Linear Algebra and its Applications

HW#8

1. Following Problem 2 of HW#5, rank the potentials of NTU, NTHU, NCTU, NCKU and NCCU teams.

2. Apply the Gram-Schmidt process to $a=[1, 1, 0]^T$, $b=[1, 0, 1]^T$ and $c=[0, 1, 1]^T$ and write the result in the form $A=QR$.

3. (a) Find the parabola: $y = C + Dt + Et^2$ fit to the following measurements by solving the normal equations:
   $y = 2$ at $t = -3$,
   $y = 0$ at $t = 0$,
   $y = 1$ at $t = 1$,
   $y = 2$ at $t = 2$.
   (b) Find your approximate solution by QR factorization and draw the observations with best-fit parabola using Excel.

4. Find the Fourier coefficients a_0, a_1, b_1, a_2, b_2 to approximate a step function $y(x)$ which equals to -1 for the interval $-\pi \le x \le 0$ and equals to 2 for interval $0 < x \le \pi$. Use Excel to plot $y(x)$ and the Fourier series on the same figure.

5. Find the closest degree-3 polynomial function to fit the same step function in Problem 4 over $-\pi \le x \le \pi$ by:
   (1) solving the normal equation
   (2) minimizing the least square
   (3) the Legendre polynomials and
   (4) Use Excel to plot the original step function and the fitted polynomial function on the same figure.

6. Find the determinants of
$$\begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 1 & 2 & 5 & 3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$$
   by eliminations and possible row exchanges.

7. Suppose you do two row operations at once, going from

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ to $\begin{bmatrix} a-mc & b-md \\ c-la & d-lb \end{bmatrix}$.

Find the determinant of the new matrix, by rules or by direct calculation.

8. If Q is an orthogonal matrix, so that $Q^{\mathrm{T}}Q = I$, prove that det$Q$ equals +1 or −1. What kind of box is formed from the rows (or columns) of Q?