1. Read the attached Chapter 4 and use the experimental results from Table 4.4(a), (b) to

(a) re-calculate the surface defect and thickness SN ratios using only Test Wafer 1 and Test Wafer 3 data.

計算Defect的S/N ratio可從下列公式得到：

$$\eta = -10\log_{10}(\frac{1}{n}\sum_{i=1}^{n} y^2)$$

根據wafer 1與wafer 3的結果可得到：

expt no.	test wafer 1			test wafer 3			SN-1&3
	top	center	bottom	top	center	bottom	
1	1	0	1	1	1	0	1.76091
2	1	2	8	126	3	1	-34.247
3	3	35	106	315	50	180	-92.414
4	6	15	6	15	40	18	-155.63
5	1720	1980	2000	2020	360	13	-671.68
6	135	360	1620	2500	270	35	-439.59
7	360	810	1215	1800	720	315	-529.04
8	270	2730	5000	9999	225	1	-502.8
9	5000	1000	1000	3000	2800	2000	-769
10	3	0	0	1	0	1	-92.414
11	1	0	1	1	0	1	1.76091
12	3	1620	90	270	8	3	-92.415
13	1	25	270	225	3	0	-43.158
14	3	21	162	63	15	39	-92.414
15	450	1200	1800	1890	180	25	-549.39
16	5	6	40	14	1	1	-139
17	1200	3500	3500	9999	600	8	-638.85
18	8000	2500	3500	5000	2000	2000	-811.87

計算thickness的S/N ratio可從下列公式可先求出兩個wafer中每個測量點得到的值的平均數與標準差。其中n為測量點總各數：

$$\mu = \frac{1}{n}\sum_{i=1}^{n} \tau_i$$

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n} (\tau_i - \mu)^2$$

根據wafer 1與wafer 3的結果可得到對thickness的S/N ratio為：

expt no.	test wafer 1			test wafer 3			mu	var	SN
	top	center	bottom	top	center	bottom			
1	2029	1975	1961	1952	1941	1949	1967.83	1032.966667	35.7389
2	5375	5191	5242	5323	5307	5091	5254.83	10577.76667	34.1672
3	5989	5894	5874	6077	5943	5962	5956.5	5300.3	38.2568
4	2118	2109	2099	2149	2130	2111	2119.33	317.0666667	41.5125
5	4102	4152	4174	5031	5040	5032	4588.5	239075.1	19.4481
6	3022	2932	2913	2934	2875	2841	2919.5	3811.5	33.4952
7	3030	3042	3028	3709	3671	3687	3361.17	129138.1667	19.4193
8	4707	4472	4336	5073	4898	45989	11579.2	284241355.8	-3.2633
9	3859	3822	3850	4110	4067	4110	3969.67	19446.66667	29.0866
10	3227	3205	3242	3599	3591	3535	3399.83	37444.96667	24.8952
11	2521	2499	2499	2551	2552	2570	2532	900.8	38.523
12	5921	5766	5844	5691	5777	5743	5790.33	6566.266667	37.0809
13	2792	2752	2716	2765	2786	2773	2764	759.6	40.0249
14	2863	2835	2859	2891	2844	2841	2855.5	418.3	42.8988
15	3218	3149	3124	3241	3189	3197	3186.33	1878.266667	37.3283
16	3020	3008	3016	3235	3162	3140	3096.83	9105.766667	30.2252
17	4277	4150	3992	4593	4298	4219	4254.83	39613.36667	26.5992
18	3125	3119	3127	4120	4088	4138	3619.5	295284.3	16.4706

(b) Plot the surface defect and thickness SN ratio effect plots using only Test Wafer 1 and Test Wafer 3 data.

