Chapter 1 Introduction to Kinematics of Mechanisms

1-1 Kinematics of Mechanisms

A rigid body is said to be under motion when it is instantaneously changing its position and/or orientation. Since the change of position can only be observed with respect to another body, the motion of a rigid body is a relative measure. Kinematics of a mechanism is the study of relative motion among the various links of a mechanism or machine by neglecting the inertia effects and the forces that cause the motion. In studying the kinematics of a mechanism, the motion of a link is often measured with respect to a fixed link or a reference frame, which may not necessarily be at rest.

Study of kinematics of machines or mechanisms can be considered from two different viewpoints, generally defined as

1. *Kinematic analysis* is the study of relative motions associated with the links of a mechanism or machine and is a critical step toward proper design of a mechanism. Specifical1y, given a mechanism and the motion of its input link(s), the relative displacement, velocity, acceleration, etc. of the other links are to be found. These characteristics can be derived by considering the constraints imposed by the joints.
2. *Kinematic synthesis* is a reverse problem to the analysis. In this case, the designer is challenged to devise a new mechanism that satisfies desired motion characteristics of an output link. Specifically, it is a process of contriving a mechanism to perform certain motion specification such as displacement, velocity or acceleration. The kinematic synthesis problem can be further divided into three interrelated phases:

a. *Type synthesis* refers to the selection of a specific type of mechanism for product development. During the conceptual design phase, the designer considers as many types of mechanism as possible and decides what type has the best potential of meeting the design objectives. The type of mechanism –cam, linkage, gear train, and so on – is determined. The selection depends to a great extent on the functional requirements of a machine and other considerations such as materials, manufacturing processes, and cost.

b. *Number synthesis* deals with the determination of the number of links, type of joint, and number of joints needed to achieve a given number of degrees of freedom of a desired mechanism. During this phase of study, the designer makes sure that a mechanism has the correct number of links that are connected with proper types of joints to ensure mobility. Number synthesis also involves the enumeration of all feasible kinematic structures or linkage topologies for a given number of degrees of freedom, number of links, and type of joints.
For this reason it is sometimes called structure synthesis or topological synthesis. Various methodologies have been developed for systematic enumeration of kinematic structure. A

thorough understanding of the structural characteristics of a given type of mechanism is critical for the development of efficient algorithm.

c. *Dimensional synthesis* deals with the determination of the dimensions or proportions of the links of a mechanism. Laying out a cam profile to meet a desired lift specification is a dimensional synthesis problem. Determination of the center distance between two pivots of a link in a bar-linkage is also a dimensional synthesis problem. Both geometric and analytical methods of synthesis may be used to perform dimensional synthesis. The Burmester theorem can be applied in the dimensional synthesis of planar linkages. Typical problems in dimensional synthesis include function generation, coupler-point curve synthesis, and rigid body guidance.

1-2 Links and Joints

· *Rigid body*: a material body if the distance between any two points of the body remains constant.

· *Links* or *members*: the individual rigid bodies that make up a mechanism or machine. Links may contain non-rigid bodies, such as cables or chains, which momentarily serve the same function as rigid bodies and are sometimes referred to as resistant bodies.

· *Joint* is the connection between two links.

· *Pair element*: The part of a link's surface that contacts another link is called a *pair element*.

· *Kinematic pair*: two paired elements constitute a *kinematic pair*.

Kinematic pairs can be classified into *higher pairs* and *lower pairs* in accordance with the contact between the pair elements. A lower pair, whose pair element not only forms the envelope of the other, but also encloses it, has surface contact. On the other hand, a higher pair, whose pair elements do not enclose each other, has line or point contact between the element surfaces.

The following are frequently used kinematic pairs (Table 1-1).

<u>Revolute or turning pair</u>: The R-pair permits only relative rotation between the two pairing elements. Hence, it has one degree of freedom. This pair is often called a pin joint.

<u>Prismatic or sliding pair</u>: The P-pair permits only relative sliding motion. It has also one degree of freedom.

<u>Helical or screw pair</u>: The H-pair allows both angular rotation and translating motion between two pairing elements. However, the translation is related to rotation by the pitch of the screw. The helical pair reverts to a revolute if the pitch is zero and to a prismatic if the pitch is infinite.

<u>Cylindrical pair</u>: The C-pair permits both angular rotation and an independent translating motion. Therefore, it has two degrees of freedom.

Spherical pair: The S-pair is a ball-and-socket joint. It has three rotational degrees of freedom.

