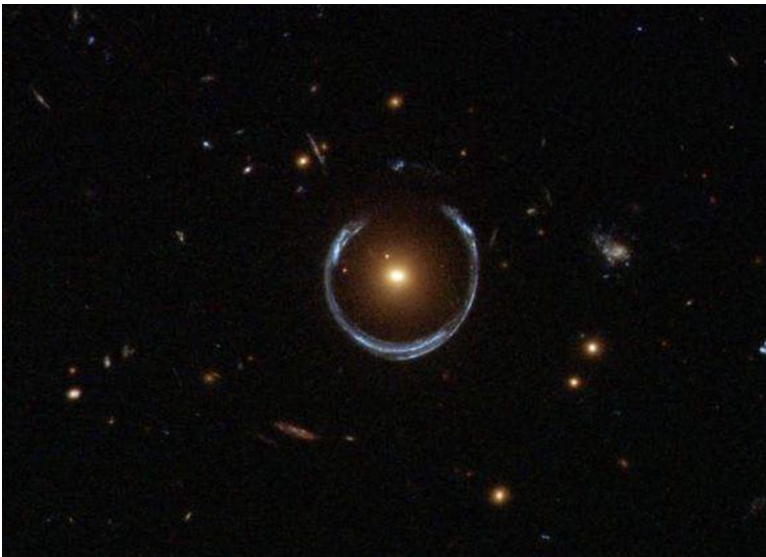


Microlensing by Galactic Center Supermassive BH

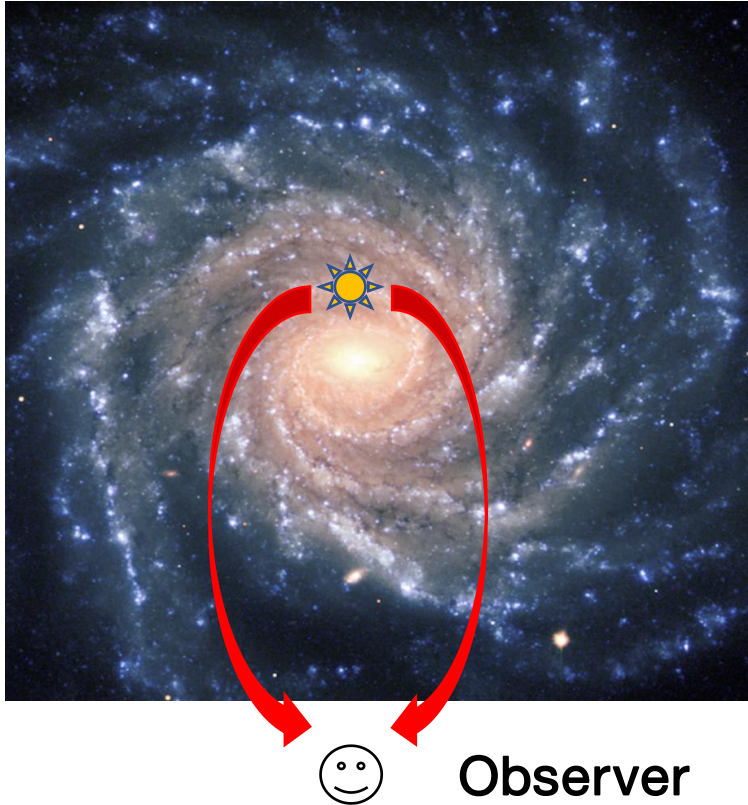
Yixian Chen
Instructor: Jessica Lu

Dep of Astronomy, UC Berkeley
Dep of Physics, Tsinghua University

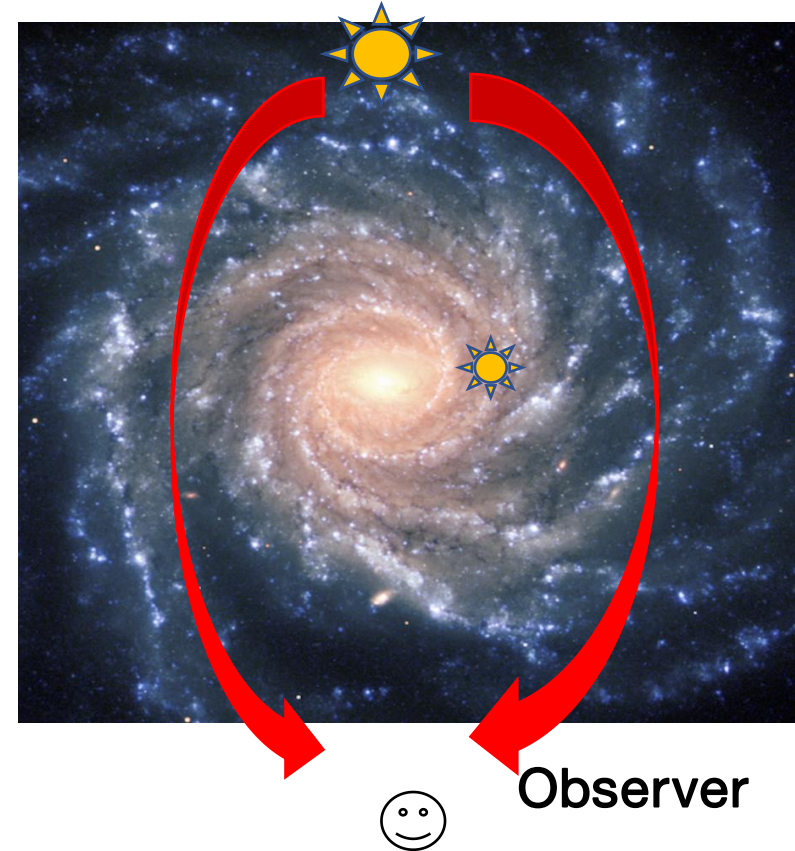


Two Cases

Alexander & Sternberg 1999: lensing of GC stellar population by the SMBH



Alexander & Loeb 2001: lensing of distant stars by SMBH perturbed by GC stars as secondary lenses



