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A new method of discrete optimization for cross-section selection of truss structures

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The main goal of this article is to develop a new method of discrete optimization for cross-section selection of truss structures. First, it introduces a parameterized description of discrete cross-section areas in an admissible list, which is an ordered list of manufacturing available cross-section areas (*i.e.* the 'available list' of cross-section areas) and constructs a discrete optimization model for truss structures. Secondly, the generalized shape function-based parameterization (GSFP) method is proposed to transform discrete variables obtained previously into continuous ones, thus transforming the discrete optimization problem into a continuous optimization problem which can be readily solved with gradient-based methods. Thirdly, by comparing the influences of different *admissible lists* formed with elements of the same *available list*, on the convergences of the optimization process, an ordering rule is proposed to determine the order of elements in the admissible list. Lastly, the proposed method is applied to several benchmark design examples, generating results with similar or improved accuracy compared to those from heuristic methods, showing significantly improved computational efficiency. The method is shown to be accurate and efficient, which would prove especially beneficial to large-scale problems.

Keywords: discrete optimization design; truss structures; discrete variables; topology optimization; multi-material topology optimization; GSFP

1. Introduction

In practical truss structure design, the cross-section parameters are usually selected from a standardized set that is normally manufactured. In this sense, the design problem of a truss structure is, in most situations, a discrete optimization problem, and it is usually more difficult to solve than optimization with continuous variables. Discrete optimization problems have as a feasible region isolated points in space (*i.e.* points that satisfy certain constraints). The discrete nature of the feasible region leads to discontinuity and non-convexity of the problem. Therefore, the effective gradient-based methods used in continuous optimization problems cannot be applied directly to solve discrete optimization problems. On the other hand, optimization with discrete variables is fundamentally combinatorial programming. This leads to the 'combinatorial explosion effect', which is often the cause of problems with commonly used enumeration methods. To illustrate with an example, in the case of a discrete optimization problem with 10 design variables, each with 10 selectable options, there are 10^{10} different design possibilities. It

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would take more than 30 years to examine all these options if 0.1 second was spent on each. This testifies to the importance of developing efficient methods to solve discrete optimization problems.

The existing algorithms for discrete optimization can be divided into two main categories (Arora, Huang, and Hsieh 1994; Thanedar and Vanderplaats 1995). The first are the *direct methods*, which include the 0–1 programming method, enumeration method, discrete complex method and heuristic search techniques. In recent years, with the development of computation capacity, *heuristic search techniques* have been widely used in discrete variable optimization for truss structures. The basic mechanisms of this group of techniques are as follows: a randomly selected subset of points from the discrete feasible region is tested with the objective function. The ones that generate the best results are accepted and are included and/or 'evolved' into the next 'generation' of subset. In contrast, the ones that generate the worst results are 'expelled' from the future generations. After sufficient iterations, the ultimate generation of subset would consist of points that generate the best results, and the optimization would be achieved with a member of this subset. Several algorithms have been developed, including the genetic algorithm, introduced by Rajeev and Krishnamoorthy (1992) to solve optimization of discrete cross-section selection for truss structure; harmony search algorithm (Lee and Geem 2004); ant colony algorithm (Camp and Bichon 2004); simulated annealing (Kripka 2004); a hybrid method involving both particle swarm and ant colony algorithms (Kaveh and Talatahari 2008) and artificial bee colony algorithm (Hadidi, Kazemzadeh Azad, and Kazemzadeh Azad 2010). Improved algorithms have also been introduced (Wu and Chow 1995; Erbatur *et al*. 2000; He *et al*. 2004; Togan and Daloglu 2006, 2008; Isaacs, Ray, and Smith 2008). Compared to combinational algorithms, these methods can be used to find a global optimal solution and facilitate the solution, and no sensitivity information on objective function or constraint is needed in the optimization process. However, these heuristic algorithms are only improvements on enumeration, and do not eliminate significantly the large number of structural analyses, great calculation workload and low calculation efficiency of large-scale problems. Moreover, a global optimal solution is not always achievable through these algorithms.

Continuous methods are another type of solution mechanism for optimization with discrete variables. The formulation of the optimization problem is modified to enable application of gradient-based methods, and the derived optimization result is then converged to discrete values through various methods. The two main challenges in such methods are, first, to realize the continuous description of discrete variables, for which parameterization methods are vital, and secondly, to transform the derived optimal design into the design of a reasonable discrete nature, namely, the *rounding-off technique*. One rounding-off approach is simply to round the design to its nearest discrete feasible solution. However, this may lead to the designs violating the constraints. Another approach is to apply the penalization or special constraints through a material interpolation scheme, as in the methods used in the topology optimization of multi-materials to make intermediate selections uneconomical in the objective function and thereby encourage discrete solutions. For most direct continuous methods, continuation is realized by directly loosening the limitation on discrete values and extending to a domain including all the discrete values of the original physical quantity, and the reference of discrete nature is realized through the rounding-off method. However, the continuous method has a great influence on the optimization results and rounding-off. Taking the cross-section design of bars in a truss structure as an example, under the direct continuous method the feasible set $A_i \in \{A_1, A_2, \ldots, A_n\}(A_1 < A_2 < \cdots < A_n)$ is transformed into $A_i \in [A_1, A_n]$. If some design values are given to continuous variables after optimization, namely, $A_i^* \in [A_k, A_{k+1}]$, a certain rule of the rounding-off process will result in A_k or A_{k+1} (no other value). In this way, the possibility of better discrete values is restricted manually. As a result, study of the appropriate parameterization method is imperative to realize continuation of the problem.

In the present work, a new method of discrete optimization for the cross-section selection of truss structures is proposed. The article is organized as follows. Section 2 states the problem of cross-section selection optimization design for the lightest truss structure. In Section 3, a generalized shape function-based parameterization (GSFP) method is formulated for discrete optimization of cross-section selection of trusses. Some classical examples are redesigned for validating the proposed method in Section 4. Finally, some conclusions are presented in Section 5.

2. Problem descriptions

In the discrete optimum design problem of the truss structure, the major task is to select an optimal cross-section of the elements from a permissible list of standard sections that minimize the weight of the structure while satisfying the design constraints (*e.g.* stress, displacement and stability). It can be stated as follows:

$$\begin{aligned} \text{find}: \quad & X = \{A_1, A_2, \ldots, A_{N_g}\} \\ \text{min}: \quad & W(X) = \sum_{e=1}^{N_b} \rho_e A_e l_e \\ \text{s.t.} \quad & g_k(X) \le 0, \; k = 1, \ldots, m \\ & A_i \in \bar{A} = \{\bar{A}_1, \bar{A}_2, \ldots, \bar{A}_{N_{mat}}\} \; i = 1, \ldots, N_g \end{aligned} \tag{1}$$

In the above formulations, $\mathbf{X} = (A_1, A_2, \ldots, A_{N_g})$ is the sizing variable vector, which is the cross-sectional area and is selected from a list of discrete values, $\bar{\boldsymbol{A}} = \{\bar{A}_1, \bar{A}_2, \ldots, \bar{A}_{N_{mat}}\}$, N_g is the total number of groups in the truss structure, N_{mat} is the number of available sections; $W(X)$ is the objective function, which is the total weight of the truss structure; ρ_e is the material density that is used for each member; A_e and l_e are the cross-sectional area and length of the eth member, respectively; N_b is the total number of members; and $g_k(X)$ is the kth constraint, *e.g.* stress, displacement, stability.

