ADCS – III
Orbital Perturbations
and
Coordinate Systems

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Questions to lecture ADCS - II

- What says the vis-viva equation?
- Show that angular momentum is a constant vector. Why has this something to do with Kepler´s second law?
- Why do we need 6 variables to define an orbit? Which set of variables do you know?
- What disturbance forces do you know? Which one is largest for low Earth orbits?
- What mathematical techniques do you know to deal with disturbances?
- Spherical harmonics do solve which differential equation?
- Spherical harmonics have angular dependence. Which one? Do you have a picture? Is there also an radial dependence?
- What do we mean with periodic and secular variation?
- For the Earth we have a J_2 term. What does it physically stand for? What is the J_2 effect? Do you know its use ? If we would not include it how large would be the error on the gravitational potential? What are the first two terms of gravitational potential?
- How do you calculate North-South acceleration? What is it for spherical Earth?

Summary of last lecture

Summary of Kepler orbits

Orbital perturbations

- Special and general perturbation

Account for gravitational perturbation to non-spherical Earth

- Gravity potential
- Laplace equation
- Spherical harmonic functions
- J_2 perturbation
  - Effect on Ω (Sun-synchronous orbits)
  - Effect on ω (Molniya orbits)
  - In first-order no effect on a, e and i

Outline

Orbital perturbations

- **Atmospheric drag**
- **Third body**
  - Restricted three body problem
  - Lagrange points
  - Halo orbits
- **Solar radiation pressure**

Geostationary orbit perturbation

Type of satellite orbits

Coordinate systems

Orbital perturbations

Perturbation of Kepler orbits due to:

Non-spherical Earth (done in lecture ADCS - II)

Earth is not point mass or perfectly spherical

Atmospheric drag

Residual atmosphere creates drag → results in gradual orbit decay

Third body

Presence of other bodies (Sun, Moon) and their gravitational fields

Solar radiation pressure

Light from Sun creates pressure on spacecraft

Caused by momentum transfer from photons to spacecraft

Atmospheric drag

Earth atmospheric drag

Earth´s atmosphere extends into space (up to several hundred km)
→ Extends into satellite orbit range

Atmosphere produces forces and torques as spacecraft travel through it

Atmospheric drag represent largest perturbation acting at altitude below $\approx$300 km
→ For low Earth orbits drag must be taken into account

Atmospheric drag force is opposite to velocity of satellite
(Same as for aircraft, but for satellites lift force is neglected in most cases)

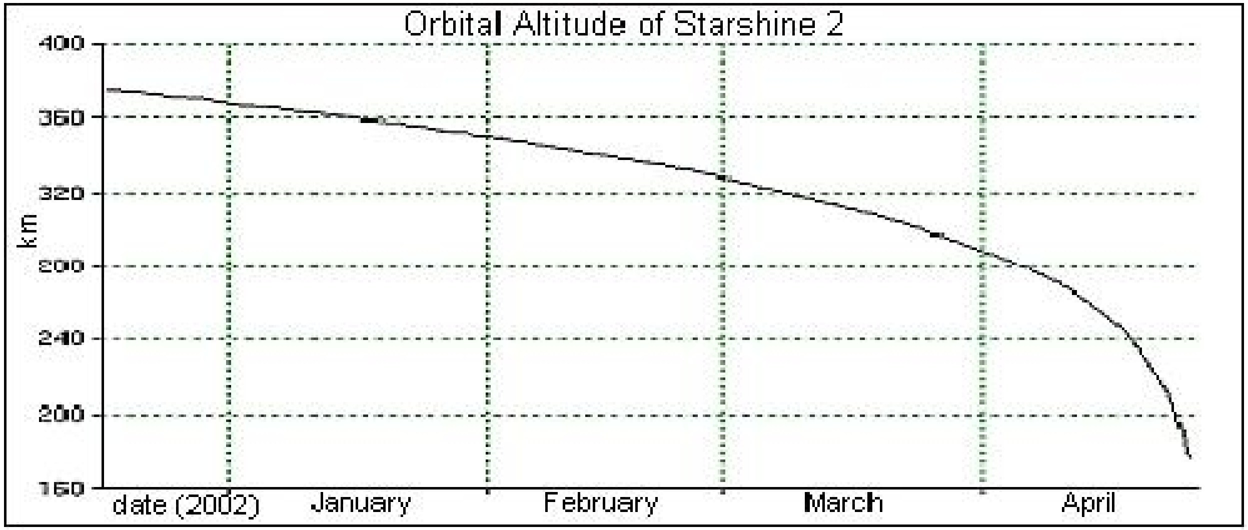
$$F_D = \frac{1}{2} C_D A \rho v^2$$

Drag force function of:

- atmosphere density ϱ (difficult to model)
- satellite velocity v
- cross sectional area A (need knowledge of attitude)
- satellite geometry C_D (non-dimensional coefficient of drag)

Typical orbital decay rate for low altitude satellite

- Atmospheric density is difficult to predict, because of solar activity
- Exist several models for atmospheric density depending on temperature, altitude, solar activity level, etc. (Jacchia, U.S. standard)
- Main problem is predicting solar activity



Altitude versus time taking into account following formulas

$$F_D = \frac{1}{2} C_D A \rho v^2$$

$$\rho = \rho_0 e^{-h/H}$$

- Satellites must maneuver to compensate for drag force
- Fuel needed to hold satellite in low Earth´s orbits (ISS and Hubble)

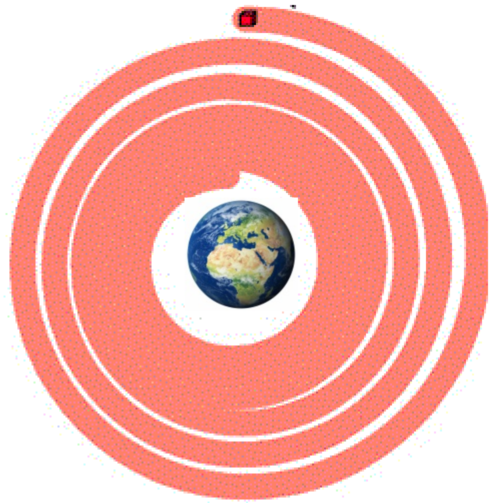
Effect of atmospheric drag for circular orbits

Principle effect of atmospheric drag is retarding force against velocity vector

$\Rightarrow$ dissipation of orbit energy

$\Rightarrow$ gradually reduce orbit radius (orbital decay)

$\Rightarrow$ until satellite crashes on Earth´s surface



Orbit decay depends on:

initial altitude, eccentricity, solar condition, etc.

Remark: Counter intuitive ("drag paradox") but effect of atmospheric drag is to increase satellite speed and kinetic energy

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} \quad \text{and} \quad \frac{2\pi a}{v} = T \Rightarrow v = \sqrt{\frac{\mu}{a}}$$

Circular orbit

Effect of atmospheric drag for eccentric orbits

Although drag occurs at perigee:

Apogee height shrinks drastically,

whereas perigee height remains relatively constant

Difficult to estimate orbit decay because if increased solar activity

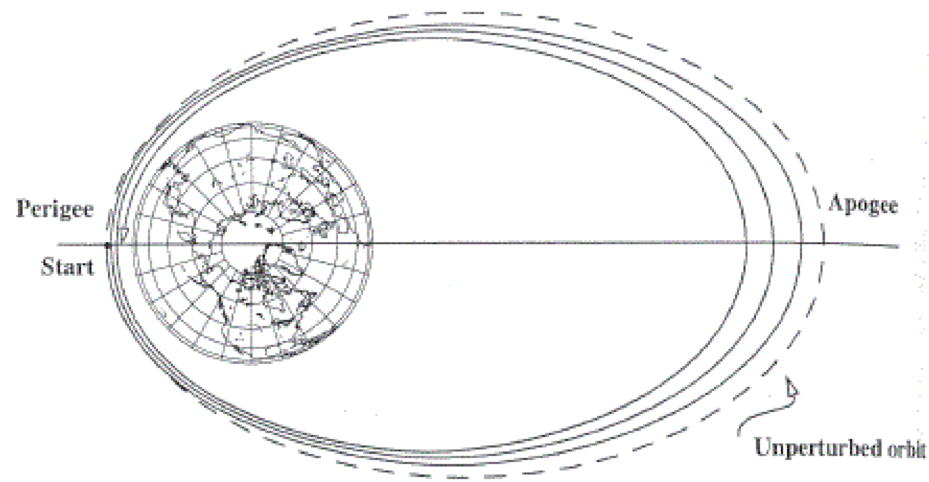
→ drastic increased drag

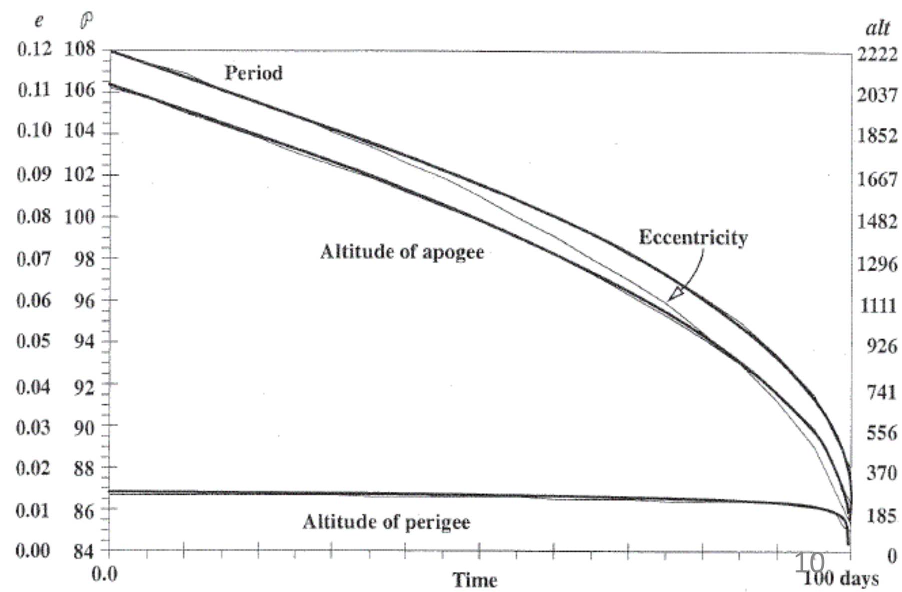
Remark:

In first approximation

orientation of orbit plane is

not changed by drag





Three-body perturbation

No exact three-body solution

Search for solutions to problem
of three mutually gravitating point masses

Three body problems involves solve system of
9 coupled second order differential equations

$$\frac{d^2\mathbf{r}_1}{dt^2} = -Gm_2\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} - Gm_3\frac{\mathbf{r}_1 - \mathbf{r}_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3}$$

$$\frac{d^2\mathbf{r}_2}{dt^2} = -Gm_3\frac{\mathbf{r}_2 - \mathbf{r}_3}{|\mathbf{r}_2 - \mathbf{r}_3|^3} - Gm_1\frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3}$$

$$\frac{d^2\mathbf{r}_3}{dt^2} = -Gm_1\frac{\mathbf{r}_3 - \mathbf{r}_1}{|\mathbf{r}_3 - \mathbf{r}_1|^3} - Gm_2\frac{\mathbf{r}_3 - \mathbf{r}_2}{|\mathbf{r}_3 - \mathbf{r}_2|^3}$$

Ten integrals of motion:
energy conservation (1 parameter)
angular momentum (3 parameters)
center of mass (6 parameters)
(CM = linear momentum + position)

- Only ten integrals known → No closed-form solution exist
- Found very few exact analytical solutions for special cases
- Most orbits unstable or even chaotic
- Chaotic means arbitrary close initial conditions
  result in orbits that are separated fare away after some time

Three-body perturbation

- Three body problem can NOT be solved as Kepler two-body problem
- Motion of satellite very different from Keplerian motion
- Orbital motion is complex and time dependent
- For Earth orbiting satellite, Sun and Moon should be modeled for accurate predictions (geometry changes continually and very precise ephemerides are needed)
- Satellite experiences gravitational pull from Sun and Moon and tend to move satellite out of orbit
- Numerical integration of equation of motion are needed
- Three body perturbation effects become noticeable when effect of atmospheric drag begin to diminish
- One largely used approach is a two-body decomposition of solar system, so called **patched conic approach** with concept of **sphere of influence** (seen in lecture ”Entorno espacial y análisis de misión”)
- However for special cases solutions exist like restricted (cricular) three body problem

Restricted Three-Body Problem

Restricted three-body problem

Assumptions:

