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Grouped random parameters bivariate probit analysis of perceived and observed aggressive driving behavior: A driving simulation study

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ABSTRACT

This paper uses driving simulation data and surveys conducted in 2014 and 2015 in Buffalo, NY, to study the factors that affect perceived (self-reported, based on surveys) and observed (as measured, based on driving simulation experiments) aggressive driving behavior. Perceived and observed aggressive driving behavior are likely to share unobserved characteristics. To simultaneously account for this cross-equation error correlation, and for unobserved heterogeneity and panel data effects, a grouped random parameters bivariate probit model is estimated. The results control and account for a number of socio-demographic, driving experience and exposure, and behavioral and other characteristics. The findings reveal that different variables play in how aggressive driving behavior is perceived and observed, and the results imply that some drivers may perceive their driving behavior as non-aggressive when it is aggressive (or the opposite). The grouped random parameters bivariate probit model results are compared to their univariate probit, full information maximum likelihood bivariate probit, bivariate probit model with random effects, and random parameters bivariate probit model counterparts, and the results reveal the statistical superiority of the former, in terms of explanatory power, model fit, and forecasting accuracy.

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1. Introduction

With active and passive safety features being affluent in modern vehicles, and with technological gadgets and social media flooding the way of life, driving behavior over the last decades has significantly been altered. The reality of driving safer vehicles in combination with attention distractions has contributed to increased aggressive driving behavior incidents, such as speeding, braking abruptly (instead of progressively), and following closely other vehicles at unsafe distances, to name a few. However, it remains unknown whether this aggressive driving behavior is performed consciously. In fact, it

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is likely that some drivers may perceive that their driving behavior is non-aggressive; however, due to the feeling of safety from the presence of modern vehicle safety features, some drivers may have formed an unconscious and habitual aggressive driving behavior. Interestingly, in a recent study that investigated the effect of driver education and police enforcement on aggressive driving behavior (Tarko et al., 2011), the majority of cited aggressive drivers were unaware that they committed a traffic violation or that they drove aggressively. This is an illustrative example that some drivers may perceive their driving behavior as non-aggressive, when in fact they are driving aggressively. And it is likely that the factors that affect the perceived and observed aggressive driving behavior may as well differ.

On the other hand, it is likely that some drivers may have been exposed to intense driving incidents, which may have inevitably altered the way they perceive driving behavior. For example, drivers that suffered a severe motor-vehicle injury or a fatality of a close family or friend (or a similarly intense driving incident), may change the way they perceive their driving behavior from non-aggressive to aggressive.

From a methodological perspective, perceived and aggressive driving behavior do not necessarily have to be affected by the same factors. At the same time, there may exist systematic variations among the unobserved characteristics of perceived and observed driving behavior. To account for such cross-equation error correlation, perceived and observed aggressive driving behavior can be simultaneously modeled as a system of equations (Anwaar et al., 2011, 2012; Washington et al., 2011; Anastasopoulos et al., 2012c; Bhat et al., 2014; Tran et al., 2015; Zhan et al., 2015; Anastasopoulos and Mannering, 2016; Zeng et al., 2016; Hong et al., 2016; Anastasopoulos, 2016; Serhiyenko et al., 2016; Shaheed et al., 2016; Sarwar and Anastasopoulos, 2016, 2017). Moreover, this paper uses driving simulation data and surveys, consisting of 41 participants, who conducted 205 driving simulation experiments. This adds an additional methodological challenge, in terms of unbalanced panel effects inherent in the data.

Taking into account all methodological considerations, this paper estimates a grouped random parameters bivariate probit model of perceived and observed (based on surveys and driving simulation experiments, respectively) aggressive driving behavior. This approach accounts for the cross-equation error correlation among the dependent variables, for panel effects by the conduction of multiple experiments by the same participant, and for other unobserved factors that may vary systematically across the participants. The results reveal that different factors affect how drivers perceive their driving behavior and how they are observed to drive, and the findings imply that some drivers may perceive their aggressive driving behavior as non-aggressive (or they may perceive their non-aggressive driving behavior as aggressive). To evaluate the underlying benefits of the proposed approach, a number of counterpart models are estimated (i.e., univariate probit, full information maximum likelihood bivariate probit, bivariate probit model with random effects, and random parameters bivariate probit), and the grouped random parameters bivariate probit model is found to be statistically superior, in terms of explanatory power and model fit.

2. Empirical setting

Several efforts have been put forth to thoroughly investigate aggressive driving behavior. For example, several studies conducted field experiments that involved data collection using moving vehicles or fixed cameras (Kaysi and Abbany, 2007; Paleti et al., 2010; Tarko et al., 2011). At the same time, a number of efforts concentrated on identifying aggressive driving behavior patterns through the use of driving simulation and survey data (Al-Shihabi and Mourant, 2007; Harder et al., 2008; AAA, 2009; Philippe et al., 2009; Rong et al., 2011; Calvi et al., 2012; Joanisse et al., 2013; Ouimet et al., 2013). In these studies, aggressive driving was identified as one of the primary factors leading to an accident or a fatality. Factors that were identified to affect the level of aggressive driving behavior included demographic and socio-economic characteristics (e.g., gender, age, income, etc.), traffic characteristics (e.g., traffic volume and traffic composition), weather conditions (e.g., rain, wind, snow, etc.), and pavement conditions (e.g., friction). The recent Strategic Highway Research Program 2 (SHRP 2) naturalistic driving study initiative is also expected to possibly reveal new factors that play in aggressive driving.

To demonstrate the potential of the grouped random parameters bivariate probit modeling approach, and to study the factors that affect perceived and observed aggressive driving behavior, driving simulation experiments were conducted and survey data were collected in 2014 and 2015 from student and employee participants at University at Buffalo. The driving simulation experiments were conducted at the New York State Center for Engineering Design and Industrial Innovation (NYSCEDII) Motion Simulation Laboratory. The used equipment included a six degrees-of-freedom motion platform with vehicle buck and surround screens. The 4-miles route design was based on the city of Buffalo, NY. The route went through local, collector and arterial roads, and included a deer-crossing zone, a school zone, and a construction zone, all noted with proper signage. During the nearly 10-min route, the participants would go through five stop signs, six traffic signals, and four speed limit changes. To follow the predetermined route, flashing guidance arrows were projected on the simulator's monitor, resembling Global Positioning System (GPS) directions.

