Estimation of Mass Moments of Inertia of Automobile

Piotr Fundowicz, Hubert Sar

Abstract— **The estimation of the values of moments of inertia of vehicles for the needs of accidents reconstruction is very important. Usually, because of the problem of costs, there is no possibility to measure these parameters on a special test rig (physical or torsional pendulum). So, it is justified to look for new methods which would allow to obtain the mass moments of inertia of automobile. Of course, estimation is always burdened with error, but somebody who uses a formula for calculation of moment of inertia should put in the results the range of error of estimated value.**

In this paper the authors presented their own formulas on the basis of which the values of moments of inertia are estimated. As the basis of their work, a set of measurement results of the moments were used. They were performed by NHTSA for passenger cars exploited in the USA mainly in the 80's and 90's. In used NHTSA database there were also the vehicles that had the identical body version as in Europe.

The presented new formulas were obtained using the procedure which was based on Monte Carlo method. However, the Authors decided to propose different formulas for each moment of inertia. They require a certain number of vehicle data, for example mass and wheelbase of a vehicle.

Index Terms— **Safety, accidents, traffic accidents, active safety, inertial parameters, Monte Carlo method, estimation.**

I. Introduction

THE Computer modelling of vehicle motion is very popular nowadays as relatively cheap method of investigating active safety of an automobile, compared to road tests. However, the problem of identifying the input parameters of a vehicle model is of great importance. Usually, if there is no possibility to obtain vehicle parameters with relatively low error, it is recommended to apply such model which requires the minimum number of them. This could be for example a single track two-degree-of-freedom model for curvilinear motion description. This paper is focused on estimation of the values of mass moments of inertia relative to main axes of inertia of a vehicle crossing its center of gravity. It is worth mentioning that if the higher the dynamics of vehicle motion in any direction (roll, pitch or yaw) is, the higher is the impact of the mass moment of inertia on the results of computer simulation.

Because of the fact that test rigs are needed to measure moments of inertia of automobile and are not easy to find, many researchers seek for indirect methods of the estimation of these moments. A majority of them apply road test measurements, often using the on-board sensors which belong to active safety systems of a vehicle. The examples of such works are [1], [2], [3] and [4]. In papers [5] and [6] there is analyzed the influence of mass moment of inertia estimation error on the results of simulation of traffic accident.

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II. Presentation of Applied Data

Table 1 presents the data of passenger cars that were selected from the group of 47 automobiles. The data of these passenger cars was selected from larger amount of the measurements whose results are enclosed in paper [7]. The vehicles were represented by different body type (sedan, hatchback, coupe and station wagon). The data includes among others the information about measured moments of inertia and basic parameters of vehicles, which are easy to find, for example vehicle real mass, wheelbase, wheel track, vehicle length. The selected data referred to the vehicles without luggage, passengers and fully tanked. These basic parameters will then be necessary to estimate the moments of inertia using proposed formulas. This is why they will be used in an optimization procedure based on Monte Carlo algorithm, where the coefficients in proposed formulas will be obtained to minimize the difference between measured and estimated value.

III. Algorithm of Obtaining the Empirical Equations

In the beginning there was assumed that such parameters like vehicle mass, its length and height, wheel base, wheel track and additionally height of the center of gravity have stronger or weaker influence on the vehicle's mass moment of inertia described by function (1). So that, it can be written as below.

$$I = f^{*}(m, l_{12}, b, H, h_{CG}, L) \tag{1}$$

where:
m - mass of a vehicle,
l_{12} – wheel base,
b – average wheel track,
H – vehicle height,
h_{CG} – center of gravity height (COG height),
L – vehicle length.

