機械所設計組 R12522615 王邑安 HW\_ID:2

**Introduction to statistical control and optimization**

Final Exam

Three design factors in this experiment:

|  |  |  |  |
| --- | --- | --- | --- |
| code | paper height(mm) | width-height ratio | leg length(mm) |
| -1 | 148.5 | 0.4 | 8 |
| 1 | 210 | 0.5 | 10 |



Unwanted noises:

1. The material property of papers may vary in each frog.
2. The humidity in air will affect the elasticity of paper.
3. The angle between the finger and a frog will vary in each jumping test.
4. The friction coefficient between frog and finger may change because of sweat on hand.
5. The finger’s force and placement on a frog will vary in each jumping test.

* X1: paper height
* X2: width-height ratio
* X3: leg length

factorial experiment:

|  |  |  |  |
| --- | --- | --- | --- |
| test | x1 | x2 | x3 |
| 1 | -1 | -1 | -1 |
| 2 | 1 | -1 | -1 |
| 3 | -1 | 1 | -1 |
| 4 | 1 | 1 | -1 |
| 5 | -1 | -1 | 1 |
| 6 | 1 | -1 | 1 |
| 7 | -1 | 1 | 1 |
| 8 | 1 | 1 | 1 |

In my experiment, I will create three identical frogs for each combination of factors. To ensure randomization, I will randomize the order of the jumping trials instead of following a fixed sequence for each run. Since there are three runs, each frog will perform three jumps.

Experiment table:

How to handle the noises:

* To prevent I using my fingernails to press paper frogs, I always cut my nails before I do experiment.
* Do more replication and randomize the experimental trials to make noises more uniformly and more randomly distributed.







Experiment details:

* Day: 6/10 Monday
* time: 11:00am~12:30pm
* place: home
* witness: 王邑宇 (reason: he is my brother)

measurement method:

* Measurement plate: Design a plate with markings ranging from 0 to 55 cm (as the picture in 4.)
* Align frogs: Position each paper frog so that its rear edge aligns with the zero line on the measurement plate before each jump.
* Execute a jump: Use fingertip to press down on the back of the frog and release it to initiate the jump
* Valid jumps: If the frog rolls or bounces twice or more, disregard the result and perform the jump again.
* Distance measurement: For each valid jump, measure the shortest distance between the zero line and the landing point of the frog. This distance is the result for that trial.

experiment result:



Parameters of this experiment:

Average and sample variance in each test:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | replicate 1 | | | replicate 2 | | | replicate 3 | | |  |  |
| test | x1 | x2 | x3 | run1 | run2 | run3 | run1 | run2 | run3 | run1 | run2 | run3 | y\_bar | s\_i^2 |
| 1 | -1 | -1 | -1 | 255 | 300 | 285 | 240 | 310 | 300 | 280 | 300 | 360 | 292.2222 | 1175.694 |
| 2 | 1 | -1 | -1 | 200 | 205 | 230 | 220 | 155 | 200 | 150 | 165 | 200 | 191.6667 | 806.25 |
| 3 | -1 | 1 | -1 | 150 | 175 | 160 | 190 | 150 | 190 | 220 | 195 | 205 | 181.6667 | 606.25 |
| 4 | 1 | 1 | -1 | 170 | 210 | 160 | 160 | 170 | 185 | 150 | 155 | 200 | 173.3333 | 431.25 |
| 5 | -1 | -1 | 1 | 300 | 290 | 345 | 230 | 285 | 270 | 275 | 295 | 385 | 297.2222 | 1994.444 |
| 6 | 1 | -1 | 1 | 140 | 150 | 145 | 200 | 150 | 185 | 145 | 140 | 135 | 154.4444 | 502.7778 |
| 7 | -1 | 1 | 1 | 175 | 210 | 230 | 230 | 210 | 245 | 245 | 210 | 275 | 225.5556 | 815.2778 |
| 8 | 1 | 1 | 1 | 180 | 175 | 205 | 200 | 250 | 205 | 200 | 175 | 205 | 199.4444 | 527.7778 |

Effect coefficient:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| E1 | E2 | E3 | E12 | E23 | E13 | E123 |
| -69.4444 | -38.8889 | 9.444444 | 52.22222 | 25.55556 | -15 | 6.111111111 |

Assuming all effects are null effect:

Degree of freedom for t-test:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | t-test | p-value | t\_64,0.025 | t\_64,0.975 |
| E1 | -69.4444 | -10.0616 | 8.28E-15 | -1.99773 | 1.997729654 |
| E2 | -38.8889 | -5.63448 | 4.23E-07 | -1.99773 | 1.997729654 |
| E3 | 9.444444 | 1.368373 | 0.17598 | -1.99773 | 1.997729654 |
| E12 | 52.22222 | 7.566296 | 1.88E-10 | -1.99773 | 1.997729654 |
| E23 | 25.55556 | 3.702656 | 0.000446 | -1.99773 | 1.997729654 |
| E13 | -15 | -2.1733 | 0.033465 | -1.99773 | 1.997729654 |
| E123 | 6.111111 | 0.885418 | 0.379246 | -1.99773 | 1.997729654 |

reject , they are significant factors.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| source of variation | SS | DOF | MS |  |  |
| E1 | 86805.56 | 1 | 86805.56 | 100.2381 | 3.986269 |
| E2 | 27222.22 | 1 | 27222.22 | 31.43468 | 3.986269 |
| E12 | 49088.89 | 1 | 49088.89 | 56.68507 | 3.986269 |
| E23 | 11755.56 | 1 | 11755.56 | 13.57465 | 3.986269 |
| E13 | 4050 | 1 | 4050 | 4.676711 | 3.986269 |
| error | 57155.56 | 66 | 865.9933 |  |  |
| total | 236077.8 | 71 |  |  |  |