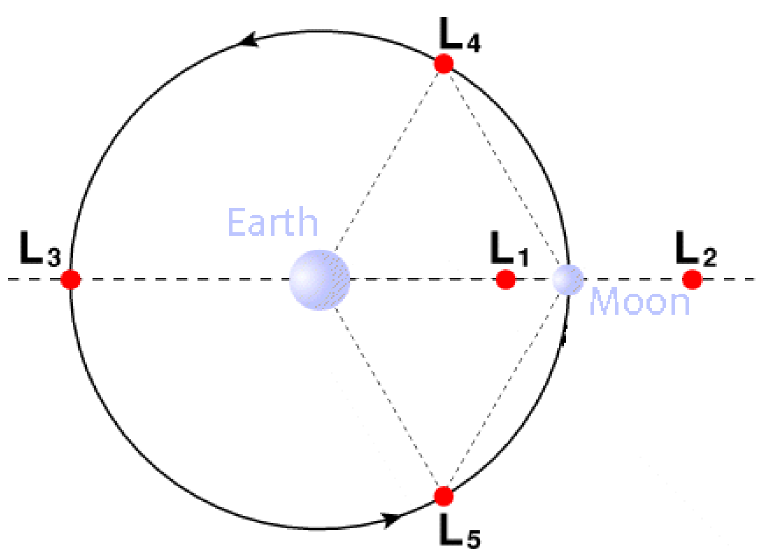
1. Gravitational effect of $\boldsymbol{m_3}$ on m_2 and m_1 is **negligible**
   → permits two-body solution for m_1 and m_2
2. Two-body motion is **circular** about their mutual center of mass
3. Initial position and velocity of m_3 are in **plane** of two-body motion
   → m_3 remains in plane

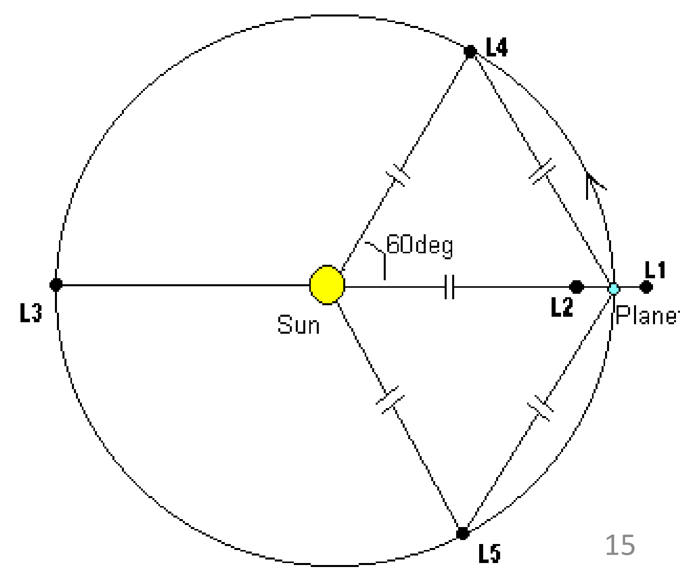
Example for Sun-planet-Moon system or Earth-Moon-spacecraft system

$\Rightarrow$ Five Lagrange points: Points where zero velocity and acceleration

$\Rightarrow$ L_1, L_2 and L_3 are unstable

$\Rightarrow$ L_4 and L_5 are stable





Restricted three body problem

Consider three masses that interact gravitationally

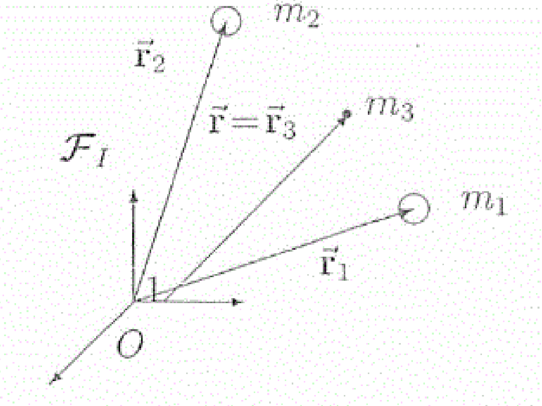


Figure 10.1 The three-body problem

Assumptions:

1. Gravitational effect of m_3 on m_2 and m_1 is **negligible**
   → permits two-body solution for m_1 and m_2
2. Two-body motion is **circular** about their mutual center of mass
3. Initial position and velocity of m_3 are in **plane** of two-body motion
   → m_3 remains in plane

Example for Sun-planet-Moon system or Earth-Moon-spacecraft system

This model is known as **restricted (planar circular) three-body problem**:

Spacecraft affected by forces from two rotating bodies, but large bodies do not feel influence of spacecraft

Restricted (planar circular) three body problem

Let´s identify m_1 with mass of Earth and m_2 with mass of Moon

Origin of following two frames are placed at center of mass (CM)

Define **X-Y inertial frame** centered at Earth-Moon center of mass

Define **x-y rotating frame** with angular velocity equivalent to mean motion of Earth-Moon system (coordinate system in which Earth and Moon are stationary)

Angular velocity given by:

$$\omega = \sqrt{G(m_1 + m_2) / r_{12}^3} \quad , \quad r_{12} = r_1 + r_2$$

(Solution to motion of m_1 and m_2 is a Kepler problem, where two bodies move in circular orbits about their center of mass)

Rotating x-y frame

m_1 and m_2 move in circular orbits around center of mass

What is motion of m_3?

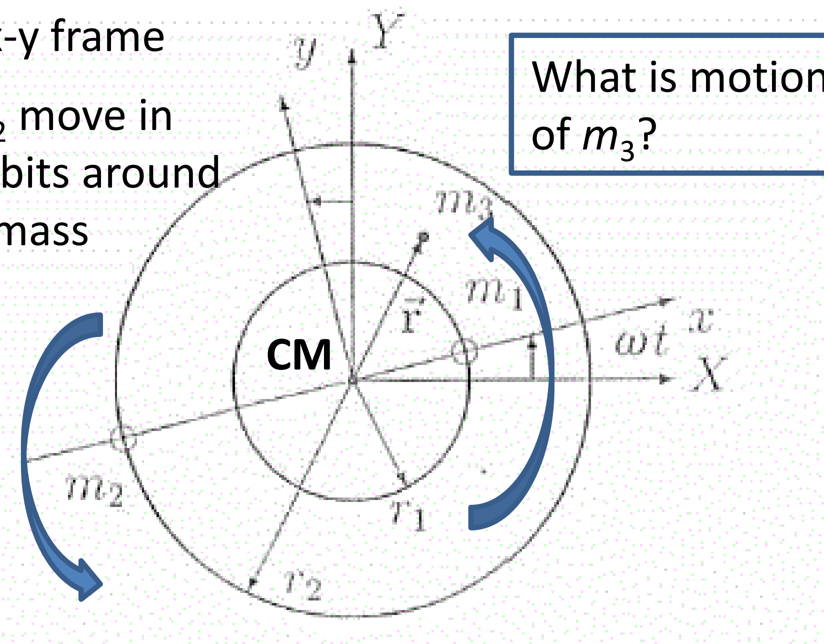


Figure 10.2 The restricted three-body problem

Position of m_1, m_2, m_3 in rotating frame

Since origin is CM

$$m_1 r_1 = m_2 r_2$$

$$r_{12} = r_1 + r_2$$

$$\Rightarrow$$

$$r_1 = \frac{m_2}{m_1 + m_2} r_{12}$$

$$r_2 = \frac{m_1}{m_1 + m_2} r_{12}$$

Position of m_3

$$\vec{\mathbf{r}} = x\vec{\mathbf{x}}_1 + y\vec{\mathbf{y}}_1$$

Angular velocity of *x*-*y* frame

$$\vec{\omega} = \omega\vec{\mathbf{z}}_1$$

Position of m_1 and m_2:
Since coordinate system rotates with ω
mass m_1 and m_2 are at rest

Acceleration of m_3 in rotating frame

Acceleration of m_3 expressed in rotating frame (will see in lecture 6)

$$\ddot{\vec{\mathbf{r}}} = \ddot{\vec{\mathbf{r}}}_{rot} + 2\vec{\omega}\times\dot{\vec{\mathbf{r}}}_{rot} + \vec{\omega}\times(\vec{\omega}\times\vec{\mathbf{r}})$$

Acceleration of m_3 in rotating frame

$$\ddot{\vec{\mathbf{r}}}_{rot} = \ddot{x}\vec{\mathbf{x}}_1 + \ddot{y}\vec{\mathbf{y}}_1$$

Velocity of m_3 in rotating frame

$$\vec{\omega}\times\dot{\vec{\mathbf{r}}}_{rot} = -\dot{y}\omega\vec{\mathbf{x}}_1 + \dot{x}\omega\vec{\mathbf{y}}_1$$

Angular velocity

$$\omega = \sqrt{G(m_1+m_2)/r_{12}^3}$$

$$\vec{\omega}\times(\vec{\omega}\times\vec{\mathbf{r}}) = -\omega^2 x\vec{\mathbf{x}}_1 - \omega^2 y\vec{\mathbf{y}}_1$$

Acceleration of m_3:

$$\ddot{\vec{\mathbf{r}}} = (\ddot{x} - 2\dot{y}\omega - \omega^2 x)\vec{\mathbf{x}}_1 + (\ddot{y} + 2\dot{x}\omega - \omega^2 y)\vec{\mathbf{y}}_1$$

Newton´s second law for m_3

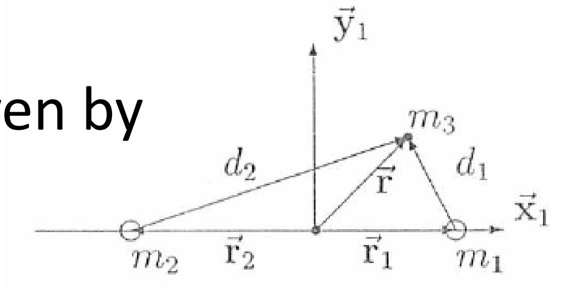
$$m_3\ddot{\vec{\mathbf{r}}} = \vec{\mathbf{f}}_1 + \vec{\mathbf{f}}_2$$

Gravitational force $\vec{\mathbf{f}}_1$ acting on m_3 due to m_1 is given by

$$\vec{\mathbf{f}}_1 = -\frac{Gm_1m_3}{d_1^3}(\vec{\mathbf{r}} - \vec{\mathbf{r}}_1) = -\frac{Gm_1m_3}{d_1^3}\left[(x - r_1)\vec{\mathbf{x}}_1 + y\vec{\mathbf{y}}_1\right]$$

Gravitational force $\vec{\mathbf{f}}_2$ acting on m_3 due to m_2 is given by

$$\vec{\mathbf{f}}_2 = -\frac{Gm_2m_3}{d_2^3}(\vec{\mathbf{r}} - \vec{\mathbf{r}}_2) = -\frac{Gm_2m_3}{d_2^3}\left[(x + r_2)\vec{\mathbf{x}}_1 + y\vec{\mathbf{y}}_1\right]$$



$$d_1 = \sqrt{(x - r_1)^2 + y^2}$$

$$d_2 = \sqrt{(x + r_2)^2 + y^2}$$

$$\ddot{\vec{\mathbf{r}}} = (\ddot{x} - 2\dot{y}\omega - \omega^2 x)\vec{\mathbf{x}}_1 + (\ddot{y} + 2\dot{x}\omega - \omega^2 y)\vec{\mathbf{y}}_1 =$$

$$= -\frac{Gm_1}{d_1^3}\left[(x - r_1)\vec{\mathbf{x}}_1 + y\vec{\mathbf{y}}_1\right] - \frac{Gm_2}{d_2^3}\left[(x + r_2)\vec{\mathbf{x}}_1 + y\vec{\mathbf{y}}_1\right]$$

Equations of motion

From previous slide get two scalar equations

$$\ddot{x} - 2\dot{y}\omega - \omega^2 x = -\frac{Gm_1}{d_1^3}(x - r_1) - \frac{Gm_2}{d_2^3}(x + r_2)$$

$$\ddot{y} - 2\dot{x}\omega - \omega^2 y = -G\left[\frac{m_1}{d_1^3} + \frac{m_2}{d_2^3}\right]y$$

These are differential equations for motion of spacecraft in rotating system

No analytical solution exist

Given initial conditions $x(0)$, $y(0)$, dx/dt and dy/dt

→ unique solution for $x(t)$ and $y(t)$ can be determined numerically

Chaotic behavior of restricted three body problem
(https://www.youtube.com/watch?v=jarcgP1rRWs)

Lagrange points

Equilibrium points to previous equations of motion can be determined

Equilibrium solutions are constant solutions for *x* and *y*

$$\dot{x} = \dot{y} = \ddot{x} = \ddot{y} = 0$$

Equilibrium points are called Lagrange points (or libration points)

$$-\omega^2 x = -G\left[\frac{m_1}{d_1^3}(x - r_1) + \frac{m_2}{d_2^3}(x + r_2)\right]$$

$$-\omega^2 y = -G\left[\frac{m_1}{d_1^3} + \frac{m_2}{d_2^3}\right] y$$

Two sets of Lagrange points

Use equilibrium points into equation of motion

→ Two nonlinear equations with two unknowns *x* and *y*

$$-\omega^2 x = -G\left[\frac{m_1}{d_1^3}(x - r_1) + \frac{m_2}{d_2^3}(x + r_2)\right]$$

$$-\omega^2 y = -G\left[\frac{m_1}{d_1^3} + \frac{m_2}{d_2^3}\right] y$$

$y = 0$ → L_1, L_2, L_3 case 1

$y \neq 0$ → L_4, L_5 case 2

Collinear Lagrange points: L_1, L_2, L_3

Case 1: $y = 0$ and
$$d_1 = \sqrt{(x-r_1)^2 + y^2}$$
$$d_2 = \sqrt{(x+r_2)^2 + y^2}$$

