2015 American Control Conference
Palmer House Hilton
July 1-3, 2015. Chicago, IL, USA

A Least-Squares Regression Based Method for Vehicle Yaw Moment of Inertia Estimation

Zitian Yu, Xiaoyu Huang, and Junmin Wang*

Abstract—**Vehicle yaw moment of inertia is an important parameter for many vehicle dynamic models and control systems yet it is usually difficult to estimate. A methodology in estimating the vehicle yaw moment of inertia is presented in this article by studying the linear relationship between vehicle lateral acceleration, yaw acceleration, and rear wheel lateral tire forces. This linear relationship is derived by manipulating the equations in the single-track model such that the front wheel force disappears in the equation. Based on the linear relationship, the common global positioning system (GPS) measurement error—antenna bias angle, can be tuned based on the symmetric assumption of vehicle left and right dynamics and a least-squares regression (LSR). A lag-like lateral tire force model is applied to capture the transient dynamics of the lateral tire force. The parameter determining relaxation length of the tire model can also be tuned based on a similar LSR method. Finally, the yaw moment of inertia can be estimated from an LSR again after knowing the estimated rear wheel lateral force. Simulation results in CarSim® show that this proposed method is capable of generating reasonable estimations of yaw moment of inertia without knowing the front road wheel steering angle.**

Keywords: Vehicle yaw moment of inertia, transient lateral force, relaxation length, estimation, least-squares, vehicle dynamics.

I. INTRODUCTION

Yaw moment of inertia is an important parameter for vehicle dynamics and controls [1]-[8]. It relates the moment applied about vehicle vertical axis to the yaw dynamics and thus is necessary for many vehicle models and yaw stability and motion control systems. The popular single-track model proposed [9] highlights the necessity of yaw moment of inertia even to the very basic modeling of the vehicle dynamics. Besides, vehicle yaw moment of inertia is a crucial parameter in the already widely commercialized Electronic Stability Control (ESC) programs, where typically preset values of the yaw moments of inertia are used.

There are generally two approaches in estimating vehicle yaw moment of inertia. Now most of the estimations need either some platform to mount the vehicle on [10], or use cables to suspend the vehicle [11]. These methods usually can be very accurate, e.g. within 1% error [12]. However, such methods demand additional efforts in mounting the vehicle to expensive facilities and cannot be used in measurement under normal operating conditions. For pickup trucks, utility trucks, and passenger vehicles, the vehicle loads are subject to changes, and the corresponding yaw moments of inertia will also vary during operations. For lightweight vehicles, the yaw moments of inertia may be more easily affected by loads. The abovementioned approaches will not suffice to capture the variations of vehicle yaw moments of inertia in daily vehicle operations.

The other category of approaches for estimating yaw moments of inertia features making the estimation based on sensor signals when the vehicles are under normal driving conditions. Sparse studies in this category have been conducted so far. Some researchers used the suspension displacement sensor signals to calculate the overall center of gravity position, then used the Monte Carlo method to estimate the payload distribution [13]. This method requires knowing the inertia information of the empty vehicle and the payload possibilities. Some other researchers established complex estimating schemes based on the single-track vehicle model [14]-[16], which require knowing the front road wheel steering angles. As stated in [15], directly measuring the front road wheel steering angle can be prohibitively expensive. Also, if one determines the road wheel steering angle from the hand-wheel steering angle, for some vehicles with less precise steering systems, the hysteresis in the steering system might lead to measurement inaccuracies [17][18].

In this study, the vehicle yaw moment of inertia is determined through a least-squares regression (LSR) to an equation without the front wheel forces. Thus, the measurement or estimation of front road wheel steering angle is avoided. Necessary variables for the linear regression are vehicle lateral acceleration, yaw acceleration, and rear wheel lateral force, with the major challenge being how to determine the rear wheel lateral force. Since the regression result is sensitive to the fast dynamics of rear wheel lateral force, steady-state value of lateral force cannot be used. The first-order model [19][20] is used to simulate the lagging effect of lateral tire force with respect to tire slip angle. The relaxation length of lateral tire force is determined by tuning based on the intercept value of the linear regression. The main contribution of this paper is that it is the first time that the front road wheel steering angle is avoided in estimating vehicle yaw moment of inertia. In addition, a method for determining the tire relaxation length effect based on vehicle lateral dynamics is proposed. As the algorithm is very simple, it has the potential to be used in online estimation under normal driving conditions.

