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Model Predictive Control for Collision Avoidance of Networked Vehicles Using Lagrangian Relaxation ⋆

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Abstract: This paper focuses on designing a steering intervention system based on Model Predictive Control. The goal is to avoid collisions between networked vehicles. Since the resulting program is non-convex, it is converted to a convex Semi-Definite Program by using Lagrangian relaxation. Furthermore, the utilized prediction vehicle model is linearized in an exact way to avoid errors due to linear approximation. Initially, the whole problem is formulated. Finally, the performance of the controller is presented in Model-in-the-Loop simulations.

Keywords: Model Predictive Control, Non-convex Optimization, Lagrangian Relaxation, Collision Avoidance, Networked Vehicles.

1. INTRODUCTION

The proposed algorithms in this paper are developed for collision avoidance of road vehicles. Nonetheless, they can be applied in other fields such as networked aerial, water and ground vehicles. In terms of road traffic, the World Health Organization published its status report on road safety in 2013. It is said that 1.24 million people die on the world's roads every yearWHO (2013). Although the mentioned number has not significantly changed since 2007, it is an important issue to find new solutions which lead to a decrease of this number, since road traffic injuries are still the eighth leading cause of death all over the world. This paper focuses on the design of a steering intervention system based on Model Predictive Control (MPC). Due to the recent developments in vehicle communication, a controller can be implemented for collision avoidance of networked vehicles. This controller should be able to solve the current optimization problem at each time step. There are two main tasks. The first task is to avoid any collision with opponent vehicles and obstacles. As a second task, the controller should keep each vehicle as close as possible to its reference trajectory (desired trajectory). Due to these partly contradictory requirements, each controlled vehicle will drive a collision-free trajectory which only deviates from the reference trajectory in case of imminent collisions. In the setup, it is distinguished between active members (vehicles) and passive ones (obstacles). Obstacles are either stationary or moving along a known path. The changes in the vehicle's steering angle with respect to the changes in the time step are penalized within the optimization problem's objective function to keep them smooth. Furthermore, a maximum absolute steering angle is defined in the input constraints to limit the lateral acceleration.

First, in section 2, the linearized and time-discretized prediction vehicle model and the optimization problem are presented. Since the vehicles and obstacles are described as circles or ellipses, the resulting collision avoidance constraints make the optimization problem a non-convex Quadratically Constrained Quadratic Program (QCQP). However, it can be converted into a convex Semi-Definite Program (SDP) through Lagrangian relaxation, which is described in section 3. An exact linearization of the vehicle model is also introduced to avoid errors due to linear approximation. Section 4 shows the simulation results which demonstrate the controller's performance. Finally, in section 5, a conclusion is given.

2. SETUP OF THE COLLISION AVOIDANCE PROBLEM

In this section, the basic problem for the MPC is formulated. The main purpose of the controller is to find appropriate steering angles for each time step such that each vehicle follows the given reference trajectory while preventing collisions at the same time. In case of an imminent collision, the reference trajectory has to be left in order to avoid the collision. After collision avoidance, the vehicle should return to the reference trajectory. In this context, a vehicle model is needed to predict the future behavior of the system. At each discrete time step, the current optimization problem has to be solved Maciejowski (2002).

The MPC has access to all relevant information about the vehicles and obstacles, their states, and their reference trajectories. The controller should solve an optimal control problem by considering the benefits of all vehicles and the defined constraints. The set of constraints considers the vehicle collision avoidance between each two vehicles, obstacle collision avoidance between each vehicle and obstacle, and the constraints on the control input. The solution of the MPC is system-wide optimal since

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the whole optimization problem is solved at once and the system feasibility collision avoidance is guaranteed through Vehicle-to-Vehicle (V2V) communication.

2.1 Vehicle Dynamics

The chosen vehicle model, which is used for the state prediction over the prediction horizon, is a nonlinear kinematic bicycle model, as introduced in Rajamani (2005), (see Fig. 1). R defines the radius of the driven circle around the center point O. For simplification, the following