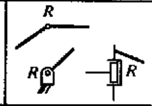
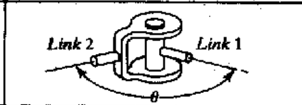
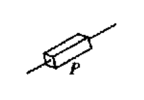
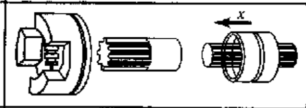
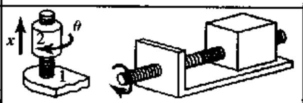
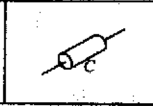
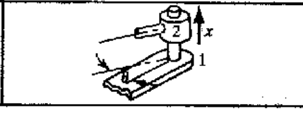
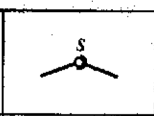
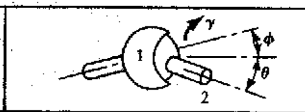
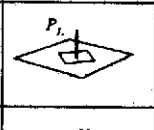
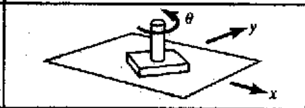
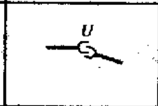
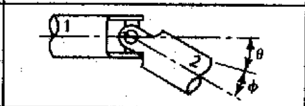
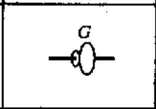
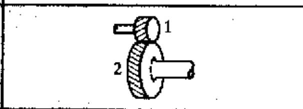
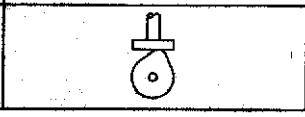
Plane pair: The plane pair permits two translational degrees of freedom along the plane of contact and a rotational degree of freedom normal to the plane of contact. Hence, it has three degrees of freedom.

Other types of pair elements are cam pair and gear pair. They belong to high pairs.
A number of common joint types are shown in Table 1-1.

1-3 Kinematic Chains

- Kinematic chain: an assembly of interconnected links is called a kinematic chain,
- Binary links: links containing only two joints.
- Ternary links: links containing three joints.
- Quaternary links: links containing four joints.
  … etc.
- Binary joint: a joint that is formed by two links.
- Multiple joint: a joint that is formed by more than two links.
- Closed loop kinematic chain: every link in the chain is connected to at least two other links; otherwise, it is referred to as open loop chain.
- Kinematic chain vs. mechanism: when one link of a kinematic chain is held fixed, the chain becomes a *mechanism*.

Table 1-1 Kinematic pairs

Type of joint (pair)	Lower pair (L) or higher pair (H)	Symbol	Degrees-of-freedom (connectivity) of the joint in a spatial linkage	Schematic representation	Possible configuration	Descriptive example
Revolute	L	R	1 θ			A pin joint that permits rotation only
Prism	L	P	1 x			A straight spline that permits sliding only
Helix	L	H	1 x or θ			Power screw or helical spline
Cylinder	L	C	2 x, θ			A sleeve that permits both rotation and sliding
Sphere	L	S	3 θ, φ, γ			A ball (and socket) joint permitting rotation in three angular directions
Plane	L	P_L	3 x, y, θ			A surface restraint permitting rotation and motion parallel to the plane of the surface
Universal joint		U	2 θ, φ			The Hooke-type universal joint that combines two revolute pairs
Gear pair	H	G	2 (rolling and sliding)			Spur gears, helical gears, and other gears
Cam pair	H	Cam	2 (rolling and sliding)			Disk cam and follower

- Planar mechanism: a mechanism whose links have planar motions all parallel to a given plane.

- Spherical mechanism: a mechanism in which all the moving links share a common stationary point; i.e. the locus of any point in a link is a curve contained in a spherical surface and the spherical surfaces defined by several arbitrarily chosen points are all concentric. In a spherical mechanism, the motion of all particles can be described by their radial projection on the surface of a sphere, usually a unit sphere, with properly chosen center. The only permissible lower-pair joint for constructing spherical mechanisms is the revolute pair and the joint axes of all revolute pairs must intersect at a point. The Hooke's universal joint is a well-known spherical four-bar linkage (Fig. 1-1).

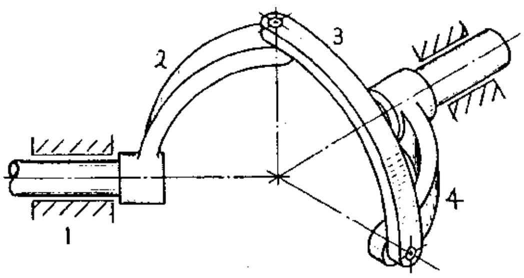


Fig. 1-1 A spherical four-link mechanism, also
called Hooke's universal joint

- Spatial mechanism: a mechanism other than planar and spherical mechanism, in other words, no specific motion characters can be identified.

