Linear Algebra and its Applications

HW#3

1. Compare the pivots produced by eliminations with and without partial pivoting for

$$A = \begin{bmatrix} .001 & 0 \\ 1 & 1000 \end{bmatrix}.$$

Based on the finding above, try to rescale the matrix before elimination to produce a better set of pivots. (Note: scaling an equation by multiplying both the left-hand side and the right-hand side by a scalar will not change the solution of $Ax=b$.)

2. Mr. Chen travels with his motorcycle. It is known that Mr. Chen's motorcycle accelerates and decelerates by a function of time equal to $\sin 2\pi t$ from time $t=0$ to $t=1$. Let time interval $h$ be 0.2 and both the initial speed and the final speed be zero. Find the distance Mr. Chen travels over t=0.0, 0.2, 0.4, 0.6, 0.8, 1.0 by formulating and solving an $Ax=b$ problem.

3. Which of the following subsets of $R^3$ are actually subspaces?
    (a) The plane of vectors $(b_1,b_2,b_3)$ with first component $b_1 = 0$.
    (b) The plane of vectors $b$ with $b_1 = 1$.
    (c) The vectors $b$ with $b_2 b_3 = 0$ (this is the union of two subspaces, the plane $b_2 = 0$ and the plane $b_3 = 0$).
    (d) All combinations of two given vectors (1,1,0) and (2,0,1).
    (e) The plane of vectors $(b_1,b_2,b_3)$ that satisfy $b_3 - b_2 + 3b_1 = 0$

4. True or false (with an explanation if true and a counterexample if false)?
    (a) The vectors $b$, that are not in the column space of $A$ except the zero vector, form a subspace.
    (b) If the column space of $A$ contains only the zero vector, then $A$ is the zero matrix.
    (c) The column space of $2A$ equals the column space of $A$.
    (d) The column space of $A-I$ equals the column space of $A$.
    (e) $R^2$ isn't a subspace of $R^3$.

5. Reduce $A$ and $B$ to echelon form. Which variables are free?

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

Find the solutions to $Ax = 0$ and $Bx = 0$.

6. The nullspace of a 3 by 4 matrix $A$ is the line through $(2,3,1,0)^T$. What is the exact echelon form $U$ of $A$ and the solution to $Ax=0$?

7. Explain why all these statements are false:

    (a) The complete solution is any linear combination of $x_{particular}$ and $x_{nullspace}$.

    (b) A system $Ax=b$ has at most one particular solution.

    (c) If $A$ is invertible there is no solution $x_{nullspace}$ in the nullsplace.

8. Find a 2 by 3 system $Ax=b$ whose complete solution is:

$$x = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + w \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

    Find a 3 by 3 system with the same solution when $b_1+b_2=b_3$.

9. Write the complete solutions $x=x_p+x_n$ to the following systems:

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$

10. True or False? (Give reason if true, or counterexample to show it is false.)

    (a) A square matrix has no free variables.

    (b) If $r<m$, there are free variables.

    (c) An $m$ by $n$ matrix has no more than $n$ pivot variables.

    (d) An $m$ by $n$ matrix has no more than $m$ pivot variables.