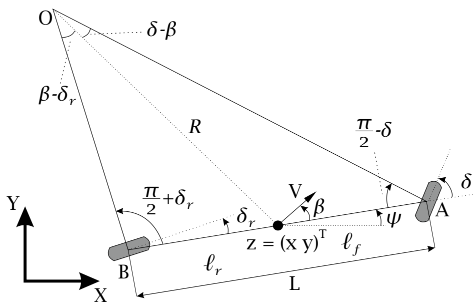

Fig. 1. Kinematic bicycle model of the vehicle

assumptions are made: (i) the height of the center of gravity (COG) is zero, hence pitch and roll dynamics are neglected, (ii) the front wheels are represented by one wheel at point A, and the rear wheels at B, (iii) rolling resistances and aerodynamic drag are neglected, (iv) the vehicle is front-wheel-only steering ($\delta_r = 0$), (v) the slip angle β between the velocity vector and longitudinal direction of the vehicle is neglected and (vi) no forces are applied neither at the front nor at the rear tire. The resulting state space system can be formulated as follows:

$$\begin{aligned} &\dot{x} = v\cos(\psi), \quad \dot{y} = v\sin(\psi), \quad \dot{\psi} = \frac{v}{L}\tan\delta, \\ &\dot{v} = a, \qquad\qquad\; \dot{a} = 0, \end{aligned} \tag{1}$$

where $z = (x\ y)^T$ indicates the location of the COG, ψ the yaw angle with respect to the X-axis, $L = l_r + l_f$ the wheelbase, v the velocity, and a the acceleration. Additionally it is assumed that $a = 0$. Therefore, $\dot{a} = 0$ is not needed. But it is introduced due to the exact linearization, which will be described in section 3.2.1. Here, $(x\ y\ \psi\ v\ a)^T$ is the state vector, z is the controlled output, and the steering angle δ is the control input. In the MPC algorithm, the nonlinear prediction model (1) is linearized using Taylor series around a particular operating point $(x_0, y_0, \psi_0, v_0, a_0, \delta_0)$ Khalil (2002). Katriniok and Abel (2011) shows that an improved control performance can be achieved using successive linearization compared to unchanged linearization. Additionally, the model is time-discretized. The resulting prediction model can be written as follows:

$$\begin{aligned} X_i(t+1) &= A_i(t)X_i(t) + B_i(t)u_i(t) + E_i(t) \\ z_i(t) &= \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}}_{C} X_i(t), \end{aligned} \tag{2}$$

where $i = 1, \ldots, N_{veh}$ denotes the vehicle's number, $X_i \in \mathbb{R}^5$, and $u_i \in \mathbb{R}$. The system matrix $A_i(t) \in \mathbb{R}^{5\times5}$, the input matrix $B_i(t) \in \mathbb{R}^5$ and the affine term $E_i(t) \in \mathbb{R}^5$, that results from the linearization of (1), are time-variant matrices. Moreover, it is $u_i \in \mathcal{U}$, where $\mathcal{U}$ indicates the set of allowed control inputs. $\mathcal{U}$ is limited in order not to exceed a maximum lateral acceleration which is defined as $a_{y,max} = 0.5g$, where $g \approx 10\frac{m}{s^2}$ is the acceleration due to gravity. According to Rajamani (2005), $\delta_{max} \approx tan^{-1}(\frac{a_{y,max}L}{v^2})$. Due to computational time issues, the kinematic model is used instead of a complex one that considers lateral forces.

2.2 Obstacle Collision Avoidance

The obstacles are assumed to be fixed or moving with a known direction and velocity. Thus, their positions and directions in the prediction can be determined exactly. Let $p_o(t+k)$ describe the position of obstacle o at time $t+k$. Assuming that the COGs are equal to the center positions of the vehicles and obstacles, the obstacle collision avoidance constraints for vehicle i can be expressed as follows:

$$\begin{aligned} \|z_i(t+k) - p_o(t+k)\|_2^{S_{e,i,o}} \geq 1,\ o &= 1, \ldots, N_o, \\ k &= 1, \ldots, H_p, \end{aligned} \tag{3}$$

where $z_i(t+k)$ denotes the predicted position of vehicle i at time $t+k$ starting from time t. N_o is the number of obstacles and H_p is the prediction horizon. Furthermore,

$$S_{e,i,o} = rot_o^T \begin{pmatrix} \dfrac{1}{\alpha_{x,i,o}^2} & 0 \\ 0 & \dfrac{1}{\alpha_{y,i,o}^2} \end{pmatrix} rot_o, \tag{4}$$

with the rotation matrix

$$rot_o = \begin{pmatrix} cos(\psi_o) & sin(\psi_o) \\ -sin(\psi_o) & cos(\psi_o) \end{pmatrix}, \tag{5}$$

specifies the ellipse around the center position of obstacle o which should not be entered by the center point of vehicle i in order to avoid a collision. The ellipse is rotated with the yaw angle ψ_o of obstacle o. The variables $\alpha_{x,i,o}$ and $\alpha_{y,i,o}$ are the ellipse constants which can also define the radius of a circle in case of $\alpha_{x,i,o} = \alpha_{y,i,o}$. Since the dimensions of each vehicle $i = 1, \ldots, N_{veh}$ have to be considered for the determination of the constants, each obstacle is described by up to N_{veh} different ellipses.

