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A Comparison of Moment of Inertia Estimation Techniques for Vehicle Dynamics Simulation

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ABSTRACT

The moments of inertia, in yaw, pitch, and roll, as well as the center of gravity height are necessary to successfully model the 3D dynamic behavior of vehicles before, during and after collision. A number of vehicle parameter estimation techniques have been developed and are currently in use in North America and Europe. Many parameters have been measured by NHTSA and others. The estimation techniques are compared to the available measured values, and recommendations are made for best estimating the parameters when measured values are not available. The sensitivity of 3D vehicle collision dynamics and trajectory simulation to variance in the moment of inertia is demonstrated.

INTRODUCTION

Early 2D collision and spin-out models required only the mass and the yaw moment of inertia. Modeling the 3D dynamic behavior of a vehicle requires a minimum of the vehicle geometry, the center of gravity height and the triaxial moments of inertia. Because moments of inertia are difficult to measure for a given vehicle, there is a need for estimation techniques that use easily obtained vehicle measurements.

More complex models will require data concerning the sprung and unsprung vehicle parameters, as well as more detailed suspension and tire modeling data. The work of this paper is limited to discussion of the whole vehicle roll, pitch, and yaw moments of inertia relative to the total vehicle centroidal axes, and the height of the whole vehicle center of gravity.

This paper briefly addresses the theory, and presents a number of estimation techniques that are available in the literature. The values measured by Garrott of the NHTSA are compared to the values determined by the estimation techniques in five different vehicle categories. Correction factors were applied to each of the estimation equations to eliminate the average error. The adjusted equations were ranked by standard deviation, and the best equations, the ones with the smallest standard deviation, were chosen and presented. The nomenclature used is described in Appendix A.

THEORY

In simple linear Newtonian mechanics, the mass of a vehicle is assumed to be concentrated at the center of gravity location. This assumption may be appropriate for the simplest dynamic models, pure central collisions, or for very coarse estimation purposes. When the vehicles are involved in eccentric collisions with rotation, as is the most common case, then the simple models are inadequate, and the distribution of the vehicle's mass about the centroid must be considered.

The resistance of an object to rotation is a function of the mass of the object, and the location of the mass with respect to the center of rotation, and is known as its moment of inertia, I.

An idealized motor vehicle can be considered a solid homogeneous slab as shown in Figure 1.

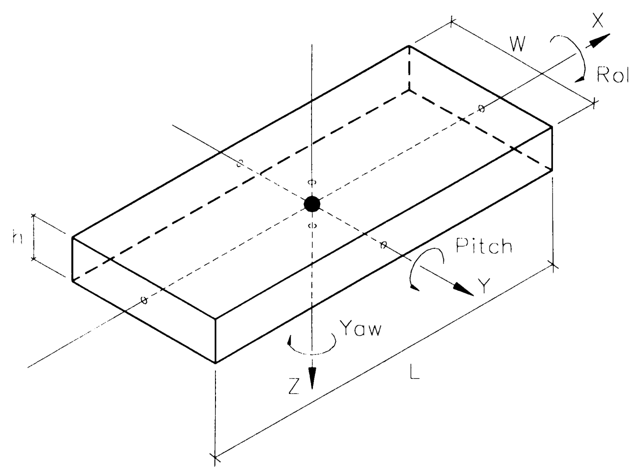

Figure 1 - The vehicle as a solid homogeneous slab or prism

The moments of inertia about the three axes of the homogeneous slab would be:[1]

$$I_{xx} = \frac{1}{12} m\left(W^2 + h^2\right) \tag{1}$$

$$I_{yy} = \frac{1}{12} m\left(L^2 + h^2\right) \quad (2)$$

$$I_{zz} = \frac{1}{12} m\left(L^2 + W^2\right) \quad (3)$$

This is termed the prism method. The yaw moment of inertia for a solid slab is a function of the length and width as well as mass, not mass alone.

A motor vehicle is not a solid homogeneous slab. It has concentrated masses at the engine, transmission, suspension, etc., as shown in Figure 2.

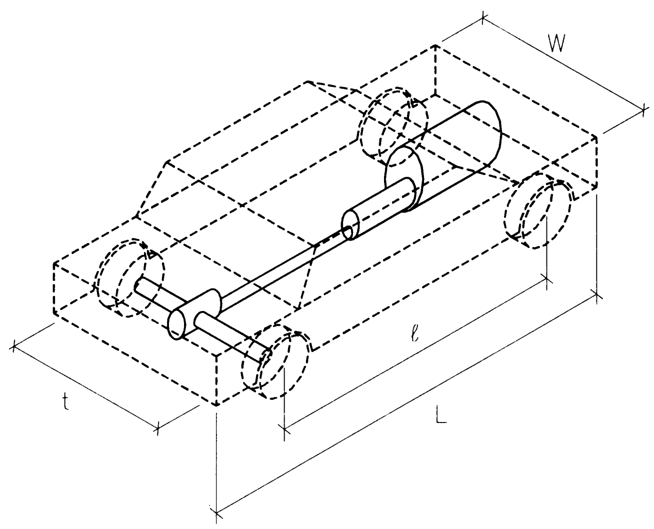

Figure 2 - The vehicle with concentrated masses

The moment of inertia for any object can be calculated, if enough is known about these distributed masses. However, the moment of inertia of a motor vehicle is more practically measured by testing.

A simplified top view of a vehicle is shown in Figure 3.

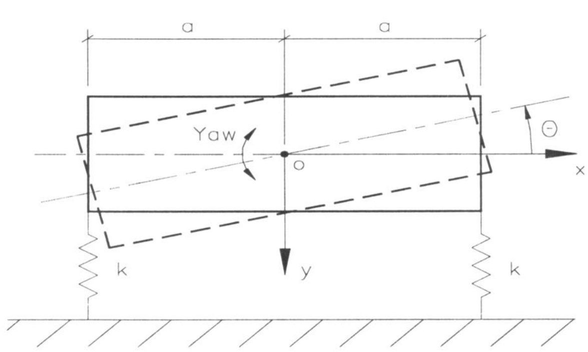

Figure 3 - Top view of a vehicle being tested for the yaw moment of inertia

The vehicle is allowed to pivot at its center of gravity, on a massless undamped frictionless frame. The fore/aft mass distribution is 50/50. The moments sum as follows:

$$\sum M = I\alpha \quad (4)$$

$$-2(ka\theta)a = I\ddot{\theta} \quad (5)$$

$$\ddot{\theta} + \left(\frac{2ka^2}{I}\right)\theta = 0 \quad (6)$$

The period of this oscillating structure is: [2]

$$\tau = 2\pi\sqrt{\frac{I}{2ka^2}} \quad (7)$$

In this idealized example, one measures the period, the geometry, and the spring constants, and the moment of inertia is readily determined:

$$I = \frac{1}{4\pi^2} 2ka^2\tau^2 \quad (8)$$

The actual measurement of the inertial properties of vehicles is far more complex than presented here, and the details and complexities have been developed by Garrott.[3,4,5]

MEASURED VALUES

The NHTSA has measured the moments of inertia for a number of vehicles, as described by Garrott (1993). These values are published and available on disk.

Burg[6] (1982) in Germany tested a number of estimation methods against the measured values for 56 contemporary European vehicles. The original data has been lost.[7] The work of Rasmussen[8] (1970) at General Motors has been superseded by later researchers, and will not be included in this paper. Reide[9] (1984) tested a large number of passenger vehicles sold commercially "somewhere in the world." The data was not included in the published work. Curzon[10] (1991) measured the inertial properties of more than 100 passenger cars (not used in his study), sport utility vehicles, pickup trucks, and vans, but has not published the data. Bixel et al.[11] (1996)) measured the parameters for a total of 313 vehicles. The data was not included with their paper.

There has been work done by others as well, but the NHTSA work is the best available current source for measured values.

ESTIMATION TECHNIQUES

A number of methods have been proposed to estimate the moments of inertia, and in some cases the height of the center of gravity. These are listed here in chronological order.

Grime[12] (1969) proposed the prism method described previously. The work of Rasmussen[13] (1970) at General Motors has been superseded by later researchers, and will not be included in this paper.

mab -- The most commonly cited estimation of the yaw moment has been with us for some time. The original source is uncertain. It has been recently cited and derived by Bastow[14]:

$$I_{zz} = mab \tag{9}$$

This method is intuitively satisfying in that it assumes the vehicle mass is concentrated at the center of the axles.

Burg -- Burg (1982) tested a number of methods against the measured values for 56 European vehicles and found that the method with the best correlation coefficient for the yaw moment for his sample was:

$$I_{zz} = 0.1269mlL \tag{10}$$

The Burg method has also been used by Reimpell.[15]

Reide -- Researchers at General Motors (1984) built on the earlier work by Rasmussen, testing a number of passenger vehicles "sold commercially somewhere in the world." No truck or bus types were included. The author describes the data as being applicable to moderate payload and driving conditions.

Body styles ranged from 2-passenger sports cars to station wagons, masses from 575 to 1761 kg, and wheelbases from 2025 to 2953 mm. Nine were front engine-rear drive, six were front engine-front drive, and two were rear engine-rear drive, for a total of 17 cars. They were tested in curb condition, no passengers and full gas tank.

The best fit for the whole vehicle moments of inertia in roll, pitch, and yaw, relative to the total vehicle centroidal axes, were:

$$I_{xx} = 0.37m - 86.4\,,\ R^2 = 0.893 \tag{11}$$

$$I_{yy} = 2.56m - 1{,}103\,,\ R^2 = 0.919 \tag{12}$$

$$I_{zz} = 2.86m - 1{,}315\,,\ R^2 = 0.920 \tag{13}$$

The sprung mass moments of inertia relative to the sprung mass centroidal axes were also given. No techniques for estimation of center of gravity heights were given.