Motivation: Discovery of IR **variable sources** in the direction of the SMBH

Basics of Micro-lensing

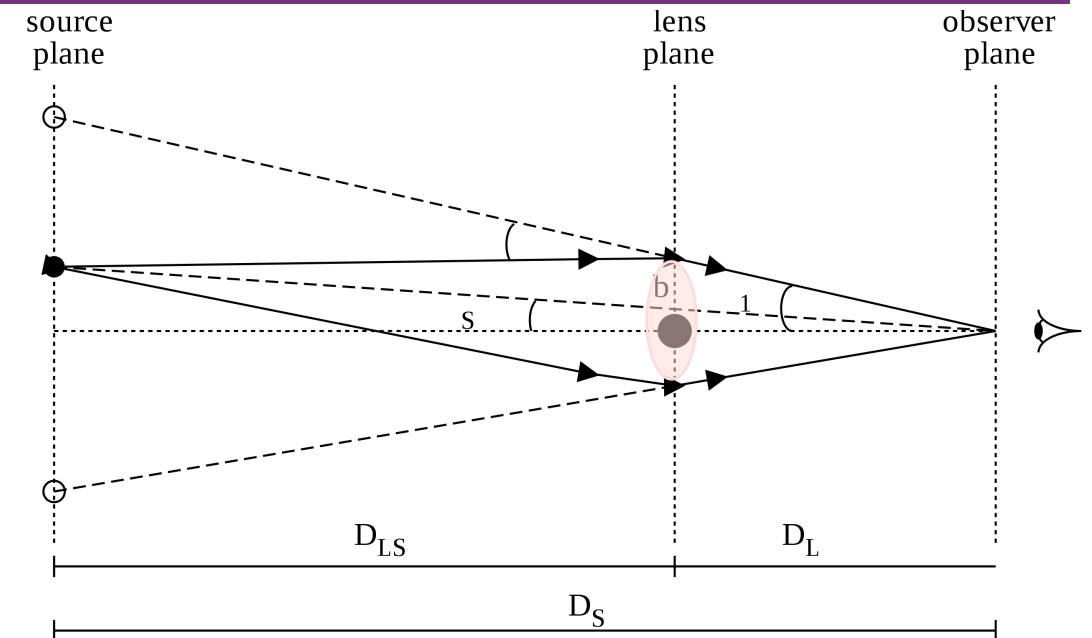
General Setup

Einstein Angle

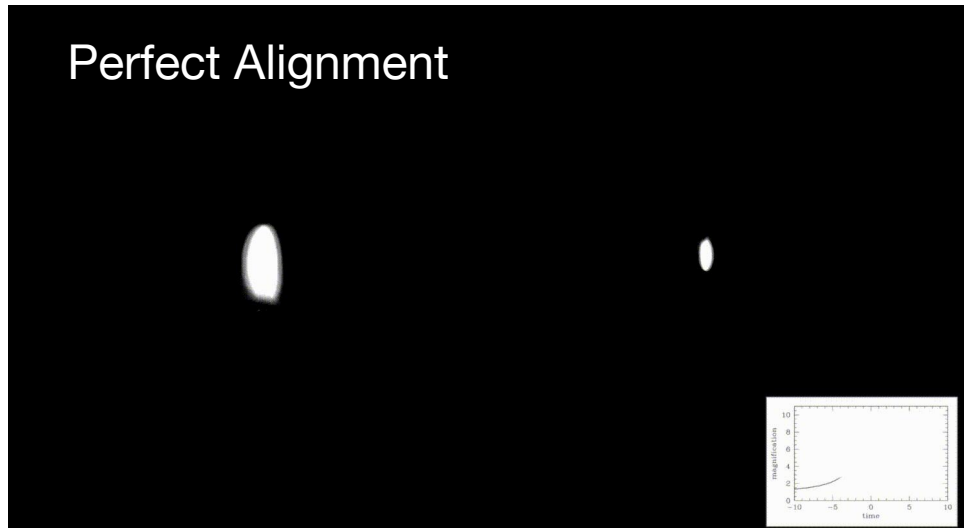
$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{d_S - d_L}{d_S d_L}}$$

Einstein Radius

$$R_E = \theta_E D_L$$



Perfect Alignment



Imperfect Alignment



Basics of Micro-lensing

Impact parameter and Magnification

$$\vec{u} = \frac{\vec{\theta}_S - \vec{\theta}_L}{\theta_E} \quad A(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}$$

Faint-star lensing $K_s \gtrsim K_0$

Need **extra amplification** to be **detected** within a timescale

$$A_s(K_s; K_0) = 10^{-0.4(K_0 - K_{s,ab} - \Delta - A_K)} = 10^{-0.4(K_0 - K_s)}$$

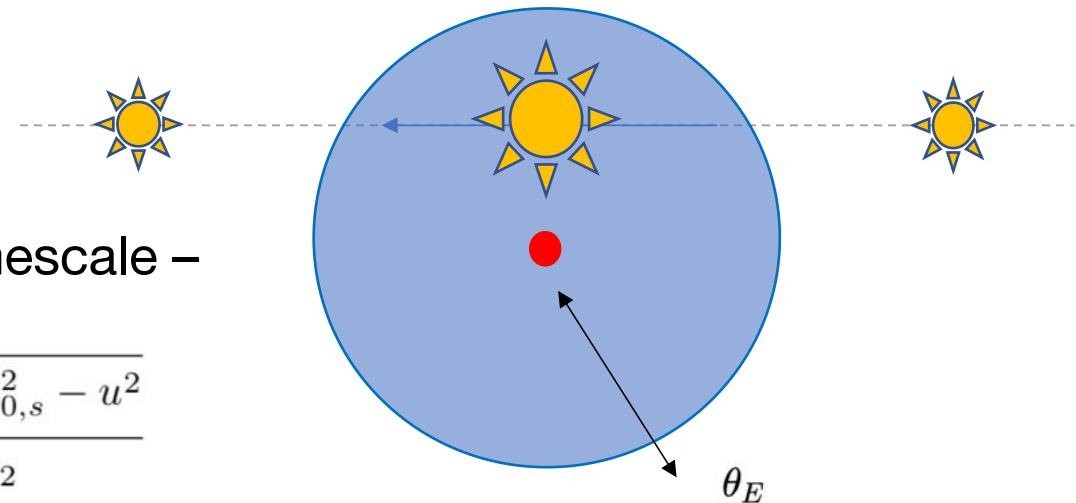
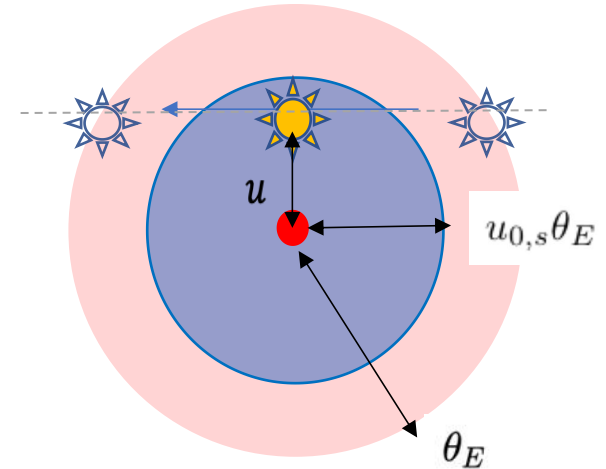
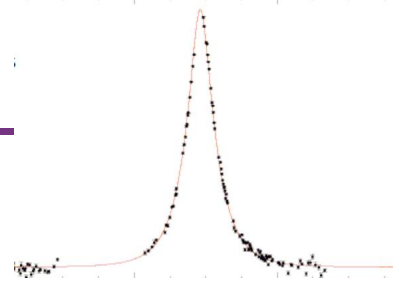
$$\longrightarrow u_{0,s}^2 = 2A(K_s; K_0) / \sqrt{A(K_s; K_0)^2 - 1} - 2$$

Bright-star lensing $K_s \lesssim K_0$

Can always be detected, care about **amplification** timescale –
Nearly exactly the time for source to pass the ER!

$$u_{0,s} \approx 1$$

$$\tau = \frac{2R_S \sqrt{u_{0,s}^2 - u^2}}{v_2}$$



Basics of Micro-lensing

Resolved Lensing

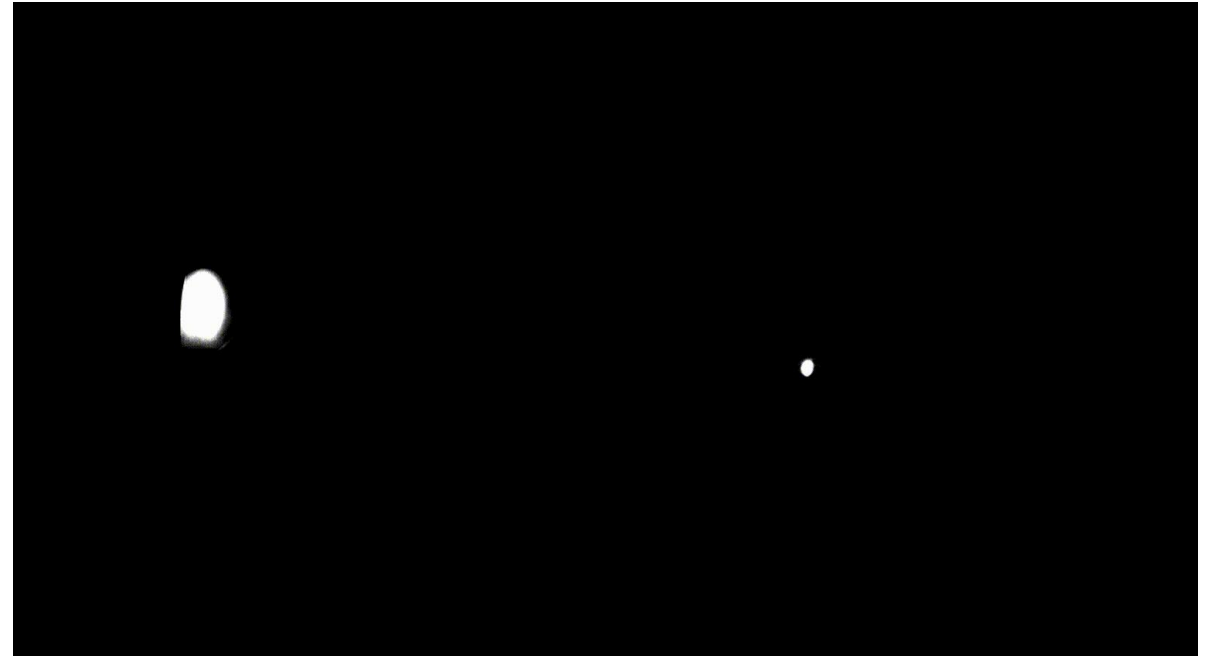
Requirement:

Angular separation between central stars
bigger than between two images

$$\Sigma_{center}(K_0)^{-1/2} \gtrsim \theta_E$$

Effect: effective amplification **halved**

$$A' = A/2$$

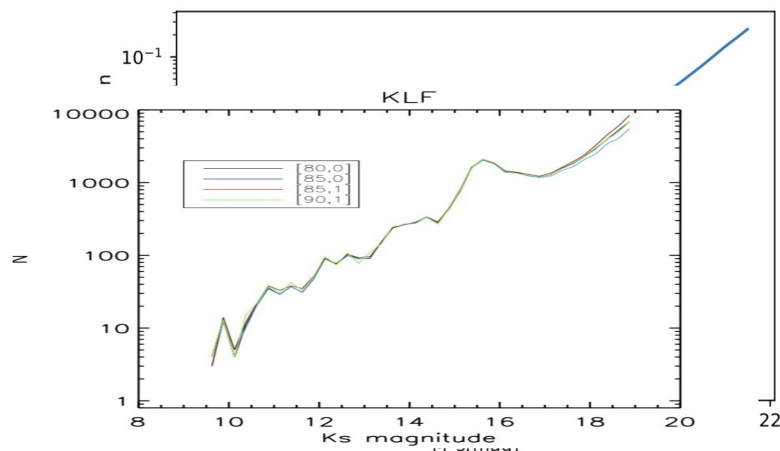


These are all models for ONE lensing event.
How about for a **population of stars** in the galactic center?

Model of Galactic Center NSC

KLF:

Luminosity
probability distribution



Gallego-Cano + 18

Velocity Distribution:

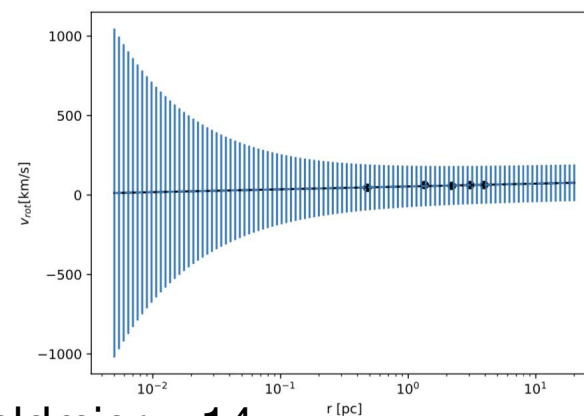
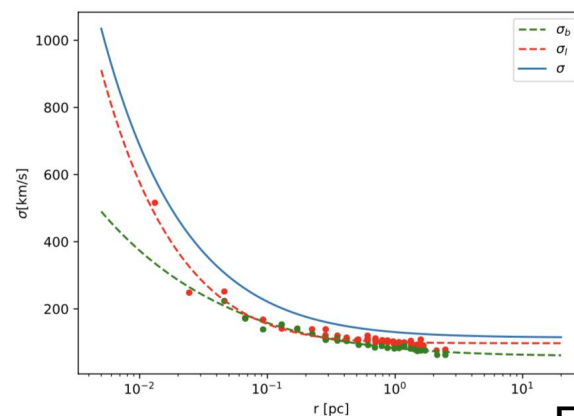
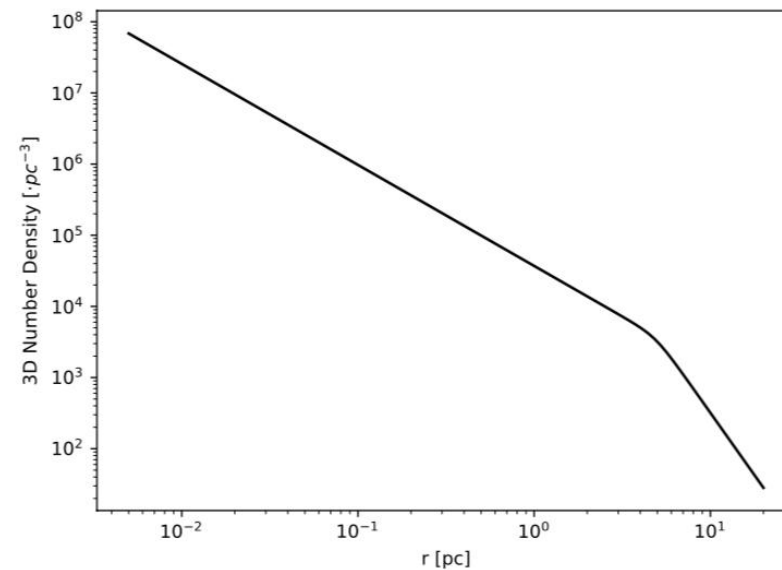
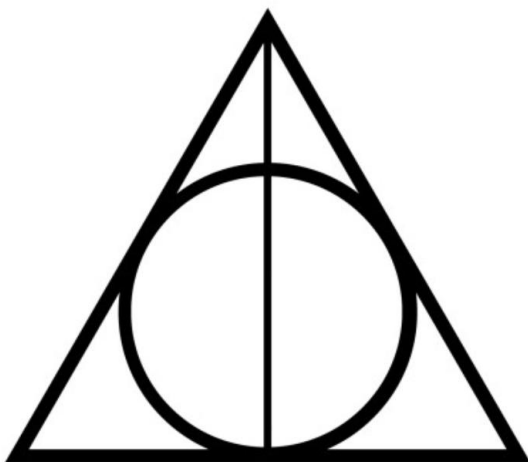
Transverse
Gaussian dispersion

$$v_{\text{rot}}(r)$$

$$\sigma^2 = \sigma_l^2 + \sigma_b^2$$

Number density: Gallego-Cano + 18

$$n_*(r)$$

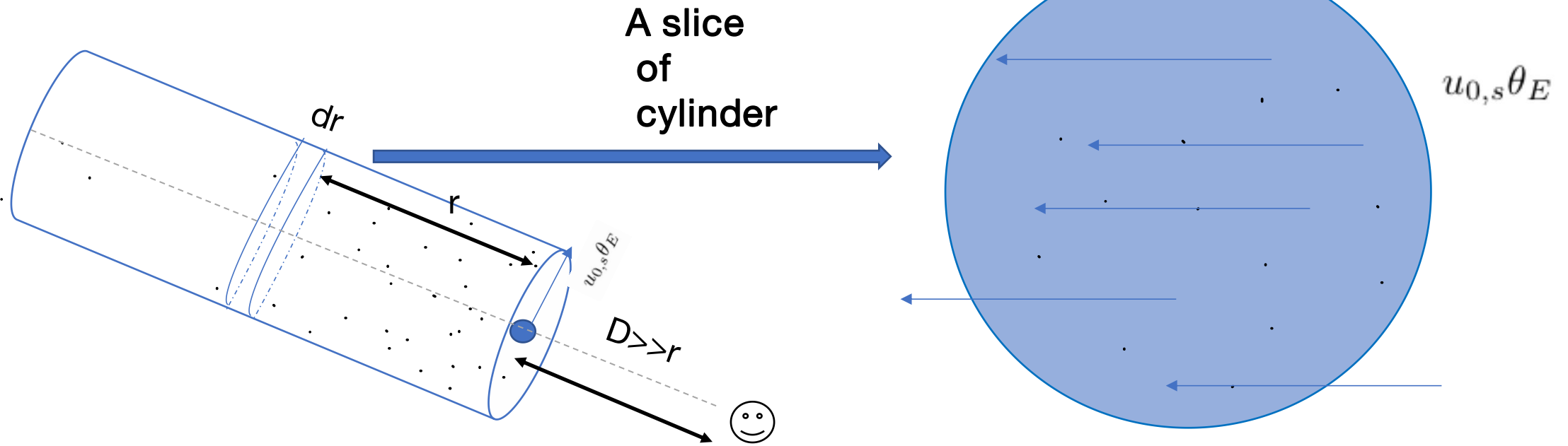


Feldmier + 14

Event Rates

$$R_E = \left(\frac{4GM_\bullet}{c^2} \cdot \frac{Dr}{D+r} \right)^{1/2} \sim 2.73 \times 10^{15} \left(\frac{r}{1 \text{ pc}} M_{4.0} \right)^{1/2} \text{ cm}$$