2.3 Vehicle Collision Avoidance

Based on the assumptions for the obstacle collision avoidance constraints, the collision avoidance constraints for the vehicles i and j can be formulated as follows:

$$\|z_i(t+k) - z_j(t+k)\|_2^{S_{c,i,j}} \geq 1,\ j > i,\ k = 1, \ldots, H_p, \tag{6}$$

where

$$S_{c,i,j} = \begin{pmatrix} \dfrac{1}{\alpha_{i,j}^2} & 0 \\ 0 & \dfrac{1}{\alpha_{i,j}^2} \end{pmatrix}, \tag{7}$$

specifies the circle with the radius $\alpha_{i,j}$ around the center position of vehicle j which should not be entered by the center point of vehicle i in order to avoid a collision. Because of the rotary symmetry of circles, $\alpha_{i,j} = \alpha_{j,i}$. Since the vehicles change their yaw angles over the prediction horizon, the trigonometric terms of their rotated ellipses are not constant. In this case, it is not possible to determine quadratic constraints for a QCQP. Hence, the vehicles are only described as circles.

2.4 Centralized Optimization Problem

Let z, r, and u indicate the position of the vehicles, the reference points, and the control input respectively. The centralized optimization problem at time t can be formulated as:

$$\min_{u(.)} \sum_{i=1}^{N_{veh}} \left(\sum_{k=1}^{H_p} \|z_i(t+k) - r_i(t+k)\|^2_{Q(i)} + \sum_{k=0}^{H_u-1} \|\Delta u_i(t+k)\|^2_{R(i)} \right) \tag{8}$$

subject to ($\forall i = 1, \dots, N_{veh}$):

$$\begin{aligned} X_i(t+k+1) = &A_i(t+k)X_i(t+k) + \\ &B_i(t+k)u_i(t+k) + \\ &E_i(t+k),\ k = 0, \dots, H_p - 1 \end{aligned} \tag{9}$$

$$z_i(t+k) = C\,X_i(t+k),\ k = 1, \dots, H_p \tag{10}$$

$$u_i(t+k) \in \mathcal{U},\ k = 0, \dots, H_u - 1 \tag{11}$$

$$\|z_i(t+k) - p_o(t+k)\|_2^{S_{e,i,o}} \geq 1, \quad o = 1, \dots, N_o,\ k = 1, \dots, H_p, \tag{12}$$

$$\|z_i(t+k) - z_j(t+k)\|_2^{S_{c,i,j}} \geq 1, \quad j > i,\ k = 1, \dots, H_p. \tag{13}$$

The objective function (8) minimizes the distance between the vehicle position and the reference trajectory over the prediction horizon. The objective function also minimizes the control input variations, where H_u indicates the control horizon. Q_i and R_i are weighting matrices. Equations (9-13) show the constraints. Equation (9) indicates the model of vehicle i. In (10), z_i represents the system output, here it is the position of vehicle i. Equation (11) specifies the control input boundary condition. Equation (12) indicates the obstacle collision avoidance and (13) the vehicle collision avoidance constraints. Since the collision avoidance between vehicles (i, j) or (j, i) is the same, the condition $j > i$ ensures to consider just one of them.

3. COLLISION AVOIDANCE USING MPC

In this section, the optimization problem is formulated as a non-convex QCQP. Then, it is converted into a convex one using Lagrangian relaxation. Furthermore, the exact linearization and the resulting input constraints formulation are presented.

3.1 Formulation of Optimization Problem and Collision Avoidance Constraints

Since the distances between the vehicles and their reference trajectories have to be considered in the objective function and since the vehicles and obstacles are described as circles and ellipses, the optimization problem can be formulated as a QCQP. As defined in Aspremont and Boyd (2003), a QCQP is characterized by a convex quadratic objective function subjected to convex quadratic constraints

$$\begin{aligned} &\min_{e} && e^T P_0 e + q_0^T e + s_0 \\ &\text{subject to:} && e^T P_i e + q_i^T e + s_i \leq 0,\ i = 1, \dots, m, \end{aligned} \tag{14}$$

