# CS233 B Linear Algebra

# Quiz 2

## Part 1 choices (30%)

1. Let A be a 2 x 2 matrix with tr(A) = 5 and det(A) =-2. The tr($A^2$)=?
   (a) 4
   (b) 21
   (c) 25
   (d) 29

2. Let $B = \begin{bmatrix} 4 & 1 & 0 & 0 & 1 \\ 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 1 & 0 & 0 & 1 & 4 \end{bmatrix}$. Which one is the eigenvalue of B?
   (a) 1
   (b) 4
   (c) 5
   (d) 6

3. Which of the following matrix has only real eigenvalues?
   (a) $\begin{bmatrix} cos45° & -sin45° \\ sin45° & cos45° \end{bmatrix}$
   (b) $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 2 \\ -2 & 0 & 3 \end{bmatrix}$
   (c) $\begin{bmatrix} 2 & -1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
   (d) $\begin{bmatrix} 4 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

4. 4-189

Given the linear operator T with standard matrix $[T]_E = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ and B-matrix

$[T]_B = \begin{bmatrix} 1 & 9 & -6 \\ 0 & 7 & -4 \\ 2 & 11 & -8 \end{bmatrix}$, which can be a basis for $B$?

(a) $\left\{\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 9 \end{bmatrix}\right\}$

(b) $\left\{\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}\right\}$

(c) $\left\{\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ -4 \end{bmatrix}\right\}$

(d) None of the above.

5. 4-190

   Let T: $R^2$->$R^2$ be linear transformation and let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \beta = \left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}\right\}$, If

   $[T]_\beta = A$, then $T\begin{bmatrix} x \\ y \end{bmatrix}$ =?

   (a) $\begin{bmatrix} -2x + y \\ 2y \end{bmatrix}$

   (b) $\begin{bmatrix} -x + 2y \\ 2x + y \end{bmatrix}$

   (c) $\begin{bmatrix} -2y \\ -x + 5y \end{bmatrix}$

   (d) $\begin{bmatrix} y \\ 2x - 3y \end{bmatrix}$

6. 3-136

   Support $S$ and $T$ are subspace of $R^{13}$, with dim(S) = 7 and dim(T) = 8. What is the smallest possible dimension of $S \cap T$?

   (a) 1
   (b) 2
   (c) 7
   (d) 8

**Part 2 True of False (30%)**

**http://people.math.harvard.edu/~knill/teaching/math21b2018/mid2/solution9.pdf**

1. If $A$ and $B$ both have $\vec{v}$ as an eigenvector, then $\vec{v}$ is an eigenvector of $AB$.
2. Similar matrices have the same eigenvectors.
3. If the rank of an $n \times n$ matrix $A$ is less than $n$, then 0 is an eigenvalue of $A$.
4. If a square matrix $A$ is diagonalizable, then $(A^T)^2$ is diagonalizable
5. If a $3 \times 3$ matrix $A$ satisfies $A^2 = I_3$ and $A$ is diagonalizable, then $A$ must be similar to the identity matrix.
6. If $A$ is invertible, then $A$ and $A^{-1}$ have the same eigenvectors.
7. If $T$: $R^n$ -> $R^m$, and $T(\vec{0})= \vec{0}$, then $T$ is linear.
8. Let $\beta$ be a basis for a finite dimensional vector space $V$. If $T$ is a linear operation on $V$ then $T$ is onto if and only if $T(\beta)$ is a basis for $V$.
9. If $E$ is an $n$ x $n$ elementary matrix, then $A$ and $EA$ must have the same row space.
10. If $V$ and $W$ are subspaces of $R^n$ having the same dimension, then $V = W$.

## Part 3 Essay (40%)

1. 5-97

   Let $T$: $P_2$->$P_2$ be the linear transformation given by $T(ax^2 + bx + c) = cx^2 + (b + c)x + a$. Is $T$ diagonalizable? If yes, find a basis $\beta$ such that $[T]_\beta$ is a diagonal matrix. (8%)

   No.

   Let $\alpha = \{x^2, x, 1\}$: basis for $P_2$

   $$A = [T]_\alpha = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

   $\det(A-\lambda I) = -(\lambda+1)(\lambda-1)^2$

   $\lambda = 1$ 重數 2 != 1=nullity(A-I)

2. 5-93

   Diagonalize the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (8%)

   A 的 eigenvalue = 1,3,3

   $$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

3. 3-141

   Let $V = \left\{\theta \in R \mid -\frac{\pi}{2} < \theta < \frac{\pi}{2}\right\}$. Define the following two operation $\oplus$ and $\odot$:

   vector addition: $\alpha \oplus \beta = tan^{-1}(tan\alpha + \tan\beta), \forall \alpha, \beta \in V$

   scalar multiplication: $r\alpha = tan^{-1}(rtan\alpha), \forall \alpha \in V, r \in R$

   Show that $V$ is a vector space. (8%)

需證 8 項定理

4. 4-106

Let $L$: $P$ -> $R^2$ defined by

$$L(p(x)) = \begin{bmatrix} \int_0^1 p(x)dx \\ p'(x) \end{bmatrix}$$

4A. Show that $L$ is a linear transformation? (4%)

需證加法封閉性、純量積封閉性

4B. Find a matrix A such that $L(ax+b) = A\begin{bmatrix} a \\ b \end{bmatrix}$. (4%)

$$A = \begin{bmatrix} \frac{1}{2} & 1 \\ 1 & 0 \end{bmatrix}$$

5. 4-121

Let $L$:$P_3$->$P_3$ defined by $L(p(x)) = xp'(x) + p''(x)$

5A. Find the matrix $A$ representing $L$ with respect to $\beta = \{1, x, x^2\}$ (2%)

5B. Find the matrix $B$ representing $L$ with respect to $\gamma = \{1, 1+x, 1+x^2\}$ (3%)

5C. Let $T$: $R^n$->$R^m$ be a linear transformation. And let $B = \{\overrightarrow{b_1}, \overrightarrow{b_2}, \dots, \overrightarrow{b_n}\}$ and $C = \{\overrightarrow{c_1}, \overrightarrow{c_2}, \dots, \overrightarrow{c_m}\}$ be bases for $R^n$ and $R^m$, respectively. The matrix $\left[[T(\overrightarrow{b_1})]_C \ [T(\overrightarrow{b_2})]_C \cdots [T(\overrightarrow{b_n})]_C\right]$ is called the matrix representation of T with respect to B and C. It is denoted by $[T]_B^C$. Find S $=[I]_\gamma^\beta$, where $I(p(x)) = p(x)$. (3%)

## Part 4 Bonus (20%)

1. 給學弟妹的一段話，請考試結束後掃描 QRcode 填寫 (10%)
2. 請說明期末學分補完計畫 (4%)
3. 請說明作業四計算距離的公式(6%)