Model residuals are the differences between predicted responses and observed responses :

|  |  |  |  |
| --- | --- | --- | --- |
| run order i |  |  |  |
| 4 | 3.9425 | 1.0925 | -0.4175 |
| 2 | -3.336666667 | -1.386666667 | -3.206666667 |
| 7 | -1.343333333 | 5.176666667 | -3.783333333 |
| 6 | 1.0475 | -1.8325 | 4.0475 |
| 1 | 4.019166667 | -0.090833333 | -5.370833333 |
| 5 | 1.33 | 2.06 | -5.26 |
| 8 | -4.215 | 0.115 | 0.875 |
| 3 | -0.374166667 | 3.115833333 | 3.795833333 |

Suppose that residuals are normal distributed. The parameters of normal distribution are and

* Rank each residual from 1~24
* The c.d.f. of effect is:
* The normal distribution value of effect:

Plot the residuals vs. run orders:

Plot the residuals vs. :

Plot the residuals vs. :

Plot the residuals vs. x1:

Plot the residuals vs. x2:

Plot the residuals vs. x3:



Bartlett’s test ():

Reject if :



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| source of variation | SS | DOF | MS | F | F\_0.1 |
| between test | 8127.225196 | 7 | 1161.032171 | 106.5757044 | 2.128 |
| pure error (within test) | 174.3034667 | 16 | 10.89396667 |  |  |
| total | 8301.528663 | 23 |  |  |  |



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| source of variation | SS | DOF | MS | F | F\_0.1 |
| E1 | 2765.55 | 1 | 2765.551 | 253.8608 | 3.04811 |
| E2 | 7.3151 | 1 | 7.315104 | 0.671482 | 3.04811 |
| E3 | 390.992 | 1 | 390.9915 | 35.89065 | 3.04811 |
| E12 | 28.0584 | 1 | 28.05844 | 2.575594 | 3.04811 |
| E23 | 4919.49 | 1 | 4919.493 | 451.5796 | 3.04811 |
| E13 | 14.5237 | 1 | 14.5237 | 1.333188 | 3.04811 |
| E123 | 1.2927 | 1 | 1.292704 | 0.118662 | 3.04811 |
| pure error | 174.303 | 16 | 10.89397 |  |  |
| total | 8301.53 | 23 |  |  |  |



There are three significant effects in ANOVA table:



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| source of variation | SS | DOF | MS | F | F\_0.1 |
| E1 | 2765.551 | 1 | 2765.551 | 253.8608 | 3.04811 |
| E3 | 390.9915 | 1 | 390.9915 | 35.89065 | 3.04811 |
| E23 | 4919.493 | 1 | 4919.493 | 451.5796 | 3.04811 |
| pure error | 174.3035 | 16 | 10.89397 |  |  |
| total | 8250.339 | 19 |  |  |  |



|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 摘要輸出 | |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| 迴歸統計 | |  |  |  |  |  |  |  |
| R 的倍數 | 0.989446 |  |  |  |  |  |  |  |
| R 平方 | 0.979003 |  |  |  |  |  |  |  |
| 調整的 R 平方 | 0.969817 |  |  |  |  |  |  |  |
|  | 標準誤(s\_p) | 3.300601 |  |  |  |  |  |  |
|  | 觀察值個數 | 24 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| ANOVA |  |  |  |  |  |  |  |  |
|  | 自由度 | SS | MS | F | 顯著值 |  |  |  |
| 迴歸 | 7 | 8127.225 | 1161.032 | 106.5757 | 3.2E-12 |  |  |  |
| 殘差 | 16 | 174.3035 | 10.89397 |  |  |  |  |  |
| 總和 | 23 | 8301.529 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | 係數 | 標準誤 | t 統計 | P-值 | 下限 95% | 上限 95% | 下限 99.0% | 上限 99.0% |
| 截距 | 49.86875 | 0.673732 | 74.01864 | 1.02E-21 | 48.4405 | 51.297 | 47.90092 | 51.83658 |
| x1 | 10.73458 | 0.673732 | 15.93301 | 3.08E-11 | 9.306335 | 12.16283 | 8.766758 | 12.70241 |
| x2 | 0.552083 | 0.673732 | 0.81944 | 0.424574 | -0.87617 | 1.980332 | -1.41574 | 2.519908 |
| x3 | -4.03625 | 0.673732 | -5.99088 | 1.88E-05 | -5.4645 | -2.608 | -6.00408 | -2.06842 |
| x1x2 | 1.08125 | 0.673732 | 1.604866 | 0.128077 | -0.347 | 2.509499 | -0.88658 | 3.049075 |
| x2x3 | 14.31708 | 0.673732 | 21.2504 | 3.75E-13 | 12.88883 | 15.74533 | 12.34926 | 16.28491 |
| x1x3 | 0.777917 | 0.673732 | 1.154638 | 0.265193 | -0.65033 | 2.206165 | -1.18991 | 2.745742 |
| x1x2x3 | -0.23208 | 0.673732 | -0.34447 | 0.734976 | -1.66033 | 1.196165 | -2.19991 | 1.735742 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 摘要輸出 | |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| 迴歸統計 | |  |  |  |  |  |  |  |
| R 的倍數 | 0.986325 |  |  |  |  |  |  |  |
| R 平方 | 0.972837 |  |  |  |  |  |  |  |
| 調整的 R 平方 | 0.968763 |  |  |  |  |  |  |  |
| 標準誤 | 3.357778 |  |  |  |  |  |  |  |
| 觀察值個數 | 24 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| ANOVA |  |  |  |  |  |  |  |  |
|  | 自由度 | SS | MS | F | 顯著值 |  |  |  |
| 迴歸 | 3 | 8076.035 | 2692.012 | 238.7663 | 7.99E-16 |  |  |  |
| 殘差 | 20 | 225.4934 | 11.27467 |  |  |  |  |  |
| 總和 | 23 | 8301.529 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | 係數 | 標準誤 | t 統計 | P-值 | 下限 95% | 上限 95% | 下限 95.0% | 上限 95.0% |
| 截距 | 49.86875 | 0.685403 | 72.75824 | 1.01E-25 | 48.43902 | 51.29848 | 48.43902 | 51.29848 |
| x1 | 10.73458 | 0.685403 | 15.6617 | 1.08E-12 | 9.304857 | 12.16431 | 9.304857 | 12.16431 |
| x3 | -4.03625 | 0.685403 | -5.88887 | 9.25E-06 | -5.46598 | -2.60652 | -5.46598 | -2.60652 |
| x2x3 | 14.31708 | 0.685403 | 20.88855 | 4.7E-15 | 12.88736 | 15.74681 | 12.88736 | 15.74681 |