Kinematic pairs

Type	low or high pair	symbo	DOF	Type	low or high pair	symbo	DOF
旋轉對	L	R	1	球面對	L	S	3
滑動對	L	P	1	平面對	L	P_l	3
螺旋對	L	H	1	universal joint 萬向接頭		U	2
滾動對	L	O	1	齒輪對	H	G	2
圓柱對	H	C	2	凸輪對	H	Cam (A)	2

Mobility (Degree of Freedom)(Kutzbact criterion)

$$\begin{aligned} F = & \quad \lambda(n-1) - (\lambda \cdot j - \sum f_i) \\ = & \quad \lambda \cdot (n-j-1) + \sum_i f_i \end{aligned}$$

Characteristics of mechanisms

重要公式

$$\begin{aligned} \sum_{i=2} i n_i = 2j & \qquad \longrightarrow i\text{接頭的桿件總和等於}2\text{倍接頭數} \\ \sum_{i=2} n_i = n & \qquad \longrightarrow i\text{接頭的桿件總和等於桿件數} \\ L = j - n + 1 & \qquad \longrightarrow loop\text{與接頭、桿件數的關係} \\ n_2 \geq 3n - 2j & \qquad \longrightarrow binarylink\text{的數量必大於一個條件} \\ F = \sum f_i - \lambda \cdot L & \qquad \longrightarrow \text{自由度與}loop\text{的關係} \end{aligned}$$

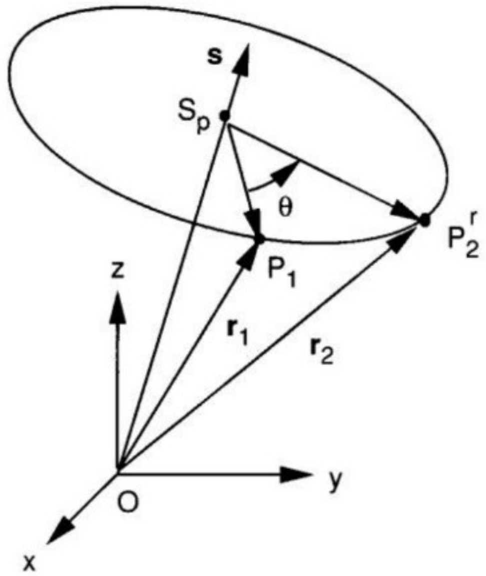
Planar Linkages with Lower-pair Joints Only (R-joints & P-joints)

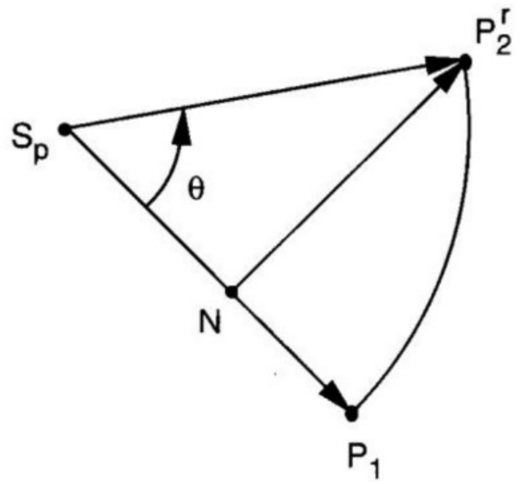
$$F = n - (2L + 1)$$

Planar Mechanisms with up to Two-d.o.f. Joints

$$F = 3n - 2j - 3 + j_2$$
$$j \leq 2n - 3$$

Scerw Axis Representation (Rodrigue's formula)





$$\mathbf{r_2} = \mathbf{r_1} cos\theta + \mathbf{s} \times \mathbf{r_1} sin\theta + \mathbf{s}(\mathbf{r_1}^T \mathbf{s})(1 - cos\theta)$$