→ $d_1 = |x - r_1|$ $d_2 = |x + r_2|$ **collinear** points

Use first equation to get $$\omega^2 x = \frac{Gm_1(x-r_1)}{\left|x-r_1\right|^3} + \frac{Gm_2(x+r_2)}{\left|x+r_2\right|^3}$$

With numerical technique solve for *x*

→ Equation has three real roots

For Earth-Moon system (use m_1 and m_2) →
$$\mathrm{L}_1 \Rightarrow x = -0.838 r_{12}$$
$$\mathrm{L}_2 \Rightarrow x = -1.156 r_{12}$$
$$\mathrm{L}_3 \Rightarrow x = 1.005 r_{12}$$

Equilateral Lagrange points: L_4, L_5

Case 2: $y \neq 0$

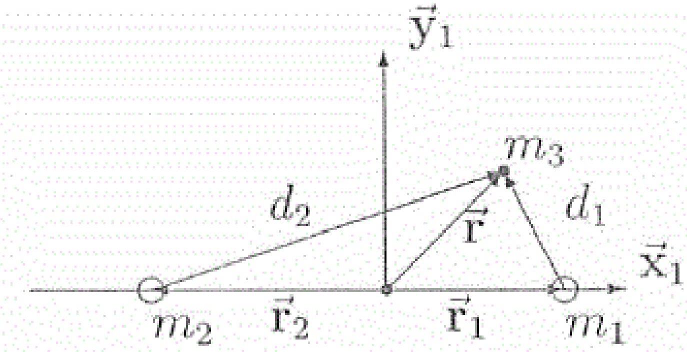
$$\frac{Gm_1}{d_1^3} + \frac{Gm_2}{d_2^3} = \omega^2 = \frac{G(m_1 + m_2)}{r_{12}^3}$$

Angular velocity of Kepler problem

If assume $d_1 = d_2$

$$\rightarrow \quad d_1^3 = d_2^3 = \frac{G(m_1 + m_2)}{\omega^2} = r_{12}^3$$

$$\rightarrow \quad d_1 = d_2 = r_1 + r_2$$



Solution corresponds to L_4 and L_5,
which are **equilateral** or triangle Lagrange points

Earth-Moon system

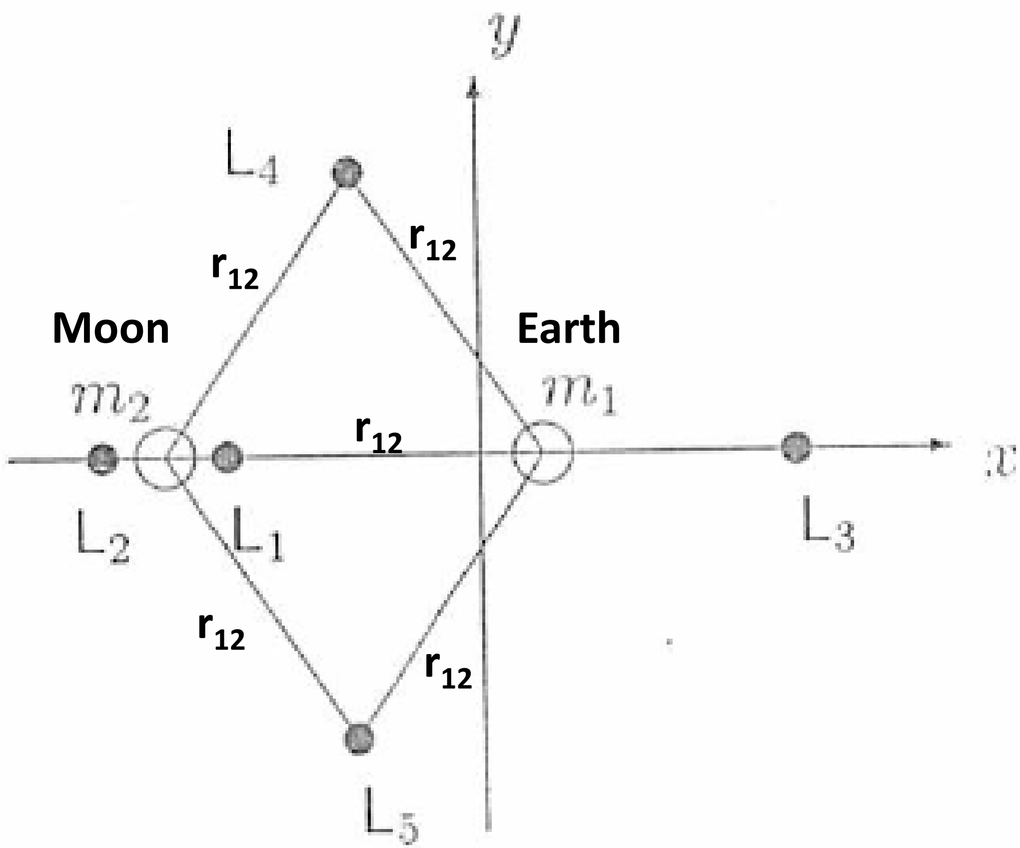


Figure 10.4 The lagrangian points

Note:
Lagrange points are points where forces are in equilibrium

Lagrange points mark positions where **gravitational attraction** of two massive bodies and **centrifugal force** are in equilibrium

Lagrange points correspond to points with zero velocity and acceleration

Small mass placed at these points remains motionless with respect to Earth and Moon (which are in circular orbits)

Stability of Lagrange points

L_1, L_2 and L_3 are unstable:

Satellite drifts away from equilibrium point for small perturbations

L_4 and L_5 are stable:

Satellite remains around equilibrium point for small perturbations

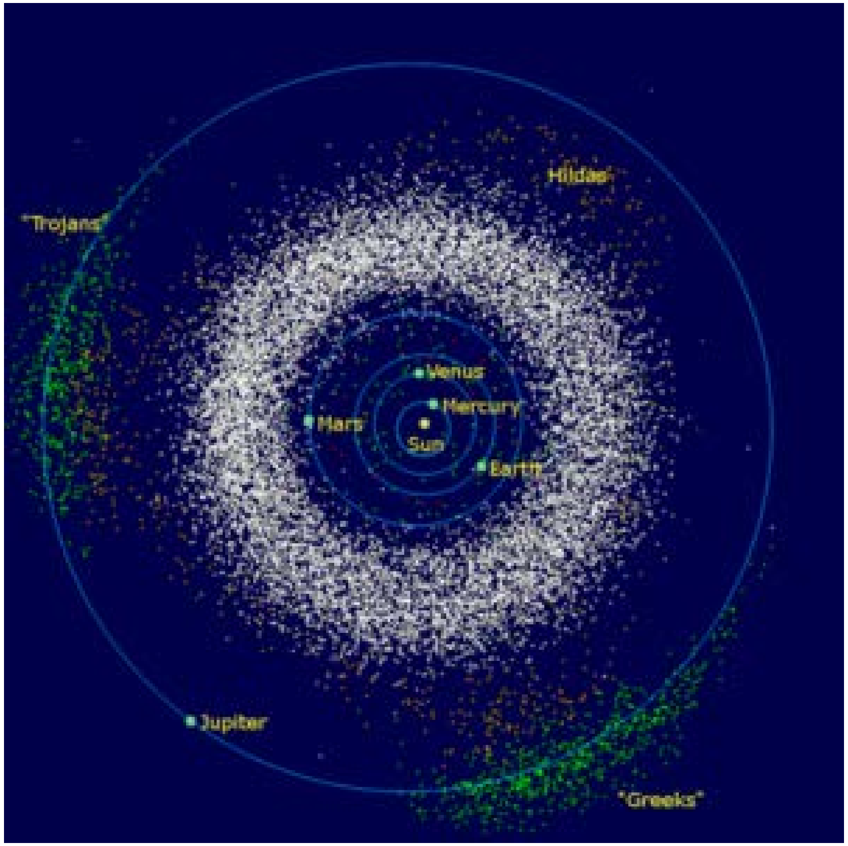
(Can be prouven mathematically similar to methods shown in lecture ADCS - IX)

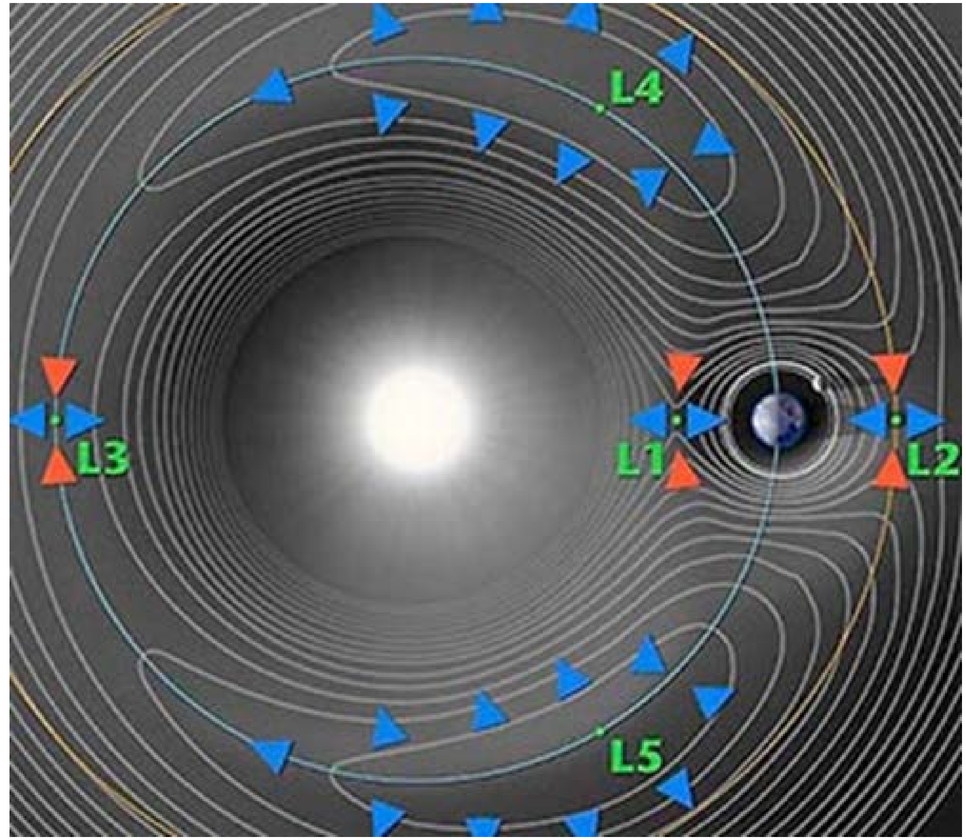
Note:

- Even if L_1, L_2 and L_3 points are unstable, amplification rate is small
- Better to locate satellite at these points and apply control system for orbital station-keeping than locate a satellite at rest at an arbitrary point in space

  → requires only small amount of fuel to keep satellite at such points
- Many space missions place satellites at Lagrange points

Sun-Jupiter system: Trojan asteroids





Jupiter trojans (green colored)

Trojans asteroids have been also found at L_4 and L_5 points for Mars and Neptune

Gravitational potential contour plot showing Earth's Lagrangian points: $L_{1,}$ $L_{2,}$ $L_{3,}$ L_4 and L_5

Jupiter's Lagrangian points are similarly situated

Halo orbit

- Halo orbit is periodic, three-dimensional circular orbit around (unstable) Lagrange points L_1, L_2 or L_3 (remark: no gravitational pull from such a point because it is only equilibrium point without mass)
- Halo orbit is result of interaction between gravitational attraction of two planetary bodies as well as centrifugal and coriolis accelerations on spacecraft
- Halo orbits are unstable and need orbital station-keeping
- Halo orbits exist in many three-body systems, such as Sun-Earth system and Earth-Moon system
- Non-periodic halo orbit is called **Lissajous orbit**

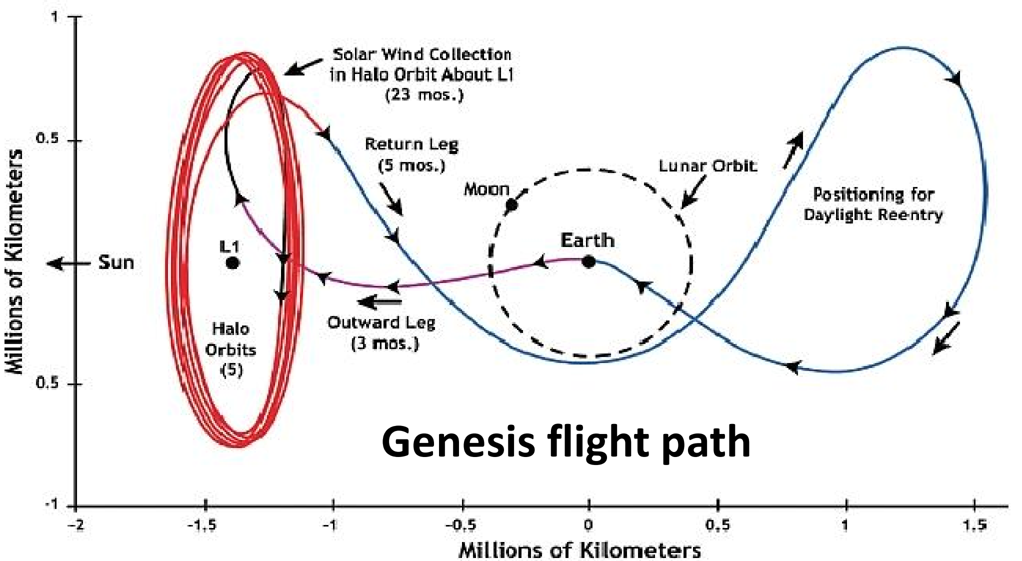
Halo orbit examples

Halo orbit:

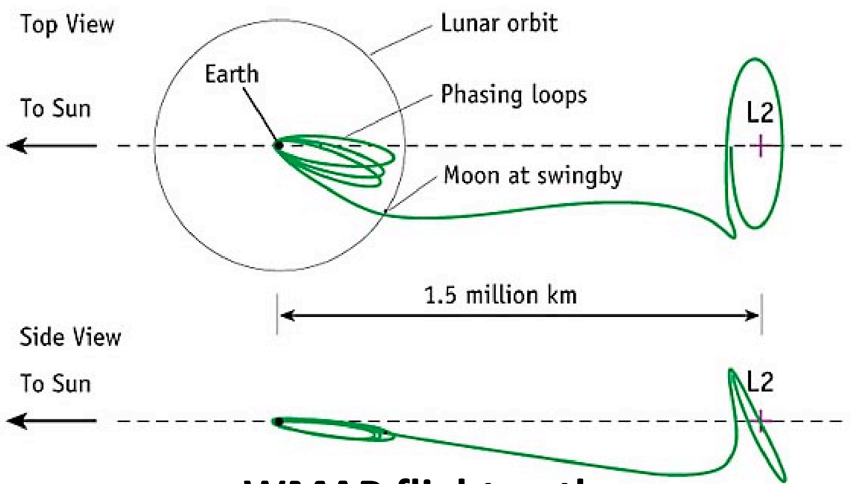
- ISEE-3 mission (1978) at Sun-Earth L_1 point for solar wind observation
- SOHO mission (1995) at Sun-Earth L_1 point for solar observation

Lissajous orbit:

- Genesis mission (2001) at Sun-Earth L_1 point for solar wind particle collection
- WMAP mission (2001) and Planck mission (2009) at Sun-Earth L_2 point to measure temperature of cosmic microwave background
- Herschel mission (2009) at Sun-Earth L_2 point with infrared telescope



Genesis flight path



WMAP flight path

Solar radiation pressure

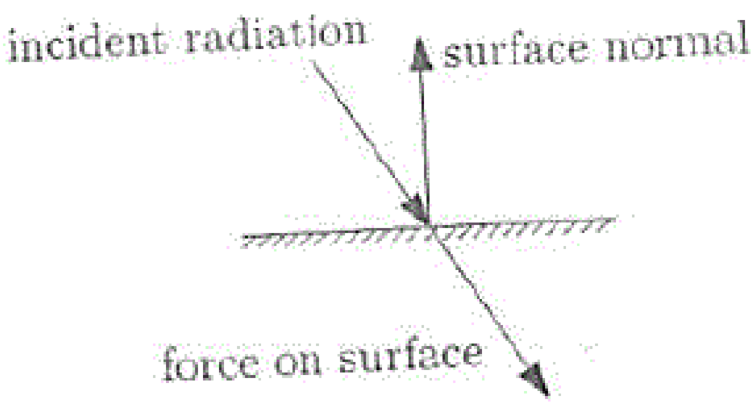
Solar radiation and solar wind

- Solar radiation NOT related to solar wind
- Solar radiation pressure induced by light (photons) momentum coming from Sun
- Solar wind continuous stream of particle coming from Sun
- Momentum flux in solar wind small compared with that due to solar radiation

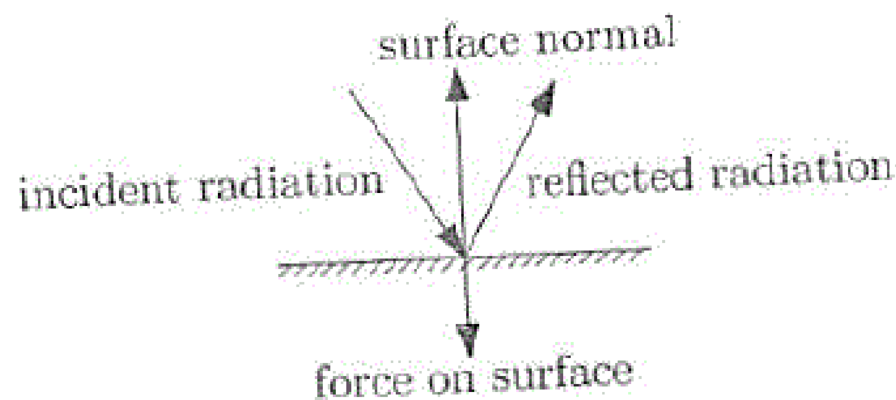
$\boxed{\text{Solar radiation (photons)}} \neq \boxed{\text{Solar wind (particles)}}$

Solar radiation pressure

- Sun light has momentum
- Sun light reflected and/or absorbed by surface
- Change in momentum generates solar pressure on satellite
- Different types of interaction between solar radiation and satellite surface, which depends on satellite surface



Absorption



Reflection

Solar radiation pressure

Solar radiation pressure causes changes in satellite orbits

$$\boxed{a_{rad} \sim \frac{A}{m}}$$

a_{rad} = solar radiation pressure acceleration
A = area of satellite surface
m = total satellite mass

Effect proportional to satellite area and inversional proportional to its mass

Important for low satellite mass with large surface area

Solar radiation pressure

Amount of energy emitted by Sun at 1 AU distance $\Phi_{rad,Earth} \approx 1362\ \mathrm{W/m^2}$

Φ_{rad} solar energy value nearly independent on solar activity (solar const.)

Φ_{rad} solar energy reduces with distance from Sun with $1/r^2$

Φ_{rad}/c = solar radiation pressure has $1/r^2$ variation

$$\boxed{a_{rad} = \frac{\Phi_{rad}(r)}{c}(1+q)\frac{A_\perp}{m}}$$

$$\boxed{\Phi_{rad}(r) = \Phi_{rad,Earth}\left(\frac{r_{Sun-Earth}}{r_{Sun-sat}}\right)^2}$$

a_{rad} = magnitude of solar radiation pressure acceleration

$\Phi_{rad}(r)$ = solar energy

c = speed of light

q = surface reflectivity (between 0 and 1)

$A_\perp$ = area of surface projected to sun line normal

m = total satellite mass

Solar radiation pressure

$$a_{rad} = \frac{\Phi_{rad}(r)}{c}(1+q)\frac{A_{\perp}}{m} \qquad \Phi_{rad}(r) = \Phi_{rad,Earth}\left(\frac{r_{Sun-Earth}}{r_{Sun-sat}}\right)^2$$

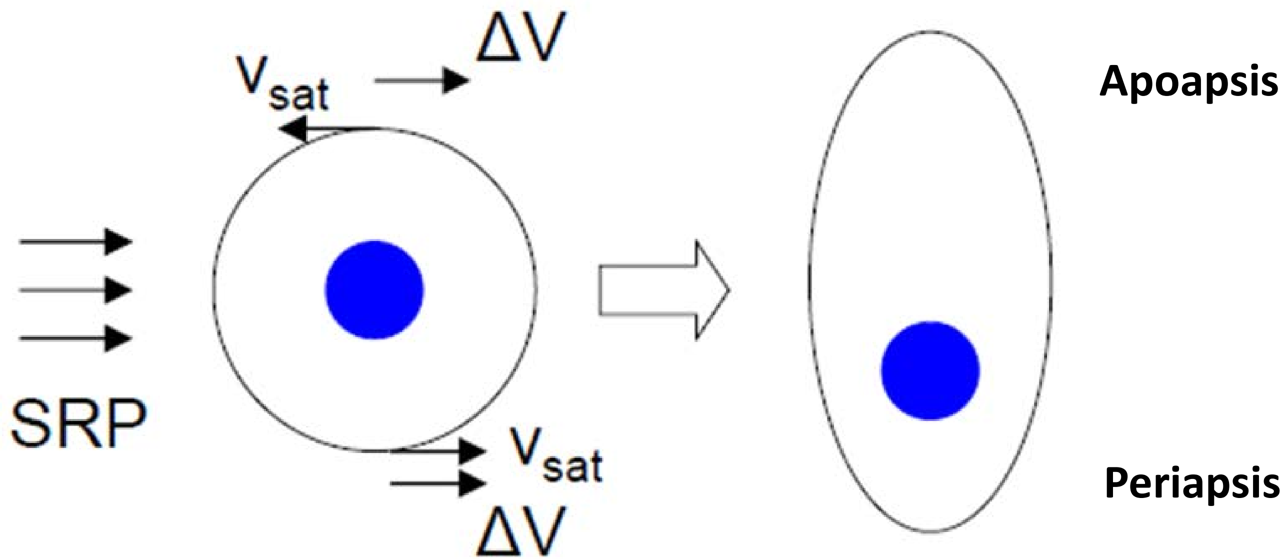
Solar radiation pressure:
- Depends on geometry and optical surface properties
- Depends on distance from Sun
- Perpendicular to Sun line
- Independent (in first approximation) of spacecraft position or velocity
- Independent on solar activity

Detailed model of solar radiation pressure should account for:
- Area exposed to Sun
- No solar radiation pressure for eclipses (Earth shadow cone where Sun is blocked)
- Reflection and absorption on each surface section of satellite
- Surface orientation and reflection from other parts of satellite

Largest acceleration for satellites with very low mass, large surface area

Remark:
Other minor sources of radiation pressure might be possible e.g. Earth albedo

Effect of solar radiation pressure on eccentricity



Solar radiation from left equivalent to

- add ΔV at lower end of orbit (increases apoapsis)
- subtract ΔV at upper end of orbit (decreases periapsis)

$\Rightarrow$ Orbit gradually transforms into more elliptical orbit

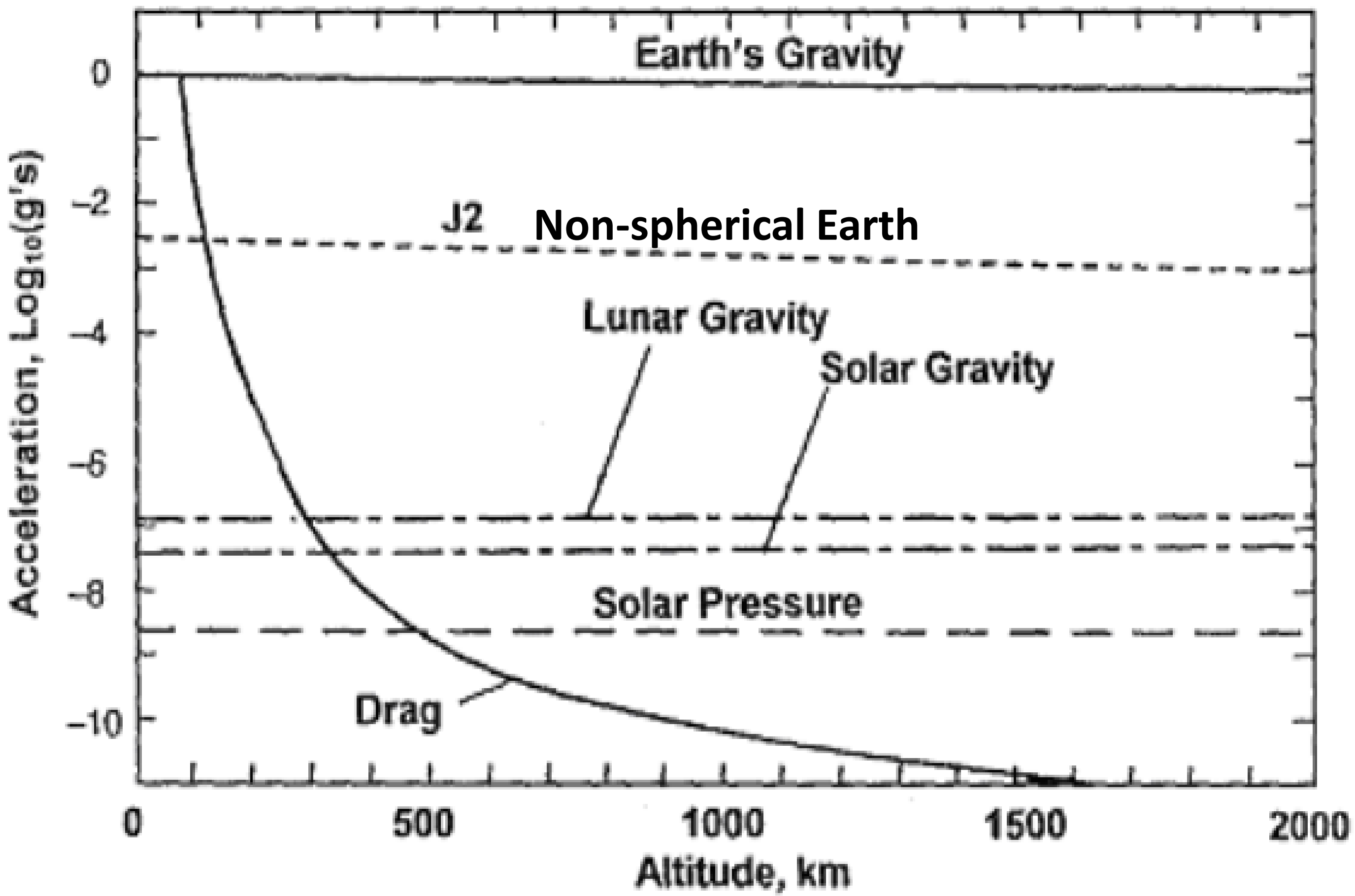
Six month later: Sun at opposite side of orbit

