Coprime Sensing for Array-InSAR Tomography

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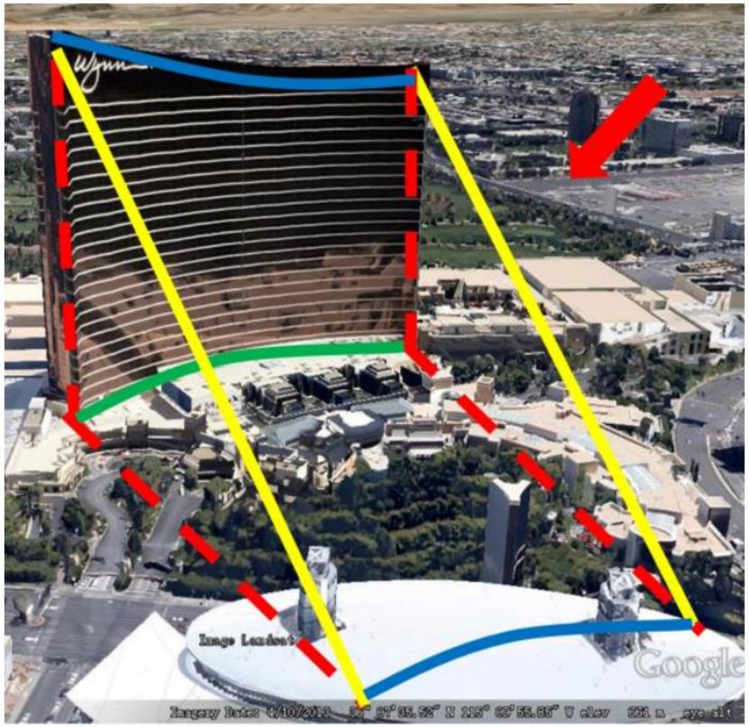
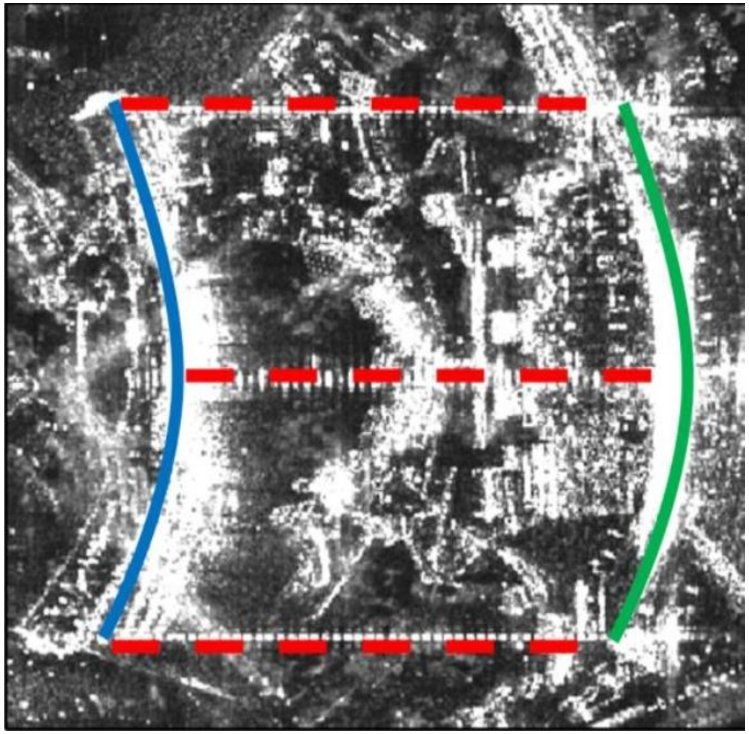
work done at EMWLab, FDU

SAR images

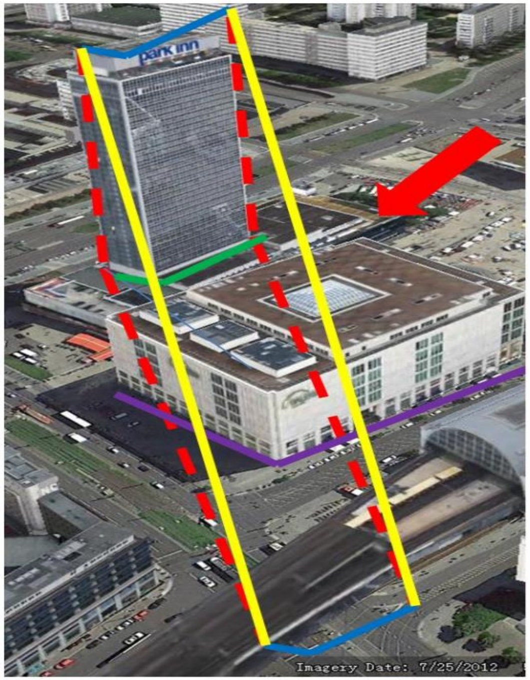


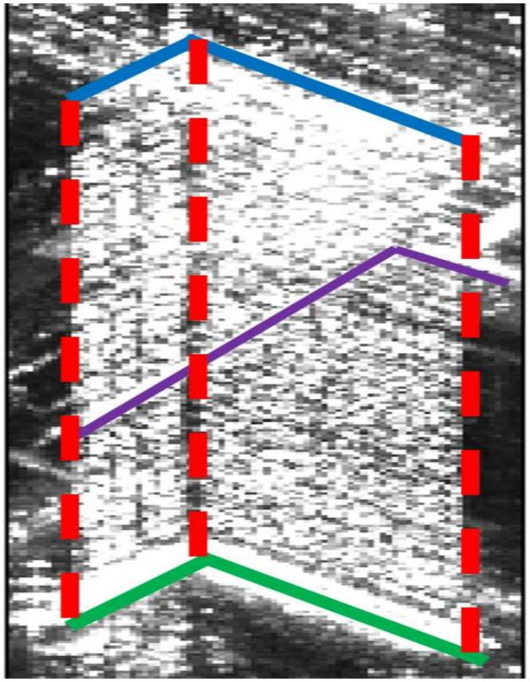


Wynn Hotel, Las Vegas [Zhu, 2010]

- Visual difficulty
- Geometry distortion

SAR images

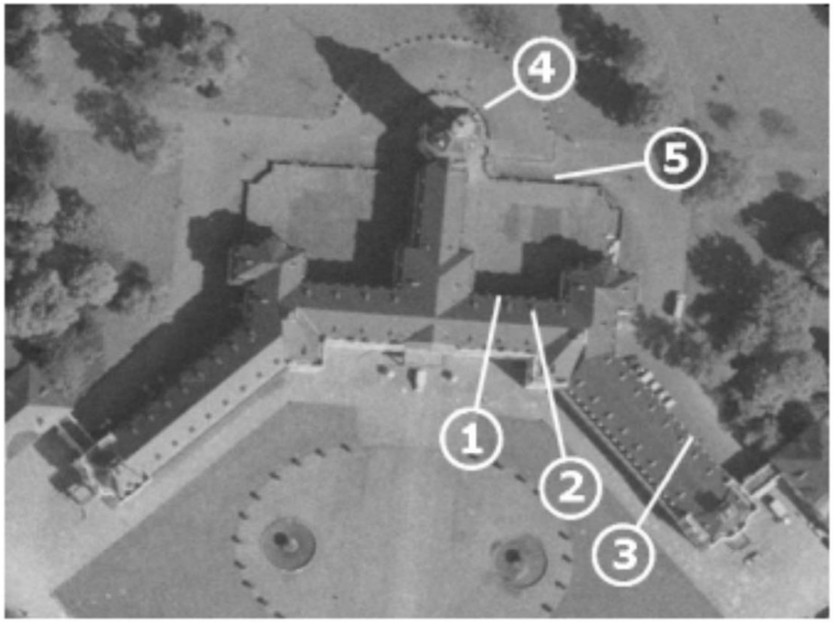


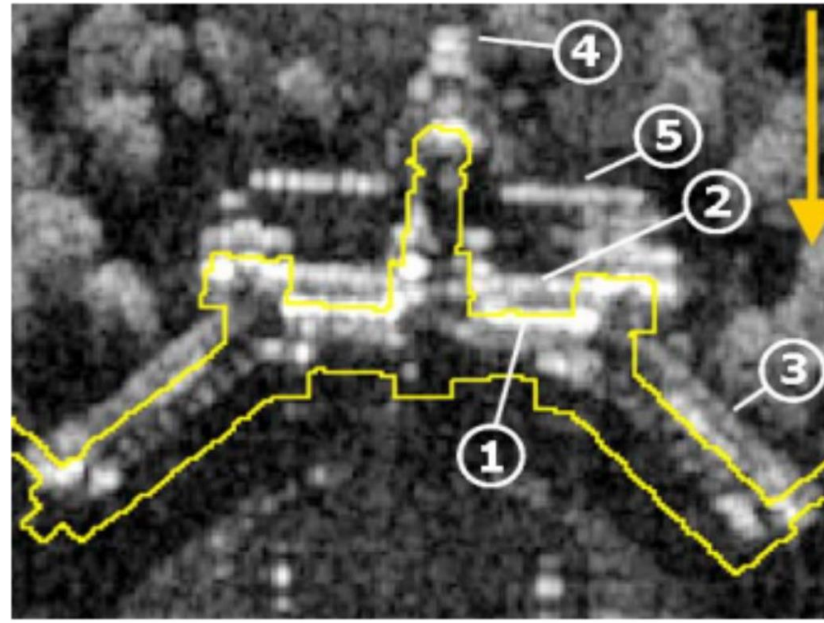


Park Inn Hotel, Berlin [Wei, 2015]

- Visual difficulty
- Geometry distortion

SAR images



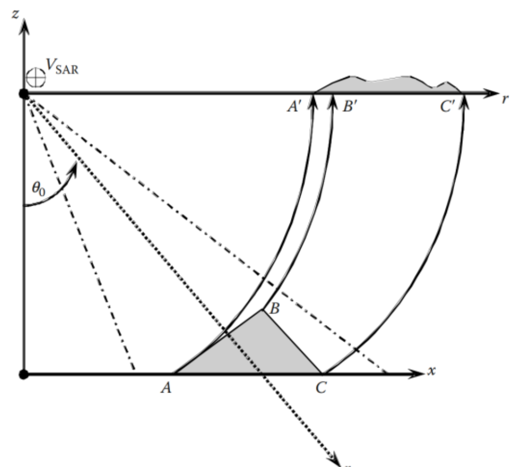


Karlsruhe castle [Timo, 2018]

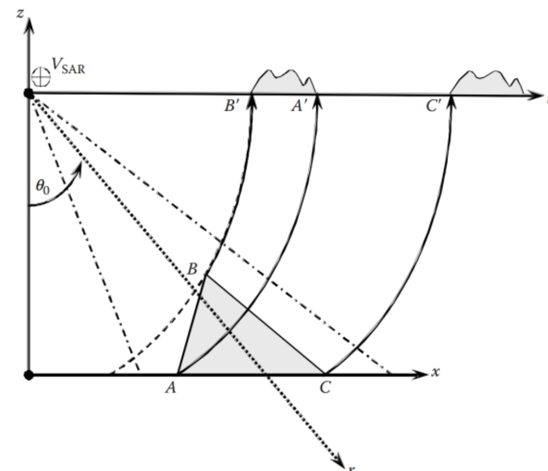
- Visual difficulty
- Geometry distortion

SAR Images

- Geometric distortion
  - Foreshortening, Layover distortion



Foreshortening distortion
[Lee, 2009]

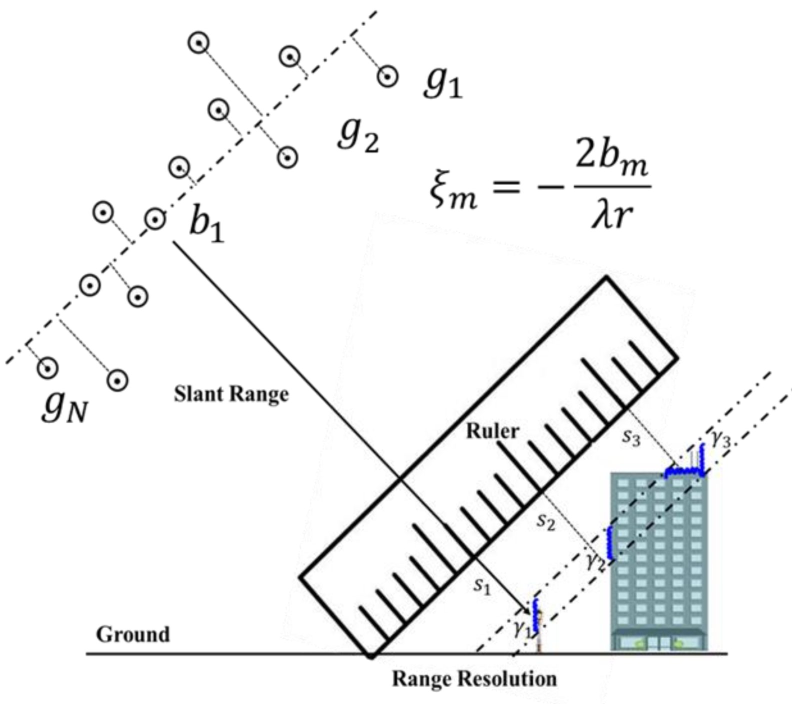


Layover distortion
[Lee, 2009]

  - We may need SAR tomography (3D imaging) to solve these inherent distortions in 2D imaging.

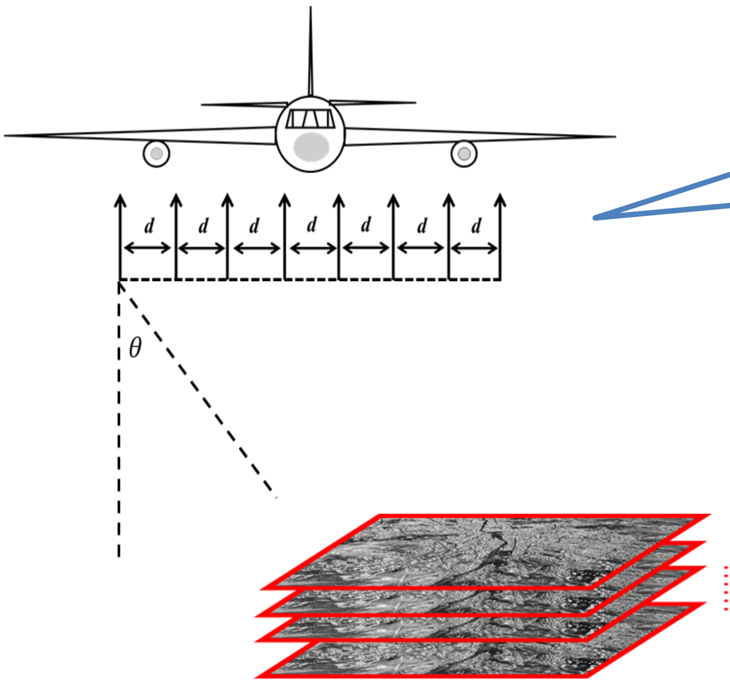
SAR Tomography

- Synthetic baseline aperture
- Imaging Model [Fornaro, 2003]
  - $g(\xi_m) = \int_{\Delta s} \gamma(s)\, exp(-j2\pi\xi_m s)\, ds$



SAR Tomography

- Repeat-pass InSAR tomography
- Single-pass Array-InSAR tomography



1) Very small baseline aperture

2) Finite number of array baseline elements

Array-InSAR system [Jiao, 2020]

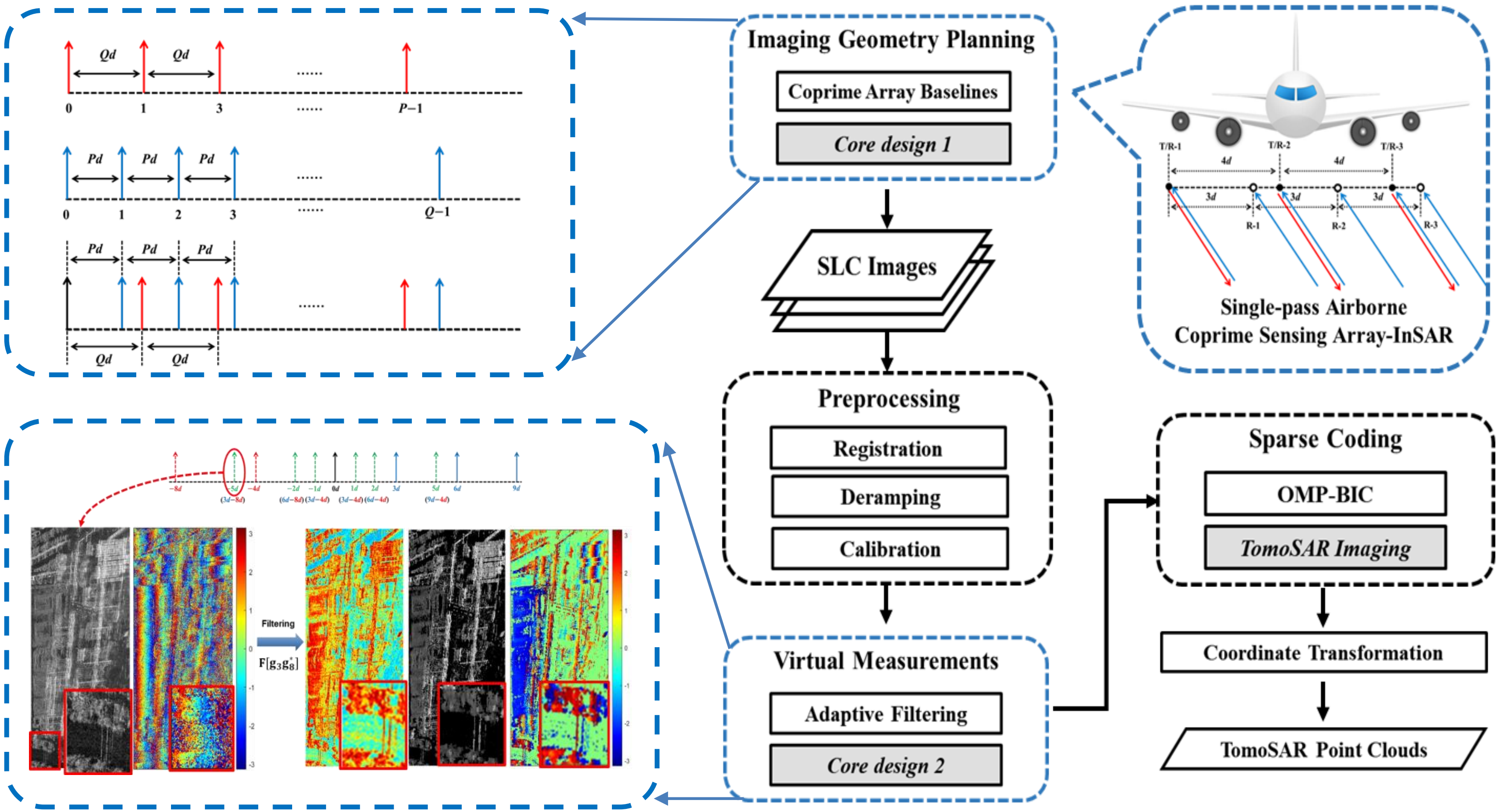
Insights into the baselines

- Coprime numbers and phase ambiguity [Wei Xu, 1994]
  - Eq: $b_2(\varphi_1 + 2k_1\pi) = b_1(\varphi_2 + 2k_2\pi)$
  - If b_1 and b_2 are coprime numbers, the ambiguity (k_1, k_2) has a unique solution
- Coprime signal processing [Vaidyanathan, 2011]
  - Coprime sampling/array can make full use of the disparity under the same number of acquisitions