The traffic during the driving simulation was designed to resemble morning uncongested conditions. The traffic was composed of approximately 90 percent passenger cars, and 10 percent trucks. Local roads were one-lane per direction, and collectors and arterials were two-lanes per direction undivided roads. At the construction zone, there was no lane closure; however, traffic cones were used to separate the lanes, and trucks were simulated to randomly enter and exit the traffic stream from the construction zone area.

For the experiments, 41 participants were randomly selected from the student and employee body of the University at Buffalo (UB), the State University of New York (several volunteers responded to a flyer and email campaign, with the final participants being randomly selected, using a stratified random sampling technique, so that a representative sample – in terms of gender, age, and ethnicity – of the UB population is used). As expected, the data sample represents only population characteristics from UB. Out of the 41 participants, 15 were female and 26 male, while 10 were undergraduate students, 25 graduate students, and 6 University employees (faculty or stuff). The youngest participant was 18 years old and the oldest 51, while the average participant age was approximately 27 years old. Finally, 4 participants were of African American origin, 14 of Asian, 15 of Caucasian, 5 of Hispanic/White, and 3 of other origin.

The participants filled out a survey upon arrival (and before using the driving simulator) to the NYSCEDII Motion Simulation Laboratory. The survey included: demographic and socio-economic characteristics (e.g., gender, age, ethnicity, marital status, household characteristics, income, level of education, etc.); traffic habitual and behavioral information (e.g., driving experience, driving frequency, driving speed given specific speed limits, reaction to traffic signal change from green to yellow, frequency of recreational trips, etc.); safety related information (e.g., accident history by injury-severity type, number of citations for driving violations by violation type, etc.); and non-traffic related habitual and behavioral information (e.g., frequency of caffeine and alcohol drinking, preference of alcoholic beverages, music preference, etc.).

All participants went through a two-minute training session before the experiments. The training session track included driving between traffic cones, and involved several exercises that would enable the participants to get themselves well familiarized with the driving simulator. Immediately before the participants began the training session, they were asked about how calm or stressed and tired or energetic they felt, and about how they felt about listening to music (for each question, a scale 1 to 5 was used). These questions aimed at helping better interpret the results, controlling for fatigue and music preference.

Following the two-minute training session, the driving simulation experiment was initiated. The experiment consisted of two phases/scenarios, one where the participants were asked to drive normally to their destination, and one where the participants were asked to rush to their destination.[1] For the second scenario (preceded by the first scenario plus a fifteen minute break), the participants were penalized for any traffic violation or aggressive driving behavior, and were informed that the fastest (with penalties accounted for) three participants would receive a prize. The prize was used as a means of motivation for the participants to get to their destination as soon as possible, without driving recklessly. This was expected to control – to some extent – extreme driving aggressiveness under this scenario.

During both driving simulation scenarios, two trained moderators were collecting aggressive driving behavior data. The data reflected the number of incidents in terms of: following too closely (tailgating) a preceding vehicle; cutting another vehicle off; not using signal indicators when making a turn or when changing lanes; violating a stop sign or a traffic signal; speeding (driving about 5 or more miles per hour over the speed limit); braking abruptly with or without cause; overpassing another vehicle in a rushed manner; and violating any other traffic regulations.

The experiment of the first scenario included: a four-minute session during which the participants drove the simulator without listening to music; a second four-minute session during which typical radio music accompanied the driving simulation; and a final two-minute session during which the music was turned off. These sessions were identified only by the moderator, and the participants realized no change other than that the music was turned on and then off (the moderator calmly notified all participants that music would be turned on/off). Throughout these three sessions, all participants were asked to drive normally and follow the projected directions to get to their destination. With the participants reaching their destination, the first phase/scenario of the driving simulation experiment was concluded, and the participants were asked again (for each question, a scale 1 to 5 was used): about how calm or stressed and tired or energetic they felt; about how they felt about listening to music; about how much they liked the music that played during their driving session; and about how they evaluated their driving simulation level of expertise. They were also asked about how aggressively they thought they drove the simulator (i.e., aggressively or non-aggressively). The participants were finally asked to fill out a detailed Motion Sickness Assessment Questionnaire (MSAQ).

Immediately before the participants began the second scenario (during which they drove the simulator while rushing to their destination), they were asked again about how calm or stressed and tired or energetic they felt, and about how they felt about listening to music (for each question, the same 1 to 5 scale was used). The experiment included a two-and-a-half-minute session without listening to music, and a second two-and-a-half-minute session during which typical radio music accompanied the driving simulation. As with the first phase/scenario, these sessions were identified only by the moderator who calmly notified all participants that music would be turned on/off, and the participants realized no change other than that. With the participants reaching their destination, the second phase/scenario of the driving simulation experiment was concluded, and the participants were asked again about how calm or stressed and tired or energetic they felt, about how they felt about listening to music, about how much they liked the music that played during their driving session, and about how they evaluated their driving simulation level of expertise (for each question, a scale 1 to 5 was used). They were again asked about how aggressively they thought they drove the simulator (i.e., aggressively or non-aggressively). And the participants

[1] It should be noted that for about 20% of the participants, the experiments were conducted in reverse order (i.e., they were first asked to rush to their destination, and then to drive normally).

Table 1
Descriptive statistics of selected variables.