1-4 Mobility (Degree of Freedom)

The mobility of a mechanism refers to the number of independent input parameter (usually pair variables) required to specify the configuration of the system. The mobility of a mechanism is equal to the number of degree of freedom of all the moving links diminished by the number of constraints imposed by the joint.

Define the following variables:

F: degree of freedom of a mechanism;

n: the number of links of a mechanism including the fixed link;

j: number of joints with each joint assumed as binary; a ternary joint is counted as 2 binary joints, etc.

f_i : the degree of freedom of relative motion at i^{th} joint.

$j_{\#}$: is the number of joints with "#" degrees of freedom. Thus j_1 denotes the number of single degree-of-freedom joints, and j_2 denotes the number of two degree-of-freedom joints, etc.

λ: the operator in which the mechanism is intended to operate: for planar and spherical motions, λ=3, and for spatial motion, λ=6.

Suppose all the moving links were unconstrained in the given working space, the total degrees of freedom would be λ(n-1); a join with f_i degree-of-freedom is equivalent to have (λ-f_i) number of constraints. Hence, the effect of j joints is to impose $\sum_{i=1}^{j}(\lambda - f_i) = \lambda \cdot j - \sum_i f_i$ number of constraints to the mechanism. Therefore

$$F = \lambda(n-1) - (\lambda \cdot j - \sum f_i)$$

$$= \lambda \cdot (n - j - 1) + \sum_i f_i \qquad (1\text{-}1)$$

Some mechanism with special dimensions may not obey the equation. These mechanisms won't be included in the following discussion.

(a) Planar mechanisms

For planar case, the total joint freedom $\sum_i f_i$ becomes

$$\sum f_i = j_1 + 2j_2 \qquad (1\text{-}2)$$

Substituting Eq.(1-2) into (1-1) with λ=3, yields

$$F = 3(n - j - 1) + j_1 + 2j_2 \qquad (1\text{-}3)$$

Note that $j = j_1 + j_2$ (1-4)

Hence $F = 3(n-1) - 2j_1 - j_2$ (1-5)

Equation (1-5) is called Kutzbach criterion for the mobility of a planar mechanism. Furthermore, if we limit ourselves to planar linkages with lower pair joints only, Eq.(1-5) becomes

$$F = 3(n-1) - 2j_1 \qquad (1\text{-}6)$$

This equation is also known as the *Grübler*'s criterion.

If the Kutzbach criterion yields F>0, the mechanism has F degrees of freedom. If the criterion yields F=0, motion is impossible and the mechanism forms a structure. If F<0, then there are redundant constraints in the chain and it forms a statically indeterminate structure.

Example 1. For the four-bar mechanism shown, (Draw Fig. 1-E1)

$n=4$, $j=j_1=4$ Hence $F=3(4-1)-2\times4=1$

Example 2. A plate cam-follower; (Draw Fig. 1-E2)

$n=3$, $j_1=2$, $j_2=1$, $F=3(3-1)-2\times2-1=1$

Example 3. Spur-gear drive ; (Draw Fig. 1-E3)

$n=3$, $j_2=1$, $j_1=2$, $F=3(3-1)-2\times2-1=1$

(b) Spatial mechanisms

We limit joint's freedom up to three degrees of freedom. Hence,

$$j = j_1 + j_2 + j_3 \qquad (1\text{-}9)$$

and

$$\sum_i f_i = j_1 + 2j_2 + 3j_3 \qquad (1\text{-}10)$$

Substituting Eq.(1-10) into (1-1), yields

$$F = 6(n - j - 1) + j_1 + 2j_2 + 3j_3$$

or
$$F = 6(n-1) - 5j_1 - 4j_2 - 3j_3$$

Example 4. An R-S-S-R mechanism; (Draw Fig. 1-E4)

F=6(4-1)-5×2-3×2=2

Example 5. As shown in Fig. E5, R-C-S-P linkage;

F=6(4-1)-5×2-4×1-3×1=18-10-4-3=1

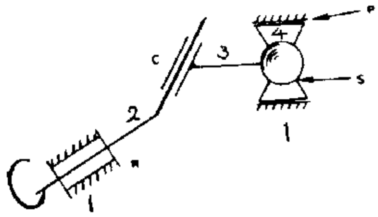


Fig. 1-E5 R-C-S-P mechanism

1-5 The Loop Mobility Criterion

A kinematic chain is an assembly of interconnected links. If every link is connected to at least two other links, then the chain forms one or more closed loops. We can observe that the four-bar linkage as shown below is a kinematic chain with one independent loop. It has four links and four joints. The five-bar linkage is a kinematic chain with one independent loop, again. It has 5 links and 5 joints. In generally, we can conclude that a simple closed-loop kinematic chain (planar, spherical or spatial configuration) consists of equal number of links and joints.

