Homework 3 - updated

Due 2023.12.8
ME7129 Optimization in Engineering,
National Taiwan University.

Problem 1 (60%)

Please find the Pareto optima of the following bi-objective design problems

$$\begin{aligned} & \min\{f_1, f_2\} \\ f_1 = x_1 + x_2 \ , f_2 = -10x_1 + x_2 & \\ \text{s. to.} \quad g_1 = \frac{x_1^2 x_2}{20} - 1 \leq 0 & \\ g_2 = \frac{(x_1 + x_2 - 5)^2}{30} - \frac{(x_1 - x_2 - 12)^2}{120} - 1 \leq 0 & \\ g_3 = \frac{80}{x_1^2 + 8x_2 + 5} - 1 \leq 0 & \\ 0 \leq \{x_1, x_2\} \leq 10 & \end{aligned} \tag{1}$$

Problem 2 (40%)

Consider the problem

$$\begin{aligned} \min f &= x_1 + x_2 \\ \text{s. to.} \quad g_1 &= 1 - \frac{x_1^2 x_2}{20} \leq 0 \\ g_2 &= 1 - \frac{(x_1 + x_2 - 5)^2}{30} - \frac{(x_1 - x_2 - 12)^2}{120} \leq 0 \\ g_3 &= 1 - \frac{80}{x_1^2 + 8x_2 + 5} \leq 0 \\ 0 &\leq x_1, x_2 \leq 10 \end{aligned} \tag{2}$$

Assume that the final optimal might have manufacturing uncertainties with $X \sim N(\mu_x, \sigma_x^2)$ where $[\mu_{X_1}, \mu_{X_2}] = [x_1^*, x_2^*]$, and $\sigma_{X_1} = \sigma_{X_2} = 0.3$.

1. Please run Monte Carlo simulations with 10 samples, what are the probability values of the optimal violating each constraints? 15%

2. Repeat previous 10-run MCS for 20 times, do you get the same results every time? Why not? 15%

3. Please run Monte Carlo simulations with 1 million samples, what are the probability values of the optimal violating each constraints? 5%

4. Repeat previous 1 million-run MCS for 20 times, do you get the same results every time? Why or why not? 10 %

5. Please explain the difference between 10 sample MCS and 1 million sample MCS. 15%