Variable description	Mean	Std. Dev.	Min.	Max.
Dependent variables				
Observed aggressive driving (OAD)	0.217	–	0	1
Perceived aggressive driving (PAD)	0.688	–	0	1
Aggressive driving behavior incidents				
Sign/Signal violation	0.818	1.274	0	5
Red signal violation	0.241	0.631	0	4
Speeding	1.754	2.268	0	11
Cutting someone off	0.251	0.644	0	4
Rushed overpass	0.364	0.801	0	4
Tailgating	0.198	0.537	0	3
Braking abruptly	1.310	1.422	0	6
Not using signal indicator	0.636	1.153	0	5
Socio-demographics				
Participant's age	26.864	6.875	18	51
Hometown indicator (1 if the participant grew up in an urban area, 0 otherwise)	0.593	–	0	1
Hometown indicator (1 if the participant grew up in a rural area, 0 otherwise)	0.153	–	0	1
Education indicator (1 if the participant has a post graduate degree, 0 otherwise)	0.317	–	0	1
Income indicator (1 if the participant's household income is less than $10,000, 0 otherwise)	0.217	–	0	1
Income indicator (1 if the participant's household income is greater than $75,000, 0 otherwise)	0.238	–	0	1
Marital status indicator (1 if the participant is married, 0 otherwise)	0.201	–	0	1
Ethnicity indicator (1 if the participant is Asian, 0 otherwise)	0.333	–	0	1
Driving experience and exposure characteristics				
Number of years the participant has had their driver's license	6.321	6.913	0	33
Driver's license indicator (1 if the participant has had their driver's license for 6 years or more, 0 otherwise)	0.429	–	0	1
Willingness to drive indicator (1 if the participant prefers to drive up to 12 hours before considering another mode of transportation, 0 otherwise)	0.175	–	0	1
Accident history indicator (1 if the participant has had no accidents in their lifetime, 0 otherwise)	0.418	–	0	1
Traffic violation indicator (1 if the participant was pulled over for more than once in the last 5 years for traffic violations, 0 otherwise)	0.138	–	0	1
Behavioral and other characteristics				
Simulation scenario indicator (1 if rushing to destination while listening to music, 0 otherwise)	0.175	–	0	1
Simulation scenario indicator (1 if rushing to destination, 0 otherwise)	0.349	–	0	1
Alcohol consumption tendency indicator (1 if the participant has 6 or more alcoholic beverages in a week, or if hard liquor is the participant's preferred alcoholic beverage, 0 otherwise)	0.275	–	0	1
Music and age indicator (1 if music type was uplifting and the age of the participant was less than 25 years, 0 otherwise)	0.550	–	0	1

were finally asked to fill out a detailed Motion Sickness Assessment Questionnaire (MSAQ). As previously, these questions aimed at helping better interpret the results, controlling for fatigue and music preference.

Note that of the 41 participants, 6 repeated the experiment on a different day. With five sessions (three sessions in phase/scenario one, and two sessions in the second phase/scenario) for every participant, there were a total of 205 data-points. After accounting for missing values, 189 data-points from 40 participants were used for the analysis. As the collected data included nearly 100 independent variables, Table 1 summarizes descriptive statistics of key parameters.

3. Method and approach

Aggressive driving behavior has been methodologically approximated using a variety of statistical, analytical, and empirical approaches. A few examples include algorithm-based approaches (Zohdy et al., 2013), empirical assessments (Ding et al., 2013; Du et al., 2014), microscopic traffic simulation (Habtemichael et al., 2013), analysis of variance (Werneke and Vollrath, 2012), structural equations modeling (Ma et al., 2010), count data and rates modeling (Tarko et al., 2011), mixed logit modeling (Goh et al., 2014), and so on. In these studies, aggressive driving behavior has been investigated from the viewpoint of either the number of aggressive driving behavior incidents, or self-reported behavior through the use of surveys.

As discussed earlier, past studies have shown the possibility that some drivers may perceive their driving behavior as non-aggressive, when in fact they are driving aggressively. At the same time, this implies that the factors that affect the perceived and observed aggressive driving behavior may differ. To that end, using the survey and driving simulation data, a grouped random parameters bivariate probit model of perceived and observed (based on the driving simulation experiments) aggressive driving behavior is estimated.

The dependent variables are: (a) the survey questions following the two phases/scenarios, "How aggressively do you think you drove the simulator?", with the answer reflecting aggressive or non-aggressive driving behavior; and (b) a

Table 2
Assigned weights used for the normalization of the various types of aggressive driving behavior incidents.

Type of aggressive driving behavior	Assigned weights	Justification
Sign/Signal violation	7	The corresponding crash modification factor (72%) is indicative of the very strong effect of this aggressive driving behavior type on fatal crashes. Also, this type of aggressive driving behavior is among the top causal factors of fatal crashes.[a,b]
Red signal violation	7	The corresponding crash modification factor (67%) is indicative of the very strong effect of this aggressive driving behavior type on fatal crashes. Also, this type of aggressive driving behavior is among the top causal factors of fatal crashes.[a,b]
Speeding	5	The corresponding crash modification factor (44%) is indicative of the strong effect of this aggressive driving behavior type on fatal crashes. Also, this type of aggressive driving behavior is one of the most common causal factors of fatal crashes.[a,b]
Cutting someone off	3	This type of aggressive driving behavior is one of the common causal factors of fatal crashes.[b]
Rushed overpass	3	This type of aggressive driving behavior is one of the common causal factors of fatal crashes.[b]
Tailgating	2	This type of aggressive driving behavior is one of the least common causal factors of fatal crashes.[b]
Braking abruptly	2	This type of aggressive driving behavior is one of the least common causal factors of fatal crashes.[b]
Not using signal indicator	1	This type of aggressive driving behavior is the least common causal factor of fatal crashes.[b]

[a] Source: Crash Modification Factors Clearinghouse (FHWA, 2009).
[b] Source: AAA (2009).

weighted average – based on the most common causal factors of fatal crashes (AAA, 2009), and on crash modification factors posted on the Crash Modification Factors Clearinghouse (FHWA, 2009) – of the aforementioned aggressive driving behavior incidents (i.e., tailgating, cutting another vehicle off, not using signal indicators, violating a stop sign, violating a traffic signal, speeding, braking abruptly, rushed over-pass, and other traffic violations), converted into the same two discrete driving behavior outcomes (i.e., driving the simulator aggressively or non-aggressively).[2] Table 2 presents the assigned weights that were used for the normalization of the various types of the aggressive driving incidents.