Exponential form of the above presented function will be

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TABLE I
EXEMPLARY DATA OF 12 AUTOMOBILES FROM 47 PASSENGER CARS ANALYZED BY THE AUTHORS (SOURCE [7])

Year	Make	Model	Type	Mass [kg]	Wheel base [m]	Average wheel track [m]	Vehicle width [m]	Roof height [m]	Vehicle length [m]	COG height [m]	Mass moment of inertia [kgm^2]		
											Roll	Pitch	Yaw
1986	BMW	325i	2S	1251	2.57	1.398	1.645	1.37	4.47	0.533	381	2011	2027
1985	Chrysler	LeBaron	4S	1238	2.623	1.441	1.727	1.39	4.57	0.583	410	2091	2160
1985	Ford	Escort	2S	1007	2.393	1.406	1.674	1.35	4.24	0.511	328	1535	1545
1988	Ford	Mustang GL	2S	1256	2.553	1.449	1.755	1.37	4.57	0.529	408	2150	2225
1988	Ford	Taurus	4S	1419	2.685	1.546	1.793	1.44	4.80	0.563	573	2553	2687
1987	Mercedes	190	4S	1301	2.664	1.427	1.706	1.39	4.45	0.559	444	2083	2095
1987	Mercedes	190 E	4S	1301	2.653	1.410	1.706	1.39	4.45	0.558	443	2123	2137
1986	Nissan	Maxima	4S	1410	2.550	1.461	1.690	1.39	4.62	0.541	514	2465	2445
1986	Hyundai	Excel	3H	939	2.381	1.368	1.603	1.36	4.09	0.540	312	1378	1434
1983	Toyota	Camry	5H	1116	2.601	1.443	1.690	1.39	4.47	0.549	429	1916	2036
1987	Toyota	Corolla FX	3H	996	2.431	1.415	1.656	1.35	4.29	0.543	324	1485	1594
1987	Yugo	GV	3H	821	2.134	1.295	1.542	1.37	3.53	0.530	251	919	940

easier for further application (see eq. (2)).

$$I = k_0 \cdot l_{12}^{k_1} \cdot b^{k_2} \cdot H^{k_3} \cdot m^{k_4} \cdot h_{CG}^{k_5} \cdot L^{k_6} \tag{2}$$

As presented in further part of this article, depending on a type of equation, some exponents will be assumed as one or zero.

The main issue is to determine the coefficients k_i by applying the Monte Carlo method where the values are narrowed down by systematic progression where the size of the range is being reduced until a final value is achieved. Using the generated random numbers, 100000 draws will be made for ‘coefficients’, which leads us to the difference between estimated and measured values of the moment of inertia and δ (see formula (4)) and thereby then the range k_i (3) of searching each of these coefficients will be narrowed.

$$k_i \in \left(k_i^{MIN}, k_i^{MAX}\right) \tag{3}$$

$$\delta = \frac{\sum_{i=1...x}\left(I_i^{(est)} - I_i^{(meas)}\right)^2}{x} \tag{4}$$

where:
$I_i^{(est)}$ – estimated value of moment of inertia for i-step of optimization,
$I_i^{(meas)}$ – measured value of moment of inertia for i-step of optimization,
x – number of optimization steps.

The Authors decided to obtain empirical equations for roll, pitch and yaw mass moments of inertia.

The proposed empirical equations estimating the mass moments of inertia of automobile are presented in the next chapter.

IV. COMPARISON OF ESTIMATED AND MEASURED VALUES OF MASS MOMENTS OF A PASSENGER CAR

A. Roll mass moment of inertia I_X

Firstly, below are presented empirical formulas for the calculation of roll mass moment of inertia I_X (in relation to x-axis of a vehicle). The rolling motion of the vehicle is critical to simulate this parameter. The coefficients were adjusted accordingly to the procedure described in the previous chapter.

$$I_X = 1.359 \cdot l_{12} \cdot b^{2.25} \cdot H^{0.178} \cdot \sqrt{m} \cdot \sqrt[4]{L} \tag{5}$$

$$I_X = 12.52 \cdot l_{12}^{1.1759} \cdot b^{2.5170} \cdot L \tag{6}$$

Then, the difference D in percentages between estimated and measured value of roll mass moment of inertia was calculated for i-number of automobiles.

$$D_i = \frac{I_i^{est} - I_i^{meas}}{I_i^{meas}} \tag{7}$$

where:
I_i^{est} – estimated value of moment of inertia for i-number of automobile,
I_i^{meas} – measured value of moment of inertia for i-number of automobile.

Figure 1 and Figure 2 show the differences between estimated and measured values of I_X. In particular, Figure 1 shows the values of roll mass moment of inertia - estimated and measured, whereas in Figure 2 are presented the differences between estimation and measurement in percentages. In the same way the results of estimation for the other directions - pitch and yaw are further presented.