透過直交表，將六個變數進行三種水準的表現進行交錯實驗，共可得到實驗數據18組。將每個變數的水準表現對應到的實驗組別所對應到的S/N ratio取平均後，根據平均繪出主因素分析圖。

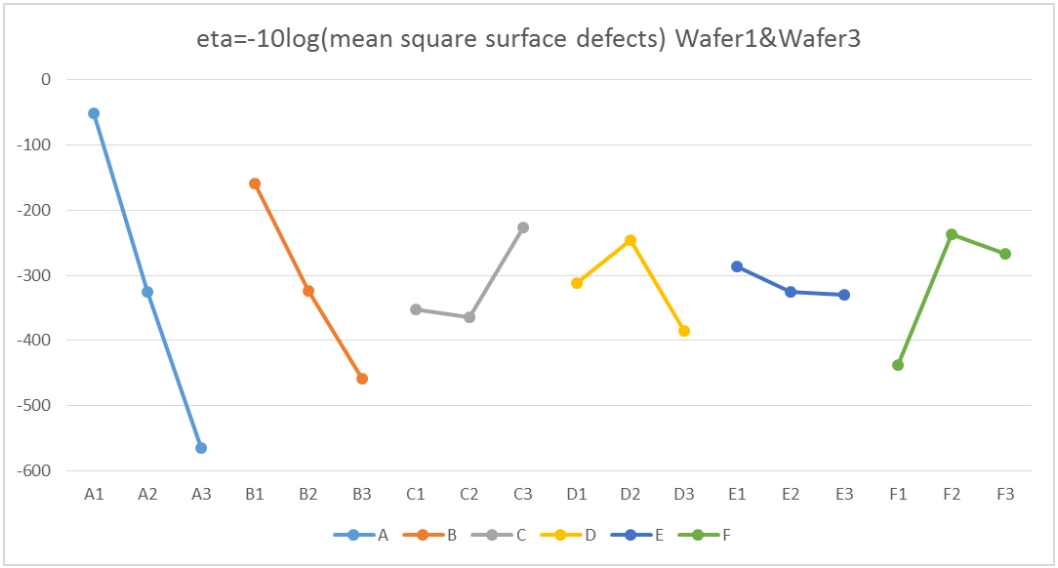
Defect的六個參數在三個水準之下的運作狀態平均S/N ratio如下表：

A1		A2		A3		B1		B2		B3		C1		C2		C3	
1	1.761	4	-155.630	7	-529.042	1	1.761	2	-34.247	3	-92.414	1	1.761	2	-34.247	3	-92.414
2	-34.247	5	-671.679	8	-502.805	4	-155.630	5	-671.679	6	-439.589	4	-155.630	5	-671.679	6	-439.589
3	-92.414	6	-439.589	9	-769.002	7	-529.042	8	-502.805	9	-769.002	9	-769.002	7	-529.042	8	-502.805
10	-92.414	13	-43.158	16	-139.002	10	-92.414	11	1.761	12	-92.415	11	1.761	12	-92.415	10	-92.414
11	1.761	14	-92.414	17	-638.847	13	-43.158	14	-92.414	15	-549.393	15	-549.393	13	-43.158	14	-92.414
12	-92.415	15	-549.393	18	-811.867	16	-139.002	17	-638.847	18	-811.867	17	-638.847	18	-811.867	16	-139.002
AVG	-51.328	AVG	-325.311	AVG	-565.094	AVG	-159.581	AVG	-323.039	AVG	-459.113	AVG	-351.558	AVG	-363.735	AVG	-226.440
D1		D2		D3		E1		E2		E3		F1		F2		F3	
1	1.761	2	-34.247	3	-92.414	1	1.761	2	-34.247	3	-92.414	1	1.761	2	-34.247	3	-92.414
6	-439.589	4	-155.630	5	-671.679	6	-439.589	4	-155.630	5	-671.679	5	-671.679	6	-439.589	4	-155.630
7	-529.042	8	-502.805	9	-769.002	8	-502.805	9	-769.002	7	-529.042	8	-502.805	9	-769.002	7	-529.042
11	1.761	12	-92.415	10	-92.414	12	-92.415	10	-92.414	11	1.761	10	-92.414	11	1.761	12	-92.415
14	-92.414	15	-549.393	13	-43.158	13	-43.158	14	-92.414	15	-549.393	15	-549.393	13	-43.158	14	-92.414
18	-811.867	16	-139.002	17	-638.847	17	-638.847	18	-811.867	16	-139.002	18	-811.867	16	-139.002	17	-638.847
AVG	-311.565	AVG	-245.582	AVG	-384.586	AVG	-285.842	AVG	-325.929	AVG	-329.962	AVG	-437.733	AVG	-237.206	AVG	-266.794