Given luminosity and distance:



For every dr :

$$d\Gamma = \frac{d\tau_*}{\bar{\tau}} = \frac{n_* \pi (R_E u_{0,s})^2 dr}{\bar{\tau}}$$

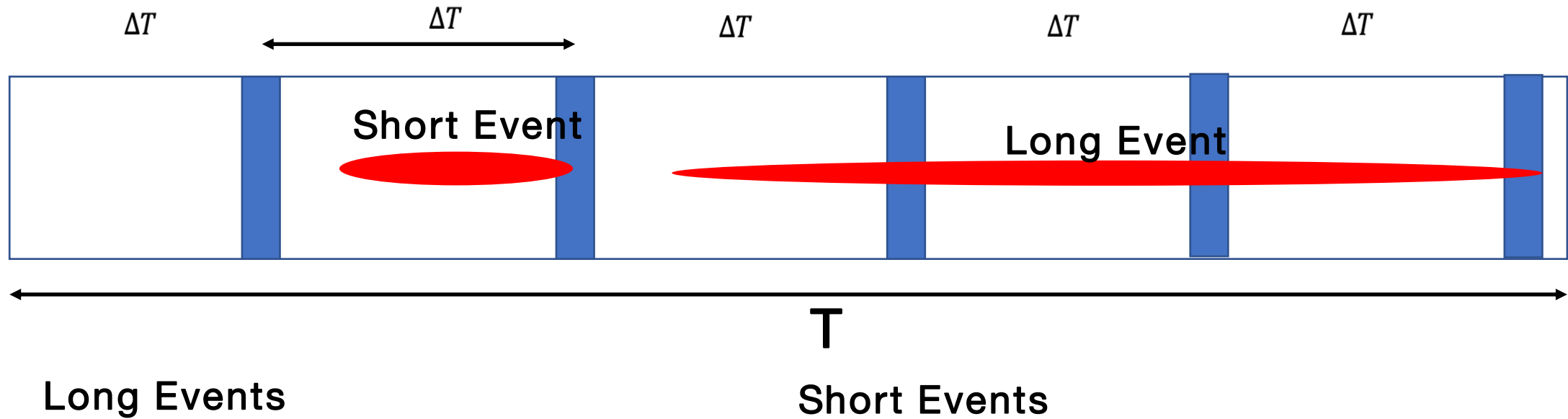
$$= 2u_{0,s} R_E \bar{v}_2 dr$$

Total Rate

$$\Gamma(K_0) = 2 \langle u_{0,s} \rangle \int_{r_1}^{r_2} R_E(r) \bar{v}_2(r) n_*(r) dr$$

0.005-20pc

For Specific Observation Campaign

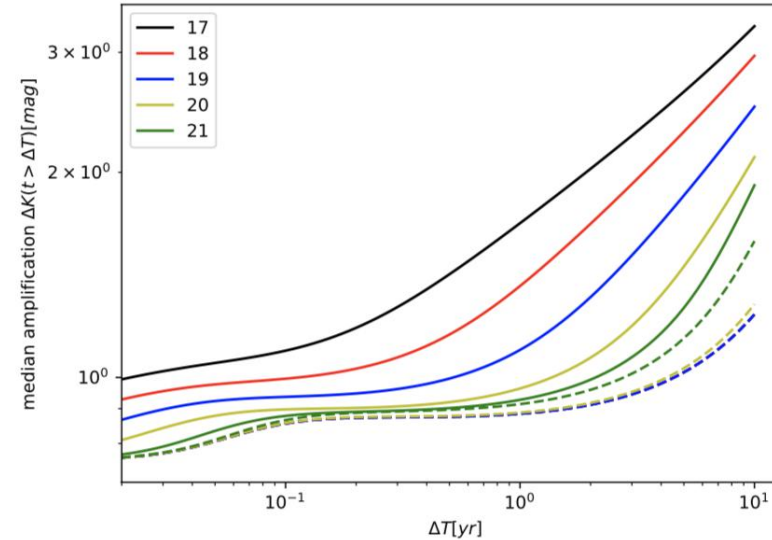
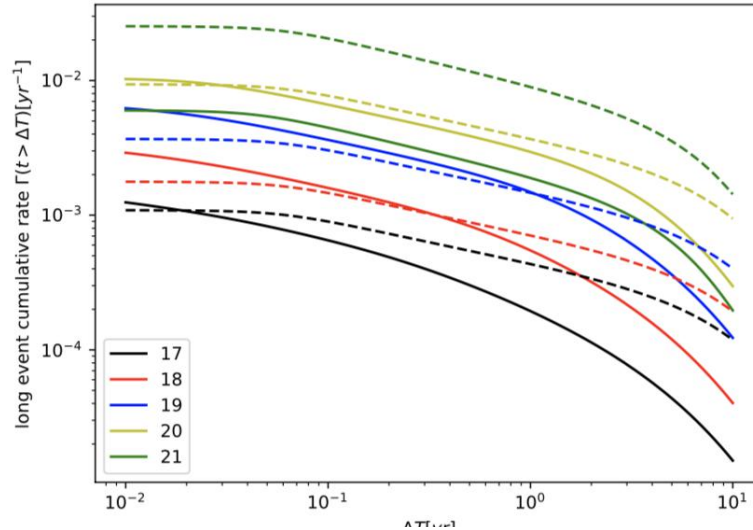


$$P = P_{short} + P_{long} = n\bar{\tau}_{short} \Gamma_{short} + T\Gamma_{long}, \Gamma_{short} + \Gamma_{long} = \Gamma_{total}$$

Results (E.g. Unresolved lensing)