where $e \in \mathbb{R}^n$, $P_i \in \mathbb{S}^n_+$ and $n, m \in \mathbb{N}$. $\mathbb{S}^n_+$ indicates positive semi-definite and symmetric $n \times n$ matrices. Furthermore, $q_i \in \mathbb{R}^n$, and $s_i \in \mathbb{R}$. If one P_i is not positive semi-definite, the QCQP is a non-convex program and can not be solved efficiently in every case. The objective function (8) is already formulated in a quadratic form according to (14). In fact, the time discrete linear state space model of the vehicle (2) is used to formulate a standard output feedback MPC using z as output. Hence, the following prediction equation is obtained:

$$\mathcal{Z}_i(t) = \underbrace{\Psi_i X_i(t) + \Upsilon_i u_i(t-1) + \Pi_i E_i(t)}_{\text{free response}} + \underbrace{\Theta_i \Delta\mathcal{U}_i(t)}_{\text{variable term}}, \tag{15}$$

where the matrices $\mathcal{Z}_i(t), \Psi_i, \Upsilon_i, \Pi_i, \Theta_i$, and $\Delta\mathcal{U}_i(t)$ are defined the same way as in Maciejowski (2002). Here, the vector $\mathcal{Z}_i(t) = [z_i(t+1), z_i(t+2), \dots, z_i(t+H_p)]^T$ contains all predicted outputs of vehicle i starting from time t over the prediction horizon H_p. The vector $\Delta\mathcal{U}_i(t) = [\Delta u_i(t), \Delta u_i(t+1), \dots, \Delta u_i(t+H_u-1)]^T$ includes the input changes and is the optimization variable. The matrices $X_i(t)$, $\Psi_i(t)$, and $\Pi_i(t)$ result from iterative use of (2). The free response term describes the vehicles movement for a constant unchanged input $u(t-1)$. In the following, the constraints in quadratic form will be derived.

Obstacle Collision Avoidance Based on (15), the prediction of vehicle i's position is:

$$\begin{aligned} z_i(t+k) = &\Psi_{i,k} X_i(t) + \Gamma_{i,k} u_i(t-1) + \Pi_{i,k} E_i(t) + \\ &\Theta_{i,k} \Delta\mathcal{U}_{i,k}(t) \\ = &b_{i,k} + \Theta_{i,k} \Delta\mathcal{U}_{i,k}(t), \end{aligned} \tag{16}$$

where the index k indicates the k-th equation. Substituting in (3):

$$\begin{aligned} &(b_{i,k} + \Theta_{i,k} \Delta\mathcal{U}_{i,k}(t) - p_o(t+k))^T S_{e,i,o} \\ &(b_{i,k} + \Theta_{i,k} \Delta\mathcal{U}_{i,k}(t) - p_o(t+k)) \geq 1, \end{aligned} \tag{17}$$

which is equivalent to:

$$\begin{aligned} &\forall i = 1, \dots, N_{veh}, \forall o = 1, \dots, N_o,\ k = 1, \cdots, H_p : \\ &\Delta\mathcal{U}_{i,k}(t)^T P_{i,k,o} \Delta\mathcal{U}_{i,k}(t) + \\ &\qquad q_{i,k,o}^T \Delta\mathcal{U}_{i,k}(t) + s_{i,k,o} \leq 0, \end{aligned} \tag{18}$$

where:

$$\begin{aligned} P_{i,k,o} &= -\Theta_{i,k}^T S_{e,i,o} \Theta_{i,k} \\ q_{i,k,o} &= -2\Theta_{i,k}^T S_{e,i,o}(b_{i,k} - p_o(t+k)) \\ s_{i,k,o} &= 1 - (b_{i,k} - p_o(t+k))^T S_{e,i,o}(b_{i,k} - p_o(t)). \end{aligned}$$

In this way, the obstacle collision avoidance constraints are written in a quadratic form. The negative quadratic term makes the constraints non-convex.