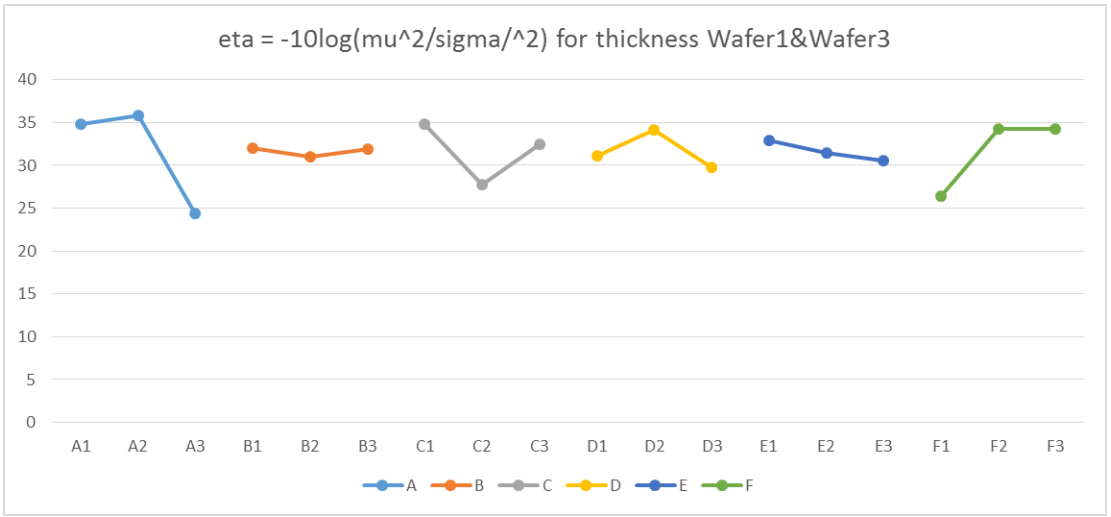
Thickness的六個參數在三個水準之下的運作狀態平均S/N ratio如下表：

A1		A2		A3		B1		B2		B3		C1		C2		C3	
1	35.739	4	41.512	7	19.419	1	35.739	2	34.167	3	38.257	1	35.739	2	34.167	3	38.257
2	34.167	5	19.448	8	24.704	4	41.512	5	19.448	6	33.495	4	41.512	5	19.448	6	33.495
3	38.257	6	33.495	9	29.087	7	19.419	8	24.704	9	29.087	9	29.087	7	19.419	8	24.704
10	24.895	13	40.025	16	30.225	10	24.895	11	38.523	12	37.081	11	38.523	12	37.081	10	24.895
11	38.523	14	42.899	17	26.599	13	40.025	14	42.899	15	37.328	15	37.328	13	40.025	14	42.899
12	37.081	15	37.328	18	16.471	16	30.225	17	26.599	18	16.471	17	26.599	18	16.471	16	30.225
AVG	34.777	AVG	35.785	AVG	24.417	AVG	31.969	AVG	31.057	AVG	31.953	AVG	34.798	AVG	27.768	AVG	32.413
D1		D2		D3		E1		E2		E3		F1		F2		F3	
1	35.739	2	34.167	3	38.257	1	35.739	2	34.167	3	38.257	1	35.739	2	34.167	3	38.257
6	33.495	4	41.512	5	19.448	6	33.495	4	41.512	5	19.448	5	19.448	6	33.495	4	41.512
7	19.419	8	24.704	9	29.087	8	24.704	9	29.087	7	19.419	8	24.704	9	29.087	7	19.419
11	38.523	12	37.081	10	24.895	12	37.081	10	24.895	11	38.523	10	24.895	11	38.523	12	37.081
14	42.899	15	37.328	13	40.025	13	40.025	14	42.899	15	37.328	15	37.328	13	40.025	14	42.899
18	16.471	16	30.225	17	26.599	17	26.599	18	16.471	16	30.225	18	16.471	16	30.225	17	26.599
AVG	31.091	AVG	34.170	AVG	29.718	AVG	32.941	AVG	31.505	AVG	30.533	AVG	26.431	AVG	34.254	AVG	34.295

