Improvement of an SSA simulator and feasibility analysis of space missions

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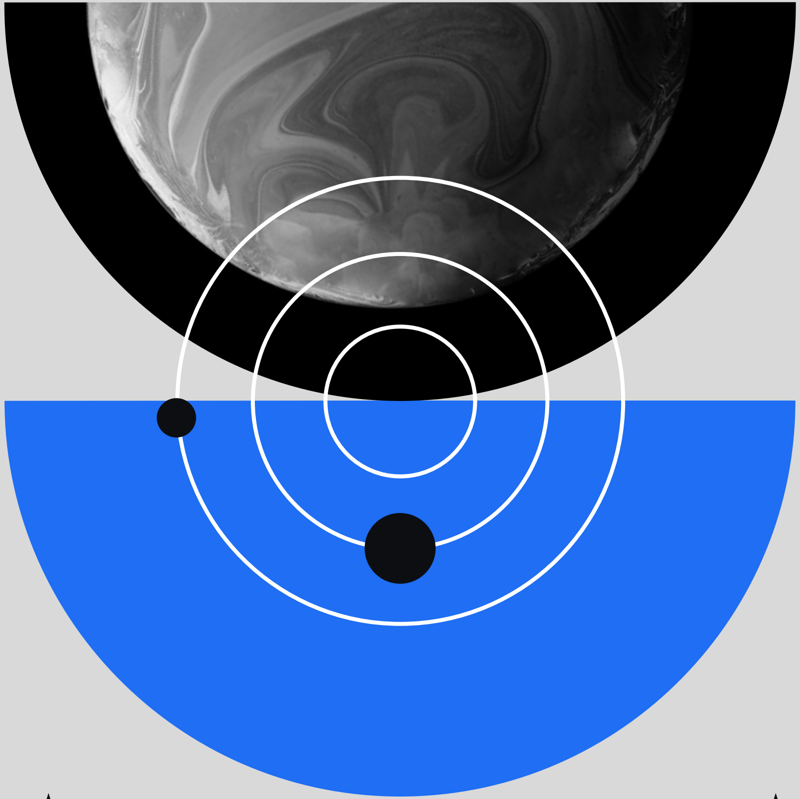
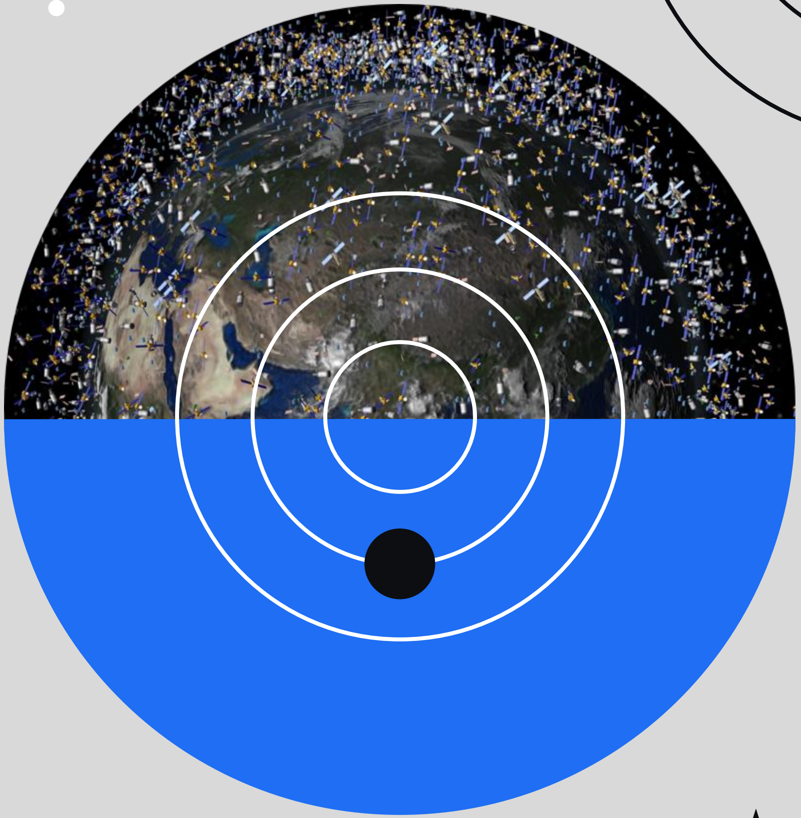


01

OVERVIEW

INTRODUCTION

- Continuous growth of space debris has become a Hazard.
- Role of Space Situational Awareness
- Space based SSA vs Earth based SSA.
- Role of SSA simulators.



MAIN OBJECTIVES

CODE

Refactor the code following a modular philosophy.

PHOTOMETRY

Get measurements of light emitted by RSO's.

RANGES OF OBSERVATION

Determine visibility windows and ranges of observation

ESTIMATOR PRECISSION

Define UKF precission

02 CODE REFACTORIZATION

INITIAL CODE

CODE STRUCTURE

- **Main Program** → **Simulation**
  - Simulation parameters
  - Simulation errors
  - Estimator structure and parameters
  - Other essential components

MAIN PROBLEMS

- Difficult to comprehend
- Spaghetti-code philosophy
- Impossible to introduce new features
- Repeated parameters

NEW CODE

STRUCTURE

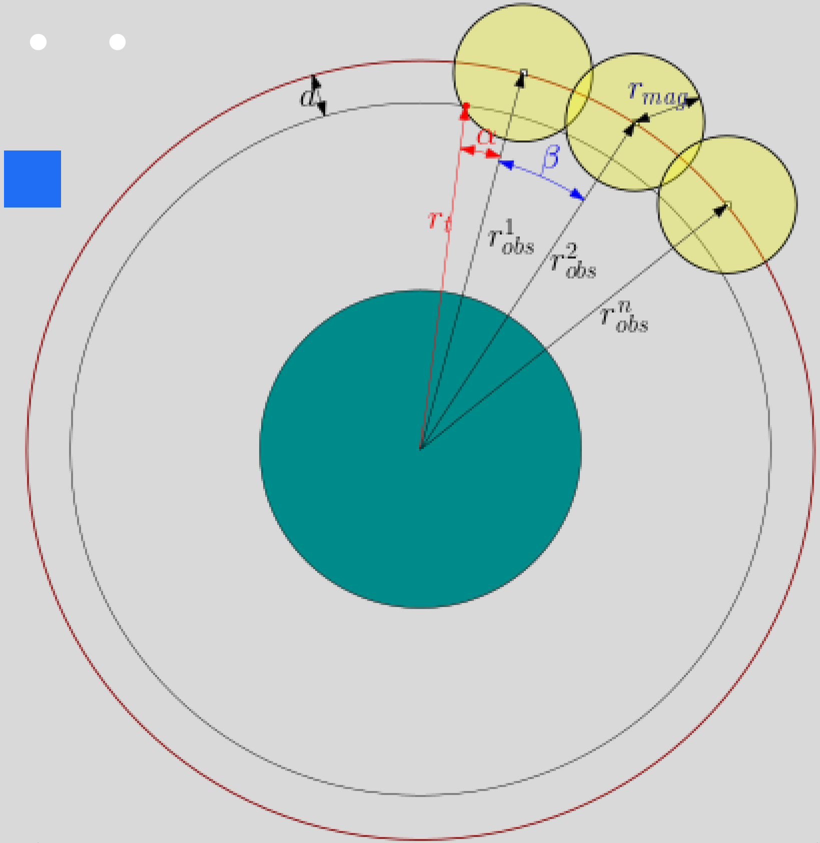
Simulator Configuration

Main Program

- **Estimation**
  - UKF Predictor
  - UKF Corrector
  - Back Propagator
- **Physics**
  - Orbit Parameters
  - Julian Date
  - Sun Ephemerids
- **Simulation**
  - Satellite
  - Data Simulation
  - Sim Measures
- **Photometry**
  - Apparent Magnitude
- **Utilities**
  - Rotation Matrix
  - Random quaternions

IMPROVEMENTS

- New structure
- **Creation of configuration file Sim_Config.m**
- **First approach to an observers constellation (BTH and ATH)**
- Implementation of a photometry module
- UKF Analysis



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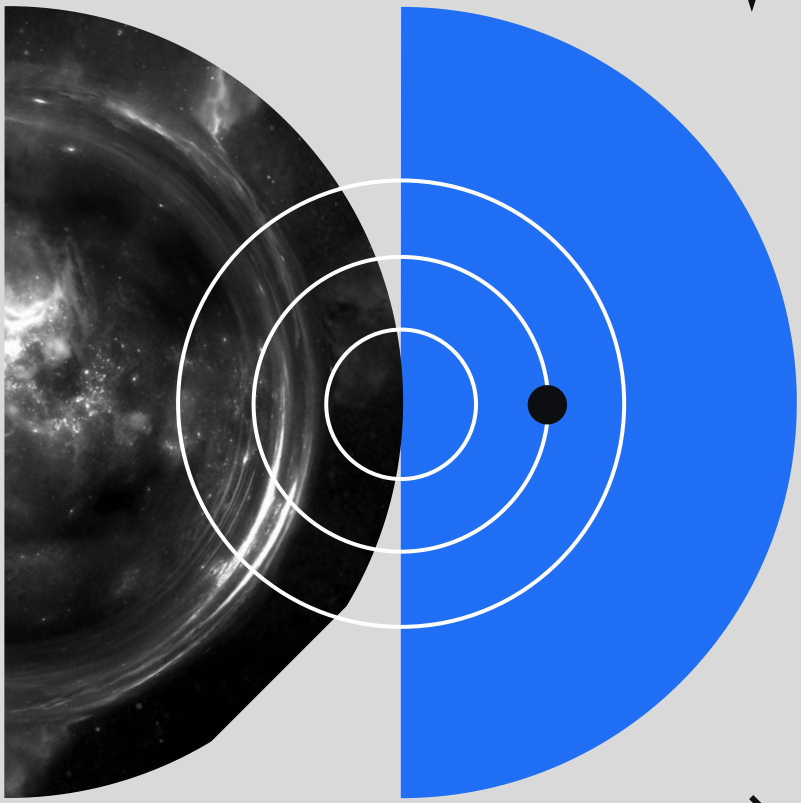
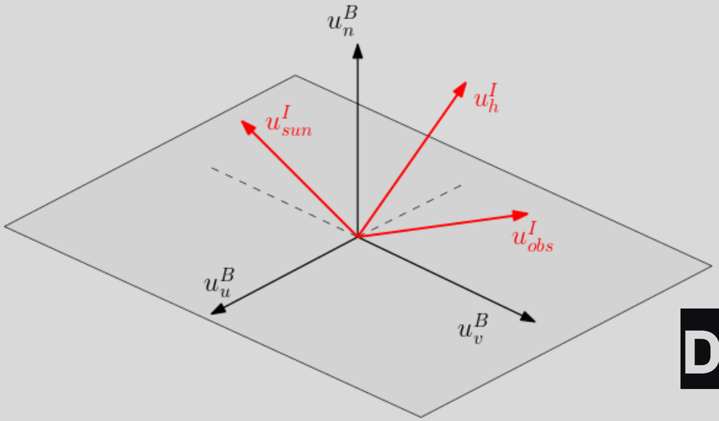
PHOTOMETRY