II. STATEMENT OF THE PROBLEM

A. Single-Track Vehicle Model

The vehicle dynamics is described based on a simple single-track model shown in Fig. 1. In this study, yaw

*Corresponding author. Zitian Yu, Xiaoyu Huang, and Junmin Wang are with the Vehicle System and Control Laboratory, Mechanical and Aerospace Engineering Department, The Ohio State University, Columbus, OH 43210 USA. (e-mail: yu.553@osu.edu; wang.1381@osu.edu). The research was partially supported by National Science Foundation CAREER Award under Grant 1149657.

moment of inertia is related to the lateral dynamics of the vehicle model through the following equations:

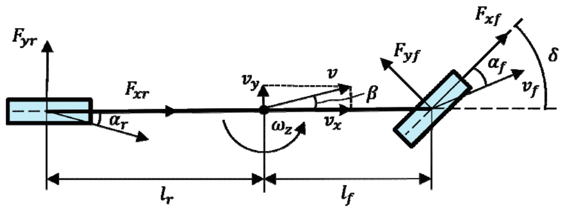

Figure 1. The single-track vehicle model.

$$ma_y = F_{yf}\cos\delta + F_{yr} + F_{xf}\sin\delta, \tag{1}$$

$$I_z\dot{\omega}_z = F_{yf}l_f\cos\delta - F_{yr}l_r + F_{xf}l_f\sin\delta, \tag{2}$$

where, m is the vehicle total mass; a_y is vehicle lateral acceleration; F_{xf} is the front tire longitudinal tire force; F_{yf} and F_{yr} are front tire and rear tire lateral forces; δ represents the front road wheel steering angle; I_z is the vehicle yaw moment of inertia; ω_z represents the vehicle yaw rate; l_f and l_r denote the distance from front axle to vehicle center of gravity (CG) and rear axle to vehicle center of gravity, respectively.

Multiplying (1) with l_f and subtracting (2) from (1) will lead to the following equation:

$$ma_y l_f = I_z\dot{\omega}_z + F_{yr}\left(l_f + l_r\right). \tag{3}$$

Since $l_f + l_r = l$, which is the distance between the front to rear axles, (3) can be written as:

$$ml_f a_y = I_z\dot{\omega}_z + lF_{yr}. \tag{4}$$

In (4), the lateral acceleration a_y and yaw acceleration $\dot{\omega}_z$ are directly measurable through a GPS (global positioning system) and IMU (inertial measurement unit) in practice [5][16], therefore they are assumed to be known variables in this paper. Now it is easy to see that the only unknown variable in (4) is the total lateral force F_{yr} generated by the rear tires and there is no front tire force involved in (4) any more.

B. Lateral Tire Force Approximation

The lateral tire force F_{yr} in (4) is calculated by summing up the estimated rear-left and rear-right lateral tire forces for a better accuracy, instead of directly calculating F_{yr} through the lumped real tire slip angle:

$$F_{yr} = F_{yrl} + F_{yrr}, \tag{5}$$

where, F_{yrl} and F_{yrr} represent the rear-left and rear-right lateral tire forces.

The driving maneuver in this study contains turnings, during which the tire slip angle changes accordingly. However, when the tire slip angle changes, the tire force does not change intermediately, but rather with a lag-like effect [19]-[21]. This transient dynamic relationship between slip angle and lateral tire force is calculated with the relaxation length model [20][21]:

$$\dot{\alpha}_i' = \left(v_{xi}/\sigma_i\right)\left(\alpha_i - \alpha_i'\right), i = rl, rr, \tag{6}$$

$$F_{yi} = f\left(F_{zi}, \alpha_i'\right), i = rl, rr, \tag{7}$$

where, σ_i is the relaxation length, which describes the distance the wheel travels for the tire force to reach a certain level; α_i' is the transient tire slip angle; α_i is the tire slip angle; f is usually an nonlinear function; F_{zi} is the tire normal force. In this study, slip angle α_i is assumed to be a known variable, as it can be calculated through GPS and IMU signals.