根據上表所繪出的Defect主因素分析圖如下：



根據上表所繪出的thickness主因素分析圖如下：



(c) Suggest the optimal factor setting with results of (a) and (b).

綜合考量Defect與thickness這兩項指標的S/N ratio，可選擇每個變數中可使S/N ratio最大者做為選項，六個變數分別挑選出來的組合及為最佳解。

根據上面兩個指標的主因素分析圖，選擇的最佳參數組合為：[A1, B1, C3, D2, E1, F2]。

(d) Use the additive model to predict the results of the suggested optimal factor settings using Test Wafer 1 and Test Wafer 3.

為驗正挑選的組合為最佳解，我們透過 addictive model 來與原先的 S/N ratio 比較。分別計算原始組合的參數水準與平均 S/N ratio 的差值，再進行相加做比較。結果如下表所示：

	start condition			optimum condition		
		contribution			contribution	
Factor	setting	surface defects	thickness	setting	surface defects	thickness
A	A2	-11.39963781	4.12490932	A1	262.5828634	3.117303746
B	B2	-9.127591353	-0.602976098	B1	154.330009	0.309620928
C	C1	-37.64740501	3.138375753	C3	87.47137921	3.138375753
D	D3	-70.67476975	-1.941225184	D2	68.32881438	-1.941225184
E	E1	28.06889595	1.28083151	E1	28.06889595	1.28083151
F	F1	-123.822005	-5.228857194	F2	76.7046741	2.593989804
overall mean		-313.9109746	31.65970545		-313.9109746	31.65970545
total		-538.5134876	32.43076355		363.5756614	40.15860201

從結果我們可以看到，透過選擇最佳組合的參數表現，在Surface defect表現中提升了近800，而在thickness表現中則提升了約8，優化效果十分顯著。

(e) Re-do (a)-(d) for Test Wafer 2 and compare and discuss the results.

僅針對wafer2的三個測量點進行分析，可得defect的S/N ratio如下表：

expt no.	test wafer 2			SN
	top	center	bottom	
1	2	0	0	-1.2494
2	180	5	0	-40.338
3	360	38	135	-46.968
4	17	20	16	-24.983
5	487	810	400	-55.454
6	2430	207	2	-62.972
7	1620	117	30	-59.443
8	360	1	2	-46.355
9	3000	1000	1000	-65.643
10	3	0	0	-4.7712
11	5	0	0	-9.2082
12	216	5	4	-41.922
13	810	16	1	-53.4
14	90	6	1	-34.333
15	2530	2080	2080	-67.005
16	54	0	8	-29.971
17	1000	3	1	-55.229
18	5000	1000	1000	-69.542