PHOTOMETRY



OBJECTIVE

- Obtain simulation-based photometric data

HYPOTHESIS

- Phong light reflexion model
- RSO has flat faces
- RSO is modeledas a satellite

DEPENDANCES

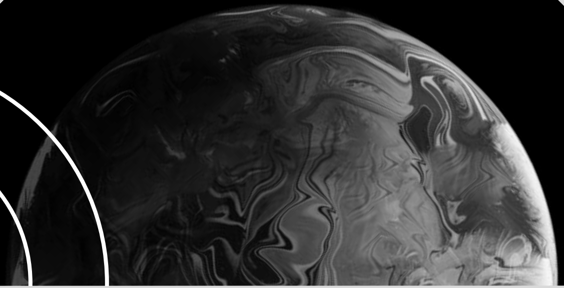
- RSO shape and materials
- Distance observer-RSO d
- Relative position RSO-Sun-Observer

RESULT

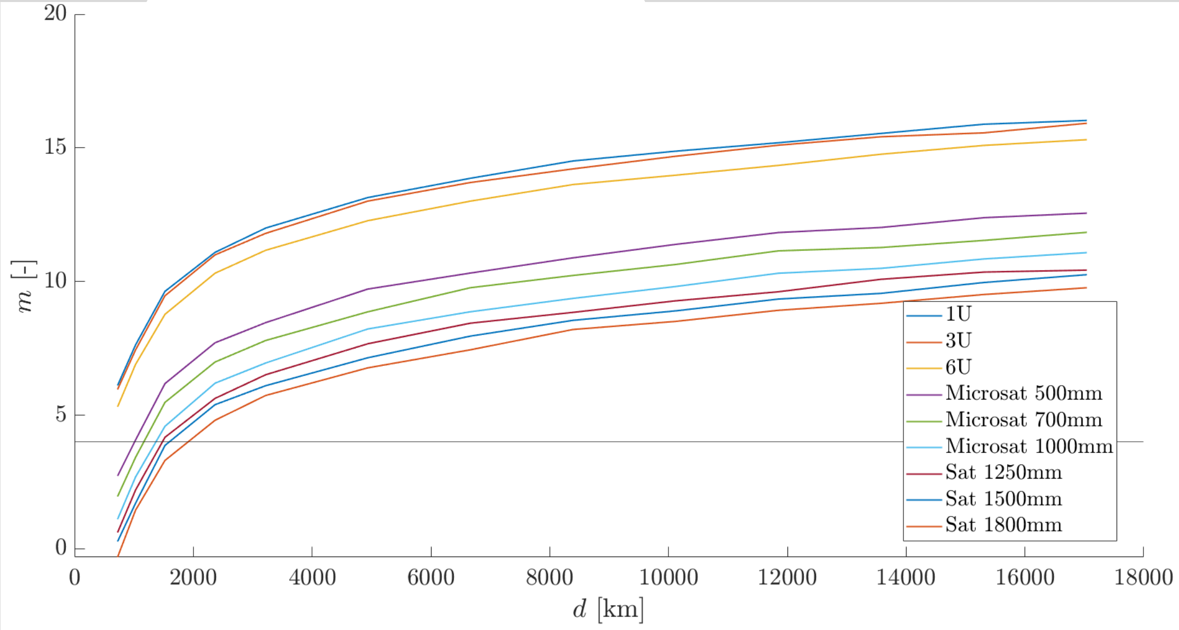
- Apparent Magnitude m
- Typical cubesat sensor $m = 4$

ANALYSIS & RESULTS

04



VISIBILITY SPHERE RADIUS



Dependance between apparent magnitude and d

RSO orbit parameters

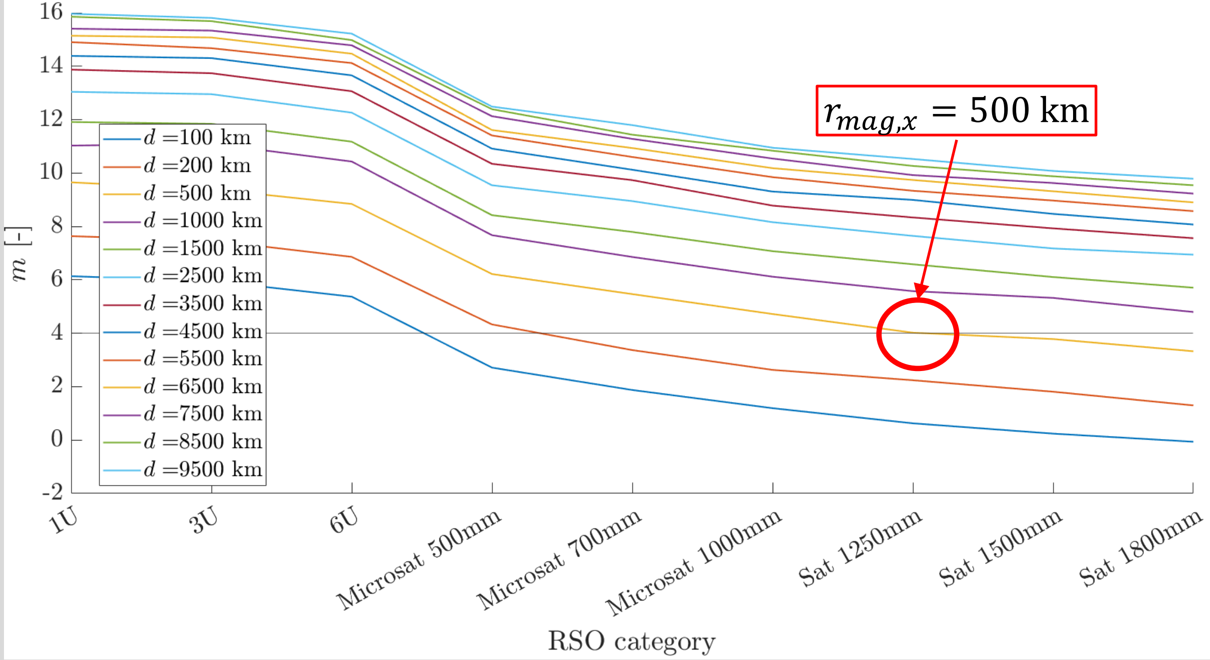
a [km]	e [-]	i [$\circ$]
6878	0	0,6

VISIBILITY SPHERE RADIUS

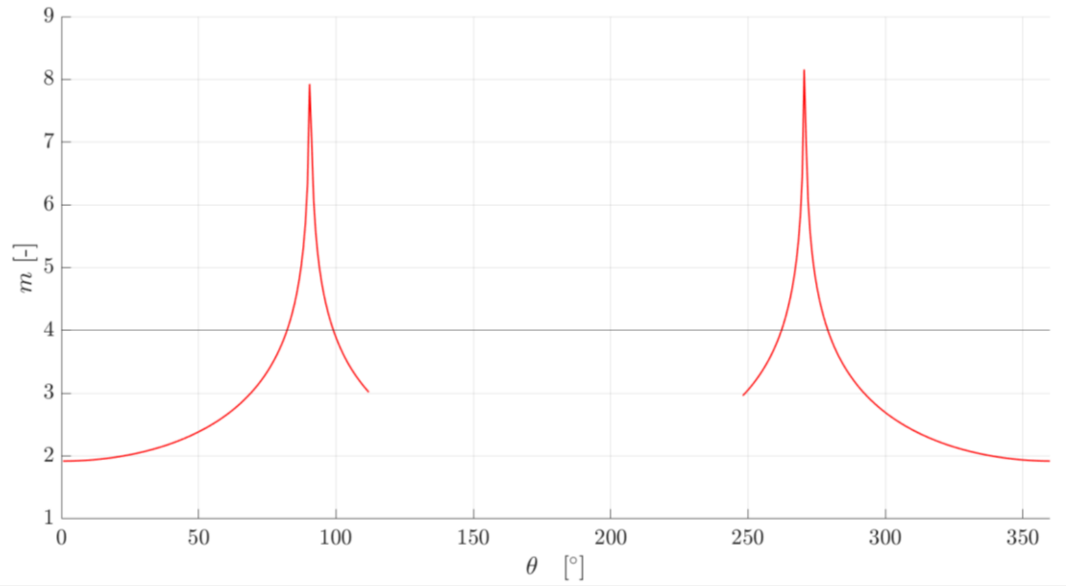
Dependance between apparent magnitude and RSO size

RSO orbit parameters

a [km]	e [-]	i [$\circ$]
6878	0	0,6



OBSERVATION WINDOWS



$\theta = \widehat{r_{RSO}, r_{Sun}}$, projected on the equatorial plane

Dependance between apparent magnitude m with and the relative positions Sun-Observer.