$\xrightarrow{\textit{inspire}}$ **Coprime Sensing Array-InSAR**

Coprime sensing technique

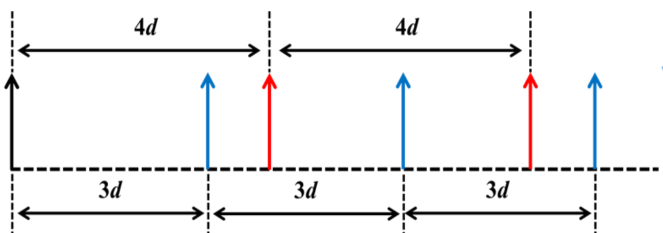
- *Two core designs*

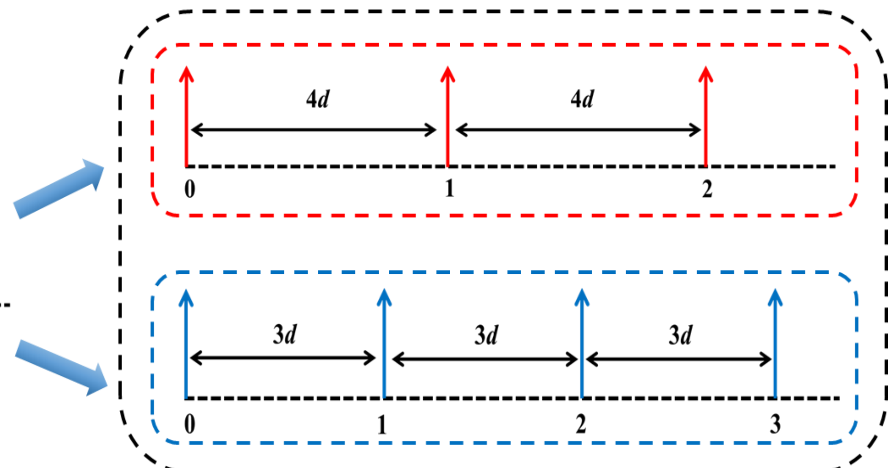


Virtual array baselines

- Coprime numbers: (P , Q)
- $P + Q - 1$ physical baselines $\Rightarrow$ PQ virtual baselines

$$\mathbf{S} = \{b \mid b = pQd, 0 \le p \le P-1\} \cup \{b \mid b = qPd, 0 \le q \le Q-1\}$$

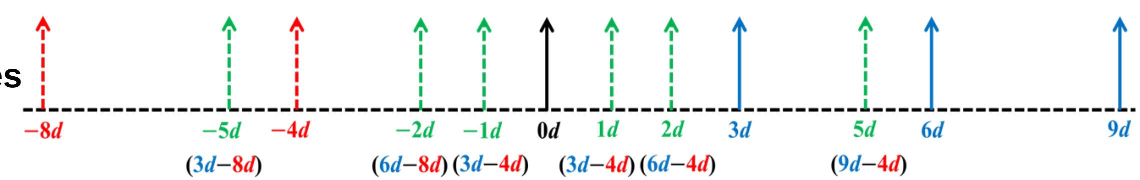




Disparity

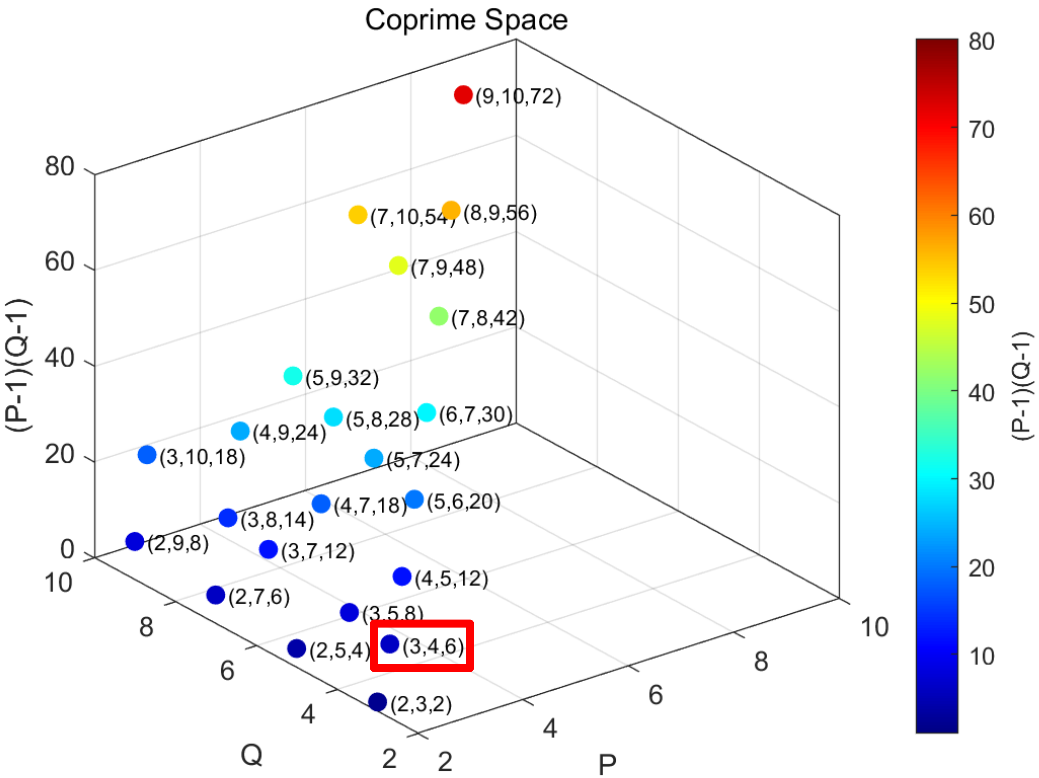
$$\mathbf{V} = \{b \mid b = (qP - pQ)d, 0 \le p \le P-1, 0 \le q \le Q-1\}$$

Virtual array baselines



Virtual array baselines

- Coprime numbers: $(P\ ,\ Q)$
- $PQ - (P + Q - 1) = (P - 1)(Q - 1)$



Virtual array processing

- Physical measurements: $\mathbf{g} = \mathbf{A} \cdot \boldsymbol{\gamma} + \boldsymbol{\varepsilon}$

(physical) sensing matrix

reflectivity distribution

- Cross-correlation matrix: $\mathbf{C} = \mathrm{E}[\mathbf{g} \cdot \mathbf{g}^H]$

Using structure redundancy information in the Array-InSAR image

- Spatial adaptive filtering: $\mathbf{C} = \mathrm{FilterKernel}[\mathbf{g} \cdot \mathbf{g}^H]$ + regularization

Expanding the measurement dimension M of **g**

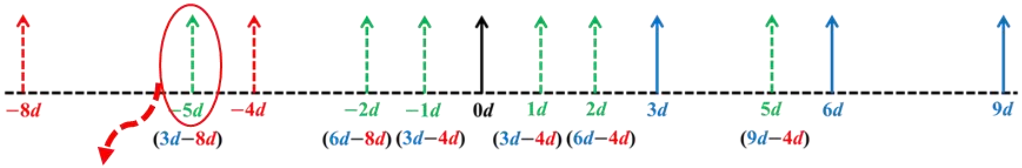
- Virtual measurements: $\mathbf{z} = \mathrm{vec}\left(\sqrt{\mathbf{C}}\right) = [\mathbf{A}^* \odot \mathbf{A}]\alpha + e$

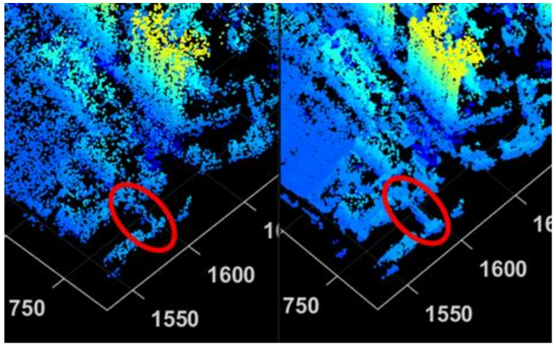
(virtual) sensing matrix

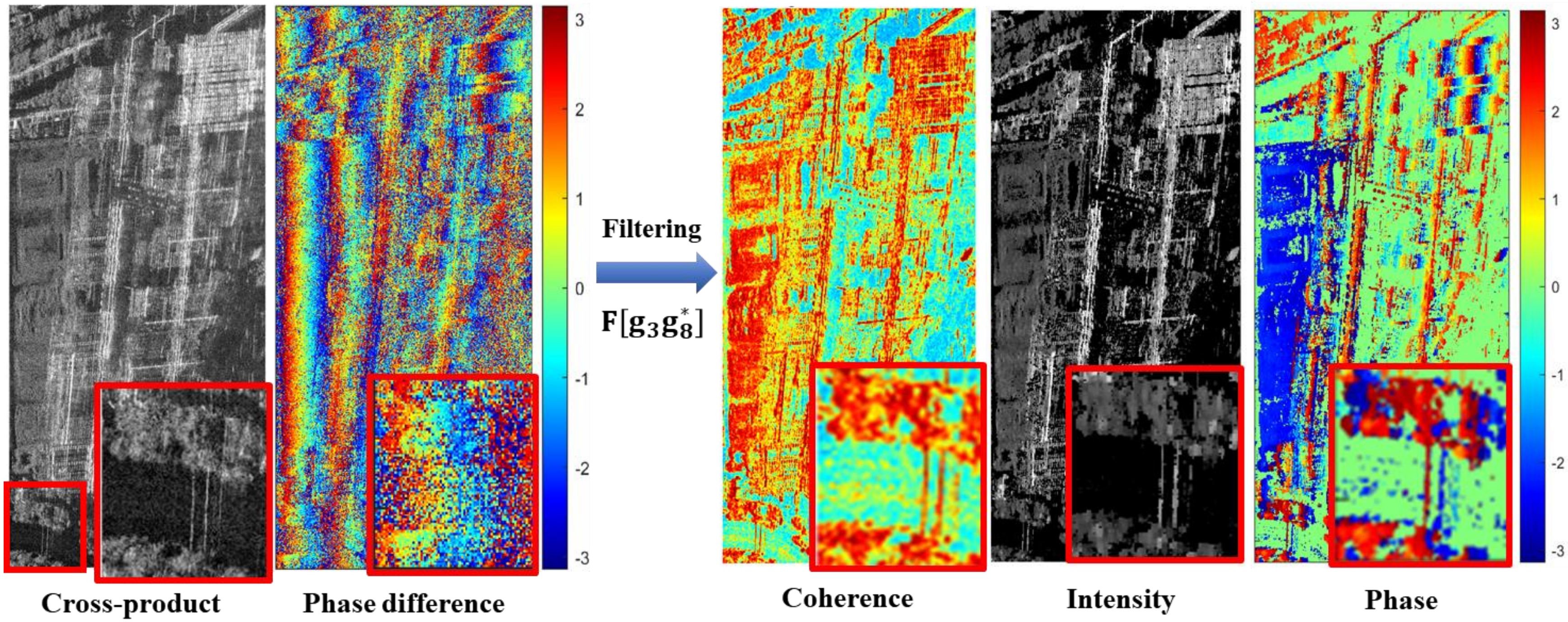
reflectivity amplitude

Spatial adaptive filtering [Ren, 2019]