Allen – Allen (1987) et al[16] proposed the rule of thumb values for the yaw moment as follows:

$$I_{zz} = mab \pm 10\% \tag{14}$$

The method proposed by Allen for the roll moment is a variant of the prism method:

$$I_{xx} = \frac{1}{12} m\left(W^2 + h^2\right) \tag{15}$$

Where,

$$h^2 = \alpha_1 h_1^{\,2} + \alpha_2 h_2^{\,2} \tag{16}$$

$$\alpha_1 + \alpha_2 = 1 \tag{17}$$

The values α_1 and α_2 shown on Figure 4 are weighting coefficients which consider the proportions of the vehicle at various heights. Allen suggests that this method of determining the roll moment is accurate to within ± 20%. However, the dimensional data to use this technique, α_1, α_2, h_1, and h_2 are not usually available in the tested or measured databases. These data are only available if the case vehicle or an exemplar is available to measure.

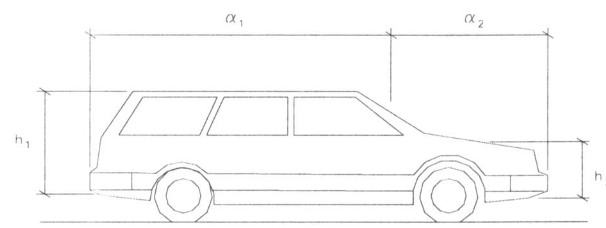

Figure 4 - The vehicle with Averaged Height

The center of gravity height is given as:

$$h_{cg} = 0.38h_{roof} \pm 5\% \tag{18}$$

Allen has offered no recommendations for the pitch moment.

Garrott -- The most complete listing of measured parameters was published by Garrott in 1988. In addition to the values measured, the authors presented a number of historic "rules of thumb", and proposed a set of revised rules of thumb (all units have been converted to metric) which are listed in Table 1.

Rules of Thumb:

$$h_{cg} = 533.4\ mm \tag{19}$$

$$h_{cg} = 0.40h_{roof} \tag{20}$$

$$I_{xx} = \frac{mt_{av}h_r}{4} \tag{21}$$

$$I_{yy} = mab \tag{22}$$

$$I_{zz} = I_{yy} \tag{23}$$

Table 1 - Garrott's Revised Rules of Thumb

Pass Cars	Light Trucks	
$h_{cg} = 540.8 \pm 38.1\ mm$	$h_{cg} = 678.4 \pm 101.6\ mm$	(24)
$h_{cg} = 0.395h_{roof} \pm 2.6\%$	$h_{cg} = 0.387h_{roof} \pm 3.5\%$	(25)
$I_{xx} = (0.73 \pm 0.13)\frac{mt_{av}h_r}{4}$	$I_{xx} = (0.67 \pm 0.16)\frac{mt_{av}h_r}{4}$	(26)
$I_{yy} = (1.07 \pm 0.17)mab$	$I_{yy} = (1.04 \pm 0.22)mab$	(27)
$I_{zz} = (1.03 \pm 0.08)I_{yy}$	$I_{zz} = (1.00 \pm 0..1)I_{yy}$	(28)

Garrott also fitted curves through the measurement data, and presented the moment of inertia parameters for passenger cars and light trucks as a function of vehicle mass.

Passenger Cars:

$$I_{xx} = 0.538m - 203\,,\ R^2 = 0.80 \qquad (29)$$

$$I_{yy} = 2.96m - 1{,}558\,,\ R^2 = 0.89 \qquad (30)$$

$$I_{zz} = 3.08m - 1{,}635\,,\ R^2 = 0.88 \qquad (31)$$

Light Trucks:

$$I_{xx} = 0.66m - 319\,,\ R^2 = 0.70 \qquad (32)$$

$$I_{yy} = 3.35m - 2{,}247\,,\ R^2 = 0.70 \qquad (33)$$

$$I_{zz} = 3.08m - 1{,}821\,,\ R^2 = 0.73 \qquad (34)$$

Curzon -- Later researchers developed the work of Garrott further. Curzon and colleagues (1991) measured the inertial properties of sport utility vehicles, pickup trucks, and vans, and developed a number of moment of inertia estimation techniques applicable to most light trucks. The authors devised a number of estimation equations and tested them against the sum total of light trucks tested by their company and also those tested by Garrott.

They found the prism method was the best estimate for the yaw moment for sport utility vehicles, using the track width instead of the vehicle width.

$$I_{zz} = \frac{1}{12} m\left(L^2 + t^2\right) \qquad (35)$$

For the yaw moment of pickup trucks, the rule of thumb was best:

$$I_{zz} = mab \qquad (36)$$

For the yaw moment for vans, they recommend either the prism method as for sport utility vehicles, or Garrott's value for light trucks:

$$I_{zz} = 3.08m - 1{,}821 \qquad (37)$$

The roll moment was best approximated by a sum of prisms, similar to the method first proposed by Allen, that account for the varying cross sections of the vehicles, as shown in Figures 5, 6, and 7.

The recommended relationship for the roll moment for sport utility vehicles is as follows:

$$I_{xx} = \frac{1}{12} m\left(\frac{L_r h_r^2}{L} + W^2\right) + \frac{1}{12} m\left(1 - \frac{L_r}{L}\right)(h_w - h_a)^2 \qquad (38)$$

Like the earlier work by Allen, the roof length is required to use this technique. The roof length is not usually available in the tested or measured databases. This data is only available if the case vehicle or an exemplar is available to measure.

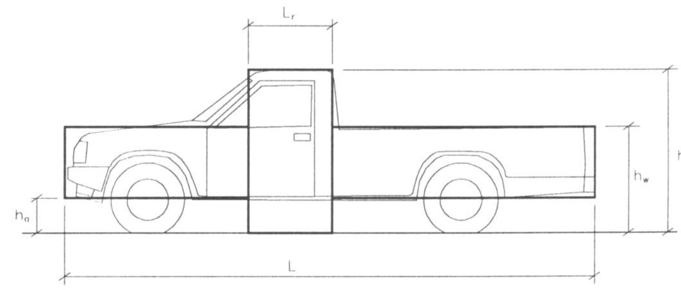

Figure 5 - Prism Models for Pickup Truck

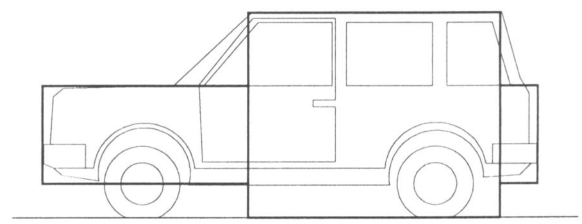

Figure 6 - Prism Models for Sport Utility

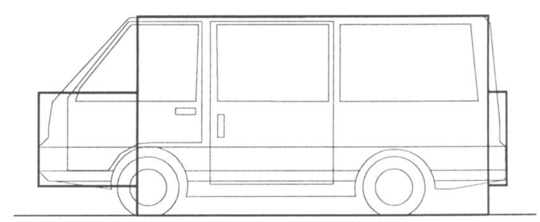

Figure 7 - Prism Models for Van

For the roll moment for vans, Curzon recommends either the method above for sport utility vehicles, or Garrott's value for light trucks:

$$I_{xx} = 0.66m - 319 \qquad (39)$$

Curzon offered no recommendations for the pitch moment or the center of gravity height.

Noon -- Noon[17] (1994) has proposed three methods to estimate the yaw moment. These methods for convenience have been called Noon-1, Noon-2, and Noon-3. The first method has the moment being a function of the mass and the overall length:

$$I_{zz} = \frac{mL^2}{12} \qquad \text{Noon-1} \qquad (40)$$

The second has the moment being a function of the mass and the wheelbase alone, assuming the masses are concentrated at half the wheelbase from the geometric center:

$$I_{zz} = \frac{ml^2}{4} \qquad \text{Noon-2} \qquad (41)$$

Noon's third method, considering the engine to be the most significant concentrated mass affecting the application of the prism method, accounts for the effect of the engine mass with a combination of methods one and two:

$$I_{zz} = 2(1-x)\frac{mL^2}{12} + (2x-1)m\frac{l^2}{4} \qquad \text{Noon-3} \quad (42)$$

where,

$x =$ fraction of vehicle mass borne by front axle

This third method assumes that the difference between the weights on the axles is due to a point mass at the heavier axle, and that the centroid is at the geometric center. Although contradictory, these assumptions are justified for an estimation technique.

Noon offered no recommendations for the pitch or roll moments or the center of gravity height.

Bixel – Bixel (1996) measured the parameters for 104 passenger cars, 84 multi-purpose vehicles, 82 pickup trucks, and 43 vans,.and recommended a number of estimation techniques. They found the roll moment for all vehicles can be estimated as:

$$I_{xx} = \frac{m(h_{roof} + h_{cg})t}{K} \qquad (43)$$

Where K is an approximation constant determined for each class of vehicle.

The pitch moment for passenger cars can be estimated as:

$$I_{yy} = \frac{m(h_{roof} + h_{cg})L}{K} \qquad (44)$$

The pitch moment for multi-purpose vehicles, pickup trucks and vans can be estimated as:

$$I_{yy} = \frac{m(h_{roof} + h_{cg})l}{K} \qquad (45)$$

The yaw moment for all vehicles can be estimated as:

$$I_{zz} = \frac{mtl}{K} \qquad (46)$$

All of the methods proposed by Bixel except yaw use the center of gravity height, a quantity that is not generally available on published databases.

