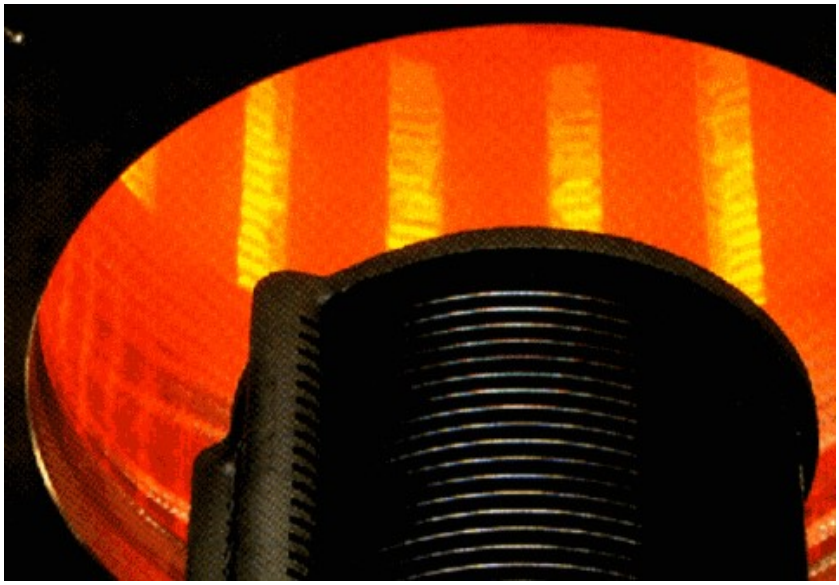
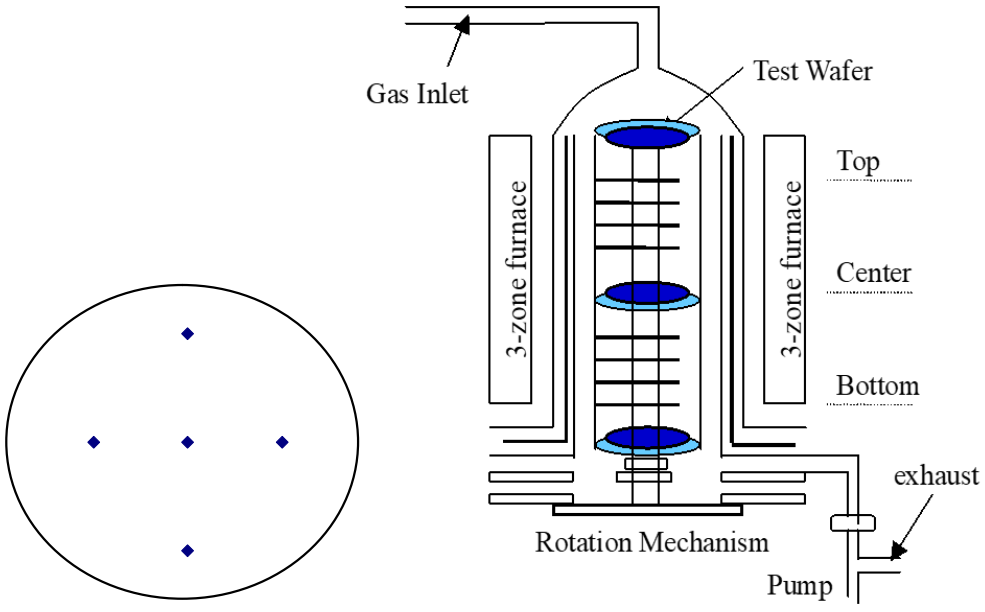
2024 Spring – SPC&O HW#4

1. In the attached excel file (24Spring-HW4.xls), we collect 5 readings of SiO_2 thickness on the test wafers from each of three heating zones: top zone, center zone and bottom zone, of the vertical thermal oxidation furnace. The 5 thickness readings are from 5 positions (up, middle, down, left and right) on each wafer.





(1) Calculate the location measures: $\overline{X}$ (average), $\widetilde{X}$ (median), $\overline{X}_e$, nd $\overline{X}_{tr(15\%)}$ and the dispersion measures: Sample Standard Deviation, Sample Range and IQR (Interquartile Range), for up, middle, down, left and right positions of the **center zone**. Discuss the differences among the 2 measures of location and 3 measures of dispersion of the 5 positions.

(2) Plot scatter plots for four pairs of thickness readings of the **center zone**: (up,

middle), (middle, down), (middle, left) and (middle, right). Calculate the sample covariance and correlation coefficients for the 4 pairs.

(3) Plot the histograms for each position's thickness readings (one histogram for each position) from the center zone.

(4) Plot the box plot for each of the five positions' thickness readings (one box for each position and put the boxes together in the same plot) from the center zone.

(5) Based on (1)-(4), describe what you find from the analysis.

(6) Plot scatter plots for five pairs of thickness readings: (up_{center}, up_{top}), ($middle_{center}$, $middle_{top}$), ($down_{center}$, $down_{top}$), ($left_{center}$, $left_{top}$), and ($right_{center}$, $right_{top}$). Calculate the sample covariance and correlation coefficients for the 5 pairs. What do you observe from the statistical analysis?

(7) Plot and compare the box plots for all thickness readings of the Top, Center and Bottom Zones (one box for each Zone and put the three boxes together in the same plot). Describe your findings.

2. Let

$\frac{\sum_i (x_i - a)^2}{n}$ be the estimate of $E[(X\text{-}a)^2]$

Show if it is an unbiased estimate. If not, find the bias.

3. The *iid* random sample $X_1, X_2, \ldots, X_n$, are taken from a Poisson distribution with parameter λ.

a) Let the estimate of the parameter be $\hat{\lambda} = \bar{X}$. Is $\hat{\lambda}$ an unbiased estimate of λ?

b) What is the standard error of the estimator $\hat{\lambda}$? When will the standard error reach the smallest? Why?

c) Assuming that the accident death numbers in regions of the Taiwan (24Spring-HW2.xls) follow the Poisson distribution model, estimate λ and corresponding standard error for each region.

d) Plot the histogram for each region and compare the histogram to the Poisson probability distribution model with the parameter estimated by (c).