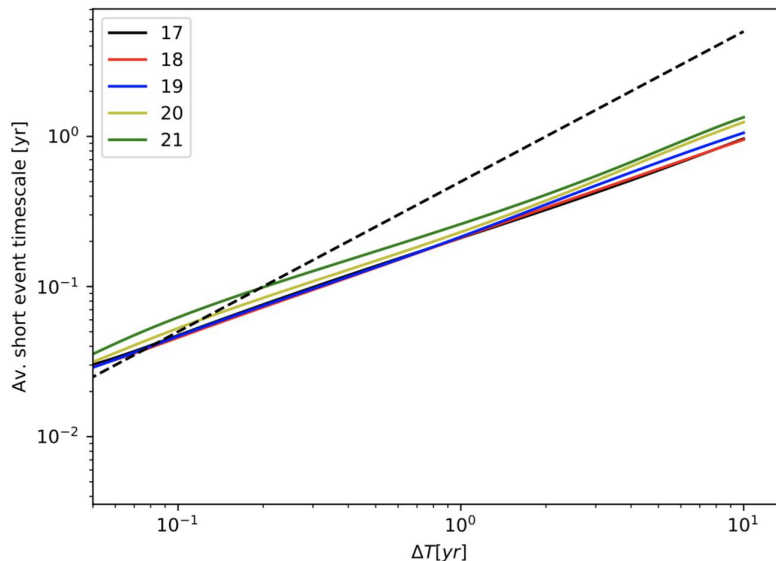
Chen+ 19 in prep

Long event
rate with
respect to
detection
interval



Long event
average
amplification
with respect
to detection
interval

Short event
average
timescale
with respect
to detection
interval



$$n\bar{\tau}_{\text{short}} \Gamma_{\text{short}} + T\Gamma_{\text{long}}$$

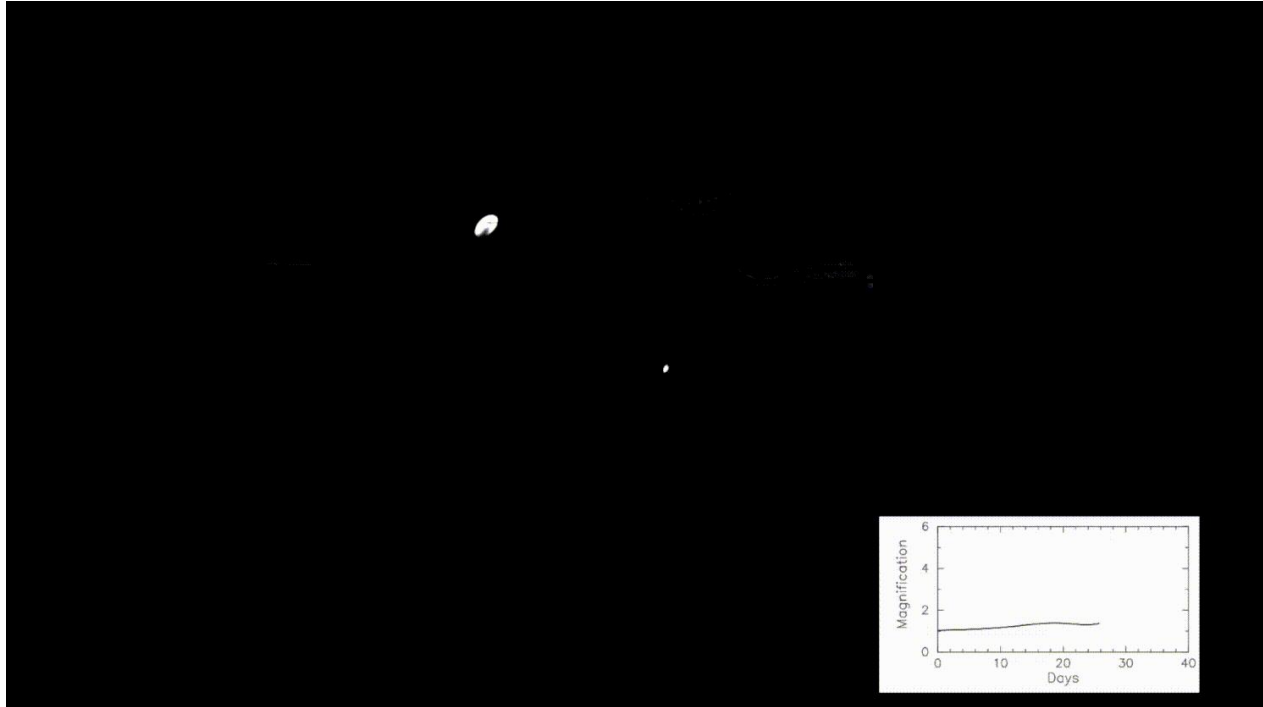
E.g. 10 yrs, 10 runs, 19 mag

$$10 \times 0.2 \times 0.004 + 10 \times 0.005 = 0.06 \text{ times}$$

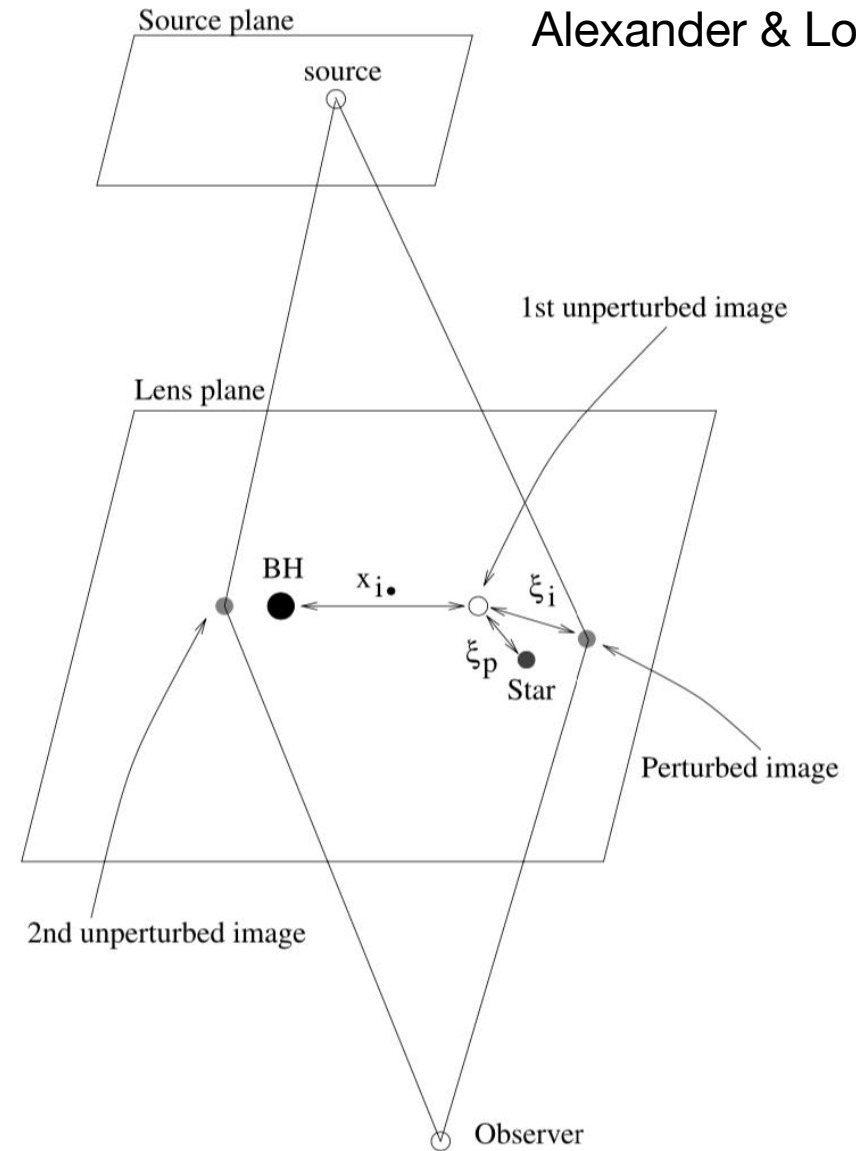
Average 1 magnitude amplification

Secondary lens

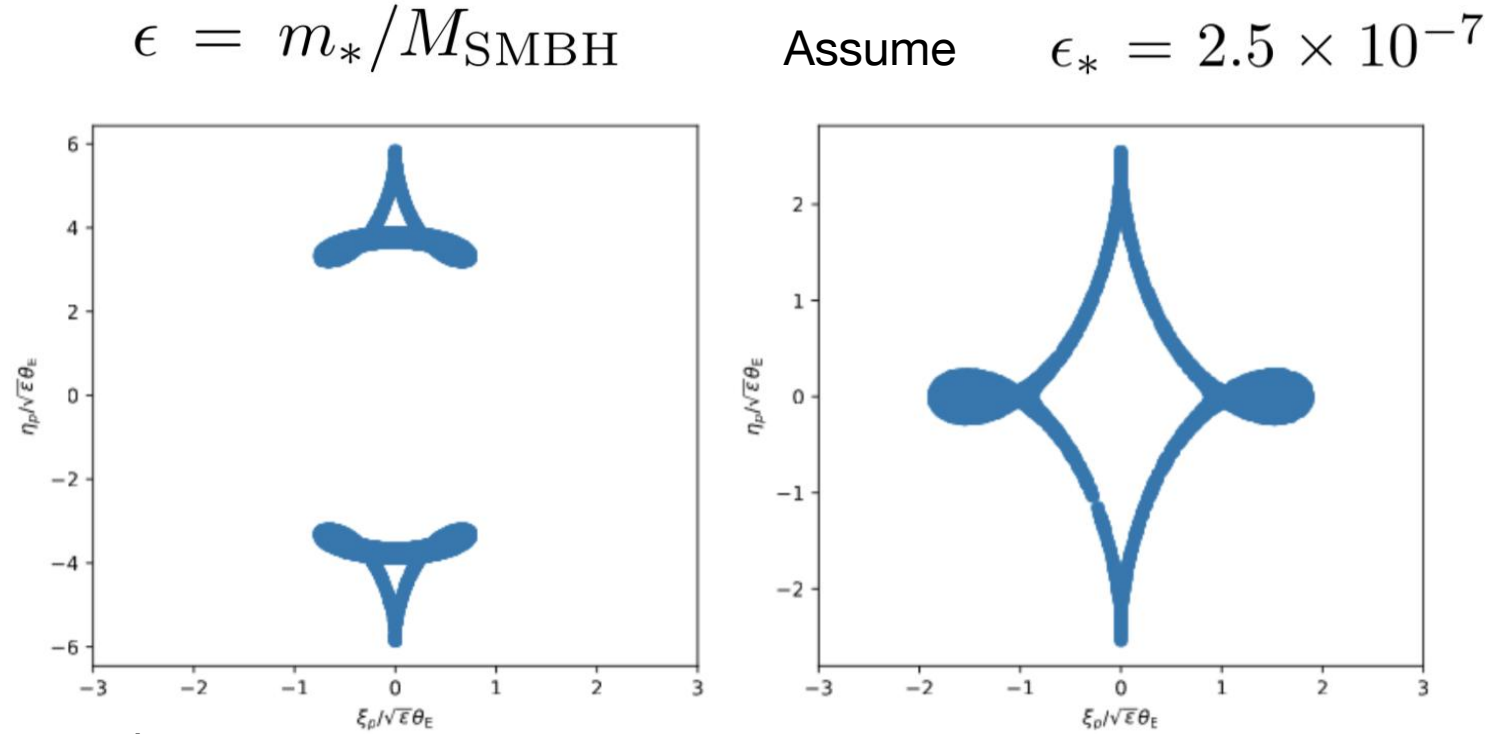
$$R_E = \left(\frac{4GM_\bullet}{c^2} \cdot \frac{Dr}{D+r} \right)^{1/2}$$



Alexander & Loeb 2001



Secondary lens