In (6), σ_i is also a function of tire slip angle and normal force [21]. However, for small tire slip angles, it is assumed independent of tire slip angle changes and only proportional to the tire normal force in this study for simplicity:

$$\sigma_i = K_1\frac{F_{zi}}{F_{zi,static}}, i = rl, rr, \tag{8}$$

where, K_1 is an unknown constant; $F_{zi,static}$ are static values of normal forces when vehicle is in stationary state; F_{zi} are normal forces and can be determined approximately as:

$$F_{zrl} \approx F_{zrl,static} + \frac{mHl_f}{2Bl}a_y, \tag{9}$$

$$F_{zrr} \approx F_{zrr,static} - \frac{mHl_f}{2Bl}a_y. \tag{10}$$

where, H is the CG height; B is the half of vehicle track width.

Several models have been established to describe the complex tire-road contact forces, like Pacejka's model [19] and Dugoff's model [22]. In this study, as the driving maneuver is relatively moderate (slip angle does not exceed 1.5 degrees), the discussion only focuses on small tire slip angle cases. It has been proposed that under small tire slip angle condition, the linear lateral tire force model is a good approximation to calculate the reference tire force F_{yi} [22]:

$$F_{yi} = C\left(F_{zi}\right)\alpha_i, i = rl, rr, \tag{11}$$

where, $C = K_2F_{zi}$ is the tire cornering stiffness, that is proportional to the normal force [23] with an unknown coefficient K_2.

A comparison between the estimated lateral force and the recorded lateral force from a vehicle model in CarSim® is provided in Fig. 2. Here, the coefficient K_1 determining the relaxation length has already been tuned by the process "Determination of Tire Relaxation Length" described in the next section of this paper.

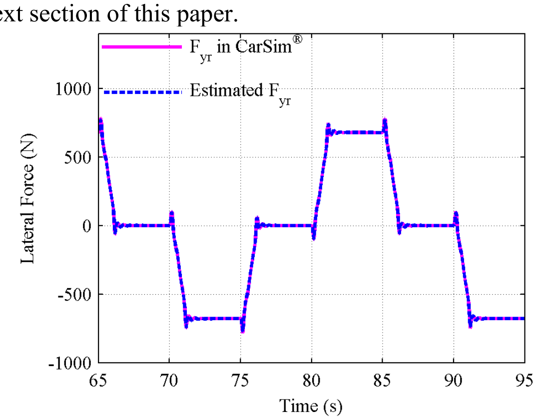

Figure 2. Estimated and CarSim® rear tire lateral force comparison.

III. Estimation Method

A. Least Square Linear Regression

For a certain value of K_1, F_{yr} can be calculated through combining (5)-(11) correspondingly. When F_{yr} is known, rewrite (4) into the following form:

$$a_y = \left(I_z / ml_f\right)\dot{\omega}_z + \left(l / ml_f\right)F_{yr}. \quad (12)$$

Then an LSR can be made based on (12) to determine the yaw moment of inertia I_z. In the linear regression, the necessary parameters to be known are vehicle mass m and longitudinal center of gravity position l_f. Vehicle length l are not needed if choosing a_y, $\dot{\omega}_z$, and F_{yr} as three independent variables in the linear regression. As a result, the unknown coefficient K_2 for cornering stiffness related to (11) is allowed to be imprecise, since the value of K_2 will not affect the regression result of I_z.

For convenience of determining the true yaw moment of inertia in the CarSim® model for comparison, (12) can be transformed into the following form:

$$ml_f a_y - lF_{yr} = I_z\dot{\omega}_z. \quad (13)$$

In (13), the terms on the left-hand side and $\dot{\omega}_z$ are all known from CarSim® simulation, therefore, the yaw moment of inertia can be easily estimated from an LSR.

All the computational methods used in this study are simple enough for implementation in the real applications in an efficient way. For a given value of K_1, the computation of lateral force F_{yr} is essentially through a low-pass filter and can be applied almost instantly. Also, the LSR algorithm is purely algebraic and only takes very short time to compute.

B. Definitions of Symmetric and Asymmetric Data Sets

For the convenience of describing in the following sections, we define the symmetric and asymmetric driving maneuvers, symmetric data set and asymmetric data set. Symmetric driving maneuver means the vehicle is undergoing a symmetric trajectory about certain center surface perpendicular to the horizontal plane. In addition, all the other variables and inputs related to vehicle dynamics are also mirror symmetric about this surface. Asymmetric driving maneuver means any driving maneuver that is not symmetric. The symmetric driving maneuver is certainly an oversimplified assumption which rarely exists. However, since the popular single-track model has already combined the left and right parts of the vehicle together, it implies the left and right difference of the vehicle is small enough to make the approximation. Thus, this paper assumes the vehicle is able to perform symmetric driving maneuver approximately.

