Linear Algebra and its Applications

HW#11

1. Find the eigenvalues and eigenvectors for

$$\frac{du}{dt} = Au = \begin{bmatrix} 0 & 3 & 0 \\ -3 & 0 & 4 \\ 0 & -4 & 0 \end{bmatrix} u.$$

Why do you know, without computing, that e^{At} will be an orthogonal matrix and $\|u(t)\|^2 = u_1^2 + u_2^2 + u_3^2$ will be constant?

2. (a) What matrix M changes the basis $V_1=(1, 1)$, $V_2=(1, 4)$ to the basis $v_1=(2, 5)$, $v_2=(1, 4)$? (Hint: the columns of M come from expressing V_1 and V_2 as combinations $\Sigma m_{ij}v_i$ of the v's.)
(b) For the same two bases, express the vector (3, 9) as a combination $c_1V_1+c_2V_2$ and also as $d_1v_1+d_2v_2$. Check numerically that M connects c to d: $Mc=d$.

3. If the transformation T is a reflection across the 45º line in the plane, find its matrix with respect to the standard basis $v_1=(1, 0)$, $v_2=(0, 1)$ and also with respect to $V_1=(1, 1,)$, $V_2=(1, -1)$. Show that those matrices are similar.

4. Write out the matrix A^H and compute $C = A^H A$ if

$$A = \begin{bmatrix} 1 & i & 0 \\ i & 0 & 1 \end{bmatrix}$$

What is the relation between C and C^H? Does the relationship hold whenever $C = A^H A$ is constructed with some different A?

5. Rewrite the following matrices in the form $\lambda_1 x_1 x_1^H + \lambda_2 x_2 x_2^H$.

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad R = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$$

6. Write one significant fact about the eigenvalues of each of the following
   (a) A real symmetric matrix
   (b) A stable matrix (solutions of $du/dt=Au$ approach zero)
   (c) A Markov matrix
   (d) A continuous Markov matrix
   (e) A defective (nondiagonalizable) matrix
   (f) A singular matrix

7. Prove the three properties of skew-Hermitian matrices on Page 10 of “matrix diagonalization”.