- Disparity reconstruction

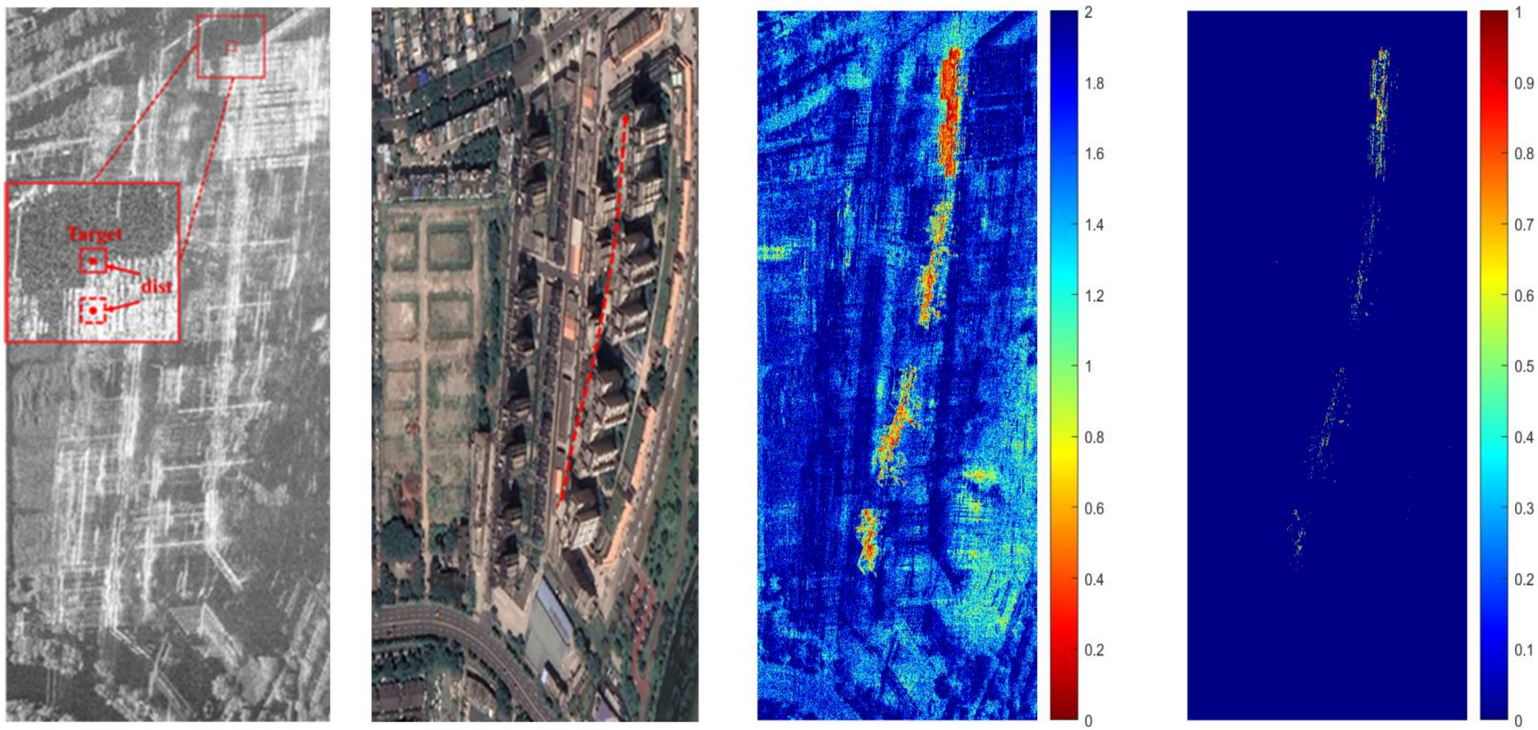






Spatial adaptive filtering

- Using spatial redundancy
  - To generate virtual measurements



Target patch | Optical image | Structural similarity | Filter kernel

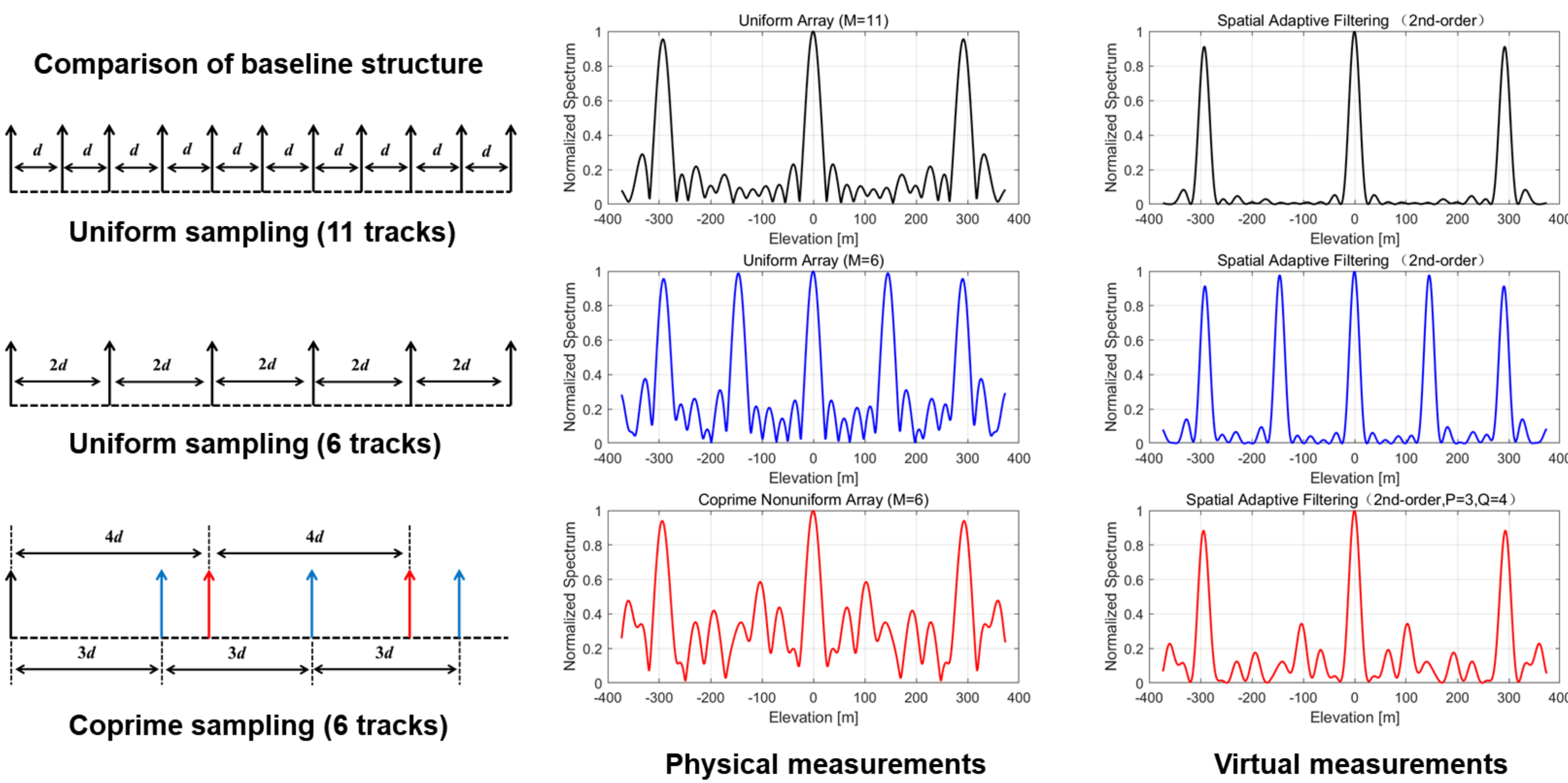
TomoSAR imaging algorithm

- OMP-BIC (very efficient, only 3 or 4 iterations per pixel)
  – Tomographic spectrum estimation: OMP
  – Model order selection: BIC

Algorithm 1:	OMP-BIC in SAR tomography
Input:	measurement vector $\mathbf{z}$, sensing matrix D,
	maximum number of scatterers $K_{max} = 3$ or 4
Initialization:	residual $\mathbf{r}_0 = \mathbf{z}$, support set $\Omega_0 = \emptyset$, iteration step $K = 0$,
	BIC list $\mathcal{B}_0$ using $2(\\|\mathbf{r}_0\\|_2^2/\sigma_\varepsilon^2) + 3K \ln M'$
While $K < K_{max}$:	
	$K = K + 1$
	Support index $\lambda_K = arg \max_{i=1,2,\cdots,L} \lvert\langle \mathbf{d}_i^H \mathbf{r}_{K-1} \rangle\rvert$
	Update support set $\Omega_K = \Omega_{K-1} \cup \{\lambda_K\}$
	Update residual $\mathbf{r}_K = \mathbf{z} - \mathbf{D}_{\Omega_K}(\mathbf{D}_{\Omega_K}^H \mathbf{D}_{\Omega_K})^{-1} \mathbf{D}_{\Omega_K}^H \mathbf{z}$
	Update BIC list $\mathcal{B}_K = \mathcal{B}_{K-1} \cup \{2(\\|\mathbf{r}_K\\|_2^2/\sigma_\varepsilon^2) + 3K \ln M'\}$
End While	
Adjust K:	$K' = arg \min_{i=1,2,\cdots,K_{max}} \mathcal{B}_i$
Output:	Support set $\Omega_{K'}$ (corresponding to elevation $s_1, s_2, \cdots, s_L$)
	Sparse coefficient $\hat{\gamma}_{\Omega_{K'}} = \left(\mathbf{D}_{\Omega_{K'}}^H \mathbf{D}_{\Omega_{K'}}\right)^{-1} \mathbf{D}_{\Omega_{K'}}^H \mathbf{z}$

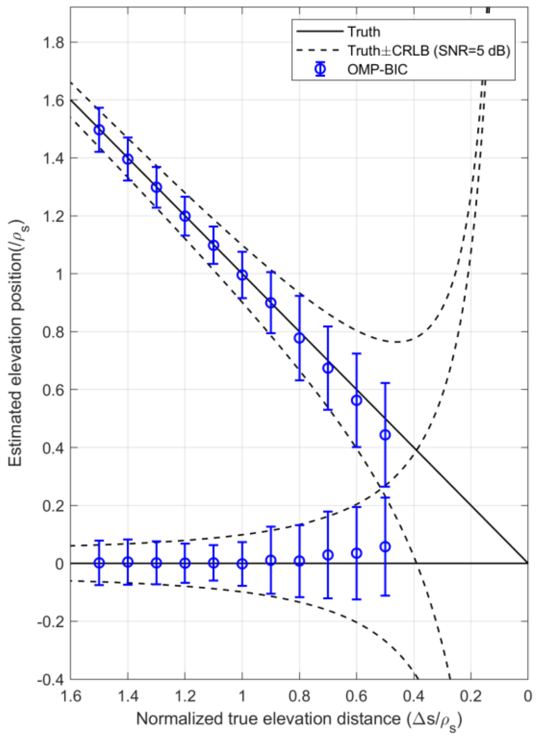
Tomographic spectrum simulation

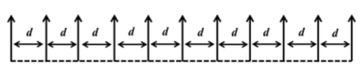
- Using coprime sampling to approximate dense uniform sampling
- Virtual measurements VS Physical measurements



CRLB simulation

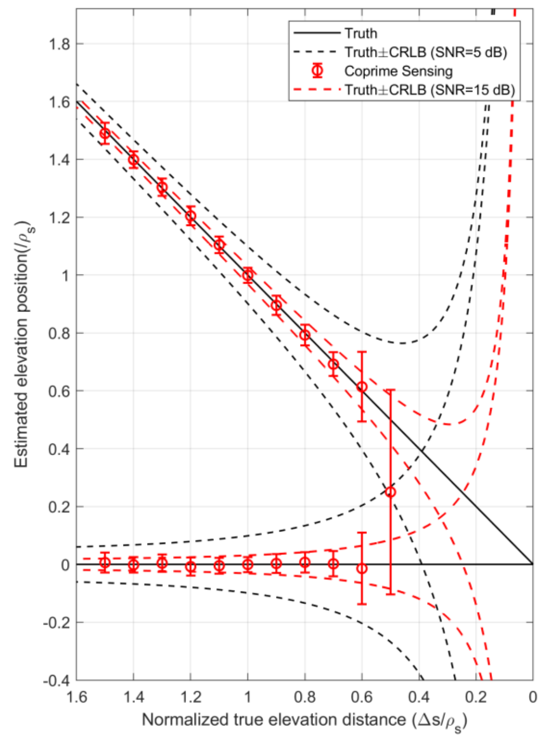
- Parameters from Array-InSAR Emei data and $SNR = 5\ \mathrm{dB}$

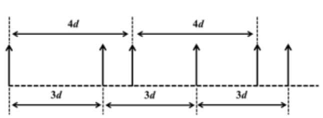




Uniform Array Baselines:11

Using Physical Measurements

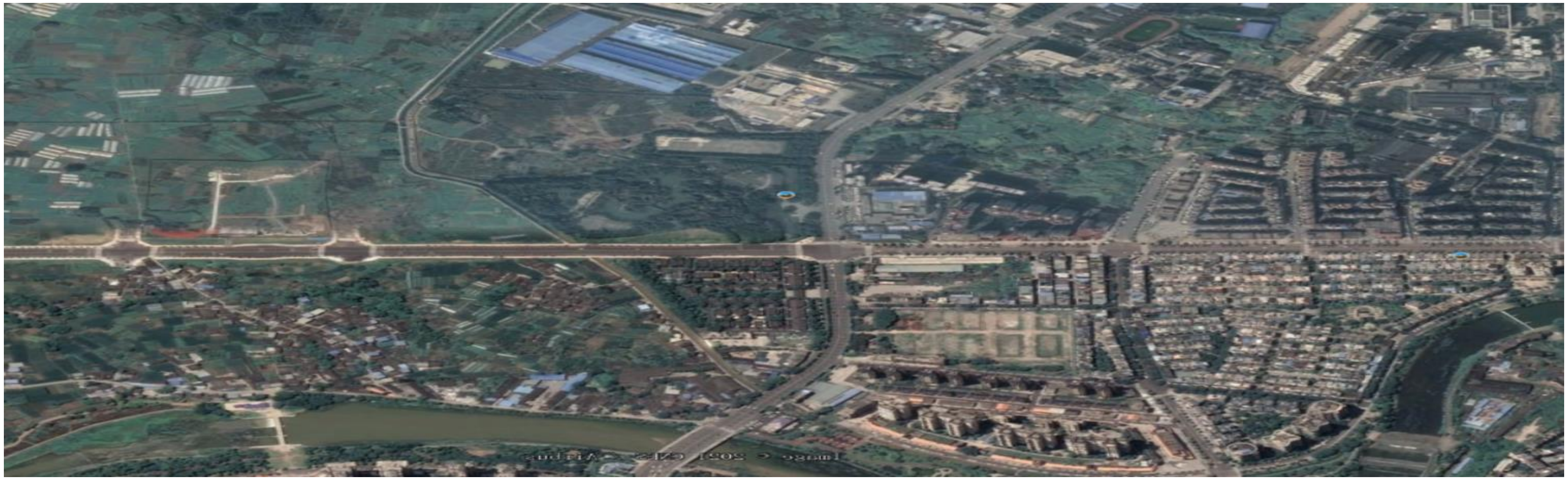




Coprime Array Baselines:6

Using Virtual Measurements (Generated by Adaptive Filter)

Experiments

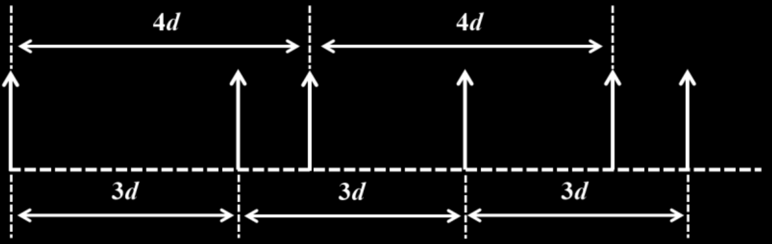


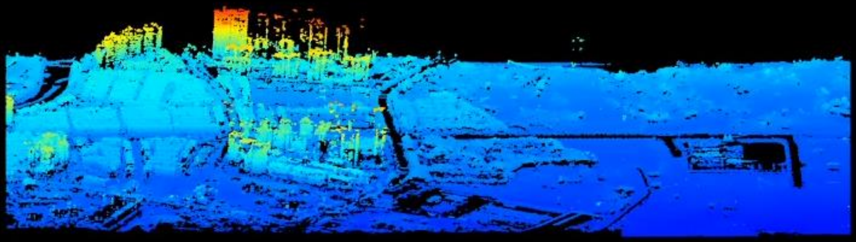
Optical image of Emei from Google Earth

AIRCAS X-band Array-InSAR data： 16384 × 5040 × 11 SLC image

Coprime Array Baselines:6
(P,Q) = (3,4)

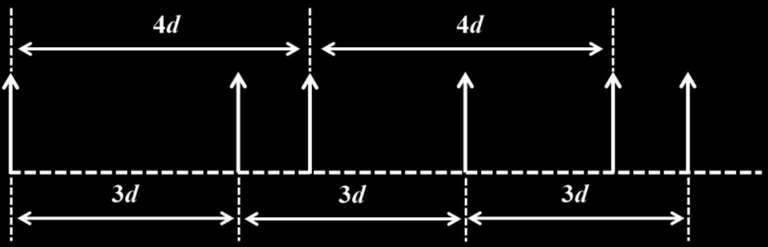
Using Virtual Measurements
(Generated by Adaptive Filter)

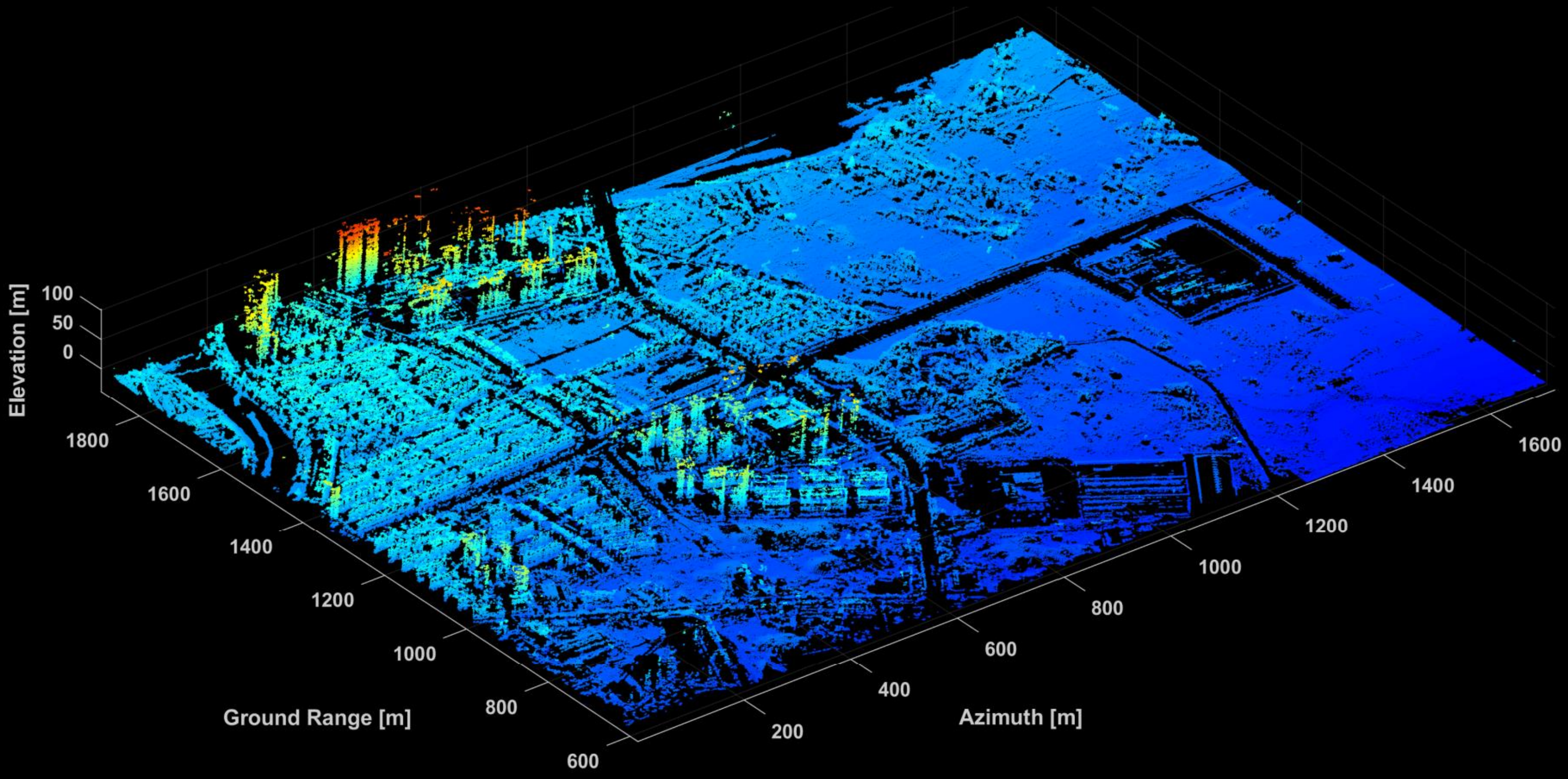




Coprime Array Baselines:6
(P,Q) = (3,4)

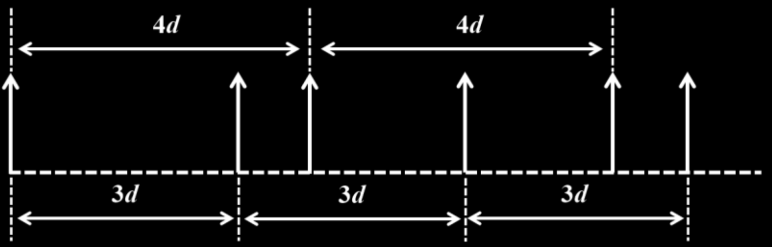
Using Virtual Measurements
(Generated by Adaptive Filter)

