# UNPA UARG - Análisis Matemático I

TERCER PARCIAL 2022 18 DE NOVIEMBRE

1. Resolver las siguientes integrales aplicando los métodos correspondientes:

   a. $\int \frac{ln(x)}{\sqrt{x}} \cdot dx$

   b. $\int 3x^6 \cdot \sqrt[3]{2-x^7} \cdot dx$

2. Resolver las siguientes integrales aplicando los métodos correspondientes:

   a. $\int \frac{2x-1}{\sqrt{x^2+2x+1}} \cdot dx$

   b. $\int \frac{2x+3}{(x-1)\cdot(x^2+2x-3)} \cdot dx$

3. Dada la superficie limitada por:

   - $y = \sqrt{x-2}$
   - $y = -\frac{1}{2}x + 5$
   - Eje y
   - Eje x

1) a $\displaystyle\int \frac{\ln(x)}{\sqrt{x}}\,dx$

Integración por partes $\rightarrow \int u\,dv = u\,v - \int v\,du$

CASO 1 (Una aplicación)

LIATE / ILATE

$u = \ln(x) \longrightarrow du = \frac{dx}{\sqrt{x}}$

$dv = \frac{dx}{\sqrt{x}} = x^{-\frac{1}{2}}\,dx \longrightarrow v = \int x^{-\frac{1}{2}}\,dx \longrightarrow \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \frac{\sqrt{x}}{\frac{1}{2}} = \boxed{2\sqrt{x}}$

$$\int \left(\ln(x)\,\frac{1}{\sqrt{x}}\right)dx = \ln(x)\,2\sqrt{x} - 2\int\left(\sqrt{x}\cdot\frac{1}{x}\right)dx$$

$$= 2\sqrt{x}\,\ln(x) - 2\underbrace{\int \frac{\sqrt{x}}{x}\,dx}_{}$$

$$\mathrm{I}\quad \int \frac{x^{\frac{1}{2}}}{x}\,dx = \int x^{\frac{1}{2}-1}\,dx \rightarrow -\tfrac{1}{2} = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \boxed{2\sqrt{x}}$$

$$= 2\sqrt{x}\,\ln(x) - 2\cdot 2\sqrt{x} \quad \mathrm{I}$$

$$\int\left(\ln(x)\cdot\frac{1}{\sqrt{x}}\right)dx = \boxed{2\sqrt{x}\,\ln(x) - 4\sqrt{x} + C}$$

b. $\int (3x^6 \cdot \sqrt[3]{2-x^7})\, dx = 3\int x^6 \cdot \sqrt[3]{2-x^7})\, dx$

$u = 2 - x^7$

$du = -7x^6\, dx$

$\frac{du}{-7} = x^6\, dx$

$= -3\int \frac{du}{7} \cdot \sqrt[3]{u}\, dx = -\frac{3}{7}\int \sqrt[3]{u} \cdot du$

$= -\frac{3}{7}\int u^{1/3}\, du = -\frac{3}{7} \cdot \frac{u^{\frac{1}{3}+1}}{\frac{1}{3}+1} = -\frac{3}{7} \cdot \frac{u^{4/3}}{\frac{4}{3}}$

$= -\frac{3}{7} \cdot \frac{3\sqrt[4]{u^3}}{4} = -\frac{9}{28} \cdot \sqrt[4]{u^3} = \boxed{-\frac{9}{28} \cdot \sqrt[4]{(2-x^7)^3} + C}$

$$2\ a)\ \int \frac{2x-1}{\sqrt{x^2+2x+1}}\,dx$$

$$\hookrightarrow (\ )' = 2x+2 \longrightarrow 2x+2 \neq 2x-1 \therefore$$

$mx+n$ no está incluido en la derivada de $ax^2+bx+c$

$$\underbrace{2x+2}_{\text{SUSTIT.}}\ \underbrace{-2+1}_{-1} \Rightarrow \int \frac{2x-1}{\sqrt{x^2+2x+1}}\,dx = \int \frac{2x+2-2-1}{\sqrt{x^2+2x+1}}\,dx$$

$$= \int \left[\frac{2x+2}{\sqrt{x^2+2x+1}} - \frac{1}{\sqrt{x^2+2x+1}}\right] dx = \underbrace{\int \frac{2x+2}{\sqrt{x^2+2x+1}}\,dx}_{\text{I}} - \underbrace{\int \frac{1}{\sqrt{x^2+2x+1}}\,dx}_{\text{II}} =$$