The Bixel K factors are summarized in Table 2. Bixel notes that these approximation constants must evolve with evolving vehicle design.

Table 2 - Bixel's Approximation Constants (K)

Vehicle Class	Pitch	Roll	Yaw
Passenger Cars	5.2901	7.9846	2.1942
Sport Utility, l<2.41 m	4.2193	9.4212	2.2048
Sport Utility, l>2.41 m	3.4510	9.4212	2.2048
Pickups	3.3783	9.4738	2.1858
Vans	3.4734	7.8854	2.2168

COMPUTER SIMULATION PROGRAMS

A number of computer programs are available to simulate the behavior of motor vehicles in collision and spin-out. The most common products are the crush-energy based progeny of the original Crash III program, EDCRASH[18] and SLAM[19], and EDSMAC[20] based on the early work of McHenry.[21] More recently PC-Crash[22], a collision and trajectory program developed by Dr. Steffan,[23] has become available.

EDCRASH and EDSMAC use a default value for the yaw moment of inertia derived from the vehicle class categories. These values can be altered by the user.

WinCRASH, the Windows version of the SLAM program, uses a default value for the radius of gyration, k, that can be used to calculate the yaw moment:

$$k_{zz} = (1.023 \pm 0.085)\sqrt{ab} \qquad \text{Slam-1} \quad (47)$$

Alternatively WinCRASH proposes:

$$k_{zz} = (0.298 \pm 0.030)L \qquad \text{Slam-2} \quad (48)$$

When expressed as the moment of inertia, the WinCRASH values are used as follows:

$$I_{zz} = mk^2 \qquad (49)$$

The WinCRASH default value can be altered by the user.

PC-Crash uses the method of Burg for the default yaw moment of inertia for passenger cars:

$$I_{zz} = 0.1269mlL \qquad (50)$$

PC-Crash uses the roll and pitch moments as functions of the Burg yaw moment:

$$I_{xx} = 0.3I_{zz} \qquad (51)$$

$$I_{yy} = I_{zz} \qquad (52)$$

For trucks and trailers, PC-Crash recommends the prism method. Because the height of a truck is similar to the width, PC-Crash reduces the roll, pitch, and yaw moments to:

$$I_{xx} = \frac{2}{12}mW^2 \qquad (53)$$

$$I_{yy} = \frac{1}{12} m\left(L^2 + W^2\right) \quad (54)$$

$$I_{zz} = I_{yy} \quad (55)$$

The PC-Crash default values can be altered by the user.

COMPARISON OF MEASURED VALUES WITH ESTIMATED VALUES

The NHTSA has measured the moment of inertia for a large number of vehicles, as described by Garrott. These values, for about 414 tests, are published and available on disk. Many of the tests were performed with multiple occupants and significant ballast to load the vehicles to the GVW. About 215 of the tests with multiple occupants or ballast appear to be repeats of tests done on the same vehicle with one or no occupants. Therefore, vehicles loaded with more than one occupant and those carrying loads have been eliminated from the samples used here. Also eliminated were multiple tests involving identical vehicles so as not to weight the sample set toward any particular vehicle.

The comparisons have been done by dividing the vehicles into five categories, as shown on Table 3, where n is the number of vehicles in a category sample. In each case the measured pitch, roll, and yaw moments of inertia and the center of gravity height are compared with the estimated values. The estimation methods compared are given in Table 4.

Table 3 - Vehicle categories for comparison

Vehicle Category	n
Passenger Car - front wheel drive	46
Passenger Car - rear wheel drive	24
Sport Utility (Multi-Purpose)	22
Pickup Truck	25
Van	17

The estimated/measured values for the 46 front wheel drive passenger cars was tested to confirm it had a nearly normal distribution. Although some of the data is skewed, the mean, median, and mode are generally in agreement.

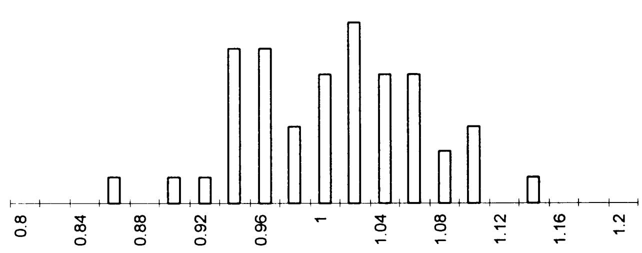

Figure 8 - Histogram of Estimated/Measured Values for FWD Passenger Cars

A histogram for the prism pitch data is as shown on Figure 8. This data set had a mean of 1.00, mode of 0.94, median of 1.01 and a skew of -0.04.

The distribution of the errors about the median shown on Figure 8 is approximately bell-shaped, thus standard statistical tests of the quality of the data will be used.

Table 4 - Estimation Methods Compared

Estimation Method	I_{xx}	I_{yy}	I_{zz}	h_{cg}
Prism	✓	✓	✓	
Burg/PC Crash	✓	✓	✓	
Reide	✓	✓	✓	
Allen				✓
Garrott	✓	✓	✓	
Rule of Thumb	✓	✓	✓	✓
Garrott's Revised Rule of Thumb	✓	✓	✓	✓
Rule of Thumb 2				✓
Garrott's Revised Rule of Thumb 2				✓
Noon 1			✓	
Noon 2			✓	
Noon 3			✓	
Bixel	✓	✓	✓	
Slam 1			✓	
Slam 2			✓	

For each vehicle in the comparison category, the estimated value has been compared to the measured value in the following way. First the individual error, E, of each estimated value compared to its measured value is calculated as:

$$E = \frac{V_e}{V_m} - 1 \quad (56)$$

Where:

V_e = estimated value

V_m = measured value

The average error, E_{ave}, is:

$$E_{ave} = \frac{\pm \sum_{1}^{n} E}{n} \quad (57)$$

Where:

n = number of vehicles in category sample

The range, R, of the errors is:

$$R = E_{\max} - E_{\min} \quad (58)$$

Where:

$E_{\max}$ = maximum positive error in sample category

$E_{\min}$ = minimum positive error in sample category

The standard deviation, SD, is:

$$SD = \sqrt{\frac{1}{n-1}\sum\left(E - E_{ave}\right)^2} \qquad (59)$$

The 68% confidence limit, L_{68}, is

$$L_{68} \cong E_{ave} \pm SD \qquad (60)$$

The 95% confidence limit, L_{95}, is;

$$L_{95} \cong E_{ave} \pm 2SD \qquad (61)$$

All the estimation methods were adjusted with a correction factor as follows:

$$V_{Correct} = CV_e \qquad (62)$$

Where the correction factor, C, is:

$$C = \frac{1}{E_{ave} + 1} \qquad (63)$$

The adjusted equations were then compared, and the best estimation equations were chosen based on the smallest standard deviation. In the event of a tie, the equation with the narrowest range was chosen.

Figure 9 shows the unadjusted values for the yaw moment of inertia for rear wheel drive passenger vehicles. The equation numbers are shown in brackets.

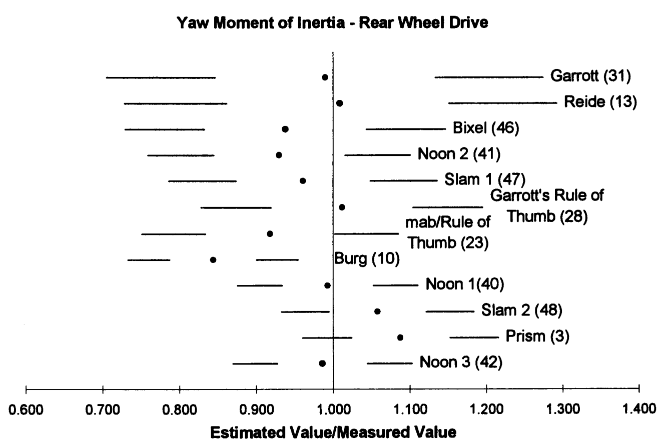

Figure 9 - Unadjusted Estimates

Figure 10 shows the adjusted values for the yaw moment of inertia for rear wheel drive passenger vehicles. The error bands on each graph connect the standard deviation to the 95th percent confidence limit, as on Figure 9. Note that the error bands for each equation have been adjusted as well.

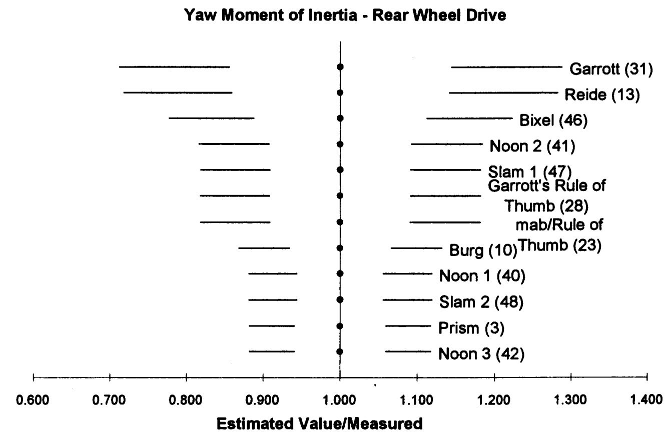

Figure 10 - Adjusted Estimates

RECOMMENDED ESTIMATION TECHNIQUES

The tables in Appendix B provide the correction factors applied to each of the estimation equations as well as statistical information indicating their accuracy. The equation numbers for each estimation technique are shown in brackets.