The optimization model (1) can be converted into the continuous optimum design problem by a continuous method which can be described as follows:

$$\begin{aligned} \text{find}: \quad & X = \{A_1, A_2, \ldots, A_{N_g}\} \\ \text{min}: \quad & W(X) = \sum_{e=1}^{N_b} \rho_e A_e l_e \\ \text{s.t.} \quad & g_k(X) \le 0, \; k = 1, \ldots, m \\ & A_i \in [\bar{A}_{\min}, \bar{A}_{\max}] \; i = 1, \ldots, N_g \end{aligned} \tag{2}$$

As mentioned earlier, a discrete solution is obtained by rounding off the continuous solution of optimization model (2) and thus it may lead far from the optimal solution. In fact, there are many ways to convert the discrete optimization problem into the continuous problem. For example, the multi-material design problem can be modelled using integer material selection variables leading to a mixed-integer problem. It can be converted to the continuous problem by the discrete material optimization (DMO) approach (Lund and Stegmann 2005; Stegmann and Lund 2005). The idea is to relax the integer constraint on the selection variables so as to allow intermediate selection variable values between 0 and 1 during the optimization. The mixed-material properties are obtained as weighted averages of the constituent properties, which can be expressed as follows:

$$C = \sum_{i=1}^{N_{\mathrm{mat}}} w_i C_i = w_1 C_1 + w_2 C_2 + \cdots + w_n C_n, \sum_{i=1}^{N_{\mathrm{mat}}} w_i = 1, \quad 0 \le w_i \le 1 \tag{3}$$

where the weighting coefficients w_i with penalization of intermediate selections control the selection of each phase. Each weighting function is affected by all design variables such that an increase in one weight results in a decrease in all other weights. The different forms of weighting coefficient may lead to the different variations. Several schemes have been proposed (Lund and Stegmann 2005; Stegmann and Lund 2005) for the defining the value of the weighting factors. In Hvejsel and Lund (2011) the new interpolation schemes as direction generalizations of the well-known SIMP and RAMP are applied for general interpolation schemes between an arbitrary number of predefined materials with given properties. In the above methods, the number of designs attached to each designable element or group just equals the number of the candidate materials.

Recently, the shape function-based parameterization (SFP) method has been described in Bruyneel (2011), in which bilinear finite element shape functions act as weights in a weighted interpolation between the candidate materials. The approach uses two variables to interpolate between four materials, leading to fewer design variables than with other approaches. A bi-value coding parameterization (BCP) scheme as an extending SFP is proposed by Gao, Zhang, and Duysinx (2012) for optimizing structures made of fibre-reinforced composite materials with n candidate ply angles.

Inspired by the idea of the SFP method, a new GSFP method is proposed for the discrete optimization of cross-section selection of truss structures, and the detailed description will be presented in the following sessions.

3. Generalized shape function-based parameterization method for discrete optimization of cross-section selection of truss structures

3.1. *Parameterization of discrete variables and formulation of discrete optimization problem*

The prescribed discrete value set of cross-sectional area, which is also called candidate cross-sectional area set, is recorded as $P_{\mathrm{List}} = \{S_1, S_2, \ldots, S_{nc}\}$, $S_1 < S_2 < \cdots < S_{nc}$. Two lists, named the available list and the admissible list, are first defined for convenience of parameterization.

The available list can be expressed as

$$S_{\mathrm{List}} = \{\overbrace{S_1, \ldots, S_1}^{N_{\mathrm{mat}}-nc}, S_1, S_2, \ldots, S_{nc}\}, S_1 \le S_2 \le \cdots \le S_{nc}, N_{\mathrm{mat}} = 2^N \tag{4}$$

The length or the total element number of the available list is N_{mat} and it is restricted to $N_{\mathrm{mat}} = 2^N$. N is an integer which will be used, in following sessions, as the number of design variables describing the cross-section of a structure component. If the number of practical candidate cross-sections, nc, is equal to 2^N, the available list is identical to P_{List}; if nc is greater than 2^{N-1} and less than 2^N, there are $2^N - nc$ elements that will be supplemented. In this article, the minimal element S_1 of P_{List} is selected as a supplementary element.

The admissible list is determined as $\mathbf{A} = \{A_1, A_2, \ldots, A_{N_{\mathrm{mat}}}\}$, which is a new permutation of the elements in the list $\mathbf{S}_{\mathrm{List}}$ according to the following rules. The parameter χ_m is employed to represent the sequence number of the element in the available list $\mathbf{S}_{\mathrm{List}}$ that corresponds to the mth element A_m of the admissible list $\mathbf{A}$. Then the admissible list $\mathbf{A}$ can be determined by the available list $\mathbf{S}_{\text{List}}$ ($A_m \in \mathbf{S}_{\text{List}}, m = 1, \ldots, N_{\text{mat}}$) and the corresponding sequence number χ_m. That is:

$$A = \{A_1, A_2, \ldots, A_m, \ldots, A_{N_{\text{mat}}} | A_m = S_{\chi_m}, \; m = 1, 2, \ldots, N_{\text{mat}}\} \tag{5}$$

The sequence number m of the cross-sections in admissible list $\mathbf{A}$ can be taken as the design variable. If so, continuation of optimization problem can be realized by directly continuing the sequence number m. However, the continuation scheme is the same as Equation (2) in nature, which continues directly the cross-section area, also leading to the rounding-off problem mentioned in the Introduction. Therefore, a new parameterization method is proposed as follows.

The sequence order m is represented by a binary number:

$$m = \sum_{i=1}^{N} a_{mi} 2^{(i-1)} + 1 \tag{6}$$

In Equation (6), a_{mi} is the ith bit in the binary of integer decimal m, equal to 0 or 1. N is the number of bits in the binary, which is determined by the number of elements in admissible list $\mathbf{A}$.

Introduce an indicating number ξ_{mi} ($i = 1, 2, \ldots N$), where the ξ_{mi} takes the value of 1 or -1:

$$\xi_{mi} = 2a_{mi} - 1 \text{ or } a_{mi} = (\xi_{mi} + 1)/2 \tag{7}$$

It can be shown that ξ_{mi} ($i = 1, 2, \ldots, N$) corresponds to the element in the admissible list by substituting the above equation into Equation (6). Therefore, ξ_{mi} ($i = 1, 2, \ldots, N$) can be regarded as descriptive parameters of elements in the admissible list.

As the value of ξ_{mi} ($i = 1, 2, \ldots, N$) is 1 or -1, ξ_{mi} ($i = 1, 2, \ldots, N$) can be regarded as a vertex coordinate of a hypercube in an N-dimensional space. The cube and vertex coordinate of $N = 3$ are shown in Figure 1.

The cross-section of a bar in a truss structure can be indicated by coordinate R_i ($i = 1, 2, \ldots, N$) in N-dimensional space. The value of the coordinate is only -1 or 1, to guarantee the vertices of the hypercube being selected. The area of the cross-section of a bar can be expressed as the weighted sum of all candidate cross-sections in the admissible list, that is:

$$A = \sum_{m=1}^{N} w_m(R_1, R_2, \ldots, R_N) A_m \tag{8}$$

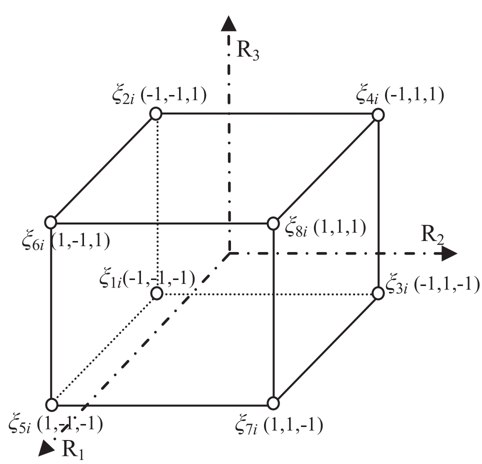

Figure 1. The coordinates of the unite hypercube ($N = 3$).