I SUSTITUCIÓN

II TCP

**I** $\int \frac{2x+2}{\sqrt{x^2+2x+1}}\, dx = \int \frac{du}{\sqrt{u}} = \int \frac{1}{\sqrt{u}} \cdot du =$

$u = x^2+2x+1$

$\rightarrow du = (2x+2)\,dx$

$\int \frac{1}{u^{1/2}}\, du = \int u^{-1/2}\, du = \frac{u^{-1/2+1}}{-\frac{1}{2}+1} = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} = \frac{\sqrt{u}}{\frac{1}{2}} = 2\sqrt{u} =$

$2 \cdot \sqrt{x^2+2x+1} = 2 \cdot \sqrt{(x+1)^2} = 2\left((x+1)^2\right)^{\frac{1}{2}} = \boxed{2(x+1)+C}$

**II** $\int \frac{1}{\sqrt{x^2+2x+1}}\, dx = \int \frac{1}{\cancel{\sqrt{}}(x+1)^{\cancel{2}}}\, dx = \int \frac{1}{x+1}\, dx =$

$u = x+1$

$du = dx$

$\int \frac{1}{u}\, du = \ln|u| + C = \boxed{\ln|x+1| + C}$

$\int \frac{2x-1}{\sqrt{x^2+2x+1}}\, dx = \boxed{2 \cdot (x+1) - \ln|x+1| + C}$

2) b. $\int \frac{2x+3}{(x-1).(x^2+2x-3)}\,dx$ $\int \frac{P(x)}{Q(x)}\,dx$

$\hookrightarrow$ $Gr\, P_{(x)} < Gr\, Q(x)$

1° Raíces $\rightarrow$ Factorizar $Q(x)$

$(x-1).(x^2+2x-3) = (x-1)(x-1)(x+3) = \boxed{(x-1)^2(x+3)}$

$\hookrightarrow x=1$ ; $x=-3$

$$\int \frac{2x+3}{(x-1)(x^2+2x-3)}\,dx = \int \frac{2x+3}{(x-1)^2(x+3)}\,dx = \int \frac{A}{(x-1)}\,dx + \int \frac{B}{(x-1)^2}\,dx + \int \frac{C}{(x+3)}\,dx$$

$$= \int \frac{9/16}{(x-1)}\,dx + \int \frac{5/4}{(x-1)^2}\,dx + \int \frac{-3/16}{(x+3)}\,dx$$

$$\boxed{\frac{2x+3}{(x-1)(x^2+2x-3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x+3)}} = \frac{A(x-1)^2(x+3)+B(x-1)\cdot(x+3)+C(x-1)(x-1)^2}{(x-1)(x-1)^2(x+3)}$$

$$2x+3 = A(x-1)^2(x+3) + B(x-1)(x+3) + C(x-1)(x-1)^2$$

$$\boxed{A = \frac{9}{16} \qquad B = \frac{5}{4} \qquad C = \frac{-3}{16}}$$

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$$\Rightarrow \frac{9}{16}\underbrace{\int \frac{dx}{x-1}} + \frac{5}{4}\underbrace{\int \frac{dx}{(x-1)^2}} - \frac{3}{16}\underbrace{\int \frac{dx}{x+3}}$$

SUSTITUCIÓN

$$\boxed{\begin{array}{l} u = x-1 \\ du = dx \end{array}} \qquad \boxed{\begin{array}{l} u = x+3 \\ du = dx \end{array}}$$

$$\Rightarrow \frac{9}{16}\int \frac{du}{u} = \frac{9}{16}\cdot \ln|u| = \boxed{\frac{9}{16}\ \ln|x-1|}$$