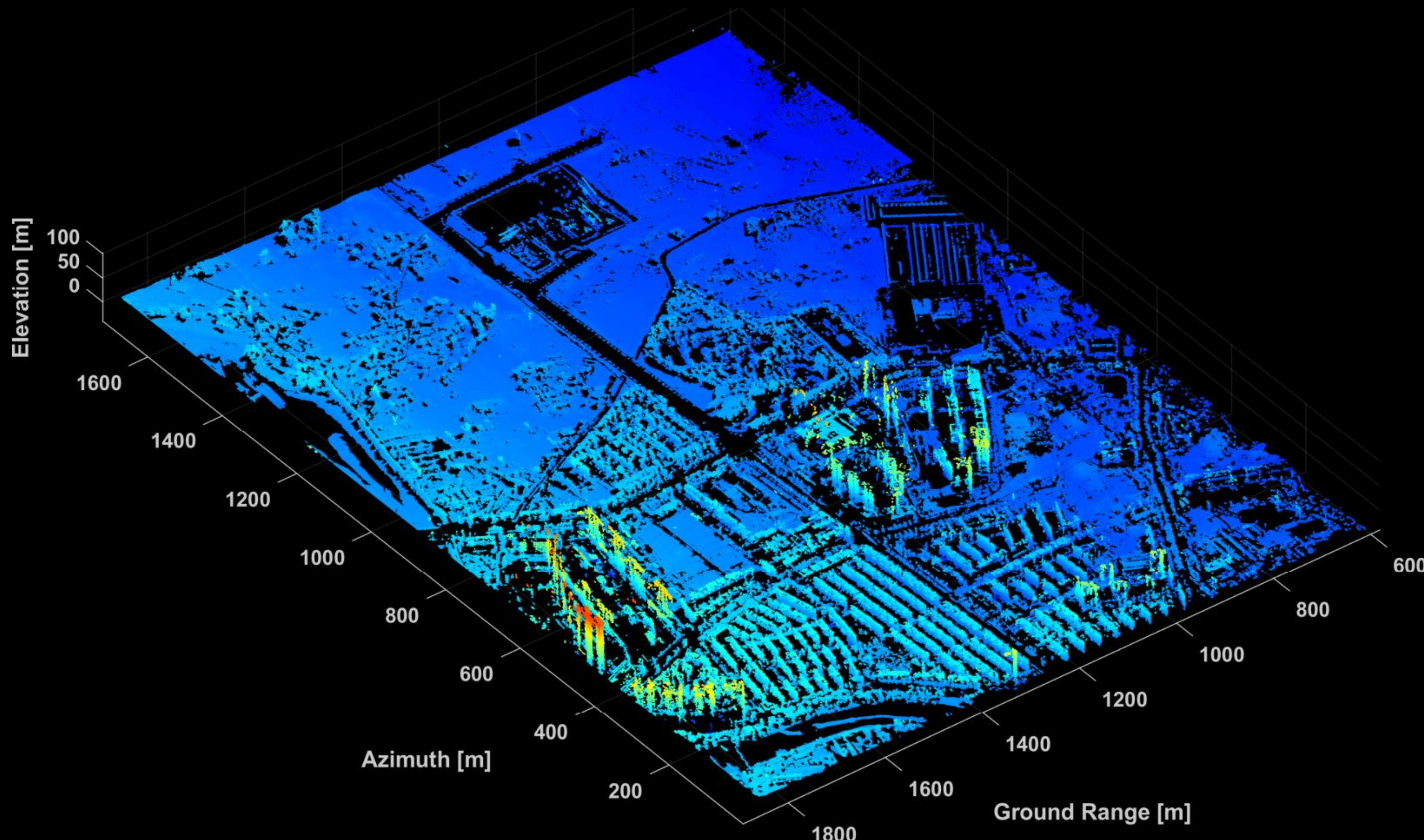




Coprime Array Baselines:6
(P,Q) = (3,4)

Using Virtual Measurements
(Generated by Adaptive Filter)

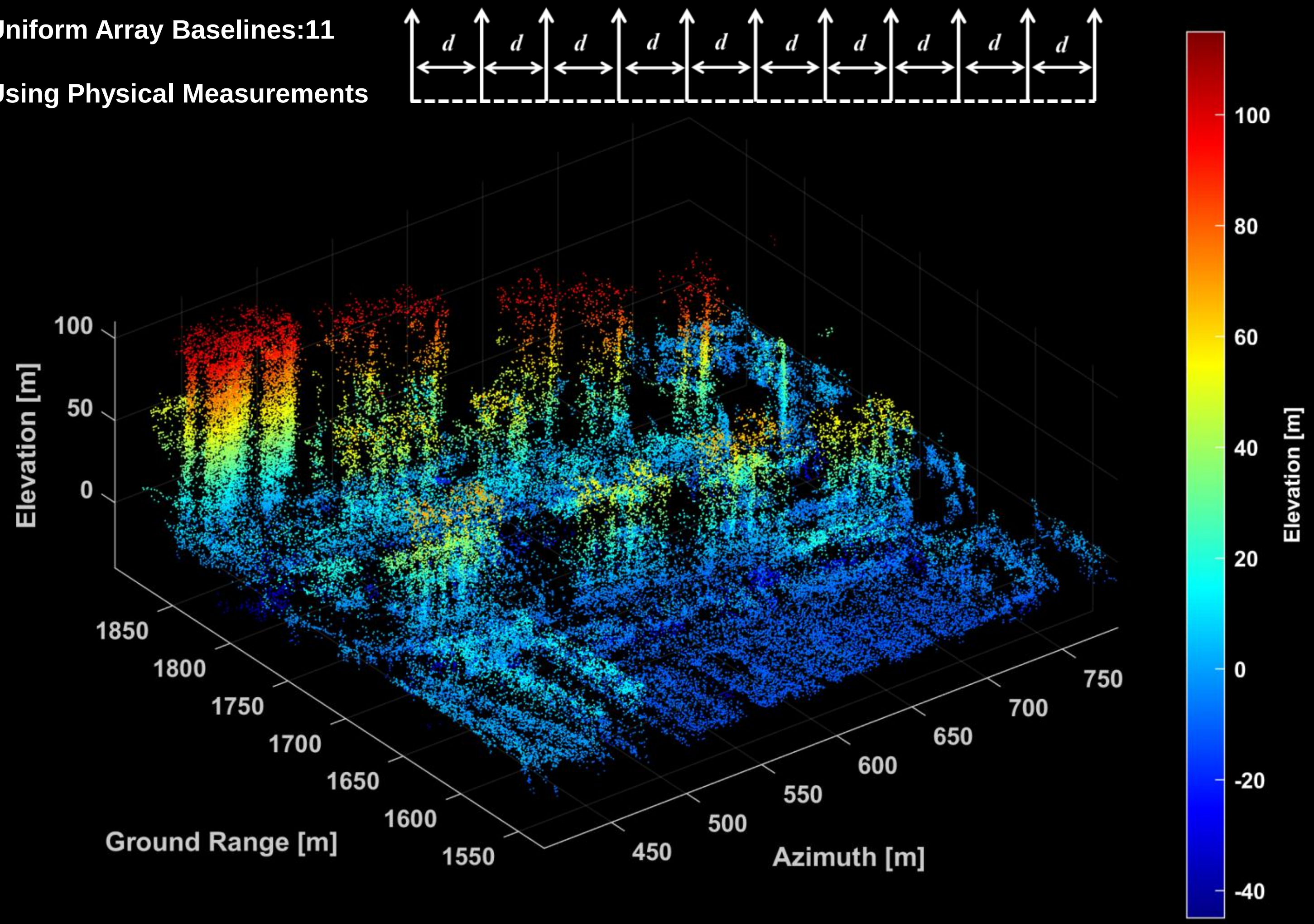




Ablation study: *Case* 1

Uniform Array Baselines:11

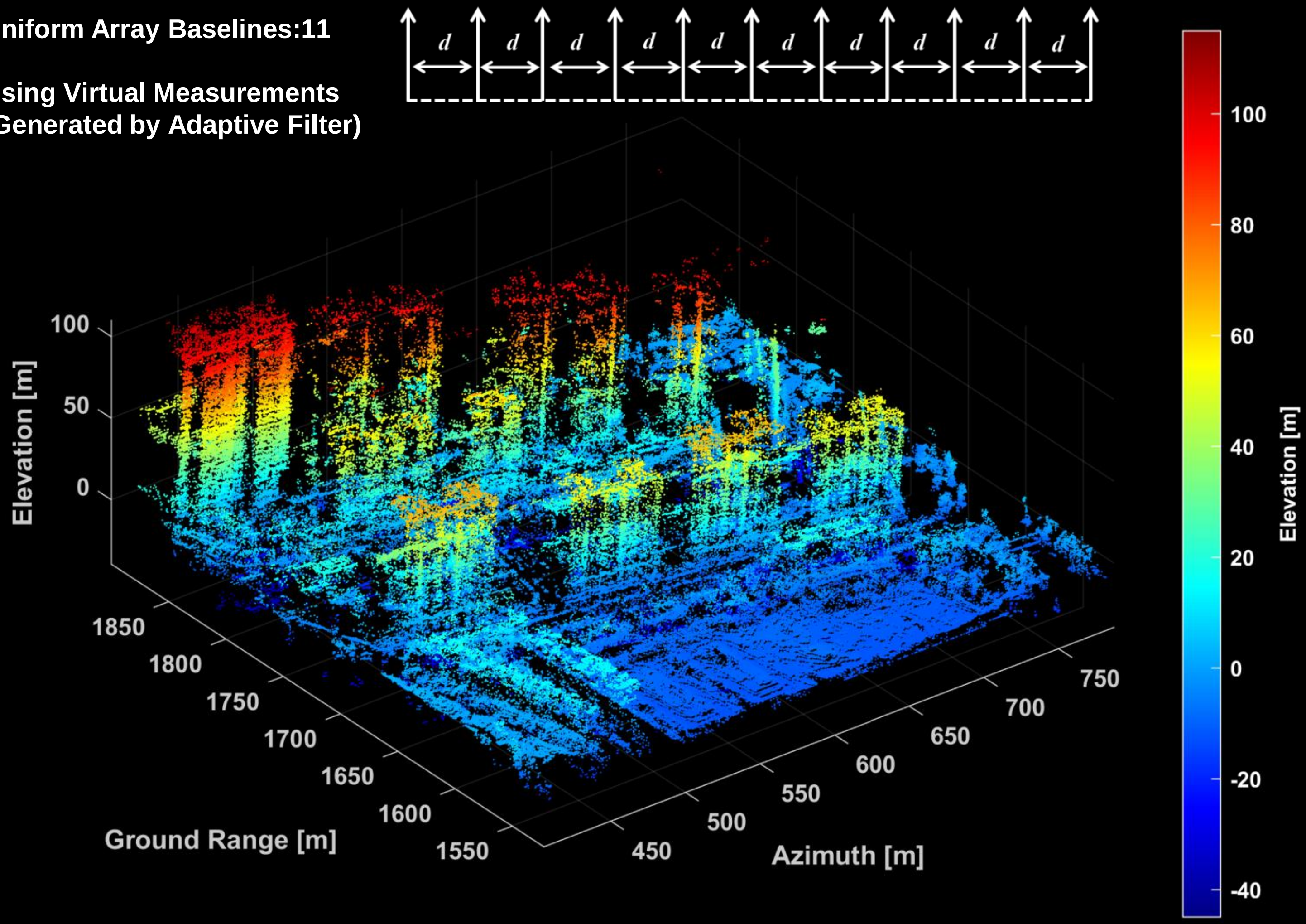
Using Physical Measurements



Ablation study: *Case* 2

Uniform Array Baselines:11

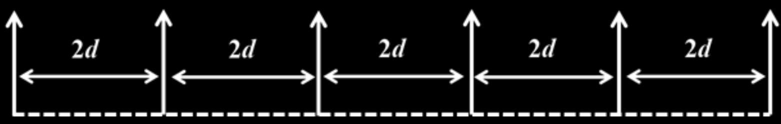
Using Virtual Measurements (Generated by Adaptive Filter)

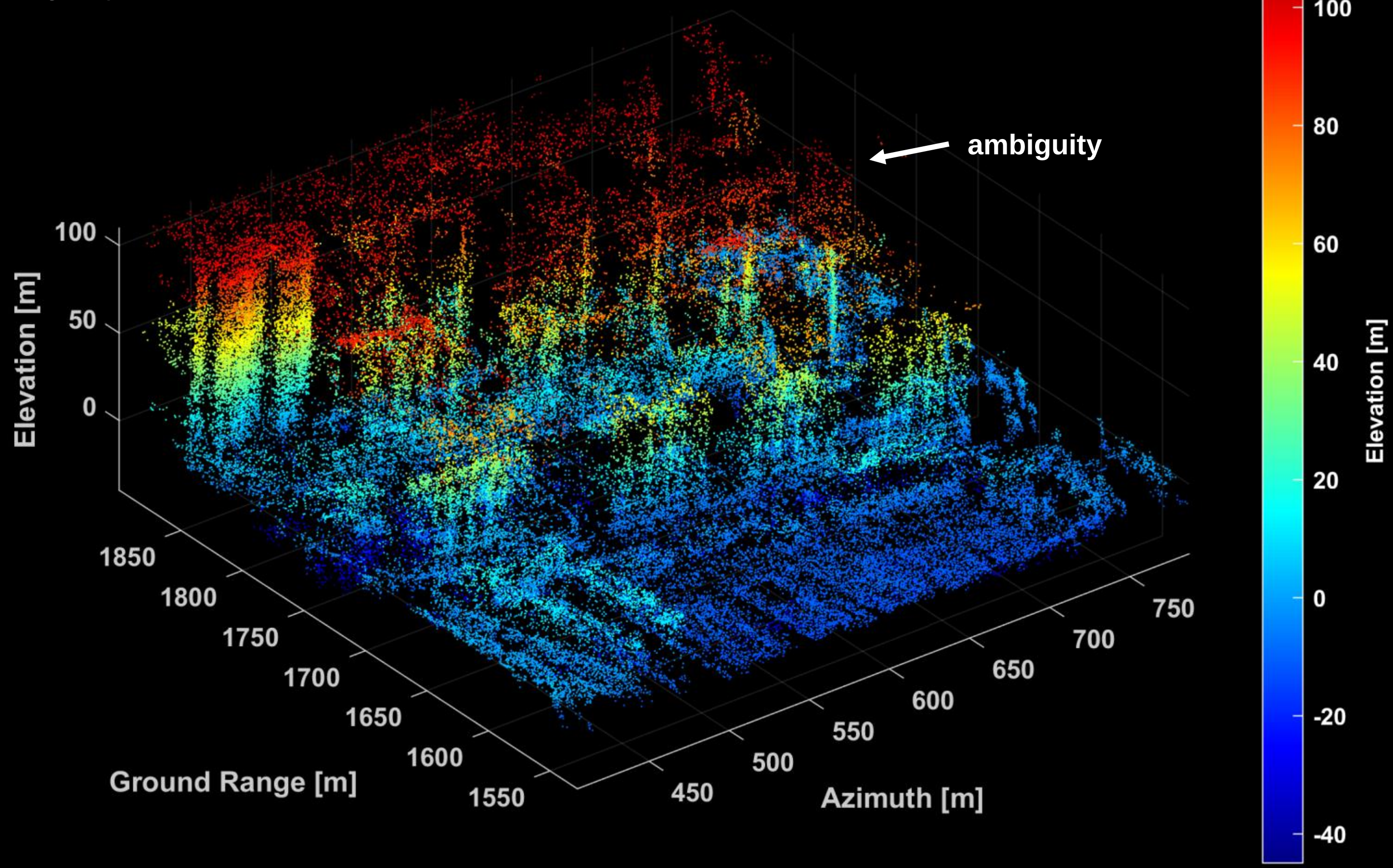


Ablation study: *Case 3*

Uniform Array Baselines:6

Using Physical Measurements

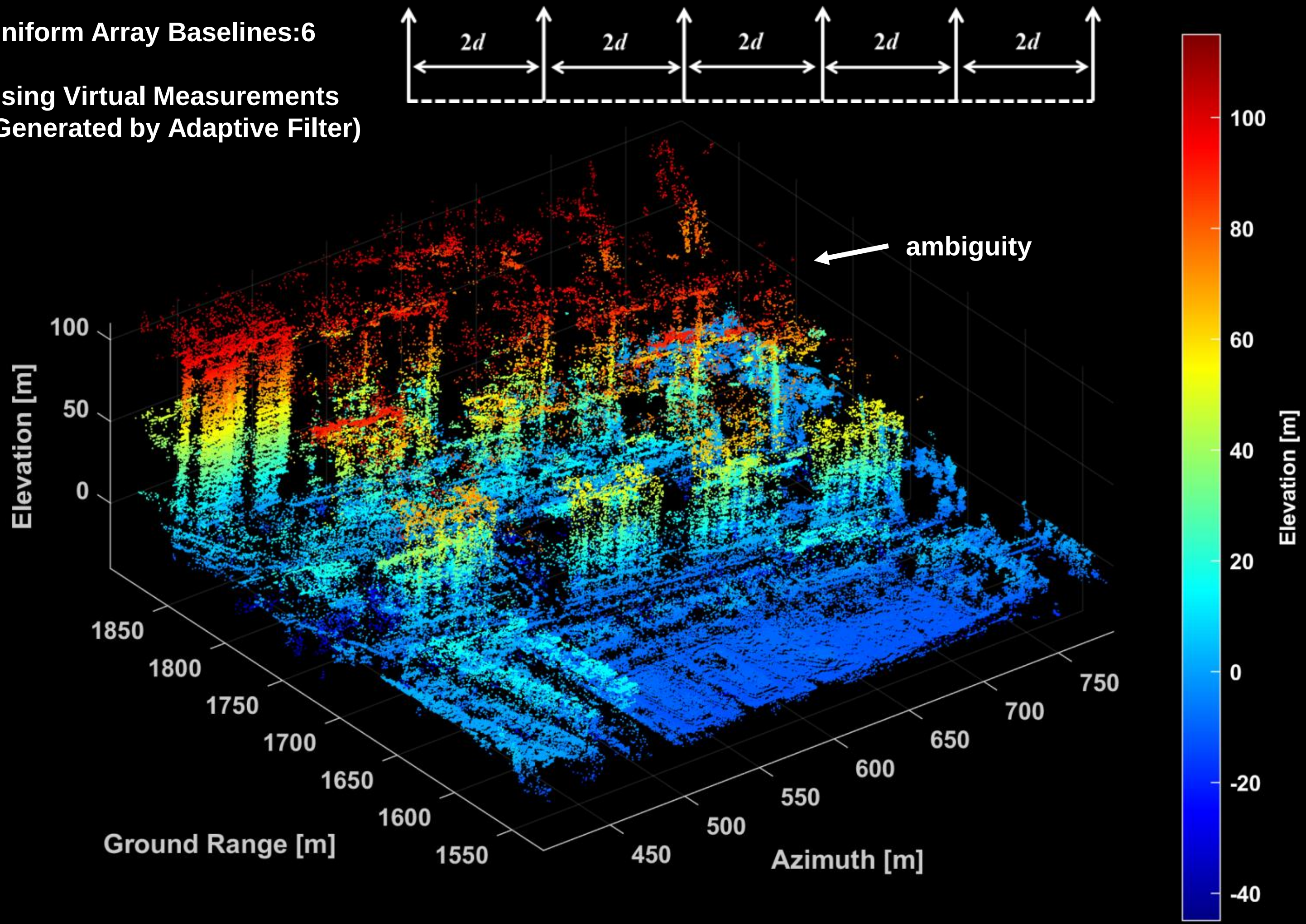




Ablation study: *Case* 4

Uniform Array Baselines:6

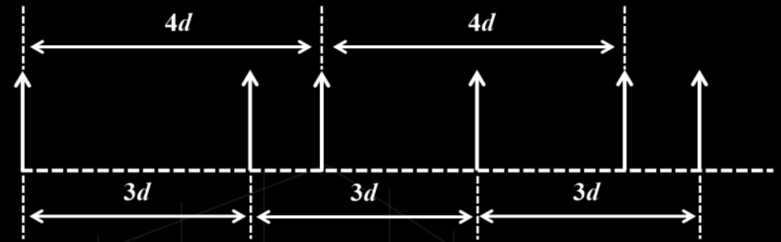
Using Virtual Measurements (Generated by Adaptive Filter)

