

Multi-Robot Formation with Obstacle Avoidance: An Improved Velocity Obstacle-Based Approach

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Abstract—This paper addressed the problem of multi-robot formation with obstacle avoidance, where the obstacles are assumed to be polyhedral. Due to the fact that the traditional velocity obstacle method cannot provide an appropriate velocity for the robots to avoid polyhedral obstacles, an improved velocity obstacle method is proposed by constructing the convex velocity obstacle region of polyhedral obstacles and planning the feasible velocity half-plane. Then, a consensus-based formation control protocol with improved velocity obstacle is proposed to avoid collisions among individuals in group formation, and to maintain formation as far as possible when to avoid obstacles. Simulations are finally provided to verify the feasibility and effectiveness of the proposed approach.

Keywords—velocity obstacle, formation obstacle avoidance, consensus-based formation, multi-robot system

I. INTRODUCTION

In recent years, cooperative control of multi-robot system has emerged as a prominent research area within the control field. Among various aspects of cooperation, consensus-based formation control has gained significant attention and finds widespread applications in both military and civilian domains [1]. In practice, multi-robot formation necessitate maintaining stability, adaptive adjustments to avoid obstacles, and fulfillment of mission requirements [2]. Consequently, the problem of obstacle avoidance and formation control for multi-robot system in practical scenarios holds immense significance.

Formation obstacle avoidance is a crucial research topic that plays a pivotal role for achieving safe and efficient task execution. The common formation obstacle avoidance methods include overall obstacle avoidance which means formation switching obstacle avoidance [3, 4], and formation splitting obstacle avoidance [5], among many others. In the complex dynamic environment, formation split obstacle avoidance has its unique advantage, while the individuals in the formation no longer maintain the original formation when to avoid obstacles [6]. In order to maintain the formation and avoid collision and obstacle, an obstacle avoidance algorithm should be introduced on the basis of the original formation control to dynamically adjust the formation for online obstacle avoidance [7, 8].

The velocity obstacle (VO) method is a popular method for obstacle avoidance that involves the construction of velocity obstacle region. It has found extensive applications in local path planning for robots [9]. However, the traditional VO method subjects to issues such as jitter and primarily focuses on collision avoidance among robots, thereby lacking the ability to effectively handle local path planning in the presence of polyhedral obstacles. In [10], an

enhancement to the traditional VO method was proposed to address the issue of jitter by introducing a feasible velocity half-plane. However, this modification still cannot adequately tackle the challenge associated with path planning in polyhedral obstacle environment. In [11], a global path planning algorithm was introduced based on the traditional VO method to address polyhedral obstacles. However, when faced with abrupt local polyhedral obstacles, it failed to provide proper avoidance strategy. In [12], a novel approach based on the VO method was presented, where a nonlinear dynamic programming regulator is utilized to navigate around partially detected obstacles by constructing a velocity feasible region delimited by boundary lines. Currently, the research on the VO method predominantly focused on the cases with regular convex obstacles. Therefore, it is necessary to propose suitable obstacle avoidance algorithms to tackle with polyhedral obstacles.

To maintain the formation during obstacle avoidance, it is necessary to incorporate formation control into obstacle avoidance. In [13], an improved artificial potential field method was developed, which includes attractive, repulsive, and potential field forces among unmanned aerial vehicles (UAV). Additionally, a consensus-based algorithm for collaborative formation trajectory planning was devised by integrating the improved artificial potential field method and consensus theory. In [14], an UAV formation technique was presented, leveraging the artificial potential field generated by precise control inputs and consensus theory. Moreover, a collaborative obstacle avoidance control strategy based on graph theory and the artificial potential field method was proposed. In [15], a formation obstacle avoidance control law was presented, using consensus-based algorithm and the artificial potential field method, assuming barrier-free motion for the leader. In [16], a combined approach of particle swarm optimization algorithm and model predictive control was proposed for obstacle avoidance. Additionally, formation control was integrated using a consensus-based algorithm. In [17], a leader-follower queue tracker is designed based on the performance function of expected relative Angle and collision avoidance, to realize obstacle avoidance and formation maintenance among robots. Based on the above, it is reliable to introduce consensus-based formation control into obstacle avoidance algorithms to achieve formation obstacle avoidance.

