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Introduction to statistical control and optimization

Homework 15

1.

To meet the requirement of Resolution V, the p in the 2^{6-p} fractional factorial design should be 1.

- $p = 1 \Rightarrow 2^5$fractional factorial design
- Design generator: $E_6 = 12345 \Rightarrow I = 123456$
- Defining relation: $I = 123456$
- Resolution: VI

Number of experimental tests in CCD:

$$2^5 + 2 \times 5 + 1 = 43$$

Number of experimental tests in BBD:

$$2^2 \times C_2^5 + 1 = 41$$

BBD						
test	x1	x2	x3	x4	x5	x6=12345
1	-1	-1	0	0	0	0
2	-1	1	0	0	0	0
3	1	-1	0	0	0	0
4	1	1	0	0	0	0
5	-1	0	-1	0	0	0
6	-1	0	1	0	0	0
7	1	0	-1	0	0	0
8	1	0	1	0	0	0
9	-1	0	0	-1	0	0
10	-1	0	0	1	0	0
11	1	0	0	-1	0	0
12	1	0	0	1	0	0
13	-1	0	0	0	-1	0
14	-1	0	0	0	1	0
15	1	0	0	0	-1	0
16	1	0	0	0	1	0
17	0	-1	-1	0	0	0
18	0	-1	1	0	0	0
19	0	1	-1	0	0	0
20	0	1	1	0	0	0

21	0	-1	0	-1	0	0
22	0	-1	0	1	0	0
23	0	1	0	-1	0	0
24	0	1	0	1	0	0
25	0	-1	0	0	-1	0
26	0	-1	0	0	1	0
27	0	1	0	0	-1	0
28	0	1	0	0	1	0
29	0	0	-1	-1	0	0
30	0	0	-1	1	0	0
31	0	0	1	-1	0	0
32	0	0	1	1	0	0
33	0	0	-1	0	-1	0
34	0	0	-1	0	1	0
35	0	0	1	0	-1	0
36	0	0	1	0	1	0
37	0	0	0	-1	-1	0
38	0	0	0	-1	1	0
39	0	0	0	1	-1	0
40	0	0	0	1	1	0
41	0	0	0	0	0	0

1.

Check the correlation coefficients among all factors, include quadratic terms.

	x1	x2	x1^2	x2^2	x1x2	y
x1	1					
x2	0	1				
x1^2	0	0	1			
x2^2	0	0	-0.13043	1		
x1x2	0	0	0	0	1	
y	-0.25697	0.190102	0.416141	0.242749	0.553519	1

It shows that there isn’t any multicollinearity effect among all factors.

Build a second-order model:

摘要輸出

迴歸統計	
R 的倍數	0.819441
R 平方	0.671483
調整的 R 平方	0.436828
標準誤	4.696548
觀察值個數	13

ANOVA

	自由度	SS	MS	F	顯著值
迴歸	5	315.5971	63.11942	2.861577	0.101537
殘差	7	154.4029	22.05756		
總和	12	470			

	係數	標準誤	t 統計	P-值	下限 95%	上限 95%	下限 95.0%	上限 95.0%
截距	41.2	2.10036	19.61569	2.23E-07	36.23344	46.16656	36.23344	46.16656
x1	-1.96967	1.66048	-1.1862	0.274238	-5.89608	1.956742	-5.89608	1.956742
x2	1.457107	1.66048	0.877521	0.409309	-2.46931	5.383519	-2.46931	5.383519
x1^2	3.7125	1.780667	2.084893	0.075537	-0.49811	7.923108	-0.49811	7.923108
x2^2	2.4625	1.780667	1.382909	0.209196	-1.74811	6.673108	-1.74811	6.673108
x1x2	6	2.348274	2.555068	0.037823	0.447215	11.55279	0.447215	11.55279

The result shows that the only significant effect is E12, therefore, the prediction model will be:

$$\hat{y} = b_0 + b_{12}x_1x_2 = 41.2 + 6x_1x_2$$

a.

Objective function: $\min \hat{y}(x_1, x_2) = 41.2 + 6x_1x_2$

Subjected to: $\begin{cases} -1.414 < x_1 < 1.414 \\ -1.414 < x_2 < 1.414 \end{cases}$

The minima is: $(x_1, x_2) = (-1.414, 1.414)$ or $(1.414, -1.414)$

$$\hat{y} = 29.2$$

b.

Objective function: $\min f(x_1, x_2) = (6x_1x_2 - 4.8)^2$

Subjected to: $\begin{cases} -1.414 < x_1 < 1.414 \\ -1.414 < x_2 < 1.414 \end{cases}$

$$f = 0 \Rightarrow \hat{y} = 46, \text{ when} \begin{cases} x_1 = c \\ x_2 = \dfrac{0.8}{c} \end{cases}$$