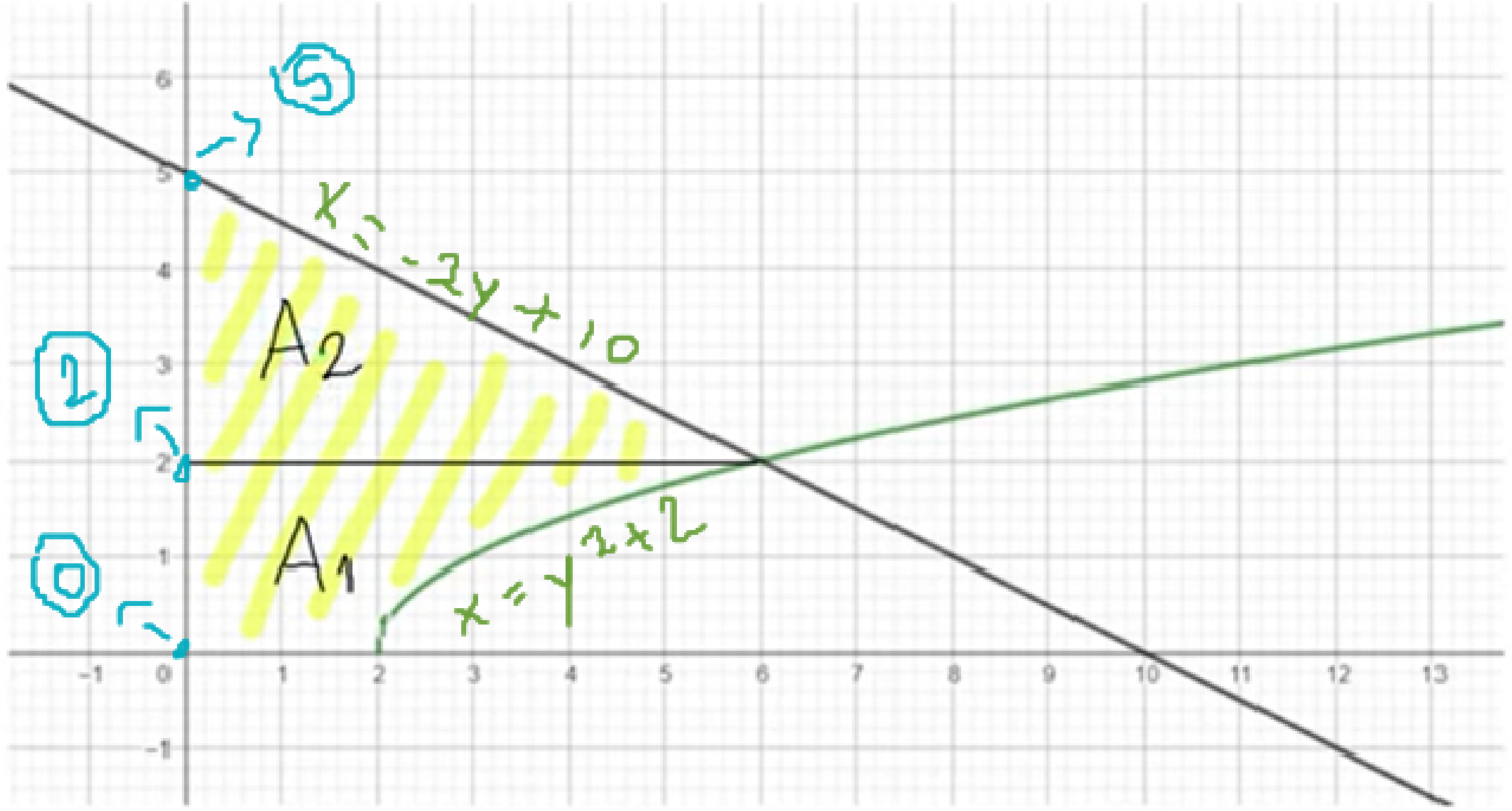
$$\Rightarrow \frac{5}{4}\int \frac{du}{u^2} = \frac{5}{4}\cdot\frac{u^{-1}}{-1} = \frac{5}{4}\cdot\left(-\frac{1}{u}\right) = -\frac{5}{4(u)} = \boxed{-\frac{5}{4(x-1)^2}}$$

$$\Rightarrow -\frac{3}{16}\int \frac{du}{u} = -\frac{3}{16}\cdot \ln|u| = \boxed{-\frac{3}{16}\cdot \ln|x+3|}$$

$$\int \frac{2x+3}{(x-1)\cdot(x^2+2x-3)}\,dx = \boxed{\frac{9}{16}\ln|x-1| - \frac{5}{4(x-1)^2} - \frac{3}{16}\ln|x+3| + C}$$

Dada la superficie limitada por:

- $y = \sqrt{x-2}$
- $y = -\frac{1}{2}x + 5$
- Eje y
- Eje x




a. Sombrear la superficie indicada en el gráfico.
b. Calcular extremos de integración.
c. Calcular el área de la superficie.

