x < 10\}$

$|\mathbb{R}| = \aleph_0$?

Assume $|\mathbb{R}| =$ ctbl ∞

Imagine $x \in [0,1]$, $b(x) = x$ in base 2,

ctbl
$0.010100\ldots$
$0.101110\ldots$
0.000111
0.110011
$\vdots$

"Flip" the diagonal $\Rightarrow$ a new #!

$\Rightarrow |\mathbb{R}| \neq \aleph_0$, $|\mathbb{R}| > \aleph_0$

$c := |\mathbb{R}|$ uncountable infinity
the cardinality of the continuum

$\max([3,5]) = 5$

$\max([3,5)) =$ d.n.e

$(-\infty, \infty) = \mathbb{R}$

same thing

Ordered Pairs

$\langle a,b \rangle := \{\{a\}, \{a,b\}\}$ (1921)

a,b in that order!

$\langle a,b \rangle \neq \langle b,a \rangle$

$\langle a,a \rangle \neq \{a\}$

Cartesian Product

$A \times B := \{\langle a,b\rangle : a \in A,\ b \in B\}$

e.g.

$A = \{1, 2\},\ B = \{3, 4\}$

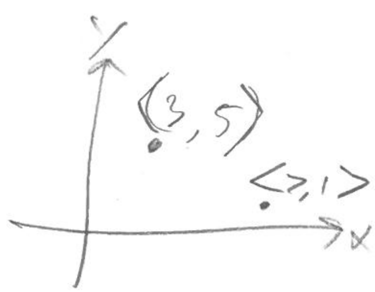
$A \times B = \underbrace{\{\langle 1,3\rangle, \langle 1,4\rangle, \langle 2,3\rangle, \langle 2,4\rangle\}}_{\text{in any order}}$

$|A \times B| = 4 = |A| \cdot |B| = 2 \cdot 2$

in general $|A \times B| = |A|\,|B|$ ... $\left|\bigtimes_{i=1}^{n} A_i\right| = \prod_{i=1}^{n} |A_i|$

$A^2 := A \times A = \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,2\rangle\}$

$A^3 := A \times A \times A$ ... , $A^n = \ldots$ $\qquad |A^n| \overset{?}{=} |A|^n$



all pts = $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$
"Cartesian plane"

Set theory

probability

Sample, experimental space

$\Omega = \{\omega_1, \omega_2, \omega_3, \ldots\}$ "lowercase Omega"

elements are called "outcomes". Each represents a possibility of an experimental trial. Assume CH sample space (horizon)

e.g.

$\Omega = \{H, T, \ldots\}$

On a coin flip, H, T could happen. Nothing else.

H, T mutually exclusive? Be careful... these aren't sets...

$\{H\}, \{T\}$ mutually exclusive

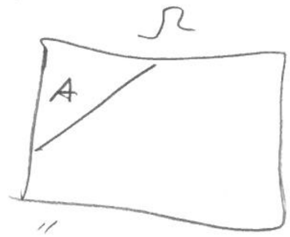
Sets s.t. all elements are outcomes are called events...

Any event is defined by $A \subseteq \Omega \Rightarrow A \in 2^{\Omega}$

Here, $2^{\Omega} = \{\emptyset, \{H\}, \{T\}, \{H,T\}\}$

the trivial events

$$P(A) := \frac{|A|}{|\Omega|}$$ working definition



proportion of sample space

"measure" of A relative to Ω.

$$P : 2^{\Omega} \longrightarrow [0,1]$$

$P(H)$ undefined!

$$P(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2}$$

$$P(\Omega) = \frac{|\Omega|}{|\Omega|} = 1 \qquad P\{H,T\} = P(\{H\} \cup \{T\}) = 1$$

$$P(\emptyset) = \frac{|\emptyset|}{|\Omega|} = 0$$

reason why $\emptyset, \Omega$ are the "trivial" events