Suppose that residuals are normal distributed. The parameters of normal distribution are and

* Rank each residual from 1~72
* The c.d.f. of effect is:
* The normal distribution value of effect:

Residual Q-Q plot:

Plot the residuals vs. run orders:

Plot the residuals vs.

Plot the residuals vs. :

Plot the residuals vs. x1:

Plot the residuals vs. x2:

Plot the residuals vs. x3:

Bartlett’s test ():

Reject if :

In the residual plots, variability of the residuals tends to increase as the predicted values () or the actual average values () rise, with this effect being particularly pronounced for the two highest values of and . This may suggest that the variation of the test 1 and test 5 is too high.



SN ratio: Larger-the-best

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | replicate 1 | | | replicate 2 | | | replicate 3 | | |  |
| test | x1 | x2 | x3 | run1 | run2 | run3 | run1 | run2 | run3 | run1 | run2 | run3 | SN |
| 1 | -1 | -1 | -1 | 255 | 300 | 285 | 240 | 310 | 300 | 280 | 300 | 360 | 49.15775 |
| 2 | 1 | -1 | -1 | 200 | 205 | 230 | 220 | 155 | 200 | 150 | 165 | 200 | 45.37338 |
| 3 | -1 | 1 | -1 | 150 | 175 | 160 | 190 | 150 | 190 | 220 | 195 | 205 | 44.96722 |
| 4 | 1 | 1 | -1 | 170 | 210 | 160 | 160 | 170 | 185 | 150 | 155 | 200 | 44.62473 |
| 5 | -1 | -1 | 1 | 300 | 290 | 345 | 230 | 285 | 270 | 275 | 295 | 385 | 49.21638 |
| 6 | 1 | -1 | 1 | 140 | 150 | 145 | 200 | 150 | 185 | 145 | 140 | 135 | 43.57489 |
| 7 | -1 | 1 | 1 | 175 | 210 | 230 | 230 | 210 | 245 | 245 | 210 | 275 | 46.87 |
| 8 | 1 | 1 | 1 | 180 | 175 | 205 | 200 | 250 | 205 | 200 | 175 | 205 | 45.85851 |

Effect coefficient:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| E1 | E2 | E3 | E12 | E23 | E13 | E123 |
| -2.694963152 | -1.250482833 | 0.349173936 | 2.017970648 | 1.219103481 | -0.631532694 | 0.297030052 |

Assuming all effects are null effects, and estimated standard error by the lowest three effects.

and are relatively small, therefore, they are used for testing the statistical significance of main effects and larger interaction effects.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | effect | t-test | p-value | t\_2,0.025 | t\_2,0.975 |
| E1 | -2.694963152 | -5.461053371 | 0.031934 | -4.30265 | 4.302653 |
| E2 | -1.250482833 | -2.533969152 | 0.126788 | -4.30265 | 4.302653 |
| E3 | 0.349173936 | 0.707563479 | 0.552555 | -4.30265 | 4.302653 |
| E12 | 2.017970648 | 4.089200775 | 0.054923 | -4.30265 | 4.302653 |
| E23 | 1.219103481 | 2.470382266 | 0.132146 | -4.30265 | 4.302653 |
| E13 | -0.631532694 | -1.279733174 | 0.329027 | -4.30265 | 4.302653 |
| E123 | 0.297030052 | 0.601899497 | 0.608386 | -4.30265 | 4.302653 |

It shows that only is significant effect to the SN ratio. Consequently, the predictive model of SN ratio is:



Based on the predictive model of jumping distance of paper frogs, a best combination of three factor can be find:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x1 | x2 | x3 |  | x1 | x2 | x3 |  |
| -1 | -1 | -1 | 300 | -1 | -1 | 1 | 289.4444 |
| 1 | -1 | -1 | 193.3333 | 1 | -1 | 1 | 152.7778 |
| -1 | 1 | -1 | 183.3333 | -1 | 1 | 1 | 223.8889 |
| 1 | 1 | -1 | 181.1111 | 1 | 1 | 1 | 191.6667 |