Each table is accompanied by a chart which graphically ranks the equations as described previously. The equations providing the smallest range of standard deviation error (68% confidence limit) are given in Tables 5 to 9. Note that these equations have been multiplied through by their individual correction factors.

If the information required to use a recommended equation is not available, another equation may be used with its correction factor taken from the tables in Appendix B. The user must then be aware of the slightly greater error being introduced.

When adjusted, it was found that Garrott's revised rules of thumb provided identical results to the original rules of thumb. This was expected as Garrott's revised rules are the result of multiplying the original rules by a constant. Allen's adjusted equation for center of gravity height also provided identical results to those of the second rule of thumb (equation 20) for the same reason.

The inertia estimation methods proposed by Garrott and Reide had greater deviations than the other methods. Both the Reide and Garrott methods consider the moments of inertia to be linearly related to the vehicle mass and do not consider the vehicle geometry.

The only available body of moment of inertia data for European vehicles has been given in the Burg study. No guidance can be offered for those European vehicles not included in the NHTSA data used for this comparison study.

Table 5 - Best Equations for Front Wheel Drive Passenger Cars

	Adjusted Equation
I_{xx}	$0.1274m(h_{roof} + h_{cg})t \pm 6.5\%$
I_{yy}	$0.1425mlL \pm 5.7\%$
I_{zz}	$0.1478mlL \pm 4.8\%$
h_{cg}	535.0 mm ± 3.0%

Table 6 - Best Equations for Rear Wheel Drive Passenger Cars

	Adjusted Equation
I_{xx}	$0.120m(h_{roof} + h_{cg})t \pm 9.5\%$
I_{yy}	$0.0733m\left(L^2 + h^2\right) \pm 5.2\%$
I_{zz}	$1.015\left[2(1-x)\frac{mL^2}{12} + (2x-1)m\frac{l^2}{4}\right] \pm 5.9\%$
h_{cg}	$0.390h_{roof} \pm 4.9\%$

Table 7 - Best Equations for Sport Utility Vehicles

	Adjusted Equation
I_{xx}	$0.145mt_{av}h_r \pm 7.6\%$
I_{yy}	$0.1561mlL \pm 5.5\%$
I_{zz}	$0.4622mtl \pm 6.7\%$
h_{cg}	$0.390h_{roof} \pm 5.4\%$

Table 8 - Best Equations for Pickup Trucks

	Adjusted Equation
I_{xx}	$0.138mt_{av}h_r \pm 7.4\%$
I_{yy}	$0.952mab \pm 5.2\%$
I_{zz}	$0.958mab \pm 4.6\%$
h_{cg}	$0.376h_{roof} \pm 3.5\%$

Table 9 - Best Equations for Vans

	Adjusted Equation
I_{xx}	$0.1212m(h_{roof} + h_{cg})t \pm 8.0\%$
I_{yy}	$0.1508mlL \pm 7.4\%$
I_{zz}	$0.1525mlL \pm 7.4\%$
h_{cg}	$0.381h_{roof} \pm 2.8\%$

EFFECT OF VEHICLE LOADING

The comparisons have been done with the NHTSA tests of unloaded vehicles (no more than one occupant). If the passenger and loading mass and distribution is significant, the estimated values must be adjusted.

Occupant positioning in a passenger car is relatively close to the radius of gyration about the centroid, thus the increase in the moments of inertia due to the passenger loading will generally be proportional to the increase in the mass.

If there is a known concentrated mass in a vehicle, due to occupant positioning or cargo loading remote from the radius of gyration, then the moments can be summed using the parallel axis theorem:

$$I' = I + mr^2 \tag{64}$$

Arndt[24] has identified the effect of passenger and cargo loading on the inertial properties, and provided methods of adjusting the estimated (or measured) moments for specific loading conditions.

SENSITIVITY ANALYSIS

The pitch and roll behavior of the vehicles in 3D simulation will be affected by the respective moments during collision and once the vehicles have separated. The inertial parameter with the greatest potential to influence a simulation is the yaw moment. Variations in the yaw moment of inertia will affect the forces developed between the two vehicles in collision and the immediate post-impact spin rate. When the vehicles have separated, the angular acceleration will be governed to a great extent by the yaw moment. A greater yaw moment of inertia will reduce the yaw angular acceleration when compared to the case of lesser yaw moment.

A test case has been selected to demonstrate the effect of variations in the yaw moment of inertia. The case uses a highly eccentric collision to maximize the sensitivity of the model to the yaw moment. The case uses a 1991 Ford Mustang and a 1989 Honda Accord, both arbitrarily chosen. The vehicle parameters used in PC-Crash are shown on Table 10.

The eastbound Mustang struck the right side at the rear of the southbound Accord, with both vehicles traveling at 50 km/h. The baseline run arbitrarily uses the moments of inertia calculated with the Burg method.

Both vehicles were permitted to spin out to rest. For the Mustang, the front wheels were locked at the instant of collision, and the rear wheels set to a drag of 5%. The Accord rear wheels were locked at impact, and the front wheels set at 5% drag. There was no steering introduced to simulate rolling damaged wheels. The force location at maximum engagement was set at the center of the overlap area (45 ms after contact), and the contact plane was set parallel to the longitudinal axis of the Accord. The collision coefficient of restitution was set at 0.1, and the inter-vehicle coefficient of friction was set at 1.00.

Table 10 - Test Vehicles

Parameter	Description	V1	V2
	Year & Make	91 Ford	89 Honda
	Model	Mustang	Accord
L	Length, m	4.56	4.57
W	Width, m	1.73	1.71
l	Wheelbase, m	2.55	2.60
h	Height, m	1.32	1.35
t	Track width, m	1.42	1.48
m	mass, kg	1270	1210
a	CG to front axle, m	1.07	0.96
I_{xx}	roll, kg-m^2	562	547
I_{yy}	pitch, kg-m^2	1874	1824
I_{zz}	yaw, kg-m^2	1874	1824
h_{cg}	CG Height	0.53	0.54
V	Speed, km/h	50	50

These simulation parameters are not intended to be representative of any particular vehicle or collision circumstance. The case was selected to illustrate the sensitivity of the collision to variance of the one variable, the yaw moment of inertia.

The yaw moment was varied from 70% to 130% of the default value. The altered values for the yaw moment of inertia are as shown in Table 11.

Table 11 - Altered Yaw Moment Values, kg-m^2

Description	Mustang	Accord
70% Value	1312	1277
80% Value	1499	1459
90% Value	1687	1642
100% Value	1874	1824
110% Value	2061	2006
120% Value	2249	2189
130% Value	2436	2371

First, the simulation was run with the default values. This run is shown graphically in Figure 11. The tire marks are drawn to simulate the actual tire marks assuming tire marks are visible if the tires are 95% saturated.

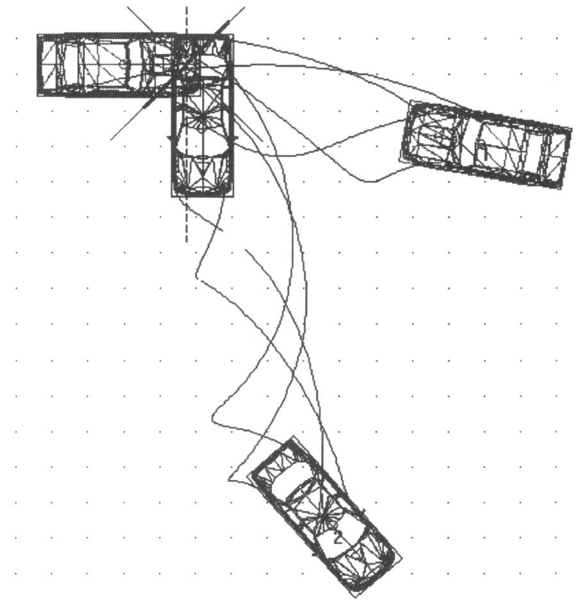

Figure 11 - Simulation with Default Yaw Moments

Next, runs were done varying only the yaw moment of inertia. The run with the yaw moment set to 130% is shown in Figure 12. The DXF drawing showing the rest positions and the tire marks for the run at 100% is plotted for comparison. The 100% run rest positions are shown as rectangles beneath the vehicles. The remainder of the runs are shown in Appendix C.

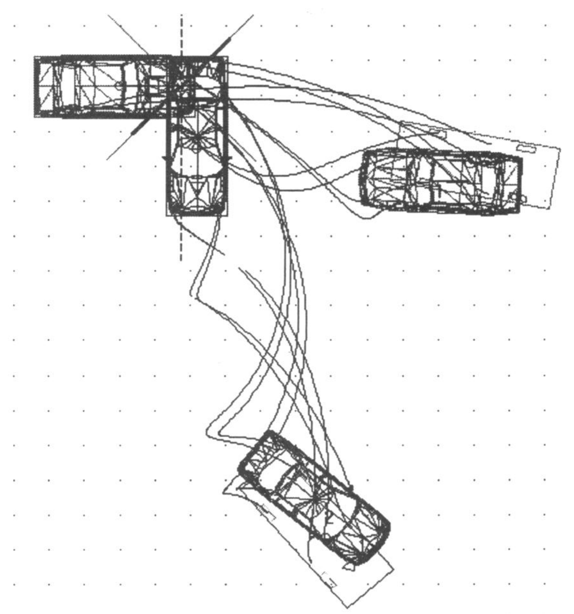

Figure 12 - Simulation with Yaw Moments increased to 130%

Finally, the 130% run was redone, varying the impact speeds and the orientation at impact to duplicate as closely as possible the rest positions for the default moments. The impact speeds were adjusted to 51.5 km/h (+3%) on the Mustang and 51 km/h (+2%) for the Accord and the Mustang was repositioned about 22 cm north at impact. This run is shown on Figure 13.

