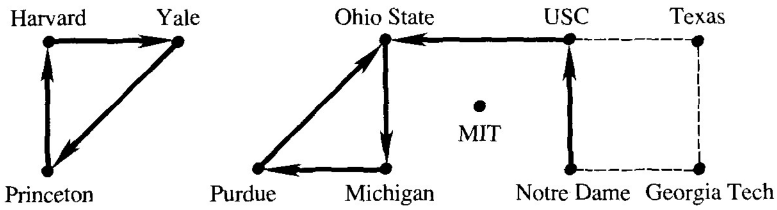
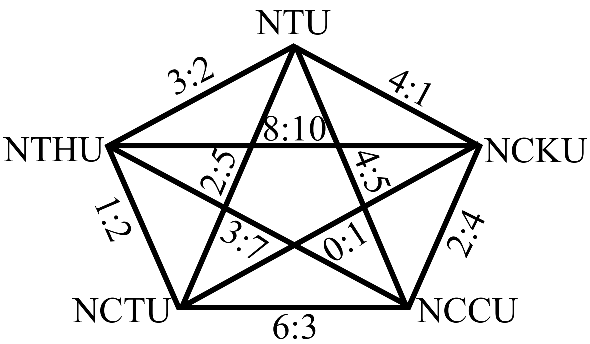
Linear Algebra and its Applications

HW#5

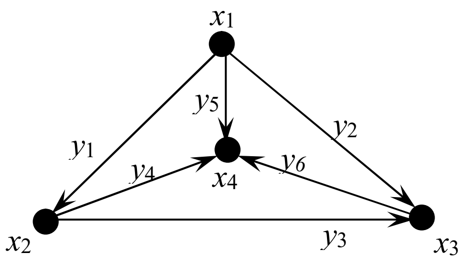
1. For the following games and teams:



   let all the games with dashed edges have been played. That is, all the dashed lines are turned into real lines in the above Football game graph,
   (a) find the edge-node incidence matrix for the games;
   (b) find the basis for the four fundamental spaces of the incidence matrix;
   (c) explain meanings of the null and left-null spaces.
   (d) show that the column space is orthogonal to the left-null space.
2. For the NTU, NTHU, NCTU, NCKU and NCCU games,



   (a) Write down an $Ax=b$ system to find potentials of the five teams (you don't need to solve the problem);
   (b) Find the dimensions for the four subspaces of the incidence matrix;
   (c) For the problem to be solvable, what constraints must be met?
3.
   (a) Write down the 6 by 4 incidence matrix A for the following graph.
   (b) Write down the dimensions of the four fundamental subspaces for this 6 by 4 incidence matrix, and the basis for each subspace.
   (c) Find vectors y that satisfy $y^{T}A=0$ and write down equations expressing Kirchhoff's Voltage Law for the graph.
   (d) Write down equations expressing Kirchhoff's Current Law for the graph.
   (e) Perform the Gaussian Elimination to A and show how the graph becomes after each step of elimination. Is the final graph a spanning tree?



4. Draw a graph with numbered and directed edges (and numbered nodes) whose incidence matrix is

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

   Perform the Gaussian Elimination and show how the graph becomes after each step of elimination and draw a spanning tree after elimination.

5. Show that $x-y$ is orthogonal to $x+y$ if and only if $\| x \| = \| y \|$

6. Find a vector x orthogonal to the row space of $\boldsymbol{A}$, and a vector y orthogonal to the column space, and a vector z orthogonal to the nullspace:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix}$$

7. Find the orthogonal complement of the plane spanned by the vectors (1,1,2) and (1,2,3), by taking these to be the rows of A and solving $Ax = 0$.

8. Draw figure of 4-subspaces of $\boldsymbol{A}$ on page 19 of class note “Graph and Orthogonality” to show each subspace for

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}.$$