A_m refers to the cross-sectional area of candidate bar m in the admissible list; w_m refers to the weighting coefficient, with the following definition:

$$w_m(R_1, R_2, \ldots, R_N) = \begin{cases} 1 & R_i = \xi_{mi} \\ 0 & R_i \neq \xi_{mi} \end{cases}, \quad i = 1, \ldots, N \tag{9}$$

From the above equation, the cross-sectional area of the bar varies with the value of R_i ($i = 1, 2, \ldots, N$). That is, R_i ($i = 1, 2, \ldots, N$) can be regarded as a descriptive parameter for the cross-section of the bar. The parameter is regarded as a design variable, so the optimization of discrete variables (1) can be rewritten into the following formulation:

$$\begin{aligned} \text{find}: \quad & X = R_{ij}, \; i = 1, 2, \ldots, N_g; \; j = 1, 2, \ldots N \\ \min: \quad & W(X) = \sum_{e=1}^{N_b} \left(\sum_{m=1}^{N_{mat}} w_m A_m \right)_e \rho_e l_e \\ \text{s.t.} \quad & g_k(X) \leq 0 \quad k = 1, \ldots, m \\ & R_{ij} \in \{-1, 1\} \; \forall (i, j) \end{aligned} \tag{10}$$

3.2. *Continuation of discrete optimization problem*

Continuation of the optimization problem expressed by (7) loosens the limitation under which design variables can only be 1 or −1. First, the feasible domain of the design variables is extended from the discrete values to the continuous domain $[-1, 1]$. Then, a penalty technique is applied to control the final result of design variables converging to be −1 or 1. The key question is to determine the cross-sectional area value of the bar when design variables are intermediate values (neither −1 nor 1). Inspired by the SFP method, an improved function (named the generalized shape function in this article) is used as the weighting factors in Equation (8), similar to the form of the shape function used as interpolation in the finite element method.

This generalized shape function is defined as:

$$w_m(R_1, R_2, \ldots, R_N) = \frac{1}{2^N} \prod_{k=1}^{N} (1 + \xi_{mk} R_k), \quad m = 1, \ldots, N_{mat} \tag{11}$$

For example, when $N = 3$, the specific form of cross-section weight coefficient is:

$$\begin{aligned} w_1 &= \frac{1}{8}(1 - R_1)(1 - R_2)(1 - R_3), \quad w_2 = \frac{1}{8}(1 - R_1)(1 - R_2)(1 + R_3) \\ w_3 &= \frac{1}{8}(1 - R_1)(1 + R_2)(1 - R_3), \quad w_4 = \frac{1}{8}(1 - R_1)(1 + R_2)(1 + R_3) \\ w_5 &= \frac{1}{8}(1 + R_1)(1 - R_2)(1 - R_3), \quad w_6 = \frac{1}{8}(1 + R_1)(1 - R_2)(1 + R_3) \\ w_7 &= \frac{1}{8}(1 + R_1)(1 + R_2)(1 - R_3), \quad w_8 = \frac{1}{8}(1 + R_1)(1 + R_2)(1 + R_3) \end{aligned} \tag{12}$$

where R_1, R_2 and R_3 are design variables.

In order to make the design variables converge as close as possible to −1 or 1 in the continuous optimization model, the quadratic penalty approach is introduced, with the form:

$$\sum_i (1 - R_i^2) = 0 \tag{13}$$

After discrete variables are continued, the optimization model (10) can be described as:

$$\begin{aligned}
\text{find}: &\quad X = R_{ij},\ i = 1, 2, \ldots, N_g;\ j = 1, 2, \ldots N \\
\min: &\quad W(X) = \sum_{e=1}^{N_b} \left(\sum_{m=1}^{N_{\text{mat}}} w_m A_m \right)_e \rho_e l_e \\
\text{s.t.} &\quad g_k(X) \le 0\ k = 1, \ldots, m \\
&\quad -1 \le R_{ij} \le 1\ \ \forall (i,j) \\
&\quad \sum_{i,j} (1 - R_{ij}^2) = 0
\end{aligned} \tag{14}$$

The continued optimization model (14) includes an equality constraint and an inequality constraint. The equality constraint can be processed by means of Lagrange multipliers. The optimization model (14) can be rewritten as:

$$\begin{aligned}
\text{find}: &\quad X = R_{ij},\ i = 1, 2, \ldots, N_g;\ j = 1, 2, \ldots N \\
\min: &\quad W(X) = \sum_{e=1}^{N_b} \left(\sum_{m=1}^{N_{\text{mat}}} w_m A_m \right)_e \rho_e l_e + \lambda \sum_{i,j} (1 - R_{ij}^2) \\
\text{s.t.} &\quad g_k(X) \le 0\ k = 1, \ldots, m \\
&\quad -1 \le R_{ij} \le 1\ \forall (i,j) \\
&\quad \lambda \ge 0
\end{aligned} \tag{15}$$

The initial value of the Lagrange multiplier has certain influences on the optimization result. The magnitude of initial λ can be chosen based on the following equation:

$$\lambda^0 \sim \frac{W(R_{ij})}{\sum_{i,j} (1 - R_{ij}^2)} \tag{16}$$

3.3. *Formation of admissible list and sequential ordering rule*

As defined in Section 3.1, the admissible list is a permutation of the elements in the previously available list. The selection of permutation may have a great influence on the convergence. It is necessary to give a rational rule to determine the correspondence between m (the sequential number of an element in the admissible list) and χ_m (the sequential number of the corresponding element in the available list) to make $A_m = S_{\chi_m}$.

Assume that the available list $\mathbf{S}_{\text{List}}$ is the element list sorted from small to large: $S_1 \le S_2 \le \cdots \le S_{N_{\text{mat}}}$. Elements in the available list $\mathbf{S}_{\text{List}}$ are put into admissible list $\mathbf{A}$ in accordance with a certain sequence. Considering the assumption that the sequential numbers of the elements in admissible list $\mathbf{A}$ are corresponding vertices of a hypercube in space consisting of indicating number ξ_{mi} $(i = 1, 2, \ldots, N)$, define the generalized distance d_m between the mth vertex and the first vertex (which is predetermined) by the following equation:

$$d_m = \frac{1}{4} \sum_{i}^{N} \left[(\xi_{mi} - \xi_{1i})^2 \left(1 + \frac{i-1}{N_{\text{mat}}} \right) \right], \quad m = 1, \ldots, N_{\text{mat}} \tag{17}$$

Table 1. The serial number m in $\mathbf{A}$ corresponding to the serial number χ_m in $\mathbf{S}_{\text{List}}$.

m $(N=3)$	1		2		3		4		5		6		7		8	
χ_m	1		4		3		7		2		6		5		8	
m $(N=4)$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
χ_m	1	5	4	11	3	10	8	15	2	9	7	14	6	13	12	16

The distances determined in Equation (17) are sorted from small to large, leading to an ordered distance list, $\mathbf{S}_{\text{Dist}}$. Denote the sequence number of the distance between the mth and the first vertices d_m in the list $\mathbf{S}_{\text{Dist}}$ as $\text{SeqD}(d_m)$, then the sequence rule of the admissible list $\mathbf{A}$ is: the sequence number χ_m in the available list $\mathbf{S}_{\text{List}}$ corresponding to the mth element in the admissible list $\mathbf{A}$ is the sequence number of the distance d_m between the mth and the first vertices in the list $\mathbf{S}_{Dist}$. That is:

$$\chi_m = \text{SeqD}(d_m) \tag{18}$$

When the number of the elements in admissible list $\mathbf{A}$ is 8 ($N = 3$) and 16 ($N = 4$), the sequence number χ_m of available list $\mathbf{S}_{\text{List}}$ corresponding to the sequence number m in admissible list $\mathbf{A}$ is shown in Table 1.

3.4. *Rule of rounding off*

Similarly to the topology optimization method, the design variables of the optimization results may have few intermediate values between -1 and 1, although the penalty constraints are imposed on the optimization model (15). Thus, it is necessary to round off the optimum results and ensure that the value of every design variable is equal to 1 or -1.

There are two ways of rounding off. One is to round off the design variables directly. However, the corresponding cross-section of the bar is a 'composite cross-section' when the design variables are intermediate values. A cross-section far beyond or far below the composite cross-section may result from such a rounding off, especially if the constraint is a tight constraint after optimization or the admissible list includes many cross-sections. The other way is to round off the cross-sectional area of the composite cross-section; this method is selected for the rounding off in this article. The conservative discrete design (CDD) approach provided by Yu, Zhang, and Johnson (2003) is employed as the specific method of rounding off.