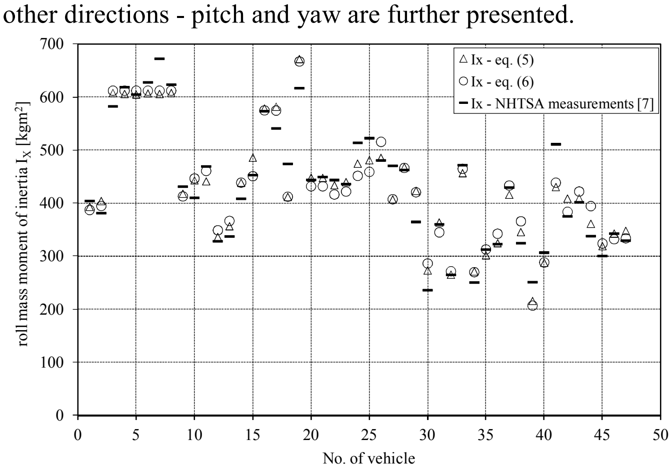

Fig. 1. Measured and estimated (formulas (5) and (6)) values of roll mass moment of inertia of automobiles.

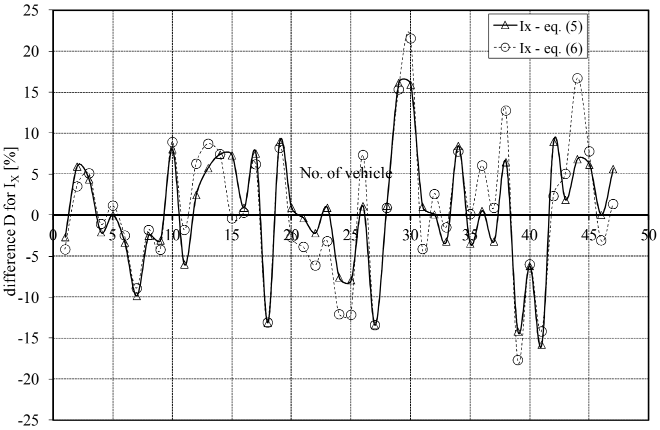

Fig. 2. Differences between measured and estimated (formulas (5) and (6)) values of roll mass moment of inertia of automobiles.

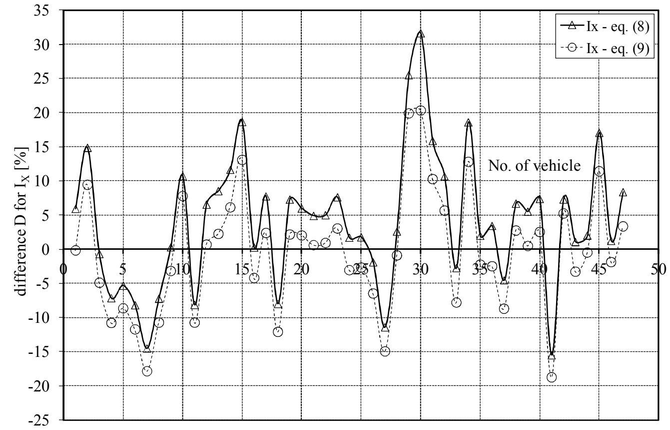

Fig. 4. Differences between measured and estimated (formulas (8) and (9) from the literature) values of roll mass moment of inertia of automobiles.

Additionally, Figure 3 and Figure 4 present the values and the differences between estimation and measurement of I_X in case of formula (8) proposed in [8] and (9) found in [9], which are cited below.

$$I_X = 0.73 \cdot \frac{m \cdot b \cdot H}{4} \tag{8}$$

$$I_X = \frac{m \cdot \left(H + h_{CG}\right) \cdot b}{7.9846} \tag{9}$$

In general, comparing equation (8) and equation (9), we note that the value of moment of inertia I_x is easier to obtain by equation 8 as compared to equation 9 due to the absence of the h_{CG} as h_{CG} may be very complicated to measure. Nevertheless, for each group of moments of inertia, there is given an alternative equation with parameters much easier to find or measure (in case of mass of automobile).