The symmetric data set refers to the collection of variables within certain time windows in a symmetric driving maneuver that can be divided into two symmetric parts about the center surface. The asymmetric data set just refers to any variable data sets that are not from a symmetric driving maneuver.

C. Determination of GPS Antenna Bias Angle

Nowadays, many vehicle dynamics studies rely on the GPS systems [16][17]. Even though these GPS systems used in research usually have very high accuracies, it is inevitable that there are measuring errors, some of which might have significant influence on the signals that are crucial to vehicle lateral dynamics. Since usually the signals related to lateral dynamics have relatively lower signal-to-noise ratios and smaller amplitudes, they are more vulnerable to systematic errors. For example, one systematic error is the GPS antenna bias angle α_0 caused by installation, as shown in Fig. 3. Next, Fig. 4 shows how different antenna bias angles affect the transversal signal readings. As we can see from Fig. 4, lateral speed and slip angle can be significantly affected by small antenna bias angle while lateral acceleration and longitudinal speed are more immune.

In this study, antenna bias angle is determined from symmetric driving maneuver and later it can help to determine the tire relaxation length when the vehicle turning maneuver is not symmetric. Once the bias angle is determined, the symmetric driving maneuver is not needed to tune the relaxation length. The procedure of this method is described as follows:

- Let the vehicle make a left turn and then right turn in symmetrical driving maneuver.
- Choose a zero antenna bias angle and a reasonable relaxation length parameter K_1 in the beginning.
- Apply the least-squares linear regression to a symmetric data set based on (12).
- While the intercept of regression is not close enough to zero, choose another antenna bias angle and repeat the above process; otherwise, stop the process.

Remark: when making the LSR to a symmetric data set based on (12), the intercept value should be zero when variables are accurately measured. The antenna bias angle searching algorithm is completed using bi-section method. It is easy to determine whether the new antenna bias angle should increase or decrease because the intercept is a monotonic function of antenna bias angle. In the result part, a fake antenna bias angle will be added on purpose in the CarSim® simulation to examine whether the algorithm can result in the expected antenna bias angle result.

D. Determination of Tire Relaxation Length

The first approach of determining relaxation length utilizes the fact that the intercept of LSR to a nonsymmetrical data set should be zero. When the GPS antenna bias angle is obtained, the LSR has already resulted in an intercept value that is close enough to zero (within a tolerance range). Now if one makes another LSR to the "asymmetric" data set, the intercept value will likely be a

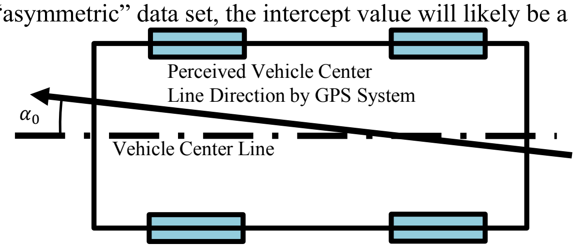

Figure 3. GPS antenna bias angle.

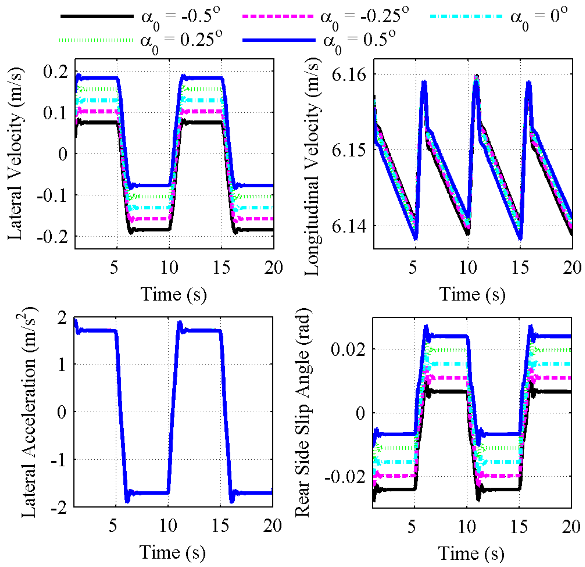

Figure 4. Influence of different values of antenna bias angle on vehicle dynamic variable signals simulated in CarSim®: the curves represent the antenna bias angle ranging from -0.5 to 0.5 degree.

non-zero value (outside the tolerance range) again. This time, the intercept value error is mainly caused by the inappropriately chosen value of K_1. An intercept that is closer to zero can be reached by tuning K_1 when making a LSR based on (12) again. Since the relaxation length roughly determines how long the lag-like effect of lateral tire force is delayed with respect to tire slip angle, changing the relaxation length parameter K_1 leads to a shifting effect to the estimated lateral tire force, causing the whole LSR surface to move and the change of intercept value.