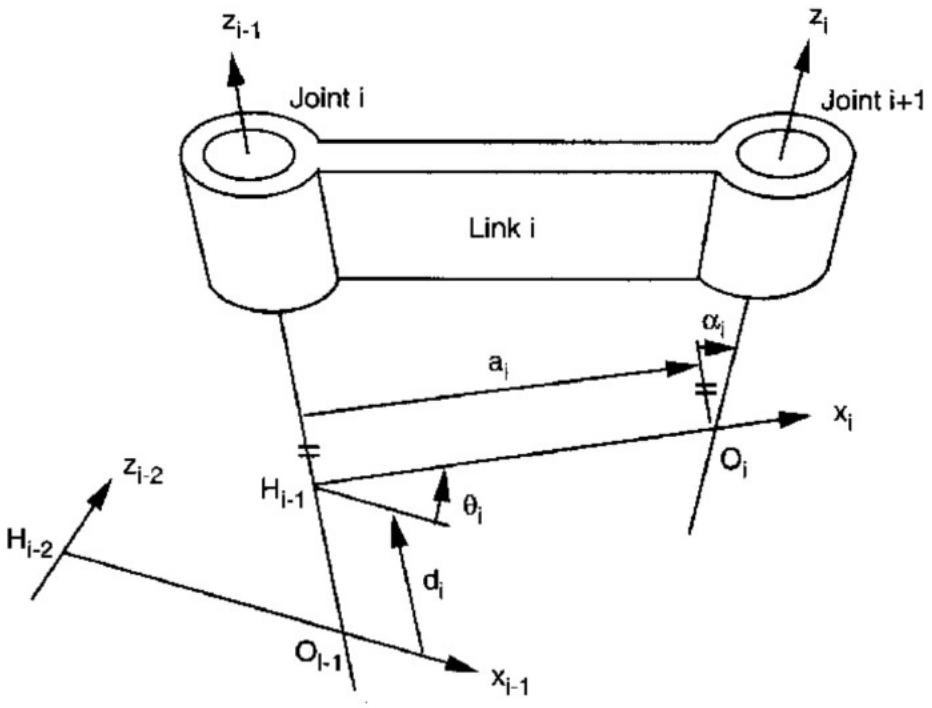
$${}^A\mathbf{p} = \mathbf{r_2}, {}^B\mathbf{p} = \mathbf{r_1} \longrightarrow {}^A\mathbf{p} = {}^A\mathbf{R}_B \cdot {}^B\mathbf{p} \gg (Rodrigue's formula)$$

$${}^A\mathbf{R}_B = \begin{bmatrix} (s_x^2 - 1)(1 - cos\theta) + 1 & s_x s_y(1 - cos\theta) - s_z sin\theta & s_x s_z(1 - cos\theta) + s_y sin\theta \\ s_y s_x(1 - cos\theta) + s_z sin\theta & (s_y^2 - 1)(1 - cos\theta) + 1 & s_y s_z(1 - cos\theta) - s_x sin\theta \\ s_z s_x(1 - cos\theta) - s_y sin\theta & s_z s_y(1 - cos\theta) + s_x sin\theta & (s_z^2 - 1)(1 - cos\theta) + 1 \end{bmatrix}$$

Homogeneous Transformation Matrix

$$
{}^{A}\mathbf{T}_{B} = \begin{bmatrix} {}^{A}\mathbf{R}_{B}(3\times 3) & : & {}^{A}\mathbf{q}(3\times 1) \\ \cdots & \cdots & \cdots \\ \gamma(1\times 3) & : & \rho(1\times 1) \end{bmatrix}
\begin{array}{r} {}^{A}\mathbf{R}_{B}(3\times 3) \longrightarrow orientationchanging(\text{旋轉}) \\ {}^{A}\mathbf{q}(3\times 1) \longrightarrow positionchanging(\text{位移}) \\ \gamma(1\times 3) \longrightarrow respectivetransformation(\text{通常記為}0) \\ \rho(1\times 1) \longrightarrow scalingfactor(\text{縮放, 通常記為}1) \end{array}
$$

Denavit-Hartenberg Notation



a_i	α_i	θ_i	d_i
z_{i-1}到z_i的距離	z_{i-1}到z_i間的夾角(繞x_i旋轉)	x_{i-1}到x_i間的夾角(繞z_i-1旋轉)	x_{i-1}到x_i的距離

$$
{}^{\mathbf{i-1}}\mathbf{T_i} = \begin{bmatrix} cos\theta_i & -sin\theta_i cos\alpha_i & sin\theta_i sin\alpha_i & a_i cos\theta_i \\ sin\theta_i & cos\theta_i cos\alpha_i & -cos\theta_i sin\alpha_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Conventional Jacobian

$$
\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{v_n} \\ \omega_\mathbf{n} \end{bmatrix} = \mathbf{J}\dot{\mathbf{q}}
$$

$$
\mathbf{J} = [\mathbf{J_1} \quad \mathbf{J_2} \quad \mathbf{J_3} \quad \cdots \quad \mathbf{J_n}],\ where \begin{aligned} \mathbf{J_i} &= \begin{bmatrix} \mathbf{z}_{i-1} \times {}^{i-1}\mathbf{p^*}_n \\ \mathbf{z}_{i-1} \end{bmatrix} && \longrightarrow for\ a\ revolute\ joint \\ \mathbf{J_i} &= \begin{bmatrix} \mathbf{z}_{i-1} \\ \mathbf{0} \end{bmatrix} && \longrightarrow for\ a\ prismatic\ joint \end{aligned}
$$

$$
\mathbf{z}_i = {}^0\mathbf{R}_i \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
$$

$$
{}^{i-1}\mathbf{p^*}_n = {}^{\mathbf{0}}\mathbf{R_{i-1}}{}^{i-1}\mathbf{r}_i + {}^{i}\mathbf{p^*}_n\ ,\ where \quad {}^{i-1}\mathbf{r}_i = \begin{bmatrix} a_i cos\theta_i \\ a_i sin\theta_i \\ d_i \end{bmatrix}
$$