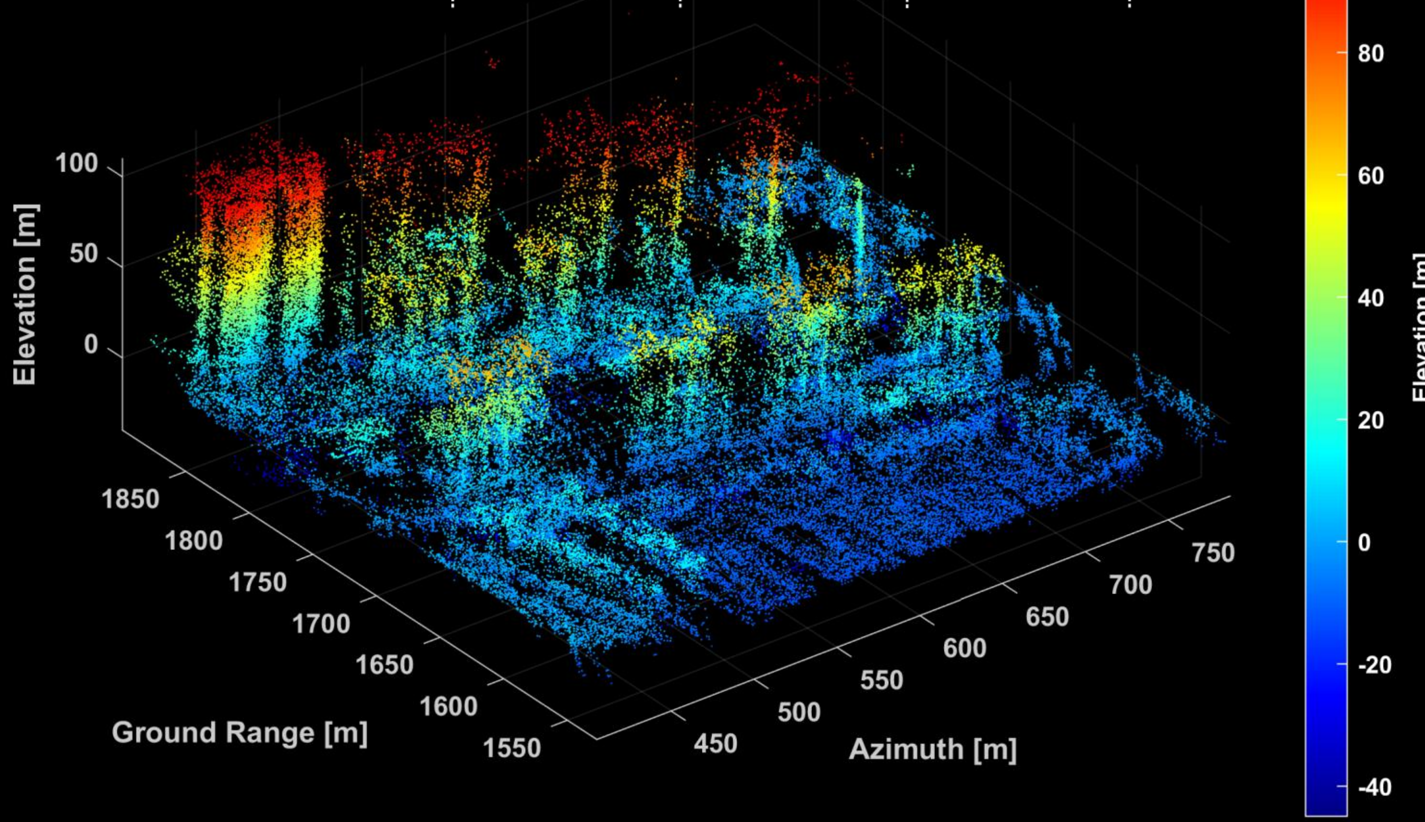


Ablation study: *Case* 5

Coprime Array Baselines:6
(P,Q) = (3,4)

Using Physical Measurements

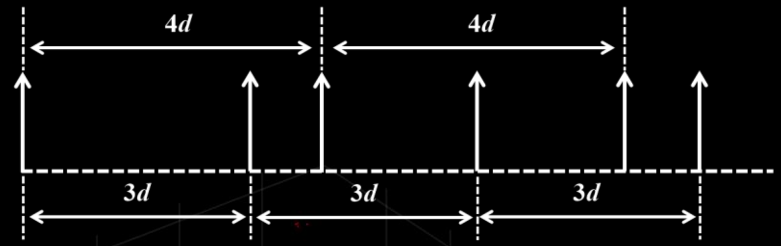


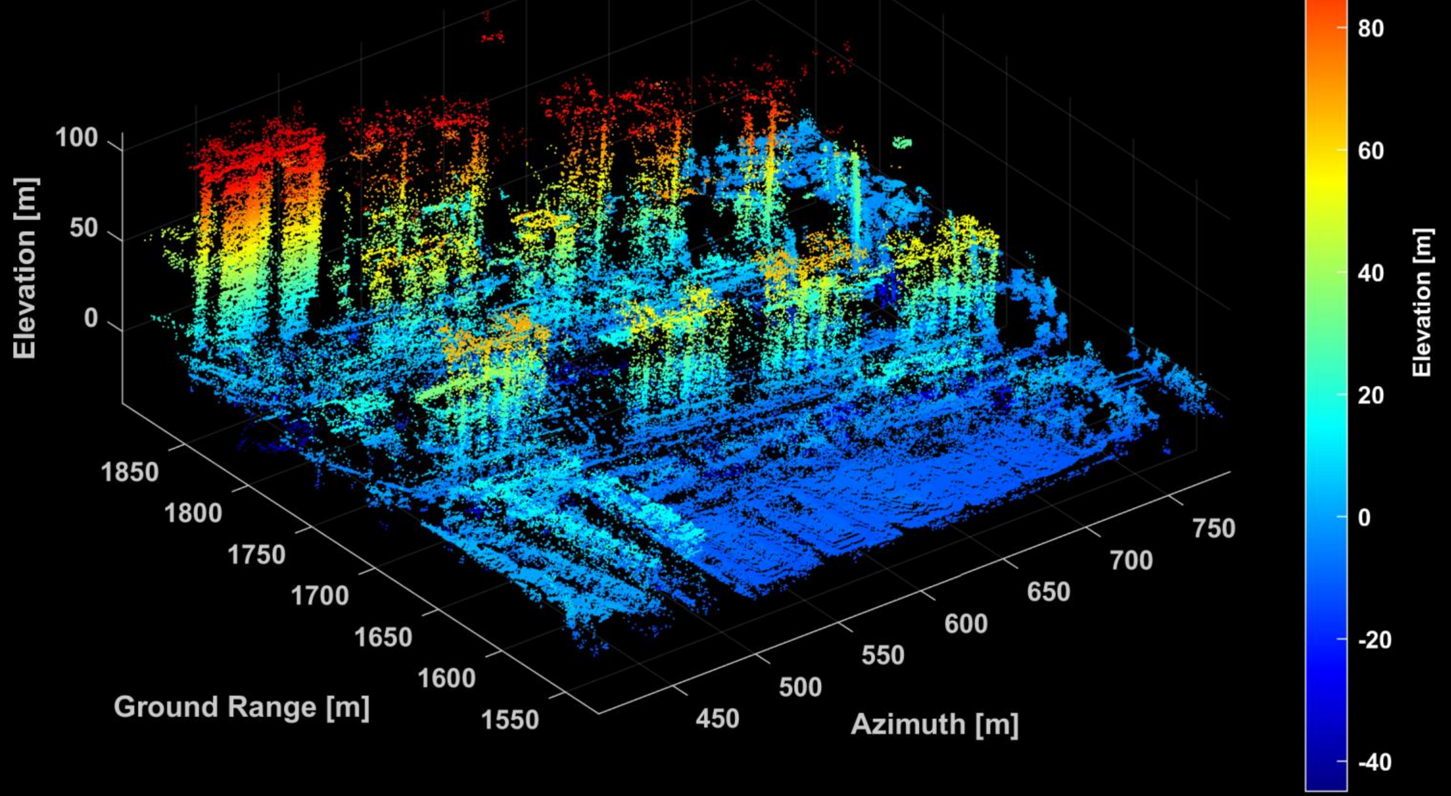


Ablation study: *Case* 6

Coprime Array Baselines:6
(P,Q) = (3,4)

Using Virtual Measurements
(Generated by Adaptive Filter)



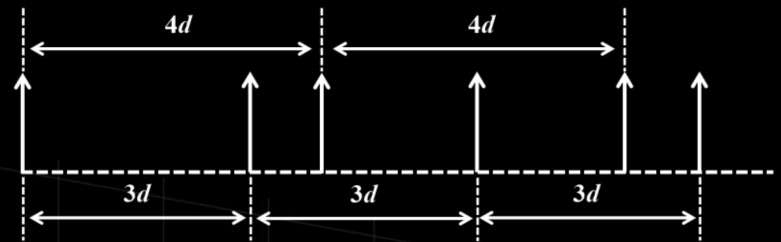


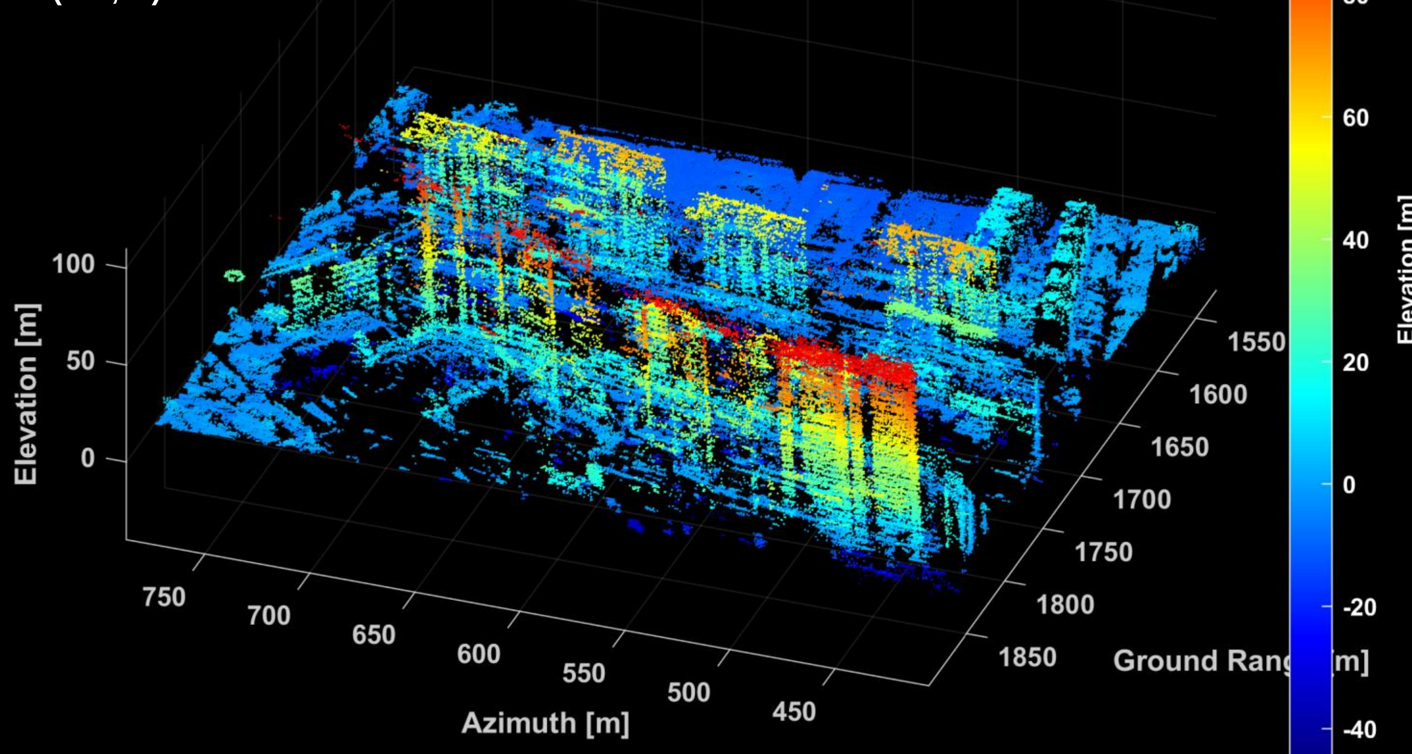
Ablation study: *Case* 6

Coprime Array Baselines:6
(P,Q) = (3,4)

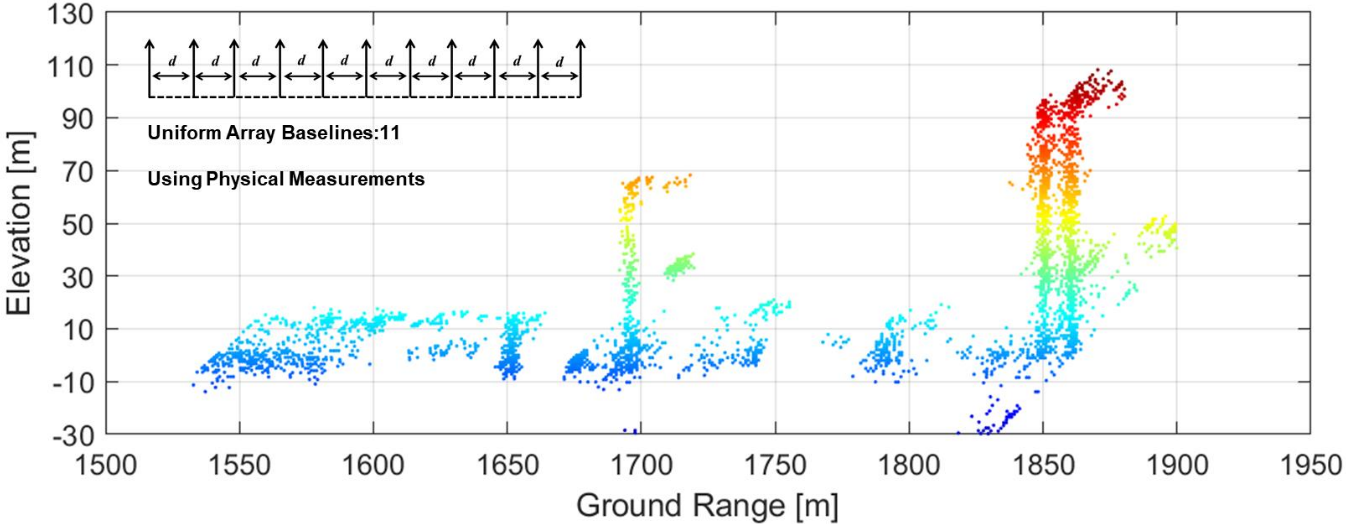
Using Virtual Measurements
(Generated by Adaptive Filter)

