1. A 2^2 factorial design has produced the following model relating machined part surface finish (y) to feed rate (x1) and nose radius (x2) of the tool ($x1$, $x2$ are in coded units of ± 1):

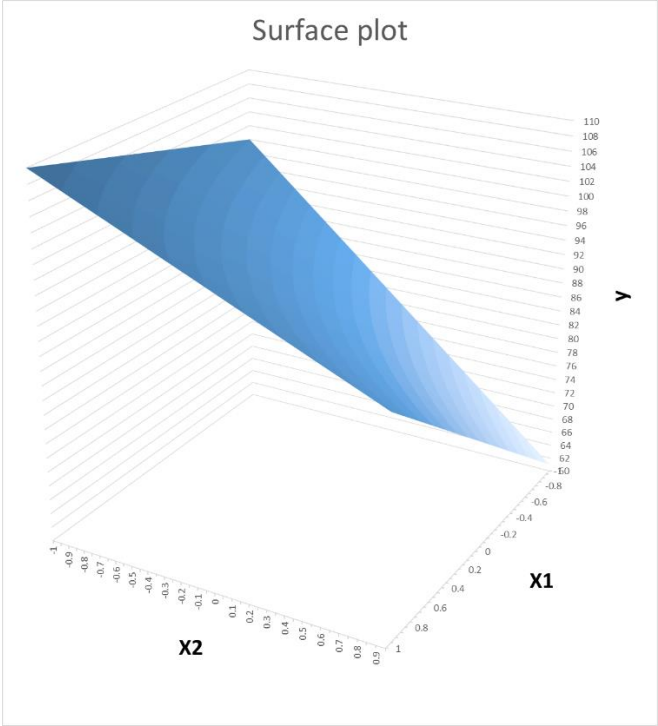
$$\hat{y} = 90 + 10x_1 - 15x_2 + 5x_1x_2$$

(a) What is the average surface finish across the experiment?
將x_1與x_2的範圍取-1~1之間以0.5為間隔，作meshing後輸出所有的x_1與x_2的組合後，計算所有輸出的平均。結果輸出如下，可得到overall的平均為90