$\Rightarrow$ ΔV reverses sign at both ends of elliptical orbit

$\Rightarrow$ Orbit will gradually be more circular

Additional information in: https://www.youtube.com/watch?v=AVO42b7QLNM

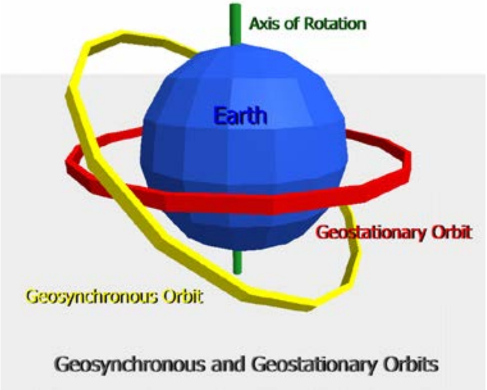
Relative importance of orbit perturbation for near Earth orbit



Logarithm of forces normalized with 1 g as function of altitude
Dominant forces: Earth´s gravity field and J_2 perturbation due to non-spherical Earth
Curve for drag has large uncertainty up to one order of magnitude (due to solar activity)

GEO perturbation

Geostationary orbit (GEO)



- Satellite that appears stationary with respect to Earth is called **geostationary**
- Note: **Geosynchronous** if rotation period equal to Earth rotation period
- Satellite appears stationary if:
  - Travel eastward at same rotational speed as Earth
  - Inclination of orbit is zero (i = 0°) and eccentricity is zero (e = 0)

    Inclination must be zero as having any inclination would lead satellite to move from North-South directions

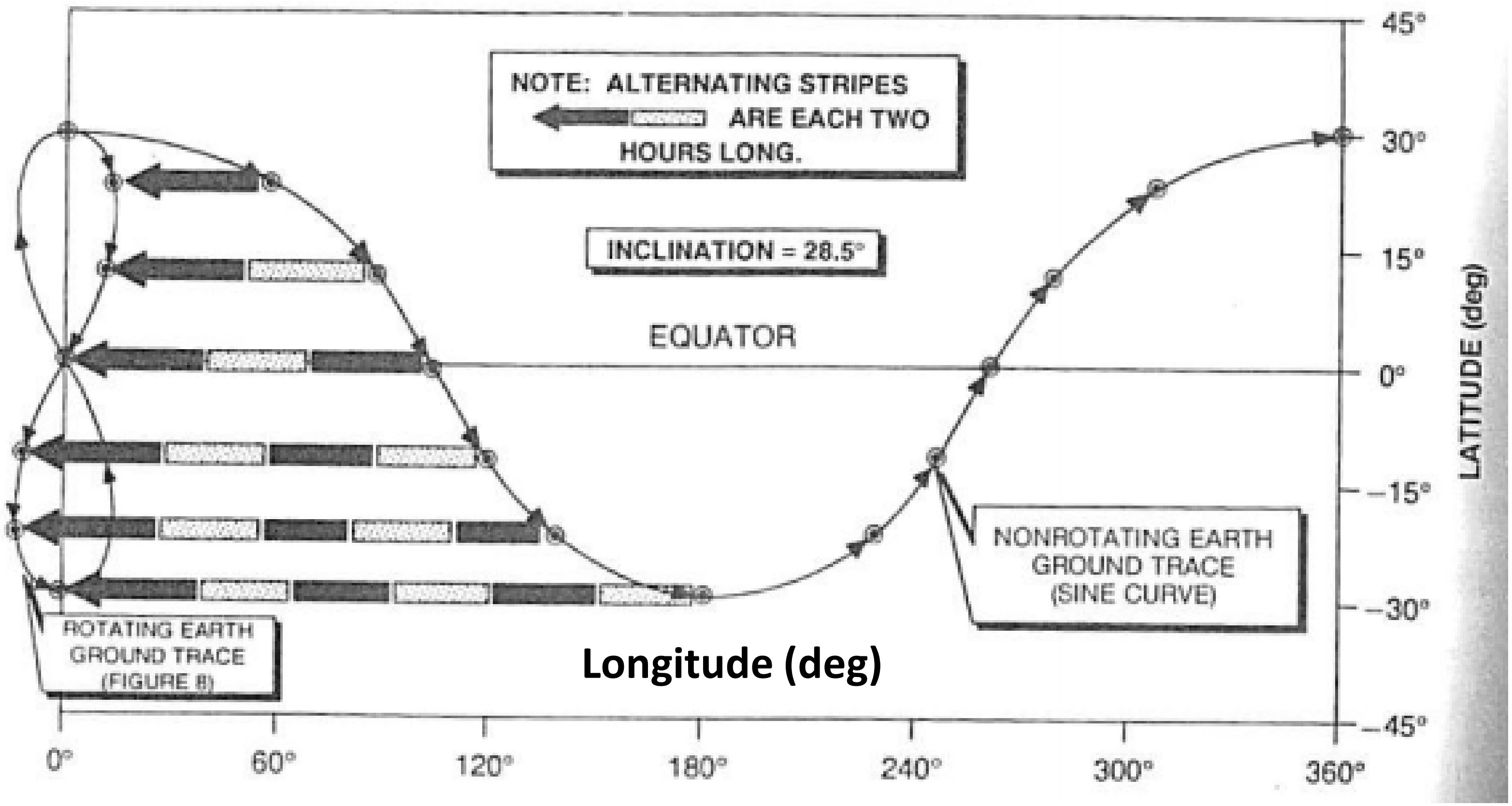
    → Orbits with zero inclination lie in the Earth´s equatorial plane
  - Orbit must be circular

    Constant speed means equal areas must be swept out at equal intervals of time

    → only possible for circular orbit
- Orbit radius ≈ 42000 km (altitude ≈ 36000 km)

$$F_{centripetal} = F_{gravitaion} \Leftrightarrow m\omega^2 r = G\frac{mM}{r^2} \Leftrightarrow r = \sqrt[3]{\frac{GM}{\omega^2}}$$

Figure-8 ground trace for an inclined geosynchronous orbit



Components of motion that make up Figure-8 ground trace for an inclined geosynchronous orbit

- Sine-curve depicts ground trace of satellite in inclined orbit over non-rotating Earth
- Each black and grey strips represent two-hours interval during which Earth rotates out from under satellite´s orbit plane at 15° per hour

→ Figure-8 ground trace results from orbit motion of satellite combined with Earth rotation

Three main perturbations for geostationary satellite

Geostationary satellites are affected by perturbations

- Non-spherical Earth (tesserial harmonic)
- Sun and Moon
- Solar radiation pressure

Effect is to slightly change velocity and thus position of satellite

East-West and North-South drift

Geostationary satellite orbit changes over time due to perturbation:

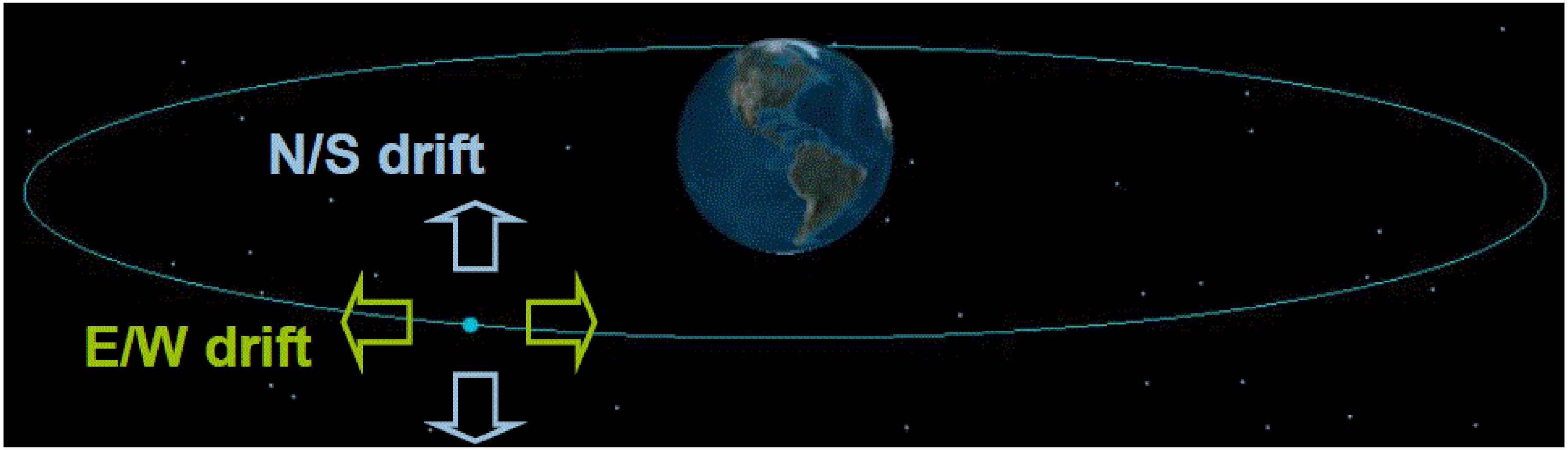
East-West drift:

Longitude modified by tesserial harmonic (non-spherical, inhomogeneous Earth)

North-South drift:

Inclination modified by Sun and Moon attraction

Change of eccentricity due to solar radiation pressure



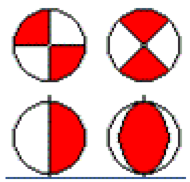
East-West drift

Geosynchronous satellite drifts in longitude due to influence of:

Inhomogeneous gravitational field of Earth

Elliptic nature of the Earth's equatorial cross-section: $\boldsymbol{J_{22}}$

(**Not** from North-South equatorial bulge term J_2)



Four spots along geosynchronous orbit where all forces balance out and no East-West drift exist → four equilibrium points

Two stable equilibrium points: 75°E and 255°E

Two unstable equilibrium points: 162°E and 348°E

Average East-West drift ≈ 0.01°/day

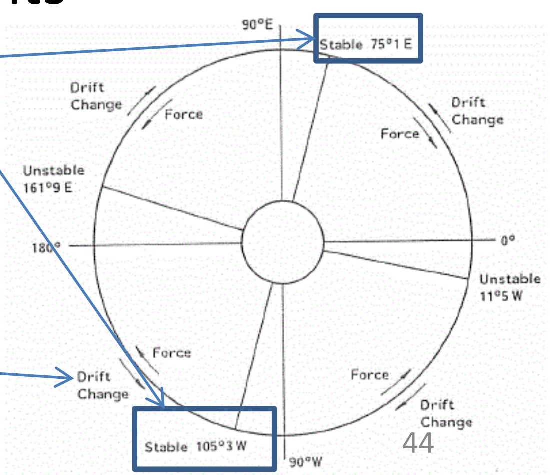
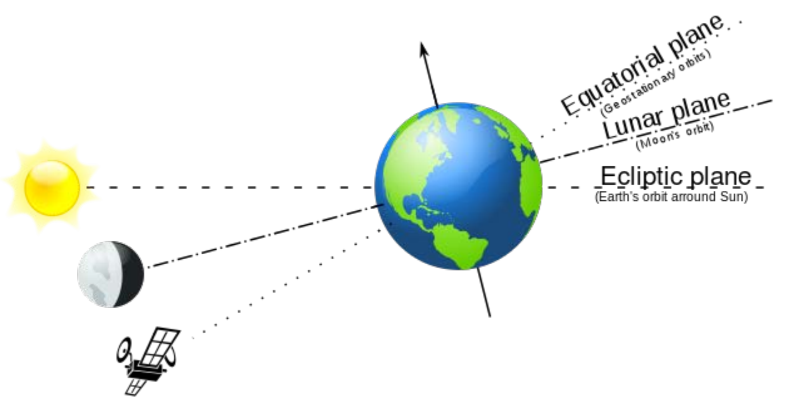