3.5. *Main procedures of GSFP method*

Solutions to the discrete optimum design problem of the truss structure described by optimization model (1) can be obtained by solving the optimization model (15). In the process of the problem continuation, a GSFP method is used. The flowchart of the proposed GSFP method is shown in Figure 2 and the detailed procedures are as follows:

(1) Obtain the structural data from the input data.
(2) Determine the number, N, of design variables needed to describe the cross-section of a group of components (bars) of the truss structure based on the number, N_{mat}, of the given candidate cross-sections of bars (the number of the elements in the available list).
(3) Form the admissible list from the available list $\mathbf{A}$ by use of the sequence rule expressed by Equation (18).
(4) Obtain the initial value of λ by Equation (16).
    (a) Define the design variables $R_{ij} = 0$.

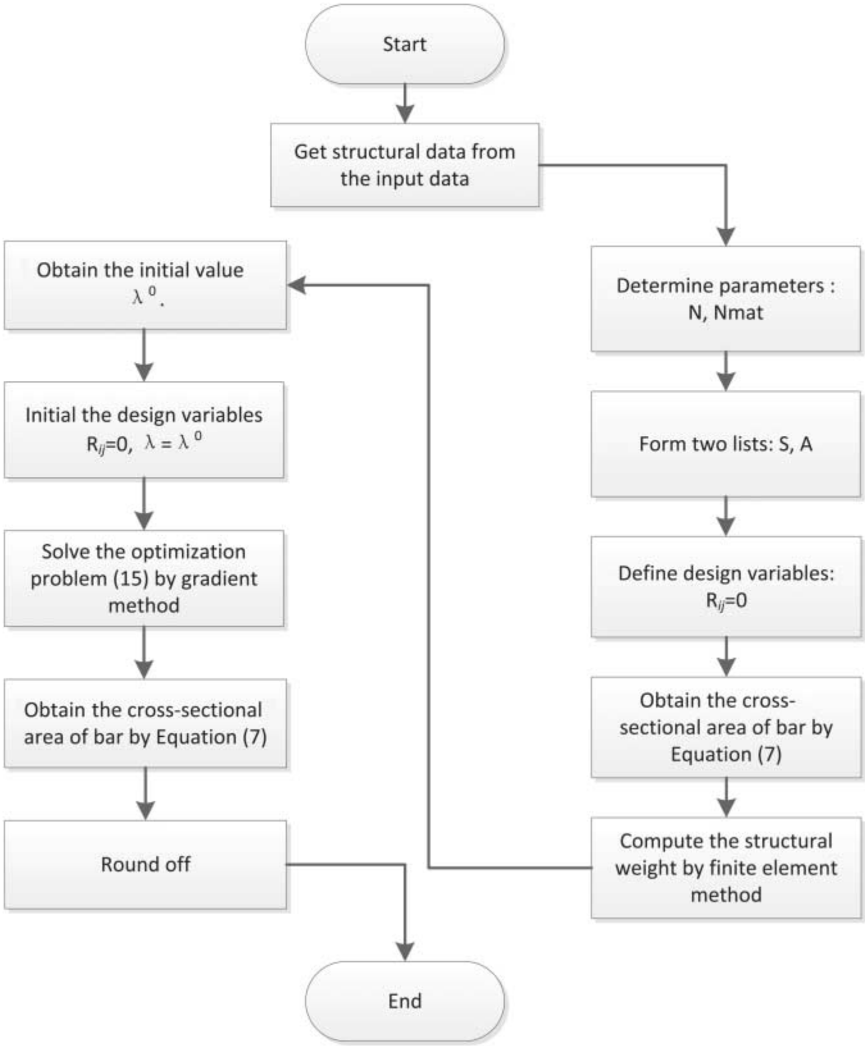

Figure 2. Flowchart of the generalized shape function-based parameterization (GSFP) method.

(b) The cross-sectional area of bars are obtained by Equation (7).
(c) The total weight of the truss structure is obtained by finite element analysis.
(d) Obtain the initial value λ^0.

(5) Initial the design variables $R_{ij} = 0$, $\lambda = \lambda^0$.
(6) Solve the optimization problem and the feasible direction method adopted in this article.
(7) Round off the design results using the rounding-off rule given in Section 3.4.

4. Examples and discussion

In order to verify the effectiveness of the present proposed method, several classic examples for discrete optimization of cross-section selection of truss structures are redesigned, and the new design results are compared to the existing results in the literature.

The ratio, CR, of the number, N_{mid}, of the design variables taking values within $(-1, 1)$ to the total number, N_{var}, of design variables is introduced to describe the convergence of the method:

$$\text{CR} = \frac{N_{\text{mid}}}{N_{\text{var}}} \tag{19}$$

Smaller CR indicates better convergence. The number of finite element analyses needed in the optimization process is given to illustrate the efficiency of the method.

The influence of the sequence order of elements in the admissible list **A** will be discussed in Section 4.1, and the availability of the rule described by Equation (18) will also be verified. The sequence ordering rule has been used for the subsequent examples. In addition, the initial values of the design variable are all set to zero and the examples are all solved by the modified feasible direction method.

4.1. *A 10-bar planar truss*

The 10-bar planar truss is shown in Figure 3. The data for the design are: $E = 10^4$ ksi; $\rho = 0.10\,\text{lb/in.}^3$; allowable stress $= \pm 25$ ksi; permissible vertical displacement $= 2$ in.; and vertical downward loads at joints 2 and 4 $= 100$ kips. The 42 available sections are given in the list P_{List}, $P_{\text{List}} = \{$(1)1.62, (2)1.80, (3)1.99, (4)2.13, (5)2.38, (6)2.62, (7)2.63, (8)2.88, (9)2.93, (10)3.09, (11)3.13, (12)3.38, (13)3.47, (14)3.55, (15)3.63, (16)3.84, (17)3.87, (18)3.88, (19)4.18, (20)4.22, (21)4.49, (22)4.59, (23)4.80, (24)4.97, (25)5.12, (26)5.74, (27)7.22, (28)7.97, (29)11.5, (30)13.5, (31)13.9, (32)14.2, (33)15.5, (34)16.0, (35)16.9, (36)18.8, (37)19.9, (38)22.0, (39)22.9, (40)26.5, (41)30.0, (42)33.5$\}$(in.^{2}).

The number of the candidate cross-sectional area set, *nc*, is 42 ($2^5 < 42 < 2^6$) in this example, and thus the element number in the available list, N_{mat}, is taken to be 64 (2^6). The first 22 elements are the minimum ones in the candidate cross-sectional area set, and the other elements are the same as the candidate cross-sectional area set. The cross-sectional area of one member will be described by six parameters and thus the total number of design variables is 60.

In order to discuss the influence of the sequence of the elements in admissible list **A**, three different admissible lists of the corresponding sequences are given as follows:

Seq 1 refers to the corresponding sequence of elements by the rule described by Equation (18).
Seq 2 refers to the corresponding sequence of elements by the rule from small to large.
Seq 3 refers to a random sequence: Seq 3 = (13, 21, 1, 1, 26, 27, 12, 1, 33, 34, 35, 36, 37, 38, 1, 1, 2, 3, 4, 5, 1, 23, 24, 25, 1, 1, 1, 1, 17, 1, 1, 22, 18, 39, 1, 1, 6, 7, 8, 40, 41, 42, 9, 10, 11, 1, 1, 1, 14, 15, 1, 1, 1, 16, 19, 20, 1, 1, 1, 28, 31, 32, 29, 30).

As shown in Figure 4, the iteration history of the optimization process is quite stable for the three sequences. Design variables optimized based on Seq 1, Seq 2 and Seq 3 are presented in

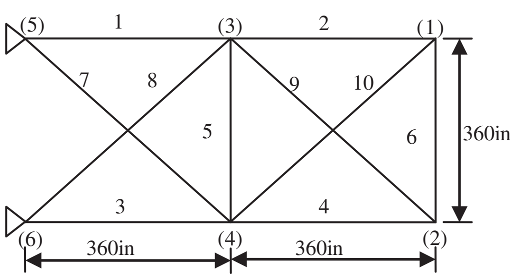

Figure 3. A 10-bar planar truss.