thickness的S/N ratio如下表：

expt no.	test wafer 2			mu	var	SN
	top	center	bottom			
1	1975	1934	1907	1938.67	1172.33	2.18452
2	5201	5254	5309	5254.67	2916.33	2.55708
3	6152	5910	5886	5982.67	21649.3	-5.5855
4	2140	2125	2108	2124.33	256.333	9.18418
5	4556	4504	4560	4540	976	6.67606
6	2833	2837	2828	2832.67	20.3333	21.4399
7	3486	3333	3389	3402.67	5992.33	-2.4578
8	4407	4156	4094	4219	27469	-8.1363
9	3871	3922	3904	3899	669	7.65527
10	3468	3450	3420	3446	588	7.67938
11	2576	2537	2512	2541.67	1040.33	3.87946
12	5780	5695	5814	5763	3757	1.85807
13	2684	2635	2606	2641.67	1554.33	2.30334
14	2829	2864	2839	2844	325	9.42046
15	3261	3205	3223	3229.67	817.333	5.96758
16	3072	3151	3139	3120.67	1812.33	2.36009
17	3888	3681	3572	3713.67	25764.3	-8.4122
18	3567	3563	3520	3550	679	7.18359

Defect的六個參數在三個水準之下的運作狀態平均S/N ratio如下表：

A1		A2		A3		B1		B2		B3		C1		C2		C3	
1	-1.249	4	-24.983	7	-59.443	1	-1.249	2	-40.338	3	-46.968	1	-1.249	2	-40.338	3	-46.968
2	-40.338	5	-55.454	8	-46.355	4	-24.983	5	-55.454	6	-62.972	4	-24.983	5	-55.454	6	-62.972
3	-46.968	6	-62.972	9	-65.643	7	-59.443	8	-46.355	9	-65.643	9	-65.643	7	-59.443	8	-46.355
10	-4.771	13	-53.400	16	-29.971	10	-4.771	11	-9.208	12	-41.922	11	-9.208	12	-41.922	10	-4.771
11	-9.208	14	-34.333	17	-55.229	13	-53.400	14	-34.333	15	-67.005	15	-67.005	13	-53.400	14	-34.333
12	-41.922	15	-67.005	18	-69.542	16	-29.971	17	-55.229	18	-69.542	17	-55.229	18	-69.542	16	-29.971
	-24.076		-49.691		-54.364		-28.970		-40.153		-59.009		-37.220		-53.350		-37.562
D1		D2		D3		E1		E2		E3		F1		F2		F3	
1	-1.249	2	-40.338	3	-46.968	1	-1.249	2	-40.338	3	-46.968	1	-1.249	2	-40.338	3	-46.968
6	-62.972	4	-24.983	5	-55.454	6	-62.972	4	-24.983	5	-55.454	5	-55.454	6	-62.972	4	-24.983
7	-59.443	8	-46.355	9	-65.643	8	-46.355	9	-65.643	7	-59.443	8	-46.355	9	-65.643	7	-59.443
11	-9.208	12	-41.922	10	-4.771	12	-41.922	10	-4.771	11	-9.208	10	-4.771	11	-9.208	12	-41.922
14	-34.333	15	-67.005	13	-53.400	13	-53.400	14	-34.333	15	-67.005	15	-67.005	13	-53.400	14	-34.333
18	-69.542	16	-29.971	17	-55.229	17	-55.229	18	-69.542	16	-29.971	18	-69.542	16	-29.971	17	-55.229
	-39.458		-41.762		-46.911		-43.521		-39.935		-44.675		-40.730		-43.589		-43.813