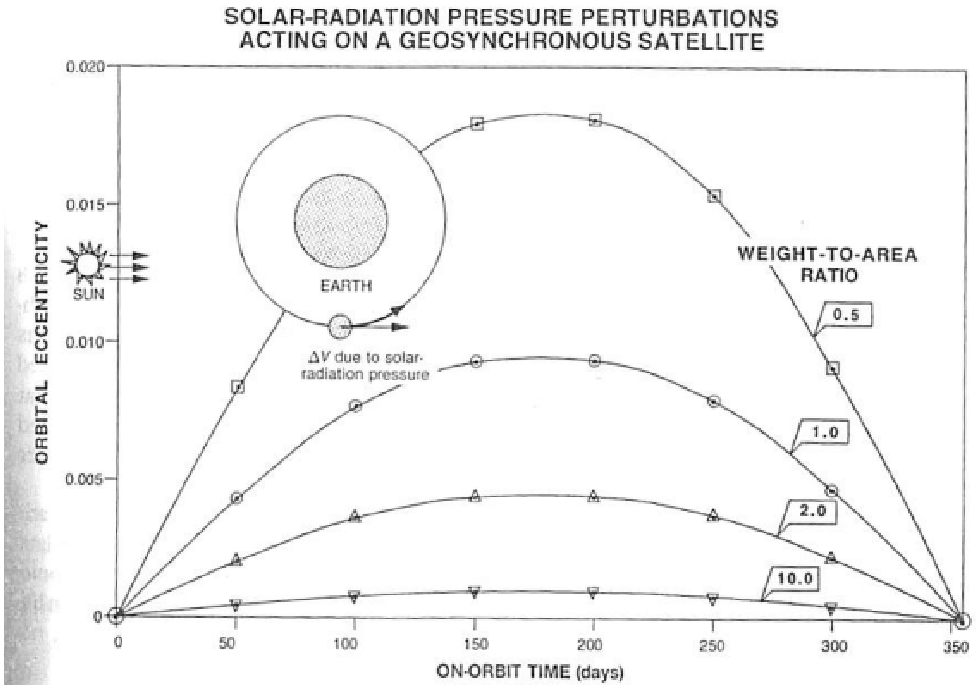
Figure 5.6 Force direction arising from J_{22} on a GEO vehicle

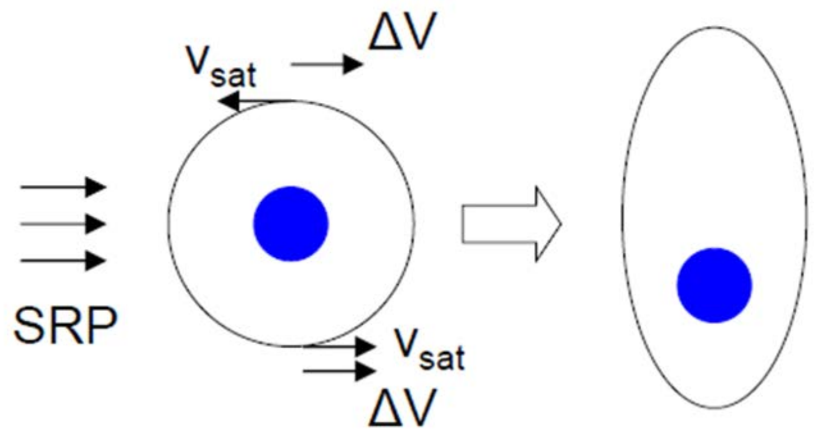
North-South drift

- Perturbations caused by Sun and Moon are predominantly out-of-plane effects causing change in inclination
- Influence of Moon is about twice that of Sun
- Average North-South drift in inclination ≈0.85°/year
- If uncorrected → satellite orbit is inclined
- Satellite describes small figure-8, which gets larger in time
- Not anymore truly geostationary, but still geosynchronous



Solar-radiation pressure perturbation





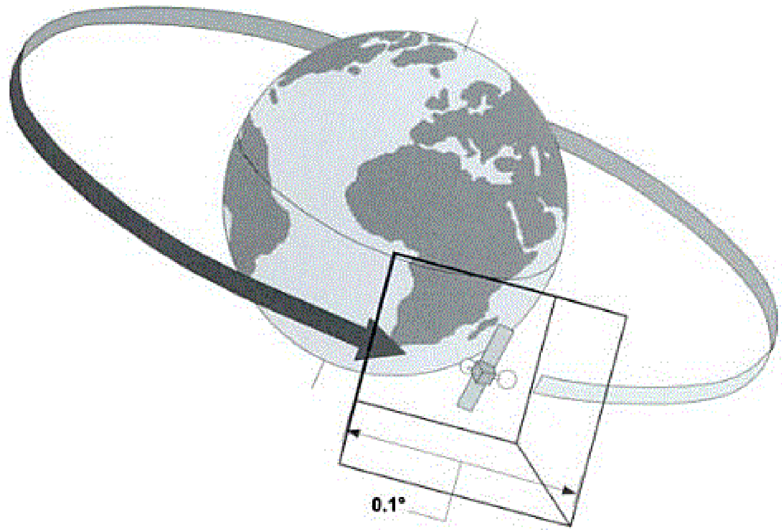
Solar-radiation pressure induced by Sun shining
Small force but steady can produce large variations in eccentricity of GEO satellite
Eccentricity of satellite orbit increases for first six months
and than gradually decreases for next six months (Sun shines from other direction)

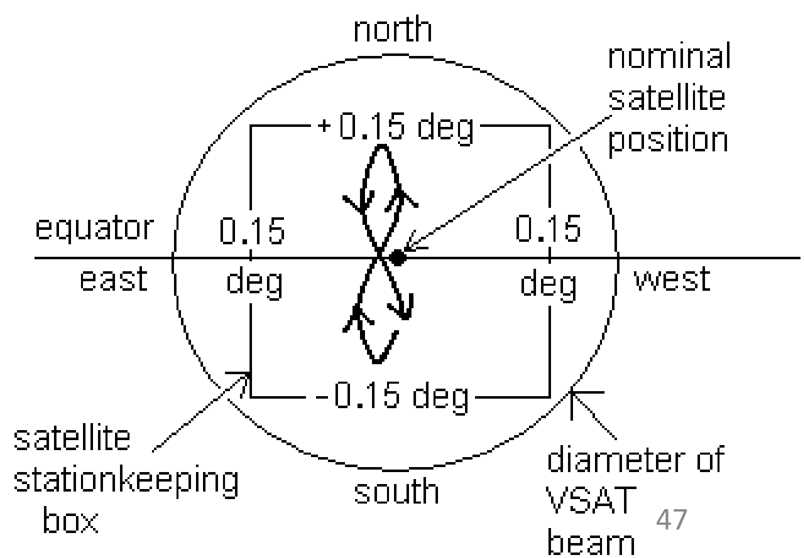
Orbit station-keeping of GEO satellite

- Effect of perturbations is to cause spacecraft to drift away from its nominal station

- If drift builds up unchecked
  → spacecraft can become useless
  Need of ΔV for corrections (fuel)

- Define orbit station-keeping box: Maximum authorized distance for actual satellite position from nominal satellite position





Magnitude of relative forces on satellite at specific heights above Earth

Table 4.3 Magnitude of disturbing accelerations acting on a space vehicle whose area-to-mass ratio is A/M

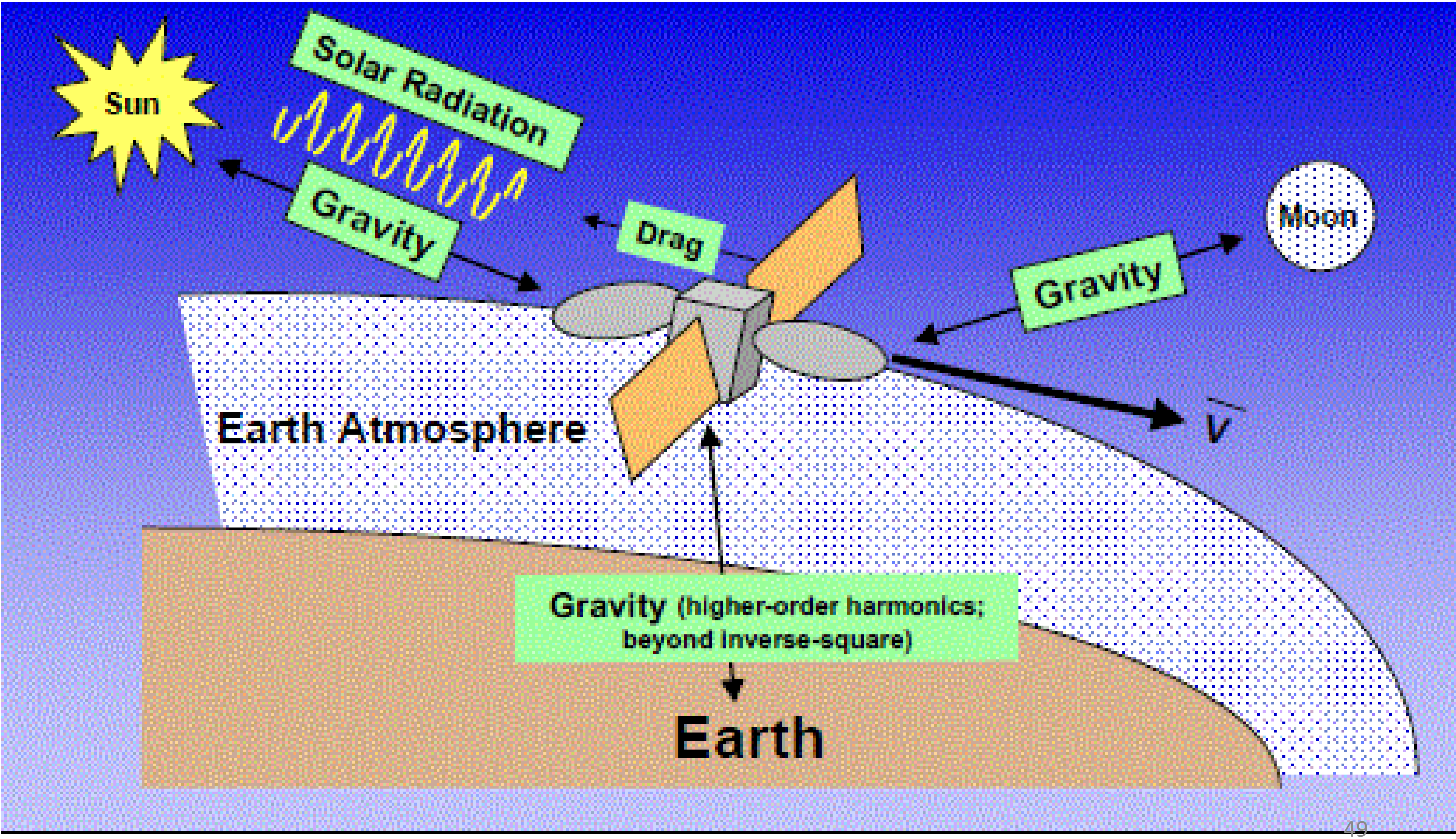
Source	Acceleration (m/s^2)	
	500 km	Geostationary orbit
Air drag	$6 \times 10^{-5}\ A/M$	$1.8 \times 10^{-13}\ A/M$
Radiation pressure	$4.7 \times 10^{-6}\ A/M$	$4.7 \times 10^{-6}\ A/M$
Sun (mean)	5.6×10^{-7}	3.5×10^{-6}
Moon (mean)	1.2×10^{-6}	7.3×10^{-6}
Jupiter (max.)	8.5×10^{-12}	5.2×10^{-11}

P. Fortescue

2/3 effect from Moon
1/3 effect from Sun

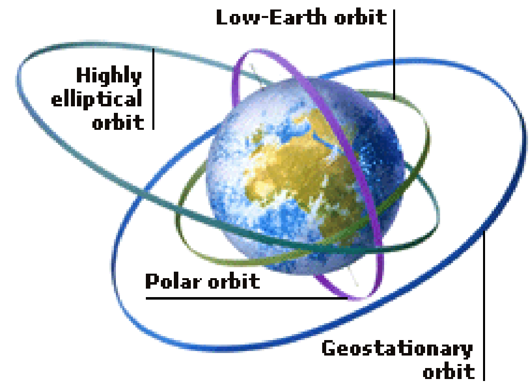
Additional information in: https://www.youtube.com/watch?v=Hcm7oQwpZfg

Summary: Disturbance forces affecting orbit



Type of satellite orbit

Type of satellite orbit



Low Earth orbit (LEO)

- Short orbital period (around 90 minutes)
- Typically 160 - 2000 km altitude
- Take into account atmospheric drag (up to 600 km)
- Frequent and lengthy passage through Earth´s shadow
- Usually circular orbit
- Each satellite is only visible to Earth station for short time per period
- Satellite required to exit protected orbit regime within 25 years after end of life

Medium Earth orbits (MEO)

- Orbital periods typically several hours
- Typically from LEO to geosynchronous altitude
- Earth station visibility typically several hours
- Common applications within MEO is constellation of 20-30 satellites providing Global Navigation Satellite System (GNSS) services

Type of satellite orbit

Geosynchronous orbit

- Orbital periods about 24 hours (sidereal rotation period)
- Geosynchronous region restricted to maximum of 15^{o} geodetic latitude and within 200 km of geostationary altitude

