The humidity and temperature can in air have an interaction effect on the speed of sound. The speed of sound in dry air could have lo

Sealed containers with different volumes are containing the same mass of gas. When those containers were heated form T1 to T2(T is temperature), the rate of pressure increase varies with different volumes. According to the ideal gas law, the relation between pressure, volume, and temperature was given by:

Where is pressure, is volume, is the amount of mole number which is fixed in this case, and and are the ideal gas constant and temperature respectively.

Because the increase rate of pressure will change when given different volume but same amount of gas and , there is an interaction effect between temperature and volume on the pressure of the gas.

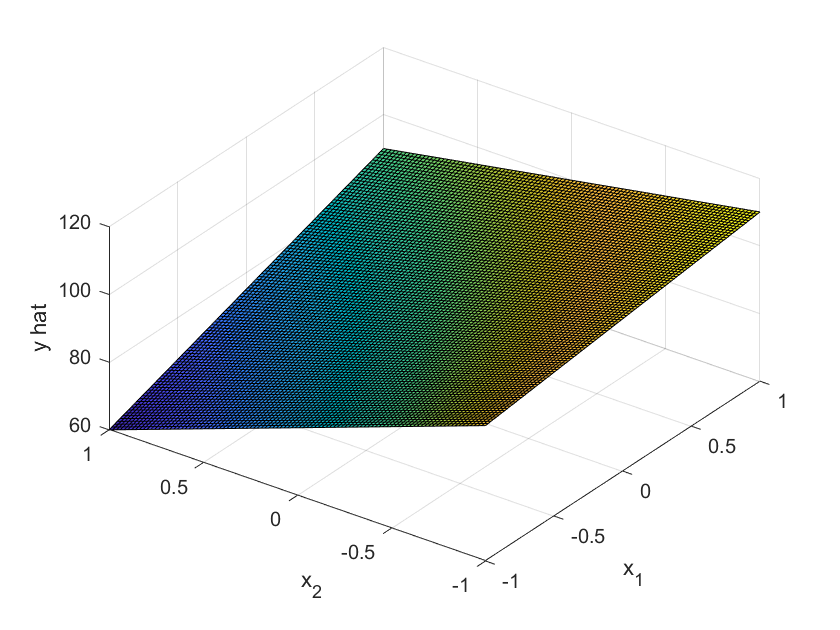


can be found by the function :

is two times of :



Surface plot:



Contour plot:





Given different values of , the impact of on will vary accordingly.

Take the function in this question for example:



Provided varying , the slop of will be different.



Set 85 as the desired , the relation between and will be:

The relation on the contour plot in (c):



There are several possible combinations of x\_1 and x\_2. Here are a few examples:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | -1 | -0.75 | -0.5 | -0.25 | 0 | 0.25 | 0.5 | 0.75 | 1 |
|  | -4 | -2.6 | -1.66667 | -1 | -0.5 | -0.11111 | 0.2 | 0.454545 | 0.666667 |
|  | 85 | 85 | 85 | 85 | 85 | 85 | 85 | 85 | 85 |



* Set temperature(T) as , 120℃ make , 80℃ make
* Set pressure(p) as , 70psi make , 50psi make
* Set Reaction time as , 15min make , 5min make

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| test | x1(T) | x2(p) | x3(t) |  |
| 1 | -1 | -1 | -1 | 59.02767 |
| 2 | 1 | -1 | -1 | 76.31333 |
| 3 | -1 | 1 | -1 | 28.87 |
| 4 | 1 | 1 | -1 | 51.41 |
| 5 | -1 | -1 | 1 | 20.3 |
| 6 | 1 | -1 | 1 | 41.62667 |
| 7 | -1 | 1 | 1 | 48.34 |
| 8 | 1 | 1 | 1 | 73.06333 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| E1 | E2 | E3 | E12 | E23 | E13 | E123 |
| 21.46892 | 1.103917 | -8.07275 | 2.16275 | 28.63442 | 1.556083 | -0.46442 |