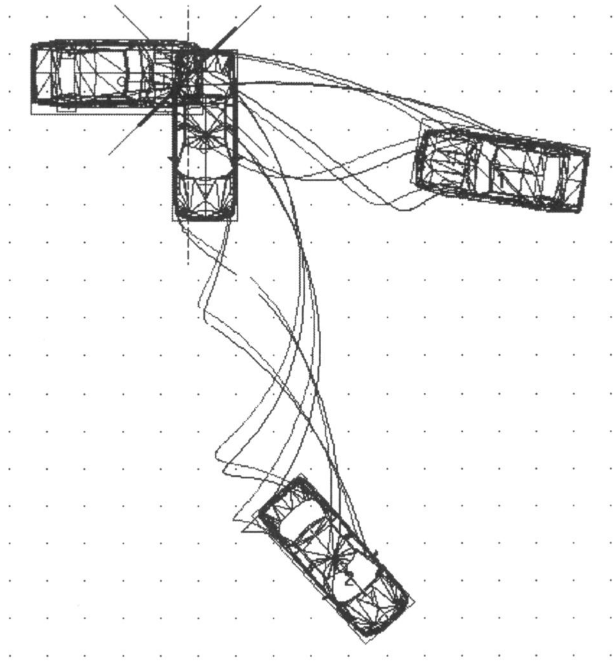

Figure 13 - 130% Simulation varied to match rest positions for 100% case

Note that although the rest positions are similar to the default case in Figure 11, the tire traces are quite different.

Runs were done to test the sensitivity of the simulation to variations in the pitch and roll moments, and the centroid height. As expected, the simulation was not very sensitive to variation in these parameters.

The simulations were very sensitive to variations in the wheel lockup factors, and steer angles on individual tires. The rest positions were more sensitive to these factors than to the yaw moments.

SUMMARY

The values measured by Garrott of the NHTSA have been compared to the values determined by the estimation techniques in five different vehicle categories. Correction factors were applied to each of the estimation equations to eliminate the average error. The adjusted equations were ranked by standard deviation, and the best equations, those with the smallest standard deviation, were chosen and presented.

The recommended equations will predict the moments of inertia and the centroid height parameters of the Garrott sample to better than ±10% with 68% confidence. The 95% confidence limit for the yaw moment was less than ±10% for front wheel drive cars and pickups, and less than ±15% for the other vehicle categories.

In some cases the differences between the recommended equations and the next best equations were very small. Where the dimensional data necessary to estimate the parameters with a recommended equation are not available, alternative equations are presented, with the expected errors given.

The inertia estimation methods proposed by Garrott and Reide consider the moments of inertia to be linearly related to the vehicle mass and do not consider the vehicle geometry. These methods generally produced greater errors than the other estimation methods.

Simulations of a highly eccentric collision and spin-out with PC-Crash have shown that if the estimated values for the yaw moment of inertia are within about ±10% of the actual values, the impact speeds in the simulation will vary less than about 1%. Variations in pitch and roll moments have been shown to have little influence on the calculated vehicle speeds for the simulated 3D planar cases.

If the selected yaw moment of inertia is greater than the actual value by about 30%, the simulation will tend to overestimate the collision speed of the vehicles by as much as 3%. If the selected yaw moment of inertia is less than the actual value by about 30%, the simulation will tend to underestimate the collision speed.

If the estimation method has 95% confidence limits greater than ±30% for the yaw moment, as could occur if the estimation method is based on mass alone, then the simulation errors could become significant.

The simulations indicate the rest positions of vehicles are more sensitive to wheel lockup factors and steer angle than to variations in moments of inertia.

The idealized vehicle prism method and "mab" rule of thumb have often yielded very good results.

This work is based on the measurement data of Garrott and the NHTSA. Others, notably Curzon and Bixel, have measured a number of vehicles. If these data were available, the results may vary.

APPENDICES

The following appendices are attached:

A. Nomenclature

B. Tables and charts of adjusted equations

C. PC-Crash runs

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APPENDIX A -- NOMENCLATURE

The following nomenclature will be used throughout:

I = moment of inertia
L = overall vehicle length
l = wheelbase
W = overall vehicle width
h = height
t = track width
a = distance center of mass to front axle
b = distance center of mass to rear axle
T = torque
ω = angular velocity
α = angular acceleration
τ = period of oscillation
k = spring constant
or radius of gyration
r = radius

Subscripts have been used as appropriate, and the meaning will generally be self-evident. The nomenclature of some cited authors has been altered for consistency. Where necessary, the units used in older cited works have been converted to metric.

Table A1 - Metric/US Conversions

1 lb force	4.448 N
1 in	25.4 mm
1 lb-ft-sec^2	1.356 kg-m2

The SAE frame of reference shown in Figure A1 is used:

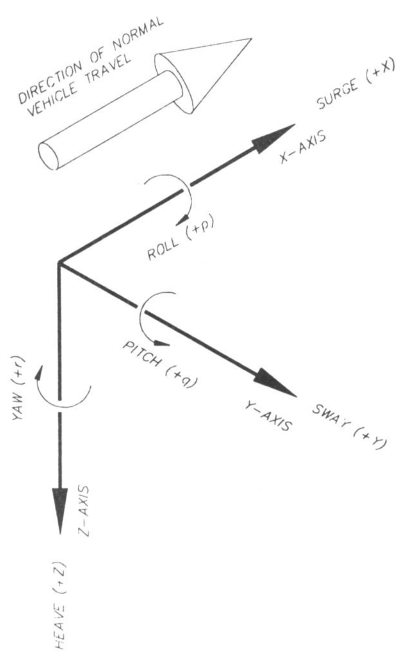


Figure A1 - The SAE frame of reference

Another common frame of reference, shown in Figure A2, has the positive direction for the Z axis directed upwards. The standard instruction in engineering schools, for example, has the X axis pointing to the right with angles about Z measured counterclockwise from the X-axis.

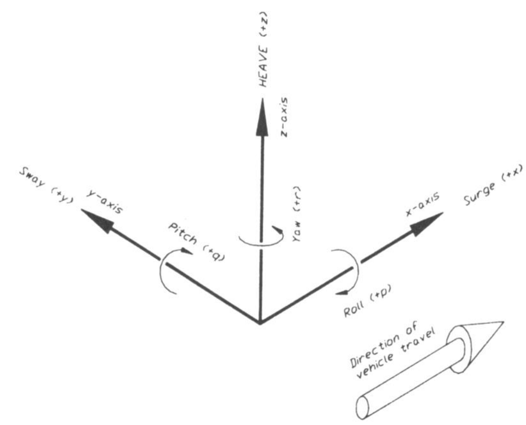


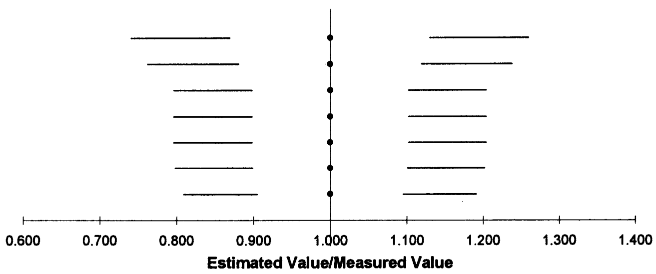
Figure A2 - A Common frame of reference

For both frames the positive angles are defined using the right hand rule.

APPENDIX B - ACCURACY OF CORRECTED EQUATIONS

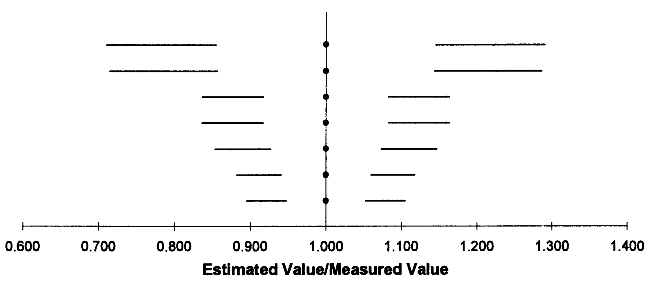
Rear Wheel Drive Passenger Cars

A) Roll Moment of Inertia



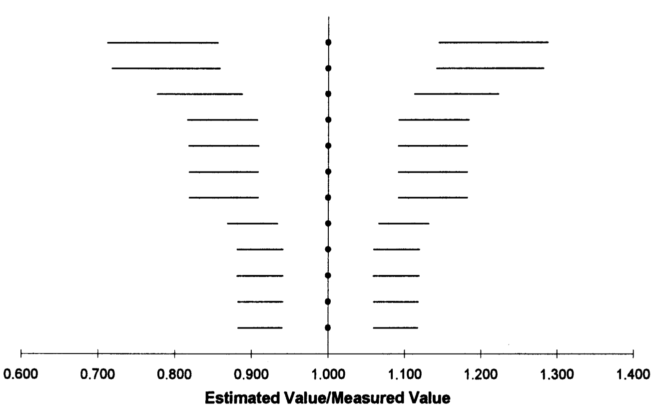
Method	Correction Factor	Range	Standard Deviation	95% Limit
Reide (11)	1.092	0.520	0.130	0.260
Garrott (29)	0.872	0.486	0.119	0.238
Prism (1)	1.000	0.497	0.102	0.204
Garrott's Rule of Thumb (26)	0.916	0.490	0.102	0.204
Rule of Thumb (21)	0.669	0.490	0.102	0.204
PC Crash (51)	0.717	0.440	0.101	0.202
Bixel (43)	0.960	0.433	0.095	0.191