To generate a kinematic chain with two independent loops by extending from a given single loop, we could take an open-ended chain and join its two ends to members of the original loop by two joints as shown in Figure 1-2. We observe that the number of joints added is more than the number of links by one in extending from a one- to two-loop chain. By induction, the difference between the number of joints and the number of links is increased by L-1. Therefore,

$$j - n = L - 1 \qquad (1\text{-}14)$$

or
$$L = j - n + 1 \qquad (1\text{-}15)$$

Combining (1-15) with (1-1), we obtain

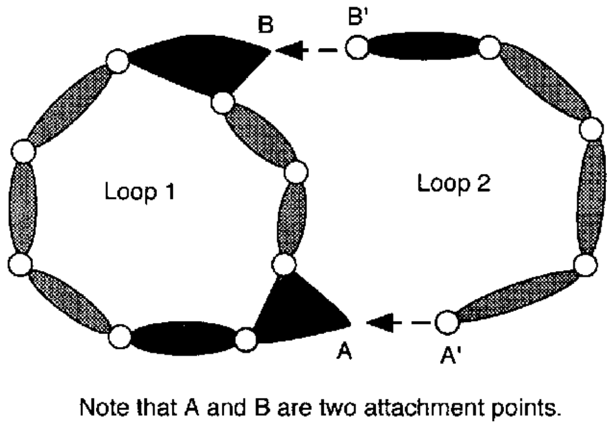


Fig. 1-2 Formation of a multiloop chain

$$\sum_i f_i = F + \lambda \cdot L \qquad (1\text{-}16)$$

Equation (1-16) is known as the loop mobility criterion. The loop mobility criterion is useful for determining the number of joint degrees of freedom needed for a kinematic chain to possess a given number of degrees of freedom.

Example 6. (Draw Fig. 1-E6) L=j-n+1=6-5+1=2

Check $F = \sum_i f_i - \lambda \cdot L$=6-3×2=0

Example 7. (Draw Fig. E7) L=9-8+1=2

Check F=(3×1+3×2+3×3)-6×2=6

1-6 Freedom and Constraint of Contact

An unconstrained rigid body, B, has six degrees of freedom, i.e. six independent parameters are required to completely specify B. If B is brought into contact with another contact rigid body that is fixed in reference frame R, the motion of B will be restricted. The degree to which the motion of B is restricted depends on the nature of the contact. If B is brought into contact with additional bodies fixed in R, the motion available to B is further reduced until B is fully constrained and unable to move in any manner relative to R. The degree to which each contact restricts motion depends not only on the nature of each contact but also on the position and orientation of the contacts relative to each other.

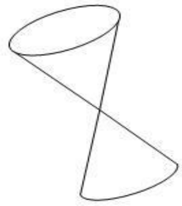
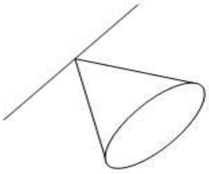
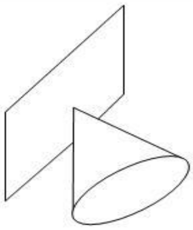
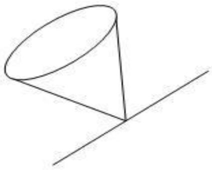
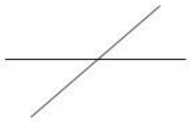
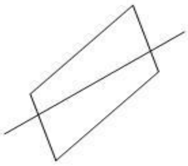
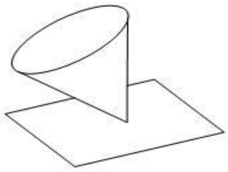
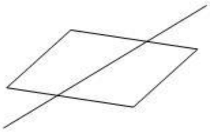
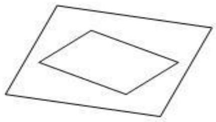
Types of contact between bodies

For each surface we will consider three possible types of contact:

- A surface has point contact if at the point under consideration both of its principal radii of curvature are very small or zero.
- A surface has line contact if one of its principal radii of curvature is very small or zero and the other radius of curvature is infinite.
- A surface has plane contact if both its principal radii of curvature are infinite.

Table 1-2 shows the 9 possible combinations of surface features between two bodies.