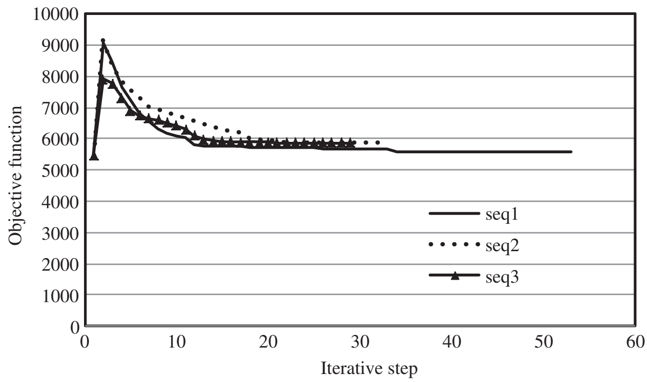

Figure 4. Iteration history of the cross-section design optimization of the 10-bar planar truss.

Table 2. Optimal design variables for Seq 1.

	Design variables					
Bar no.	R_1	R_2	R_3	R_4	R_5	R_6
1	1.00	1.00	1.00	1.00	1.00	1.00
2	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
3	1.00	1.00	1.00	−1.00	1.00	1.00
4	1.00	1.00	1.00	1.00	1.00	−0.17
5	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
6	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
7	1.00	1.00	−0.45	−1.00	1.00	1.00
8	1.00	1.00	1.00	−1.00	1.00	1.00
9	1.00	1.00	1.00	−1.00	1.00	1.00
10	−1.00	−1.00	−1.00	1.00	−1.00	−1.00

Table 3. Optimal design variables for Seq 2.

	Design variables					
Bar no.	R_1	R_2	R_3	R_4	R_5	R_6
1	1.00	1.00	1.00	1.00	1.00	1.00
2	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
3	1.00	1.00	1.00	1.00	1.00	1.00
4	1.00	1.00	1.00	−1.00	−1.00	−1.00
5	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
6	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00
7	−0.93	1.00	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00	1.00	1.00
9	1.00	1.00	1.00	−1.00	−1.00	−1.00
10	−1.00	−1.00	−1.00	−1.00	−1.00	−1.00

Tables 2–4 respectively. It can be seen that the case of Seq 1 or Seq 2 has a good CR on the whole and nearly all design variables tend to 1 or −1. The CR values of Seq 2, Seq 1 and Seq 3 are 0.017, 0.033 and 0.083, respectively. These values indicate that the proposed method has good numerical convergence.

The design results for the cases of Seq 1, Seq 2 and Seq 3 are obtained as represented in Table 5. The tables also provide a comparison between the optimal design results reported in the literature and the current work. In the present literature, the lightest weight of structure created by Togan

Table 4. Optimal design variables for Seq 3.

Bar no.	R_1	R_2	R_3	R_4	R_5	R_6
	Design variables					
1	1.00	−1.00	1.00	−1.00	−1.00	1.00
2	−1.00	1.00	−1.00	−1.00	1.00	−1.00
3	1.00	−1.00	1.00	−0.53	−1.00	1.00
4	1.00	−1.00	1.00	0.27	−1.00	1.00
5	−1.00	1.00	−1.00	−1.00	1.00	−1.00
6	−1.00	1.00	−1.00	−1.00	1.00	−1.00
7	1.00	−1.00	1.00	0.60	−1.00	1.00
8	1.00	−1.00	1.00	−0.32	−1.00	1.00
9	1.00	−1.00	1.00	−0.18	−1.00	1.00
10	1.00	1.00	−1.00	1.00	1.00	−1.00

Table 5. Comparison of optimal design for the 10-bar planar truss.

	M1	M2	M3	M4	M5	M6	M7	M8	M9
A1	33.50	33.50	30.00	33.50	33.50	33.50	33.50	33.50	33.50
A2	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
A3	22.00	22.90	22.90	22.90	26.50	22.90	22.90	22.90	22.90
A4	15.50	15.50	13.50	15.50	13.50	15.50	15.50	14.20	14.20
A5	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
A6	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
A7	14.20	7.97	13.90	7.97	7.22	7.22	7.97	7.97	7.97
A8	19.90	22.00	22.00	22.00	22.90	22.90	22.00	22.90	22.90
A9	19.90	22.00	22.00	22.00	22.00	22.00	22.00	22.00	22.00
A10	2.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
Weight (lb)	5613.84	5491.71	5586.59	5491.71	5556.95	5499.30	5491.71	5490.74	5490.74
	M10	M11	M11	M12	M13	Seq 1	Seq 2	Seq 3	
A1	33.50	33.50	33.50	33.50	33.50	33.50	33.50	33.50	
A2	2.13	1.62	1.62	1.62	1.62	1.62	1.62	2.13	
A3	22.00	22.90	22.90	22.90	22.00	22.00	33.50	26.50	
A4	13.90	13.90	16.00	14.20	14.20	16.00	16.90	13.50	
A5	1.62	1.62	1.62	1.62	1.62	1.62	1.62	2.13	
A6	1.99	1.62	1.62	1.62	1.62	1.62	1.62	2.13	
A7	7.97	7.97	7.97	7.97	7.97	7.97	4.22	7.97	
A8	22.90	22.90	22.90	22.90	22.90	22.00	33.5	22.90	
A9	22.90	22.00	19.90	22.00	22.00	22.00	16.90	19.90	
A10	1.62	1.62	1.62	1.62	1.62	1.62	1.62	4.18	
Weight (lb)	5525.04	5479.94	5448.62	5490.74	5458.30	5479.07	6058.73	5660.51	
Modify	–	–	–	–	–	**5477.32**	**6058.63**	**5673.64**	
CR	–	–	–	–	–	0.033	0.017	0.083	
Max. def.	–	2.004	2.0173	–	2.0123	2.0047	–	–	

Note: M1 (Rajeev and Krishnamoorthy 1992); M2 (Cai and Thiereuf 1993a); M3 (Coello 1994); M4 (Camp, Pezeshk, and Cao 1998); M5 (Tong and Liu 2001); M6 (Nanakorn and Meesomklin 2001); M7 (Turkkan 2003); M8 (Camp and Bichon 2004); M9 (Kripka 2004); M10 (Sousa and Takahashi 2005); M11 (Togan and Daloglu 2008); M12 (Sonmez 2011); M13 (Galante 1996).

and Daloglu (2008) is 5448.62 lb, but its maximum displacement is 2.0173 in., which slightly violates the constraint value of 2.00 in., by about 1%. Another result, also provided by Togan and Daloglu (2008), is regarded as the lightest design in the existing results because its maximum displacement is 2.004 in., which is much closer to the constraint value.

The rounded structural weight for Seq 1 is 5477.32 lb and it is lighter than the lightest design, 5479.94 lb. However, other results for Seq 2 and Seq 3 are apparently the local optimum solution. In particular, Seq 3 is only a random sequence. If sorted by another random sequence, the optimum result would be different. More numerical experiments have been completed and in most cases the objection function values are higher than the result in the existing literature, and the convergence of design variables is worse than Seq 1. Therefore, the ordering rule for the admissible list enables the proposed method to converge well with the optimal result. Hence, in the following examples, all cross-sections in admissible list **A** are sorted based on Equation (18), without any further description.