	-1	-0.95	-0.9	-0.85	-0.8	-0.75	-0.7	-0.65	-0.6	-0.55	-0.5	-0.45	-0.4	-0.35	-0.3	-0.25	-0.2	-0.15	-0.1	-0.05	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
-1	100	100.25	100.5	100.75	101	101.25	101.5	101.75	102	102.25	102.5	102.75	103	103.25	103.5	103.75	104	104.25	104.5	104.75	105	105.25	105.5	105.75	106	106.25	106.5	106.75	107	107.25	107.5	107.75	108	108.25	108.5	108.75	109	109.25	109.5	109.75	110
-1	99	99.2625	99.525	99.7875	100.05	100.3125	100.575	100.8375	101.1	101.3625	101.625	101.8875	102.15	102.4125	102.675	102.9375	103.2	103.4625	103.725	103.9875	104.25	104.5125	104.775	105.0375	105.3	105.5625	105.825	106.0875	106.35	106.6125	106.875	107.1375	107.4	107.6625	107.925	108.1875	108.45	108.7125	108.975	109.2375	109.5
-0.9	98	98.275	98.55	98.825	99.1	99.375	99.65	99.925	100.2	100.475	100.75	101.025	101.3	101.575	101.85	102.125	102.4	102.675	102.95	103.225	103.5	103.775	104.05	104.325	104.6	104.875	105.15	105.425	105.7	105.975	106.25	106.525	106.8	107.075	107.35	107.625	107.9	108.175	108.45	108.725	109
-0.9	97	97.2875	97.575	97.8625	98.15	98.4375	98.725	99.0125	99.3	99.5875	99.875	100.1625	100.45	100.7375	101.025	101.3125	101.6	101.8875	102.175	102.4625	102.75	103.0375	103.325	103.6125	103.9	104.1875	104.475	104.7625	105.05	105.3375	105.625	105.9125	106.2	106.4875	106.775	107.0625	107.35	107.6375	107.925	108.2125	108.5
-0.8	96	96.3	96.6	96.9	97.2	97.5	97.8	98.1	98.4	98.7	99	99.3	99.6	99.9	100.2	100.5	100.8	101.1	101.4	101.7	102	102.3	102.6	102.9	103.2	103.5	103.8	104.1	104.4	104.7	105	105.3	105.6	105.9	106.2	106.5	106.8	107.1	107.4	107.7	108
-0.8	95	95.3125	95.625	95.9375	96.25	96.5625	96.875	97.1875	97.5	97.8125	98.125	98.4375	98.75	99.0625	99.375	99.6875	100	100.3125	100.625	100.9375	101.25	101.5625	101.875	102.1875	102.5	102.8125	103.125	103.4375	103.75	104.0625	104.375	104.6875	105	105.3125	105.625	105.9375	106.25	106.5625	106.875	107.1875	107.5
-0.7	94	94.325	94.65	94.975	95.3	95.625	95.95	96.275	96.6	96.925	97.25	97.575	97.9	98.225	98.55	98.875	99.2	99.525	99.85	100.175	100.5	100.825	101.15	101.475	101.8	102.125	102.45	102.775	103.1	103.425	103.75	104.075	104.4	104.725	105.05	105.375	105.7	106.025	106.35	106.675	107
-0.7	93	93.3375	93.675	94.0125	94.35	94.6875	95.025	95.3625	95.7	96.0375	96.375	96.7125	97.05	97.3875	97.725	98.0625	98.4	98.7375	99.075	99.4125	99.75	100.0875	100.425	100.7625	101.1	101.4375	101.775	102.1125	102.45	102.7875	103.125	103.4625	103.8	104.1375	104.475	104.8125	105.15	105.4875	105.825	106.1625	106.5
-0.6	92	92.35	92.7	93.05	93.4	93.75	94.1	94.45	94.8	95.15	95.5	95.85	96.2	96.55	96.9	97.25	97.6	97.95	98.3	98.65	99	99.35	99.7	100.05	100.4	100.75	101.1	101.45	101.8	102.15	102.5	102.85	103.2	103.55	103.9	104.25	104.6	104.95	105.3	105.65	106
-0.6	91	91.3625	91.725	92.0875	92.45	92.8125	93.175	93.5375	93.9	94.2625	94.625	94.9875	95.35	95.7125	96.075	96.4375	96.8	97.1625	97.525	97.8875	98.25	98.6125	98.975	99.3375	99.7	100.0625	100.425	100.7875	101.15	101.5125	101.875	102.2375	102.6	102.9625	103.325	103.6875	104.05	104.4125	104.775	105.1375	105.5
-0.5	90	90.375	90.75	91.125	91.5	91.875	92.25	92.625	93	93.375	93.75	94.125	94.5	94.875	95.25	95.625	96	96.375	96.75	97.125	97.5	97.875	98.25	98.625	99	99.375	99.75	100.125	100.5	100.875	101.25	101.625	102	102.375	102.75	103.125	103.5	103.875	104.25	104.625	105
-0.5	89	89.3875	89.775	90.1625	90.55	90.9375	91.325	91.7125	92.1	92.4875	92.875	93.2625	93.65	94.0375	94.425	94.8125	95.2	95.5875	95.975	96.3625	96.75	97.1375	97.525	97.9125	98.3	98.6875	99.075	99.4625	99.85	100.2375	100.625	101.0125	101.4	101.7875	102.175	102.5625	102.95	103.3375	103.725	104.1125	104.5
-0.4	88	88.4	88.8	89.2	89.6	90	90.4	90.8	91.2	91.6	92	92.4	92.8	93.2	93.6	94	94.4	94.8	95.2	95.6	96	96.4	96.8	97.2	97.6	98	98.4	98.8	99.2	99.6	100	100.4	100.8	101.2	101.6	102	102.4	102.8	103.2	103.6	104
-0.3	87	87.4125	87.825	88.2375	88.65	89.0625	89.475	89.8875	90.3	90.7125	91.125	91.5375	91.95	92.3625	92.775	93.1875	93.6	94.0125	94.425	94.8375	95.25	95.6625	96.075	96.4875	96.9	97.3125	97.725	98.1375	98.55	98.9625	99.375	99.7875	100.2	100.6125	101.025	101.4375	101.85	102.2625	102.675	103.0875	103.5
-0.3	86	86.425	86.85	87.275	87.7	88.125	88.55	88.975	89.4	89.825	90.25	90.675	91.1	91.525	91.95	92.375	92.8	93.225	93.65	94.075	94.5	94.925	95.35	95.775	96.2	96.625	97.05	97.475	97.9	98.325	98.75	99.175	99.6	100.025	100.45	100.875	101.3	101.725	102.15	102.575	103
-0.2	85	85.4375	85.875	86.3125	86.75	87.1875	87.625	88.0625	88.5	88.9375	89.375	89.8125	90.25	90.6875	91.125	91.5625	92	92.4375	92.875	93.3125	93.75	94.1875	94.625	95.0625	95.5	95.9375	96.375	96.8125	97.25	97.6875	98.125	98.5625	99	99.4375	99.875	100.3125	100.75	101.1875	101.625	102.0625	102.5
-0.2	84	84.45	84.9	85.35	85.8	86.25	86.7	87.15	87.6	88.05	88.5	88.95	89.4	89.85	90.3	90.75	91.2	91.65	92.1	92.55	93	93.45	93.9	94.35	94.8	95.25	95.7	96.15	96.6	97.05	97.5	97.95	98.4	98.85	99.3	99.75	100.2	100.65	101.1	101.55	102
-0.1	83	83.4625	83.925	84.3875	84.85	85.3125	85.775	86.2375	86.7	87.1625	87.625	88.0875	88.55	89.0125	89.475	89.9375	90.4	90.8625	91.325	91.7875	92.25	92.7125	93.175	93.6375	94.1	94.5625	95.025	95.4875	95.95	96.4125	96.875	97.3375	97.8	98.2625	98.725	99.1875	99.65	100.1125	100.575	101.0375	101.5
-0.1	82	82.475	82.95	83.425	83.9	84.375	84.85	85.325	85.8	86.275	86.75	87.225	87.7	88.175	88.65	89.125	89.6	90.075	90.55	91.025	91.5	91.975	92.45	92.925	93.4	93.875	94.35	94.825	95.3	95.775	96.25	96.725	97.2	97.675	98.15	98.625	99.1	99.575	100.05	100.525	101
-0	81	81.4875	81.975	82.4625	82.95	83.4375	83.925	84.4125	84.9	85.3875	85.875	86.3625	86.85	87.3375	87.825	88.3125	88.8	89.2875	89.775	90.2625	90.75	91.2375	91.725	92.2125	92.7	93.1875	93.675	94.1625	94.65	95.1375	95.625	96.1125	96.6	97.0875	97.575	98.0625	98.55	99.0375	99.525	100.0125	100.5
0	80	80.5	81	81.5	82	82.5	83	83.5	84	84.5	85	85.5	86	86.5	87	87.5	88	88.5	89	89.5	90	90.5	91	91.5	92	92.5	93	93.5	94	94.5	95	95.5	96	96.5	97	97.5	98	98.5	99	99.5	100
0.05	79	79.5125	80.025	80.5375	81.05	81.5625	82.075	82.5875	83.1	83.6125	84.125	84.6375	85.15	85.6625	86.175	86.6875	87.2	87.7125	88.225	88.7375	89.25	89.7625	90.275	90.7875	91.3	91.8125	92.325	92.8375	93.35	93.8625	94.375	94.8875	95.4	95.9125	96.425	96.9375	97.45	97.9625	98.475	98.9875	99.5
0.1	78	78.525	79.05	79.575	80.1	80.625	81.15	81.675	82.2	82.725	83.25	83.775	84.3	84.825	85.35	85.875	86.4	86.925	87.45	87.975	88.5	89.025	89.55	90.075	90.6	91.125	91.65	92.175	92.7	93.225	93.75	94.275	94.8	95.325	95.85	96.375	96.9	97.425	97.95	98.475	99
0.15	77	77.5375	78.075	78.6125	79.15	79.6875	80.225	80.7625	81.3	81.8375	82.375	82.9125	83.45	83.9875	84.525	85.0625	85.6	86.1375	86.675	87.2125	87.75	88.2875	88.825	89.3625	89.9	90.4375	90.975	91.5125	92.05	92.5875	93.125	93.6625	94.2	94.7375	95.275	95.8125	96.35	96.8875	97.425	97.9625	98.5
0.2	76	76.55	77.1	77.65	78.2	78.75	79.3	79.85	80.4	80.95	81.5	82.05	82.6	83.15	83.7	84.25	84.8	85.35	85.9	86.45	87	87.55	88.1	88.65	89.2	89.75	90.3	90.85	91.4	91.95	92.5	93.05	93.6	94.15	94.7	95.25	95.8	96.35	96.9	97.45	98
0.25	75	75.5625	76.125	76.6875	77.25	77.8125	78.375	78.9375	79.5	80.0625	80.625	81.1875	81.75	82.3125	82.875	83.4375	84	84.5625	85.125	85.6875	86.25	86.8125	87.375	87.9375	88.5	89.0625	89.625	90.1875	90.75	91.3125	91.875	92.4375	93	93.5625	94.125	94.6875	95.25	95.8125	96.375	96.9375	97.5
0.3	74	74.575	75.15	75.725	76.3	76.875	77.45	78.025	78.6	79.175	79.75	80.325	80.9	81.475	82.05	82.625	83.2	83.775	84.35	84.925	85.5	86.075	86.65	87.225	87.8	88.375	88.95	89.525	90.1	90.675	91.25	91.825	92.4	92.975	93.55	94.125	94.7	95.275	95.85	96.425	97
0.35	73	73.5875	74.175	74.7625	75.35	75.9375	76.525	77.1125	77.7	78.2875	78.875	79.4625	80.05	80.6375	81.225	81.8125	82.4	82.9875	83.575	84.1625	84.75	85.3375	85.925	86.5125	87.1	87.6875	88.275	88.8625	89.45	90.0375	90.625	91.2125	91.8	92.3875	92.975	93.5625	94.15	94.7375	95.325	95.9125	96.5
0.4	72	72.6	73.2	73.8	74.4	75	75.6	76.2	76.8	77.4	78	78.6	79.2	79.8	80.4	81	81.6	82.2	82.8	83.4	84	84.6	85.2	85.8	86.4	87	87.6	88.2	88.8	89.4	90	90.6	91.2	91.8	92.4	93	93.6	94.2	94.8	95.4	96
0.45	71	71.6125	72.225	72.8375	73.45	74.0625	74.675	75.2875	75.9	76.5125	77.125	77.7375	78.35	78.9625	79.575	80.1875	80.8	81.4125	82.025	82.6375	83.25	83.8625	84.475	85.0875	85.7	86.3125	86.925	87.5375	88.15	88.7625	89.375	89.9875	90.6	91.2125	91.825	92.4375	93.05	93.6625	94.275	94.8875	95.5
0.5	70	70.625	71.25	71.875	72.5	73.125	73.75	74.375	75	75.625	76.25	76.875	77.5	78.125	78.75	79.375	80	80.625	81.25	81.875	82.5	83.125	83.75	84.375	85	85.625	86.25	86.875	87.5	88.125	88.75	89.375	90	90.625	91.25	91.875	92.5	93.125	93.75	94.375	95
0.55	69	69.6375	70.275	70.9125	71.55	72.1875	72.825	73.4625	74.1	74.7375	75.375	76.0125	76.65	77.2875	77.925	78.5625	79.2	79.8375	80.475	81.1125	81.75	82.3875	83.025	83.6625	84.3	84.9375	85.575	86.2125	86.85	87.4875	88.125	88.7625	89.4	90.0375	90.675	91.3125	91.95	92.5875	93.225	93.8625	94.5
0.6	68	68.65	69.3	69.95	70.6	71.25	71.9	72.55	73.2	73.85	74.5	75.15	75.8	76.45	77.1	77.75	78.4	79.05	79.7	80.35	81	81.65	82.3	82.95	83.6	84.25	84.9	85.55	86.2	86.85	87.5	88.15	88.8	89.45	90.1	90.75	91.4	92.05	92.7	93.35	94
0.65	67	67.6625	68.325	68.9875	69.65	70.3125	70.975	71.6375	72.3	72.9625	73.625	74.2875	74.95	75.6125	76.275	76.9375	77.6	78.2625	78.925	79.5875	80.25	80.9125	81.575	82.2375	82.9	83.5625	84.225	84.8875	85.55	86.2125	86.875	87.5375	88.2	88.8625	89.525	90.1875	90.85	91.5125	92.175	92.8375	93.5
0.7	66	66.675	67.35	68.025	68.7	69.375	70.05	70.725	71.4	72.075	72.75	73.425	74.1	74.775	75.45	76.125	76.8	77.475	78.15	78.825	79.5	80.175	80.85	81.525	82.2	82.875	83.55	84.225	84.9	85.575	86.25	86.925	87.6	88.275	88.95	89.625	90.3	90.975	91.65	92.325	93
0.75	65	65.6875	66.375	67.0625	67.75	68.4375	69.125	69.8125	70.5	71.1875	71.875	72.5625	73.25	73.9375	74.625	75.3125	76	76.6875	77.375	78.0625	78.75	79.4375	80.125	80.8125	81.5	82.1875	82.875	83.5625	84.25	84.9375	85.625	86.3125	87	87.6875	88.375	89.0625	89.75	90.4375	91.125	91.8125	92.5
0.8	64	64.7	65.4	66.1	66.8	67.5	68.2	68.9	69.6	70.3	71	71.7	72.4	73.1	73.8	74.5	75.2	75.9	76.6	77.3	78	78.7	79.4	80.1	80.8	81.5	82.2	82.9	83.6	84.3	85	85.7	86.4	87.1	87.8	88.5	89.2	89.9	90.6	91.3	92
0.85	63	63.7125	64.425	65.1375	65.85	66.5625	67.275	67.9875	68.7	69.4125	70.125	70.8375	71.55	72.2625	72.975	73.6875	74.4	75.1125	75.825	76.5375	77.25	77.9625	78.675	79.3875	80.1	80.8125	81.525	82.2375	82.95	83.6625	84.375	85.0875	85.8	86.5125	87.225	87.9375	88.65	89.3625	90.075	90.7875	91.5
0.9	62	62.725	63.45	64.175	64.9	65.625	66.35	67.075	67.8	68.525	69.25	69.975	70.7	71.425	72.15	72.875	73.6	74.325	75.05	75.775	76.5	77.225	77.95	78.675	79.4	80.125	80.85	81.575	82.3	83.025	83.75	84.475	85.2	85.925	86.65	87.375	88.1	88.825	89.55	90.275	91
0.95	61	61.7375	62.475	63.2125	63.95	64.6875	65.425	66.1625	66.9	67.6375	68.375	69.1125	69.85	70.5875	71.325	72.0625	72.8	73.5375	74.275	75.0125	75.75	76.4875	77.225	77.9625	78.7	79.4375	80.175	80.9125	81.65	82.3875	83.125	83.8625	84.6	85.3375	86.075	86.8125	87.55	88.2875	89.025	89.7625	90.5
1	60	60.75	61.5	62.25	63	63.75	64.5	65.25	66	66.75	67.5	68.25	69	69.75	70.5	71.25	72	72.75	73.5	74.25	75	75.75	76.5	77.25	78	78.75	79.5	80.25	81	81.75	82.5	83.25	84	84.75	85.5	86.25	87	87.75	88.5	89.25	90

