Linear Algebra and its Applications

2023 Fall HW#1

1. For the equations $x + y = 4$, $2x - 2y = 4$, draw the row picture (two intersecting lines) and the column picture (combination of two columns equal to the column vector (4, 4) on the right side).

2. Describe the intersection of the three planes $u+v+w+z=6$ and $u+w+z=4$ and $u+w=2$ (all in four-dimensional space). Is it a line or a point or an empty set? What is the intersection if the fourth plane $u=-1$ is included? Find a fourth equation that leaves us with no solution.

3. Explain why the system:

   $u + v + w = 2$

   $u + 2v + 3w = 1$

   $v + 2w = 0$

   is singular (a) by finding a combination of the three equations that adds up to $0 = 1$ (b) by showing that the three columns lie in the same plane (hint: expressing the third column as a combination of the first two). Are there solutions? If yes, try to express the solutions. If not, give a b such that there exist solutions.

   If b is the zero vector (0,0,0), what are the solutions (u,v,w)?

4. Under what condition on y_1, y_2, y_3 do the points $(0,y_1)$, $(1,y_2)$, $(2,y_3)$ lie on a straight line?

5. Solve to find a combination of the columns that equals b for the following triangular system:

   $u - v - w = b_1$

   $v + w = b_2$

   $w = b_3$