4.2. *A 25-bar spatial truss*

The 25-bar spatial truss is shown in Figure 5. The members are divided into eight groups, presented in Table 6. The assumed data are: $E = 10^4$ ksi and $\rho = 0.10\,\mathrm{lb/in.}^3$, with the applied loads listed in Table 7. The objective function of the problem is set to minimize the weight of the structure. The stress is constrained to ± 40 ksi and only the displacement at joints 1 and 2 are restricted,

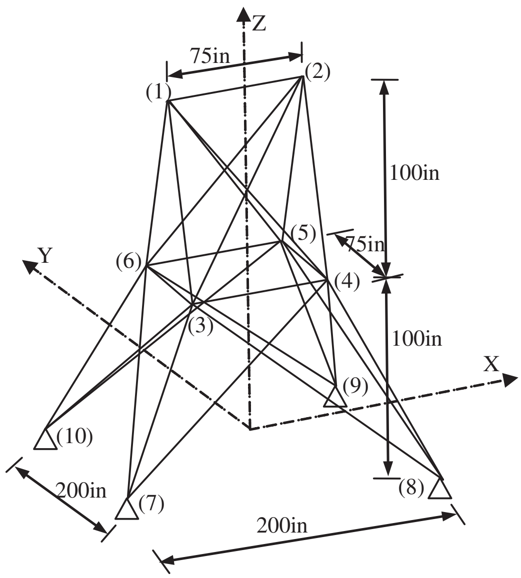

Figure 5. A 25-bar spatial truss.

Table 6. Group membership for the 25-bar spatial truss.

Group no.	Members	Group no.	Members
1	1–2	5	3–4, 5–6
2	1–4, 2–3, 1–5, 2–6	6	3–10, 6–7, 4–9, 5–8
3	2–5, 2–4, 1–3, 1–6	7	3–8, 4–7, 6–9, 5–10
4	3–6, 4–5	8	3–7, 4–8, 5–9, 6–10

Table 7. Loading conditions for the 25-bar spatial truss.

Node	F_x (kips)	F_y (kips)	F_z (kips)
1	1.0	−10.0	−10.0
2	0.0	−10.0	−10.0
3	0.5	0.0	0.0
6	0.6	0.0	0.0

Table 8. Optimal design variables for the 25-bar spatial truss.

Bar no.	Design variables R_1	R_2	R_3	R_4	R_5
1	−1.00	−1.00	−1.00	−1.00	−1.00
2	−1.00	−1.00	−1.00	−1.00	1.00
3	1.00	1.00	1.00	1.00	1.00
4	−1.00	−1.00	−1.00	−1.00	−1.00
5	−1.00	−1.00	1.00	1.00	0.62
6	−1.00	−1.00	−1.00	0.14	1.00
7	−1.00	−1.00	−1.00	−1.00	1.00
8	1.00	1.00	1.00	1.00	1.00

Table 9. Comparison of optimal design for the 25-bar spatial truss.

Method	W (lb)	A1 (in.^{2})	A2 (in.^{2})	A3 (in.^{2})	A4 (in.^{2})	A5 (in.^{2})	A6 (in.^{2})	A7 (in.^{2})	A8 (in.^{2})	Num. FEA	*CR*	Max. def.
M1	562.93	0.1	1.8	2.6	0.1	0.1	0.8	2.1	2.6	–	–	–
M2	546.01	0.1	1.8	2.3	0.2	0.1	0.8	1.8	3.0	–	–	–
M3	487.41	0.1	0.1	3.4	0.1	2.0	1.0	0.7	3.4	–	–	–
M4	539.78	1.5	0.7	3.4	0.7	0.4	0.7	1.5	3.2	–	–	–
M5	486.29	0.1	0.5	3.4	0.1	1.5	0.9	0.6	3.4	–	–	–
M6	493.80	0.1	1.2	3.2	0.1	1.1	0.9	0.4	3.4	–	–	–
M7	485.05	0.1	0.5	3.4	0.1	1.9	1.0	0.4	3.4	–	–	–
M8	484.85	0.1	0.3	3.4	0.1	2.1	1.0	0.5	3.4	–	–	–
M9	484.33	0.1	0.4	3.4	0.1	2.2	1.0	0.4	3.4	39,201	–	–
M10	484.85	0.1	0.3	3.4	0.1	2.1	1.0	0.5	3.4	–	–	–
M11	484.85	0.1	0.3	3.4	0.1	2.1	1.0	0.5	3.4	–	–	–
M12	484.85	0.1	0.3	3.4	0.1	2.1	1.0	0.5	3.4	–	–	–
M13	483.354	0.1	0.3	3.4	0.1	2.0	1.0	0.5	3.4	–	–	0.3505
Present work	481.59 **484.33**	0.1	0.4	3.4	0.1	2.2	1.0	0.4	3.4	223	0.05	0.3499

Note: M1 (Zhu 1986); M2 (Rajeev and Krishnamoorthy 1992); M3 (Cai and Thiereuf 1993a); M4 (Coello 1994); M5 (Wu and Chow 1995); M6 (Erbatur *et al*. 2000); M7 (Tong and Liu 2001); M8 (Camp and Bichon 2004); M9 (Kripka 2004); M10 (Lee *et al*. 2005); M11 (Li, Huang, and Liu 2009); M12 (Sonmez 2011); M13 (Togan and Daloglu 2008).

both to less than ±0.35 in. in the x and y directions. The available discrete values of the variables are S = {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.0, 3.2, 3.4}(in.^{2}).

There are 30 ($2^4 < 30 < 2^5$) available cross-sections in the example; each bar cross-section will be described by five parameters, resulting in 40 design variables. The optimal design variables are shown in Table 8. Refer to Table 9 for the optimal structural weight, rounded objective function value (bold script in the table), cross-section selection of each group, number of called finite element analyses during the analysis process ($k = 0$) and CR.

In this example, CR = 0.05, which means that only two design variables fail to tend to 1 or −1, resulting in a good convergence effect. The iteration history is also quite stable, as shown in Figure 6. The structural weight is equal to the optimal result in M9. Although the optimal result in M13 is also 483.354 lb, the maximum displacement is 0.3505 in., which slightly violates the constraint. Finite element analysis is called for 223 times in the first optimization step and no more than 800 times in the overall optimization circulation. Simulated annealing algorithm is applied by Kripka (2004), in which the finite element analysis is called for 39,201 times. The method in the article increases efficiency by 50 times compared to the simulated annealing algorithm.

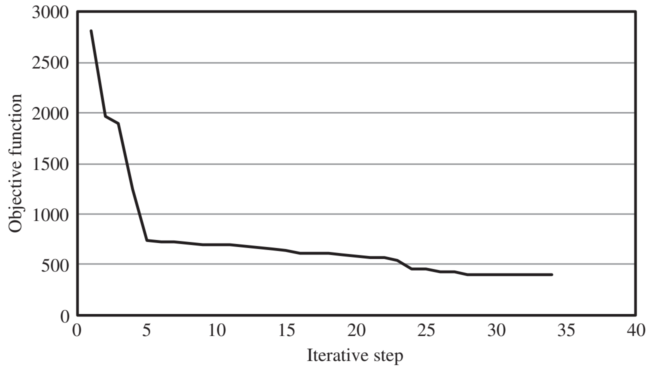

Figure 6. Iteration history of the cross-section design optimization of the 25-bar planar truss.

4.3. *A 72-bar spatial truss*

For the 72-bar spatial truss structure shown in Figure 7, the members are divided into 16 groups, according to Table 10. The assumed data are: $E = 10^4$ ksi and $\rho = 0.10\,\text{lb/in.}^3$, the applied loads at joints 17 and $P_x = 5.0$ kips, $P_y = 5.0$ kips, $P_z = -5.0$ kips. The objective function of the problem is set to minimize the weight of the structure W. The members are subjected to stress limits of ± 25 ksi. The nodes are subjected to displacement limits of ± 0.25 in. The available discrete values of the variables are S = {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1, 3.2}(in.^{2}).