$$y = \sqrt{x-2}$$

$$y^2 = \left(\sqrt{x-2}\right)^2$$

$$y^2 = x - 2$$

$$y^2 + 2 = x \Rightarrow \boxed{x = y^2 + 2}$$

$$y = -\frac{1}{2}x + 5$$

$$\frac{1}{2}x = 5 - y$$

$$\boxed{x = -2y + 10}$$

Cálculo de extremos de integración

$$-2y+10=0$$

$$-2y=-10$$

$$y=-\frac{10}{-2} \Rightarrow \boxed{y=5}$$

$$y^2+2=-2y+10$$

$$y^2+2y-8=0 \quad \begin{cases} \boxed{y_1=2} \\ y_2=-4 \end{cases}$$

$$A_1=\int_0^2 (y^2+2)\cdot dy=\left[\frac{y^3}{3}+2y\right]_0^2=\left(\frac{2^3}{3}+2\cdot2\right)-\cancel{\left(\frac{0^3}{3}+2\cdot0\right)}=\frac{8}{3}+4=\boxed{\frac{20}{3}\ us}$$

$$A_2=\int_2^5 (-2y+10)\cdot dy=\left[-\frac{\cancel{2}y^2}{\cancel{2}}+10y\right]_2^5=\left[-y^2+10y\right]_2^5=\left(-5^2+10\cdot5\right)-\left(-2^2+10\cdot2\right)=$$