When the intercept value is tuned to be small enough, the corresponding relaxation length parameter K_1 could be treated as a good approximation to the true value. It can be seen in Fig. 2, the estimated lateral force can be very close to the true value given by CarSim®. When K_1 has been estimated, it can be used to estimate the rear tire force F_{yr}.

IV. Simulation Results and Analysis

The above mentioned method has been tested in the CarSim® environment with a sample European van vehicle model. The vehicle configuration parameters are listed in Table I. A simulation of 100 seconds is performed with steering angle signal as the input to the system. The vehicle is in the front wheel drive mode, with constant speed of about 22km/h.

A. Vehicle Maneuver

When determining GPS antenna bias angle, it is preferable that the vehicle is making symmetrical left and right turns. Such maneuvers can be realized by using an automatic steering system [16] in real practice. If the left and right turns are not symmetrical, or the vehicle turns in an arbitrary manner to only one side, this method is still applicable if the GPS antenna bias angle is known.

TABLE I. European van configuration parameters

Vehicle mass	1300 kg
Sprung mass yaw moment of inertia	2975 kg·m^2
Distance of CG from front axle	1.35 m
Distance of CG from rear axle	1.225 m
Half of front track width	0.75 m
Half of rear track width	0.75 m
Tire effective radius	0.355 m

B. Determination of Antenna Bias Angle

When the antenna bias angle is changed, the tire slip angle will change accordingly. Then this adjustment of tire slip angle can cause the estimated lateral tire force to change, and consequently the regression plane will shift. According to the symmetry assumption, a zero intercept value should be achieved when the signals presented by GPS are correctly tuned, especially the lateral velocity signal.

An antenna bias angle is set to be 0.1 degree on purpose. Fig. 5 shows the automatic tuning process for antenna bias angle. It can be seen that the antenna bias angle approaches very closely to the set value in approximately 10-15 steps of iteration using the aforementioned bi-section algorithm. And if the tolerance range for interception value is chosen as [-0.001, 0.001] (m/s^2), the intercept value falls inside this tolerance range in about 19 steps. At the 21st step, the antenna bias angle α_0 is tuned to be almost exactly 0.1 degree. The LSR result for a zero antenna bias angle is shown in Fig. 6. As can be seen in Fig. 6, the trajectories of variables basically lie along or near the LSR surface.

C. Determination of Tire Relaxation Length

When the antenna bias angle is determined, it can always be used to correct the received signals from GPS, and the symmetric driving maneuver is not needed any more for tuning the relaxation length. The corrected signals from GPS should be the same as the signals measured when antenna bias angle is zero. Ideally, if the antenna bias angle is zero and relaxation length is accurate, the regression should have zero intercept and the data points should all lie exactly onto the regression surface. In Fig. 6 the data points

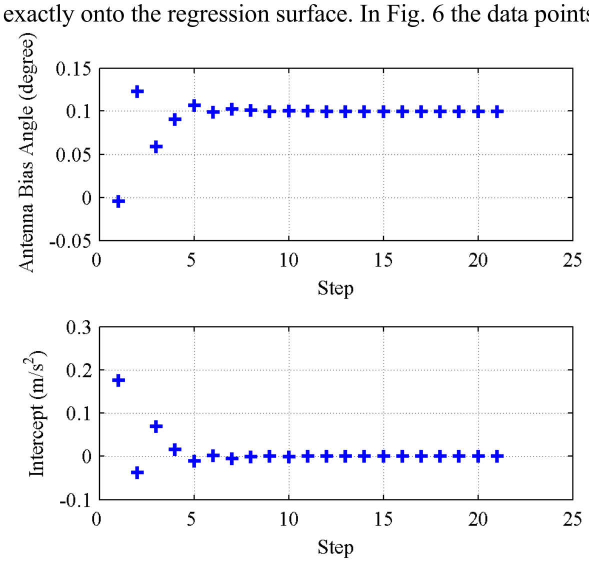

Figure 5. Convergence of antenna bias angle and the LSR intercept.

