CS233 B Linear Algebra

Quiz 1

Part 1 choices (30%)

1. Find x such that $\begin{bmatrix} 2x & 7 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$

   (A) 1
   (B) 2
   (C) 3
   (D) 4 送分

2. Let $AX = B$ be a system of linear equations with solution X_1 and X_2. What is incorrect step about the following proof?

   (A) $AX_1 = B$ and $AX_2 = B$
   (B) $AX_1 = AX_2$
   (C) $X_1 = X_2$
   (D) Thus every system of linear equations has at most one solution

3. Consider the partitioned matrix $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ O & O & A_{23} \end{bmatrix}$, where A_{11} is invertible. rank(A)=?

   (A) rank(A_{11})
   (B) rank(A_{11})+ rank(A_{23})
   (C) rank(A_{23})
   (D) rank(A_{12})+ rank(A_{23})

4. Solve the linear system

$$\begin{cases} 2x + 8y + 4z = 2 \\ 2x + 5y + z = 5 \\ 4x + 10y - z = 1 \end{cases}$$

   z =?

   (A) 1
   (B) 2
   (C) 3
   (D) 4

5. If A is a n x n matrix and $A = -A^T$, that trace(A) = ?

   (A) 0
   (B) 1
   (C) n
   (D) n/2

6. Find the largest possible number of independent vectors from the following vector set: $v_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$, $v_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$, $v_5 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$, $v_6 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$
   (A) 3
   (B) 4
   (C) 5
   (D) 6

Part 2 True of False (30%)

1. If *A* and *B* are diagonal matrices of the same size, that *AB* = *BA*. T
2. Let *A* and *B* be *n* x *n* matrices. The operator [*A*, *B*] is defined as [*A*, *B*] = *AB* – *BA*, that [*A*, [*B*, *C*]]+ [*B*, [*C*, *A*]]+ [*C*, [*A*, *B*]] = *O* T
3. Let A and *B* be *m* x *n* matrices. If both the system of linear equations *Ax*=0 and *Bx* =0 have nontrivial solution, i.e. $\exists x \neq 0, Ax = 0$, then (*A*+*B*)*x* =0 has nontrivial solutions. F
4. If [A|b] is in row echelon form, then the system Ax = b must have a solution. F
5. Every matrix can be transformed into a unique matrix in row echelon form by a sequence of elementary row operations. *F*
6. trace((*A*+*B*)*C*)) = trace(*AC*) + trace(*BC*) T
7. trace(*AB*) = trace(A) * trace(B) F
8. Given any vectors $x_1, x_2 \dots x_l \in R^n$. Define an *l* x *l* matrix A with $A_{ij} = x_i^T x_j, i, j = 1,2, \dots, l$. Then *A* is invertible matrix. F
9. If w_1 ,w_2 ,w_3 are independent vectors, the differences $v_1 = w_2 - w_3$ and $v_2 = w_1 - w_3$ and $v_3 = w_1 - w_2$ are independent. F
10. If v_i and v_j are linear independent for $i, j = 1,2,3, i \neq j$, then v_1 ,v_2 ,v_3 are linear independent. F

Part 3 Essay (40%)

1. Consider the systems of linear equations defined by augmented matrices *P* of the following sizes. The ranks of the augmented matrix and matrix of coefficient *Q* are given in each case. Will the systems have a single, many, or no solutions? (6%)
   (a) P is 4 x 5. rank(P) = 4, rank(Q) = 4
      single
   (b) P is 3 x 4. rank(P) = 3, rank(Q) = 2
      no
   (c) P is 4 x 4. rank(P) = 2, rank(Q) = 2
      many

2. $A = \begin{bmatrix} -1 & 0 & -1 & -1 \\ -3 & -1 & 0 & -1 \\ 5 & 0 & 4 & 3 \\ 3 & 0 & 3 & 2 \end{bmatrix}$,

Please using the Gauss elimination to determine the inverse of A (8%)

$$\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & -1 & -3 & 4 \\ 1 & 0 & -1 & 2 \\ -3 & 0 & 0 & -1 \end{bmatrix}$$

3. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, and $AM = BA$, where M is a 3x3 matrix. Find M^5. (10%)

$M = A^{-1}BA$

$M^5 = A^{-1}B^5A$

$$\begin{bmatrix} 32 & 0 & 0 \\ -64 & -32 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

4. Let $k \in R$, and define $S = \{(1,0,0), (0,1,1), (1, k, 1)\}$. For which values of k is it true that $\text{span}(S) = R^3$ (6%)

K=2

5. Support the complete solution to the equation $Ax = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$ is $x = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Find A. (10%)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 3 & 0 & -3 \end{bmatrix}$$

Part 4 Bonus (20%)

1. Prove or disprove if $G = AA^T$ *and* $P = A^TA$, for any arbitrary matrix A, that G and P are both symmetric matrices. (10%)

$$G^T = (AA^T)^T = AA^T = G$$

$$P^T = (A^TA)^T = A^TA = P$$

2. 對本門課的建議 (10%)