2019 Spring –SPCO HW#3

1. Use the Excel
   (1) Plot probability density distribution and cumulated probability distribution curves for Gamma distributions with (α, β)=(1, 2), (2, 1), (4, ½), (6, 1/3) normal distributions with (mean, variance)=(2, 4), (2, 2), (2, 1), (2, 2/3).
   (2) Find the cumulated probability distribution function in Excel to calculate P($X \le \mu - 0.5\sigma$), P($X \le \mu - 1.5\sigma$), P($X \le \mu - 2.0\sigma$), P($X \le \mu - 2.5\sigma$), P($X \le \mu - 3\sigma$) and P($\mu - 3\sigma \le X \le \mu + 3\sigma$) where X follows a normal distribution with mean=μ and standard deviation=σ.

2. X_i (i=1, and 2) are iid uniformly distributed over the range [a, b]. it is known that $Y = X_1 + X_2$ follows a triangular distribution. Derive the probability density function of the triangular distribution by calculation the convolution function.

3. Let $Y = \sum_{i=1}^{n} X_i$ where random variables X_i i=1, 2,…, n are independent and following the identical geometric distribution with parameter p. It is known that Y will follow the Negative Binomial distribution NB(n, p)
   (1) Derive the mean and variance of the Negative Binomail Distribution based on $Y = \sum_{i=1}^{n} X_i$.
   (2) With p=0.1, calculate the means and variances of the summation of the geometric distributed random variables, i.e., Negative Binomial distributed random variables, with n=1, 5, 10, 20 and 50 and use Excel to plot the probability density function.
   (3) Use the means and variances calculated in (2) as the means and variances of normal distributions and plot the normal probability density functions on the same chart of (2). Describe your observations.

4. Derive the mean and the variance of the exponential distribution.

5. Let $Y = \sum_{i=1}^{n} X_i$ where random variables X_i i=1, 2,…, n are independent and following the identical exponential distribution with parameter λ. It is known that Y will follow the Gamma distribution with $\alpha = n$ and $\beta = 1/\lambda$.
   (1) Derive the mean and variance of the Gamma Distribution based on $Y = \sum_{i=1}^{n} X_i$.
   (2) With λ=1, calculate the means and variances of the summation of the exponentially distributed random variables, i.e., Gamma distributed

random variables, with n=1, 5, 10, 20 and 50 and use Excel to plot the probability density function.

(3) Use the means and variances calculated in (2) as the means and variances of normal distributions and plot the normal probability density functions on the same chart of (2). Describe your observations.