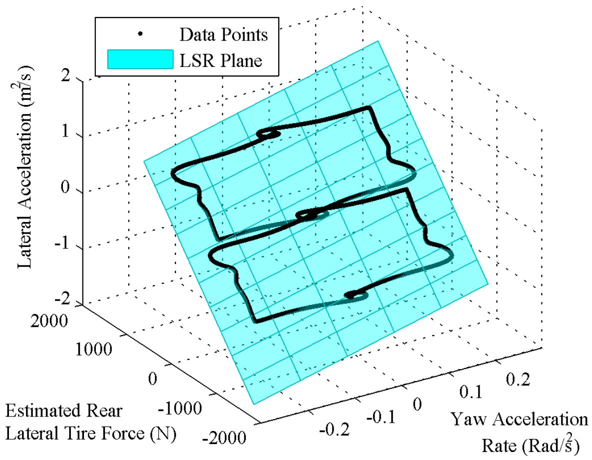

Figure 6. LSR plane when the antenna bias angle is correctly tuned.

may not lie exactly on the regression surface, this is because the assumed relaxation length parameter K_1 is incorrect.

For the same reason, although the correct antenna bias angle is obtained, if asymmetric data points are used here instead, incorrect relaxation length parameter K_1 will lead to an intercept usually outside the tolerance range again. In sum, even though the antenna bias angle is correct, if relaxation length parameter K_1 is not correct, asymmetric data points will always give an erroneous intercept again. So the relaxation length parameter K_1 should be tuned correctly such that the intercept value of LSR is back to the tolerance range again.

In this study, the relaxation length parameter K_1 is tuned using the dataset from a one-side turning. The illustration of the automatic tuning process is shown in Fig. 7, using the bi-section method. As can be seen in Fig. 7, the intercept value falls back into the tolerance zone in about 16 steps. The resulting tuned relaxation length parameter K_1 is 0.5903m. A comparison between the simulated rear wheel lateral tire force and the estimated rear wheel lateral force based on (5) using this relaxation length parameter K_1 is plotted in Fig. 2. As can be seen in Fig. 2, the true and estimated lateral tire force curves are very close in phase.

D. Determination of Yaw Moment of Inertia

When both the antenna bias angle and relaxation length parameter K_1 are tuned, the yaw moment of inertia can be derived according to (12). If all the necessary parameters and variables are accurate enough; the regression result from different combinations of data sets should be quite close. The yaw moment of inertia estimation results from different maneuvers are listed in Table II.

In Table II, results from different data sets are derived by using the same relaxation length parameter K_1 tuned above. LSR is made to a left turn, the combination of a left turn and right turn, and other driving maneuvers. The left turn and combination of left and right turn data sets correspond to the steering input as shown in Fig. 8. The triangular and sine wave form steering inputs are shown in Fig. 9 and Fig. 10. All these steering signals have the same maximum amplitude. The estimated yaw moment of inertia is within 1% compared to value 3488 kg·m^2 calculated based on the

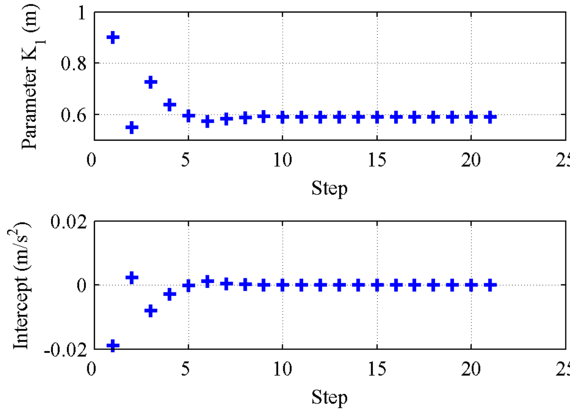

Figure 7. Convergence of parameter K_1 and the LSR intercept.

CarSim® simulated rear wheel lateral force. The value 3488 kg·m^2 may be regarded as the ideal result based on this method; however, this value might not be the true yaw moment of inertia.