B) Pitch Moment of Inertia



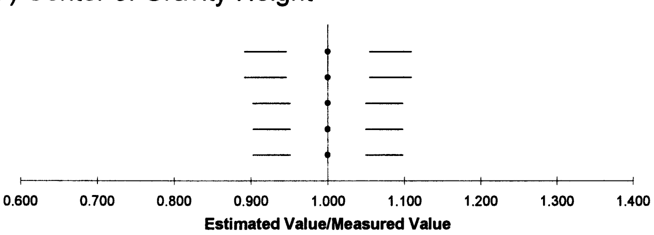
Method	Correction Factor	Range	Standard Deviation	95% Limit
Reide (12)	1.015	0.620	0.145	0.290
Garrott (30)	0.992	0.592	0.143	0.286
Garrott's Rule of Thumb (27)	0.968	0.273	0.082	0.164
Rule of Thumb (22)	1.035	0.273	0.082	0.164
Bixel (44)	1.026	0.250	0.073	0.147
PC Crash (52)	1.126	0.256	0.059	0.118
Prism (2)	0.880	0.201	0.052	0.105

C) Yaw Moment of Inertia



Method	Correction Factor	Range	Standard Deviation	95% Limit
Garrott (31)	1.010	0.608	0.144	0.288
Reide (13)	0.991	0.583	0.141	0.282
Bixel (46)	1.066	0.474	0.112	0.223
Noon 2 (41)	1.076	0.319	0.092	0.184
SLAM 1 (47)	1.041	0.315	0.091	0.182
Garrott's Rule of Thumb (28)	0.988	0.315	0.091	0.182
mab/Rule of Thumb (23)	1.089	0.315	0.091	0.182
Burg/PC Crash (10)	1.185	0.264	0.066	0.132
Noon 1 (40)	1.007	0.242	0.059	0.119
SLAM 2 (48)	0.945	0.242	0.059	0.119
Prism (3)	0.919	0.252	0.059	0.118
Noon 3 (42)	1.015	0.221	0.059	0.118

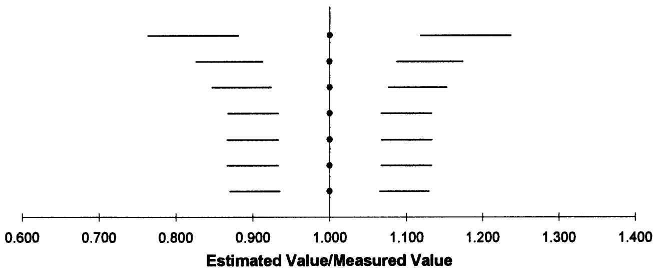
D) Center of Gravity Height



Method	Correction Factor	Range	Standard Deviation	95% Limit
Garrott's Rule of Thumb 1 (24)	0.996	0.202	0.054	0.109
Rule of Thumb 1 (19)	1.010	0.202	0.054	0.109
Allen (18)	1.027	0.265	0.049	0.098
Garrott's Rule of Thumb 2 (25)	0.988	0.265	0.049	0.098
Rule of Thumb 2 (20)	0.976	0.265	0.049	0.098

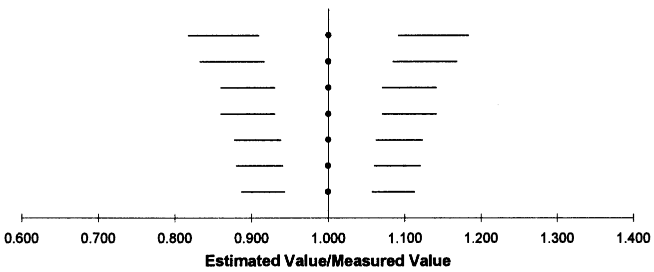
Front Wheel Drive Passenger Cars

A) Roll Moment of Inertia



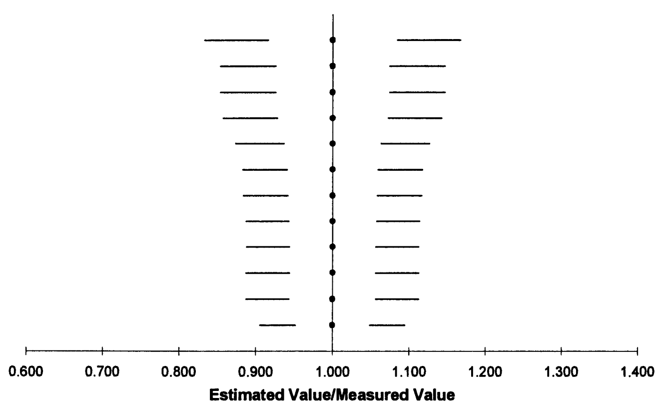
Method	Correction Factor	Range	Standard Deviation	95% Limit
PC Crash (51)	0.822	0.425	0.118	0.237
Garrott (29)	0.961	0.336	0.087	0.174
Reide (11)	1.177	0.309	0.076	0.153
Garrott's Rule of Thumb (26)	0.971	0.331	0.067	0.134
Rule of Thumb (21)	0.709	0.331	0.067	0.134
Prism (1)	1.061	0.329	0.067	0.133
Bixel (43)	1.017	0.321	0.065	0.130

B) Pitch Moment of Inertia



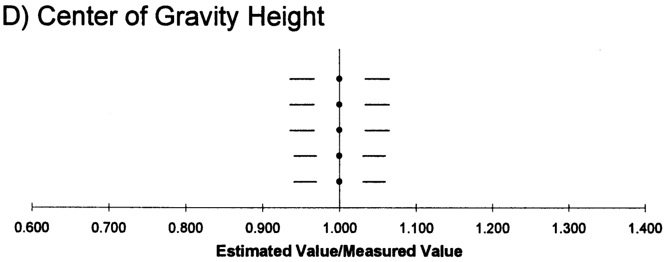
Method	Correction Factor	Range	Standard Deviation	95% Limit
Garrott (30)	0.985	0.377	0.091	0.183
Reide (12)	0.984	0.362	0.084	0.168
Garrott's Rule of Thumb (27)	0.986	0.297	0.070	0.141
Rule of Thumb (22)	1.055	0.297	0.070	0.141
Bixel (44)	0.998	0.289	0.062	0.123
Prism (2)	0.896	0.293	0.060	0.120
PC Crash (52)	1.123	0.247	0.057	0.113

C) Yaw Moment of Inertia



Method	Correction Factor	Range	Standard Deviation	95% Limit
Garrott (31)	0.989	0.316	0.084	0.167
Noon 1 (40)	1.021	0.319	0.074	0.147
SLAM 2 (48)	0.958	0.319	0.074	0.147
Reide (13)	0.953	0.301	0.072	0.143
Prism (3)	0.922	0.279	0.063	0.127
Bixel (46)	0.991	0.331	0.059	0.118
Noon 3 (42)	1.023	0.267	0.058	0.117
Noon 2 (41)	1.025	0.233	0.057	0.113
SLAM 1 (47)	1.046	0.233	0.056	0.113
Garrott's Rule of Thumb (28)	0.993	0.233	0.056	0.113
mab/Rule of Thumb (23)	1.095	0.233	0.056	0.113
Burg/PC Crash (10)	1.165	0.204	0.048	0.095

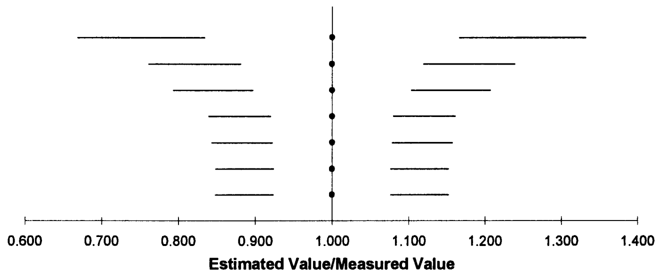
D) Center of Gravity Height



Method	Correction Factor	Range	Standard Deviation	95% Limit
Allen (18)	1.027	0.136	0.033	0.065
Garrott's Rule of Thumb 2 (25)	0.988	0.136	0.033	0.065
Rule of Thumb 2 (20)	0.976	0.136	0.033	0.065
Garrott's Rule of Thumb 1 (24)	0.989	0.146	0.030	0.060
Rule of Thumb 1 (19)	1.003	0.146	0.030	0.060

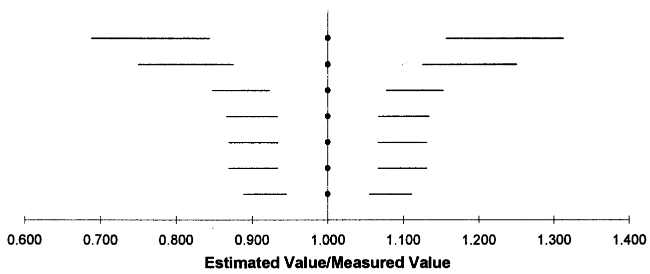
Sport Utility Vehicles

A) Roll Moment of Inertia



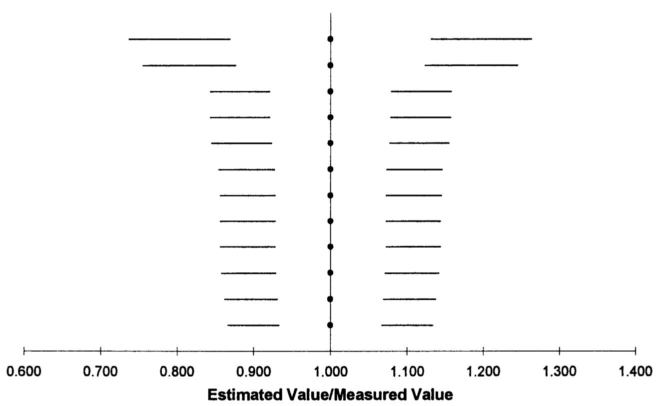
Method	Correction Factor	Range	Standard Deviation	95% Limit
PC Crash (51)	0.893	0.637	0.166	0.332
Garrott (32)	0.786	0.425	0.119	0.239
Reide (11)	1.143	0.379	0.103	0.207
Prism (1)	0.858	0.283	0.080	0.161
Bixel (43)	0.983	0.293	0.078	0.157
Garrott's Rule of Thumb (26)	0.867	0.273	0.076	0.152
Rule of Thumb (21)	0.581	0.273	0.076	0.152