Note that the categorization for the observed aggressive driving behavior was based on AASHTO's Highway Safety Manual (2009) excess expected average crash frequency approach. In this approach, the number of aggressive driving incidents (normalized by the trip duration) are first multiplied by their corresponding weights in Table 2, and are aggregated by participant-trip. Next, an aggressive driving norm is defined as the average (i.e., a single value) of all weighted and normalized aggressive driving incidents. The aggressive driving norm is then multiplied with the duration of each trip, to give the typical number of aggressive driving incidents for each observation. Finally, the product of the aggressive driving norm with each trip duration is subtracted from the weighted aggregated number of all aggressive driving incidents by participant-trip, which gives the excess over the aggressive driving norm for each participant-trip. To define the observed aggressive driving behavior variable as dichotomous (aggressive, or non-aggressive outcomes), the median of the excess over the aggressive driving norm is used as a threshold.[3]

The first dependent variable represents the participants' perceived and self-reported level of aggressive driving behavior; whereas, the second represents the observed level of aggressive driving behavior from the driving simulation experiment. Fig. 1 presents a histogram of the perceived and observed aggressive driving behavior of the data sample. The figure shows that 31% of the participants perceived their driving behavior as non-aggressive, while 21% of the participants were observed to drive aggressively. On the other hand, 79% of the participants were observed to drive non-aggressively, but only 69% perceived their driving behavior as aggressive.[4]

From a statistical modeling viewpoint, the perceived and observed aggressive driving behavior level can be modeled simultaneously, with a bivariate probit model, as it is assumed that the error terms of the two latent variables of the binary models are significantly correlated and may be capturing the same or similar unobserved characteristics. The bivariate binary probit model can be defined as (Greene and Hensher, 2009; Anastasopoulos et al., 2012a; Greene, 2012a),

[2] For simplicity, the weighted observed aggressive driving behavior derived from the driving simulation experiment measurements will – from this point onward – be referred to as observed aggressive driving behavior. While, perceived aggressive driving behavior will be used as a term derived from the survey question.

[3] It should be noted that several other thresholds (based on other statistical measures, or data distributions) can be used to define the observed aggressive driving behavior variable.

[4] This is an interesting finding, when compared to past studies (e.g., Tarko et al., 2011) that have found that drivers are generally more likely to perceive their driving behavior as less aggressive -compared to their measured driving behavior. The current finding may be attributed to the nature of the perceived aggressive driving behavior variable, which has a dichotomous outcome (i.e., aggressive or non-aggressive), as compared to other studies that investigated the level of aggressiveness (and had more than two levels of aggressive driving behavior). As such, it is likely that some participants may have thought that they drove somewhat aggressively, in which case they reported their answer as driving aggressively. This effect may be possibly captured – to some extent – by the proposed grouped random parameters bivariate probit model (through the cross-equation error correlation and the random parameters). As a direction for future work, studying various levels of perceived and observed aggressive driving behavior through the use of multiple ranked outcomes (e.g., non-aggressive, somewhat aggressive, aggressive, and very aggressive), may reveal additional insights (Yasmin et al., 2015; Eluru and Yasmin, 2015).

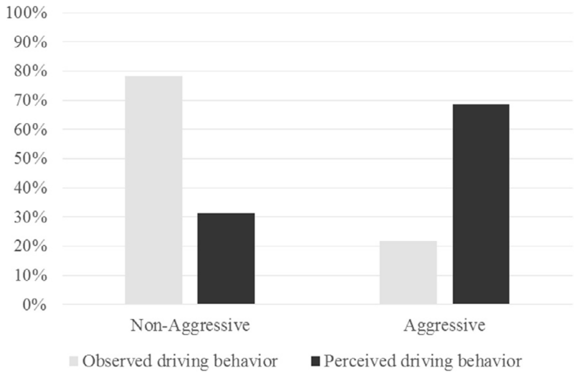

Fig. 1. Histogram of the observed and perceived aggressive driving behavior.

$$\begin{aligned} Z_{i,1} &= \beta_{i,1}X_{i,1} + \varepsilon_{i,1}, \quad y_{i,1} = 1 \text{ if } Z_{i,1} > 0, \text{ and } y_{i,1} = 0 \text{ otherwise} \\ Z_{i,2} &= \beta_{i,2}X_{i,2} + \varepsilon_{i,2}, \quad y_{i,2} = 1 \text{ if } Z_{i,2} > 0, \text{ and } y_{i,2} = 0 \text{ otherwise} \end{aligned} \tag{1}$$

with the cross-equation correlated error terms,

$$\begin{pmatrix} \varepsilon_{i,1} \\ \varepsilon_{i,2} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right] \tag{2}$$

where, X is a vector of explanatory variables that determine perceived and observed aggressive driving behavior characteristics of observation i, β is a vector of estimable parameters, y corresponds to integer binary outcome (zero or one for both dependent variables), ε is a random error (assumed to be normally distributed with zero mean and a variance of one), and ρ the cross equation correlation coefficient of the error terms. The bivariate probit model and corresponding log-likelihood function are then respectively defined as (Greene, 2012a),

$$\Phi(Z_1, Z_2, \rho) = \frac{\exp\left[-0.5(Z_1^2 + Z_2^2 - 2\rho Z_1 Z_2)/(1-\rho^2)\right]}{\left[2\pi\sqrt{(1-\rho^2)}\right]} \tag{3}$$

and

$$\begin{aligned} \sum_{i=1}^{N} [& y_{i,1}y_{i,2} \ln \Phi(\beta_{i,1}X_{i,1}, \beta_{i,2}X_{i,2}, \rho) + (1-y_{i,1})y_{i,2} \ln \Phi(-\beta_{i,1}X_{i,1}, \beta_{i,2}X_{i,2}, -\rho) \\ & + (1-y_{i,2})y_{i,1} \ln \Phi(\beta_{i,1}X_{i,1}, -\beta_{i,2}X_{i,2}, -\rho) + (1-y_{i,1})(1-y_{i,2}) \ln \Phi(-\beta_{i,1}X_{i,1}, -\beta_{i,2}X_{i,2}, \rho)] \end{aligned} \tag{4}$$

where, $\Phi(.)$ is the cumulative distribution function of the bivariate normal distribution.