The optimum is 300mm when the combination of three factors is

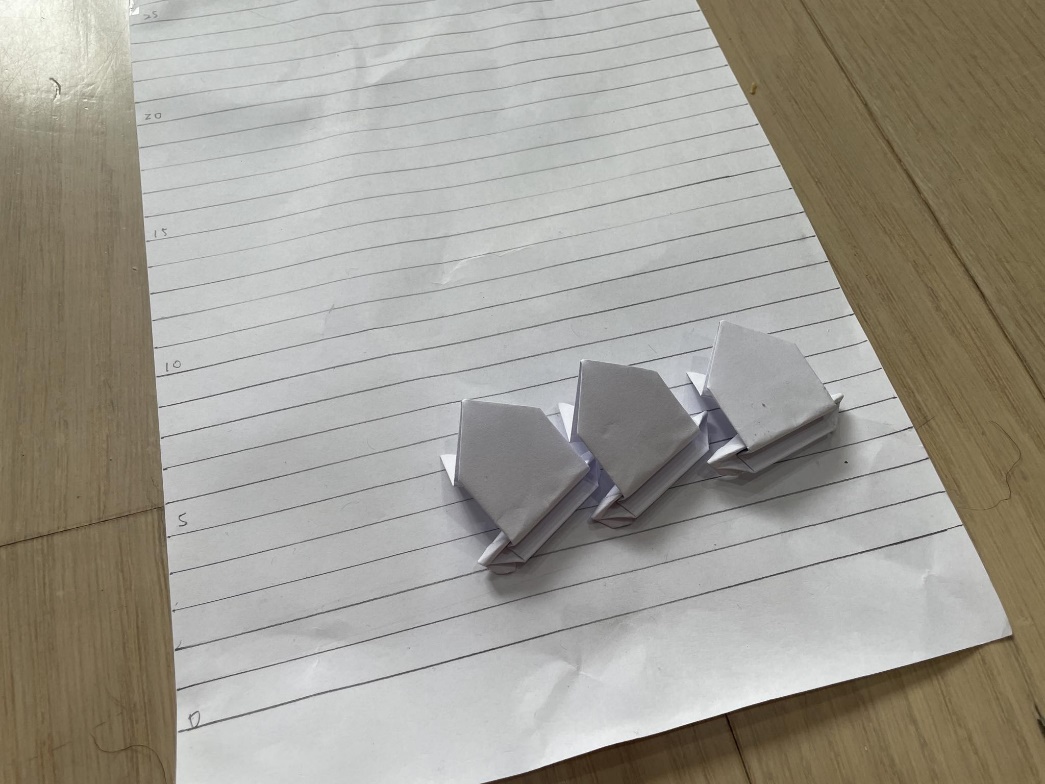
On the other hand, the predictive model of SN ratio is:

|  |  |
| --- | --- |
| x1 |  |
| -1 | 47.55283923 |
| 1 | 44.85787608 |

It shows that when , the jumping distance SN ratio of paper frogs is largest.

As a result, there is no conflict between the predictive model of jumping distance and the predictive model of the SN ratio for jumping distance. The estimated indicates that the SN ratio performs best when . Similarly, the estimated shows that the optimal jumping distance occurs when .



Make three optimum frogs and measure their jumping distance:

|  |  |  |  |
| --- | --- | --- | --- |
| verify | | | |
|  | replication1 | replication2 | replication3 |
| run1 distance(mm) | 270 | 330 | 360 |
| run2 distance(mm) | 290 | 275 | 340 |
| run3 distance(mm) | 355 | 310 | 330 |
| average | 317.7777778 | SN | 49.90908236 |

In the verification, the average jumping distance of the frogs at the optimal conditions was 317.78 mm. This value is higher than the most of the jumping distance tested in the previous experiment, suggesting that the regression model shows the tendency of the jumping distance in optimum factor combination. However, the average jumping distance of paper frogs in this verification is slightly larger than the predictive distance(300mm). This indicates that the regression model is likely to underestimate the performance of paper frogs.

Moreover, the SN ratio of jumping distance in the verification is 49.90908236. This value is relatively high comparing to the previous experiment results. This means that the predictive model shows the trend of SN ratio of paper frogs’ jumping distance. Nevertheless, the SN ratio value in verification still has some difference comparing to the value of the predictive model (-47.55283923).



To measure error, the test of center point (test 15) should be executed more than once.

|  |  |  |  |
| --- | --- | --- | --- |
| test | x1 | x2 | x3 |
| 1 | -1 | -1 | -1 |
| 2 | 1 | -1 | -1 |
| 3 | -1 | 1 | -1 |
| 4 | 1 | 1 | -1 |
| 5 | -1 | -1 | 1 |
| 6 | 1 | -1 | 1 |
| 7 | -1 | 1 | 1 |
| 8 | 1 | 1 | 1 |
| 9 | -1.682 | 0 | 0 |
| 10 | 1.682 | 0 | 0 |
| 11 | 0 | -1.682 | 0 |
| 12 | 0 | 1.682 | 0 |
| 13 | 0 | 0 | -1.682 |
| 14 | 0 | 0 | 1.682 |
| 15 | 0 | 0 | 0 |
| Measure error | | | |
| 15--1 | 0 | 0 | 0 |
| 15--2 | 0 | 0 | 0 |
| 15--3 | 0 | 0 | 0 |
| 15--4 | 0 | 0 | 0 |





Experiment details:

* Day: 6/10 Monday
* time: 1:30pm~2:30pm
* place: home
* witness: 王邑宇 (reason: he is my brother)

measurement method: same method in (5)

experiment result:





Parameters of this experiment:

Average and sample variance in each test:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | replicate 1 | | | replicate 2 | | | replicate 3 | | |  |  |
| test | x1 | x2 | x3 | run1 | run2 | run3 | run1 | run2 | run3 | run1 | run2 | run3 | y\_bar | s\_i^2 |
| 1 | -1 | -1 | -1 | 255 | 300 | 285 | 240 | 310 | 300 | 280 | 300 | 360 | 292.2222 | 1175.694 |
| 2 | 1 | -1 | -1 | 200 | 205 | 230 | 220 | 155 | 200 | 150 | 165 | 200 | 191.6667 | 806.25 |
| 3 | -1 | 1 | -1 | 150 | 175 | 160 | 190 | 150 | 190 | 220 | 195 | 205 | 181.6667 | 606.25 |
| 4 | 1 | 1 | -1 | 170 | 210 | 160 | 160 | 170 | 185 | 150 | 155 | 200 | 173.3333 | 431.25 |
| 5 | -1 | -1 | 1 | 300 | 290 | 345 | 230 | 285 | 270 | 275 | 295 | 385 | 297.2222 | 1994.444 |
| 6 | 1 | -1 | 1 | 140 | 150 | 145 | 200 | 150 | 185 | 145 | 140 | 135 | 154.4444 | 502.7778 |
| 7 | -1 | 1 | 1 | 175 | 210 | 230 | 230 | 210 | 245 | 245 | 210 | 275 | 225.5556 | 815.2778 |
| 8 | 1 | 1 | 1 | 180 | 175 | 205 | 200 | 250 | 205 | 200 | 175 | 205 | 199.4444 | 527.7778 |
| 9 | -1.682 | 0 | 0 | 210 | 220 | 215 | 145 | 160 | 150 | 165 | 175 | 175 | 179.4444 | 815.2778 |
| 10 | 1.682 | 0 | 0 | 195 | 205 | 215 | 170 | 190 | 165 | 155 | 165 | 170 | 181.1111 | 429.8611 |
| 11 | 0 | -1.682 | 0 | 205 | 185 | 180 | 175 | 185 | 225 | 225 | 180 | 205 | 196.1111 | 379.8611 |
| 12 | 0 | 1.682 | 0 | 200 | 190 | 205 | 225 | 195 | 220 | 240 | 235 | 220 | 214.4444 | 315.2778 |
| 13 | 0 | 0 | -1.682 | 205 | 190 | 220 | 210 | 230 | 200 | 175 | 155 | 170 | 195 | 606.25 |
| 14 | 0 | 0 | 1.682 | 195 | 220 | 210 | 230 | 245 | 245 | 225 | 230 | 240 | 226.6667 | 275 |
| 15 | 0 | 0 | 0 | 250 | 230 | 260 | 210 | 260 | 225 | 200 | 200 | 220 | 228.3333 | 562.5 |
| 15--1 | 0 | 0 | 0 | 220 | 220 | 215 | 205 | 260 | 250 | 200 | 255 | 245 | 230 | 512.5 |
| 15--2 | 0 | 0 | 0 | 205 | 245 | 200 | 235 | 255 | 250 | 230 | 230 | 220 | 230 | 362.5 |
| 15--3 | 0 | 0 | 0 | 215 | 245 | 185 | 250 | 210 | 205 | 200 | 215 | 200 | 213.8889 | 448.6111 |
| 15--4 | 0 | 0 | 0 | 235 | 230 | 205 | 255 | 250 | 225 | 200 | 205 | 210 | 223.8889 | 411.1111 |

Check the correlation coefficient between each factor (including quadratic terms):

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | x1x2 | x2x3 | x1x3 | x1x2x3 | x1^2 | x2^2 | x3^2 | y |
| x1 | 1 |  |  |  |  |  |  |  |  |  |  |
| x2 | 0 | 1 |  |  |  |  |  |  |  |  |  |
| x3 | 0 | 0 | 1 |  |  |  |  |  |  |  |  |
| x1x2 | 0 | 0 | 0 | 1 |  |  |  |  |  |  |  |
| x2x3 | 0 | 0 | 0 | 0 | 1 |  |  |  |  |  |  |
| x1x3 | 0 | 0 | 0 | 0 | 0 | 1 |  |  |  |  |  |
| x1x2x3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |  |  |  |
| x1^2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |  |  |
| x2^2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.12814 | 1 |  |  |
| x3^2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.12814 | -0.12814 | 1 |  |
| y | -0.39964 | -0.18126 | 0.132315 | 0.396679 | 0.194119 | -0.11394 | 0.04642 | -0.23466 | -0.03296 | 0.011866 | 1 |

The result shows that there is little correlation between each factor, therefore, it can be considered as no multicollinearity effect among this regression models:

Regression analysis results:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 摘要輸出 | |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 迴歸統計 | |  |  |  |  |  |
| R 的倍數 | 0.692915 |  |  |  |  |  |
| R 平方 | 0.480131 |  |  |  |  |  |
| 調整的 R 平方 | 0.447639 |  |  |  |  |  |
| 標準誤 | 31.83754 |  |  |  |  |  |
| 觀察值個數 | 171 |  |  |  |  |  |
|  |  |  |  |  |  |  |
| ANOVA |  |  |  |  |  |  |
|  | 自由度 | SS | MS | F | 顯著值 |  |
| 迴歸 | 10 | 149783.7 | 14978.37 | 14.77697 | 2.02E-18 |  |
| 殘差 | 160 | 162180.6 | 1013.629 |  |  |  |
| 總和 | 170 | 311964.3 |  |  |  |  |
|  |  |  |  |  |  |  |
|  | 係數 | 標準誤 | t 統計 | P-值 | 下限 95% | 上限 95% |
| 截距 | 224.557 | 4.740528 | 47.36962 | 4.68E-96 | 215.1949 | 233.9191 |
| x1 | -20.1325 | 2.871579 | -7.01095 | 6.28E-11 | -25.8036 | -14.4614 |
| x2 | -9.1314 | 2.871579 | -3.17992 | 0.001769 | -14.8025 | -3.46031 |
| x3 | 6.665651 | 2.871579 | 2.321249 | 0.021535 | 0.994565 | 12.33674 |
| x1x2 | 26.11111 | 3.75209 | 6.959084 | 8.33E-11 | 18.7011 | 33.52112 |
| x2x3 | 12.77778 | 3.75209 | 3.405509 | 0.000835 | 5.367769 | 20.18779 |
| x1x3 | -7.5 | 3.75209 | -1.99889 | 0.047314 | -14.91 | -0.08999 |
| x1x2x3 | 3.055556 | 3.75209 | 0.814361 | 0.416649 | -4.35445 | 10.46556 |
| x1^2 | -12.2111 | 2.871912 | -4.2519 | 3.59E-05 | -17.8828 | -6.53933 |
| x2^2 | -3.37442 | 2.871912 | -1.17497 | 0.241751 | -9.04616 | 2.297324 |
| x3^2 | -1.41072 | 2.871912 | -0.49121 | 0.62395 | -7.08246 | 4.261025 |

First, the table shows that reject the null hypothesis with . Therefore, they are significant effects. Second, look closely on the ANOVA table, the p-value(“顯著值”) of F-test is significantly small. This indicates that the observed differences between tests are not caused by noise. Finally, the and are 0.480131 and 0.447639 respectively. This implies that the proportion of the variation explained by model is almost half.



Based on the first regression analysis table, a predictive regression model of the paper frogs’ jumping distance can be constructed:

Final regression analysis:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 摘要輸出 | |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 迴歸統計 | |  |  |  |  |  |
| R 的倍數 | 0.687861 |  |  |  |  |  |
| R 平方 | 0.473153 |  |  |  |  |  |
| 調整的 R 平方 | 0.450528 |  |  |  |  |  |
| 標準誤 | 31.75417 |  |  |  |  |  |
| 觀察值個數 | 171 |  |  |  |  |  |
|  |  |  |  |  |  |  |
| ANOVA |  |  |  |  |  |  |
|  | 自由度 | SS | MS | F | 顯著值 |  |
| 迴歸 | 7 | 147607 | 21086.71 | 20.91256 | 6.29E-20 |  |
| 殘差 | 163 | 164357.4 | 1008.327 |  |  |  |
| 總和 | 170 | 311964.3 |  |  |  |  |
|  |  |  |  |  |  |  |
|  | 係數 | 標準誤 | t 統計 | P-值 | 下限 95% | 上限 95% |
| 截距 | 220.6764 | 3.158602 | 69.86522 | 2.1E-123 | 214.4393 | 226.9134 |
| x1 | -20.1325 | 2.864059 | -7.02935 | 5.39E-11 | -25.7879 | -14.477 |
| x2 | -9.1314 | 2.864059 | -3.18827 | 0.001716 | -14.7868 | -3.47596 |
| x3 | 6.665651 | 2.864059 | 2.327344 | 0.021177 | 1.010209 | 12.32109 |
| x1x2 | 26.11111 | 3.742265 | 6.977355 | 7.17E-11 | 18.72154 | 33.50068 |
| x2x3 | 12.77778 | 3.742265 | 3.41445 | 0.000807 | 5.388209 | 20.16735 |
| x1x3 | -7.5 | 3.742265 | -2.00413 | 0.046712 | -14.8896 | -0.11043 |
| x1^2 | -11.5979 | 2.809925 | -4.12747 | 5.84E-05 | -17.1464 | -6.04935 |

The analysis table shows that all effects in the predictive model are significant. The F-test value in ANOVA table is still large and the p-value of F-test is relatively small, implying that the observed differences between vary factors are statistically significant and not merely due to random noise. The and are 0.473153 and 0.450528, respectively. This suggests that almost half of the variation could be explained by this model.

Compare to the predictive model built in (6), several similarities and differences are observed.