RSO orbit parameters

	Start	End
Date	1 Jan 2023	16 June 2023

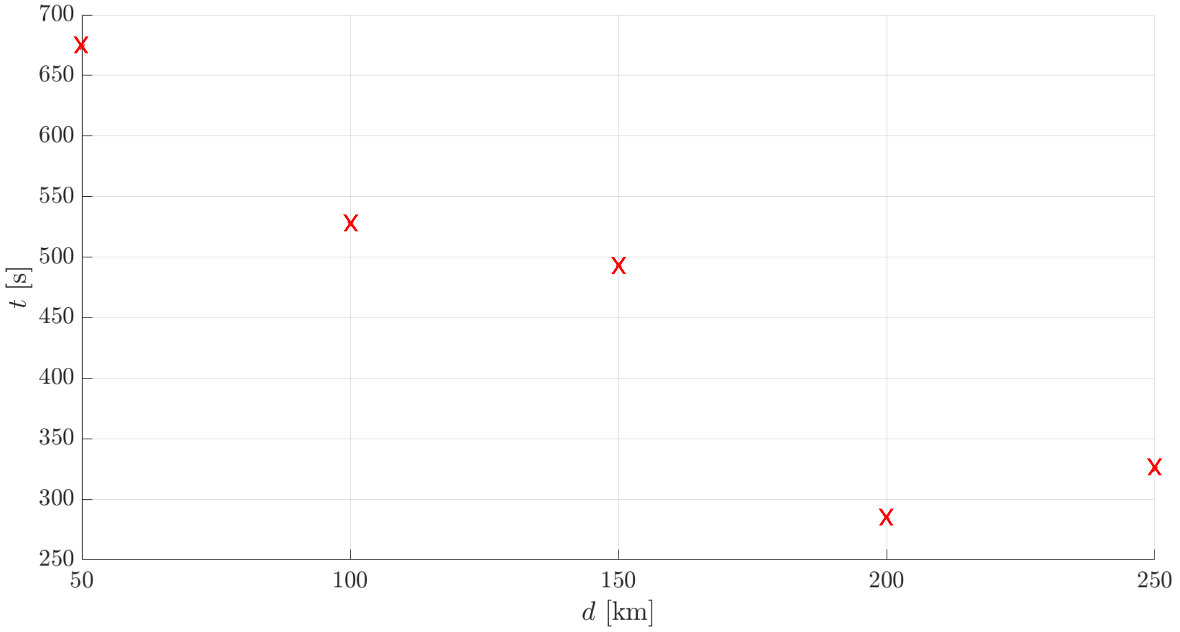
VISIBILITY TIMES

Dependance between visibility time and distance

- RSO is a 1.2 m satellite
- 5 observers

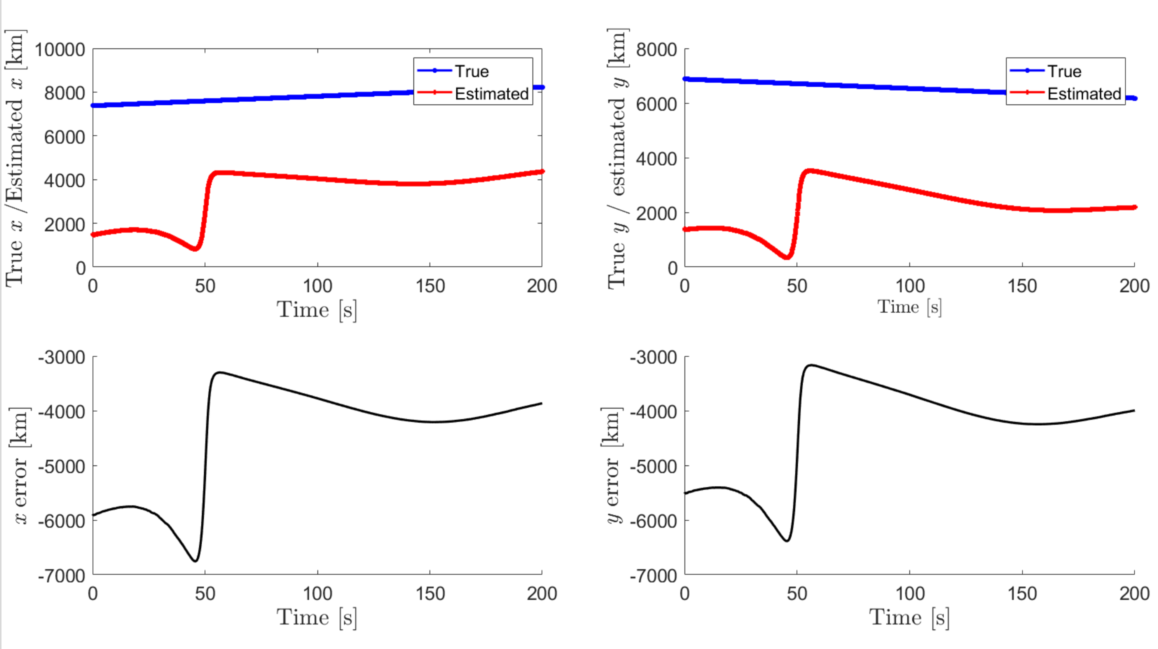
Average visibility times [s]

Max	*Min*
678,7	289,4



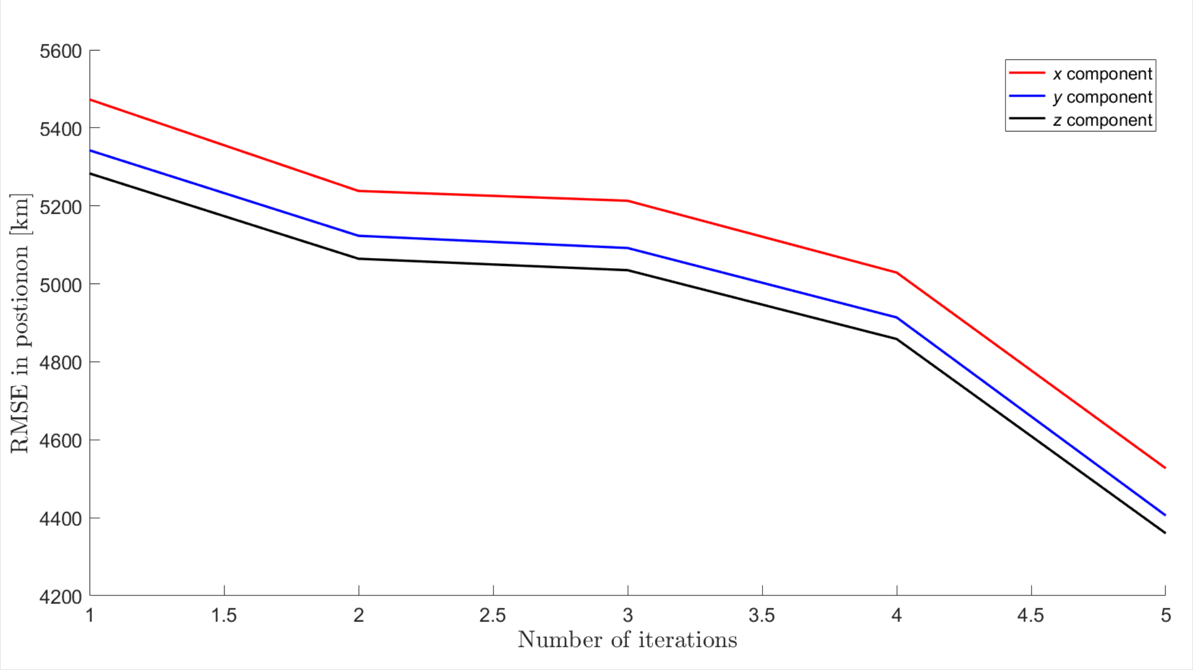
UKF PRECISSION

XY error for d = 50 km and t = 200 s with 1 observer.



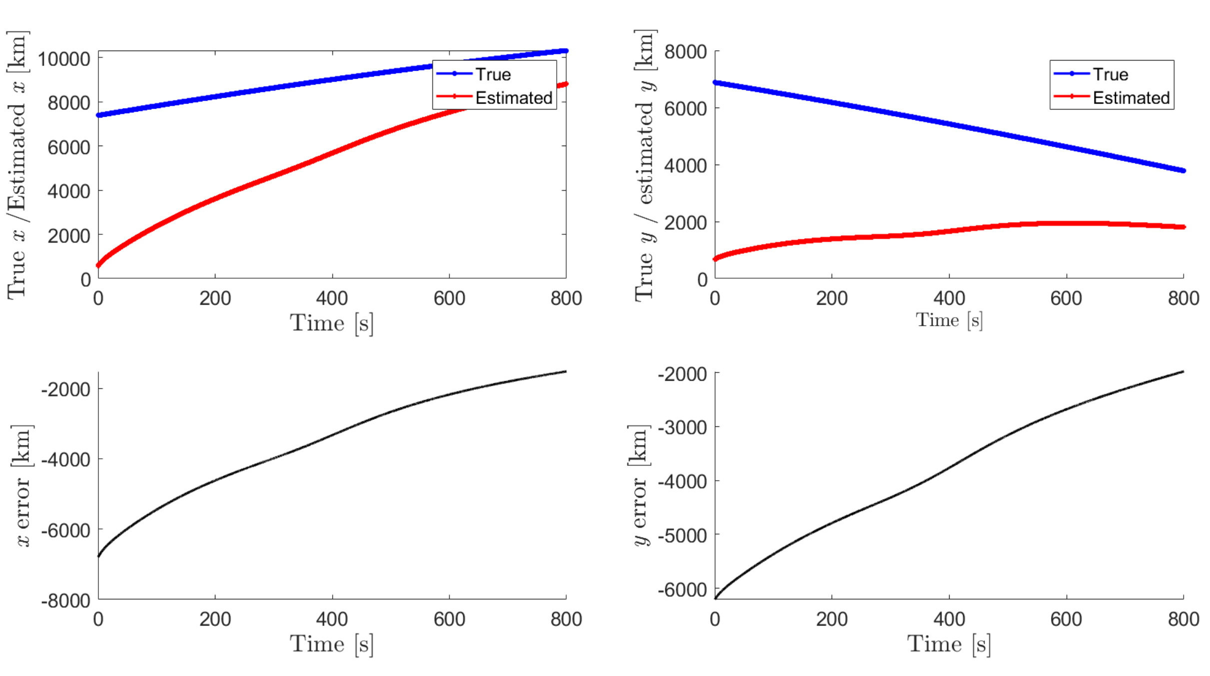
UKF PRECISSION

RMSE error for d = 50 km and t = 200 s with 1 observer.



