Linear Algebra and its Applications

2023 Fall HW#2

1. Starting from a 3 by 3 matrix $\boldsymbol{A}$ with pivots 2, 7, 6, add a fourth row and column to produce $\boldsymbol{M}$. What are the first three pivots for $\boldsymbol{M}$, and why? What fourth row and column are sure to produce 9 as the fourth pivot?

2. True or false; explain your answers.
   (a) If the fist and third columns of $\boldsymbol{B}$ are the same, so are the first and third columns of $\boldsymbol{AB}$.
   (b) If the first and third rows of $\boldsymbol{B}$ are the same, so are the first and third rows of $\boldsymbol{AB}$.
   (c) $(\boldsymbol{AB})^2=\boldsymbol{A}^2\boldsymbol{B}^2$
   (d) The product of two lower triangular matrices is again lower triangular.

3.

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 \\ 4 & 0 & -\frac{3}{4} & 1 \end{bmatrix}$$

For A_iB, explain what each A_i does to B and find the inverse of each A_i, i=1,2..

4. Perform row exchanges to $\boldsymbol{A}$ to produce $\boldsymbol{A'}$ and fine $\boldsymbol{A'=LU}$ factorization for

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

   (a) What are $\boldsymbol{L}$ and $\boldsymbol{U}$?
   (b) What is $\boldsymbol{L}^{-1}$?

5.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} 5 & 7 & 8 \\ 0 & 2 & 3 \\ 0 & 0 & 6 \end{bmatrix}$$

   (1) Describe all the Gauss elimination steps that lead $\boldsymbol{A}$ to an upper triangular matrix.

(2) Will a row exchange be required?

(3) Let b=[1 2 3]^T and solve $Ax=b$ by solving two triangular systems.

6. Find the $PA=LDU$ factorization for

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

7. Compare and discuss the operations (division or multiplication-subtraction) required for solving $Ax=b$ by Gaussian elimination and back substitution and for solving $Ax=b$ by f Gauss-Jordan Method and $A^{-1}b$.

8. Find a 3 by 3 permutation matrix with $\boldsymbol{P}^3=\boldsymbol{I}$ (but $\boldsymbol{P}\neq\boldsymbol{I}$). Find a 4 by 4 permutation matrix $\boldsymbol{P}$ with $\boldsymbol{P}^4\neq\boldsymbol{I}$.

9. Find an $\boldsymbol{A}^{-1}$ formula from $\boldsymbol{PA}=\boldsymbol{LDU}$.

10. If $\boldsymbol{A} = \boldsymbol{A}^T$ and $\boldsymbol{B} = \boldsymbol{B}^T$, which of these matrices are certainly symmetric?
(a) $\boldsymbol{A}^2 - \boldsymbol{B}^2$ (b) $(\boldsymbol{A}+\boldsymbol{B})(\boldsymbol{A}-\boldsymbol{B})$ (c) $\boldsymbol{ABA}$ (d) $\boldsymbol{ABAB}$.