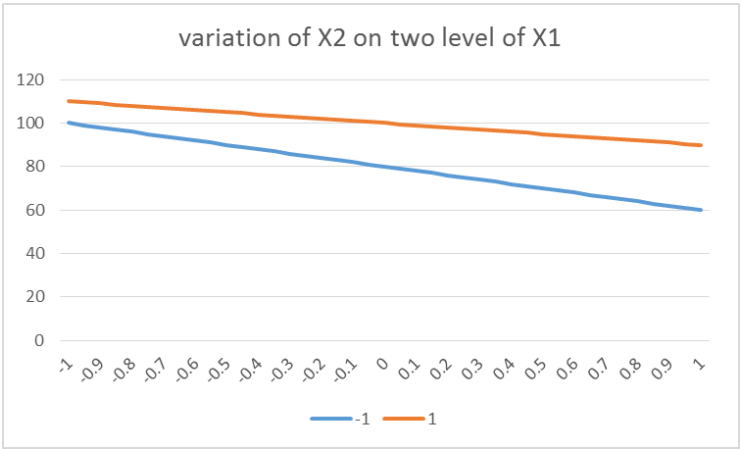
(b) What are the values for E1, E2, and E12 as obtained from the experiment?
從effect推論係數數的過程為：
B0 = overall mean
B1 = E1/2,
B2 = E2/2
B3 = E12/2
因此可以推論
E1 = 2*B1 = 20
E2 = 2*B2 = -30
E12 = 2*B3 = 10