Motivated by the above works, this paper focuses on the problem of multi-robot formation with polyhedral obstacle avoidance. The main contributions are as follows. On the one hand, an improved velocity obstacle method is proposed by constructing the convex region of the velocity obstacle for multi-robot system and utilizing linear programming, removing the limitation of the traditional velocity obstacle method in path planning within polyhedral obstacle

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environment. On the other hand, a consensus-based formation control protocol with improved velocity obstacle is proposed. This protocol is able to rectify position and velocity errors in multi-robot system based on desired states, enabling collision avoidance, obstacle avoidance, and achieving the desired formation configuration during operation.

This paper is structured as follows. In Sect. II, the system model and the communication topology are described. In Sect. III, an improved velocity obstacle method is presented. In Sect. IV, a consensus-based formation obstacle avoidance protocol is designed and analyzed. In Sect. V, the effectiveness of the obstacle avoidance algorithm and the consensus-based formation obstacle avoidance protocol are further verified by simulation. In Sect. VI, the conclusion is drawn.

II. PROBLEM DESCRIPTION

A. System Model

Firstly, we consider a formation model of multi-robot system in two dimensions. In this model, the kinematic equations of i -th robot is described by the following second-order integral dynamics:

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = u_i, \end{cases} \quad i \in \{1, 2, \dots, n\}, \quad (1)$$

where $x_i = [x_{ix}, x_{iy}]^T$, $v_i = [v_{ix}, v_{iy}]^T$, $u_i = [u_{ix}, u_{iy}]^T$ denote the position state, velocity state and control input of robot i , respectively. The state vector of the multi-robot system is denoted as:

$$\begin{cases} x = [x_1^T & x_2^T & \dots & x_n^T]^T \in R^{n \times 2}, \\ v = [v_1^T & v_2^T & \dots & v_n^T]^T \in R^{n \times 2}. \end{cases} \quad (2)$$

Assumed that the required formation center trajectory $x_L = [x_{Lx}, x_{Ly}]^T$ is generated by $\dot{x}_L = v_L = [v_{Lx}, v_{Ly}]^T$, with $\dot{v}_L = u_L = [0, 0]^T$. And the formation pattern is determined by vector $\Delta_i = [\Delta_{ix}, \Delta_{iy}]^T$, which point to formation center.

The multi-robot system is said to accomplish formation if the following statements hold:

$$\begin{cases} \lim_{t \rightarrow \infty} \|x_i(t) - x_j(t) - (\Delta_i - \Delta_j)\| = 0, \\ \lim_{t \rightarrow \infty} \|v_i(t) - v_j(t)\| = 0, \end{cases} \quad \forall i \neq j \in \{1, 2, \dots, n\}. \quad (3)$$

and

$$\lim_{t \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n x_i(t) = \lim_{t \rightarrow \infty} x_L(t). \quad (4)$$

B. Communication Topology

The network topology for information exchange among the multi-robot system is usually represented by an undirected graph $G = \{V, E, A\}$. $V = \{1, 2, \dots, n\}$ is the set of nodes of the graph, and $E \subseteq V \times V$ is the set of edges of the graph. $A = [a_{ij}]$ is the adjacency matrix of graph G , and a_{ij} is the weight of edge satisfying:

$$a_{ij} = \begin{cases} 1, & (i, \sim j) \in E, \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

The required formation center is denoted by a virtual leader. Let $B = [b_i]$ represent the relationship between the robot i and the leader. Note that $b_i > 0$, if the robot i has access to the information of leader and, $b_i = 0$, otherwise.

There are n robots in the system, each robot is considered as a node, and the edges represent the information intersection between robots. For formation obstacle avoidance control, assume that there is no time delay and data loss in the data transfer between robots.

III. IMPROVED VELOCITY OBSTACLE METHOD

A. Traditional Velocity Obstacle Method

The velocity obstacle algorithm was first proposed in [9], which accomplishes obstacle avoidance by excluding the velocity of possible future collisions, and it is mainly used for obstacle avoidance between robots. The main idea of this method is shown in Fig. 1. The velocity obstacle region is shown in (6) and (7).

$$D(p, r) = \{q \mid \|q - p\| < r\}, \quad (6)$$

$$VO_{AB}^r = \{v \mid \exists t \in [0, \tau] :: tv \in D(p_B - p_A, r_A + r_B)\}, \quad (7)$$

where $D(p, r)$ is denoted as a hollow disk with p as the center and radius r . And r_i , v_i , p_i denote the radius, current velocity and current position of the robot i , respectively.