Note that as multiple data-points are generated for each participant, panel data effects need to be accounted for (Washington et al., 2011). At the same time, relatively recent research in random parameters modeling (Anastasopoulos and Mannering, 2009, 2011; Washington et al., 2011; Anastasopoulos et al., 2012d, 2016; Behnood et al., 2014; Russo et al., 2014; Venkataraman et al., 2014; Coruh et al., 2015; Barua et al., 2016; Mannering et al., 2016; Sarwar et al., 2016; Behnood and Mannering, 2016) has shown that the effect of the explanatory parameters may vary across the observations due to unobserved heterogeneity (i.e., unobserved factors that may vary systematically across the observations). These are two misspecification concerns that can be accounted for simultaneously. To that end, we account for unobserved heterogeneity within each group of observations generated by each participant, by defining the estimable parameters as grouped random parameters (Wu et al., 2013),

$$\beta_j = \beta + u_j \tag{5}$$

where, β is the vector of estimable parameters and u_j is a randomly distributed term for each participant j with mean zero and variance σ^2. In such grouped random parameters modeling scheme, each β is defined for each participant j, as opposed to the traditional random parameters scheme where each β is defined for each observation i. In other words, Eq. (5) demonstrates that each of the 41 participants will have their own β; while, in traditional random parameters modeling, each of the 205 observations would have their own β.

To evaluate the grouped random parameters bivariate probit model (GRP), four other modeling approaches are explored: (a) univariate probit models (UP) for each of the two dependent variables; (b) a full information maximum likelihood bivariate probit (FIML) model; (c) a bivariate probit model with individual (i.e., group) level random effects (RE); and (d) a random parameters bivariate probit (RP). The following paragraphs include a brief description of these model formulations (for the basic univariate probit model, interested readers are referred to Washington et al., 2011).

The full information maximum likelihood bivariate probit (FIML) captures panel effects in a system of equation setting, without, however, accounting for unobserved heterogeneity (Mannering et al., 2016). The model can be defined as (Greene, 2012b):

$$P(y_{it}) = g(\beta_i,\ x_{it}, \varepsilon_{it}) \tag{6}$$

where, $P(y_{it})$ is the probability density function of the observed random variable, y_{it}; i denotes the i-th group or individual; t denotes the t-th observation of individual i; y_{it} is the observed dependent variable; x_{it} is a vector of independent variables; β_i is a vector of estimable parameters; ε_{it} are the error terms, and $g(.)$ is the density of the observed random variable.

The bivariate probit model with individual (i.e., group) level random effects (RE) accounts for both panel effects across groups, and for unobserved heterogeneity captured only by the constant term (β_o); however, it does not account for systematic variations of the non-constant variables across the observations. The formulation changes as follows (Greene, 2012b):

$$g(\beta_i, x_{it}, \varepsilon_{it}) = g(\beta_0 + u_{it}, \beta' x_{it}, \varepsilon_{it}) \tag{7}$$

where, u_{it} is the randomly distributed term to estimate a different constant, β_o, for each observation, and everything else as previously defined.

Finally, the random parameters bivariate probit (RP) model captures cross-equation error correlation, and unobserved heterogeneity – by allowing the effect of all (or some) coefficients to vary across the observations – without, however, accounting for group-specific heterogeneity (as the GRP model). The model is defined as (Greene, 2012b):

$$g(\beta_i, x_{it}, \varepsilon_{it}) = g\left[\beta_i(z_i, v_i)' x_{it}, \varepsilon_{it}\right] \tag{8}$$

where, z_i is the observable heterogeneity and v_i is the unobserved heterogeneity.

Finally, for model estimation, a simulated maximum likelihood estimation with 200 Halton draws approach (which provided parameter stability) was utilized for the random parameter models (Halton, 1960; Washington et al., 2011; Anastasopoulos et al., 2012b; Mannering and Bhat, 2014).

4. Model estimation results

The model estimation results and pseudo-elasticities[5] of the grouped random parameters bivariate probit model (GRP) for perceived and observed aggressive driving behavior are presented in Tables 3 and 4, respectively, along with the model estimation results of its four counterparts (univariate probit models – UP, full information maximum likelihood bivariate probit model – FIML, bivariate probit model with individual level random effects – RE, and random parameters bivariate probit – RP). For model interpretation, a positive value of β implies that an increase in the explanatory parameter will increase the probability of the "aggressive driving behavior" response and will decrease the probability of the "non-aggressive driving behavior" response. To better interpret the results, pseudo-elasticities (averaged over all observations) are computed for each independent variable and each category as (Washington et al., 2011),

$$E = \phi\left(\frac{\beta_j X_{j,1}}{\sigma}\middle| X_i = 1\right) - \phi\left(\frac{\beta_j X_j}{\sigma}\middle| X_i = 0\right) \tag{9}$$

where, ϕ (.) is the probability mass function of the standard normal distribution.

In terms of statistical fit, Table 3 shows the statistical superiority of the grouped random parameters bivariate probit model over its counterparts, as indicated by the Akaike information criterion, AIC = $2k–2LL(\beta)$. This result supports the effort to simultaneously account for cross-equation error correlation, panel effects, and group-level heterogeneity, and is in line with Wu et al. (2013) that identified that the grouped random parameters approach provides a superior statistical fit when compared to the traditional random parameters approach. This is further supported by the computed (for each observation) outcome probabilities. For example, the grouped random parameters bivariate probit model correctly predicts 84.7% of the observed aggressive and non-aggressive driving behavior outcomes; the random parameters bivariate probit model 84.4%; the random effects bivariate probit model 83.9%; the full information maximum likelihood bivariate probit model 83.9% (also); and the univariate probit models correctly predicts 83.6% of the observed aggressive and non-aggressive driving behavior outcomes.