Predictive model in (6):

Predictive model in (15):

* Similarity:
* Both models identify (length of papers), (width ratio of papers), (interaction effect between length and width ratio), (interaction effect between width ratio and leg length), and (interaction effect between length and leg length) as significant effects. This suggests that these main effects and interactions effects are crucial in determining the jumping distance of the paper frogs.
* Difference:
* The current model includes (length of the paper frog’s legs) as a significant factor. This highlights that the leg length of the paper frog plays a vital role in influencing the jumping distance, which was not directly considered in the previous model.
* The current model considers the quadratic term of , indicating a nonlinear relationship between frogs’ jumping distance and papers length.
* Comparing to the previous model, the coefficients for and are significantly reduced in the current model. This suggests that, while and remain important, other factors or interactions might play a more dominant role in determining the jumping distance in the current context.



Suppose that residuals are normal distributed. The parameters of normal distribution are and

* Rank each residual from 1~171
* The c.d.f. of effect is:
* The normal distribution value of effect:

Residual Q-Q plot:

Plot the residuals vs. run orders:

Plot the residuals vs.

Plot the residuals vs. :

Plot the residuals vs. x1:

Plot the residuals vs. x2:

Plot the residuals vs. x3:

Problem observation:

* In the residual plots, variability of the residuals tends to increase as the predicted values () or the actual average values () rise, with this effect being particularly pronounced for the two highest values of and .
* In the “residual vs. x1”, “residual vs. x2”, and “residual vs. x3” plots, the residuals show greater variability around the center of the graphs compared to the two sides.



SN ratio: Larger-the-best

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | replicate 1 | | | replicate 2 | | | replicate 3 | | |  |
| test | x1 | x2 | x3 | run1 | run2 | run3 | run1 | run2 | run3 | run1 | run2 | run3 | SN |
| 1 | -1 | -1 | -1 | 255 | 300 | 285 | 240 | 310 | 300 | 280 | 300 | 360 | 49.15775 |
| 2 | 1 | -1 | -1 | 200 | 205 | 230 | 220 | 155 | 200 | 150 | 165 | 200 | 45.37338 |
| 3 | -1 | 1 | -1 | 150 | 175 | 160 | 190 | 150 | 190 | 220 | 195 | 205 | 44.96722 |
| 4 | 1 | 1 | -1 | 170 | 210 | 160 | 160 | 170 | 185 | 150 | 155 | 200 | 44.62473 |
| 5 | -1 | -1 | 1 | 300 | 290 | 345 | 230 | 285 | 270 | 275 | 295 | 385 | 49.21638 |
| 6 | 1 | -1 | 1 | 140 | 150 | 145 | 200 | 150 | 185 | 145 | 140 | 135 | 43.57489 |
| 7 | -1 | 1 | 1 | 175 | 210 | 230 | 230 | 210 | 245 | 245 | 210 | 275 | 46.87 |
| 8 | 1 | 1 | 1 | 180 | 175 | 205 | 200 | 250 | 205 | 200 | 175 | 205 | 45.85851 |
| 9 | -1.682 | 0 | 0 | 210 | 220 | 215 | 145 | 160 | 150 | 165 | 175 | 175 | 44.79915 |
| 10 | 1.682 | 0 | 0 | 195 | 205 | 215 | 170 | 190 | 165 | 155 | 165 | 170 | 45.01357 |
| 11 | 0 | -1.682 | 0 | 205 | 185 | 180 | 175 | 185 | 225 | 225 | 180 | 205 | 45.74185 |
| 12 | 0 | 1.682 | 0 | 200 | 190 | 205 | 225 | 195 | 220 | 240 | 235 | 220 | 46.5465 |
| 13 | 0 | 0 | -1.682 | 205 | 190 | 220 | 210 | 230 | 200 | 175 | 155 | 170 | 45.60465 |
| 14 | 0 | 0 | 1.682 | 195 | 220 | 210 | 230 | 245 | 245 | 225 | 230 | 240 | 47.04125 |
| 15 | 0 | 0 | 0 | 250 | 230 | 260 | 210 | 260 | 225 | 200 | 200 | 220 | 47.0485 |
| 15--1 | 0 | 0 | 0 | 220 | 220 | 215 | 205 | 260 | 250 | 200 | 255 | 245 | 47.12242 |
| 15--2 | 0 | 0 | 0 | 205 | 245 | 200 | 235 | 255 | 250 | 230 | 230 | 220 | 47.15169 |
| 15--3 | 0 | 0 | 0 | 215 | 245 | 185 | 250 | 210 | 205 | 200 | 215 | 200 | 46.49718 |
| 15--4 | 0 | 0 | 0 | 235 | 230 | 205 | 255 | 250 | 225 | 200 | 205 | 210 | 46.90821 |

Check the correlation coefficient between each factor (including quadratic terms):