Geostationary orbit (GEO)

- About 36000 km altitude using Earth radius and about 24 hours period
- Idealized circular orbit with zero (or very small) inclination
- Always visible to Earth station in interesting area
- GEO appears stationary to observer on ground
- Most important orbit for telecommunications and Earth observation applications

Type of satellite orbit

High Earth orbit

- Orbit with apoapsis (greatest radial distance from focus) altitude more than 200 km beyond that of geostationary orbit

Highly elliptical orbit (HEO)

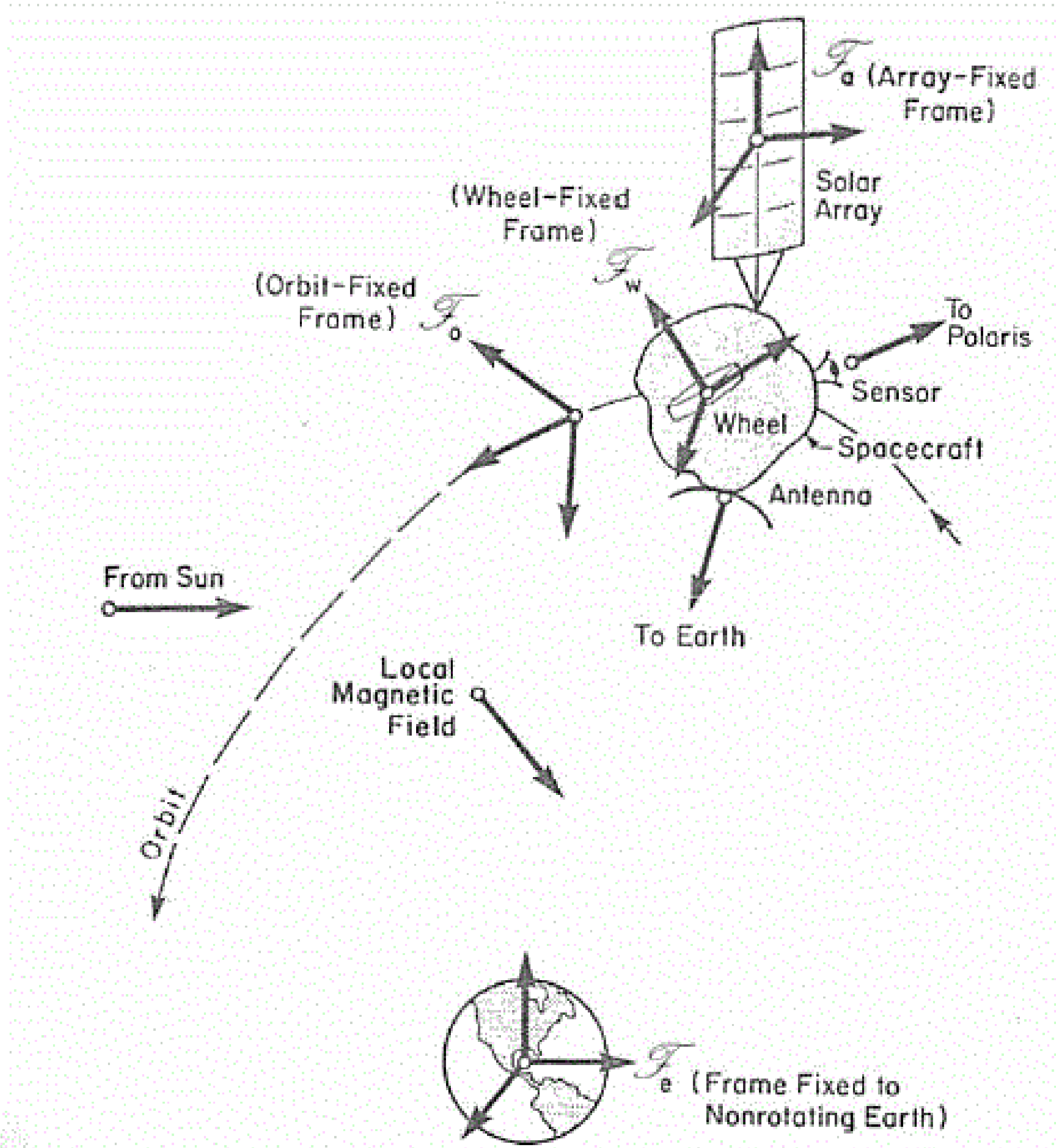
- HEO is subset of high Earth orbit with large eccentricity
- Long times around apoapsis (Kepler second law)
- Typical special cases: Molniya orbit (critical inclination orbits)
- Molniya orbit has zero rotation of perigee (63.4^o inclination)
- Used for coverage of higher latitudes which cannot be served by geostationary satellites (USSR)

Sun-Synchronous orbit

- Fixed orientation with respect ton Sun-line through full year
- Used for Earth observation

Coordinate systems

Typical attitude dynamics analysis



How to describe orientation of spacecraft or part of spacecraft?

Note:

Payload and sensor fixed coordinate systems are parameterized with respect to body coordinate systems

Alignment between different coordinate system is measured on ground but may shift during lunch and other disturbances

Precision attitude knowledge requires on orbit calibration of alignments shift and distortions

Typical coordinate systems

A coordinate system is a set of three mutually perpendicular unit vectors (orthonormal)

Typical coordinate system of interest for ADCS include:

- Heliocentric (Sun centered **inertial**)
- ECI (Earth centered **inertial**)
- Perifocal (Earth centered, orbital-based, **inertial**)
- ECEF (Earth-centered, Earth-fixed, **rotating**)
- Orbital (Earth-center orbital-based, **rotating**)
- Body-fixed (satellite-fixed, **rotating**)
- Etc.

You already have seen proper definition of coordinate systems and reference frames in “Entorno espacial y análisis de misión”, but short reminder

Inertial coordinate system

Inertial coordinate system is defined as a system that is **neither rotating nor accelerating** with respect to any other inertial origin

Alternative definition:

Inertial coordinate system is a system for which Newton’s laws are true

No known inertial coordinate system exist, but for most problems an inertial system can be found that is “inertial enough”

For some problems an Earth-fixed coordinate system is sufficient, whereas for some other problems rotation of Earth must be taken into account

Other coordinate systems

After proper definition of an inertial coordinate system, other coordinate systems can be defined according to need of problem

Three basic types of coordinate system:

- fixed in inertial space
- fixed relative to orbit
- fixed relative body of spacecraft

Heliocentric coordinate system

Heliocentric coordinate system (assumed to be inertial = fixed respect to stars)

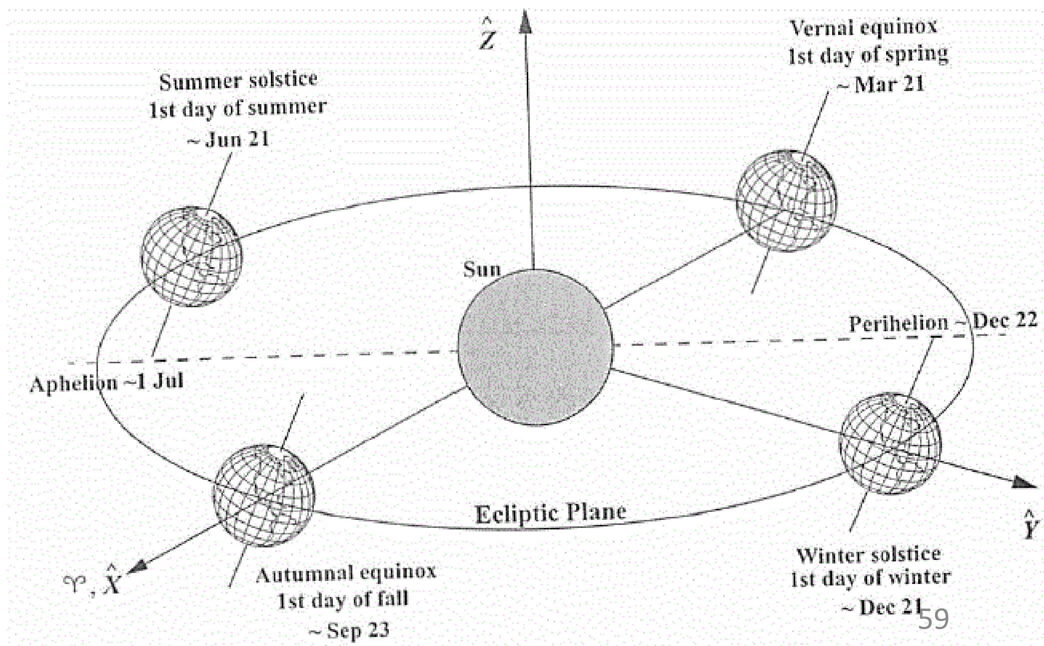
- Origin of system at center of Sun
- Primary direction defined by intersection of ecliptic and equatorial planes
- Equatorial plane inclined at around 23.5^o with respect to ecliptic plane
- Ecliptic plane contains Sun and Earth orbit
- Typically used for interplanetary mission

Season for northern hemisphere

Equinox:
Two equinoxes in year (spring and fall)

Length of day and night is same everywhere on Earth

Note: Equinox moves slowly over time

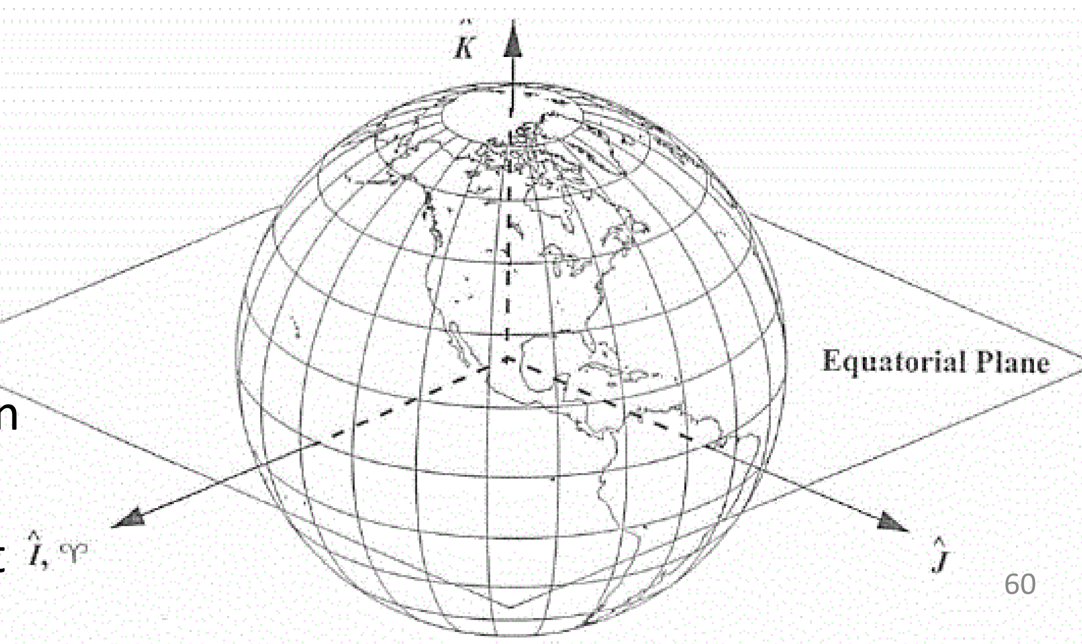


Earth centered inertial (ECI)

- Origin at center of mass of Earth
- Coordinate system defined by Earth´s equator and axis of rotation
- I-axis in vernal equinox direction
- K-axis = Earth’s rotation axis, which is perpendicular to equatorial plane
- J-axis in equatorial plane and defined by right hand rule
- Also called **geocentric equatorial coordinate system**
- ECI frames are inertial in contrast to Earth Centered Earth Fixed (ECEF) frames which rotate in inertial space

Earth centered inertially fixed coordinate system used to describe satellite orbital position and orientation

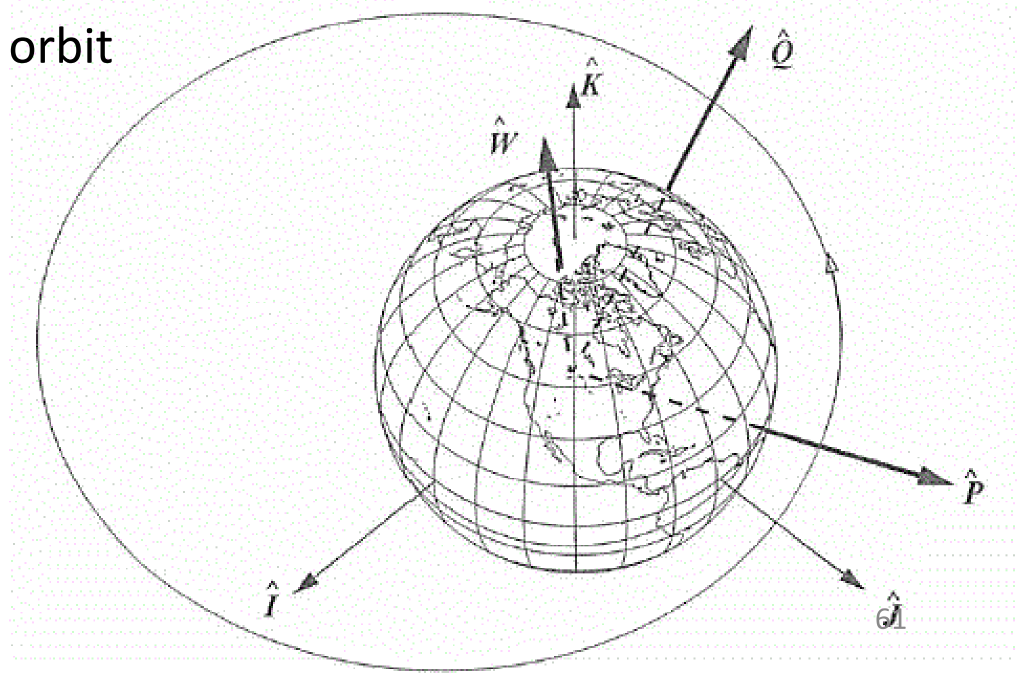
Since Earth axis move, reference frame specified with respect to an epoch date (J2000)