The parameters of coded predictive model:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| b0 | b1 | b2 | b3 | b12 | b23 | b13 | b123 |
| 49.86888 | 10.73446 | 0.551958 | -4.03638 | 1.081375 | 14.31721 | 0.778042 | -0.23221 |

Coded predictive model:



The regression model:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 摘要輸出 | |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| 迴歸統計 | |  |  |  |  |  |  |  |
| R 的倍數 | 1 |  |  |  |  |  |  |  |
| R 平方 | 1 |  |  |  |  |  |  |  |
| 調整的 R 平方 | 65535 |  |  |  |  |  |  |  |
| 標準誤 | 0 |  |  |  |  |  |  |  |
| 觀察值個數 | 8 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| ANOVA |  |  |  |  |  |  |  |  |
|  | 自由度 | SS | MS | F | 顯著值 |  |  |  |
| 迴歸 | 7 | 2709.093 | 387.0133 | #NUM! | #NUM! |  |  |  |
| 殘差 | 0 | 0 | 65535 |  |  |  |  |  |
| 總和 | 7 | 2709.093 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | 係數 | 標準誤 | t 統計 | P-值 | 下限 95% | 上限 95% | 下限 95.0% | 上限 95.0% |
| 截距 | 49.86888 | 0 | 65535 | #NUM! | 49.86888 | 49.86888 | 49.86888 | 49.86888 |
| x1(T) | 10.73446 | 0 | 65535 | #NUM! | 10.73446 | 10.73446 | 10.73446 | 10.73446 |
| x2(p) | 0.551958 | 0 | 65535 | #NUM! | 0.551958 | 0.551958 | 0.551958 | 0.551958 |
| x3(t) | -4.03638 | 0 | 65535 | #NUM! | -4.03638 | -4.03638 | -4.03638 | -4.03638 |
| x1x2 | 1.081375 | 0 | 65535 | #NUM! | 1.081375 | 1.081375 | 1.081375 | 1.081375 |
| x2x3 | 14.31721 | 0 | 65535 | #NUM! | 14.31721 | 14.31721 | 14.31721 | 14.31721 |
| x1x3 | 0.778042 | 0 | 65535 | #NUM! | 0.778042 | 0.778042 | 0.778042 | 0.778042 |
| x1x2x3 | -0.23221 | 0 | 65535 | #NUM! | -0.23221 | -0.23221 | -0.23221 | -0.23221 |



(100-yield) is a smaller-the-better performance measure. The SN ratio will be:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| test | temperature | pressure | time | 1 | 2 | 3 | average | variance | σ | SN |
| 1 | 80 | 50 | 5 | 61.433 | 58.58 | 57.07 | 59.02767 | 4.909246 | 2.215682 | -32.2583 |
| 2 | 120 | 50 | 5 | 75.62 | 77.57 | 75.75 | 76.31333 | 1.188633 | 1.090245 | -27.4962 |
| 3 | 80 | 70 | 5 | 27.51 | 34.03 | 25.07 | 28.87 | 21.4576 | 4.632235 | -37.0533 |
| 4 | 120 | 70 | 5 | 51.37 | 48.49 | 54.37 | 51.41 | 8.6448 | 2.940204 | -33.7415 |
| 5 | 80 | 50 | 15 | 24.8 | 20.69 | 15.41 | 20.3 | 22.1571 | 4.707133 | -38.0393 |
| 6 | 120 | 50 | 15 | 43.58 | 44.31 | 36.99 | 41.62667 | 16.25723 | 4.032026 | -35.3381 |
| 7 | 80 | 70 | 15 | 45.2 | 49.53 | 50.29 | 48.34 | 7.5391 | 2.745742 | -34.2713 |
| 8 | 120 | 70 | 15 | 70.51 | 74 | 74.68 | 73.06333 | 5.005233 | 2.237238 | -28.6268 |