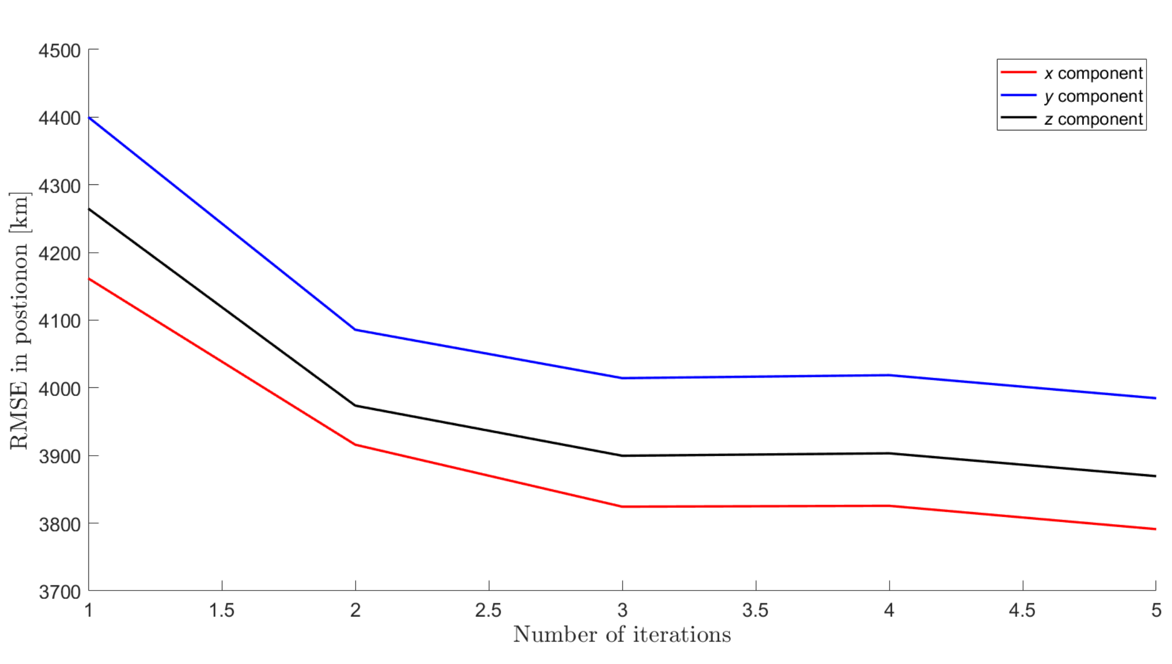
UKF PRECISSION

XY error for d = 50 km and t = 800 s with 1 observer.



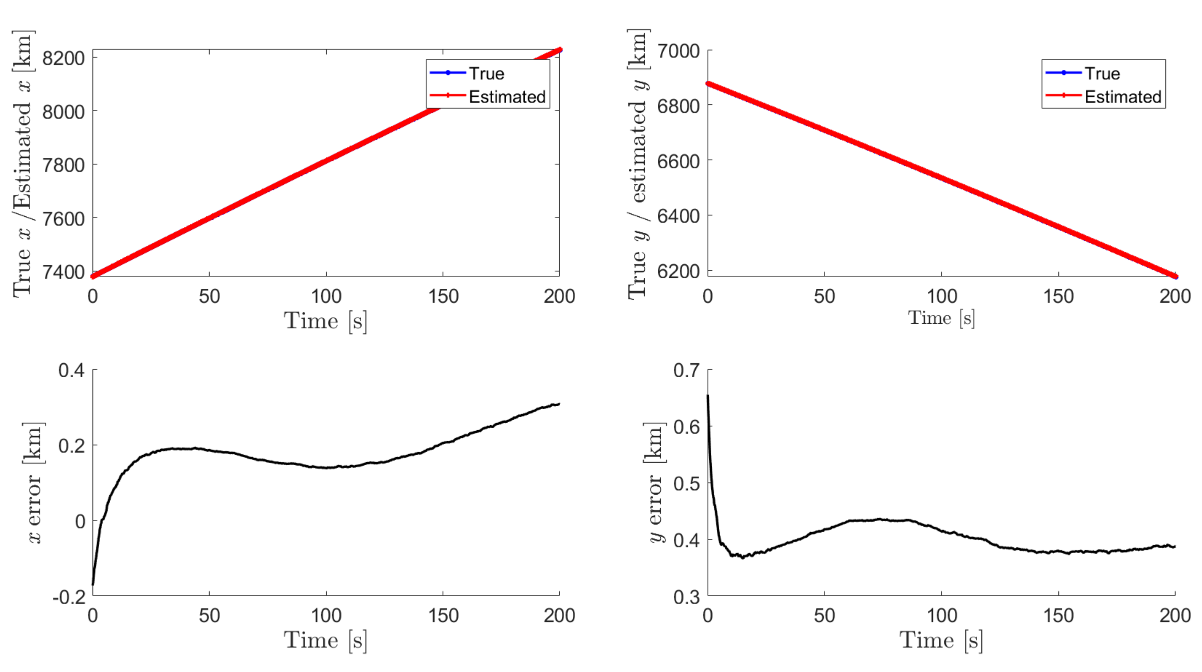
UKF PRECISSION

RMSE error for d = 50 km and t = 800 s with 1 observer.



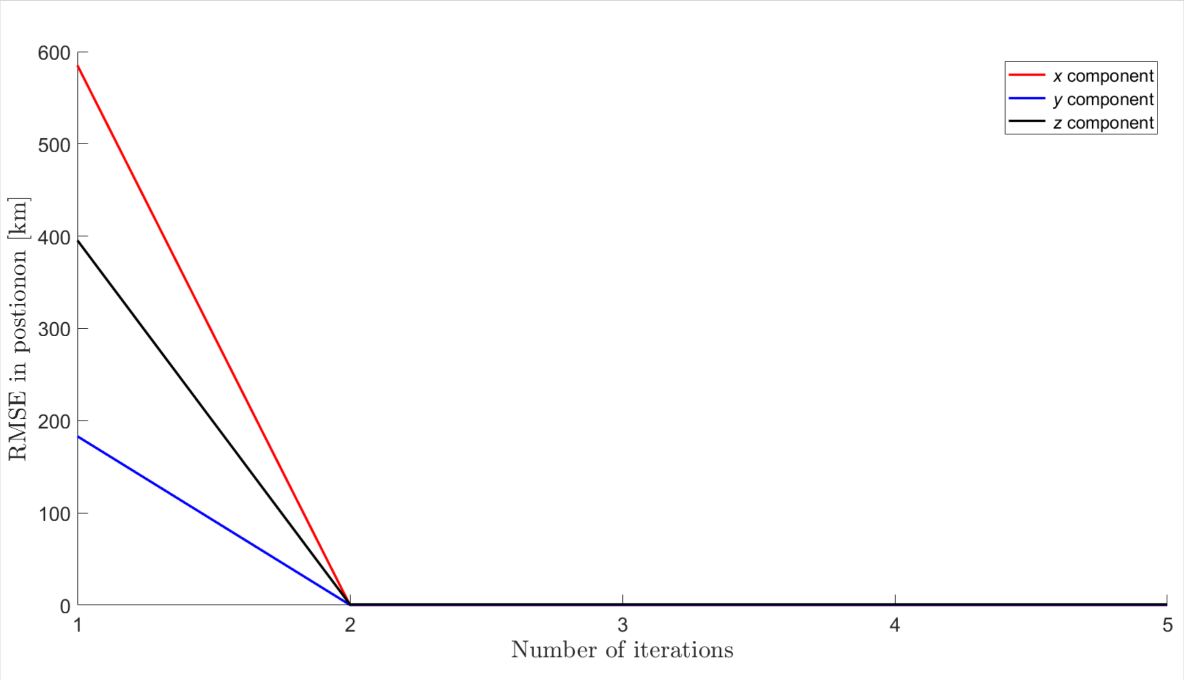
UKF PRECISSION

XY error for d = 50 km and t = 200 s with 2 observers.