Vehicle Collision Avoidance The vehicle collision avoidance constraints are similar to the obstacle collision avoidance constraints. The difference is that the states of the corresponding vehicles are both variables in the prediction. As shown in (13), the vehicle collision avoidance is defined such that the distance between vehicles i and j should be greater than $\alpha_{i,j}$. Similar to (16), the positions of vehicles i and j at time $t+k$ can be determined as:

$$\begin{aligned} z_i(t+k) &= b_{i,k} + \Theta_{i,k} \Delta\mathcal{U}_{i,k}(t) \\ z_j(t+k) &= b_{j,k} + \Theta_{j,k} \Delta\mathcal{U}_{j,k}(t). \end{aligned} \tag{19}$$

Substituting in (13) and rearranging to:

$$\begin{aligned}&\forall i, j \in \{1,\ldots,N_{veh}\},\ j>i,\ k=1,\ldots,H_p:\\ &\Delta\mathcal{U}_{i,j,k}(t)^T P_{i,j,k}\Delta\mathcal{U}_{i,j,k}(t)+\\ &\qquad q_{i,j,k}^T\Delta\mathcal{U}_{i,j,k}(t)+s_{i,j,k}\le 1,\end{aligned} \tag{20}$$

where

$$\begin{aligned}P_{i,j,k} &= \begin{pmatrix}-\Theta_{i,k}^T S_{c,i,j}\Theta_{i,k} & \Theta_{i,k}^T S_{c,i,j}\Theta_{j,k}\\ \Theta_{j,k}^T S_{c,i,j}\Theta_{i,k} & -\Theta_{j,k}^T S_{c,i,j}\Theta_{j,k}\end{pmatrix},\\ q_{i,j,k} &= \begin{pmatrix}-2\Theta_{i,k}^T S_{c,i,j}(b_{i,k}-b_{j,k})\\ 2\Theta_{j,k}^T S_{c,i,j}(b_{i,k}-b_{j,k})\end{pmatrix},\\ s_{i,j,k} &= 1-(b_{i,k}-b_{j,k})^T S_{c,i,j}(b_{i,k}-b_{j,k}).\end{aligned}$$

As for the obstacle avoidance, the quadratic term makes the constraints non-convex.

Relaxation As shown in S. Boyd and L. Vandenberghe (1997), the constraints in (14) can be combined with the objective function to the Lagrangian:

$$\begin{aligned}L(e,\lambda) =& e^T\left(P_0+\sum_{i=1}^m\lambda_i P_i\right)e+\Bigg(q_0+\\ &\sum_{i=1}^m\lambda_i q_i\Bigg)^T e+s_0+\sum_{i=1}^m\lambda_i s_i,\end{aligned} \tag{21}$$

where $\lambda_i \ge 0, \forall i=1,\ldots,m$ are *dual variables.* In the following, the infimum of the Lagrangian over e has to be found which is the *Lagrangian dual function* $g(\lambda)$:

$$\begin{aligned}g(\lambda) =& \inf_x L(e,\lambda)\\ =& -\frac{1}{4}\Big(q_0+\sum_{i=1}^m\lambda_i q_i\Big)^T\Big(P_0+\sum_{i=1}^m\lambda_i P_i\Big)^\dagger\Big(q_0+\\ &\sum_{i=1}^m\lambda_i q_i\Big)+s_0+\sum_{i=1}^m\lambda_i s_i\ .\end{aligned} \tag{22}$$

Here, it is assumed that $(P_0+\sum_{i=1}^m\lambda_i P_i)\in\mathbb{S}_+^n$, which means symmetric positive semi-definite. The † sign indicates a pseudo-inverse. According to Boyd and Vandenberghe (2009), this dual function describes the point-wise infimum of a family of affine functions of λ. Hence, $g(\lambda)$ is a concave function even when the original problem in (14) is not convex. Let L^* be the optimal value of the original problem. It is obvious that the Lagrangian $L(e,\lambda)$ represents a lower bound, so that:

$$g(\lambda)\le L(e,\lambda)\le L^*. \tag{23}$$

The target is now to determine a vector λ for that $g(\lambda)$ is as close to L^* as possible. The *dual problem* can be expressed as follows:

$$\begin{aligned}&\max && g(\lambda)\\ &\text{subject to:} && \lambda_i\ge 0,\ i=1,\ldots,m.\end{aligned} \tag{24}$$

This problem contains the maximization of the concave dual function and is subject to convex constraints. Thus, the whole problem is convex and can be solved efficiently. The dual problem can be converted into another but equivalent dual problem for avoiding the pseudo inverse formulation. For the formulation of the alternative dual problem, a further scalar dual variable γ is defined:

$$\gamma\le g(\lambda). \tag{25}$$

The new dual problem is:

$$\begin{aligned}&\max && \gamma\\ &\text{subject to:} && g(\lambda)-\gamma\ge 0\\ &&& \lambda_i\ge 0,\ i=1,\ldots,m.\end{aligned} \tag{26}$$