The true yaw moment of inertia in CarSim® can be computed approximately by adding up the contribution of sprung mass and unsprung mass components, with unsprung mass being approximated as mass points. The calculation gives 3420 kg·m^2. Another way of estimating the true yaw moment of inertia in CarSim® is to make a regression between the total moment including tire self-aligning moment applied to the vehicle and the vehicle yaw acceleration, this method yields a result of 3434.6 kg·m^2. These true values of yaw moment of inertia are about 1.5% lower than the results from (12) and (13), which suggest more detailed methods should consider the self-aligning moments for a better accuracy.

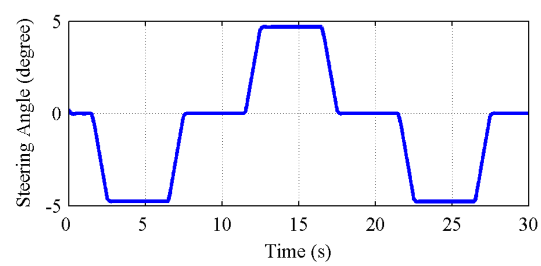

Figure 8. Steering angle input for tuning relaxation length parameter.

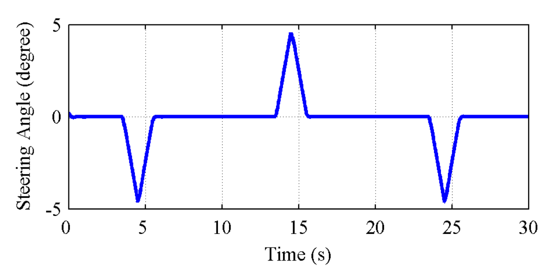

Figure 9. Triangular steering angle input waveform.

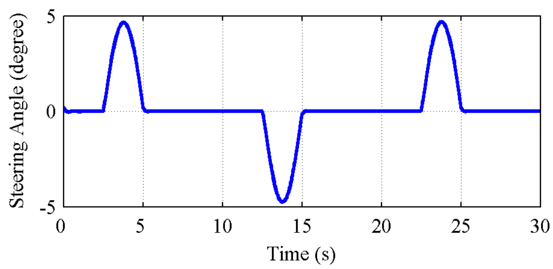

Figure 10. Semi-sine steering angle input waveform.

TABLE II. Results of yaw moment of inertia estimation from LSR

Data sets	Estimated yaw moment of inertia I_z (kg·m^2)	I_z (kg·m^2) based on F_{yr} [a]
left turn	3496.3	3488.3
combination of left and right turn	3496.6	3488.2
steering input: triangular wave form	3501.1	3488.7
steering input: sine wave form	3506.7	3488.7

a. True F_{yr} means the result is from a LSR using the CarSim® simulated rear wheel lateral tire force based on (13).

V. Conclusion

In this paper, a methodology for estimating the vehicle yaw moment of inertia is presented. By utilizing the linear relationship between the vehicle lateral acceleration, yaw acceleration, and rear wheel lateral tire forces, the method does not require knowing front wheel steering angle and front wheel tire force. Based on the linear relationship, An LSR is designed to estimate the yaw moment of inertia using the estimated rear wheel lateral force relaxation length. Simulation results based on a full-vehicle model in CarSim® show that this proposed method is capable of generating reasonable estimations of vehicle yaw moment of inertia without knowing the front ground wheel steering angle.

References

[1] X. Huang and J. Wang. "Adaptive vehicle planar motion control with fast parameter estimation," *IEEE 51st Annual Conference on Decision and Control*, Dec. 2012.

[2] R. Rajamani, *Vehicle Dynamics and Control*. New York: Springer-Verlag, 2012.

[3] J. Wang and R. G. Longoria, "Coordinated and Reconfigurable Vehicle Dynamics Control," *IEEE Transactions on Control Systems Technology*, Vol. 17, No. 3, pp. 723 – 732, 2009.

[4] X. Huang and J. Wang, "Real-Time Estimation of Center of Gravity Position for Lightweight Vehicles Using Combined AKF-EKF Method," *IEEE Transactions on Vehicular Technology*, Vol. 63, No. 9, pp. 4221 - 4231, 2014.

[5] X. Huang and J. Wang, "Longitudinal Motion based Lightweight Vehicle Payload Parameter Real-Time Estimations," *ASME Transactions Journal of Dynamic Systems, Measurement, and Control*, Vol. 135, Issue 1, 011015 (10 pages), 2013.