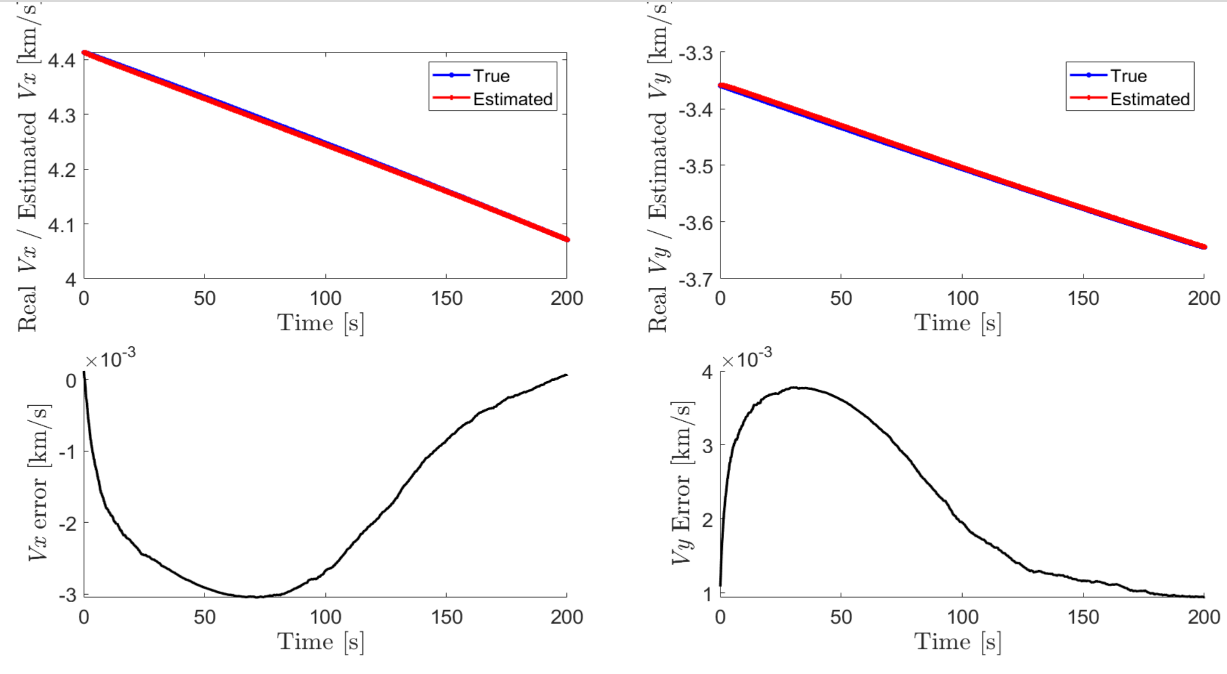
UKF PRECISSION

RMSE error for d = 50 km and t = 200 s with 2 observers.



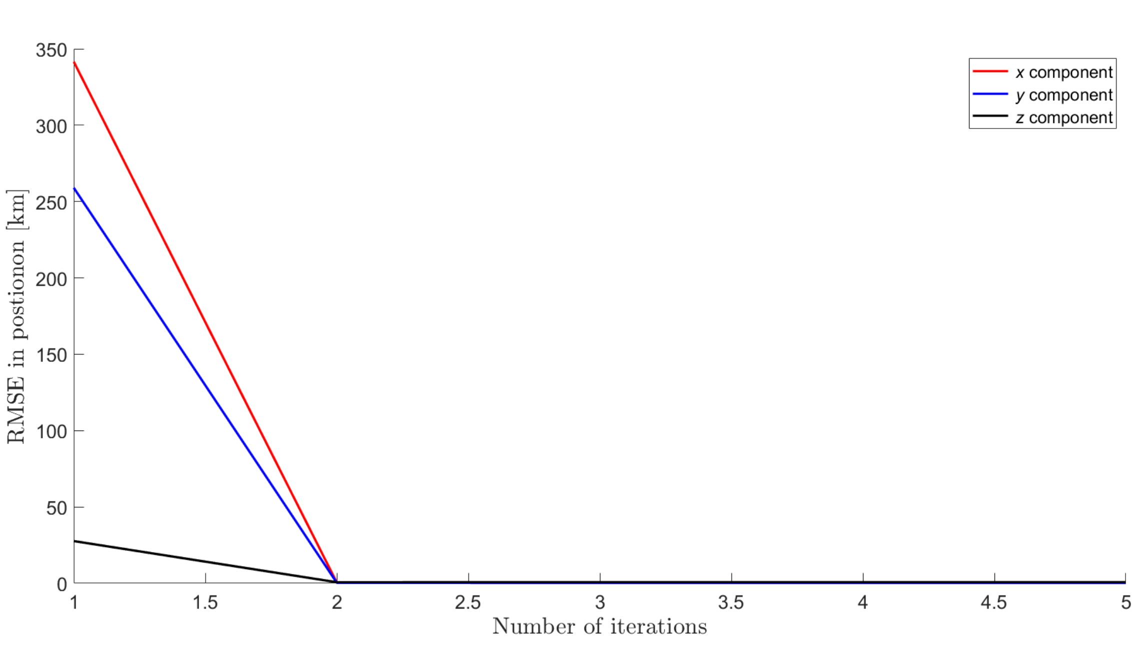
UKF PRECISSION

XY error for d = 50 km and t = 200 s with 5 observers.

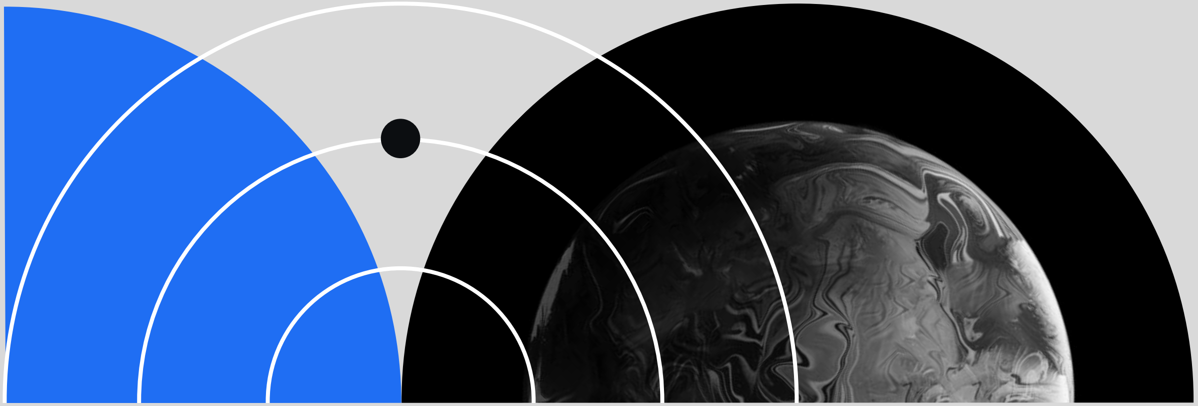


UKF PRECISSION

RMSE error for d = 50 km and t = 200 s with 1 observer.



05 CONCLUSIONS



CONCLUSIONS

- Code refactored for improved structure and modularity
- User-friendly script for simulation execution developed
- Photometry module calculates apparent magnitude, providing insights for future work
- Initial observation constellation prototype developed, requiring further refinement
- Sensitivity analysis identifies the number of observers as a critical variable for future improvements

FUTURE WORK

- The implementation of various types of constellations, so that the RSO
- is always within the visibility sphere of at least two satellites
- The expansion of functionalities within the photometry module to allow for diverse modeling of the RSO, for example, using light curves.
- The incorporation of RSO modeling into the estimator. This involves modifying the estimator so that magnitude becomes the input, rather than the position vector.
- The execution of extensive analyses to obtain a diverse population, facilitating more nuanced
- and quantitative results.

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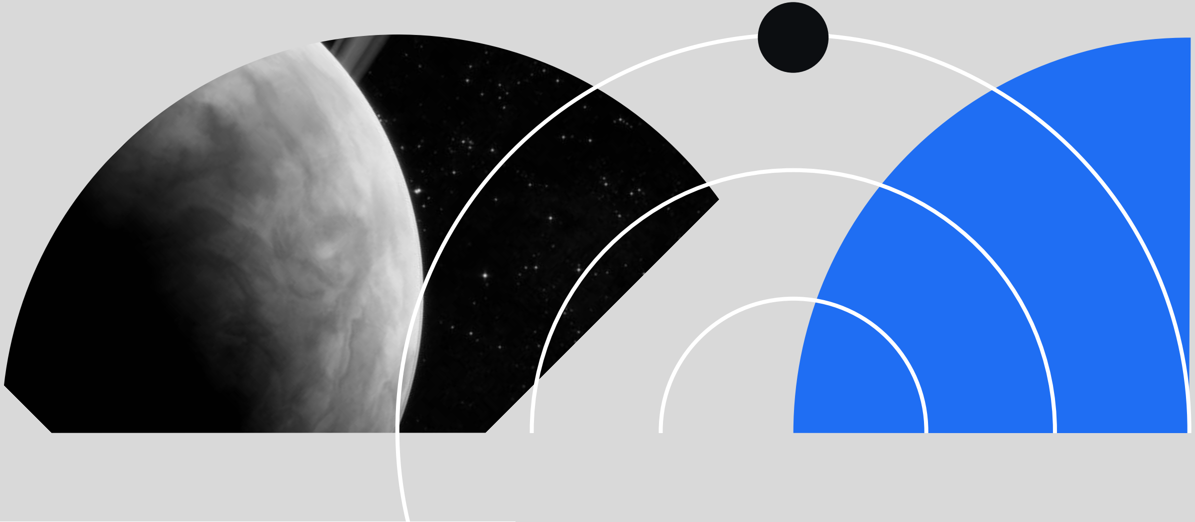
QUESTIONS?

