

2024 Spring – SPC&O HW#5

1. To estimate the parameter of Poisson distribution, n experiments are performed to obtain the random sample observations: X_1, X_2, \dots, X_n , where X_i is the observed value of the i th experiment. Show that the MLE of λ is the average \bar{X} .
2. To estimate the probability of success (p) of the Binomial distribution, k samples are taken: $(x_1, n_1), (x_2, n_2), \dots, (x_k, n_k)$, where n_i is the number of trials of the i th sample and x_i is the number of successes in n_i trials.

a) Derive, with the detailed procedure, the MLE of p (\hat{p}_{MLE}).

b) Let the other estimator of p be:

$$\hat{p} = \frac{\sum_i (x_i/n_i)}{k}$$

Evaluate and compare this \hat{p} and \hat{p}_{MLE} in terms of their biases and standard errors

3. Using 24Spring-HW4.xls,
 - a) Assuming normal distributions, use Excel to plot the (Q-Q) plots for thickness at up, middle, down, left, and right positions for the center zone.
 - b) Assuming exponential distribution, use the moment estimator to estimate the exponential distribution model and then use Excel to plot the (Q-Q) plots for thickness at up, middle, down, left, and right positions for the center zone.
 - c) Assuming Gamma probability distribution, use the moment estimator to estimate the parameters of the Gamma distributions for thickness at up, middle, down, left, and right positions for the center zone and then use Excel to plot the (Q-Q) plots for thickness at up, middle, down, left, and right positions for the center zone.
 - d) Which distribution, Exponential, Gamma or Normal, can better describe the random natures of the thickness of different positions for the center zone.
4. In Q-Q plot, $[F^{-1}(\text{estimated cumulated probability of the } i\text{th smallest observation.}), \text{the } i\text{th smallest observed value}]$ are plotted on the scatter chart. Another plot is called P-P plot in which $[F(\text{the } i\text{th smallest observed value}), \text{the estimated cumulated probability of the } i\text{th smallest observation}]$ are plotted on the scatter chart. With the accident death data of the first six regions in 24Spring-HW2.xls,
 - a) assuming that the numbers of accident deaths are discrete random variables possibly following the Geometric, Negative Binomial or Poisson distribution, use the moment estimators to estimate the model parameters, respectively, for each of

- the first six regions.
- b) Plot the P-P plots for each regions.
 - c) Which distribution model fits best to each region's numbers of accident deaths?
5. Use the thickness data provided in 24Spring-HW#4.xls.
- a) Calculate 90% confidence intervals for the thickness mean at the left and right positions of the center zone using z-statistics with σ estimated by s .
 - b) Calculate 90% confidence intervals for the thickness mean at the left and right positions of the center zone using t -statistic and compare the results with that of (a).
 - c) With the 90% confidence interval of the left position in calculated in (b), do you think the thickness mean of the left position of the center zone is actually off from the target 350? With the confidence intervals of the left and right positions, do you think the two thickness means are equal?