Table 1-2 Possible combinations of surface features between two bodies

		Body B		
		point	line	plane
Body R	point			
	line			
	plane			

Of these pairing, there are three transient conditions—a point on a point, a point on a line and a line on a point. The constraints imposed by such matings are not stable and will be ignored in the analysis. It can be concluded that there are essentially three types of stable pairings between the bodies B and R. These are called:

i. Point contact (point on plane,)
ii. Line contact (line on plane or plane on line), and
iii. Plane contact (plane on plane)

Effect of single contacts between bodies without looking at the details of each contact type, we can classify them into one of 5 categories according to the relative freedom of motion allowed between the bodies. A 1-degree-of-freedom contact implies that only one parameter is needed to specify the relative motions of the bodies if contact is maintained. A 2-degree-of-freedom contact requires two parameters to specify subsequent relative motions, and so on up to five parameters for a 5-degree of-freedom contact.

Here we'll look at the effect of various contact types on the relative mobility between contacting objects.

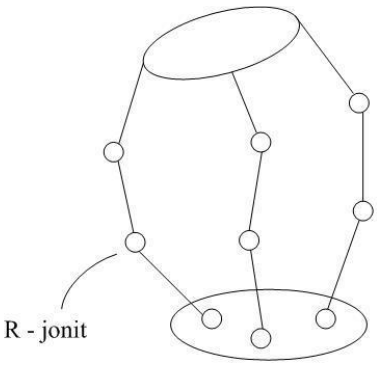
The relative freedom of motion allowed by each type of contact depends on whether we consider frictional forces to be significant. Table 1-3 shows the relative number of freedoms possible for the basic contact geometries described in the previous section.

Table 1-3 Relative number of freedoms possible for the basic contact geometries

Freedom of motion for basic contact geometries		
	Without Friction	With Friction
point contact	5	3
line contact	4	1
plane contact	3	0

There is another contact type frequently encountered which we'll call a soft finger. It behaves to the first order as a point contact with friction, except that its contact area is large enough that it is able to resist moments about the contact normal. It is a 2-dof contact. The analysis of constraint of contact has been applied to the design of multi-finger dexterous hand design. The following example illustrates such application.

Example 8.



A mechanical hand is designed with the following features.

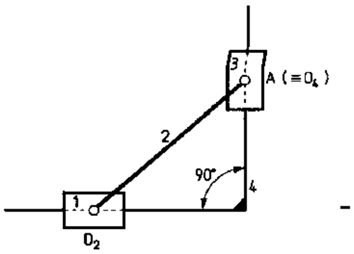
- Three fingers, each finger has three r-joints.
- The contact between each finger tip and the object surface is point contact with friction.

(1) Find the mobility of the grasped object. (2) How do you describe the independent parameters for the mechanical hand?

1-7 Kinematic Inversion

When different links are chosen as the reference frame for a given kinematic chain, the relative motions between the various links are not altered but their absolute motion may be changed. The processing of choosing different links of a kinematic chain for the frame is known as kinematic inversion.

Example 9. Identify the mechanisms shown below (Fig. 1-E8 (d) & (e)) belong to which related kinematic inversion ((b) & (c)). (b→ d, c→ e)



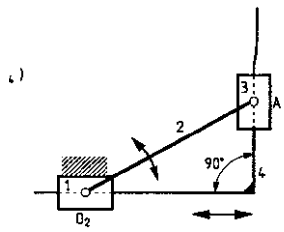
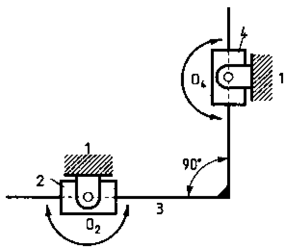
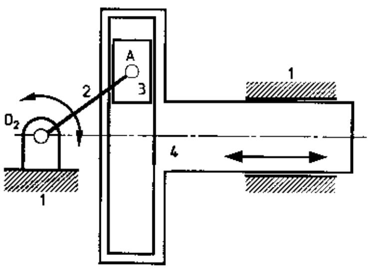
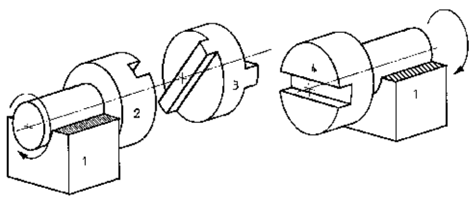


Fig. 1-E8 (a)An R-R-P-P chain (b) A kinematic inversion of RRPP chain



(c) Another kinematic inversion of RRPP chain





(d) The Scotch-yoke mechanism (e) An Oldham's coupling