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | x1 | x2 | x3 | x1x2 | x2x3 | x1x3 | x1x2x3 | x1^2 | x2^2 | x3^2 | SN |
| x1 | 1 |  |  |  |  |  |  |  |  |  |  |
| x2 | 0 | 1 |  |  |  |  |  |  |  |  |  |
| x3 | 0 | 0 | 1 |  |  |  |  |  |  |  |  |
| x1x2 | 0 | 0 | 0 | 1 |  |  |  |  |  |  |  |
| x2x3 | 0 | 0 | 0 | 0 | 1 |  |  |  |  |  |  |
| x1x3 | 0 | 0 | 0 | 0 | 0 | 1 |  |  |  |  |  |
| x1x2x3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |  |  |  |
| x1^2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |  |  |
| x2^2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.12814 | 1 |  |  |
| x3^2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -0.12814 | -0.12814 | 1 |  |
| SN | -0.45901 | -0.16073 | 0.16798 | 0.464637 | 0.280698 | -0.14541 | 0.068391 | -0.35546 | -0.05274 | -0.00902 | 1 |

The result shows that there is little correlation between each factor, therefore, it can be considered as no multicollinearity effect among this regression models:

Regression analysis results:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 摘要輸出 | |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 迴歸統計 | |  |  |  |  |  |
| R 的倍數 | 0.852273 |  |  |  |  |  |
| R 平方 | 0.726369 |  |  |  |  |  |
| 調整的 R 平方 | 0.38433 |  |  |  |  |  |
| 標準誤 | 1.135935 |  |  |  |  |  |
| 觀察值個數 | 19 |  |  |  |  |  |
|  |  |  |  |  |  |  |
| ANOVA |  |  |  |  |  |  |
|  | 自由度 | SS | MS | F | 顯著值 |  |
| 迴歸 | 10 | 27.40242 | 2.740242 | 2.123645 | 0.149064 |  |
| 殘差 | 8 | 10.32279 | 1.290349 |  |  |  |
| 總和 | 18 | 37.72521 |  |  |  |  |
|  |  |  |  |  |  |  |
|  | 係數 | 標準誤 | t 統計 | P-值 | 下限 95% | 上限 95% |
| 截距 | 46.92693 | 0.507413 | 92.48262 | 2.09E-13 | 45.75683 | 48.09703 |
| x1 | -0.76285 | 0.307366 | -2.48189 | 0.037999 | -1.47164 | -0.05406 |
| x2 | -0.26713 | 0.307366 | -0.86909 | 0.410114 | -0.97592 | 0.44166 |
| x3 | 0.279175 | 0.307366 | 0.908282 | 0.39027 | -0.42961 | 0.987963 |
| x1x2 | 1.008985 | 0.401614 | 2.512328 | 0.036239 | 0.082862 | 1.935108 |
| x2x3 | 0.609552 | 0.401614 | 1.517756 | 0.167554 | -0.31657 | 1.535675 |
| x1x3 | -0.31577 | 0.401614 | -0.78624 | 0.454381 | -1.24189 | 0.610357 |
| x1x2x3 | 0.148515 | 0.401614 | 0.369796 | 0.721132 | -0.77761 | 1.074638 |
| x1^2 | -0.61766 | 0.307402 | -2.00931 | 0.079363 | -1.32653 | 0.091205 |
| x2^2 | -0.18014 | 0.307402 | -0.586 | 0.574033 | -0.88901 | 0.528733 |
| x3^2 | -0.11695 | 0.307402 | -0.38044 | 0.713525 | -0.82582 | 0.591923 |

With , assuming all main effects, interaction effects, and quadratic effects are null. The table shows that only and reject and could be considered as significant effects. Look closely on the ANOVA table, the p-value of F-test is 0.149064, which is relatively high. This suggests that the mean square (MS) between tests is similar to the MS of error and the difference between tests could be caused by noises and therefore can’t explained by the regression model.

In this analysis, is 0.726369, indicating that approximately 72.64% of the variation in jumping distance is explained by the model. However, adjusted is significantly lower at 0.38433. The low adjusted suggests that the model may overestimate its explanatory power and may not be able to capture the variation of jumping distance SN ratio.



Base on the previous regression analysis, the final regression model to predict the SN ratio of jumping distance is:

The final regression analysis:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 摘要輸出 | |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 迴歸統計 | |  |  |  |  |  |
| R 的倍數 | 0.653128 |  |  |  |  |  |
| R 平方 | 0.426577 |  |  |  |  |  |
| 調整的 R 平方 | 0.354899 |  |  |  |  |  |
| 標準誤 | 1.162769 |  |  |  |  |  |
| 觀察值個數 | 19 |  |  |  |  |  |
|  |  |  |  |  |  |  |
| ANOVA |  |  |  |  |  |  |
|  | 自由度 | SS | MS | F | 顯著值 |  |
| 迴歸 | 2 | 16.0927 | 8.046349 | 5.9513 | 0.01169 |  |
| 殘差 | 16 | 21.63251 | 1.352032 |  |  |  |
| 總和 | 18 | 37.72521 |  |  |  |  |
|  |  |  |  |  |  |  |
|  | 係數 | 標準誤 | t 統計 | P-值 | 下限 95% | 上限 95% |
| 截距 | 46.26936 | 0.266758 | 173.451 | 1.25E-27 | 45.70386 | 46.83486 |
| x1 | -0.76285 | 0.314627 | -2.42462 | 0.027535 | -1.42983 | -0.09587 |
| x1x2 | 1.008985 | 0.411101 | 2.454349 | 0.025946 | 0.13749 | 1.88048 |

First, this table shows that all terms in the final regression model are significant. Look closely on the ANOVA table, the p-value of F-test is 0.01169. It is considerably lower than the previous analysis. This indicates that current model has higher explanatory power on variation between tests. However, the R^2 and adjusted R^2 are 0.426577 and 0.354899, respectively, meaning that the regression model can only explain part of variability of jumping distance SN ratio, and remains over half of variation unexplained.