The first constraint of the problem can be treated like a Schur complement S which is introduced in Boyd and Vandenberghe (2009). It is:

$$S=C_s-B_s^T A_s^{-1}B_s, \tag{27}$$

with $A_s=(P_0+\sum_{i=1}^m\lambda_i P_i)$, $B_s=(q_0+\sum_{i=1}^m\lambda_i q_i)/2$ and $C_s=(r_0+\sum_{i=1}^m\lambda_i r_i-\gamma)$. The given constraint $S\ge 0$ and the assumption $A_s>0$ ($A_s\in\mathbb{S}_{++}^n$), which logically leads to a bigger minimum of $g(\lambda)$ than $A_s\ge 0$ ($A\in\mathbb{S}_+^n$), can be replaced by the following constraint:

$$\begin{pmatrix}A_s & B_s^T\\ B_s & C_s\end{pmatrix}\ge 0. \tag{28}$$

The final dual problem is:

$$\begin{aligned}&\max\ \gamma\\ &\text{subject to:}\\ &\begin{pmatrix}(P_0+\sum_{i=1}^m\lambda_i P_i) & (q_0+\sum_{i=1}^m\lambda_i q_i)/2\\ (q_0+\sum_{i=1}^m\lambda_i q_i)^T/2 & \sum_{i=1}^m\lambda_i r_i+r_0-\gamma\end{pmatrix}\ge 0\\ &\lambda_i\ge 0,\ i=1,\ldots,m.\end{aligned} \tag{29}$$

Now, it is obvious that Lagrangian relaxation in fact leads to a SDP. Due to its convexity, it is easy to solve compared to the original non-convex QCQP. Note that the results from the relaxation are feasible, which means that the collision avoidance constraints are satisfied.

3.2 Exact Linearization and Input Constraints

As mentioned in section 2, the vehicle model is linearized by using the Taylor series (conventional linearization). The exact linearization provides an approach which avoids errors due to linear approximation.

Exact Linearization The exact linearization is performed for the kinematic vehicle model (1). According to Meurer (2013), the suitable choice of outputs, which fulfills certain criteria, has to be made. One of the criteria is the *flatness.* In the context of this paper, (x,y) are considered as flat outputs, if the state vector of the nonlinear kinematic vehicle model is reduced to $[x,y,\psi,v]^T$ and the input vector is defined as $[tan(\delta),a]^T$. The reason is that all other states and inputs can be expressed with the help of (x,y) and their time-derivatives as follows:

$$\begin{aligned}\psi &= tan^{-1}\left(\frac{\dot y}{\dot x}\right)\\ v &= \frac{\dot y}{sin(tan^{-1}(\frac{\dot y}{\dot x}))}=\frac{\dot x}{cos(tan^{-1}(\frac{\dot y}{\dot x}))}\\ a &= \dot\psi\\ tan(\delta) &= \frac{L}{v}\dot\psi.\end{aligned} \tag{30}$$

Note that $\dot\psi$ can also be expressed with the flat outputs since ψ has already been expressed by them. The acceleration is now defined as an input because the exact linearization requires a number of outputs that is equal to the number of inputs. Furthermore, an input-affine system is needed. Hence, $\tan(\delta)$ is defined as an input. After some equivalent rearrangements, a new state space system can be created with flat outputs and their first time-derivatives as state vector:

$$\begin{pmatrix}\dot{x}\\\dot{y}\\\ddot{x}\\\ddot{y}\end{pmatrix} = \begin{pmatrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{pmatrix}\begin{pmatrix}x\\y\\\dot{x}\\\dot{y}\end{pmatrix} + \begin{pmatrix}0&0\\0&0\\ & C_{conv} & \end{pmatrix}\begin{pmatrix}\tan(\delta)\\a\end{pmatrix}, \tag{31}$$

with

$$C_{conv} = \begin{pmatrix}-\frac{v^2}{L}\sin(\psi) & \cos(\psi)\\ \frac{v^2}{L}\cos(\psi) & \sin(\psi)\end{pmatrix}. \tag{32}$$

The exact linearization is accomplished by substituting the original inputs with new defined inputs

$$w = \begin{pmatrix}w_1\\w_2\end{pmatrix} = C_{conv}\begin{pmatrix}\tan(\delta)\\a\end{pmatrix}, \tag{33}$$

where C_{conv} can be interpreted as a conversion matrix, so that it is:

$$\begin{pmatrix}\dot{x}\\\dot{y}\\\ddot{x}\\\ddot{y}\end{pmatrix} = \begin{pmatrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{pmatrix}\begin{pmatrix}x\\y\\\dot{x}\\\dot{y}\end{pmatrix} + \begin{pmatrix}0&0\\0&0\\1&0\\0&1\end{pmatrix}\begin{pmatrix}w_1\\w_2\end{pmatrix}. \tag{34}$$

It has to be mentioned, that no assumption and negligence was made. The system is the result of equivalent rearrangements and a substitution. Since the new inputs can not be interpreted in a physical way, the inputs have to be constrained indirectly.