[6] X. Huang and J. Wang, "Lightweight Vehicle Control-Oriented Modeling and Payload Parameter Sensitivity Analysis," *IEEE Transactions on Vehicular Technology*, Vol. 60, Issue 5, pp. 1999-2011, 2011.

[7] X. Huang and J. Wang, "Center of Gravity Height Real-Time Estimation for Lightweight Vehicles Using Tire Instant Effective Radius," *Control Engineering Practice*, Vol. 21, Issue 4, pp. 370 - 380, 2013.

[8] J. Wang and M.-F. Hsieh, "Vehicle Yaw-Inertia- and Mass-Independent Adaptive Steering Control," *Journal of Automobile Engineering, Proceedings of the Institution of Mechanical Engineers, Part D*, Vol. 223, No. 9, pp. 1101 – 1108, 2009.

[9] L. Segel, "Theoretical prediction and experimental substantiation of the response of the automobile to steering control," *Proceedings of the Institue of Mechnical Engineers, Automobile Division*, vol. 7, pp. 310-330, 1956.

[10] G. J. Heydinger, N. J. Durisek, D. A. Coovert, D. A. Guenther, and S. J. Novak, "The design of a vehicle inertia measurement facility," *SAE Technical Paper*, 950309, Feb. 1995.

[11] A. A. Woodfield, "Measurement of the yawing moment and product of inertia of an aircraft by the single point suspension method: theory and rig design," *Aeronautical Research Council Reports and Memoranda*, Great Britain, No. 3607, 1969.

[12] G. J. Heydinger, R. A. Bixel, W. R. Garrott, M. Pyne, J. G. Howe, and D. A. Guenther, "Measured Vehicle Inertial Parameters-NHTSA's data through November 1988," *SAE Technical Paper*, 1999-01-1336, Mar. 1999.

[13] S. D. Bruyne, H. V. Auweraer, P. Diglio, J. Anthonis, "Online estimation of vehicle inertial parameters for improving chassis control systems," *Proceedings of the 18th IFAC World*, vol. 18, no. 1, pp. 1814-1819, Sep. 2011.

[14] C. S. Liu, H. Peng, "A state and parameter identification scheme for linearly parameterized systems," *Journal of Dynamic System, Measurement, and Control*, vol. 120, no. 4, pp. 524-528, Dec. 1998.

[15] J. Limroth, "Real-time vehicle parameter estimation and adaptive stability control," Ph.D. dissertation, Dept. Auto. Eng., Clemson University, Clemson, SC, 2009.

[16] J. Ryu, "State and parameter estimation for vehicle dynamics control using GPS," Ph.D. dissertation, Dept. Mech. Eng., Stanford University, Stanford, CA, 2004.

[17] X. Huang, H. Zhang, G. Zhang, and J. Wang, "Robust Weighted Gain-Scheduling H_∞ Vehicle Lateral Motion Control With Considerations of Steering System Backlash-Type Hysteresis," *IEEE Transactions on Control Systems Technology*, vol. 22, no. 5, pp. 1740–1753, Sep. 2014.

[18] D. Feng, J. Wang, and D. Huang, "Hand-Wheel Steering Signal Estimation and Diagnosis Approaches for Ground Vehicles," *Control Engineering Practice*, Vol. 20, Issue 7, pp. 654 - 662, 2012.

[19] E. Bakker, and H. B. Pacejka, "A new tire model with an application in vehicle dynamics studies," *SAE Technical Paper*, 890087, Apr. 1989.

[20] P. Bolzern, F. Cheli, G. Falciola and F. Resta, "Estimation of the nonlinear suspension tire cornering forces from experimental road test data," *Vehicle System Dynamics*, vol. 31, pp. 23-24, 1999.

[21] H. B. Pacejka, *Tire and Vehicle Dynamics*. Oxford: Butterworth-Heinemann, 2012.

[22] H. Dugoff, P. S. Fancher, and L. Segal, "Tire performance characteristics affecting vehicle response to steering and braking control inputs," Report for Nat. Bureau Standards CST-460, 1969.

[23] G. Baffet, A. Charara, and J. Stéphant, "Sideslip angle, lateral tire force and road friction estimation in simulations and experiments," *Proceedings of the IEEE International Conference on Control Applications*, pp. 903-908, Oct. 2006.