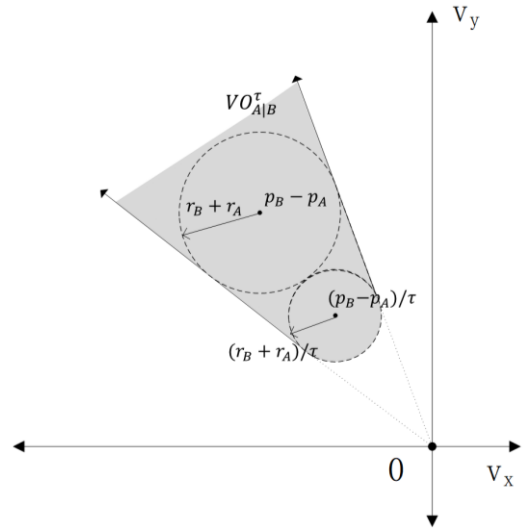


Fig. 1. Diagram of velocity obstacle method.

However, the traditional velocity obstacle method cannot avoid polyhedral obstacles, so an improved velocity obstacle algorithm is proposed in the following.

B. Improved Velocity Obstacle Algorithm

In this paper, based on [10] and the traditional velocity obstacle method, we model the division of irregular obstacles, achieve the effective division of velocity feasible half-plane by constructing convex regions, and then apply the improved velocity obstacle method into the design of formation control protocol of multi-robot system.

Suppose the radius of robot A is r_A , the position is p_A , and the polyhedral obstacle it faces is O . For the detected obstacle O , the approximate geometry is processed according to its facade characteristics and the set of line segments is modeled, as shown in Fig. 2.

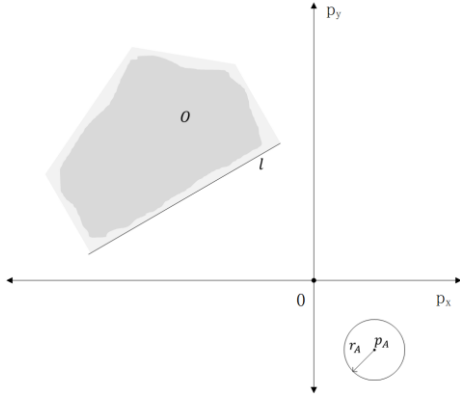


Fig. 2. Diagram of obstacle division.

Assuming that one of the sets of line segments approximated by the polyhedral obstacle O is denoted as l . To construct the convex region, l is extended to an ellipse, denoted as O' , which is shown in the white area in Fig. 3. The constraints for the long axis a and short axis b of the constructed convex packet ellipse are shown in (8), i.e. the shortest distance between the expanded convex packet and the original line segment should be greater than r_A , otherwise there is a risk of causing a collision.

$$d = \min(|l - O' \oplus D(p_A, r_A)|) > r_A, \quad (8)$$

where $O' \oplus D(p_A, r_A)$ means minkowski sum between O' and $D(p_A, r_A)$.

At this point, the velocity obstacle $IVO_{A|O'}^\tau$ for the obstacle O' in time τ is shown in Fig. 3. The expression is as follows:

$$IVO_{A|O'}^\tau = \{v \mid \exists t \in [0, \tau] :: tv \in O' \oplus D(p_A, r_A)\}, \quad (9)$$

where the time window τ is denoted as $\tau = \text{dist}_{A|O'} / v_A^{\text{opt}}$. It means that moving at the current velocity, the robot will collide within this time.

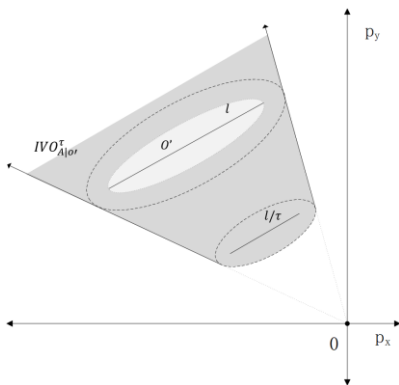


Fig. 3. Schematic diagram of the improved velocity obstacle area.