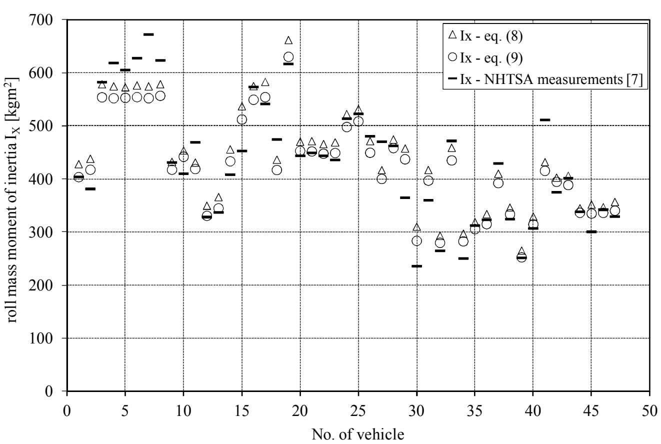

Fig. 3. Measured and estimated (formulas (8) and (9) from the literature) values of roll mass moment of inertia of automobiles.

Roll mass moment of inertia I_X plays an important role in simulating rollover traffic accidents, in particular the dynamics of roll motion of a vehicle. It may be also necessary when modeling vertical vibrations of a vehicle.

B. Pitch mass moment of inertia I_Y

Similar to roll mass moment of inertia, the value of pitch mass moment of inertia I_Y (in relation to y-axis) is necessary for simulation regarding vertical vibrations of the vehicle body.

The forms of equations describing pitch mass moment of inertia are as follows. The structure of equation (10) is similar to equation (8), but differs in the value of its coefficients.

$$I_Y = 0.284 \cdot l_{12}^{0.347} \cdot b^{-0.453} \cdot H^{0.469} \cdot m^{0.819} \cdot h_{CG}^{-0.0869} \cdot L^{1.790} \tag{10}$$

$$I_Y = 0.121 \cdot \sqrt[3]{l_{12}} \cdot \sqrt{\frac{H}{b}} \cdot m \cdot L^{1.550} \tag{11}$$

Figure 5 shows the estimated and measured values of I_Y and Figure 6 shows the difference between them.

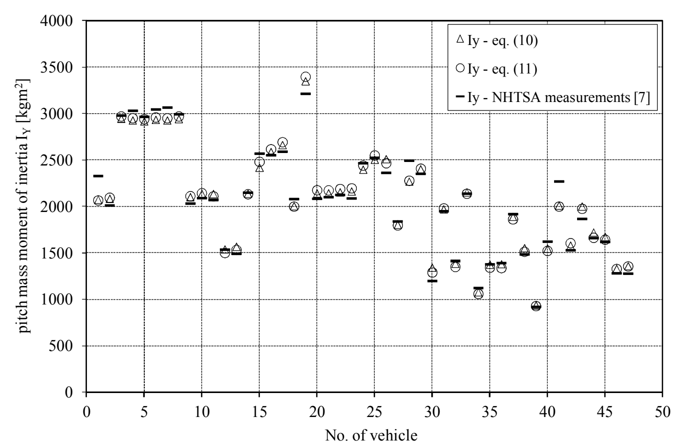

Fig. 5. Measured and estimated (authorial formulas (10) and (11)) values of pitch mass moment of inertia of automobiles.

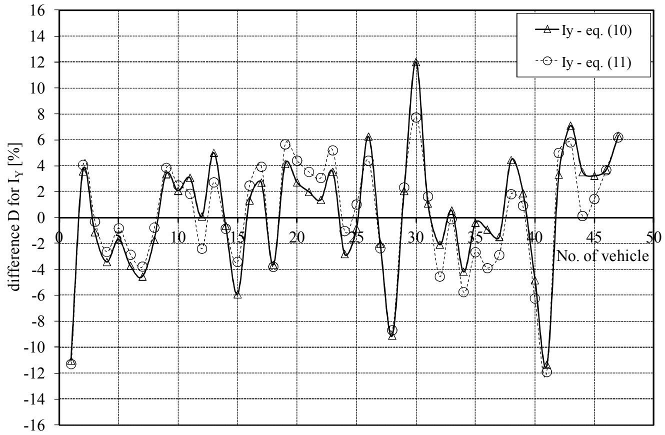

Fig. 6. Differences between measured and estimated (authorial formulas (10) and (11)) values of pitch mass moment of inertia of automobiles.