APPENDIX



A1. SIMULATOR

- Newton Equation propagated with RK4

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{Bmatrix} = \begin{Bmatrix} \dot{\boldsymbol{r}} \\ -\frac{\mu}{r^3}\boldsymbol{r} + a_p \end{Bmatrix}.$$

$$X_{n+1} = X_n + \frac{\Delta t}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right),$$

$$t_{n+1} = t_n + \Delta t, \quad , withn = 0, 1, 2, 3...,$$

$$k_1 = f(t_n, X_n),$$

$$k_2 = f\left(t_n + \frac{\Delta t}{2}, X_n + \Delta t\frac{k_1}{2}\right),$$

$$k_3 = f\left(t_n + \frac{\Delta t}{2}, X_n + \Delta t\frac{k_2}{2}\right),$$

$$k_4 = f(t_n + \Delta t, y_n + \Delta t k_3).$$

A2. UKF FORMULATION

Consider this non-linear system

$$\boldsymbol{x}_{k+1} = f(\boldsymbol{x}_k, \boldsymbol{v}_k)$$

$$\boldsymbol{y}_{k+1} = h(\boldsymbol{x}_{k+1}, u_k)$$

Assuming a known (or accurately estimated) covariance matrix of the state variables (Pk) the sigma points are:

$$\chi_k^0 = x_k$$

$$\chi_k^i = \boldsymbol{x}_k + \left(\sqrt{(n+\lambda)\mathbf{P}_k}\right)_i$$

$$\chi_k^i = x_k + \left(\sqrt{(n+\lambda)\mathbf{P}_k}\right)_{i-n}$$

And the weights associated to these points

$$W^{0,m} = \frac{\lambda}{\lambda + n}$$

$$W^{0,c} = \frac{\lambda}{\lambda + n} + 1 - \alpha^2 + \beta \quad ,$$

$$W^{i,m} = W^{i,c} = \frac{\lambda}{2(\lambda + n)} \quad i = 1...2n$$

Then, the predicted state and covariance matrix are:

$$\mathbf{x}_{k+1}^- = \sum_{i=0}^{2n} W_i^m f(\chi_k^i)$$

$$\mathbf{P}_{k+1}^- = \sum_{i=0}^{2n} W_i^c \left(\chi_{k+1}^{-,i} - \mathbf{x}_{k+1}^-\right)\left(\chi_{k+1}^{-,i} - \mathbf{x}_{k+1}^-\right)^T + \mathbf{Q}$$

A2. UKF FORMULATION

This first prediction step keeps estimating the
estate variables until there is a new experimental (or simulated) measurement available, starting the correction step. When this happens, the expected state is calculated as follows

$$Y_{k+1}^{-,i} = h(\chi_{k+1}^{-,i})$$

$$y_{k+1}^{-} = \sum_{i=0}^{2n} W_i^m h(\chi_{k+1}^{-,i}).$$

Having the expected measurement, the corrected state can be calculated as:

$$\mathrm{P}_{\boldsymbol{y}\mathbf{y}} = \sum_{i=0}^{2n} W_i^c \left(Y_{k+1}^{-,i} - \boldsymbol{y}_{k+1}^{-}\right)\left(Y_{k+1}^{-,i} - \boldsymbol{y}_{k+1}^{-}\right)^{\mathrm{T}} + \mathbf{R}$$

$$\mathbf{P}_{xy} = \sum_{i=0}^{2n} W_i^c \left(\chi_k^i - \boldsymbol{x}_{k+1}^{-}\right)\left(Y_{k+1}^{-,i} - \boldsymbol{y}_{k+1}^{-}\right)^{\mathrm{T}}$$

$$K = \mathbf{P}_{xy}\mathbf{P}_{yy}^{-1}$$

$$x_{k+1} = x_{k+1}^{-} + K(y_{k+1} - y_{k+1}^{-})$$

$$\mathbf{P}_{k+1} = \mathbf{P}_{k+1}^{-} - K\mathbf{P}_{yy}K^{\mathrm{T}}.$$

$$\lambda = \alpha^2(n + \kappa) - n,$$

with $\alpha \approx 10^{-3}, \kappa \approx 0, \ \beta = 2$

A3. M.APP. CALCULATION

Vector from the observer to the RSO

$$\boldsymbol{d} = \boldsymbol{r_{RSO}} - \boldsymbol{r_{obs}}$$

Decompose the BDRF (Phong reflexión model

$$\rho_{tot}(i) = \rho_{spec}(i) + \rho_{diff}(i) \quad i = 1 \ldots n_c,$$

where:

$$\rho_{\mathrm{spec}}(i) = C_{\mathrm{spec}} \frac{(\mathbf{u}^{\mathbf{I}}_{\mathbf{obs}} \cdot \mathbf{u}^{\mathbf{I}}_{\mathbf{spec}})}{(\mathbf{u}^{\mathbf{I}}_{\mathbf{sun}} \cdot \mathbf{u}^{\mathbf{I}}_{\mathbf{n}})},$$

$$\rho_{diff}(i) = \frac{C_{diff}}{\pi}.$$

The fraction of (visible) light reaching the RSO is:

$$F_{\mathrm{sun}}(i) = \Phi_{\mathrm{sun,vis}} \rho_{\mathrm{total}}(i)(\mathbf{u}^I_n(i) \cdot \mathbf{u}^I_{\mathrm{sun}}),$$

Only a fraction of the light reflected by the RSO is visible to the observer:

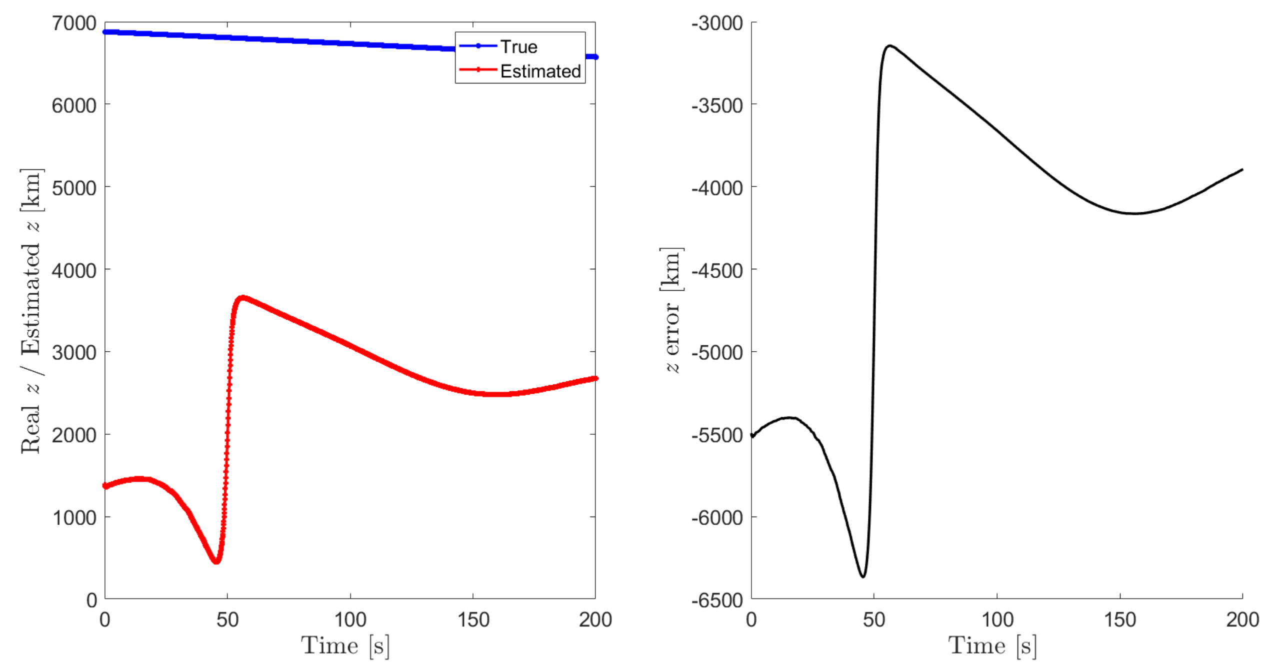
$$F_{\mathrm{obs}}(i) = \frac{F_{\mathrm{sun}}(i)\mathcal{A}(i)(\mathbf{u}^I_n(i) \cdot \mathbf{u}^I_{\mathrm{obs}})}{\|\mathbf{d}^I\|^2}.$$

Finally, the apparent magnitude is given by:

$$m_{\mathrm{app}} = -26.7 - 2.5 \log_{10} \left| \sum_{i=1}^{N_F} \frac{F_{\mathrm{obs}}(i)}{\Phi_{\mathrm{sun,vis}}} \right|.$$

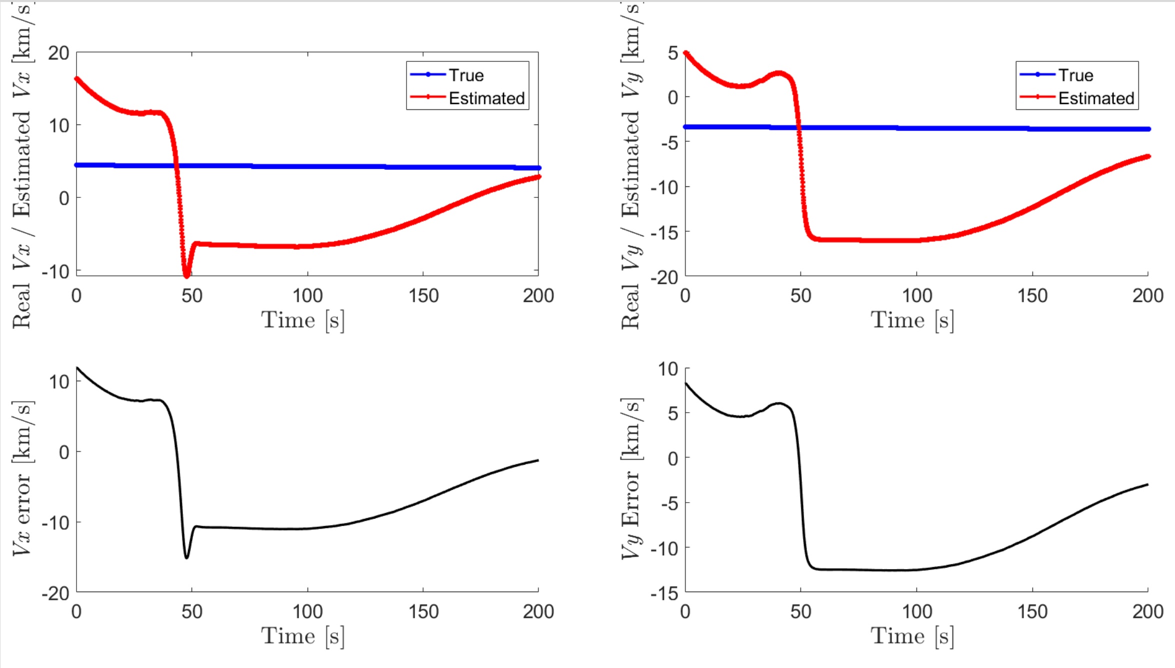
A4. UKF ANALYSIS

Z error for d = 50 km and t = 200 s with 1 observer.