$$\left(-25+50\right)-\left(-4+20\right)=25-16=\boxed{9us}$$

$$A_t=A_1+A_2=\frac{20}{3}us+9us=$$

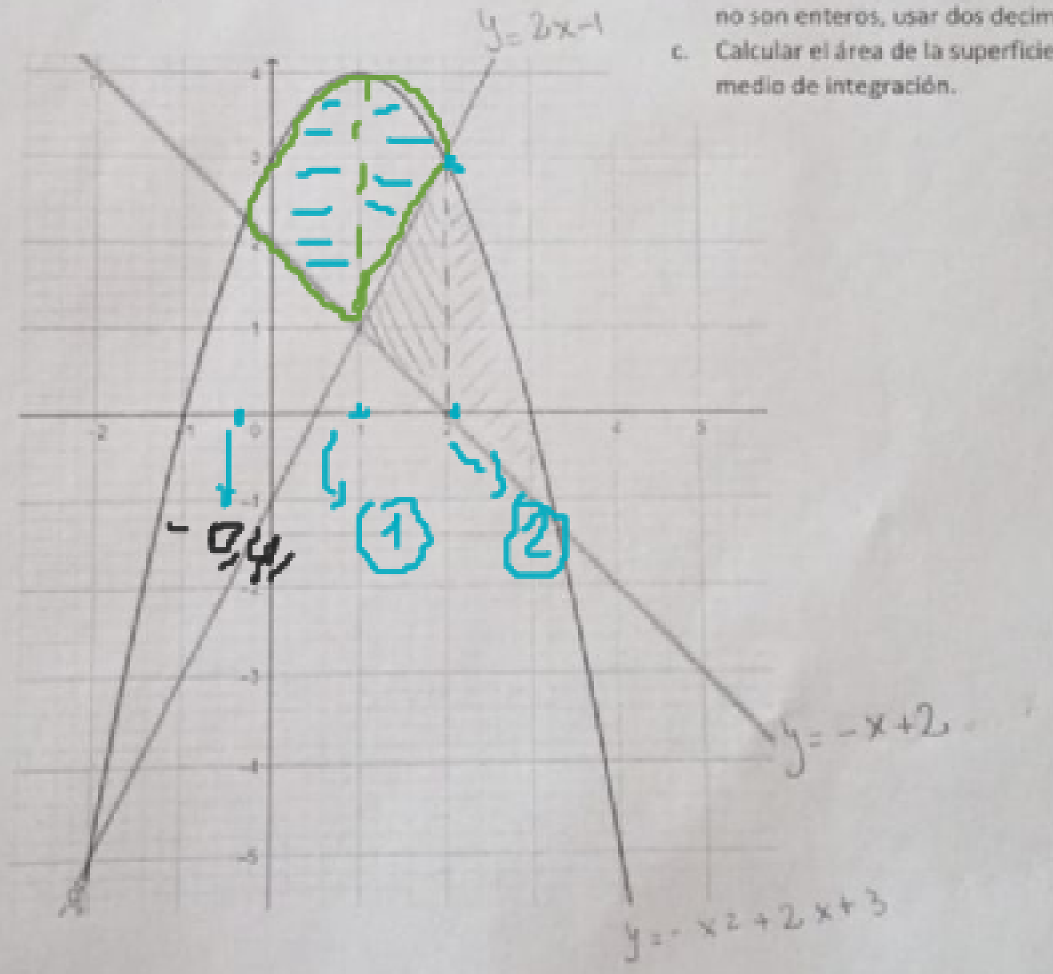
$$\boxed{\frac{47}{3}\ u.s}$$

# UNPA UARG - Análisis Matemático I

RDCUPERATORIO TERCER PARCIAL 2022 — 25 DE NOVIEMBRE

1. Resolver las siguientes integrales aplicando los métodos correspondientes:
   a. $\int arcsen(x) \cdot dx$
   b. $\int \frac{4-10x^4}{\sqrt[3]{2x-x^5}} \cdot dx$
2. Resolver las siguientes integrales aplicando los métodos correspondientes:
   a. $\int \sqrt{x^2+2x+2} \cdot dx$
   b. $\int \frac{x-9}{x^2-3x-4} \cdot dx$
3. Dada la superficie limitada por:
   - $y = -x^2 + 2x + 3$
   - $y = 2x - 1$
   - $y = -x + 2$

   a. Sombrear la superficie indicada en el gráfico (elegir una de las que cumplen con la consigna)
   b. Calcular extremos de integración (si no son enteros, usar dos decimales)
   c. Calcular el área de la superficie por medio de integración.




1 a) $\int arcsen(x)\, dx$

$$\boxed{\int u\, dv = u.v - \int v\, du}$$

$$\boxed{\begin{array}{l} u = arcsen(x) \\ dv = dx \\ du = \frac{1}{\sqrt{1-x^2}} dx \\ v = \int dx = x \end{array}}$$

$$= arcsen\,x \,.\, x - \int x \,.\, \frac{1}{\sqrt{1-x^2}}\, dx$$

$$= x\,.\, arcsen\,x - \int x \,.\, \frac{1}{(1-x^2)^{\frac{1}{2}}}\, dx$$

$$= x\,.\, arcsen\,x - \left(-\frac{1}{2}\right) \int (1-x^2)^{-\frac{1}{2}} (-2)\, x\, dx$$

$$= x\,.\, arcsen\,x + \frac{1}{2}\, \frac{(1-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= x\, arcsen\,x + \frac{1}{2}\, \frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= x\,.\, arcsen\,x + \frac{1}{\cancel{2}}\, \frac{\cancel{2}\,(1-x^2)^{\frac{1}{2}}}{1} + C = \boxed{x\, arcsen\,x + \sqrt{1-x^2} + C}$$

b. $\int \frac{4-10x^4}{\sqrt[3]{2x-x^5}} \cdot dx$

SUSTITUCIÓN

$u = 2x - x^5$

$du = (2-5x^4)\,dx$

$$= \int \frac{2 \cdot (2-5x^4)\,dx}{\sqrt[3]{2x-x^5}} dx = 2\int \frac{1}{\sqrt[3]{u}}\,du = 2\int \frac{1}{u^{1/3}}\,du = 2\int u^{-1/3}\,du = 2\,\frac{u^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} =$$

$$2\,\frac{u^{2/3}}{\frac{2}{3}} = 3\sqrt[3]{u^2} = \boxed{3\sqrt[3]{(2x-x^5)^2}+C}$$

2. a) $\int \sqrt{\underbrace{x^2+2x+2}_{}} \cdot dx$

NO ES TCP

$= \int \sqrt{(x+1)^2+1}\, dx$

$= \int \sqrt{u^2+1^2}\, du$

$= \frac{1}{2} \cdot \left[ (x+1) \cdot \sqrt{(x+1)^2+1^2} + 1^2 \cdot \ln\left|(x+1) + \sqrt{(x+1)^2+1^2}\right| \right] + C$

$$\boxed{= \frac{1}{2} \cdot \left[ (x+1) \cdot \sqrt{(x+1)^2+1} + \ln\left|x+1+\sqrt{(x+1)^2+1}\right| \right] + C}$$