Presented below is equation (12) and (13) which can be found in [9] and in [8] respectively. Figure 7 and 8 depicts the differences in pitch moments of inertia between estimated and measured values.

$$I_Y = \frac{m \cdot \left(H + h_{CG}\right) \cdot L}{5.2901} \tag{12}$$

$$I_Y = 1.07 \cdot m \cdot l_1 \cdot l_2 \tag{13}$$

Figure 7 shows the estimated values through the application of the formulas (12) and (13) of yaw moment of inertia on the background of measured values, whereas in Figure 8 the differences between measurement and estimation in percentages are shown.

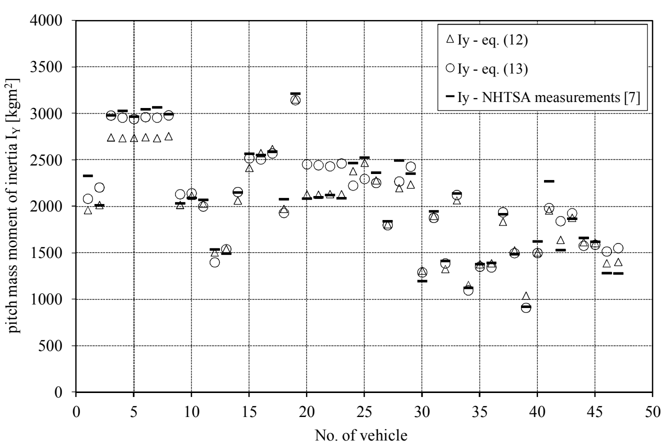

Fig. 7. Measured and estimated (formulas (12) and (13) from the literature) values of pitch mass moment of inertia of automobiles.

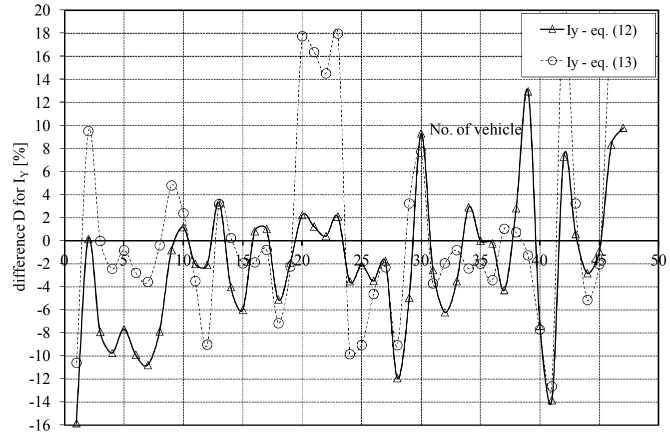

Fig. 8. Differences between measured and estimated (formulas (12) and (13) from the literature) values of pitch mass moment of inertia of automobiles.

C. Yaw mass moment of inertia I_Z

Yaw mass moment of inertia I_Z (in relation to *z*-axis) is one of the most important inertial parameters necessary to perform simulations for a vehicle undergoing curvilinear motion, as the influence of tire slip has to be taken into account. Equations 14, 15 and 16 represent optimized equations necessary for the calculation of moment of inertia.

$$I_Z = 0.279 \cdot l_{12}^{0{,}166} \cdot b^{0.042} \cdot H^{0.357} \cdot m^{0.798} \cdot h_{CG}^{-0.343} \cdot L^{1.829} \tag{14}$$

$$I_Z = 0.401 \cdot l_{12}^{0.138} \cdot b^{0.042} \cdot \sqrt[3]{H} \cdot m^{0.822} \cdot L^{1.641} \tag{15}$$

$$I_Z = 0.17 \cdot \sqrt[3]{H} \cdot m \cdot L^{1.46} \tag{16}$$

Figure 9 plots the estimated and measured values (proposed by formulas (14), (15) and (16)) of I_Z and Figure 10 shows the differences between them.

