Linear Algebra and Its Applications

線性代數與應用

Course#: 546 U6040 Credits: 3

Time: Tuesday 2:20-5:10 PM Classroom: 共同 301

Original Screenplay: a novel by Gilbert Strang

Linear Algebra and Its Applications (歐亞 Tel: 02-8912-1188)

http://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/video-lectures/

Movie directed by 陳正剛(Argon Chen) achen@ntu.edu.tw (國青 111)

Assistant Directors (助教)

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Class website: https://cool.ntu.edu.tw/courses/31968

Course Outline

1. Introduction - Importance of Linear Algebra
2. $\boldsymbol{Ax=b}$, Matrix, Gaussian Elimination and Application
3. Applications: Differential Equation
4. Vector Spaces and Linear Equation
5. Applications: Graphs and Networks
6. Orthogonality and Least Squares
7. Applications: Multiple Regression
8. Application: Linear Programming and Karmarkar Method (if time allowed)
9. Determinant and its Applications
10. Eigenvalues and Eigenvectors
11. A^k and Difference Equations
12. e^A and Differential Equations
13. Similarity Transformation and Spectral Theorem
14. Positive Definite Matrices and Minimum Principles
15. Applications: Multivariate Analysis and Principal Component Analysis
16. Singular Value Decomposition and its Applications

Evaluation (*subject to minor changes)

Mid-term	30% *
Final	30% *
Term report	25%*
Homework	15%

Linear Algebra and Its Applications

Motivation behind this course:

I am trying to direct a drama series about the *STORY* of

Linear Algebra

- **The story is *clean* and *beautiful***
  - **A mathematician's statement?**
  - **Linear algebra is one of very few mathematical subjects that can really make you feel that way**
  - **This is exactly one of this class's goals: tell you something exciting and beautiful.**
- **Things learned from the story are *needed* and *used***
  - **The subject has been taught too abstractly.**
  - **Crucial importance of the story was often missed**
  - **An essential sophomore course but not really appreciated**
  - **We are here trying to pick up this subject again and hopefully this time we can have different perspectives not only on this subject but also on any subject that uses linear algebra.**
- **Less on rigor and much more on *understanding*.**
  - ***Explain* rather than *deduce***
  - **Ideas come with examples**
  - **Ability to *reason mathematically* will naturally develop**
  - **The story moves simply and naturally from a line or a plane to the *n*-dimensional space. That step is mathematics at its best and every one of you should be able to take it.**

Read the Novel and Watch the Episodes

Linear Equations (n=2)

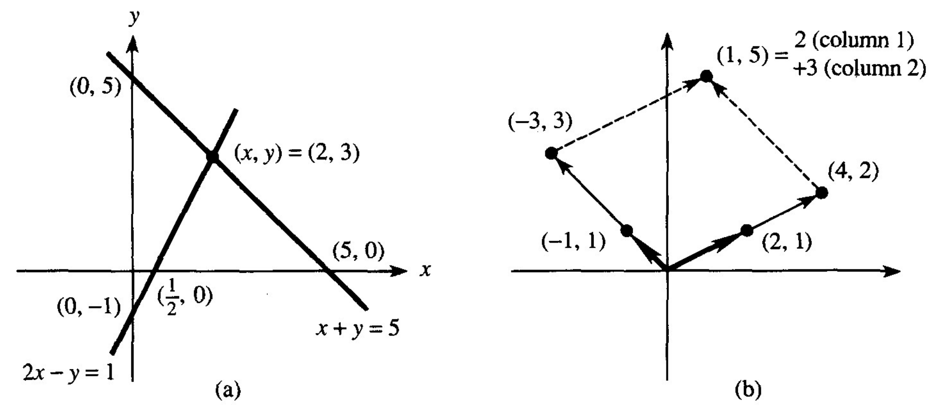
Example:

$$
\begin{aligned}
2x - y &= 1 \\
x + y &= 5
\end{aligned}
$$

Two ways to look at the system:

1. by equation or *by row*

Each equation represents a straight line on the *x*-*y* plane and the intersection point is the only point on both lines and therefore the solution of the system



2. *by column*, two equations are one *vector equation*

$$x\begin{bmatrix}2\\1\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}1\\5\end{bmatrix}$$

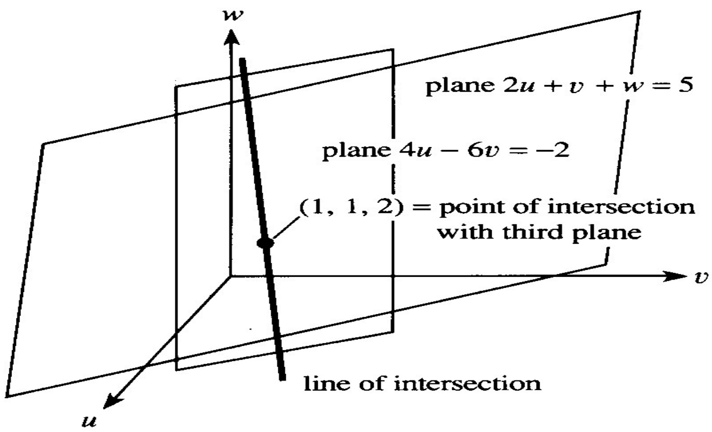
To find the combination of the column vectors on the left side which produces the vector on the right side

Linear Equations (n=3 and beyond) – the rows

Example

$$\begin{aligned} 2u + v + w &= 5 \\ 4u - 6v \quad &= -2 \\ -2u + 7v + 2w &= 9 \end{aligned}$$

By rows:



In *three* dimensions a *line* requires *two* equations; in n dimensions it will require n-1

- **Extended into n dimensions?**

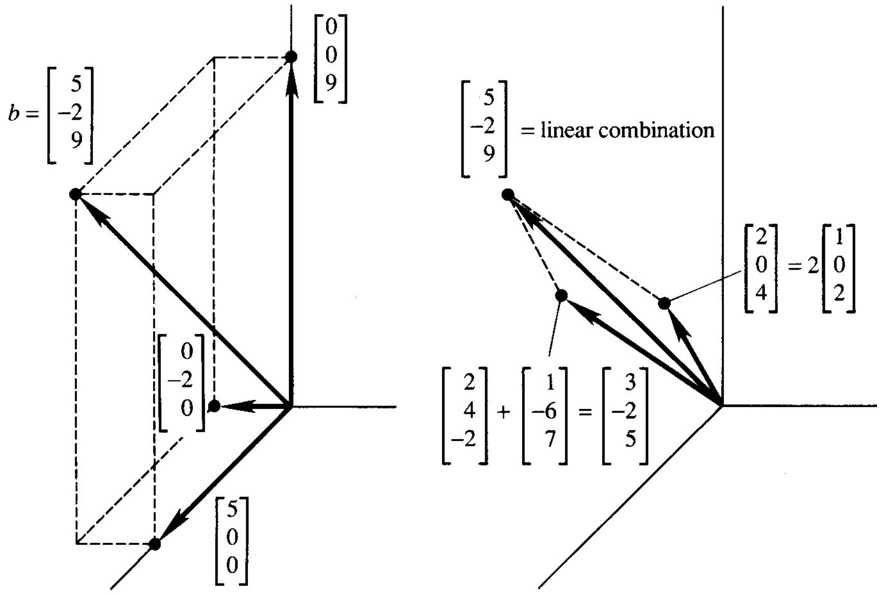
  n equations and they contain n unknowns….