A4. UKF ANALYSIS

Velocity XY error for d = 50 km and t = 200 s with 1 observer.



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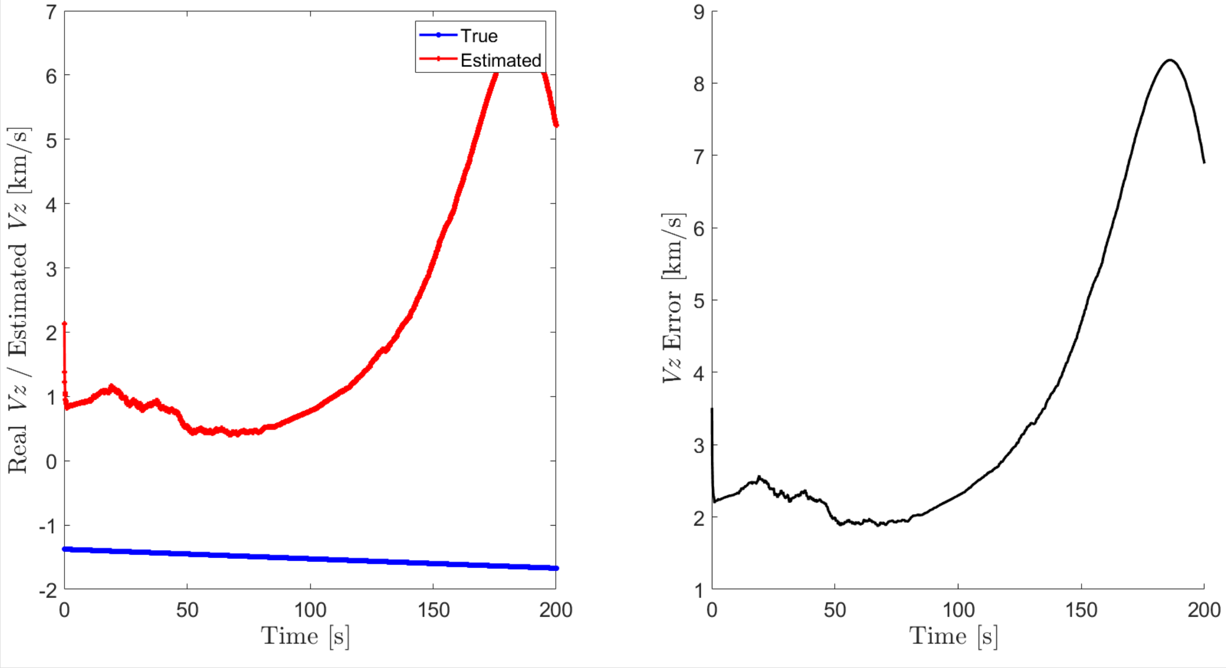
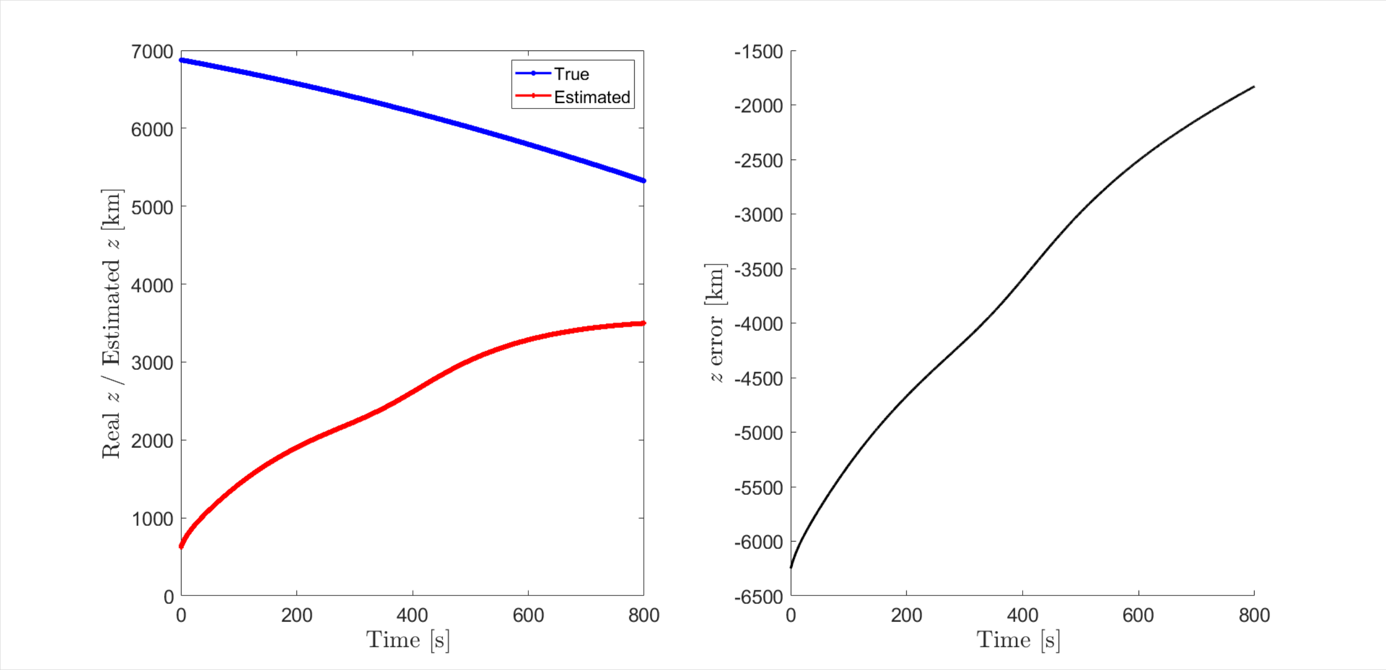


UKF PRECISSION

Z error for d = 50 km and t = 800 s with 1 observer.



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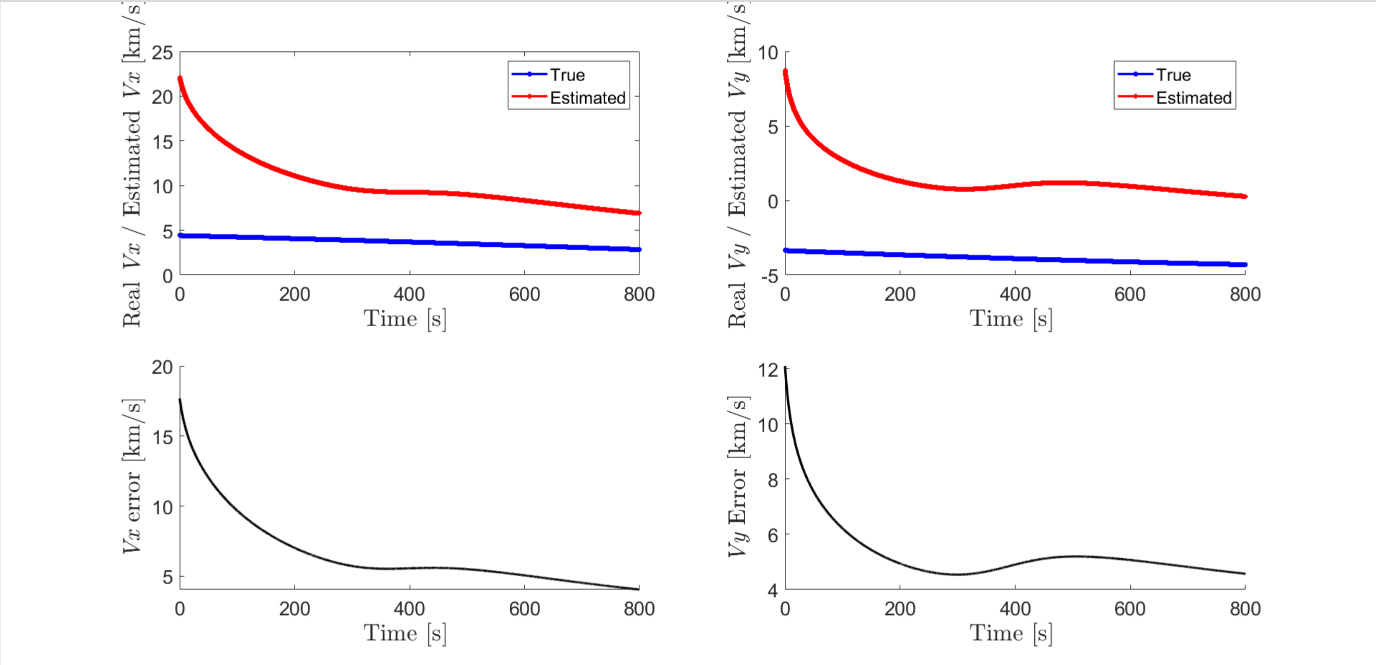