B) Pitch Moment of Inertia



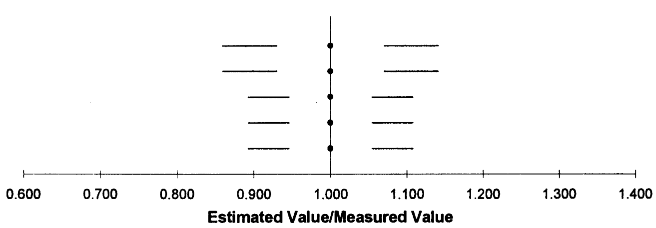
Method	Correction Factor	Range	Standard Deviation	95% Limit
Reide (12)	0.870	0.720	0.156	0.312
Garrott (33)	0.850	0.560	0.125	0.250
Bixel (45)	1.017	0.308	0.077	0.153
Prism (2)	0.959	0.238	0.067	0.134
Garrott's Rule of Thumb (27)	1.002	0.269	0.066	0.131
Rule of Thumb (22)	1.042	0.269	0.066	0.131
PC Crash (52)	1.228	0.182	0.055	0.111

C) Yaw Moment of Inertia



Method	Correction Factor	Range	Standard Deviation	95% Limit
Reide (13)	0.828	0.601	0.131	0.263
Garrott (34)	0.875	0.523	0.123	0.245
Noon 1 (40)	1.158	0.274	0.079	0.157
SLAM 2 (48)	1.087	0.274	0.079	0.157
Noon 3 (42)	1.153	0.270	0.077	0.155
Noon 2 (41)	1.071	0.289	0.073	0.146
SLAM 1 (47)	1.028	0.280	0.072	0.144
Garrott's Rule of Thumb (28)	1.034	0.280	0.072	0.144
mab/Rule of Thumb (23)	1.076	0.280	0.072	0.144
Prism (3)	1.032	0.237	0.071	0.142
Burg/PC Crash (10)	1.267	0.255	0.069	0.138
Bixel (46)	1.019	0.267	0.067	0.134

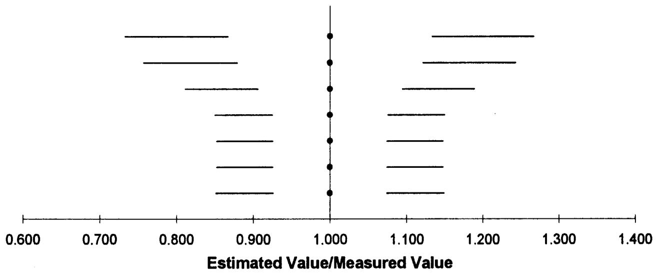
D) Center of Gravity Height



Method	Correction Factor	Range	Standard Deviation	95% Limit
Garrott's Rule of Thumb 1 (24)	0.982	0.249	0.070	0.141
Rule of Thumb 1 (19)	1.249	0.249	0.070	0.141
Allen (18)	1.025	0.187	0.054	0.108
Garrott's Rule of Thumb 2 (25)	1.006	0.187	0.054	0.108
Rule of Thumb 2 (20)	0.974	0.187	0.054	0.108

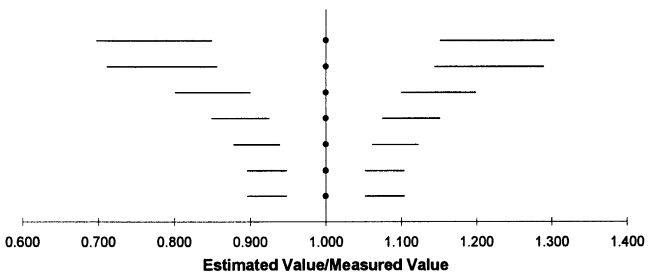
Pickup Trucks

A) Roll Moment of Inertia



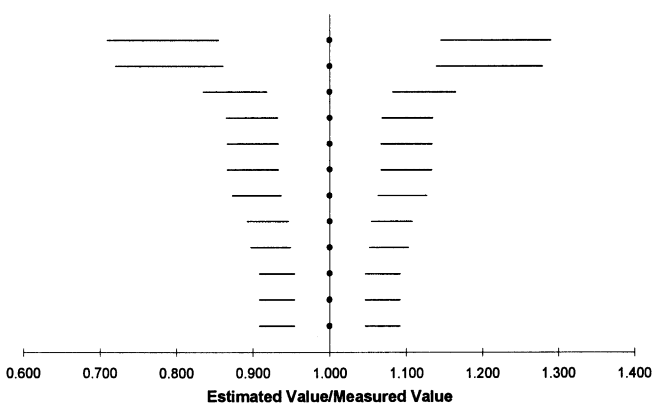
Method	Correction Factor	Range	Standard Deviation	95% Limit
PC Crash (51)	0.613	0.502	0.133	0.267
Reide (11)	1.138	0.392	0.121	0.243
Garrott (32)	0.777	0.328	0.094	0.189
Bixel (43)	0.947	0.317	0.075	0.150
Prism (1)	0.821	0.304	0.074	0.149
Garrott's Rule of Thumb (26)	0.822	0.306	0.074	0.148
Rule of Thumb (21)	0.551	0.306	0.074	0.148

B) Pitch Moment of Inertia



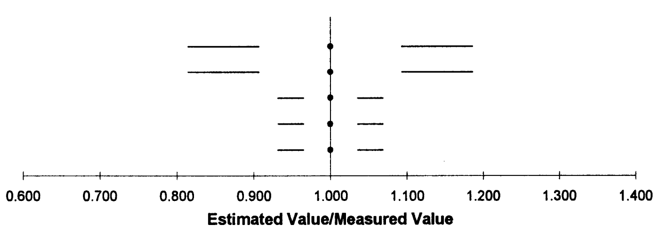
Method	Correction Factor	Range	Standard Deviation	95% Limit
Reide (12)	1.125	0.517	0.151	0.303
Garrott (33)	1.074	0.495	0.144	0.289
Bixel (45)	1.005	0.315	0.100	0.199
Prism (2)	0.898	0.274	0.075	0.151
PC Crash (52)	1.094	0.250	0.061	0.122
Garrott's Rule of Thumb (27)	0.916	0.253	0.052	0.104
Rule of Thumb (22)	0.952	0.253	0.052	0.104

C) Yaw Moment of Inertia



Method	Correction Factor	Range	Standard Deviation	95% Limit
Reide (13)	1.037	0.465	0.145	0.290
Garrott (34)	1.085	0.443	0.139	0.279
Bixel (46)	1.004	0.283	0.082	0.165
Prism (3)	0.922	0.289	0.068	0.135
Noon 1 (40)	1.007	0.285	0.067	0.134
SLAM 2 (48)	0.945	0.285	0.067	0.134
Noon 3 (42)	0.993	0.255	0.063	0.127
Burg/PC Crash (10)	1.100	0.211	0.054	0.108
Noon 2 (41)	0.928	0.198	0.051	0.103
SLAM 1 (47)	0.915	0.174	0.046	0.092
Garrott's Rule of Thumb (28)	0.921	0.174	0.046	0.092
mab/Rule of Thumb (23)	0.958	0.174	0.046	0.092

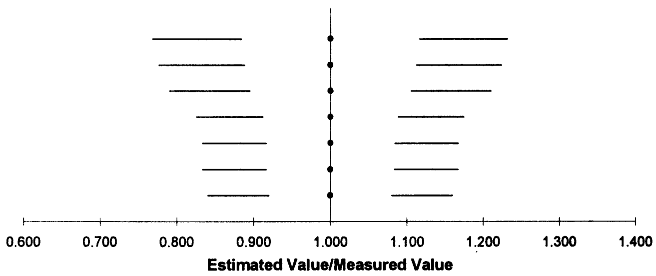
D) Center of Gravity Height



Method	Correction Factor	Range	Standard Deviation	95% Limit
Garrott's Rule of Thumb 1 (24)	0.948	0.326	0.093	0.186
Rule of Thumb 1 (19)	1.206	0.326	0.093	0.186
Allen (18)	0.990	0.124	0.035	0.069
Garrott's Rule of Thumb 2 (25)	0.972	0.124	0.035	0.069
Rule of Thumb 2 (20)	0.940	0.124	0.035	0.069