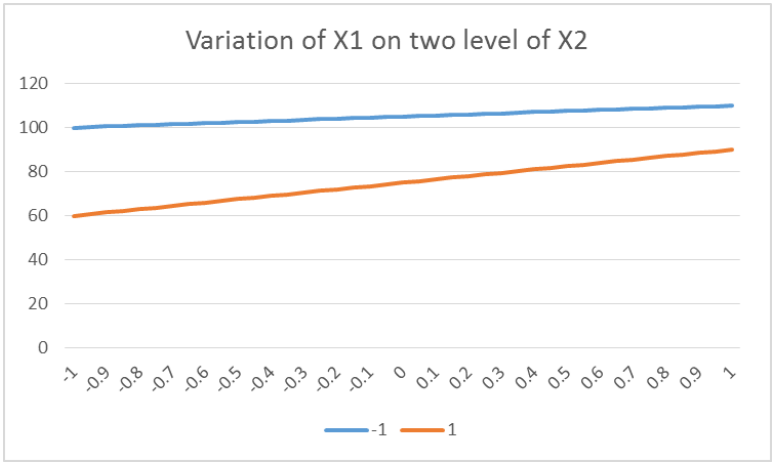
(c) Use Excel to sketch the 3D or contour response surface in the x1, x2, y coordinate space.
根據(a)所得結果會出曲面圖如下：



(d) Explain the meaning of the interaction effect.

Interaction effect指的是一個參數如何影響另一個參數對於結果的影響。在這個題目中，x2的改變會影響y，但x1的改變會影響x2對y的值，這種非完全獨立的參數關係則稱為interaction effect。





(e) If a surface finish of 100 is desired, what values of feed rate and nose radius (coded units) should be used? Identify the possible values of the feed rate and the nose radius to achieve the desired surface finish on the response surface in (c).

設feed rate為x1，nose radius為x2，從regression model中可得到y與x1和x2的關係。當y鎖定為100時，可得到x1與x2的關係式為：

$$x_1 = \frac{10 + 15x_2}{10 + 5x_2}$$

將x2從-1~1的區間以0.05為單位，找出對應的x1(x1須介於-1~1之間才有意義)，並輸出對應的y，所得結果如下：

x2	x1	y
-1	-1	100
-0.95	-0.8095238	100
-0.9	-0.6363636	100
-0.85	-0.4782609	100
-0.8	-0.3333333	100
-0.75	-0.2	100
-0.7	-0.0769231	100
-0.65	0.037037	100
-0.6	0.1428571	100
-0.55	0.2413793	100
-0.5	0.3333333	100
-0.45	0.4193548	100
-0.4	0.5	100
-0.35	0.5757576	100
-0.3	0.6470588	100
-0.25	0.7142857	100
-0.2	0.7777778	100
-0.15	0.8378378	100

-0.1	0.8947368	100
-0.05	0.9487179	100
0	1	100

2. Give an example where a dependent variable (output) y is affected by two independent variables x1 and x2 with interaction effect. Explain why there exists the interaction effect.
車輛在超車的時候，在不當的操作下，會有一定的翻車機率(y)，其中影響最顯著的兩個因子為超車時的速度(x1)與超車路徑(通常是用貝茲曲線來表示超車的猛烈度)的前進距離與橫移長度的比例(x2)。另外，超車時的翻車機率也會受到其他因素影響(noise)。
前進距離與橫移長度的比例(x2)會顯著的影響到翻車時的機率，但若將車速(x1)的高低也加入考量，則會使x2對超車時的翻車機率影響的顯著程度加劇；同樣的，超車時的速度(x1)也會顯著的影響超車時的翻車機率，但超車時的猛烈程度(x2)的高低表現也會影響到超車速度高低對翻車機率的影響程度。

3. For the problem of "Glove Box Door Alignment" experiment to achieve parallelism equal to zero,
(a) Calculate the SN ratios of the parallelisms of the experimental results;
S/N ratio的計算公式如下：