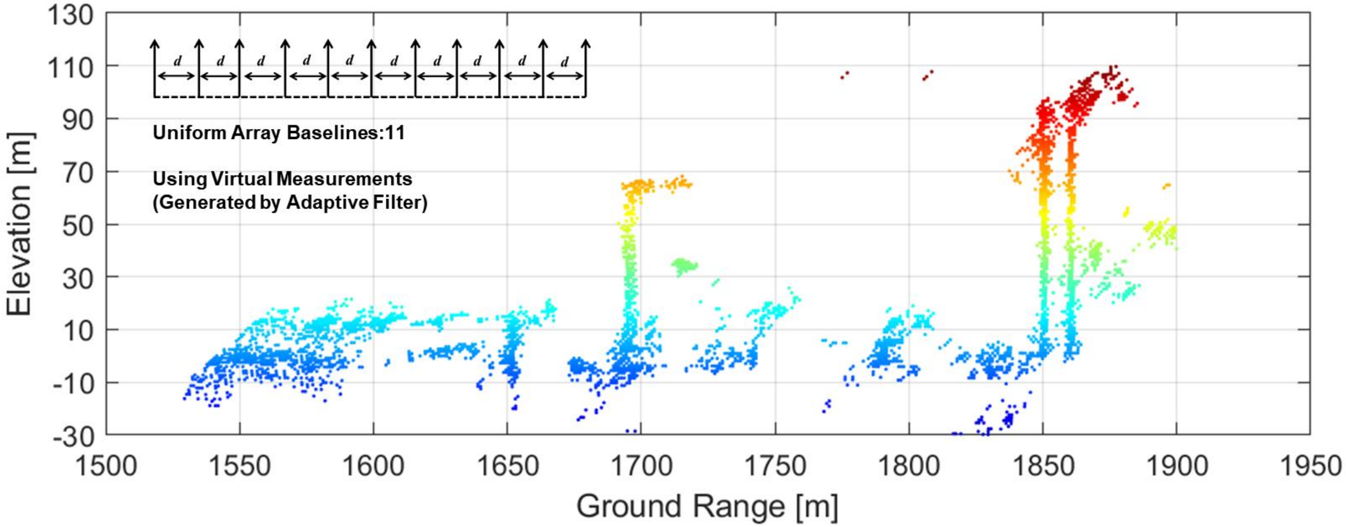
View(200,30)



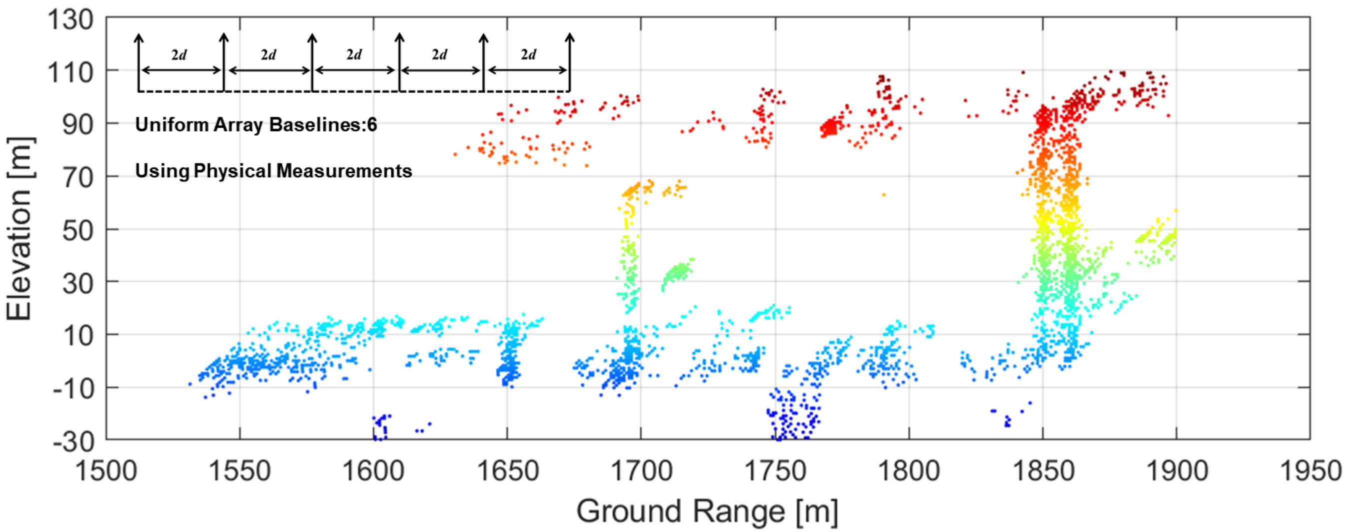


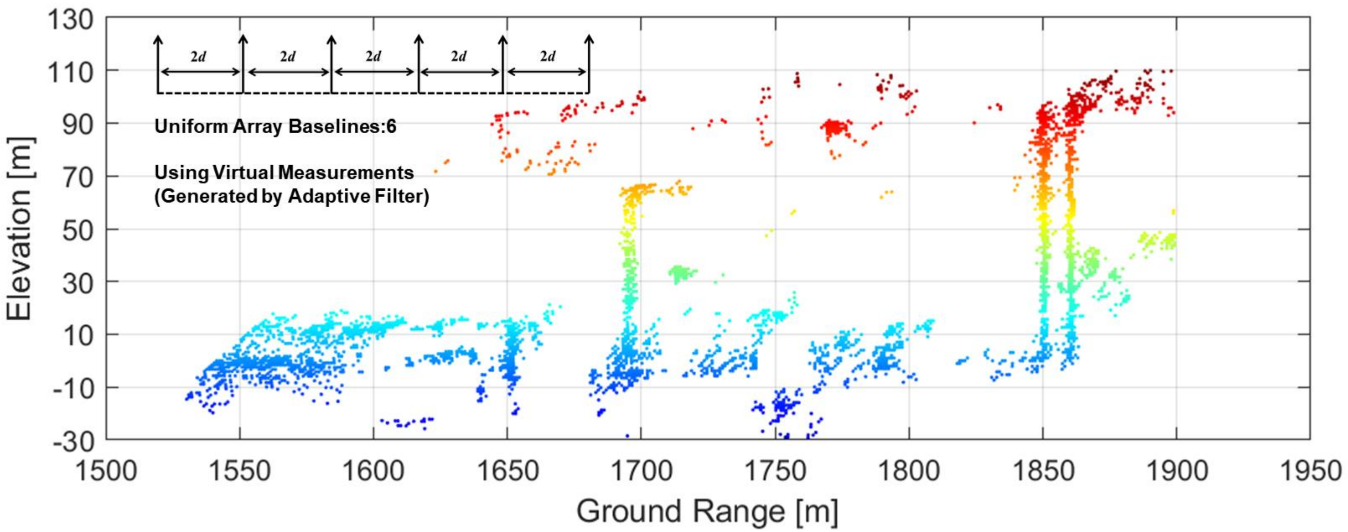
Profiles: *Case* 1 and *Case* 2



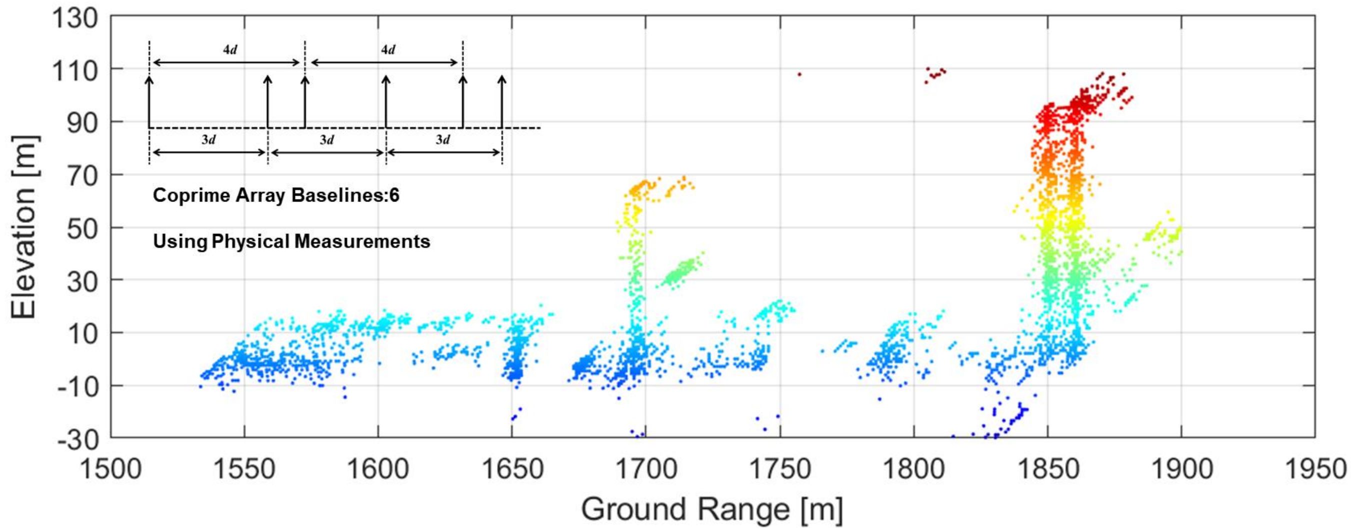


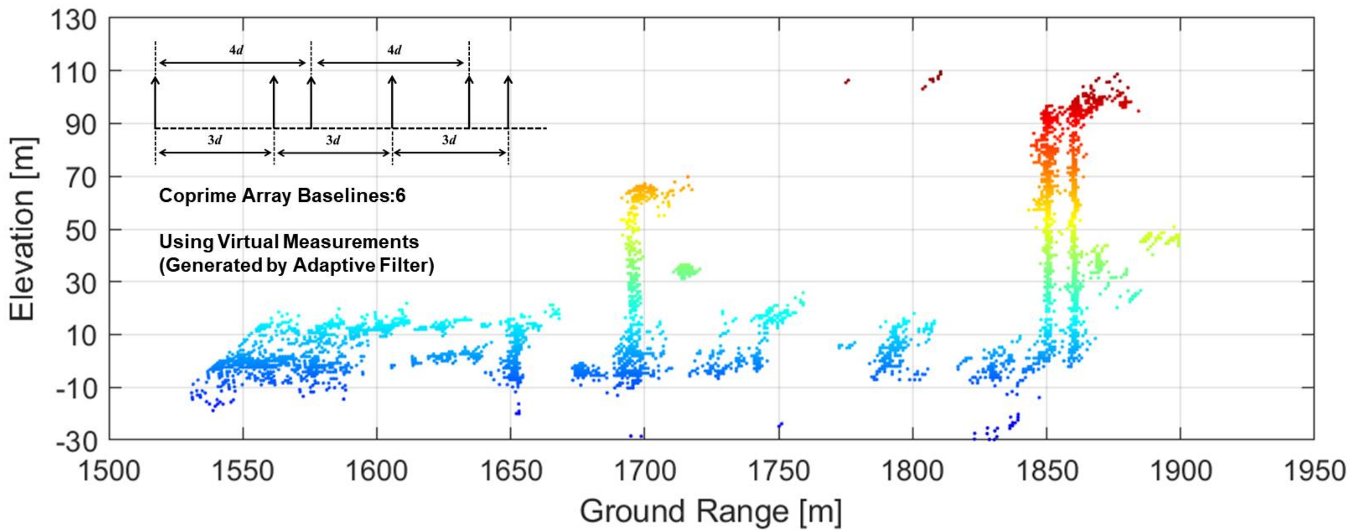
Profiles: *Case* 3 and *Case* 4





Profiles: *Case* 5 and *Case* 6



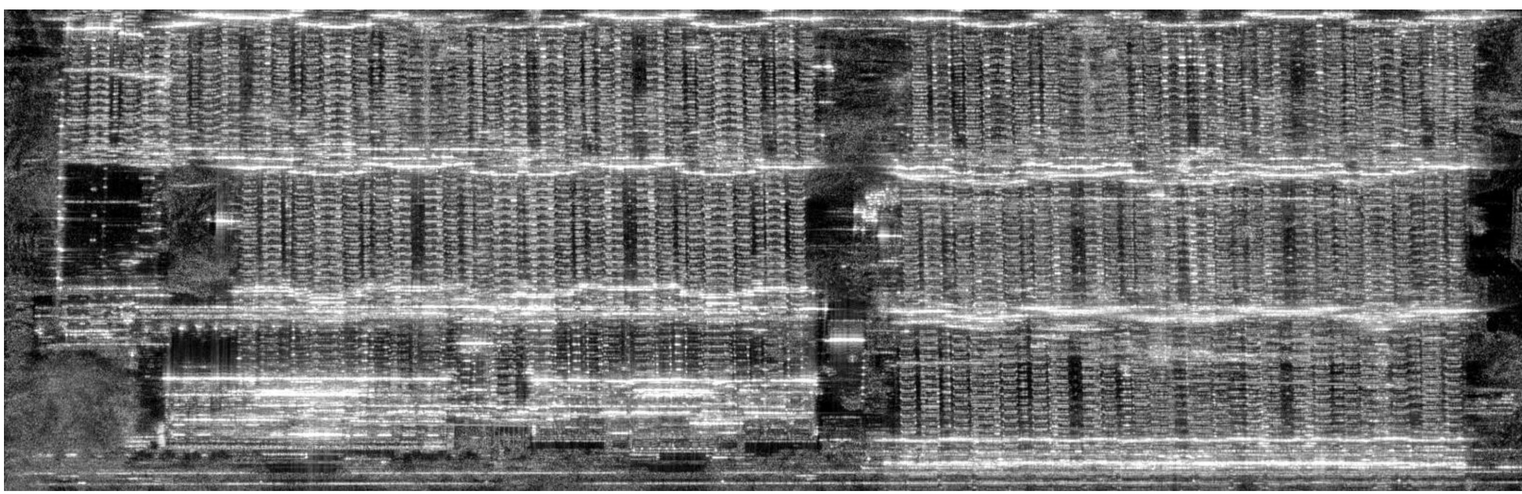


Ablation study:

- *Virtual measurements* VS Physical measurements:
  - A higher SNR
  - More degrees of freedom
- *Coprime baselines* VS Uniform baselines:
  - Reducing the number of required acquisitions via virtual baselines
  - A larger elevation measurement range

Experiments

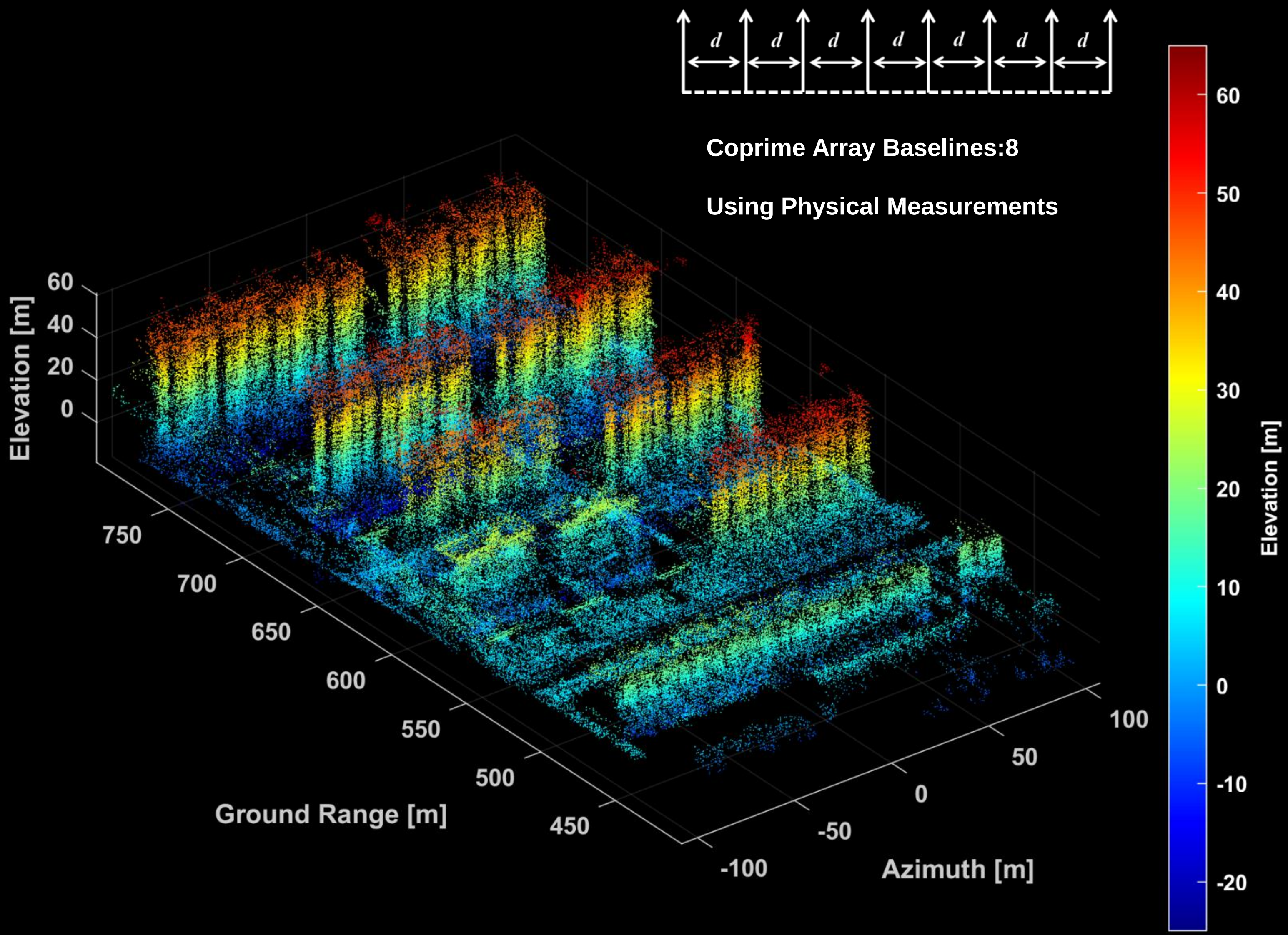
Yuncheng： Optical image from google earth