* Set temperature(T) as , 120℃ make , 80℃ make
* Set pressure(p) as , 70psi make , 50psi make
* Set Reaction time as , 15min make , 5min make

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| test | x1(T) | x2(p) | x3(t) | SN |
| 1 | -1 | -1 | -1 | -32.2583 |
| 2 | 1 | -1 | -1 | -27.4962 |
| 3 | -1 | 1 | -1 | -37.0533 |
| 4 | 1 | 1 | -1 | -33.7415 |
| 5 | -1 | -1 | 1 | -38.0393 |
| 6 | 1 | -1 | 1 | -35.3381 |
| 7 | -1 | 1 | 1 | -34.2713 |
| 8 | 1 | 1 | 1 | -28.6268 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| E1 | E2 | E3 | E12 | E23 | E13 | E123 |
| 4.104871 | -0.14027 | -1.431518 | 0.373253 | 5.3799089 | 0.067943 | 1.098389 |



The parameter of the SN ratio predictive model:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| d0 | d1 | d2 | d3 | d12 | d23 | d13 | d123 |
| -33.3531 | 2.0524355 | -0.07014 | -0.715759 | 0.186626 | 2.6899545 | 0.033972 | 0.549194 |

SN ratio predictive model:



|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 摘要輸出 | |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| 迴歸統計 | |  |  |  |  |  |  |  |
| R 的倍數 | 1 |  |  |  |  |  |  |  |
| R 平方 | 1 |  |  |  |  |  |  |  |
| 調整的 R 平方 | 65535 |  |  |  |  |  |  |  |
| 標準誤 | 0 |  |  |  |  |  |  |  |
| 觀察值個數 | 8 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| ANOVA |  |  |  |  |  |  |  |  |
|  | 自由度 | SS | MS | F | 顯著值 |  |  |  |
| 迴歸 | 7 | 98.4254 | 14.06077 | #NUM! | #NUM! |  |  |  |
| 殘差 | 0 | 0 | 65535 |  |  |  |  |  |
| 總和 | 7 | 98.4254 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | 係數 | 標準誤 | t 統計 | P-值 | 下限 95% | 上限 95% | 下限 95.0% | 上限 95.0% |
| 截距 | -33.3531 | 0 | 65535 | #NUM! | -33.3531 | -33.3531 | -33.3531 | -33.3531 |
| x1(T) | 2.052435 | 0 | 65535 | #NUM! | 2.052435 | 2.052435 | 2.052435 | 2.052435 |
| x2(p) | -0.07014 | 0 | 65535 | #NUM! | -0.07014 | -0.07014 | -0.07014 | -0.07014 |
| x3(t) | -0.71576 | 0 | 65535 | #NUM! | -0.71576 | -0.71576 | -0.71576 | -0.71576 |
| x1x2 | 0.186626 | 0 | 65535 | #NUM! | 0.186626 | 0.186626 | 0.186626 | 0.186626 |
| x2x3 | 2.689954 | 0 | 65535 | #NUM! | 2.689954 | 2.689954 | 2.689954 | 2.689954 |
| x1x3 | 0.033972 | 0 | 65535 | #NUM! | 0.033972 | 0.033972 | 0.033972 | 0.033972 |
| x1x2x3 | 0.549194 | 0 | 65535 | #NUM! | 0.549194 | 0.549194 | 0.549194 | 0.549194 |



There are two objective functions:

“Solver Options” in Excel can be used to find the optimal values in those functions:

|  |  |  |  |
| --- | --- | --- | --- |
|  | x1 | x2 | x3 |
| 76.31333 | 1 | -1 | -1 |

|  |  |  |  |
| --- | --- | --- | --- |
|  | x1 | x2 | x3 |
| -27.4962 | 1 | -1 | -1 |

Both of the result show that can achieve the highest and stable yield.