$$\eta = -10\log(\frac{\text{mean}^2}{var})$$

其中平均值為兩次run的平均，標準差為兩次run的樣本標準差。
計算S/N ratio的結果如下表所示：

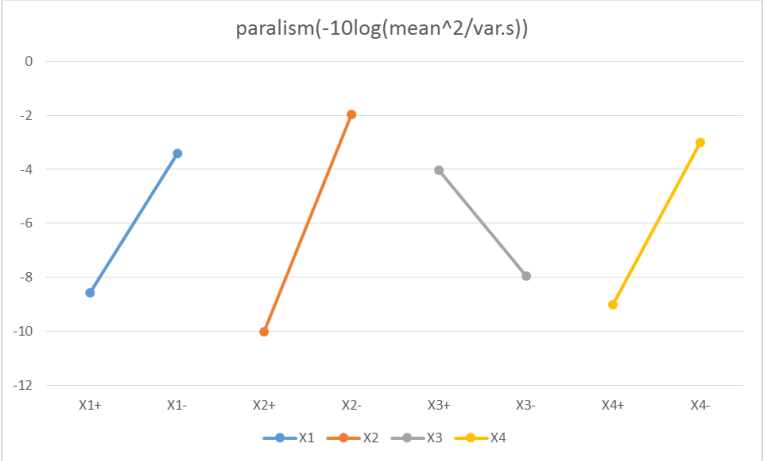
test	X1	X2	X3	X4	Run1	Run2	mean	var	S/N
1	-1	-1	-1	-1	-1.44	-0.08	-0.76	0.9248	2.044206365
2	1	-1	-1	-1	-1.79	-1.01	-1.4	0.3042	-8.090968616
3	-1	1	-1	-1	0.39	0.17	0.28	0.0242	-5.105006967
4	1	1	-1	-1	-0.5	-0.24	-0.37	0.0338	-6.074867479
5	-1	-1	1	-1	-0.2	0.17	-0.015	0.06845	24.83190934
6	1	-1	1	-1	-0.79	-0.64	-0.715	0.01125	-16.57459561
7	-1	1	1	-1	1.22	0.28	0.75	0.4418	-1.048968152
8	1	1	1	-1	0.21	0.28	0.245	0.00245	-13.89166084
9	-1	-1	-1	1	-0.4	-0.65	-0.525	0.03125	-9.454685851
10	1	-1	-1	1	-0.63	-1.19	-0.91	0.1568	-7.227367263
11	-1	1	-1	1	0.47	0.44	0.455	0.00045	-26.6281028
12	1	1	-1	1	-0.01	-0.03	-0.02	0.0002	-3.010299957
13	-1	-1	1	1	1.29	0.64	0.965	0.21125	-6.442579091
14	1	-1	1	1	-1.17	0.14	-0.515	0.85805	5.098981376
15	-1	1	1	1	0.48	1.06	0.77	0.1682	-5.471554589
16	1	1	1	1	0.4	0.34	0.37	0.0018	-18.81130943

(b) Plot the effect plot and build an additive model;

計算出每個參數的高水準表現及低水準表現的Effect，可整理於下表

X1+		X1-		X2+		X2-		X3+		X3-		X4+		X4-	
2	-8.091	1	2.04421	3	-5.105	1	2.04421	5	24.832	1	2.04421	9	-9.4547	1	2.04421
4	-6.0749	3	-5.105	4	-6.0749	2	-8.091	6	-16.57	2	-8.091	10	-7.2274	2	-8.091
6	-16.575	5	24.8319	7	-1.049	5	24.8319	7	-1.049	3	-5.105	11	-26.628	3	-5.105
8	-13.892	7	-1.049	8	-13.892	6	-16.575	8	-13.89	4	-6.0749	12	-3.0103	4	-6.0749
10	-7.2274	9	-9.4547	11	-26.628	9	-9.4547	13	-6.443	9	-9.4547	13	-6.4426	5	24.8319
12	-3.0103	11	-26.628	12	-3.0103	10	-7.2274	14	5.099	10	-7.2274	14	5.09898	6	-16.575
14	5.09898	13	-6.4426	15	-5.4716	13	-6.4426	15	-5.472	11	-26.628	15	-5.4716	7	-1.049
16	-18.811	15	-5.4716	16	-18.811	14	5.09898	16	-18.81	12	-3.0103	16	-18.811	8	-13.892
AVG	**-8.5728**		**-3.4093**		**-10.005**		**-1.9769**		**-4.039**		**-7.9434**		**-8.9934**		**-2.9887**

根據所計算出來的Effect，繪製出來的effect plot如圖所示：



Addictive model則針對所有參數的低水準與高水準建立，並從effect plot中找出最佳參數組合為(x1, x2, x3, x4) = (-1, -1, 1, -1)，完成optimal的最佳解的addictive model，也確實可以看見最佳解在Effect上比起最高與最低水準的組合都表現得更好。

	all low level			all high level			optimal	
		contribution			contribution			contribution
Factor	setting		Factor	setting		Factor	setting	
X1	X1-	2.58170663	X1	X1+	-2.58170663	X1	X1-	2.58170663
X2	X2-	4.014166929	X2	X2+	-4.014166929	X2	X2-	4.014166929
X3	X3-	-1.952332223	X3	X3+	1.952332223	X3	X3+	1.952332223
X4	X4-	3.002310352	X4	X4+	-3.002310352	X4	X4-	3.002310352
overall mean		-5.991054348	overall mean		-5.991054348	overall mean		-5.991054348
total		1.654797342	total		-13.63690604	total		5.559461787

(c) Use the effect plot to determine the optimal setting for the process and use the additive model to predict the optimal parallelism;

從effect plot中找出最佳參數組合為(x1, x2, x3, x4) = (-1, -1, 1, -1)，完成optimal的最佳解的addictive model，也確實可以看見最佳解在Effect上比起最高與最低水準的組合都表現得更好。

	all low level			all high level			optimal	
		contribution			contribution			contribution
Factor	setting		Factor	setting		Factor	setting	
X1	X1-	2.58170663	X1	X1+	-2.58170663	X1	X1-	2.58170663
X2	X2-	4.014166929	X2	X2+	-4.014166929	X2	X2-	4.014166929
X3	X3-	-1.952332223	X3	X3+	1.952332223	X3	X3+	1.952332223
X4	X4-	3.002310352	X4	X4+	-3.002310352	X4	X4-	3.002310352
overall mean		-5.991054348	overall mean		-5.991054348	overall mean		-5.991054348
total		1.654797342	total		-13.63690604	total		5.559461787

從實驗中即可以得到在(x1, x2, x3, x4) = (-1, -1, 1, -1)時的參數組合的實驗的結果，如下

optimized set									
test	X1	X2	X3	X4	Run1	Run2	mean	var	S/N ratio
5	-1	-1	1	-1	-0.2	0.17	-0.015	0.06845	24.83191

(d) Calculate all the main factor and two-factor interaction effects on the SN ratio and build a regression predictive model;
對應於單一參數對結果的影響及兩個參數的對結果的影響，可從每個參數組合的高底水準(-1&1)與對應的輸出結果的sum product進行平均，可得到每個參數組合所對應到的Effect。Regression predictive model的係數則可透過利用effect到迴歸至平面的最佳參數關係來得到。所得到的各參數組合對應到的effect及regression predictive model的係數呈現於下表：