For model estimation, all possible variables and variable interactions were explored, and only the statistically significant (at 0.90 level of confidence) variables are included in the grouped random parameters model. In comparing the model estimation results of all five models, the parameter estimates and pseudo-elasticities are relatively close in terms of sign and magnitude. However, three variables are statistically insignificant in the grouped random parameters model counterparts: the standard deviation of the parameter density function of the constant for the bivariate probit model with individual level random effects of perceived aggressive driving behavior; the music and age indicator variable for the univariate probit model

[5] Note that pseudo-elasticities are estimated, as all explanatory parameters are indicator (binary) variables. Pseudo-elasticities measure the effect on the aggressive driving behavior outcome probabilities, due to a change from zero to one in the explanatory variable.

Table 3
Model estimation results of the grouped random parameters bivariate probit model, and of its model counterparts (*t*-statistics are in parentheses).

Variables	**GRP**[a]		**RP**[b]		**RE**[c]		**FIML**[d]		**UP**[e]	
	PAD[f] **Coeff.**[h] **(*t*-stat)**	**OAD**[g] **Coeff. (*t*-stat)**	**PAD Coeff. (*t*-stat)**	**OAD Coeff. (*t*-stat)**	**PAD Coeff. (*t*-stat)**	**OAD Coeff. (*t*-stat)**	**PAD Coeff. (*t*-stat)**	**OAD Coeff. (*t*-stat)**	**PAD Coeff. (*t*-stat)**	**OAD Coeff. (*t*-stat)**
Constant	1.046 (4.30)	−1.961 (−4.81)	1.006 (3.84)	−2.090 (−6.26)	1.002 (3.43)	−1.948 (−5.59)	0.989 (3.35)	−1.907 (−5.45)	0.956 (3.63)	−1.604 (−4.90)
Standard deviation of parameter density function					*0.169 (1.48)*	*0.209 (1.86)*				
Socio-demographics										
Hometown indicator (1 if the participant grew up in an urban area, 0 otherwise)		1.376 (3.04)		1.381 (3.82)		1.366 (3.65)		1.336 (3.55)		1.069 (3.41)
Standard deviation of parameter density function		*0.376 (2.46)*		*0.714 (3.97)*						
Hometown indicator (1 if the participant grew up in a rural area, 0 otherwise)	−0.758 (−3.55)		−0.681 (−2.27)		−0.700 (−2.26)		−0.690 (−2.23)		−0.775 (−2.15)	
Education indicator (1 if the participant has a post graduate degree, 0 otherwise)		−1.123 (−2.35)		−1.228 (−2.81)		−1.035 (−2.43)		−1.013 (−2.39)		−0.951 (−2.69)
Income indicator (1 if the participant's household income is less than $10,000, 0 otherwise)		1.036 (3.49)		1.098 (3.27)		1.006 (2.82)		0.984 (2.74)		0.737 (2.30)
Income indicator (1 if the participant's household income is greater than $75,000, 0 otherwise)	−0.637 (−2.75)		−0.621 (−1.99)		−0.634 (−1.92)		−0.625 (−1.88)		−0.695 (−2.51)	
Marital status indicator (1 if the participant is married, 0 otherwise)	2.250 (2.04)		1.024 (2.32)		1.043 (2.33)		1.027 (2.30)		0.094 (2.92)	
Standard deviation of parameter density function	*2.613 (1.98)*		*0.011 (0.03)*							
Ethnicity indicator (1 if the participant is Asian, 0 otherwise)		−1.009 (−2.35)		−1.119 (−2.81)		−1.000 (−2.59)		−0.978 (−2.55)		−0.926 (−2.78)
Driving experience and exposure characteristics										
Driver's license indicator (1 if the participant has had their driver's license for 6 years or more, 0 otherwise)	−1.336 (−5.53)		−1.436 (−4.88)		−1.436 (−4.43)		−1.417 (−4.34)		−1.493 (−5.16)	
Willingness to drive indicator (1 if the participant prefers to drive up to 12 hours before considering another mode of transportation, 0 otherwise)	−0.764 (−3.77)		−0.652 (−2.43)		−0.648 (−2.25)		−0.639 (−2.22)		−0.730 (−2.35)	
Accident history indicator (1 if the participant has had no accidents in their lifetime, 0 otherwise)		0.727 (2.11)		0.700 (2.64)		0.631 (2.34)		0.618 (2.29)		0.439 (1.75)
Traffic violation indicator (1 if the participant was pulled over for more than once in the last 5 years for traffic violations, 0 otherwise)		−1.787 (−2.12)		−1.925 (−2.59)		−1.735 (−2.38)		−1.698 (−2.33)		−1.500 (−2.37)
Behavioral and other characteristics										
Simulation scenario indicator (1 if rushing to destination while listening to music, 0 otherwise)		0.764 (1.93)		0.897 (2.92)		0.839 (2.83)		0.821 (2.78)		0.927 (3.24)
Simulation scenario indicator (1 if rushing to destination, 0 otherwise)	1.441 (4.99)		1.379 (3.66)		1.425 (3.70)		1.405 (3.66)		1.772 (5.22)	
Alcohol consumption tendency indicator (1 if the participant has 6 or more alcoholic beverages in a week, or if hard liquor is the participant's preferred alcoholic beverage, 0 otherwise)	0.566 (2.57)		0.764 (2.32)		0.789 (2.32)		0.779 (2.29)		0.817 (2.70)	
Music and age indicator (1 if music type was uplifting and the age of the participant was less than 25 years, 0 otherwise)		0.594 (1.78)		0.696 (2.44)		0.535 (1.92)		0.523 (1.89)		0.386 (1.40)