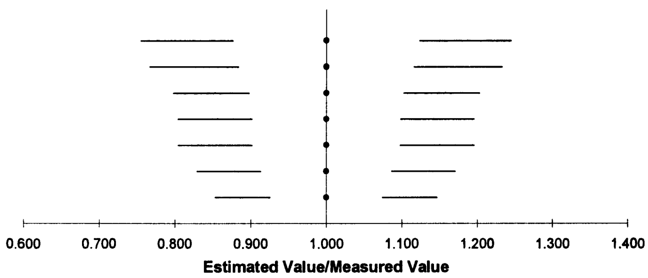
Vans

A) Roll Moment of Inertia



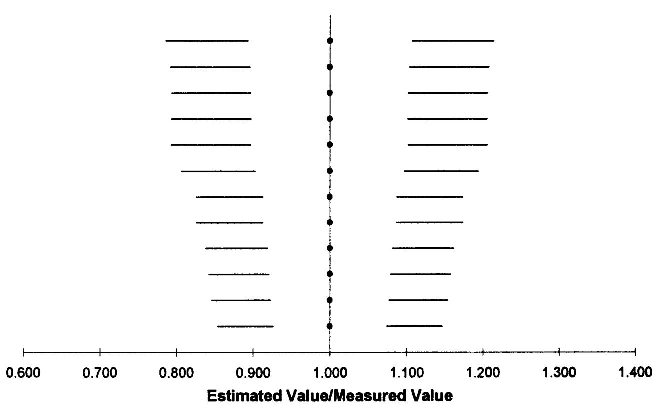
Method	Correction Factor	Range	Standard Deviation	95% Limit
PC Crash (51)	0.896	0.411	0.116	0.232
Reide (11)	1.465	0.335	0.112	0.224
Garrott (32)	0.981	0.318	0.105	0.210
Prism (1)	0.996	0.317	0.088	0.175
Garrott's Rule of Thumb (26)	1.000	0.305	0.084	0.167
Rule of Thumb (21)	0.670	0.305	0.084	0.167
Bixel (43)	0.956	0.291	0.080	0.160

B) Pitch Moment of Inertia



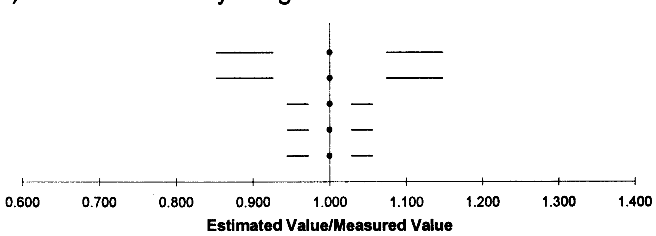
Method	Correction Factor	Range	Standard Deviation	95% Limit
Reide (12)	1.067	0.461	0.123	0.245
Garrott (33)	0.993	0.418	0.116	0.233
Bixel (45)	0.988	0.338	0.102	0.203
Garrott's Rule of Thumb (27)	0.934	0.319	0.098	0.196
Rule of Thumb (22)	0.972	0.319	0.098	0.196
Prism (2)	0.999	0.373	0.086	0.171
PC Crash (52)	1.188	0.304	0.074	0.147

C) Yaw Moment of Inertia



Method	Correction Factor	Range	Standard Deviation	95% Limit
Reide (13)	0.989	0.399	0.107	0.214
Garrott (34)	1.021	0.375	0.104	0.208
SLAM 1 (47)	0.939	0.328	0.103	0.206
Garrott's Rule of Thumb (28)	0.945	0.328	0.103	0.206
mab/Rule of Thumb (23)	0.982	0.328	0.103	0.206
Noon 2 (41)	0.960	0.340	0.097	0.194
Noon 1 (40)	1.160	0.333	0.087	0.174
SLAM 2 (48)	1.088	0.333	0.087	0.174
Prism (3)	1.040	0.318	0.081	0.162
Bixel (46)	0.998	0.257	0.079	0.158
Noon 3 (42)	1.126	0.321	0.077	0.154
Burg/PC Crash (10)	1.202	0.309	0.074	0.147

D) Center of Gravity Height



Method	Correction Factor	Range	Standard Deviation	95% Limit
Garrott's Rule of Thumb 1 (24)	1.001	0.214	0.074	0.148
Rule of Thumb 1 (19)	1.273	0.214	0.074	0.148
Allen (18)	1.003	0.118	0.028	0.056
Garrott's Rule of Thumb 2 (25)	0.985	0.118	0.028	0.056
Rule of Thumb 2 (20)	0.953	0.118	0.028	0.056

APPENDIX C - PC-CRASH RUNS

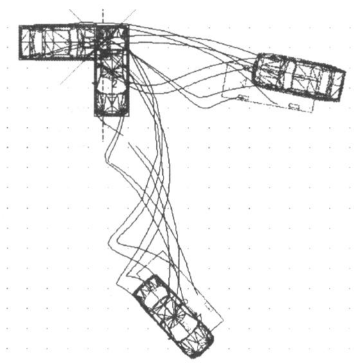


Figure C1 - Simulation with Yaw Moments decreased to 70%

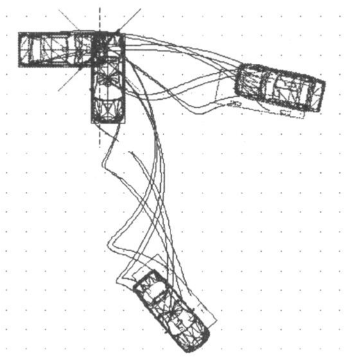


Figure C2 - Simulation with Yaw Moments decreased to 80%

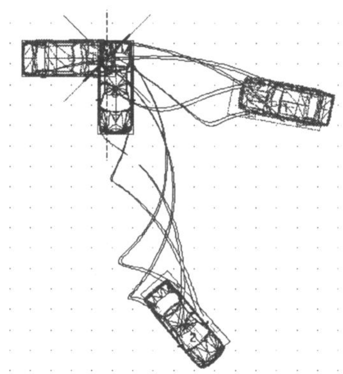


Figure C3 - Simulation with Yaw Moments decreased to 90%

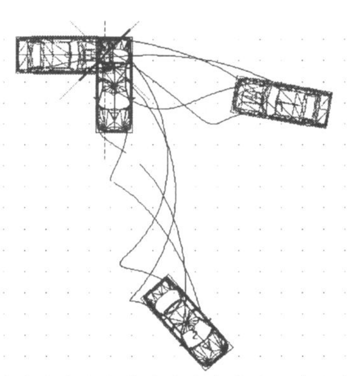


Figure C4 - Simulation with Yaw Moments increased to 100%

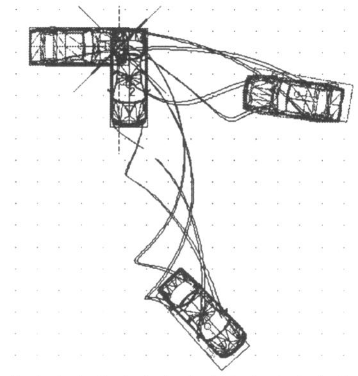


Figure C5 - Simulation with Yaw Moments increased to 110%

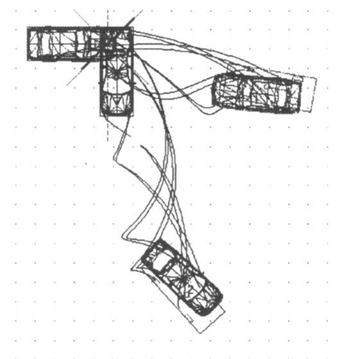


Figure C6 - Simulation with Yaw Moments increased to 120%

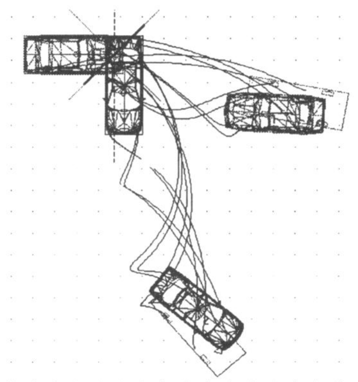


Figure C7 - Simulation with Yaw Moments increased to 130%

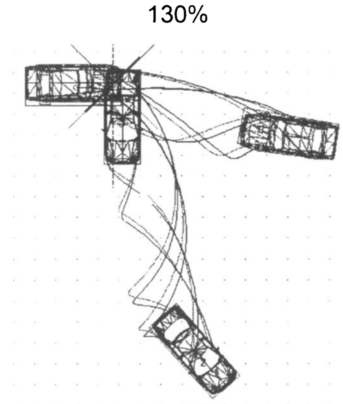


Figure C8 - Simulation with Yaw Moments increased to 130%, and corrected.

SAE # 970951
Reviewer's Discussion
by Bruce Main

A Comparison of Moment of Inertia Estimation Techniques for Vehicle Dynamics Simulation

William E. Cliff, Duane D. MacInnis, Craig A. Brown, Colin W. Blair, Authors

In "A Comparison of Moment of Inertia Estimation Techniques for Vehicle Dynamics Simulation", MacInnis et. al. provide a useful summary of the various vehicle parameter estimation techniques developed over the years. Their review of the vehicle dynamics models assists readers to compare the various techniques and methods used. The authors present the distribution for one vehicle category which is approximately normal. The use of standard statistical tests appear appropriate. However, the reader is left to assume that the other vehicle categories are also approximately normally distributed.

The authors' analysis indicate that earlier estimation techniques for the yaw moment of inertia were quite robust. It is encouraging to find that early and perhaps less sophisticated estimation efforts provide robust results when compared with current analytical capabilities.

MacInnis et. al. accurately use the term "corrected" in the sense that the work of prior authors has been adjusted to minimize the deviation from a calculated value to the measured value. This should not be interpreted as a criticism of the prior work or that the prior work is "incorrect". Rather, the current authors present an analysis to reduce the level of variation between the estimated and measured values.

MacInnis et. al. provide a concise summary of the estimation techniques for moments of inertia for vehicle dynamics simulations. Their analyses of the techniques which provide the smallest error will provide engineers a useful resource for reconstruction analyses.