Velocity constraints In this paper, it is intended to keep the acceleration equal to $0\frac{m}{s^2}$. Since a cannot be directly expressed in connection with a QCQP, the speed can be kept close to a constant value instead. Referring to the kinematic vehicle model (1), it is obvious that the squared velocity v can be directly expressed by the sum $\dot{x}^2+\dot{y}^2$. For the speed constraints, the output in (2) has to be changed to

$$z_i^*(t) = \underbrace{\begin{pmatrix}0&0&1&0\\0&0&0&1\end{pmatrix}}_{C^*} X_i^*(t), \tag{35}$$

where the star $*$ indicates the new vectors and matrices that result from the exact linearized system. For a desired velocity $v_{i,des}$ and an accepted deviation Δv_i, the constraints can be formulated as:

$$\begin{aligned}&\forall i = 1,\dots,N_{Veh}:\\ &\|z_{i,k}^*\|_2 \leq (v_{i,des}+\Delta v_i)^2\\ &\|z_{i,k}^*\|_2 \geq (v_{i,des}-\Delta v_o)^2, k=1,\dots,H_p.\end{aligned} \tag{36}$$

As for the collision avoidance constraints, the resulting constraints are quadratic.

Steering angle constraints For the design of the steering angle constraints, the interpretation problem of the input vector w has to be considered. But considering the graphical description of the input conversion and the assumption that the velocity v is known and $a = 0\frac{m}{s^2}$, the steering angle can be easily constrained. Fig. 2 gives an overview over the situation. The value d^* describes the distance to the axis of the scaled input $\tan(\delta)\frac{v^2}{L}$ which is approximately equal

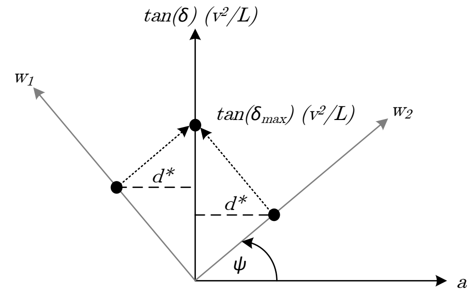

Fig. 2. Determination of the steering angle constraints

for both inputs of w_1, w_2, if the acceleration is zero. In this case, the constraint for the steering angle is:

$$\begin{aligned}&\forall i = 1,\dots,N_{veh}:\\ &\|w_i(t+k)\|_2 \leq \left(\tan(\delta_{max})\frac{v^2}{L}\right)^2, \; k=1,\dots,H_p.\end{aligned} \tag{37}$$

The resulting constraints are quadratic.

4. SIMULATION RESULTS

In Fig. 3, the controller's performance with prediction models based on conventional and exact linearization are presented. In this scenario, one vehicle drives with a velocity of $v = 5\frac{m}{s}$ towards a static obstacle. Both have the dimensions of a BMW 5 Series. For the exact linearized system, a velocity deviation of $\Delta v = 0.05\frac{m}{s}$ is accepted. The obstacle is described as an ellipse. Furthermore, $H_p = H_u = 10$ and $\delta_{max} = 10°$. A sampling time of $\Delta t = 0.5s$ was chosen. The weights for the input changes within the cost function are equal to 500. Distances to the reference trajectory are weighted with a factor of 80 for $k = 1,\dots,H_p-1$ and a factor of 160 for $k = H_p$. In both cases, Model-in-the-Loop (MIL) and Software-in-the-Loop (SIL) simulation results are shown. For the MIL simulation the vehicle's reaction is represented by the kinematic vehicle model. The SIL simulation shows the movement of a complex vehicle model which can be considered as more realistic. CarMaker from IPG Automotive was the integrated software. It is obvious that the controller with the exact linearized prediction model has a better performance. Here, the differences between the MIL and the SIL simulations are small. The more symmetric shape