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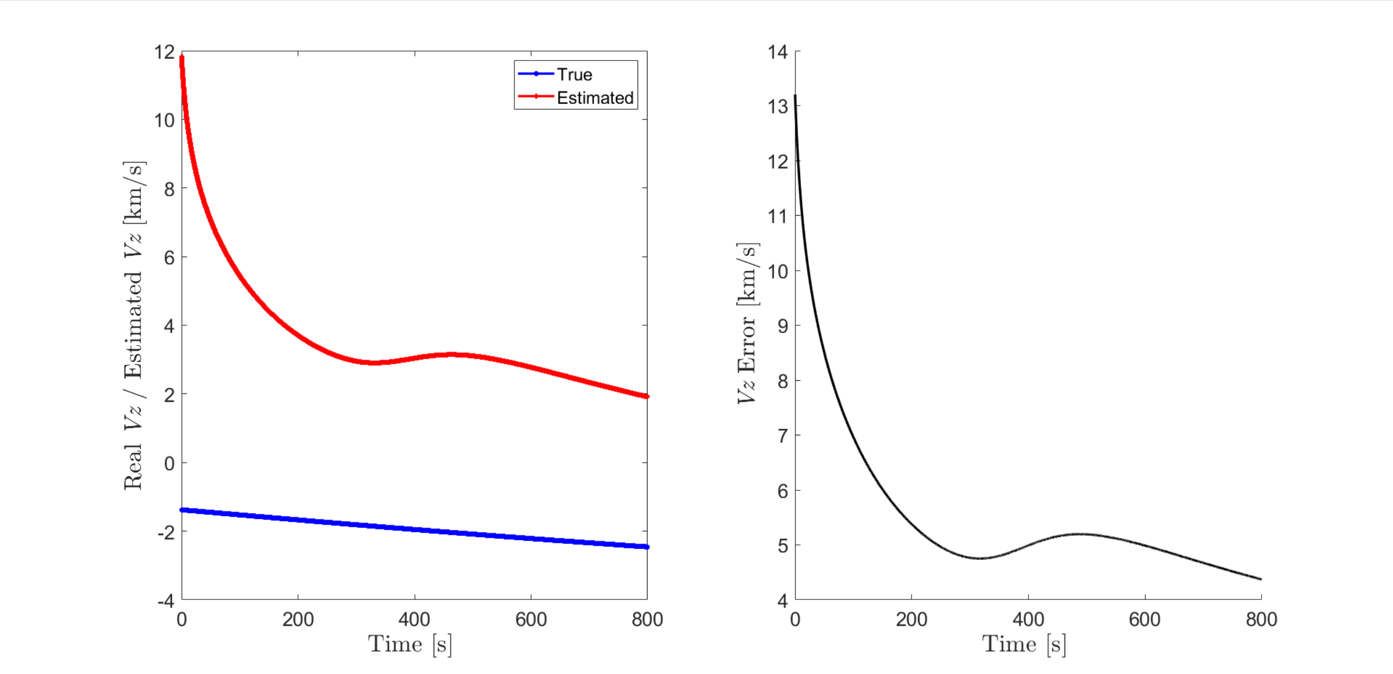
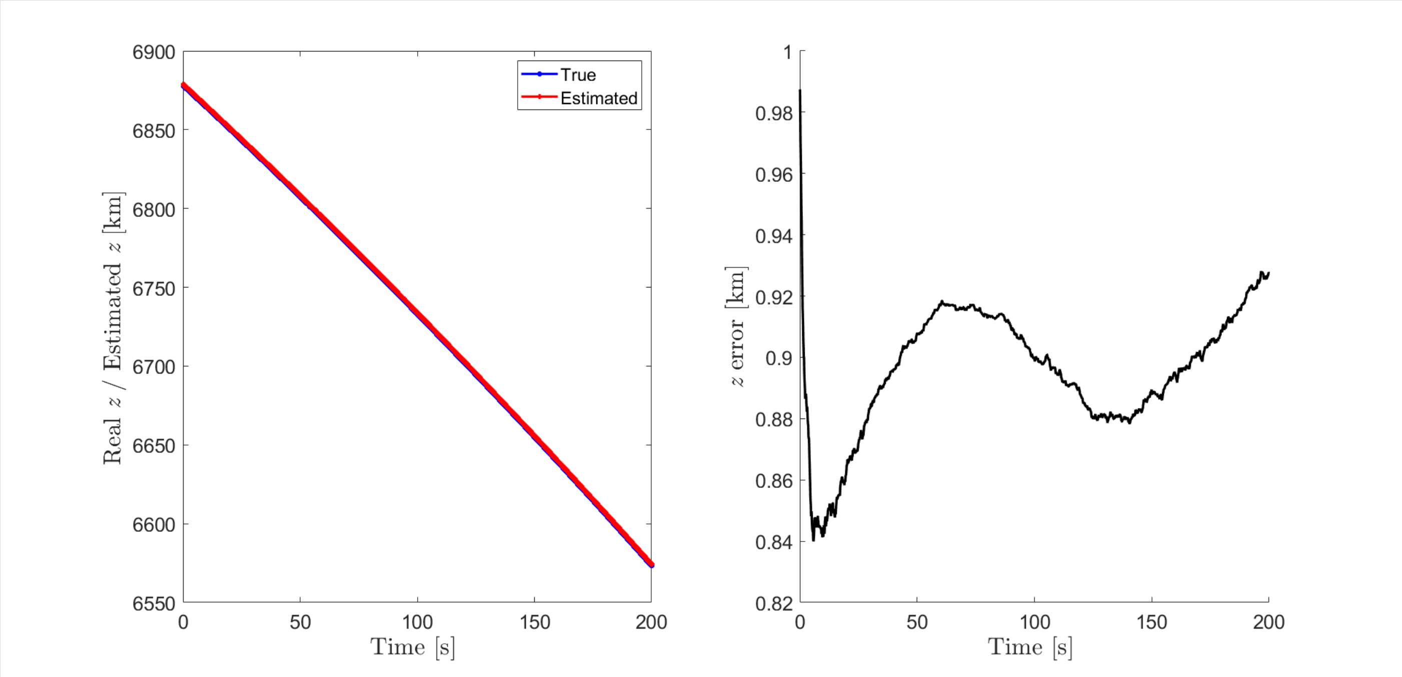


A4. UKF ANALYSIS

Z error for d = 50 km and t = 200 s with 2 observer.



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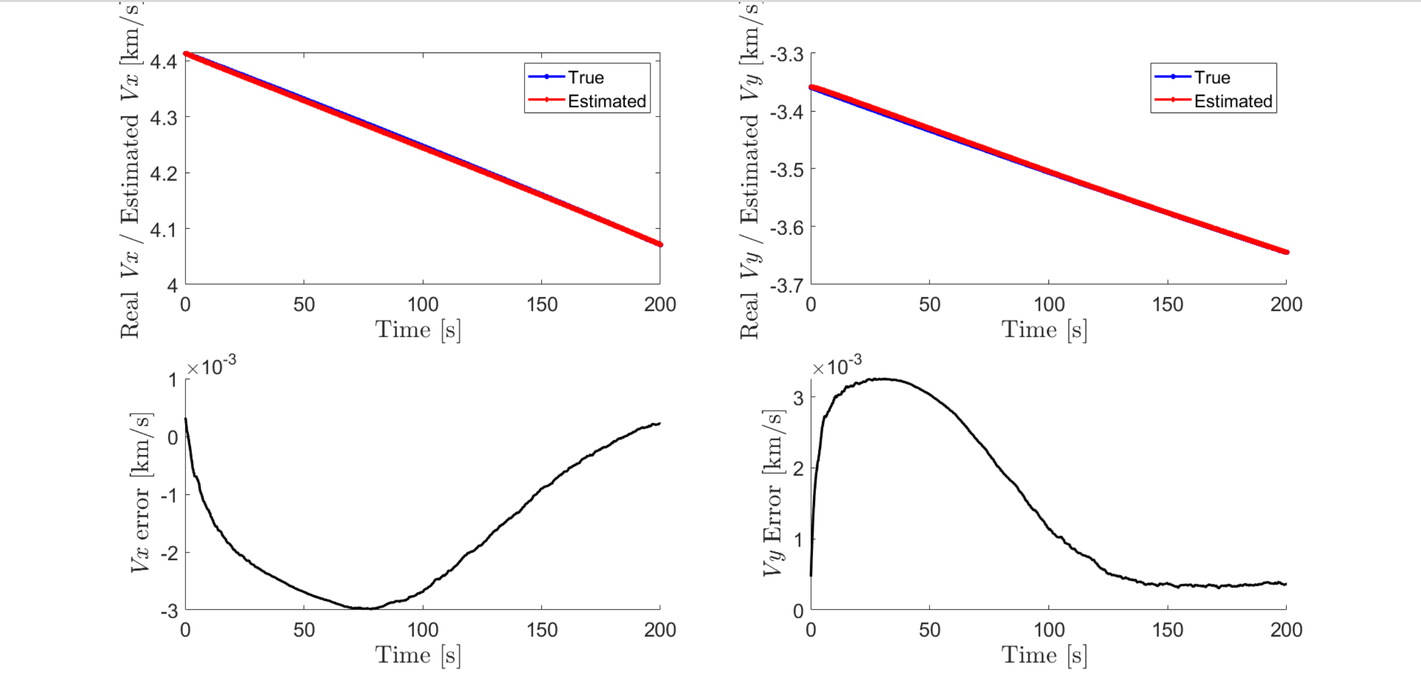
Velocity XY error for d = 50 km and t = 200 s with 2 observer.



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Velocity Z error for d = 50 km and t = 200 s with 2 observer.