

2019 Spring –SPC&O HW#13

1. The following table provides the design matrix for a certain two-level fractional factorial experimental design.

Test	1	2	3	4	5
1	-	-	-	+	-
2	+	-	-	-	+
3	-	+	-	-	+
4	+	+	-	+	-
5	-	-	+	+	+
6	+	-	+	-	-
7	-	+	+	-	-
8	+	+	+	+	+

- Write down the generators and the defining relation for this design.
 - What is the resolution of this design?
 - Write down all of the linear combinations of confounded effects that can be estimated from the result of this experiment. Assume that all interaction effects may be important.
 - Suppose that it was decided to run only a four-variable experiment using eight tests. What generator would you propose for this design? How would the design resolution differ as compared with the original design?
2. An import/export company is working with a construction engineering firm to study the tensile strength of various concrete formulations. Five variables of interest have been identified, and are given in the first table:

Variable	Low Level	High Level
Specimen size (ft ³)	2	4
Amount of water	Low	high
Curing time (hr)	24	48
Mixing technique	by hand	machine
Aggregate size	Fine	coarse

A replicated $L_8 (2^7)$ orthogonal array was performed to study the effects of these factors, and the results of this experiment are summarized in the second table:

Test	1	2	3	4	5	Y
1	1	1	1	2	2	2.3
2	2	1	1	1	1	3.5
3	1	2	1	1	2	3.0
4	2	2	1	2	1	2.1
5	1	1	2	2	1	3.5
6	2	1	2	1	2	2.6
7	1	2	2	1	1	2.9
8	2	2	2	2	2	3.9

- a. Assuming all interaction effects are insignificant, estimate all the main effects.
 - b. Assuming all interaction effects are insignificant, construct an ANOVA table to identify which of the main effects determined in part (a) are significant and build a regression model.
3. Ozzie Cadenza, owner and manager of Ozzie's Bar and Grill, recently decided to study the factors that influence the amount of business done at his bar. At first, he did not know which factors were important and which were not, but he drew up a list of six variables, which he decided to investigate by means of a fractional factorial experiment:
- (1) The amount of lighting in the bar
 - (2) The presence of free potato chips and chip dip at the bar
 - (3) The volume of the jukebox
 - (4) The presence of Ozzie's favorite customer, a young lady by the name of Rapunzel Freeny. Miss Freeny is a real "life of the party," continually chatting with the customers, passing around the potato chips, and so on, all of which make Ozzie feel that she has a real effect on the amount of bar business.
 - (5) The presence of a band of roving Gypsies, who have formed a musical group called the Roving Gypsy Band, and who have been hired by Ozzie to play a limited engagement at the bar.
 - (6) The effect of a particular bartender who happens to be on duty. There were originally three bartenders: Tom, Dick, Harry. Ozzie fired Harry, however, so that each of the factors for the experiment would have only two levels.

Low and high levels were selected for each of the variables as follows:

Variable	Low	High
1	Lights dim	Lights bright
2	No chips	Free chips
3	Jukebox playing softly	Jukebox blaring loudly
4	Miss Freeny stays home	Miss Freeny at the bar
5	No gypsy band	Gypsy band playing
6	Tom is the bartender	Dick is the bartender

Ozzie decided to perform one “run” of the experiment every Friday night during the happy hour (4:30-6:30). He thought that he should try a fractional factorial with as few runs as possible, since he was never quite sure when the band of gypsies would pack up and leave town. He finally decided to use a member of the family of the 2^{6-3} designs with generators of $I=\pm 124$, $I=\pm 135$ and $I=\pm 236$. He had wanted to find a resolution III design, in which the jukebox would never be blaring away while the gypsies were playing, but he found this requirement impossible.

a) Why is the statement above true? Carefully explain.

Ozzie did insist, however, that no run of the experiment could simultaneously have variables 1, 3 and 5 at their high levels. This restriction was made necessary by the annoying tendency of all light fuses to blow whenever the gypsies played their electric zither at the same time the lights were turned up brightly and the jukebox was playing at full volume. Note that this restriction makes it impossible for the principal fraction generators of the specified family to be used.

b) What members of the given family of generators does the specified restriction above allow?

Ozzie finally settled on a 2^{6-3} fractional factorial design, the recipe matrix for which is given in the table. The response (income in dollars) associated with each run is also given.

1	2	3	4	5	6	Y
+	+	+	+	–	+	3150
+	+	–	+	+	–	2050
+	–	+	–	–	–	2050
+	–	–	–	+	+	1550
–	–	–	+	–	+	2650
–	+	+	–	+	+	1250
–	+	–	–	–	–	1350

-	-	+	+	+	-	1950
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- c) What are the generators and defining relation for this design?
- d) Assuming that third- and higher-order interactions are negligible, write down the estimates obtained from this experiment and tell what they estimate (e.g., “1+24-35 estimated to be 400”)

Being partial to Miss Freeny, and encouraged by the results of the first fraction, Ozzie chose a second fraction that would give unconfused estimates of her and each of her two-factor interactions. The results of this second fraction in standard order are 1350, 1650, 2850, 1750, 2050, 1950, 2950 and 1450.

- e) What are the estimates obtained by combining the results of both fractions?
- f) Offer a brief conjecture that might explain the presence and direction of the interactions involving Miss Freeny.

4. Use a 2^{6-p} fractional factorial design with Resolution V to design a 6-factor CCD experiment. To minimize the number of the experimental tests, what would be p ? How many experimental tests are required for this CCD design? Design a 6-factor BBD experiment and compare it with the CCD.