There are 32 available cross-sections in this example; each bar cross-section will be described by five parameters, resulting in 80 design variables. Please refer to Table 11 for optimal design

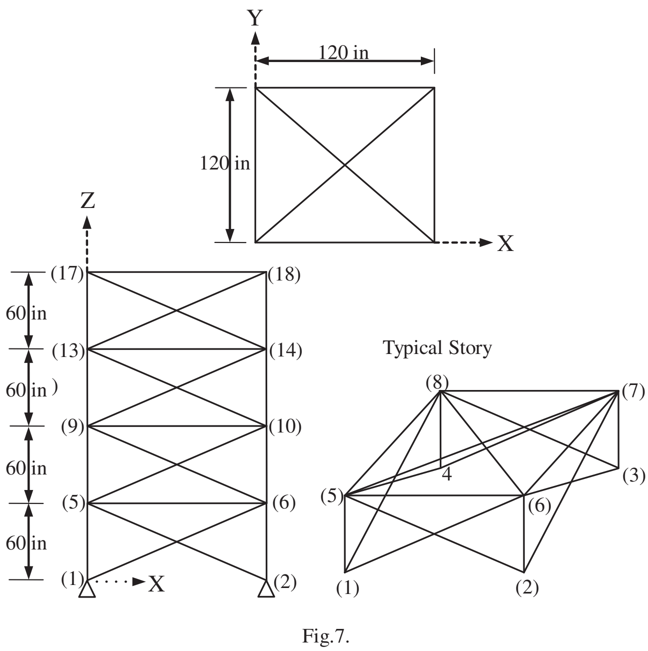

Figure 7. A 72-bar spatial truss.

Table 10. Group membership.

Group no.	Members
1	1–5, 2–6, 3–7, 4–8
2	2–5, 1–6, 2–7, 3–6, 3–8, 4–7, 1–8, 4–5
3	5–6, 6–7, 7–8, 8–5
4	6–8, 5–7
5	4–9, 6–10, 7–11, 8–12
6	6–9, 5–10, 6–11,7–10, 7–12, 8–11, 5–12, 8–9
7	9–10, 10–11, 11–12, 12–9
8	10–12, 9–11
9	8–13, 10–14, 11–15, 12–16
10	10–13, 9–14, 10–15, 11–14, 11–16, 12–15, 9–16, 12–13
11	13–14, 14–15, 15–16, 16–13
12	14–16, 13–15
13	12–17, 14–18, 15–19, 16–20
14	14–17, 13–18, 14–19, 15–18, 15–20, 16–19, 13–20, 16–17
15	17–18, 18–19, 19–20, 20–17
16	18–20, 17–19

Table 11. Optimal design variables for the 72-bar spatial truss.

	Design variables				
Bar no.	R_1	R_2	R_3	R_4	R_5
1	1.00	1.00	1.00	−1.00	−1.00
2	−1.00	−1.00	−1.00	−1.00	1.00
3	−1.00	−1.00	−1.00	−1.00	−1.00
4	−1.00	−1.00	−1.00	−1.00	−1.00
5	−1.00	−1.00	−1.00	1.00	1.00
6	−1.00	−1.00	1.00	−1.00	−1.00
7	−1.00	−1.00	−1.00	−1.00	−1.00
8	−1.00	−1.00	−1.00	−1.00	−1.00
9	−1.00	−1.00	−1.00	−1.00	1.00
10	−1.00	1.00	−1.00	−1.00	−0.80
11	−1.00	−1.00	−1.00	−1.00	−1.00
12	−1.00	−1.00	−1.00	−1.00	−1.00
13	−1.00	−1.00	−1.00	−1.00	−1.00
14	−1.00	−1.00	−1.00	−1.00	1.00
15	−1.00	−1.00	−1.00	−1.00	1.00
16	−1.00	−1.00	−1.00	−1.00	1.00

variables and Table 12 for optimal structural weight, rounded structural weight (bold script in the table), cross-section selection of each group, number of called finite element analyses during the analysis process ($k = 0$) and CR.

In the example, CR $= 0.0125$, which means that only one design variable fails to tend to 1 or -1, resulting in a good convergence effect. The iteration history is also quite stable, as shown in Figure 8. The objective function value is slightly better than the optimal result in the existing literature.

4.4. *A 200-bar planar truss*

The configuration of the 200-bar planar truss structure is given in Figure 9. Not all element numbers are given, to improve the clarity of the figure. This truss structure is designed by using different types of constraints under different numbers of design variables in the literature. In this study, the

Table 12. Comparison of optimal design for the 72-bar spatial truss.

	M1	M2	M3	M4	M5	M6	Present work
A1	1.5	1.9	2.6	3.0	2.1	1.9	1.7
A2	0.7	0.5	1.5	1.4	0.6	0.5	0.6
A3	0.1	0.1	0.3	0.2	0.1	0.1	0.1
A4	0.1	0.1	0.1	0.1	0.1	0.1	0.1
A5	1.3	1.4	2.1	2.7	1.4	1.3	1.6
A6	0.5	0.6	1.5	1.9	0.5	0.5	0.4
A7	0.2	0.1	0.6	0.7	0.1	0.1	0.1
A8	0.1	0.1	0.3	0.8	0.1	0.1	0.1
A9	0.5	0.6	2.2	1.4	0.5	0.6	0.6
A10	0.5	0.5	1.9	1.2	0.5	0.5	0.4
A11	0.1	0.1	0.2	0.8	0.1	0.1	0.1
A12	0.2	0.1	0.9	0.1	0.1	0.1	0.1
A13	0.2	0.2	0.4	0.4	0.2	0.2	0.1
A14	0.5	0.5	1.9	1.9	0.5	0.6	0.6
A15	0.5	0.4	0.7	0.9	0.3	0.4	0.6
A16	0.7	0.6	1.6	1.3	0.7	0.6	0.6
Weight (lb)	400.66	387.94	1089.88	1069.79	388.94	385.54	384.16 **384.41**
Num. FEA	–	–	–	–	–	–	504
CR	–	–	–	–	–	–	0.0125

Note: M1 (Wu and Chow 1995); M2 (Lee *et al*. 2005); M3, M4, M5 (Li, Huang, and Liu 2009); M6 (Kaveh and Talatahari 2009).

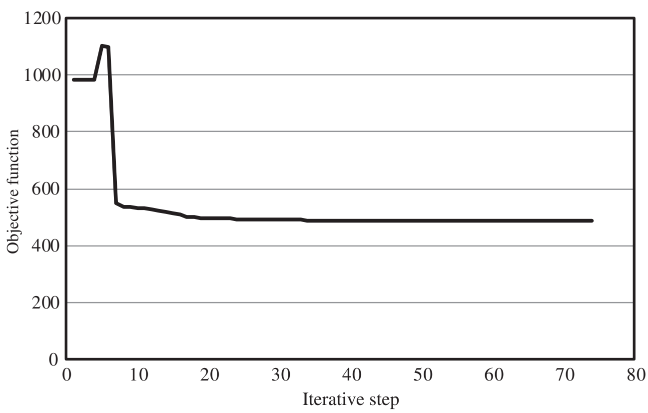

Figure 8. Iteration history of the cross-section design optimization of the 72-bar planar truss.

members of this structure are categorized in to 96 groups. The data on these groups are given in Table 13. Material properties and constraints used in this study are as follows: modulus of elasticity, $E = 3 \times 10^4$ ksi and density of material, $\rho = 0.283\,\text{lb/in.}^3$ The allowable displacement is limited to 0.5 in. and the allowable stress is ± 30 ksi. The following is a list of 30 discrete values of design variables adopted to solve this problem: S = {0.100, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180, 23.680, 28.080, 33.700}(in.^{2}). This structure is subjected to three different load cases, as follows:

Load case 1: 1kip acting in the positive *x* direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62 and 71.