Let u be the minimum variable required to escape $IVO_{A|O'}^\tau$, and n be the normal vector of u . The feasible velocity half-plane formed at this point is noted as the improved velocity obstacle avoidance touch, i.e. $IVOA_{A|O'}^\tau$ is shown in Fig. 4. The expression is shown below:

$$IVOA_{A|O'}^\tau = \{v \mid [v - (v_A^{\text{opt}} + \alpha u)] \cdot n \geq 0\}, \quad (10)$$

where v_A^{opt} is the adaptive velocity for optimal collision avoidance, which ranges between $[0, v_A^{\text{pref}}]$ and is selected according to the complexity of the environment, usually v_A^{opt} is the current velocity of the robot. the coefficient α of u is determined by the state of the obstacle, usually the following case is a static obstacle in the direction of the polyhedral obstacle O , when the robot A needs to pay full responsibility for obstacle avoidance, at this time $\alpha = 1$.

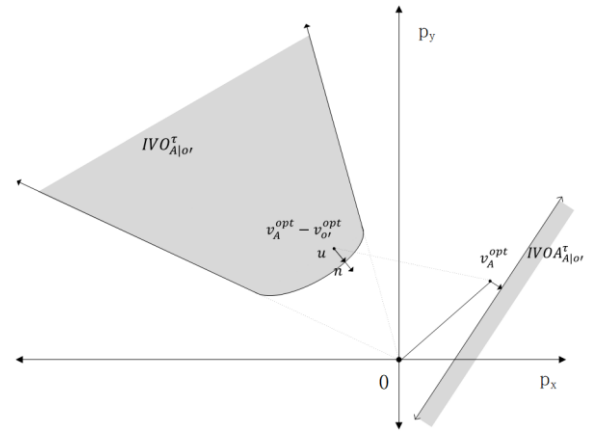


Fig. 4. Diagram of IVO feasible speed area.

For a large number of polyhedral obstacle environments, the set of allowable velocities of robot A with respect to all polyhedral obstacles is the intersection of the half-planes of all induced allowable velocities, as follows:

$$IVOA_A^\tau = D(0, v_A^{\text{max}}) \cap \bigcap_{i \in N_O} IVOA_{A|O_i}^\tau, \quad (11)$$

where $D(0, v_A^{\text{max}})$ denotes the range of the area to be considered for obstacle avoidance or not. N_O means the collection of obstacles within the obstacle avoidance range.

Since the convex package is constructed for the obstacle, the feasible velocity half-plane formed at this time is still a convex region, and for robot A to choose its new velocity, the velocity closest to the ideal velocity v_A^{pref} can be chosen from the range of velocities allowed by $IVOA_A^\tau$ using linear programming, as shown in (12). The ideal velocity v_A^{pref} is usually directed to the target point, and its form under formation obstacle avoidance will be given in (17).

$$v_A^{\text{new}} = \arg \min_{v \in IVOA_A^\tau} \|v - v_A^{\text{pref}}\|. \quad (12)$$

The improved velocity obstacle algorithm mainly focuses on the avoidance of polyhedral obstacles, but in practical applications, there are not only static polyhedral obstacles but also dynamical ones, so linear programming can be used

to combine the feasible velocity half-plane divided between the two plane to achieve the problem of obstacle avoidance in complex dynamic environments. The new velocity feasible region is represented as follows:

$$IVO_{A^{\tau+}} = D(0, v_A^{\max}) \cap \bigcap_{i \in N_O} IVO_{A|O_i}^{\tau} \cap \bigcap_{B \neq A} ORCA_{A|B}^{\tau}, \quad (13)$$

where B refers to the robot in the formation other than A . The improved velocity obstacle method at this point can achieve real-time obstacle avoidance for polyhedral obstacles as well as robots.

IV. CONSENSUS-BASED FORMATION OBSTACLE AVOIDANCE CONTROLLER DESIGN

A. Formation Controller Design

In order to achieve convergence and consensus between multi-robot formation in any formation state, the virtual leader is introduced as the center of the formation in the basic consensus-based control law [18], which is proposed as follows:

$$u_{ci} = -k_1 \left\{ \sum_{j=1}^n a_{ij} (x_i - x_j - \Delta_i + \Delta_j) + b_i (x_i - x_L - \Delta_i) \right\} - k_2 \left[\sum_{j=1}^n a_{ij} (v_i - v_j) + b_i (v_i - v_L) \right], i \in \{1, 2, \dots, n\}. \quad (14)$$

where, $k_1 > 0$ and $k_2 > 0$ are control gains.

