Linear Algebra and its Applications

HW#07

1. If V is the subspace spanned by (1, 1, 0, 1) and (0, 0, 1, 0), find
   (a) a basis for the orthogonal complement $V^{\perp}$
   (b) the projection matrix P onto $V^{\perp}$
   (c) the vector in V closest to the vector b = (0, 1, 0, -1) in $V^{\perp}$

2. (a) Find the bases for the null space and the row space of

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \end{bmatrix}.$$

   (b) Split $x = (3, 3, 3)^T$ into a row-space component x_r and a null-space component x_n.
   (c) Find the pseudoinverse A^+ such that $A^+Ax=x_r$.
   (d) Let $Ax=(9, 21)^T$. Recover the row space component of x.
   (e) Show that the pseudoinverse found in (c) is the right inverse of A.

3. Find the best straight-line fit to the following measurements, and sketch your solution:
   $y = -2$ at $t = -1$, $y = 0$ at $t = 0$,
   $y = -3$ at $t = 1$, $y = -5$ at $t = 2$.

4. Suppose that instead of a straight line, we fit the data in Problem 3 by a parabola: $y=C+Dt+Et^2$. Formulate the problem into the $Ax=b$ system and find the least-squares solution of x if the system is not solvable.

5. Project the vector b=(1, 2) onto a 2-dimensional space with two basis vectors, (1, 0) and (1, 1), and show that, unlike the orthogonal basis, the sum of the two projections does not equal to b.

6. If Q_1 and Q_2 are orthogonal matrices, so that $Q^TQ = I$, show that Q_1Q_2 is also orthogonal.

7. Find a third column so that the following matrix is orthogonal

$$Q = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{14} & \\ 1/\sqrt{3} & 2/\sqrt{14} & \\ 1/\sqrt{3} & -3/\sqrt{14} & \end{bmatrix}.$$

It must be a unit vector that is orthogonal to the other columns; how much freedom does this leave? Verify that the rows automatically become orthonormal at the same time.

8. Show that an orthogonal matrix that is upper triangular must be diagonal.