test	X1	X2	X3	X4	X1X2	X1X3	X1X4	X2X3	X2X4	X3X4	S/N
1	-1	-1	-1	-1	1	1	1	1	1	1	**2.044206365**
2	1	-1	-1	-1	-1	-1	-1	1	1	1	**-8.090968616**
3	-1	1	-1	-1	-1	1	1	-1	-1	1	**-5.105006967**
4	1	1	-1	-1	1	-1	-1	-1	-1	1	**-6.074867479**
5	-1	-1	1	-1	1	-1	1	-1	1	-1	**24.83190934**
6	1	-1	1	-1	-1	1	-1	-1	1	-1	**-16.57459561**
7	-1	1	1	-1	-1	-1	1	1	-1	-1	**-1.048968152**
8	1	1	1	-1	1	1	-1	1	-1	-1	**-13.89166084**
9	-1	-1	-1	1	1	1	-1	1	-1	-1	**-9.454685851**
10	1	-1	-1	1	-1	-1	1	1	-1	-1	**-7.227367263**
11	-1	1	-1	1	-1	1	-1	-1	1	-1	**-26.6281028**
12	1	1	-1	1	1	-1	1	-1	1	-1	**-3.010299957**
13	-1	-1	1	1	1	-1	-1	-1	-1	1	**-6.442579091**
14	1	-1	1	1	-1	1	1	-1	-1	1	**5.098981376**
15	-1	1	1	1	-1	-1	-1	1	1	1	**-5.471554589**
16	1	1	1	1	1	1	1	1	1	1	**-18.81130943**

regression model										
	E1(X1)	E2(X2)	E3(X3)	E4(X4)	E5(X1X2)	E6(X1X3)	E7(X1X4)	E8(X2X3)	E9(X2X4)	E10(X3X4)
	-5.163	-8.028	3.905	-6.005	4.280	-8.848	11.175	-3.506	-0.946	1.269
b0	b1(X1)	b2(X2)	b3(X3)	b4(X4)	b5(X1X2)	b6(X1X3)	b7(X1X4)	b8(X2X3)	b9(X2X4)	b10(X3X4)
-5.991	-2.582	-4.014	1.952	-3.002	2.140	-4.424	5.588	-1.753	-0.473	0.634

與Excel中的回歸求解功能所得到的答案也是一致的。

	係數	標準誤	t 統計	P-值	下限 95%	上限 95%	下限 95.0%	上限 95.0%
截距	-5.99105	2.345115	-2.5547	0.050974	-12.0194	0.037255	-12.0194	0.037255
X1	-2.58171	2.345115	-1.10089	0.321099	-8.61002	3.446603	-8.61002	3.446603
X2	-4.01417	2.345115	-1.71171	0.14763	-10.0425	2.014142	-10.0425	2.014142
X3	1.952332	2.345115	0.83251	0.443055	-4.07598	7.980642	-4.07598	7.980642
X4	-3.00231	2.345115	-1.28024	0.256636	-9.03062	3.025999	-9.03062	3.025999
X1X2	2.139893	2.345115	0.91249	0.403363	-3.88842	8.168203	-3.88842	8.168203
X1X3	-4.42422	2.345115	-1.88657	0.117878	-10.4525	1.604092	-10.4525	1.604092
X1X4	5.587573	2.345115	2.382644	0.062958	-0.44074	11.61588	-0.44074	11.61588
X2X3	-1.75298	2.345115	-0.7475	0.488402	-7.78129	4.275325	-7.78129	4.275325
X2X4	-0.47279	2.345115	-0.2016	0.848172	-6.50109	5.555524	-6.50109	5.555524
X3X4	0.634417	2.345115	0.270527	0.797567	-5.39389	6.662726	-5.39389	6.662726

(e) Use the regression model to determine the optimize setting and predict the resulting optimal parallelism. Compare the results with (c).
使用excel內建的規劃求解功能，其最佳化列式如下：

$$\text{Max} \quad (y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_1x_2 + b_6x_1x_3 + b_7x_1x_4 + b_8x_2x_3 + b_9x_2x_4 + b_{10}x_3x_4)$$
$$\text{w.r.t} \quad x_1, x_2, x_3, x_4$$

s.t

$$-1 \le x_1 \le 1$$
$$-1 \le x_2 \le 1$$
$$-1 \le x_3 \le 1$$
$$-1 \le x_4 \le 1$$

所得到的最佳化結果為：當x1=-1, x2=-1, x3=1, x4=-1時，所得到的S/N ratio 最佳值為18.35692724。相比於(c)中所求得的實驗結果(S/N ratio=24.8319)來的大，推測是實驗的誤差以及模型本身在迴歸時就忽略了高次的影響，也會影響到收斂結果。但在兩個組合之中的參數設定於最佳解時是一致的，因此以應用層面上可以得知該組參數組合仍然是具有相當高可信度的最佳解。

4. The yield (as large as possible with the maximum at 100%) of a semiconductor fabrication process has been studied as a function of three factors, at the following experimental levels:

Variable	Low Level	High Level
1. Temperature (°C)	80	120
2. Pressure (psi)	50	70
3. Reaction time (min)	5	15

A 2^3 factorial design was performed. Each test, or unique combination of the three variables, was performed three times. The table provides the results of the tests.

Test	Temperature	Pressure	Time	Trial			Average
				1	2	3	
1	80	50	5	61.43(14)	58.58(18)	57.07(16)	59.03
2	120	50	5	75.62(6)	77.57(11)	75.75(2)	76.31
3	80	70	5	27.51(21)	34.03(1)	25.07(19)	28.87
4	120	70	5	51.37(10)	48.49(15)	54.37(9)	51.41
5	80	50	15	24.80(3)	20.69(7)	15.41(5)	20.30
6	120	50	15	43.58(17)	44.31(22)	36.99(13)	41.63
7	80	70	15	45.20(24)	49.53(12)	50.29(20)	48.34
8	120	70	15	70.51(8)	74.00(4)	74.68(23)	73.07

在下列的作答中，為了方便表達，令temperature的低水準表現(80)為-1，高水準表現(120)為1、壓力的低水準表現(50)為-1，高水準表現(70)為1、時間的低水準表現(5)為-1，高水準表現(15)為1。在regression model、predicted model中的x所代表的範圍皆在-1~1之間，於求得x後在等比例換算回所對應到的時計物理量。

(a) Estimate the variance of the noises.

在有多次實驗下的replicate noise to estimate overall variance of noise, 當一組實驗的重複次數相同時($\nu_1 = \nu_2 = \nu_3 = \cdots = \nu_m$)時，

$$\hat{\sigma}_\epsilon^2 = s_p^2 = \frac{s_1^2 + s_2^2 + \cdots + s_m^2}{m}$$

在本次的數據中，每次實驗皆重複三次，計算三次實驗平均、樣本變異數、及S/N ratio。其中，S/N ratio所使用公式如下：

$$\eta = -10\log(\frac{\text{mean}^2}{var.s})$$

整理計算結果於下表中：

test	temp	pres	time	trial_1	trial_2	trial_3	average	var	S/N ratio
1	-1	-1	-1	61.43	58.58	57.07	59.0267	4.9020	-28.5172
2	1	-1	-1	75.62	77.57	75.75	76.3133	1.1886	-36.9015
3	-1	1	-1	27.51	34.03	25.07	28.8700	21.4576	-15.8931
4	1	1	-1	51.37	48.49	54.37	51.4100	8.6448	-24.8534
5	-1	-1	1	24.8	20.69	15.41	20.3000	22.1571	-12.6948
6	1	-1	1	43.58	44.31	36.99	41.6267	16.2572	-20.2770
7	-1	1	1	45.2	49.53	50.29	48.3400	7.5391	-24.9129
8	1	1	1	70.51	74	74.68	73.0633	5.0052	-30.2797

將所得到的variance平均後，可得到variance of the noises = 10.89397

(b) Estimate the variance of effects.