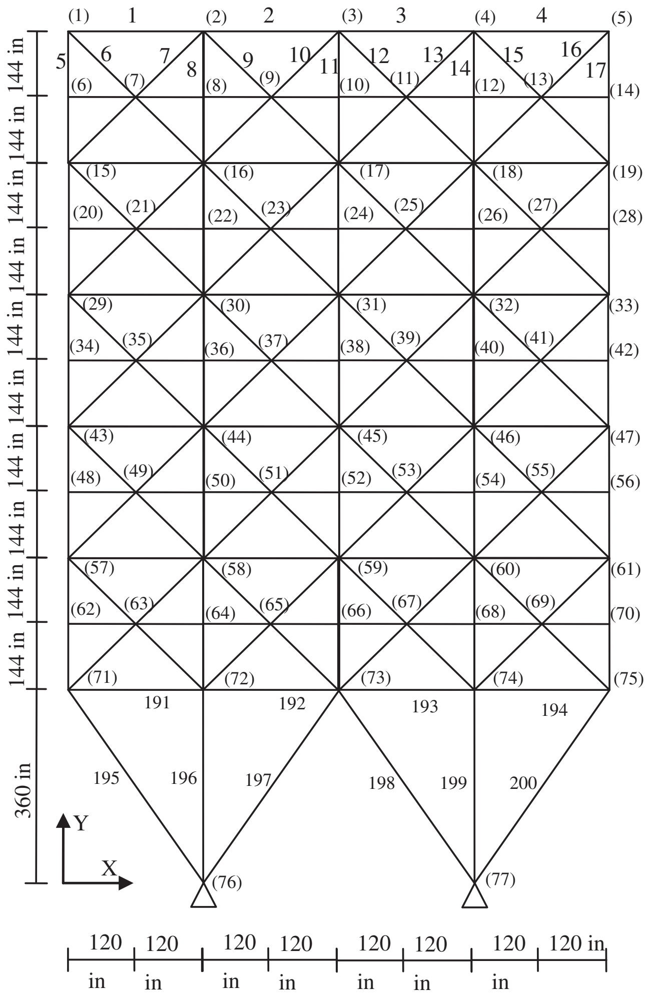

Figure 9. A 200-bar planar truss.

Load case 2: 10kips acting in the negative y direction at nodes 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 54, 56, 57, 58, 59, 60, 61, 62, 64, 66, 68, 70, 71, 72, 73, 74 and 75.

Load case 3: case 1 and case 2 combined.

There are 30 available cross-sections in the example; each bar cross-section will be described by five parameters, resulting in 498 design variables. The cross-section selection of each group after optimization is shown in Table 14. The iteration history is quite stable, as shown in Figure 10.

In this example, the number of called finite element analyses in the optimization ($k = 0$) is 1681, CR = 0.051. The optimal structural weight is 29,586.05 lb and the rounded structural weight is 29,826.02 lb. Using the same data for this example, Dede, Bekiroğlu, and Ayvaz (2011) found the minimum weight of this structure to be 30,868 lb. Ghasemi, Hinton, and Wood (1999) found the minimum weight of this structure to be 30,905 lb with GA2-800 and 31,109 lb with GA2-100, and Cai and Thiereuf (1993b) found the minimum weight of this structure to be 31,014 lb. Compared to these, a lighter structural weight is given in this article.

Table 13. Group membership for the 200-bar planar truss.

Group no.	Members	Group no.	Members	Group no.	Members	Group no.	Members
1	1, 4	25	46, 52	49	102, 114	73	146
2	2, 3	26	47, 51	50	103, 113	74	153, 156
3	5, 17	27	48, 50	51	104, 112	75	154, 155
4	6, 16	28	49	52	105, 111	76	157, 169
5	7, 15	29	57, 58, 61, 62	53	106, 110	77	158, 168
6	8, 14	30	59, 60	54	107, 109	78	159, 167
7	9, 13	31	64, 76	55	108	79	160, 166
8	10, 12	32	65, 75	56	115, 118	80	161, 165
9	11	33	66, 74	57	116, 117	81	162, 164
10	132, 139, 170, 177, 18, 25, 56, 63, 94, 101	34	67, 73	58	119, 131	82	163
11	19, 20, 23, 24	35	68, 72	59	120, 130	83	171, 172, 175, 176
12	21, 22	36	69, 71	60	121, 129	84	173, 174
13	26, 38	37	70	61	122, 128	85	178, 190
14	27, 37	38	77, 80	62	123, 127	86	179, 189
15	28, 36	39	78, 79	63	124, 126	87	180, 188
16	29, 35	40	81, 93	64	125	88	181, 187
17	30, 34	41	82, 92	65	133, 134, 137, 138	89	182, 186
18	31, 33	42	83, 91	66	135, 136	90	183, 185
19	32	43	84, 90	67	140, 152	91	184
20	39, 42	44	85, 89	68	141, 151	92	191, 194
21	40, 41	45	86, 88	69	142, 150	93	192, 193
22	43, 55	46	87	70	143, 149	94	195, 200
23	44, 54	47	95, 96, 99, 100	71	144, 148	95	196, 199
24	45, 53	48	97, 98	72	145, 147	96	197, 198

Table 14. Optimal design results for the 200-bar planar truss.

Group no.	Area	Group no.	Area	Group no.	Area	Group no.	Area
1	0.347	25	4.805	49	10.850	73	8.525
2	0.100	26	0.100	50	0.100	74	0.100
3	3.813	27	0.100	51	0.100	75	0.100
4	0.100	28	4.805	52	6.572	76	13.330
5	0.100	29	0.100	53	0.100	77	0.100
6	3.131	30	0.100	54	1.018	78	0.100
7	0.100	31	8.525	55	6.572	79	9.300
8	0.100	32	0.100	56	0.100	80	3.131
9	3.131	33	0.539	57	0.100	81	0.100
10	0.100	34	4.805	58	10.850	82	8.525
11	0.100	35	0.100	59	0.100	83	0.100
12	0.100	36	0.100	60	0.100	84	0.100
13	5.952	37	5.952	61	8.525	85	13.330
14	0.100	38	0.100	62	0.100	86	0.100
15	0.100	39	0.100	63	3.131	87	0.100
16	3.131	40	13.330	64	6.572	88	10.850
17	0.100	41	0.100	65	0.100	89	0.100
18	0.100	42	0.100	66	0.100	90	3.131
19	3.131	43	5.952	67	10.850	91	8.525
20	0.100	44	1.018	68	0.100	92	9.300
21	0.100	45	0.100	69	0.100	93	9.300
22	7.192	46	5.952	70	8.525	94	17.170
23	0.539	47	0.100	71	3.131	95	14.290
24	0.100	48	0.100	72	0.100	96	8.525

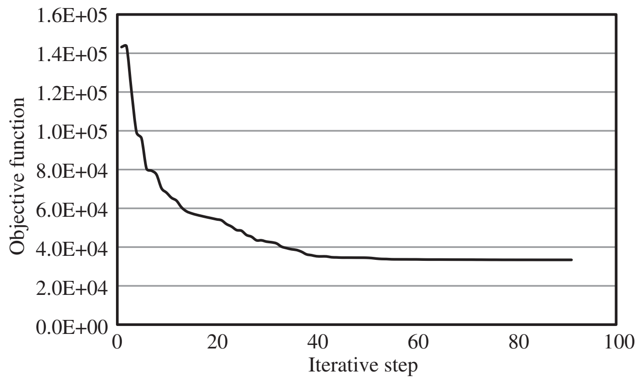

Figure 10. Iteration history of the cross-section design optimization of the 200-bar planar truss.

The numerical examples above demonstrate that the computational efficiency is high in the method presented in this article, because the optimization method based on gradient information has relatively high efficiency. By penalizing design variables through Equations (11) and (17), design variables tend towards 1 or -1 as far as possible, which guarantees relatively good convergence. Only very few design variables did not tend towards 1 or -1. Compared to the existing results in the literature, better or compatible objective function values can be obtained through the proposed method, which verifies the calculation reliability.

5. Conclusions

In this article, a GSFP method for discrete optimization of cross-section selection of truss structures is proposed. The parameterized description of a discrete cross-section admissible list is established. Furthermore, the sequence of admissible list has a significant influence on the optimal design result. The ordering rule for the admissible list enables the proposed method to converge well with the optimal result. Several representative numerical examples demonstrate the high efficiency of this method.

Moreover, the discrete optimization of cross-section selection of trusses is transformed into a continuous optimization problem, so it can be solved by gradient optimization algorithms. Computational efficiency can be improved by one to three orders of magnitude compared to the heuristic method; therefore, this method is suitable for tackling large-scale optimization design with a number of discrete variables. Finally, the present method has good numerical convergence and the design results have only a few middle values, so it is easy to round off.

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