Towards Sun at vernal equinox

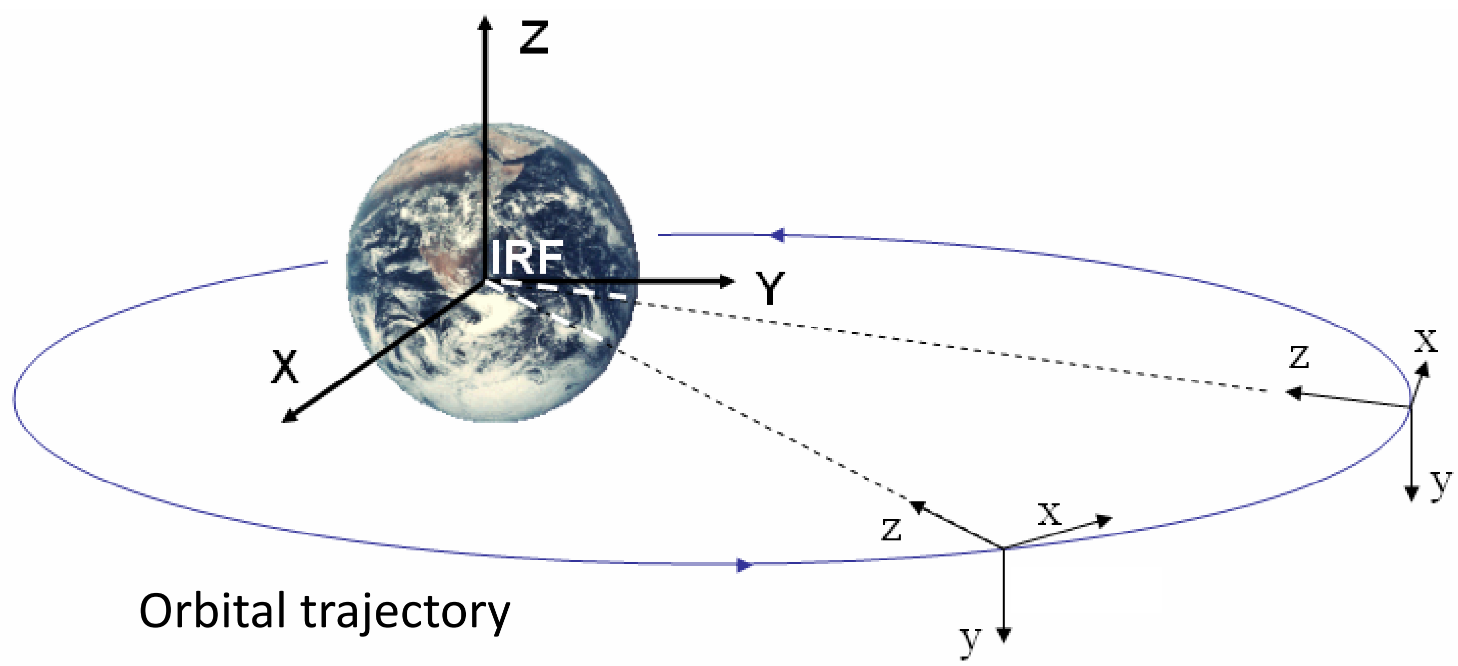
Perifocal coordinate system

- Origin at center of Earth
- Fundamental plane given by satellite orbit
- Earth-centered, orbit-based, inertial
- P-axis points toward perigee
- Q-axis 90^{o} from P-axis in direction of satellite motion in orbital plane
- W-axis perpendicular to orbital plane
- Note: PQW system maintains orientation towards perigee
  → not suitable for perfectly circular orbit



Orbital coordinate system

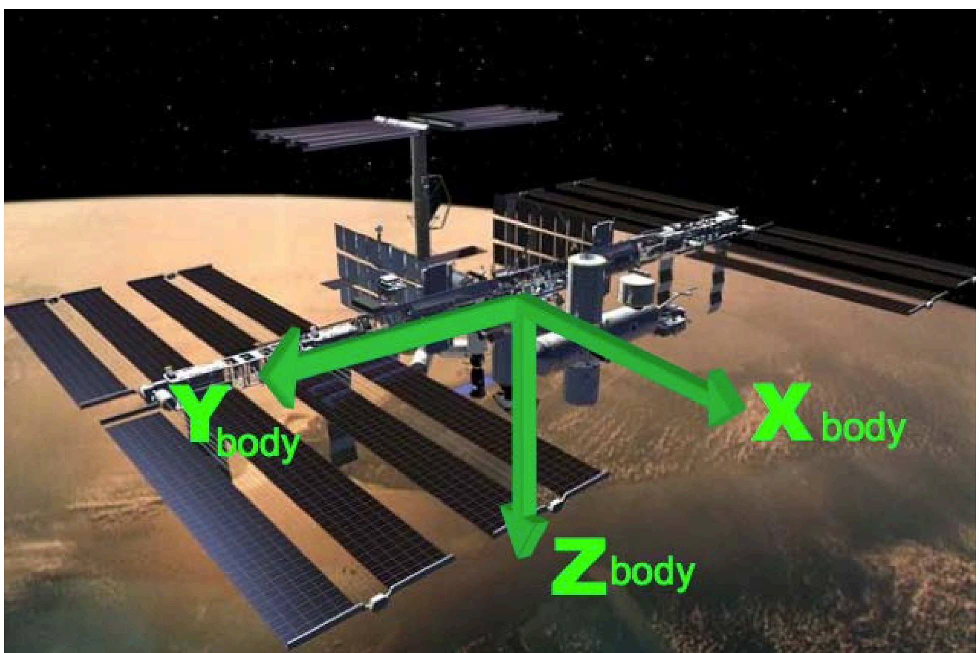
- Origin at center of mass of satellite
- Coordinate system rotates as satellite orbits
- z-axis always points to center of Earth (nadir direction)
- y-axis always in negative orbit normal direction ("pointing down")
- x-axis points in direction of motion (velocity vector direction for circular orbits)
- Typically notation for z-y-x frame is also o_3-o_2-o_1 frame
- In spacecraft dynamics these direction are given names "roll-pitch-yaw"

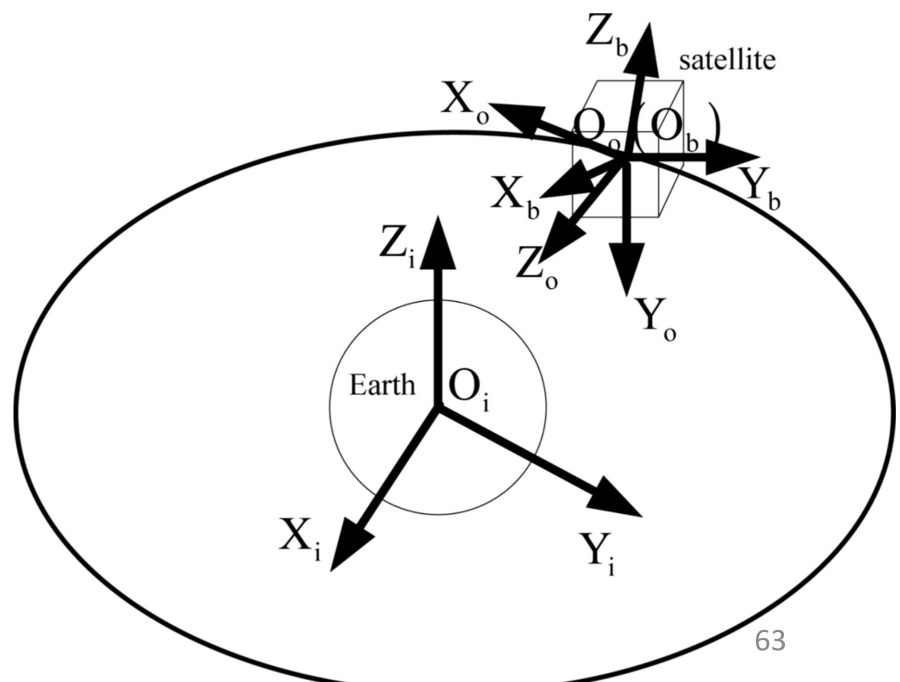


Orbital trajectory

Body fixed coordinate system

- Body fixed coordinate system is fixed with respect to satellite body
- Many different coordinate system adapted for specific satellite missions
- Assume body fixed reference frame only slightly displaced from orbiting frame
- z-axis in nadir direction
- y-axis in negative orbit normal direction
- x-axis points in direction of motion





Transformation between coordinate systems

Coordinate systems form reference for position/angular measurement

Coordinate transformation between two coordinate systems involve rotation and translation

Relationships between coordinate systems can be characterized in different ways:

- Direction cosine matrices
- Euler angle rotations
- Euler parameters, quaternion

Knowledge of relationship between coordinate systems is required for attitude determination and control

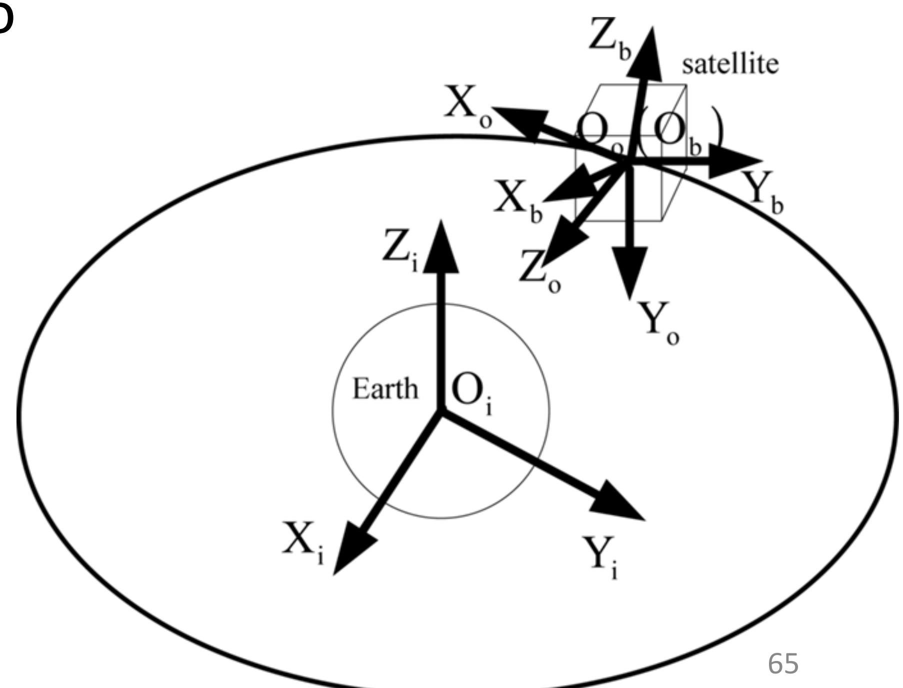
More details about coordinate transformation in lecture ADCS - VI

Transformation between successive coordinate systems

Transformation between successive frames can be determined from a series of matrix multiplication

E.g. transformation from inertial to body frame is given by

1. Inertial to Earth fixed transformation multiplied by
2. Earth fixed to orbit frame transformation multiplied by
3. Orbit to body frame transformation



Self control

Please have look to following videos:

“Spacecraft stabilization and control 1968”

https://www.youtube.com/watch?v=NJL1ey0zpZg

https://www.youtube.com/watch?v=NROrBUp96o0

Write short summary

After final lecture in January please have look again to these videos

Write short summary

Compare two summaries

Summary

Disturbance forces

- Atmospheric drag, Solar radiation pressure, Third body
- Restricted three body problem → Lagrange points → Halo orbits

GEO perturbation

Conclusion for near Earth orbits:

- J_2 acceleration is dominant perturbation for low Earth orbits
- Atmospheric drag dominant perturbation at very low altitudes

For geostationary orbit:

- J_2 less important than solar or moon perturbation (North-South drift)
- Tesserial harmonic (East-West drift) and solar radiation (change of e)

Type of satellite orbits

Coordinate systems (Reference frames)