Considering the collision and obstacle avoidance in formation, the improved velocity obstacle method is incorporated into the consensus-based protocol (14), and the position and velocity information is updated in real time with a vector of desired distance representations. The improved consensus-based formation obstacle avoidance protocol is designed as follows:

$$\begin{aligned} \hat{u}_i &= u_{ci} - k_3 (\hat{v}_i - V_{IVO}) \\ &= k_1 \left[\sum_{j=1}^n a_{ij} (\hat{x}_i - \hat{x}_j) + b_i \hat{x}_i \right] \\ &\quad - k_2 \left[\sum_{j=1}^n a_{ij} (\hat{v}_i - \hat{v}_j) + b_i \hat{v}_i \right] \\ &\quad - k_3 (\hat{v}_i - V_{IVO}), \end{aligned} \quad (15)$$

where k_3 is control gain greater than k_1, k_2 . $\hat{v}_i = v_i - v_L$, and $\hat{x}_i = x_i - x_L - \Delta_i$.

V_{IVO} provides the feasible velocity of obstacle avoidance for the improved velocity obstacle algorithm. The implementation of multi-robot formation obstacle avoidance includes the avoidance of collision between robots in the formation and the avoidance of both static and dynamic obstacles in the environment. At this point, V_{IVO} is chosen as follows:

$$V_{IVO} = \arg \min_{v \in IVO_{A^{\tau+}}} \|v - v_A^{pref}\|, \quad (16)$$

where the ideal velocity v_A^{pref} points to the desired position of the robot at the current moment, as shown in (17). v^{\max} is the maximum velocity allowed.

$$v_A^{pref} = v^{\max} \frac{x_i - x_L - \Delta_i}{\|x_i - x_L - \Delta_i\|}. \quad (17)$$

B. Stability Analysis

Considering the generality, this paper assumes that the formation system consists of n robots, and the robots communicate well with each other. The following theorem is given:

Theorem 1: Consider a system consisting of n robots with a connected communication topology. The cooperative formation with obstacle avoidance can be achieved with the control protocol in (15).

Proof:

The Lyapunov function is established as follows:

$$V = \frac{1}{2} \sum_{i=1}^n (\hat{v}_i)^2 + \frac{k_1}{4} \sum_{i=1}^n \sum_{j=1}^n a_{ij} (\hat{x}_i - \hat{x}_j)^2 + \frac{k_1}{2} \sum_{i=1}^n b_i (\hat{x}_i)^2 + k_3 \sum_{i=1}^n \int_0^t \hat{v}_i (\hat{v}_i - V_{IVO}) dt. \quad (18)$$

Taking the derivative of V , we have:

$$\begin{aligned} \dot{V} &= \sum_{i=1}^n \hat{v}_i \hat{u}_i + \frac{k_1}{2} \sum_{i=1}^n \sum_{j=1}^n a_{ij} (\hat{x}_i - \hat{x}_j) (\hat{v}_i - \hat{v}_j) \\ &\quad + k_1 \sum_{i=1}^n b_i \hat{x}_i \hat{v}_i + k_3 \sum_{i=1}^n \hat{v}_i (\hat{v}_i - V_{IVO}). \end{aligned} \quad (19)$$

Upon completion:

$$\dot{V} = -\frac{k_2}{2} \sum_{i=1}^n \sum_{j=1}^n a_{ij} (\hat{v}_i - \hat{v}_j)^2 - k_2 \sum_{i=1}^n b_i (\hat{v}_i)^2. \quad (20)$$

It can be seen from (20) that $\dot{V} \leq 0$ and V is a decreasing function. Consider a set of $\Omega = \{(\hat{v}, \hat{x}) | V \leq V_0\}$ is bounded, and define $S = \{(\hat{v}, \hat{x}) | \dot{V} = 0\}$. \hat{v}_i is bounded, and according to $(\hat{v}_i)^2 \leq \sum_{i=1}^n (\hat{v}_i)^2 \leq 2V \leq 2V_0$, we can get Ω is a compact set. When $t \rightarrow \infty$, according to the LaSalle's invariant principle, the states will converge to the largest invariant set contained in S . It means $\dot{V} = 0$, then $\hat{v}_i = 0$, $\hat{v}_i - \hat{v}_j = 0$. According to (1) and (15), we can get $\hat{x}_i - \hat{x}_j = 0$, that means system can keep the formation stable.