Thickness的六個參數在三個水準之下的運作狀態平均S/N ratio如下表：

A1		A2		A3		B1		B2		B3		C1		C2		C3	
1	2.185	4	9.184	7	-2.458	1	2.185	2	2.557	3	-5.585	1	2.185	2	2.557	3	-5.585
2	2.557	5	6.676	8	-8.136	4	9.184	5	6.676	6	21.440	4	9.184	5	6.676	6	21.440
3	-5.585	6	21.440	9	7.655	7	-2.458	8	-8.136	9	7.655	9	7.655	7	-2.458	8	-8.136
10	7.679	13	2.303	16	2.360	10	7.679	11	3.879	12	1.858	11	3.879	12	1.858	10	7.679
11	3.879	14	9.420	17	-8.412	13	2.303	14	9.420	15	5.968	15	5.968	13	2.303	14	9.420
12	1.858	15	5.968	18	7.184	16	2.360	17	-8.412	18	7.184	17	-8.412	18	7.184	16	2.360
	2.096		9.165		-0.301		3.542		0.997		6.420		3.410		3.020		4.530
D1		D2		D3		E1		E2		E3		F1		F2		F3	
1	2.185	2	2.557	3	-5.585	1	2.185	2	2.557	3	-5.585	1	2.185	2	2.557	3	-5.585
6	21.440	4	9.184	5	6.676	6	21.440	4	9.184	5	6.676	5	6.676	6	21.440	4	9.184
7	-2.458	8	-8.136	9	7.655	8	-8.136	9	7.655	7	-2.458	8	-8.136	9	7.655	7	-2.458
11	3.879	12	1.858	10	7.679	12	1.858	10	7.679	11	3.879	10	7.679	11	3.879	12	1.858
14	9.420	15	5.968	13	2.303	13	2.303	14	9.420	15	5.968	15	5.968	13	2.303	14	9.420
18	7.184	16	2.360	17	-8.412	17	-8.412	18	7.184	16	2.360	18	7.184	16	2.360	17	-8.412
	6.942		2.298		1.719		1.873		7.280		1.807		3.592		6.699		0.668

為驗正挑選的組合為最佳解，我們透過 addictive model 來與原先的 S/N ratio 比較。分別計算原始組合的參數水準與平均 S/N ratio 的差值，再進行相加做比較。結果如下表所示：

start condition			optimum condition		
	contribution			contribution	
setting	surface defects	thickness	setting	surface defects	thickness
A2	-6.980958871	5.51207	A1	18.63435885	-1.5577
B2	2.557577827	-2.6557	B1	13.74078022	-0.1109
C1	5.490873856	-0.2434	C3	5.148548355	0.87648
D3	-4.200486963	-1.9338	D1	3.25229637	-1.9338
E1	-0.810786184	-1.7803	E2	2.775368898	-1.7803
F1	1.980876377	-0.0607	F2	-0.878209303	-0.0607
ean	-42.71044813	3.65318		-42.71044813	3.65318
	-44.67335209	2.49135		-0.037304743	-0.9137

從表中我們可看到，選擇最佳組合會使surface defects提升約44，但在thickness上卻是退步了大約3.5。雖然沒有在兩個指標都成長，但在總合評估下仍然是較原來的進步。

2. Please find an orthogonal array suitable for an experiment for three 2-level factors and nine 3-level factors. Provide the experimental matrix.

For three 2-level factor, an $L_4(2^3)$ is sufficient.

	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

For nine 3-level factors, an $L_{27}(3^{13})$is sufficient.

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2
3	1	1	1	1	3	3	3	3	3
4	1	2	2	2	1	1	1	2	2
5	1	2	2	2	2	2	2	3	3
6	1	2	2	2	3	3	3	1	1
7	1	3	3	3	1	1	1	3	3
8	1	3	3	3	2	2	2	1	1
9	1	3	3	3	3	3	3	2	2
10	2	1	2	3	1	2	3	1	2
11	2	1	2	3	2	3	1	2	3
12	2	1	2	3	3	1	2	3	1
13	2	2	3	1	1	2	3	2	3
14	2	2	3	1	2	3	1	3	1
15	2	2	3	1	3	1	2	1	2
16	2	3	1	2	1	2	3	3	1
17	2	3	1	2	2	3	1	1	2
18	2	3	1	2	3	1	2	2	3
19	3	1	3	2	1	3	2	1	3
20	3	1	3	2	2	1	3	2	1
21	3	1	3	2	3	2	1	3	2
22	3	2	1	3	1	3	2	2	1
23	3	2	1	3	2	1	3	3	2
24	3	2	1	3	3	2	1	1	3
25	3	3	2	1	1	3	2	3	2
26	3	3	2	1	2	1	3	1	3
27	3	3	2	1	3	2	1	2	1