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2024 Spring –SPCO HW#3

1. Let X be a continuous random variable with the following cdf:

$$F(x) = \begin{cases} 0 & x \le 0 \\ \dfrac{x}{5}(1 + \ln\dfrac{5}{x}) & 0 < x \le 5 \\ 1 & x > 5 \end{cases}$$

   (1) $P(X \leqq 1)$? $P(2 \leqq X \leqq 3)$?

   (2) Find the probability density function (Remember: cdf is the integral of pdf)? expected value? and variance?

   (3) What could be a possible parameter in the pdf that can be used to construct a distribution family.

   (4) Use excel to plot the pdf and cdf of the distribution families with different parameter values.

2. Use the Excel to plot the probability density distribution and cumulated probability distribution curves for normal distributions with (mean, standard deviation)=(0, 1), (0, 5), (5, 5), (5, 10) and to calculate P($X \le \mu - 0.5\sigma$), P($X \le \mu - 1.5\sigma$), P($X \le \mu - 2.0\sigma$), P($X \le \mu - 2.5\sigma$), P($X \le \mu - 3\sigma$) and P($\mu - 3\sigma \le X \le \mu + 3\sigma$) for X following a normal distribution with mean=μ and standard deviation=σ.

3. X_i (i=1, 2) are iid uniformly distributed over the range [a, b]. It is known that $Y=X_1+X_2$ follows a triangular distribution. Derive the probability density function of the triangular distribution by calculating the convolution function and plot the probability density function and cumulated distribution function using Excel.

4. Let $Y = \sum_{i=1}^{n} X_i$ where random variables X_i i=1, 2,…, n are independent and following the identical Bernoulli distribution with parameter p. It is known that Y will follow the Binomial distribution $b(n, p)$.

   (1) Explain why Y follows the Binomial distribution and derive the mean and variance of Binomial distribution based on $Y = \sum_{i=1}^{n} X_i$

   (2) With p=0.4, calculate the means and variances of the summation of the Bernoulli distributed random variables, i.e., Binomial distributed random variables, with n=1, 5, 20, 50 and 100 and use Excel to plot the probability density function.

   (3) Use the means and variances calculated in (1) as the means and variances of normal distributions and plot the normal probability density functions on the same chart of (2). Describe your observations.

5. Let $Y = \sum_{i=1}^{n} X_i$ where random variables X_i i=1, 2,…, n are independent and following the identical exponential distribution with parameter λ. It is known that Y will follow the Gamma distribution with α=n and β=1/λ.

   (1) Derive the mean and variance of the Gamma Distribution based on $Y = \sum_{i=1}^{n} X_i$.

   (2) With λ=2, calculate the means and variances of the summation of the exponentially distributed random variables, i.e., Gamma distributed random variables, with n=2, 10, 20 and 50 and use Excel to plot the probability density function. Describe how the distribution shape changes as n becomes larger.

   (3) Use the means and variances calculated in (2) as the means and variances of normal distributions and plot the normal probability density functions on the same chart of (2). Describe your observations.