INTEGRACIÓN DE POLINOMIOS CUADRÁTICOS

$a^2 + 2ab + b^2 = (a+b)^2$

$x^2 + 2x + 2 =$

$a^2 = x^2 \Rightarrow a = x$

$2ab = 2x \Rightarrow b = \frac{2x}{2a} = \frac{2x}{2x} = 1$

$b = 1 \Rightarrow b^2 = 1^2 = 1$

$\underbrace{x^2+2x+1}_{TCP} \underbrace{-1+2}_{}$

$\boxed{(x+1)^2+1}$

MÉTODO DE COMPLETAR CUADRADOS

b) $\int \frac{x-9}{x^2-3x-4}\,dx$

$= \int\left(\frac{1}{(x-4)} - \frac{2}{(x+1)}\right)dx$

$= \int \frac{1}{(x-4)}\,dx - \int \frac{2}{(x+1)}\,dx$

$= \int \frac{1}{(x-4)}\,dx - 2\int \frac{1}{(x+1)}\,dx$

$= \int \frac{1}{u}\,du - 2\int \frac{1}{u}\,du$

$= \boxed{\ln|x-4| - 2\ln|x+1| + C}$

# MÉTODO DE FRACCIONES SIMPLES

$\int \frac{P(x)}{Q(x)}\,dx \qquad gr(P(x)) < gr(Q(x))$

$1 < 2$ ✓

CASO $\boxed{I}$

PASO 1

$x^2 - 3x - 4 = 0 \begin{cases} x_1 = 4 \\ x_2 = -1 \end{cases}$

Raíces reales distintas

$(x-4).(x+1)$

PASO 2

$\frac{x-9}{(x-4)(x+1)} = \frac{A}{(x-4)} - \frac{B}{(x+1)}$

$\boxed{\frac{x-9}{(x-4)(x+1)} = \frac{A(x+1) - B(x-4)}{(x-4)(x+1)}}$

PASO 3

- Si $x=4 \Rightarrow 4-9 = A(4+1) - B(\cancel{4-4})$

  $5 = A(5) \Rightarrow \frac{5}{5} = A \Rightarrow \boxed{A=1}$

- Si $x=-1 \Rightarrow -10 = B(-5)$

  $\Rightarrow \boxed{2 = B}$

3. Dada la superficie limitada por:

$$Y = -X^2 + 2X + 3$$

$$Y = 2X - 1$$

$$Y = -X + 2$$


$$-X + 2 = -X^2 + 2X + 3$$

$$-(-x+2) = -\left(-[(x-3)(x+1)]\right)$$

$$x - 2 = (x-3)(x+1)$$

$$X = (x-3).(X+1) + 2$$

$$X = X^2 + x - 3x - 3 + 2$$

$$X = X^2 - 2X - 1$$

$$\begin{cases} X_1 = 1 + \sqrt{2} \\ x_2 = 1 - \sqrt{2} \end{cases}$$

$\rightarrow -0{,}41$ ✓

$$\boxed{X = 1 - \sqrt{2} = -0{,}41}$$

$$-X + 2 = 2X - 1$$

$$2 + 1 = 2X + X$$

$$3 = 3x$$

$$\frac{3}{3} = X$$

$$\boxed{1 = X}$$

$$2x - 1 = -x^2 + 2x + 3$$

$$\cancel{2x} + x^2 - \cancel{2x} = 3 + 1$$

$$X^2 = 4$$

$$X = \sqrt{4}$$

$$\boxed{X = 2}$$

$A_1 = 2{,}1$ aprox.

$A_2 = 1{,}6$ aprox.

$$A_1 = \int_{-0,41}^{1} \left[(-x^2+2x+3)-(-x+2)\right] dx = \int_{-0,41}^{1} (-x^2+2x+3+x-2)\, dx = \int_{-0,41}^{1} (-x^2+3x+1)\, dx =$$

$$\left[-\frac{x^3}{3}+\frac{3x^2}{2}+x\right]_{-0,41}^{1} = \left(-\frac{1^3}{3}+3\frac{1^2}{2}+1\right)-\left(-\frac{(-0,41)^3}{3}+\frac{3(-0,41)^2}{2}+(-0,41)\right)$$

$$= \frac{13}{6} - (-0,134876333\overline{3}) = \boxed{2,03 \text{ u.s}}$$

$$A_2 = \int_{1}^{2} \left[(-x^2+2x+3)-(2x-1)\right] dx = \int_{1}^{2} \left[-x^2+\cancel{2x}+3-\cancel{2x}+1\right] dx = \int_{1}^{2} (-x^2+4)\, dx$$

$$= \left[-\frac{x^3}{3}+4x\right]_{1}^{2} = \left(-\frac{2^3}{3}+4.2\right)-\left(-\frac{1^3}{3}+4.1\right) = \frac{16}{3}-\frac{11}{3} = \frac{5}{3} = \boxed{1,\widehat{6}7 \text{ u.s}}$$

$$A_T = A_1 + A_2 = 2,03 \text{ u.s} + 1,\widehat{6}7 \text{ u.s} = \boxed{3,69\widehat{6}7 \text{ u.s}}$$