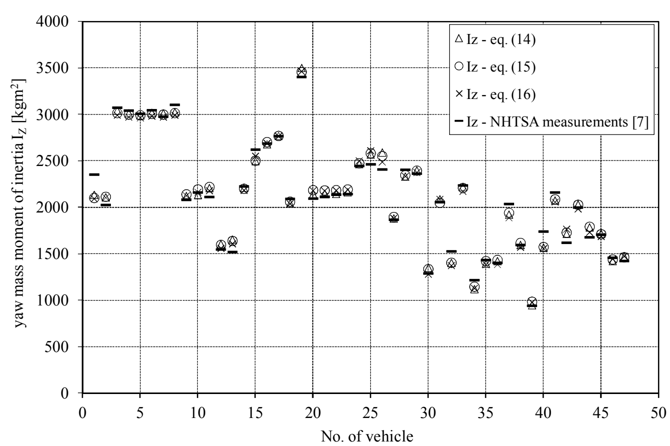

Fig. 9. Measured and estimated (authorial formulas (14), (15) and (16)) values of yaw mass moment of inertia of automobiles.

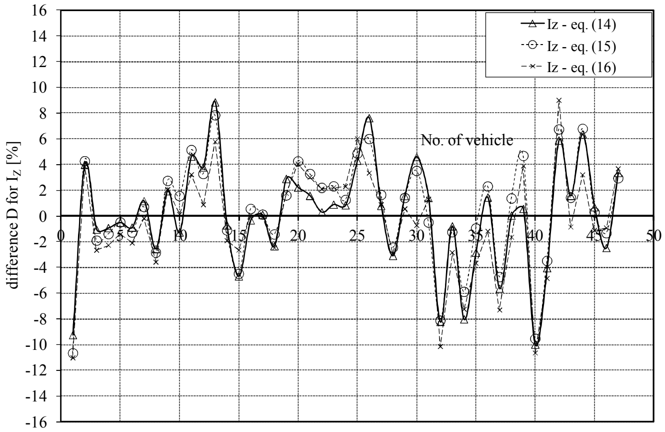

Fig. 10. Differences between measured and estimated (authorial formulas (14), (15) and (16)) values of yaw mass moment of inertia of automobiles.

Many empirical equations can be found in literature which are similar to equations presented in this article. Cited below are three of them. Equation (17) is proposed by Noon [10]. Equations (18a) and (18b), were proposed by MacInnis et al. [11] which are suitable for front and rear wheel drive.

$$I_Z = \frac{m \cdot L^2}{12} \tag{17}$$

$$I_Z = 0.1478 \cdot m \cdot l_{12} \cdot L \tag{18a}$$

$$I_Z = 1.015 \cdot \left(2 \cdot \left(1 - \frac{l_2}{l_{12}} \right) \frac{m \cdot L^2}{12} + \left(2 \cdot \frac{l_2}{l_{12}} - 1 \right) \cdot m \frac{l_{12}}{4} \right) \tag{18b}$$

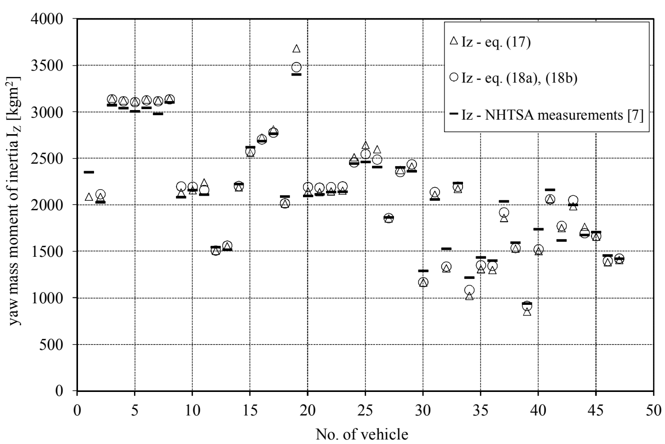

Fig. 11. Measured and estimated (formulas (17), (18a) and (18b) from the literature) values of yaw mass moment of inertia of automobiles.

Looking at the plots presenting the values of pitch and yaw mass moment of inertia, it can be concluded that the calculated and measured values for a particular vehicle are very close to each other.