(*continued on next page*)

Table 3 (*continued*)

Variables	**GRP**[a]		**RP**[b]		**RE**[c]		**FIML**[d]		**UP**[e]	
	PAD[f] **Coeff.**[h] **(*t*-stat)**	**OAD**[g] **Coeff. (*t*-stat)**	**PAD Coeff. (*t*-stat)**	**OAD Coeff. (*t*-stat)**	**PAD Coeff. (*t*-stat)**	**OAD Coeff. (*t*-stat)**	**PAD Coeff. (*t*-stat)**	**OAD Coeff. (*t*-stat)**	**PAD Coeff. (*t*-stat)**	**OAD Coeff. (*t*-stat)**
Cross equation correlation, ρ	0.999 (126.57))		0.960 (10.49		0.800 (3.36)		0.773 (2.94)		-	-
Number of participants	40[*]		40		40		40		40	40
Number of observations	189		189		189		189		189	189
Log-likelihood at convergence	−137.944		−140.262		−140.278		−140.290		−68.90	−74.76
Log-likelihood at zero	−205.483		−205.483		−205.483		−205.483		−117.3	−98.84
Akaike information criterion (AIC)	315.9		320.5		320.6		316.6		153.8	167.5

[a] GRP: Grouped random parameters bivariate probit with random effects model.
[b] RP: Random parameters bivariate probit model.
[c] RE: Random effects bivariate probit model.
[d] FIML: Full information maximum likelihood bivariate probit or bivariate pooled probit model.
[e] UP: Univariate probit model.
[f] PAD: Perceived aggressive driving behavior.
[g] OAD: Observed aggressive driving behavior.
[h] Coeff.: Parameter estimates (Coefficient).
[*] After accounting for missing data, 189 observations from 40 participants are used for the analysis.

of observed aggressive driving behavior; and the standard deviation of the parameter density function of the marital status indicator for the random parameters bivariate probit model of perceived aggressive driving behavior. This is another indication of the statistical superiority of the grouped random parameters model over its counterparts, in terms of explanatory power.

Table 3 shows that two different sets of socio-demographic, driving experience and exposure, behavioral, and other characteristics are found to affect the perceived and observed driving behavior. In fact, each of $Z_{i,1}$ and $Z_{i,2}$ (see Eq. (1)) have nine and ten unique statistically significant variables, respectively, including the constants and the standard deviation of the parameter density function of the grouped random parameters. And the nine variables defined for the perceived aggressive driving behavior are different from the ten variables that are defined for the observed aggressive driving behavior. Also, note that the cross equation correlation coefficient of the error terms, ρ, is statistically significant, indicating the appropriateness of the bivariate modeling scheme.

Moving to the modeling results, a number of different socio-demographic characteristics are found to affect the perceived and the observed driving behavior. For example, Asian participants and participants who hold a postgraduate degree are less likely (by −0.210 and −0.181, respectively, as indicated by the pseudo-elasticities in Table 4) to drive aggressively. Low-income participants – with household annual income less than $10,000 – are more likely (by 0.250, as indicated by the pseudo-elasticity in Table 4) to drive aggressively. On the other hand, high-income participants – with household annual income greater than $75,000 – are less likely to perceive that they drove aggressively. Young participants – 25 years old or younger – who listen to uplifting music while driving are more likely (by 0.125, as indicated by the pseudo-elasticity in Table 4) to drive aggressively. The majority (80.5%) of married participants are more likely to perceive that they drove aggressively, while the remaining 19.5% are less likely to perceive that they drove aggressively – as indicated by the mean (2.250) and standard deviation (2.613) of the random parameter's density function. The vast majority (99.9%) of participants whose hometowns were located in urban areas are more likely to drive the simulator aggressively, while the remaining 0.1% are less likely to drive aggressively – as indicated by the mean (1.376) and standard deviation (0.376) of the random parameter's density function.[6] On the other hand, participants whose hometowns were located in rural areas are less likely to perceive that they drove aggressively.

Turning to the driving experience and exposure variables, it is found that participants who had no accident in their lifetime, are more likely (by 0.159, as indicated by the pseudo-elasticity in Table 4) to drive aggressively. On the contrary, participants with multiple traffic violations over the last five years are less likely to drive aggressively. At the same time, participants with significant driving experience and exposure – in terms of holding a driver's license for six or more years, and in terms of willingness to drive up to twelve hours before considering another mode of transportation – are less likely (by −0.272 and −0.161, respectively, as indicated by the pseudo-elasticity in Table 4) to perceive that they drove aggressively.

Finally, a number of behavioral and other characteristics are found to affect aggressive driving behavior. For example, participants rushing to the destination while listening to music (any kind) are more likely (by 0.185, as indicated by the pseudo-elasticity in Table 4) to drive aggressively. In contrast, participants rushing to the destination are found to be more likely (by 0.282, as indicated by the pseudo-elasticity in Table 4) to perceive that they drove aggressively, irrespective of listening to

[6] Note that, even though the distributional parameters indicate that nearly all coefficients are positive, the variable effect of the coefficients across the groups is expressed within the coefficients' magnitude (in words, each group will have a different – but likely positive – coefficient).

Table 4
Pseudo-elasticities (averaged over all observations).