Chen+ 19 in prep

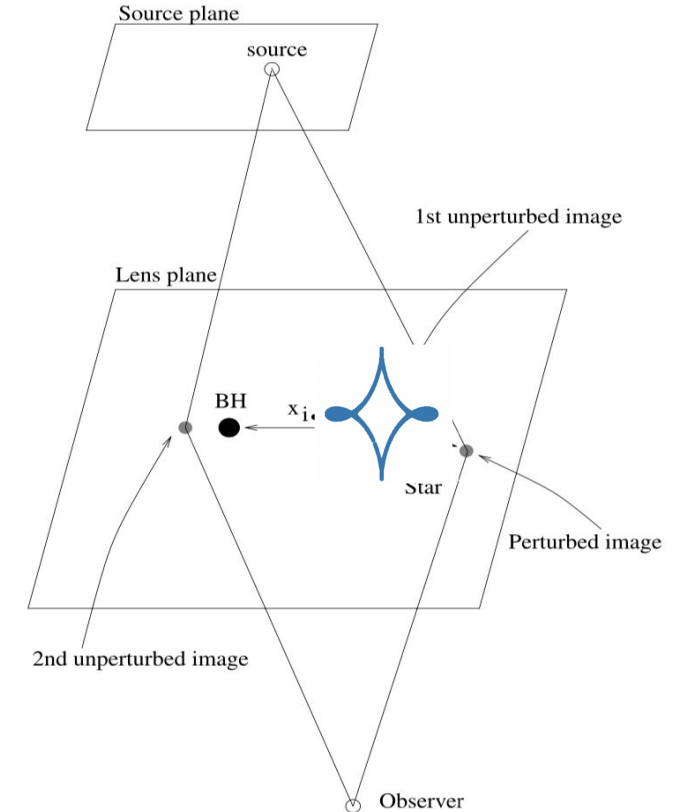
Figure 8. Areas in the (ξ_p, η_p) plane that satisfy the constraint of $A > 300\%$ (for *any* of the perturbed images generated), left panel: $\gamma = 1.3$; right panel: $\gamma = 0.6$. The intrinsic shapes differ for γ larger and smaller than 1, and approaches infinity for γ approaching 1, these images are consistent with (Gould & Loeb 1992)

Optical Depth (Lens)

$$\tau_*(> A, x_{BH}) = \sigma_*(> A, x_{BH}) \Sigma_*(x_{BH}) \ll 1$$

$$\Sigma_*(r) = 2 \int_r^\infty \frac{x n_*(x) dx}{\sqrt{x^2 - r^2}}$$

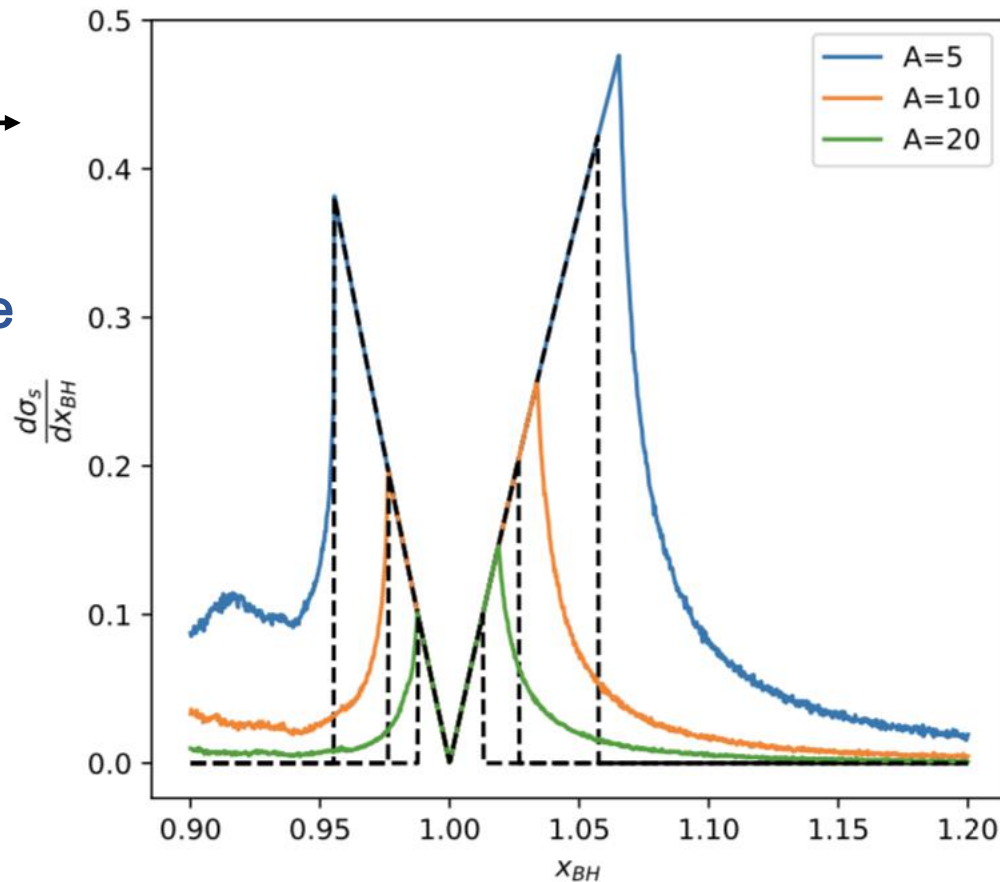
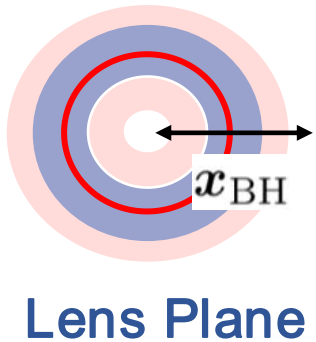
Higher order terms are neglected
(lower probability for TWO secondary lenses)