Compare to the predictive model construct in (9):

Regression model in (9):

Regression model in (18):

* Similarity:
* Both models identify (length of papers) as significant effect. This suggests that the effect of are crucial in determining the jumping distance of the paper frogs.
* Difference:
* The previous model only considers the direct linear impact of on SN ratio of paper frogs’ jumping distance. On the other hand, the current model incorporates the interaction effect between and into consideration, providing a deeper understanding of how the combined influence of paper length and width affects the outcome.
* In the predictive model in (9), has a stronger negative effect on SN ratio with a coefficient of −1.347481576. However, the main effect in the current model is less negative (), and has a interaction effect with . This suggesting that the impact of on SN ratio is depended on .



There are two objective functions:

* For the paper frog’s jumping distance:

* For the SN ratio of paper frog’s jumping distance

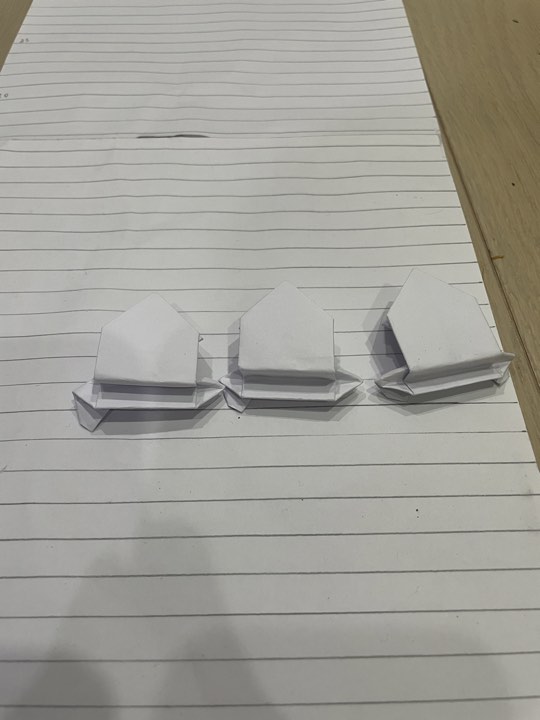
In the first objective function, an optimum can be find when :

In the second objective function, an optimum can be find when :

As a result, the both predictive models in (15) and (18) suggests that the paper frog will have a best jumping performance ( when .



Make three optimum frogs and measure their jumping distance:



|  |  |  |  |
| --- | --- | --- | --- |
| verify | | | |
|  | replication1 | replication2 | replication3 |
| run1 distance(mm) | 290 | 265 | 310 |
| run2 distance(mm) | 260 | 275 | 270 |
| run3 distance(mm) | 300 | 295 | 280 |
| average | 282.777778 | SN | 48.9876096 |

The average jumping distance of paper frogs in this verification is relatively larger than the most of the results in the previous experiment. This indicates that the regression model accurately captures the underlying trend of the paper frogs’ jumping performance. Nonetheless, the average distance result in this verification (282.777778mm) is slightly higher than the predictive jumping distance (265.8413578mm) in the regression model built in (15), showing that this predictive model may underestimate the performance of reality paper frog.

In addition, the SN ratio of jumping distance in the verification is quite large comparing to the previous experiment results, suggesting that the regression model have the ability to predict the tendency of paper frogs’ jumping distance SN ratio. Moreover, the SN value in the verification is 48.9876096, and the maximize SN ratio in the model is 48.04119433. Although there is a slightly difference, this shows that the predictive model indeed reflects the SN ratio in the reality.

Compare the result in (20) to the result in (11):

* The regression models of jumping distance of paper frogs in (11) and (20) shows the different optimum setting. The current one predicts that paper frogs will have the best performance (with distance =265.8413578mm) when . While the previous one predicts that when , paper frog will jump farthest (distance=300mm). Both regression models show the trend that shorter paper length and width will have a positive impact on frogs’ jumping performance. But they have different point of views on the effect of frogs’ leg length.

In addition, the predictive model of SN ratio in (11) only considers the effect of paper length. In contrast, the current regression model of SN ratio considers not only the main effect of but also the interaction effect between paper length and width ratio. However, both regression models exclude the impact of the length of paper frogs’ leg.

This problem may due to the too small difference of the experiment parameters. provided the variation of the leg length and the width of the paper could be enlarged, this problem will be likely to be solved.

* Although both models underestimate the result in the verifications, the current model of jumping distance tends to predicts a lower optimum comparing to the model in (11).