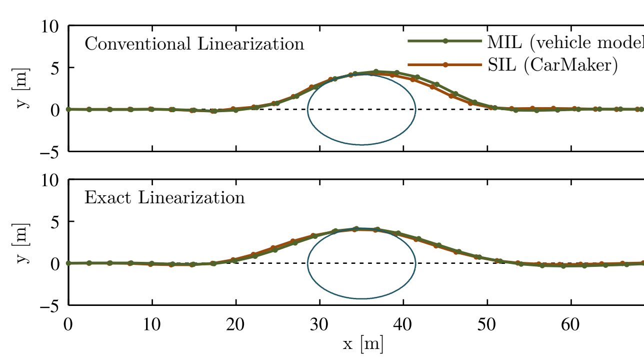

Fig. 3. Complete trajectory for scenario with one vehicle and one static obstacle

is caused by the different influence of the input change weights (R-Matrix in (8)). For the exact linearization, the new defined inputs w_1, w_2 are considered instead of the steering angle.

A more complex MIL scenario result is shown in Fig. 4. Here, the prediction model is linearized in an exact way. Two vehicles ($v = 5\frac{m}{s}$) and two obstacles ($v = 1\frac{m}{s}$) drive towards the zero point. All of them are described as circles. The rest of the configuration is equal to the previously shown scenario. For different time steps, the predicted trajectory based on the optimized steering angles over the prediction horizon is shown. Additionally, the reference trajectories are shown. At the 13th and 24th time step, it can be seen that a collision is avoided because the center points of the vehicles do not enter the opponent vehicles' and obstacles' circles. After the collision avoidance, the vehicles get back to the reference trajectory. Since this scenario is symmetric, the velocities of the vehicles and the steering angles are equal over the time. It is shown that neither the velocity values nor the steering angles break the rules given by the constraints.

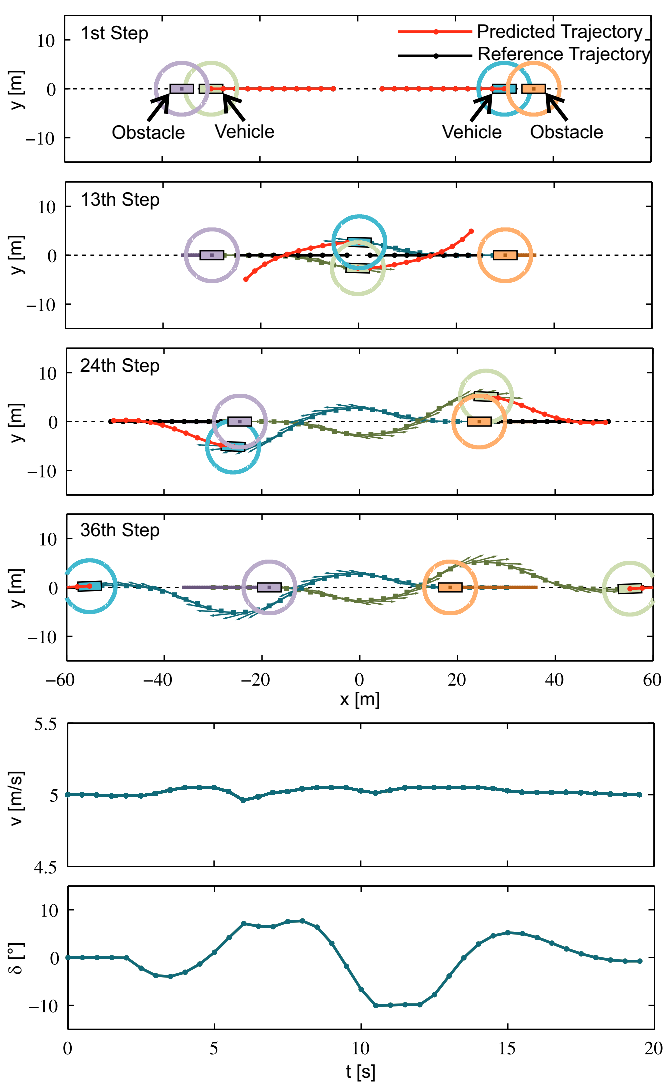

Fig. 4. Scenario with two vehicles and two moving obstacles

5. CONCLUSIONS

An MPC, which optimizes the steering angles of the controlled vehicles, was implemented. Since opponent vehicles and obstacles were described as ellipses and circles, the collision avoidance constraints became non-convex. But by using Lagrangian relaxation, the non-convex QCQP could be converted to a convex SDP so that the determination of the problem's optimum can be done efficiently. Furthermore, it could be shown that compared to the conventional linearization, the exact linearization of the prediction model leads to a better prediction of the controlled vehicle's movement.

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