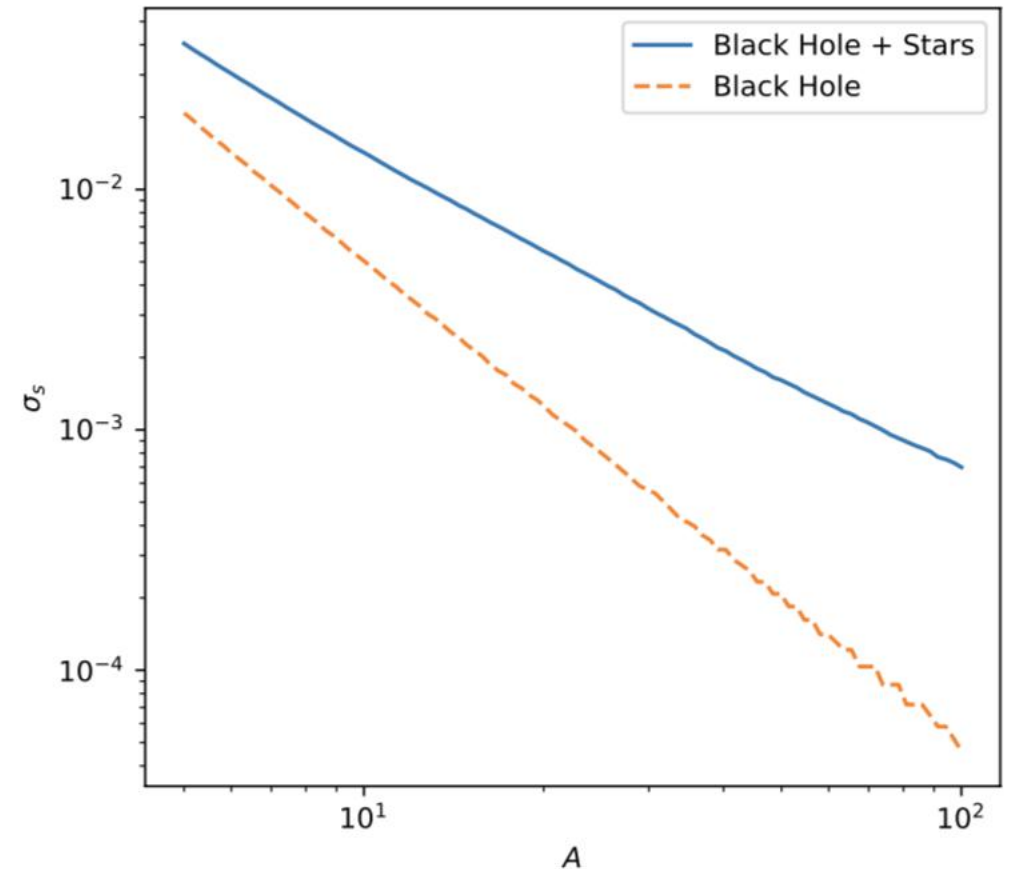
Cross Section in Source Plane

$$P(> A, \mathbf{x}_{\text{BH}}) \simeq \tau_*(> A, x_{\text{BH}}) + \Theta(A_{\text{BH}} - A)$$

$$\sigma_s(> A) = 2 \int P(> A, x_{i\text{BH}}) A_{\text{BH}}^{-1}(x_{i\text{BH}}) x_{i\text{BH}} dx_{i\text{BH}}$$



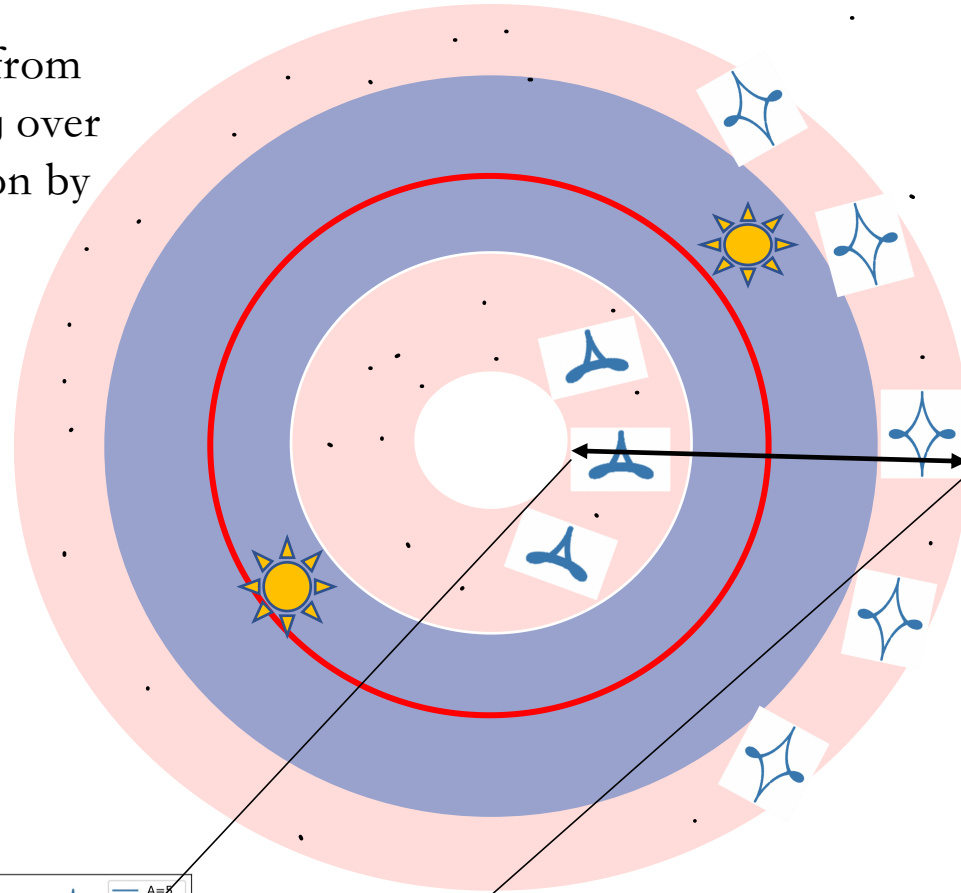
Heaviside step function:
Only when the BH isn't enough to give the amplification needed does secondary lens optical depth play a role in putting a probability weight $\ll 1$



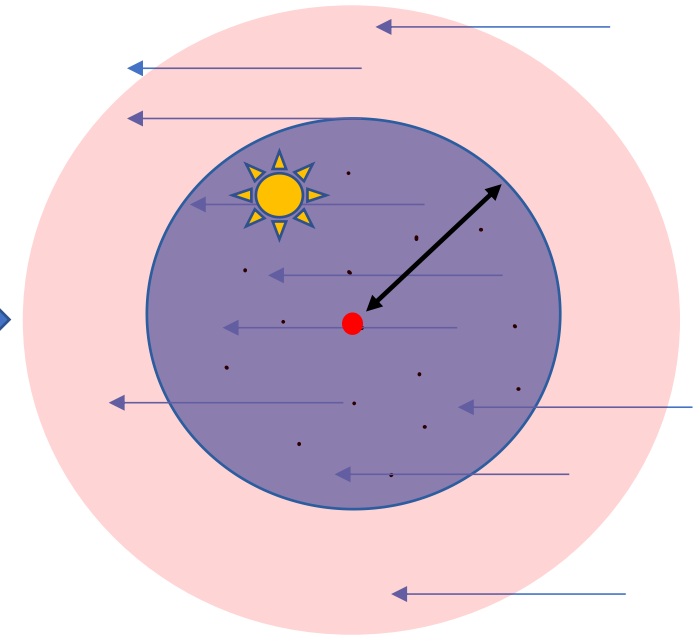
Cross Section in Source Plane

$$\sigma_s(> A) = 2 \int P(> A, x_{i\text{BH}}) A_{\text{BH}}^{-1}(x_{i\text{BH}}) x_{i\text{BH}} dx_{i\text{BH}}$$