Variables	GRP[a]		RP[b]		RE[c]		FIML[d]		UP[e]	
	PAD[f]	OAD[g]	PAD	OAD	PAD	OAD	PAD	OAD	PAD	OAD
Socio-demographics										
Hometown indicator (1 if the participant grew up in an urban area, 0 otherwise)		0.273		0.256		0.269		0.268		0.222
Hometown indicator (1 if the participant grew up in a rural area, 0 otherwise)	−0.161		−0.153		−0.156		−0.156		−0.161	
Education indicator (1 if the participant has a post graduate degree, 0 otherwise)		−0.210		−0.208		−0.192		−0.192		−0.184
Income indicator (1 if the participant's household income is less than $10,000, 0 otherwise)		0.250		0.251		0.243		0.241		0.182
Income indicator (1 if the participant's household income is greater than $75,000, 0 otherwise)	−0.132		−0.142		−0.144		−0.143		−0.149	
Marital status indicator (1 if the participant is married, 0 otherwise)	0.309		0.203		0.205		0.204		0.208	
Ethnicity indicator (1 if the participant is Asian, 0 otherwise)		−0.181		−0.184		−0.178		−0.178		−0.178
Driving experience and exposure characteristics										
Driver's license indicator (1 if the participant has had their driver's license for 6 years or more, 0 otherwise)	−0.272		−0.333		−0.329		−0.327		−0.326	
Willingness to drive indicator (1 if the participant prefers to drive up to 12 hours before considering another mode of transportation, 0 otherwise)	−0.161		−0.150		−0.147		−0.147		−0.156	
Accident history indicator (1 if the participant has had no accidents in their lifetime, 0 otherwise)		0.159		0.141		0.136		0.136		0.098
Traffic violation indicator (1 if the participant was pulled over for more than once in the last 5 years for traffic violations, 0 otherwise)		−0.244		−0.232		−0.232		−0.233		−0.215
Behavioral and other characteristics										
Simulation scenario indicator (1 if rushing to destination while listening to music, 0 otherwise)		0.185		0.208		0.206		0.204		0.246
Simulation scenario indicator (1 if rushing to destination, 0 otherwise)	0.282		0.293		0.298		0.297		0.350	
Alcohol consumption tendency indicator (1 if the participant has 6 or more alcoholic beverages in a week, or if hard liquor is the participant's preferred alcoholic beverage, 0 otherwise)	0.112		0.160		0.163		0.163		0.160	
Music and age indicator (1 if music type was uplifting and the age of the participant was less than 25 years, 0 otherwise)		0.125		0.135		0.112		0.111		0.084

[a] GRP: Grouped random parameters bivariate probit with random effects model.
[b] RP: Random parameters bivariate probit model.
[c] RE: Random effects bivariate probit model.
[d] FIML: Full information maximum likelihood bivariate probit or bivariate pooled probit model.
[e] UP: Univariate probit model.
[f] PAD: Perceived aggressive driving behavior.
[g] OAD: Observed aggressive driving behavior.

music while driving. Lastly, participants who are prone to alcohol – who consume six or more alcoholic beverages per week, or hard liquor is their preferred alcoholic beverage – are more likely (by 0.112, as indicated by the pseudo-elasticity in Table 4) to perceive that they drove aggressively.

Even though the specific model estimation results may be data specific, they are indicative of the different factors that affect perceived and observed aggressive driving behavior. A possible direction for future work is to expand the data sample with more representative and non-location specific population characteristics, and study various levels of observed and perceived aggressive driving behavior. In all, the analysis demonstrates the potential of grouped random parameters bivariate probit modeling in terms of model fit and explanatory power, by accounting for cross-equation correlation, unbalanced panel effects, and group-level unobserved heterogeneity.

Table 5
Summary of the variable effect on perceived and observed aggressive driving behavior (parentheses reveal mixed effects).

Variables	Grouped random parameters bivariate probit model	
	Perceived aggressive driving behavior	Observed aggressive driving behavior
Socio-demographics		
Urban hometown		(↑)
Rural hometown	↓	
Post graduate degree		↓
Low income (less than $10,000)		↑
High income (greater than $75,000)	↓	
Married	(↑)	
Asian ethnicity		↓
Driving experience and exposure characteristics		
Experienced driver (had driver's license for 6 or more years)	↓	
Willingness to drive (preference to drive up to 12 hours before considering another mode of transportation)	↓	
No accidents (throughout lifetime)		↑
Frequent traffic violations (two or more in the last 5 years)		↓
Behavioral and other characteristics		
Rushing to destination with music		↑
Rushing to destination	↑	
Alcohol tendencies (consumption of 6 or more alcoholic beverages in a week, or preference of hard liquor as preferred alcoholic beverage)	↑	
Music (uplifting music) and age (less than 25 years old)		↑

5. Summary and conclusion

Several studies have sought to identify the factors that cause drivers to drive aggressively. However, past research has shown that drivers are frequently unaware of the fact that they are driving aggressively, when indeed they do. On the contrary, it is possible that drivers who experienced in their lifetime a motor-vehicle injury or a fatality of a close relative or friend (or a similarly intense driving incident) are likely to shift their driving behavior perception from non-aggressive to aggressive, and thus drive non-aggressively and perceive their driving behavior as aggressive. At the same time, the factors that affect the perceived and observed aggressive driving behavior are also likely to differ. Using driving simulation data and surveys, a grouped random parameters bivariate probit model was estimated – accounting simultaneously for unobserved heterogeneity, unbalanced panel data effects, and cross-equation error correlation – to study the factors that increase or decrease observed and perceived aggressive driving behavior likelihood. Table 5 summarizes the results, which show that different socio-demographic (level of education, household income level, ethnicity, and hometown), driving experience and exposure (accident and traffic violation history, driver experience, and willingness to drive), and behavioral and other characteristics (alcohol consumption, rushing to destination, and music) affect how the participants' driving behavior is perceived and observed.

To evaluate the proposed approach, the grouped random parameters bivariate probit model results were compared to their four model counterparts: univariate probit, full information maximum likelihood bivariate probit, bivariate probit model with random effects, and random parameters bivariate probit model counterparts. The comparison highlights the statistical superiority of the grouped random parameters bivariate probit model, in terms of model fit, explanatory power, and forecasting accuracy.

Even though the nature of this empirical study is exploratory, the results show that different factors affect the perceived and the observed driving behavior. At the same time, the findings imply that drivers are likely to perceive their driving behavior as aggressive (or non-aggressive), when in reality it is non-aggressive (or aggressive). Finally, the proposed approach demonstrates the potential of the grouped random parameters bivariate probit model to simultaneously account for group-level unobserved heterogeneity, unbalanced panel data effects, and cross-equation error correlation.

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