對於Effect而言，可以下列式子計算：

$$\mathrm{E}_1 = \left(\frac{\sum y_{ij}}{N/2}\right)$$

對於variance of effects的計算，在假設每個實驗的noise都相同的條件下，可使用下列公式：

$$\mathrm{var(E)} = \mathrm{var}\left[\frac{\sum y_{ij}}{\frac{N}{2}}\right] = \left(\frac{2}{N}\right)^2 var\left(\sum y_{ij}\right) = \left(\frac{2}{N}\right)^2 \times N \times \sigma_\epsilon^2 = \frac{4}{N} \times \sigma_\epsilon^2$$

其中，N為所進行的實驗總次數，在這裡N=8*3=24，因此可得知

variance of effect = $\frac{4}{24} \times 10.89397 = 1.81566$

(c) With t test statistics and a=0.05, which effects are statistically significant?

使用test, 自由度為$\nu = \sum_{test}(\#replicates - 1) = m(n-1) = 8 \times 2 = 16$。

計算一個、兩個、參個參數組合時的effect後，可從回歸的最佳模型中得到regression model中的係數bi。其結果如下：

	avg	X1	X2	X3	X1X2	X1X3	X2X3	X1X2X3
		-1	-1	-1	1	1	1	-1
		1	-1	-1	-1	-1	1	1
		-1	1	-1	-1	1	-1	1
		1	1	-1	1	-1	-1	-1
		-1	-1	1	1	-1	-1	1
		1	-1	1	-1	1	-1	-1
		-1	1	1	-1	-1	1	-1
		1	1	1	1	1	1	1
Ei	49.86875	21.46917	1.104167	-8.0725	2.1625	28.63417	1.555833	-0.46417
bi	49.86875	10.73458	0.552083	-4.03625	1.08125	14.31708	0.777917	-0.23208

進行t檢定的目的為要檢測該係數與0的差異，因此在考量noise時，無效假設為$H_0: E_i = 0$。所使用的test statistic為

$$t = \frac{E_i - 0}{S_{effect}} \sim t_{\nu=16}$$

其中，在信心水準為95%下，上下界的範圍應為

$$E_i \pm t_{16,0.975}(S_{effect})$$

因此可從regression model的係數中整理出信賴水準95%的信賴區間。並可將其信賴區間與0做比較，若該信賴區間沒有包含0，則檢測其p-value，若p-value小於0.05，則可以認定有足夠的信心水準推翻無效假設，也可進一步認定該係數的影響為顯著的。

	B0	**B1**	**B2**	**B3**	**B4(X1X2)**	**B5(X2X3)**	**B6(X1X3)**	**B7(X1X2X3)**
upper limit	52.72524755	12.16283211	1.98033211	-2.608001224	2.509498776	15.74533211	2.206165443	1.196165443
lowe limit	47.01225245	9.306334557	-0.876165443	-5.464498776	-0.346998776	12.88883456	-0.65033211	-1.66033211
t=(Ei-0)/s	37.00931838	15.93300864	0.819440145	-5.990880514	1.604865793	21.25040212	1.154637547	-0.344474083
p-value	0	3.08109E-11	0.424574047	1.999981156	0.128077371	3.74811E-13	0.265193298	1.265024133
t test result	H1 stands	H1 stands	H0 stands	H1 stands	H0 stands	H1 stands	H0 stands	H0 stands
significant?	**significant**	**significant**	**N**	**significant**	**N**	**significant**	**N**	**N**

從表中可以看到，除了截距項之外，B1、B3、與B5都是顯著的項目，分別是對應到E_1、E_3與E_{23}。

(d) Use the significant effects to construct a mathematical prediction model.

承上題，保留檢測為顯著的係數，整理預測的反應曲面可得到下列方程式：

$$y = 49.86875 + 10.73458x_1 - 4.03625x_3 + 14.31708x_2x_3$$

(e) Use Excel's regression analysis to build the regression model and compare it to (c).

將實驗結果與組合整理如下表所示：

test	X1	X2	X3	X1X2	X1X3	X2X3	X1X2X3	output
1_1	-1	-1	-1	1	1	1	-1	**61.43**
1_2	1	-1	-1	-1	-1	1	1	**75.62**
1_3	-1	1	-1	-1	1	-1	1	**27.51**
1_4	1	1	-1	1	-1	-1	-1	**51.37**
1_5	-1	-1	1	1	-1	-1	1	**24.8**
1_6	1	-1	1	-1	1	-1	-1	**43.58**
1_7	-1	1	1	-1	-1	1	-1	**45.2**
1_8	1	1	1	1	1	1	1	**70.51**
2_1	-1	-1	-1	1	1	1	-1	**58.58**
2_2	1	-1	-1	-1	-1	1	1	**77.57**
2_3	-1	1	-1	-1	1	-1	1	**34.03**
2_4	1	1	-1	1	-1	-1	-1	**48.49**
2_5	-1	-1	1	1	-1	-1	1	**20.69**
2_6	1	-1	1	-1	1	-1	-1	**44.31**
2_7	-1	1	1	-1	-1	1	-1	**49.53**
2_8	1	1	1	1	1	1	1	**74**
3_1	-1	-1	-1	1	1	1	-1	**57.07**
3_2	1	-1	-1	-1	-1	1	1	**75.75**
3_3	-1	1	-1	-1	1	-1	1	**25.07**
3_4	1	1	-1	1	-1	-1	-1	**54.37**
3_5	-1	-1	1	1	-1	-1	1	**15.41**
3_6	1	-1	1	-1	1	-1	-1	**36.99**
3_7	-1	1	1	-1	-1	1	-1	**50.29**
3_8	1	1	1	1	1	1	1	**74.68**

使用excel迴歸求解工具，可得到回歸結果如下：

迴歸統計	
R 的倍數	0.989446
R 平方	0.979003
調整的 R	0.969817
標準誤	3.300601
觀察值個	24

ANOVA

	自由度	SS	MS	F	顯著值
迴歸	7	8127.225	1161.032	106.5757	3.2E-12
殘差	16	174.3035	10.89397		
總和	23	8301.529			