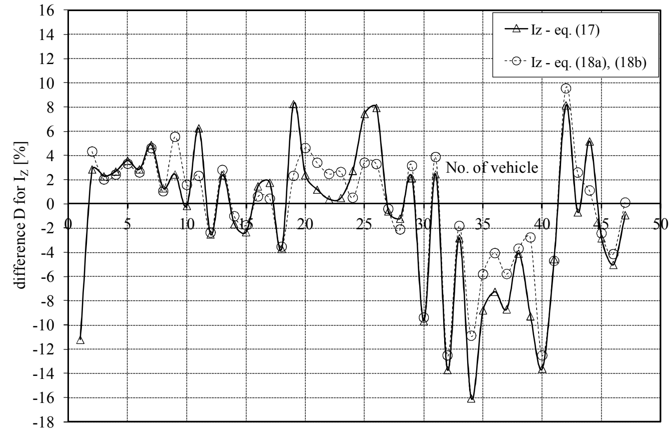

Fig. 12. Differences between measured and estimated (authorial formulas (17), (18a) and (18b) from the literature) values of yaw mass moment of inertia of automobiles.

V. Estimation Errors of Empirical Equations of Moments of Inertia

Estimation of error for moments of inertia was obtained through the calculations of standard deviation calculated as the difference D_i between estimated and measured values described by equation (7). Standard deviation was further multiplied by t-Student coefficient which equaled 3, for 99.7% of confidence interval. The assumption here was that the data used in calculations was characterized by normal distribution.

$$\Delta I_{est\%} = 3 \cdot \sigma_{I\,est\%} \tag{19}$$

where $\sigma_{Iest\%}$ is the standard deviation of the differences between measured and estimated value of moment I, calculated for standard automobiles and is expressed in percentages.

Hence, it was then possible to obtain an estimation error (14)

$$\Delta I_{est} = \Delta I_{est\%} \cdot I_{est} \tag{20}$$

where I_{est} is current value of moment of inertia in one of three directions

In Tables 2, 3 and 4 the estimation errors of the values of I_{est} is expressed in percentages (ΔI_{est}) suitably for roll, pitch and yaw mass moment of inertia.

TABLE II

Estimation error for new formulas and equations found in the literature – roll mass moment of inertia I_X

No. of equation	$\Delta I_{X\%}$ [%]
eq. (5)	22
eq. (6)	25
eq. (8) [8]	**29**
eq. (9) [9]	**26**

In case of empirical formulas for roll moment I_X, there are relatively high estimation errors. The value of I_X may be the most problematic for estimation, because the values of this moment are the smallest compared to pitch and yaw. Additionally, equation (6) does not include mass of a vehicle. This may be the reason for higher value of error. Generally, all the other empirical equations for moments of inertia include mass of a vehicle. In this case, another issue is to properly estimate or, if possible, measure real mass of automobile, which for example took part in traffic incident.

TABLE III

ESTIMATION ERROR FOR NEW FORMULAS AND EQUATIONS FOUND IN THE LITERATURE – PITCH MASS MOMENT OF INERTIA I_Y

No. of equation	$\Delta I_{Y\%}$ [%]
eq. (10)	14
eq. (11)	13
eq. (12) [9]	**18**
eq. (13) [8]	**26**

TABLE IV

ESTIMATION ERROR FOR NEW FORMULAS AND EQUATIONS FOUND IN THE LITERATURE – YAW MASS MOMENT OF INERTIA I_Z

No. of equation	$\Delta I_{Z\%}$ [%]
eq. (14)	12
eq. (15)	12
eq. (16)	13
eq. (17) [10]	**18**
eq. (18a) front driven [11]	**11**
eq. (18b) rear driven [11]	**19**

VI. CONCLUSION

Because of the fact that test rigs for the measurement of mass moments of inertia are very expensive, which makes them usually difficult to find, it is justified to look for other methods to estimate moment of inertia. This article presents new formulas to obtain these moments and compares the calculations with the values of moments using equations known from literature.

The new formulas for estimation of mass moments of inertia are characterized by lower error of estimation compared to the formulas from the literature. There is one exception, where equation (18a) [11] gave the error lower than the formulas proposed in this paper. However, it may result from the fact that for equations (18a) and (18b) the error was calculated for lower number of vehicles (front and rear driven respectively).

All new formulas are characterized by comparable estimation error, as it is depicted in Tables 2, 3, 4.

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