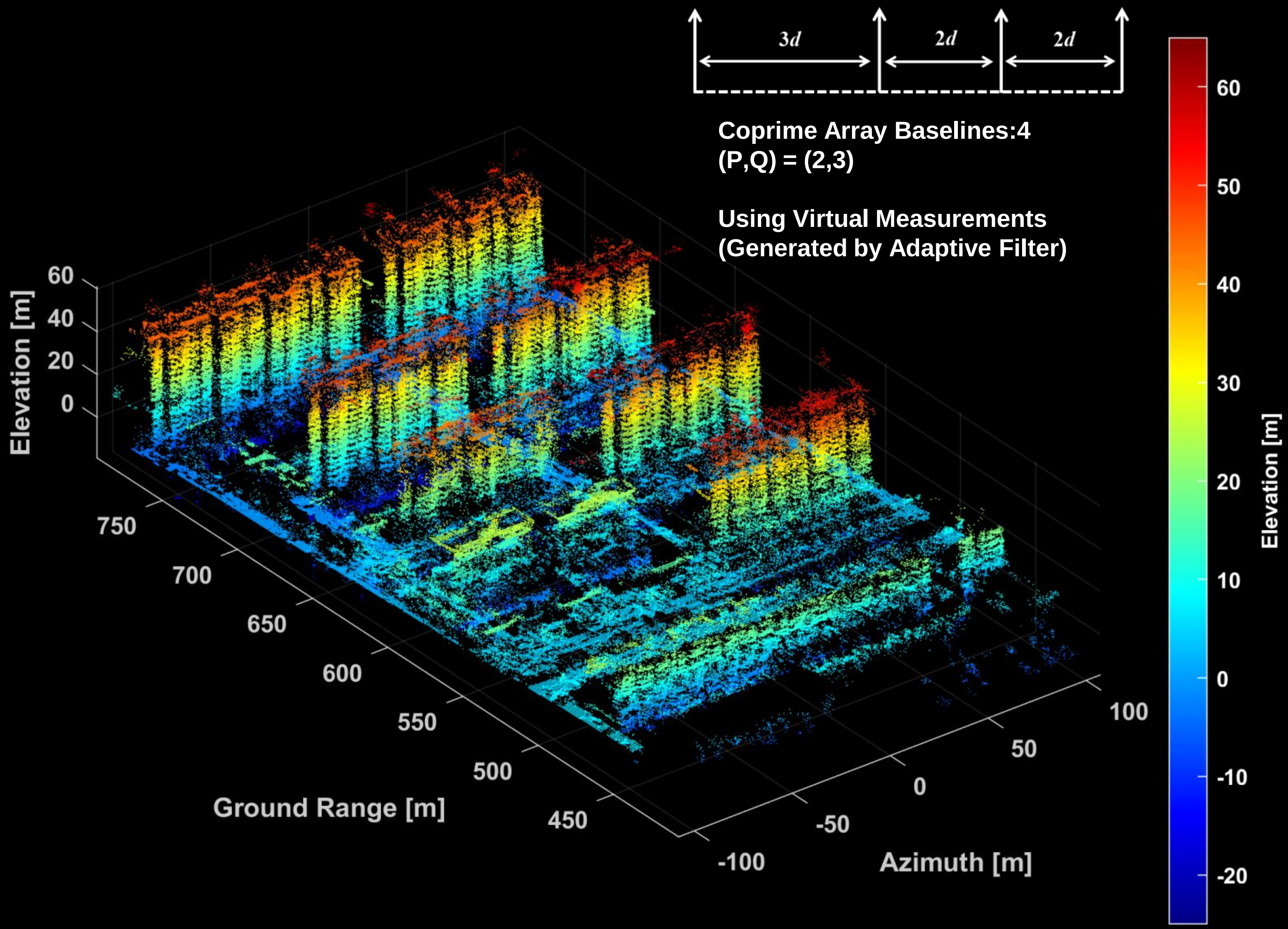


Range

Azimuth

AIRCAS Ku band Array-InSAR： 3110 × 1220 × 8 SLC image

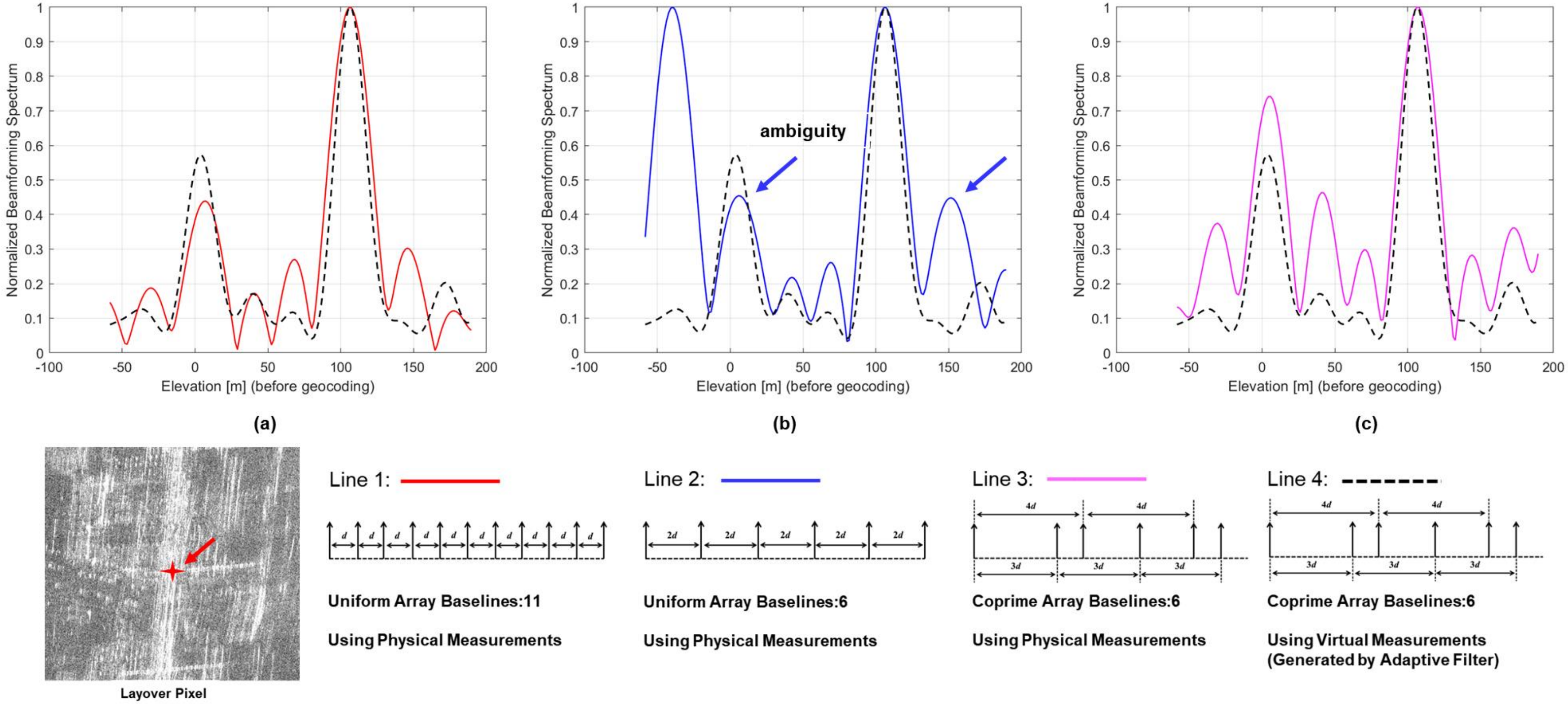

d
Coprime Array Baselines:8
Using Physical Measurements
Elevation [m]
60
40
20
0
750
700
650
600
550
500
450
Ground Range [m]
-100
-50
0
50
100
Azimuth [m]
Elevation [m]
60
50
40
30
20
10
0
-10
-20



3d
2d
2d
Coprime Array Baselines:4
(P,Q) = (2,3)
Using Virtual Measurements
(Generated by Adaptive Filter)
Elevation [m]
60
40
20
0
750
700
650
600
550
500
450
Ground Range [m]
-100
-50
0
50
100
Azimuth [m]
60
50
40
30
20
10
0
-10
-20
Elevation [m]

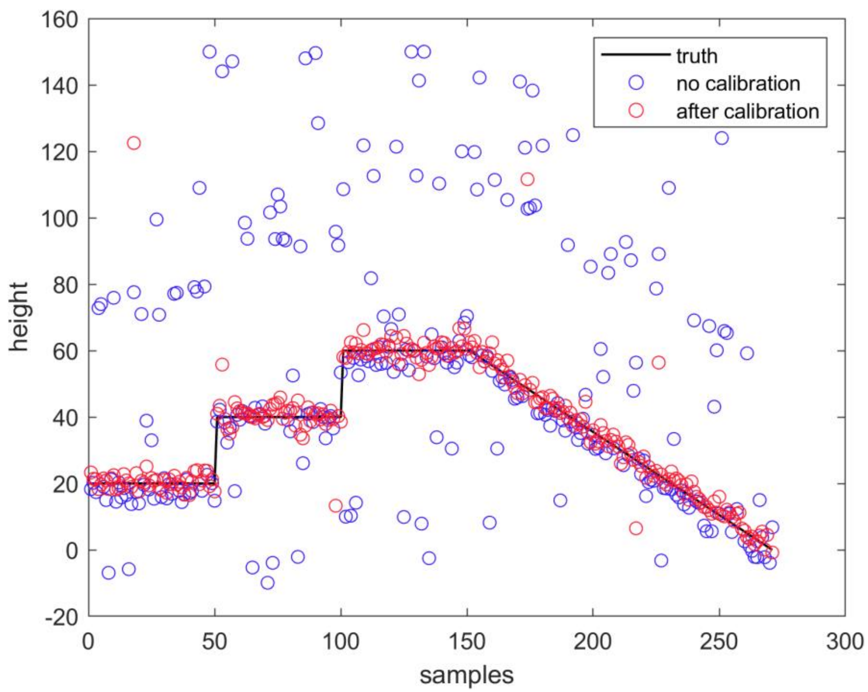
Tomographic spectrum analysis

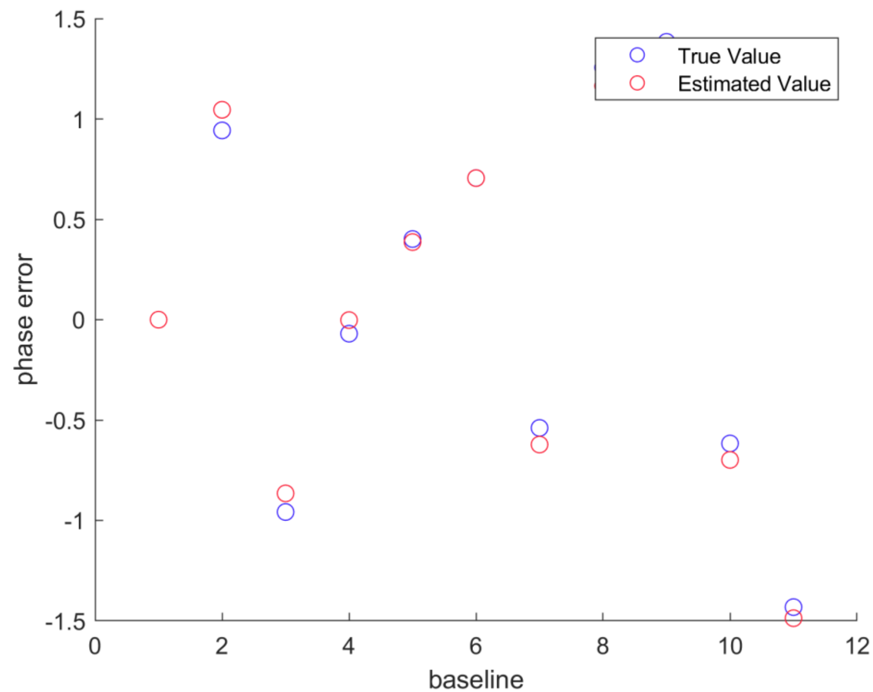
- Select a layover pixel from the Emei data
  - Solve layover with fewer physical acquisitions



Challenges

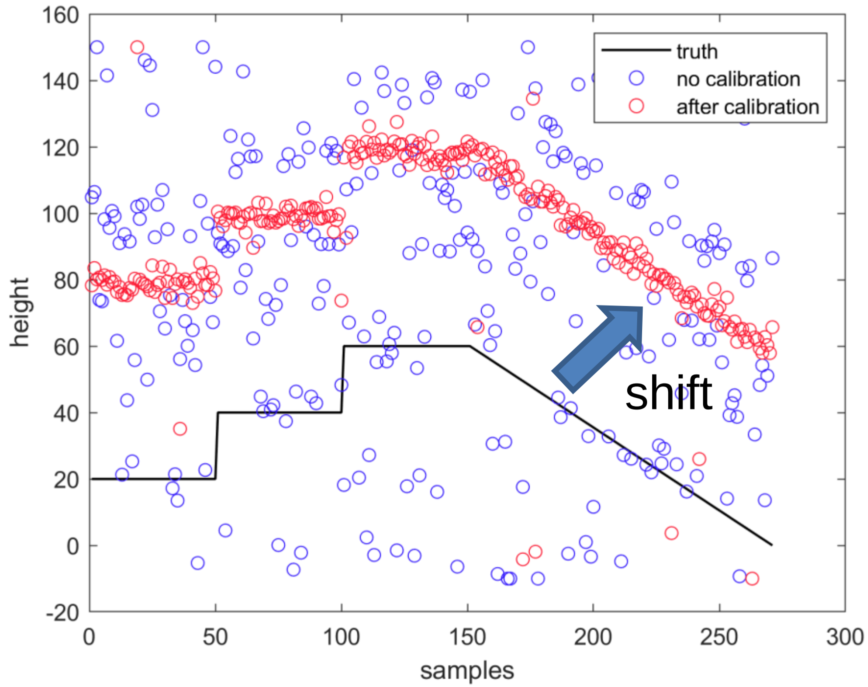
- Phase calibration
  - Surveying adjustment and kernel trick (work well in a “pi”)

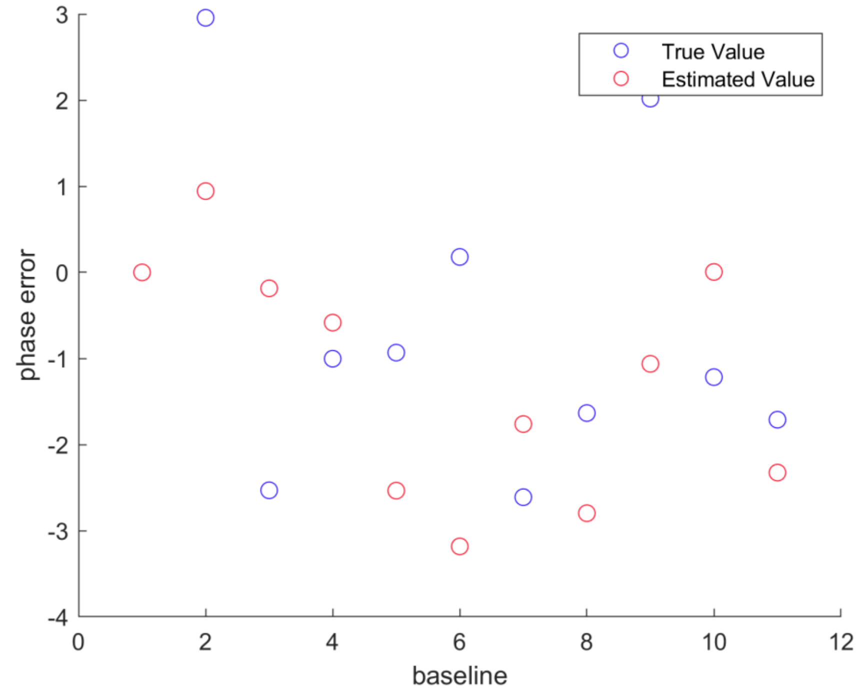




Challenges

- Phase calibration
  - When the phase error exceeds range (-pi/2, pi/2)