	係數	標準誤	t 統計	P-值	下限 95%	上限 95%	下限 95.0%	上限 95.0%
截距	49.86875	0.673732	74.01864	1.02E-21	48.4405	51.297	48.4405	51.297
X1	10.73458	0.673732	15.93301	3.08E-11	9.306335	12.16283	9.306335	12.16283
X2	0.552083	0.673732	0.81944	0.424574	-0.87617	1.980332	-0.87617	1.980332
X3	-4.03625	0.673732	-5.99088	1.88E-05	-5.4645	-2.608	-5.4645	-2.608
X1X2	1.08125	0.673732	1.604866	0.128077	-0.347	2.509499	-0.347	2.509499
X1X3	0.777917	0.673732	1.154638	0.265193	-0.65033	2.206165	-0.65033	2.206165
X2X3	14.31708	0.673732	21.2504	3.75E-13	12.88883	15.74533	12.88883	15.74533
X1X2X3	-0.23208	0.673732	-0.34447	0.734976	-1.66033	1.196165	-1.66033	1.196165

從迴歸的結果中，選出p-value小於0.05的項，認定他為顯著。從結果中可看到被認定為顯著的項有截距項、x1、x3、及x2x3項。所得到的regression model如下

$$y = 49.86875 + 10.73458x_1 - 4.03625x_3 + 14.31708x_2x_3$$

與(c)小題比較，會發現篩選出來的顯著項都是相同的，因此檢話過後的regression model會相同。

(f) Calculate all the main and interaction effects on the SN ratio.
S/N ratio所使用公式如下：

$$\eta = -10\log(\frac{\text{mean}^2}{var.s})$$

整理計算結果於下表中：

test	temp	pres	time	trial_1	trial_2	trial_3	average	var	S/N ratio
1	-1	-1	-1	61.43	58.58	57.07	59.0267	4.9020	-28.5172
2	1	-1	-1	75.62	77.57	75.75	76.3133	1.1886	-36.9015
3	-1	1	-1	27.51	34.03	25.07	28.8700	21.4576	-15.8931
4	1	1	-1	51.37	48.49	54.37	51.4100	8.6448	-24.8534
5	-1	-1	1	24.8	20.69	15.41	20.3000	22.1571	-12.6948
6	1	-1	1	43.58	44.31	36.99	41.6267	16.2572	-20.2770
7	-1	1	1	45.2	49.53	50.29	48.3400	7.5391	-24.9129
8	1	1	1	70.51	74	74.68	73.0633	5.0052	-30.2797

將main effect、interaction effect的參數組合分別乘上計算出來的S/N ratio後做sum product在除以4，及可得到每一個不同的S/N ratio effect.
整理於下表：

	X1	X2	X3	X1X2	X1X3	X2X3	X1X2X3
1	28.51720257	28.51720257	28.51720257	-28.51720257	-28.51720257	-28.51720257	28.51720257
2	-36.90152942	36.90152942	36.90152942	36.90152942	36.90152942	-36.90152942	-36.90152942
3	15.89312423	-15.89312423	15.89312423	15.89312423	-15.89312423	15.89312423	-15.89312423
4	-24.85340258	-24.85340258	24.85340258	-24.85340258	24.85340258	24.85340258	24.85340258
5	12.69479158	12.69479158	-12.69479158	-12.69479158	12.69479158	12.69479158	-12.69479158
6	-20.27696632	20.27696632	-20.27696632	20.27696632	-20.27696632	20.27696632	20.27696632
7	24.91293788	-24.91293788	-24.91293788	24.91293788	24.91293788	-24.91293788	24.91293788
8	-30.27974636	-30.27974636	-30.27974636	-30.27974636	-30.27974636	-30.27974636	-30.27974636
Ei	**E1**	**E2**	**E3**	**E12**	**E23**	**E13**	**E123**
	-7.573397103	**0.612819713**	**4.500204165**	**0.409853691**	**1.098905498**	**-11.72328288**	**0.69782944**

(g) Build a predictive model for the SN Ratio.
將S/N ratio的參數組合列進行線性迴歸，可得到結果如下：

Ei	0.000	E1	E2	E3	E12	E13	E23	E123
		-7.573	0.613	4.500	0.410	1.099	-11.723	0.698
Bi	**-24.291**	**-3.787**	**0.306**	**2.250**	**0.205**	**0.549**	**-5.862**	**0.349**

以Excel的迴歸工具驗證結果，會得到相同的係數：

	係數	標準誤	t 統計	P-值	下限 95%	上限 95%	下限 95.0%	上限 95.0%
截距	-24.2912	0	65535	#NUM!	-24.2912	-24.2912	-24.2912	-24.2912
X1	-3.7867	0	65535	#NUM!	-3.7867	-3.7867	-3.7867	-3.7867
X2	0.30641	0	65535	#NUM!	0.30641	0.30641	0.30641	0.30641
X3	2.250102	0	65535	#NUM!	2.250102	2.250102	2.250102	2.250102
X1X2	0.204927	0	65535	#NUM!	0.204927	0.204927	0.204927	0.204927
X1X3	0.549453	0	65535	#NUM!	0.549453	0.549453	0.549453	0.549453
X2X3	-5.86164	0	65535	#NUM!	-5.86164	-5.86164	-5.86164	-5.86164
X1X2X3	0.348915	0	65535	#NUM!	0.348915	0.348915	0.348915	0.348915

因此，根據S/N ratio所建立的反應曲面如下：

$$y = -24.291 - 3.7876x_1 + 0.30641x_2 + 2.250102x_3 + 0.204927x_1x_2 + 0.549453x_1x_3 - 5.86164x_2x_3 + 0.348915x_1x_2x_3$$

(h) Suggest an optimum setting for the process to achieve the highest, stable yield based on results of (d) and (g).
利用規劃求解功能，將x1, x2, x3的參數範圍設於-1~1之間，可得到下列結果

regression on average	X1	X2	X3	Y
	1	-1	-1	78.95666667
	120	50	5	
regression on S/N ratio	X1	X2	X3	Y
	-1	-1	1	-12.69479158
	80	50	15	

從表中可以看到，在兩個不同的模型中所最佳化出來的結果參數組合並不一樣。在[temp, pressure, time]=(120,50,5)時可讓平均達到最大，而在[temp, pressure, time]=(80,50,15)時可讓S/N ratio達最大。由於在兩個模型中，最佳化的目標函數並不一樣，一個是可以讓平均最高，一個是讓品質(S/N)一致性最高，因此優化的參數組合結果不同也是可以解釋的。