V. SIMULATION VERIFICATION

In this paper, we mainly simulate in a two-dimensional polyhedral obstacle environment, analyze the results of the improved velocity obstacle algorithm and consensus-based formation obstacle avoidance algorithm, and further verify the effectiveness of the proposed algorithm.

A. Simulation of Improved Velocity Obstacle

Two robots are considered in the experiments, whose initial positions as well as state information are shown in Table I. The maximum velocity is $[3, 3]^T$ and the expected velocity direction is pointing to the target position.

TABLE I. INITIAL STATE OF ROBOT

Robot	The initial position	The target position	The initial velocity	Radius
robot1	$[0,0]^T$	$[25,13]^T$	$[0,0]^T$	0.5
robot2	$[24,14]^T$	$[10,0]^T$	$[0,0]^T$	0.5

The result of the conventional optimal reciprocal collision avoidance (ORCA) algorithm [10] is shown in Fig. 5, Fig. 5(b) indicates the minimum distance from the obstacle. When the conventional ORCA algorithm encounters a polyhedral obstacle, it can only ensure that the robot will not collide with the obstacle, but cannot move around it, i.e. bypass the obstacle to reach the target point.

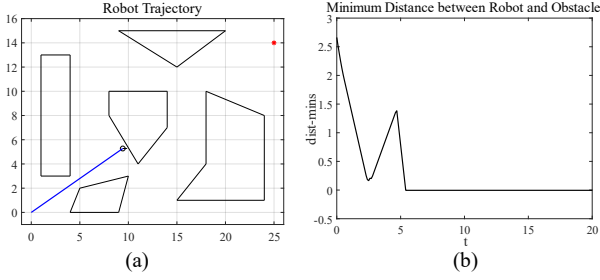


Fig. 5. Experimental results of ORCA method.

For polyhedral obstacle avoidance, this paper proposes to improve velocity obstacle as shown in Fig. 6. Fig. 6(b) shows the minimum distance between the robot and the obstacle, and Fig. 6(c) represents the minimum distance between two robots, and they are both greater than 0. In the constructed complex obstacle environment, the robot can successfully reach the target point by bypassing the obstacles and can avoid dynamic obstacles while avoiding polyhedral obstacles.

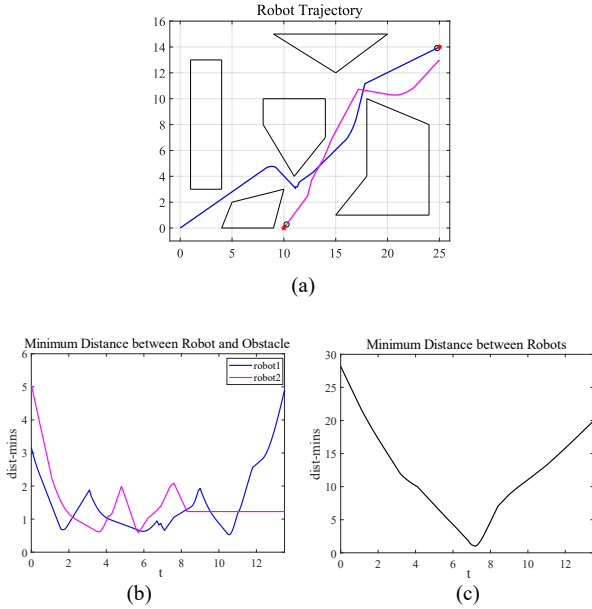


Fig. 6. Experimental results of improved velocity obstacle method.

B. Simulation of Consensus-Based Formation Obstacle Avoidance Algorithm

A total of four robots are considered in the experiment, where robot 0 is the virtual leader and the remaining three robots form a formation system with the communication

topology shown in Fig. 7. The communication topology is as follows:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \quad (21)$$

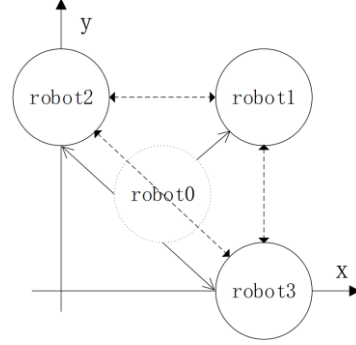


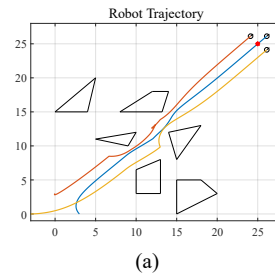
Fig. 7. Communication topology diagram with desired formation position.