  - ***The 1st equation:* an n-1-dimensional plane in n dimensional space**
  - ***The 2nd equation:* another plane, and hopefully intersect with the 1st plane in a smaller set of dimension n-2**
  - ***Every new equation (plane):* reduces dimension by one.**
  - ***At the end:* intersection of all n planes has dimension *zero – a point***

Linear Equations (n=3 and beyond) – the columns

$$u\begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix} + v\begin{bmatrix} 1 \\ -6 \\ 7 \end{bmatrix} + w\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}$$

Descarte's ideas: *vectors*



Addition of vectors:

Multiplication by a scalar:

Linear combination:

Summary:

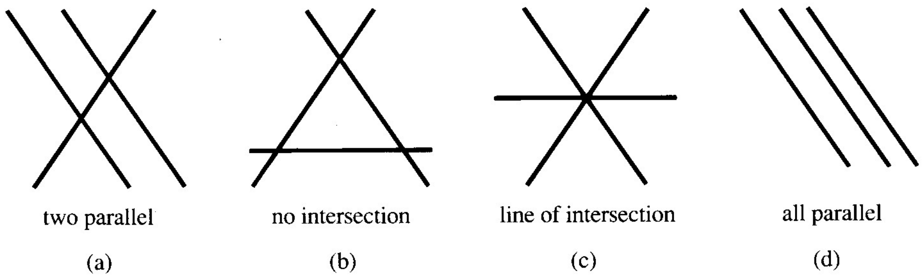
***Row picture*: intersection of *n* planes**

***Column picture*: the right side *b* is a combination of the column vectors**

***Solution*: intersection point planes = coefficients in the combination of columns**

The Singular Case

- **Two-dimensional problem:**
  Lines parallel: singular
  Lines meet: nonsingular
- **How about three-dimensional singular problem by row picture?**



- **How about four-dimensional singular cases by row picture?**
- **How about singular problem by column picture?**

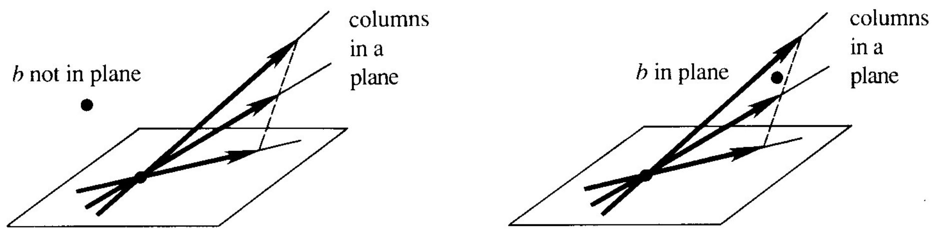
$$u\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + v\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + w\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = b$$

For *b*=(2, 5, 7), the above is possible but not for *b*=(2, 5, 6). Why?

***The three column vectors actually lie in the same 2-dimenstional*! Any linear combination of the three column vectors must be in the same plane. That is, <u>*b* must also lie in the same plane in order to make the equation possible!!</u>**

The Singular Case by Column Picture

- **Chance for *b* to be in the plane is much less than the chance to be not in the plane --- *a singular system generally has no solution*.**



(a) no solution (b) infinity of solutions

- **What if *b* is in the plane? Infinitely many solutions!! How?**

Because $3\begin{bmatrix}1\\2\\3\end{bmatrix}-1\begin{bmatrix}1\\0\\1\end{bmatrix}-2\begin{bmatrix}1\\3\\4\end{bmatrix}=0,$

$$b=\begin{bmatrix}2\\5\\7\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}+\begin{bmatrix}1\\3\\4\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}+\begin{bmatrix}1\\3\\4\end{bmatrix}+c\left(3\begin{bmatrix}1\\2\\3\end{bmatrix}-1\begin{bmatrix}1\\0\\1\end{bmatrix}-2\begin{bmatrix}1\\3\\4\end{bmatrix}\right)$$

c **can be any number and b has infinitely many solutions.**

- ***n*-dimensional singular problem?**
- **Row picture? @#$&@·········**

  n planes have no point in common

- **Column picture? *the n columns lie in the n*-1 (*or less*) *-dimensional subspace!***
- **We study nonsingular case first and singular case later.**