Obtained from
integrating over
contribution by
annulus

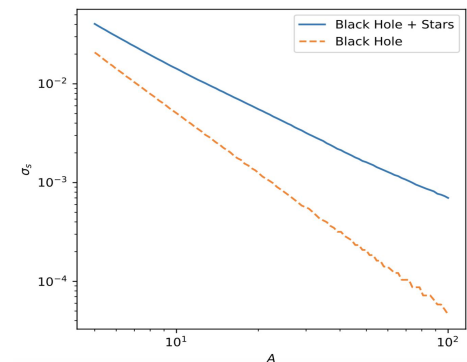
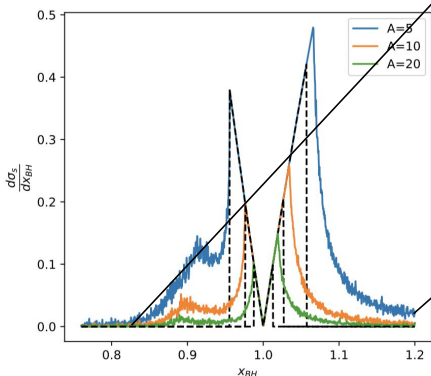


Lens Plane



Source Plane

- **Annulus** around ER on lens plane = one point behind lens the source plane
- **Contribution by BH alone** – projected back with weight 1
- **Contribution by secondary lenses** – projected back with weight optical depth



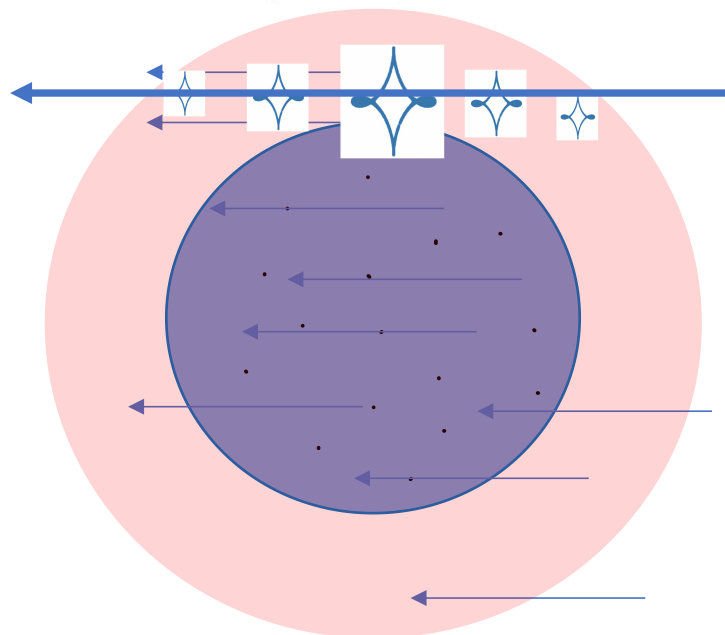
Results

Background star surface density (in KLF form)

$$\Sigma_s(< K) = \hat{\Sigma}_s 10^{bK}$$

$$\langle N_i(> A; K_0) \rangle = \int_A^\infty dA' \int_{-\infty}^{K_0} dK \left| \frac{d\sigma_s}{dA} \right|_{A'} \left| \frac{d\Sigma_s}{dK} \right|_{K+K_{A'}}$$

Alexander & Loeb 2001

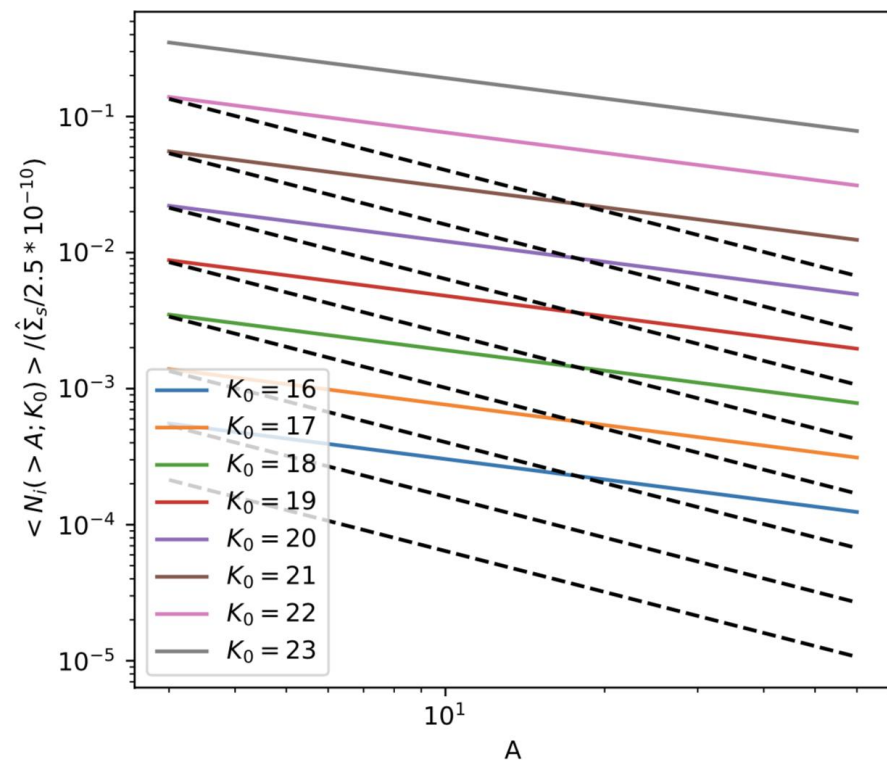


$$\Gamma = N_i / \bar{\tau}?$$

Velocity brings too much complication into RATE and timescale calculation

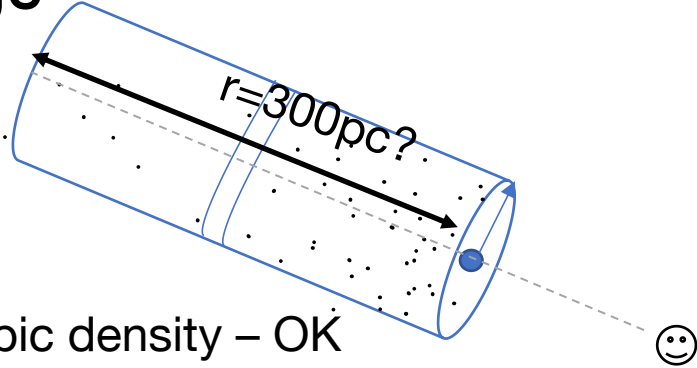
Snapshot of the Source Plane

How many events can we see looking at the GC 2'' ANY TIME?



Future Prospects

- **Density profiles for GC Disk and Bulge**



Anisotropic density – OK
Anisotropic velocity - complicated

- **Background stellar density calls for better model**

$$\Sigma_s(< K) = \hat{\Sigma}_s 10^{bK}$$

Uncertainty in velocity, density, extinction.....

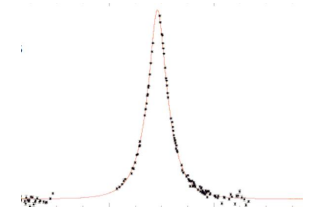
- **Distribution of Secondary lens mass**

Assumed uniformly solar-mass

Different $\epsilon = m_*/M_{\text{SMBH}}$ might have different probability weight at different annulus.

This has potentials to solve problems for microlensing in **other galaxies** and **asteroid in planetary systems**

- **Analysis of survey-data: Hypothesis test**



Non periodic variable K-band source with predicted typical magnification and timescale in direction of SMBH (**0.01-0.1''**)?

Microlensing events from campaigns Within **central 2''? VVV, OGLE...**