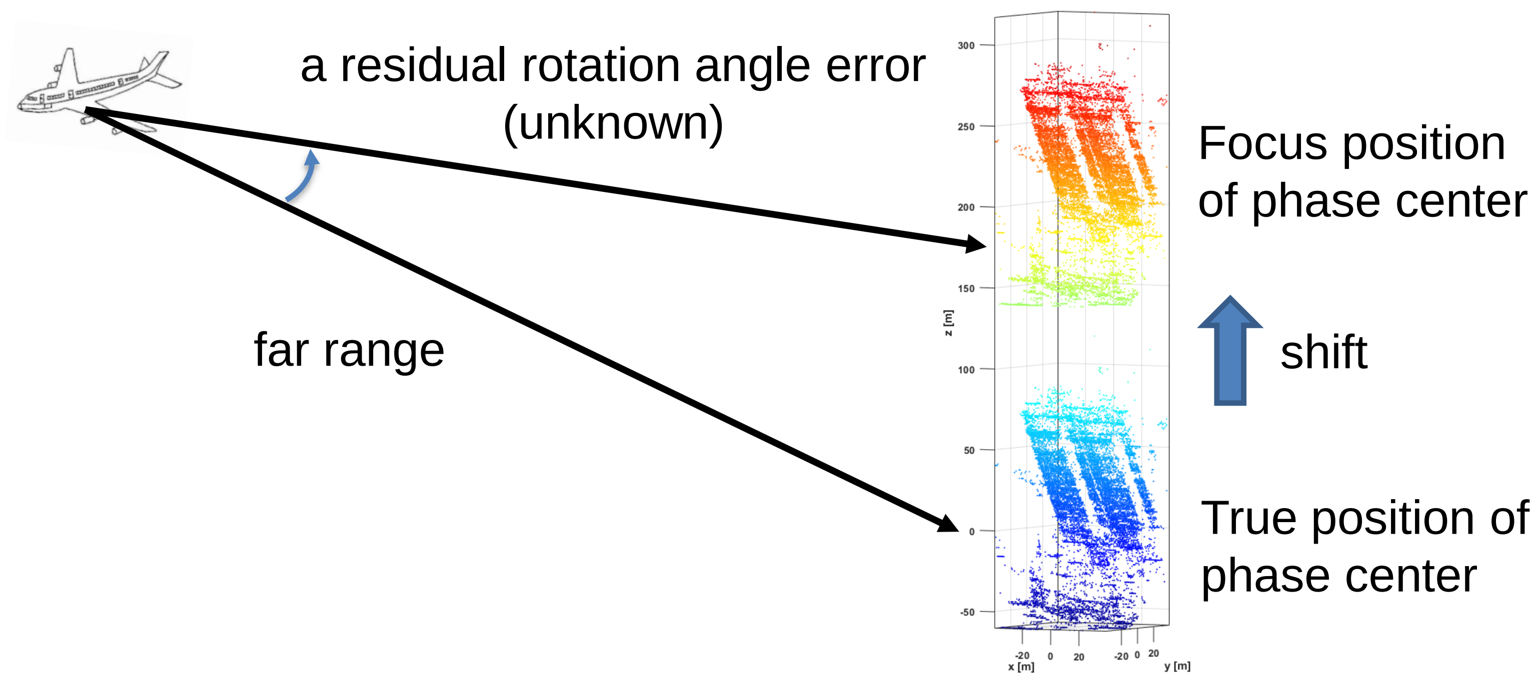


Convergence to a wrong phase center interval after more than 50 iterations

Miscalculation

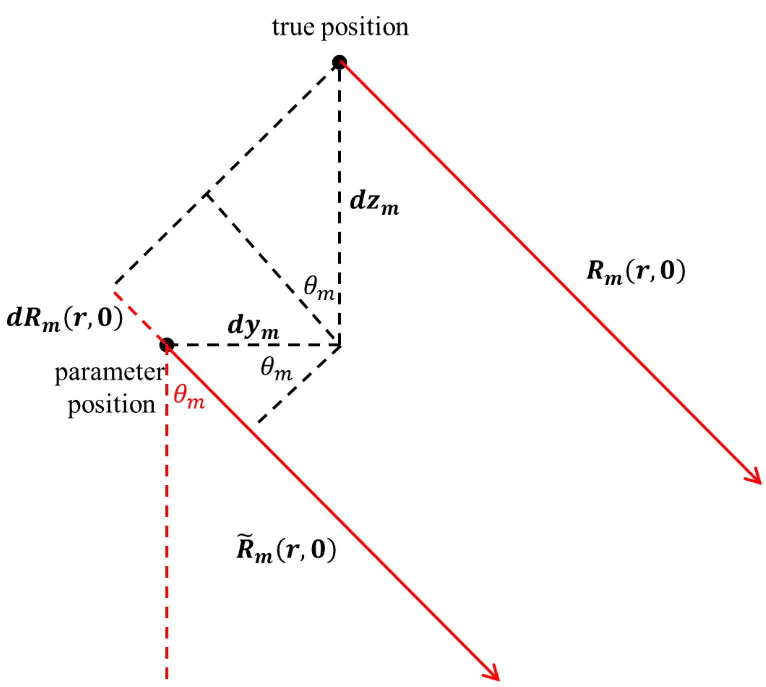
Challenges

- Phase calibration
  - Residual phase error (= an unknown rotation angle)



Challenges

- Phase calibration
  - Space-varying phase screen: $\exp(\mathrm{j}\alpha_m)$ [Tebaldini, PCDL, 2016]



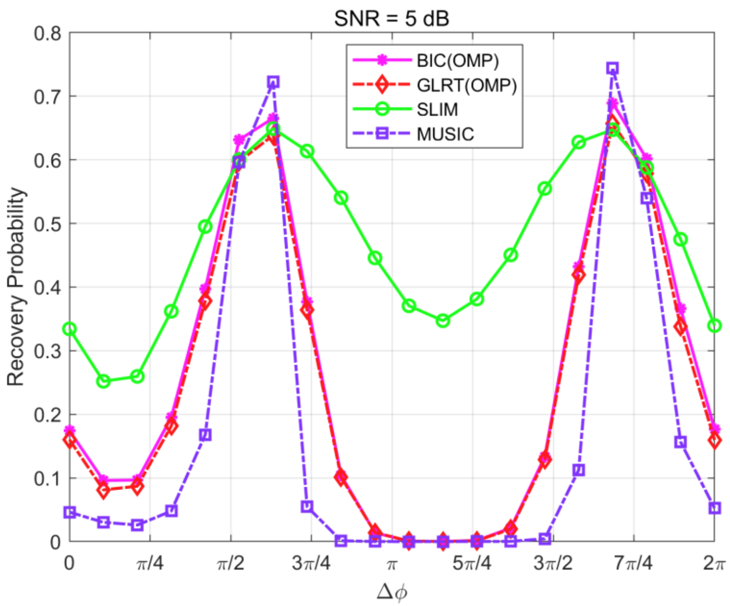
$$\tilde{R}_m(r,0) \simeq R_m(r,0) + dR_m(r,0)$$

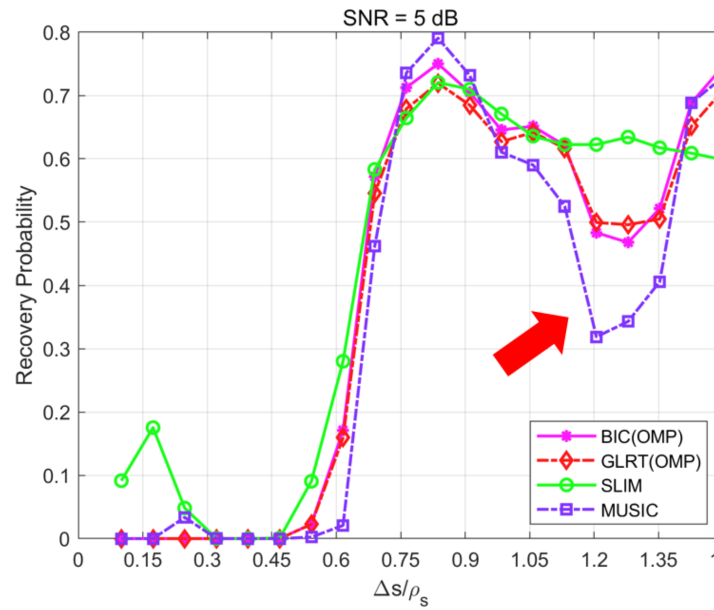
$$dR_m(r,0) \simeq -\sin\theta_m dy_m + \cos\theta_m dz_m$$

$$\alpha_m \simeq \frac{4\pi}{\lambda}(-\sin\theta_m dy_m + \cos\theta_m dz_m)$$

Challenges

- Interfering effects of multiple scatterers in one pixel



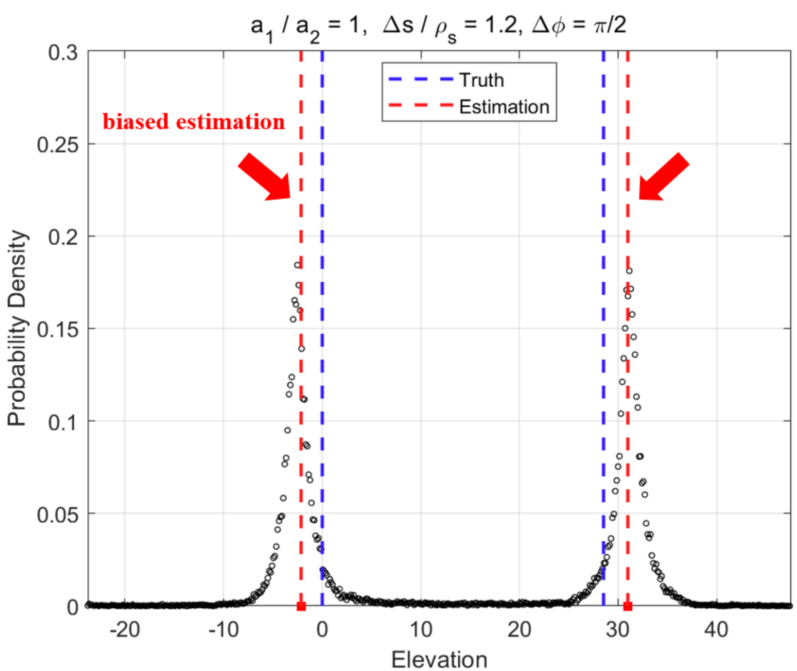


$\Delta\phi$: Phase difference between the multiple scatterers
$a_1/a_2 = 1$, $\Delta s/\rho_s = 0.7$

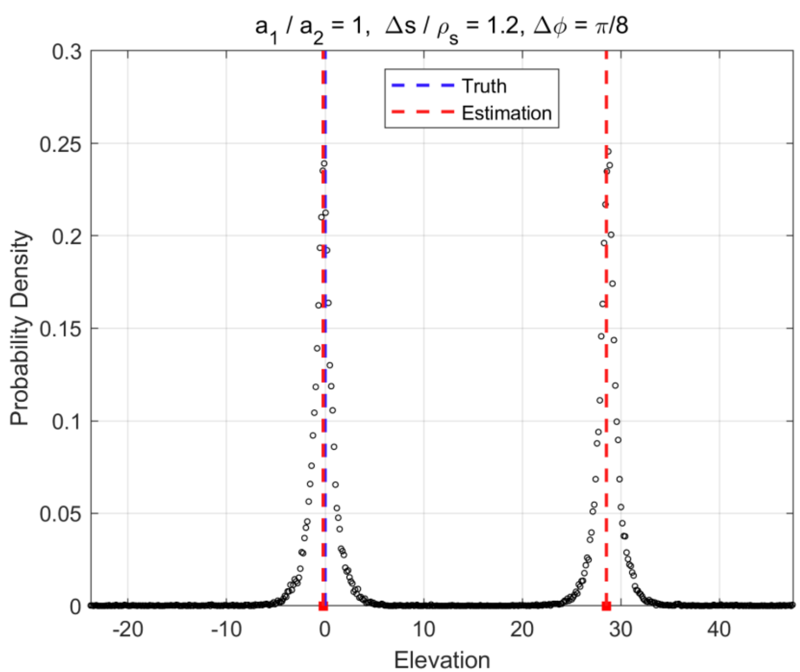
$\Delta s/\rho_s$: Normalized distance between the multiple scatterers
$a_1/a_2 = 1$, $\Delta\phi = \pi/2$
$\boldsymbol{\Delta s/\rho_s = 1.2}$**: Abnormal decline**

Challenges

- Interfering effects of multiple scatterers in one pixel
  - $\Delta s/\rho_s$ and $\Delta\phi$ (elevation phase and scattering phase) interfere with each other



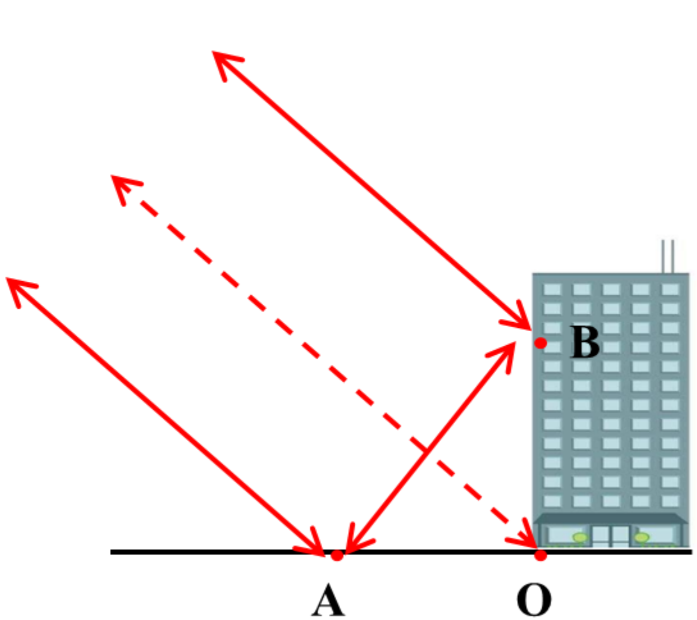
(a) Biased estimation (MUSIC)



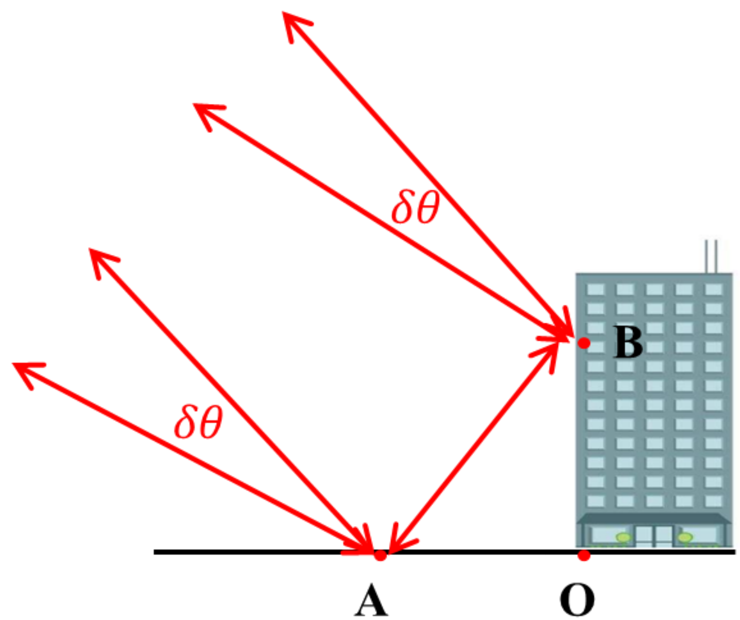
(b) Unbiased estimation (MUSIC)

Challenges

- Multipath scattering in Array-InSAR Tomography
  - Schematic diagram:



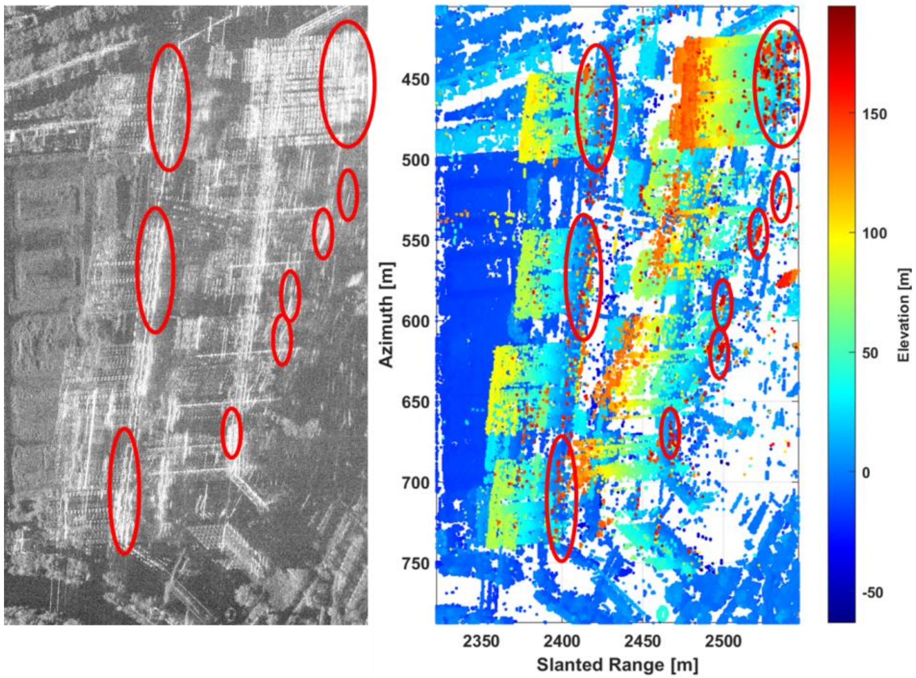
Repeat-pass InSAR tomography
single transmit and single receive



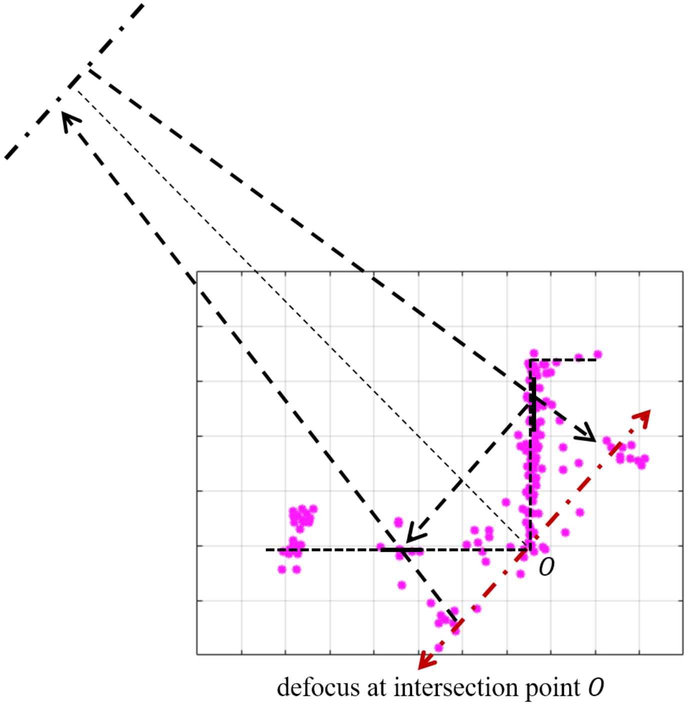
Array-InSAR tomography
For example:
single transmit and double receive

Challenges

- Multipath scattering in Array-InSAR Tomography
  - Example: defocusing



Marking of defocused point clouds



Defocusing phenomenon of double-bounce scattering in Array- InSAR tomography

Summary

- *Coprime sensing technique*
  - An efficient sensing strategy
  - Fully using virtual baselines
  - Fully using spatial redundancy

Thank you!

Ren et al. Coprime Sensing for Airborne Array Interferometric SAR Tomography, *TGRS*, vol. 60, pp. 1-15, 2022