The initial position state information of robot 0 and three robots is shown in Table II, where robot 0 moves toward the target point at a uniform velocity, where velocity is $[3,3]^T$ and the target point position of robot 0 is $[25,25]^T$. Set the desired formation position relationship to $\Delta_1 = [1,1]^T$, $\Delta_2 = [-1,1]^T$, $\Delta_3 = [1,-1]^T$ and the desired position of the virtual leader is located in the center of the formation. The controller parameters are set to $k_1 = 0.25$, $k_2 = 0.45$, $k_3 = 37.5$.

TABLE II. INITIAL STATE OF ROBOTS

Robot	The initial position	The initial velocity	Radius
robot0	$[0,0]^T$	$[0,0]^T$	0.5
robot1	$[3,0]^T$	$[0,0]^T$	0.5
robot2	$[0,3]^T$	$[0,0]^T$	0.5
robot3	$[-3,0]^T$	$[0,0]^T$	0.5

The simulation results of the three robots described in (1) from the initial position in the constructed complex obstacle environment using the consensus-based formation obstacle avoidance algorithm proposed in this paper are shown in Fig. 8(a)-(e).



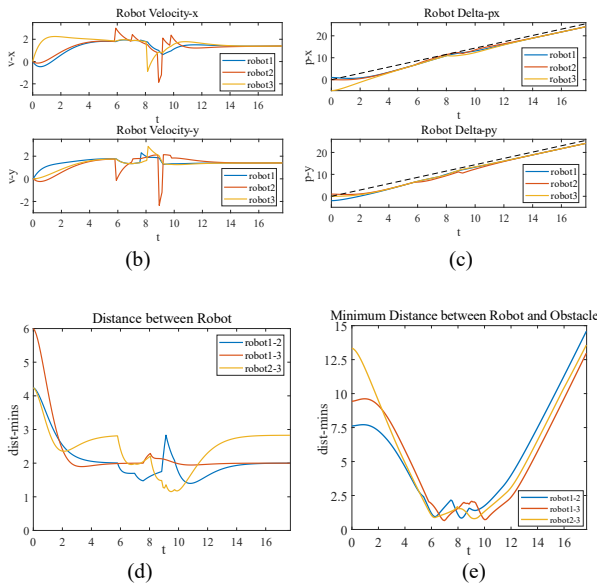


Fig. 8. Experimental results of consensus formation obstacle avoidance protocol

In Fig. 8(c), the velocity components of the multi-robot systems can also converge to the desired velocity under the control strategy of this paper. However, when just passing through the obstacle area, the velocity components of each robot will have the problem of partial jitter, which is caused by the over-sensitive obstacle avoidance algorithm, and the solution scheme of this problem will continue to be studied subsequently, and the controller will improve the convergence ability.

From Fig. 8(d), it can be seen that the spacing between robots is always greater than 0, which indicates that the controller can guarantee the safety of operation within the formation and achieve great intra-team collision avoidance. When the multi-robot formation safely passes the obstacle zone, the multi-robot formation can quickly converge to the desired formation. In Fig. 8(e), according to the consensus-based formation obstacle avoidance protocol proposed in this paper, three robots can achieve formation obstacle avoidance and finally reach the target position.

VI. CONCLUSION

In this paper, an improved velocity obstacle method has been proposed by constructing convex regions for addressing the problem that the traditional velocity obstacle method cannot avoid obstacles in complex environments containing polyhedral obstacles. For the second-order model, a consensus-based formation avoidance control protocol and improved velocity obstacle method have been proposed where the stability and consistency of the formation system are discussed. The results show that under the improved velocity obstacle method, multiple robots can drive safely. And, the consensus-based formation obstacle avoidance protocol can successfully avoid obstacles and achieve the desired formation.

In the future research, the jitter problem of the control protocol in resuming the formation during the formation process will be considered to adapt to the more practical situation.

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