Dust Diffusion in Protostellar Disks and its Effect on Planet Formation

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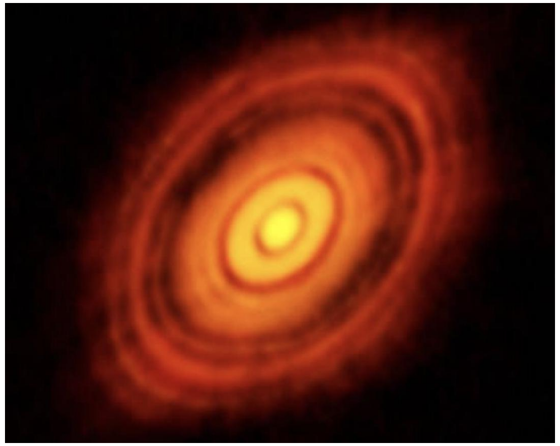
PART 1:
Dust Diffusion in Protostellar Disks

Background

Protostellar Disk



Composition:99% gas 1% dust
Timescale:1-10 mil yrs

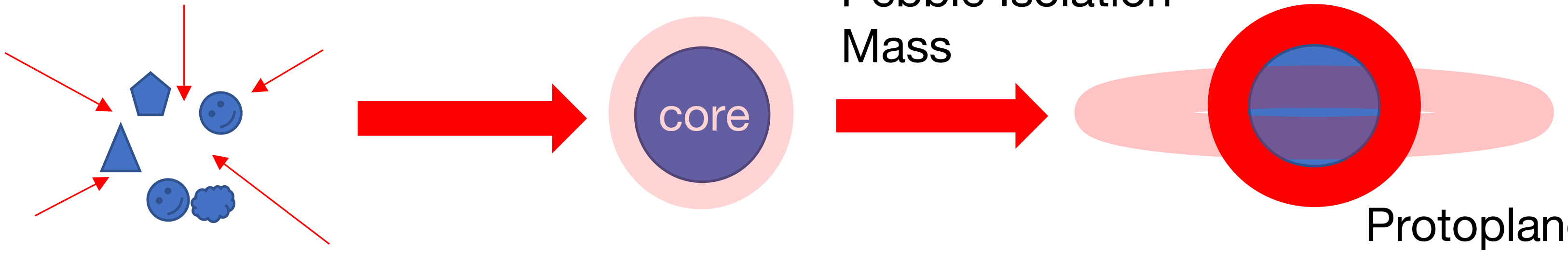
Standard Process of Planet Formation:

- **1. Accretion of pebbles & planetesimals into a SOLID CORE**
- **2. Accretion of GAS envelope**

Planets:

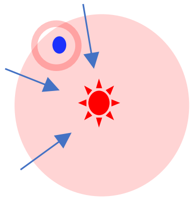
- **Ice Giant**
- **Terrestrial planets**

- **Super Earth**(one of many explanations)
- **Gas Giant**



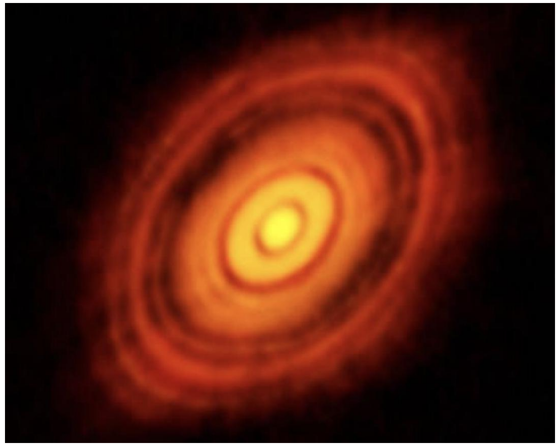
Reaching Pebble Isolation Mass

Protoplanetary disk



Background

Protostellar Disk



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- 1. Accretion of pebbles
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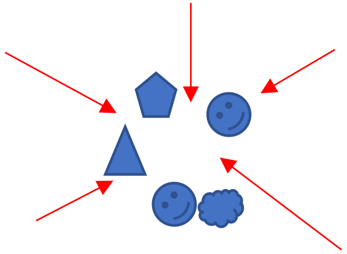
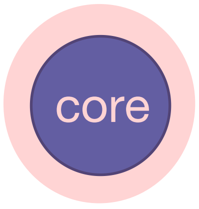
Planets:

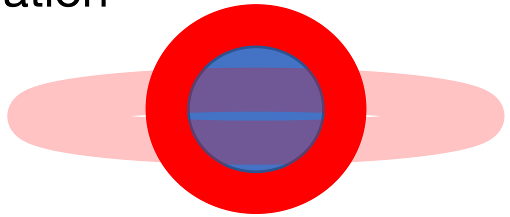
- Ice Giant
- Terrestrial planets

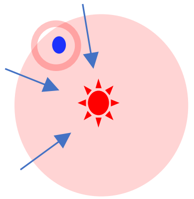
- Super Earth(in situ accretion)
- Gas Giant





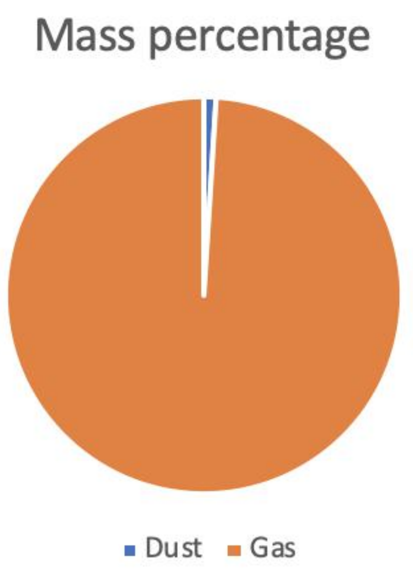
Reaching
Pebble Isolation
Mass

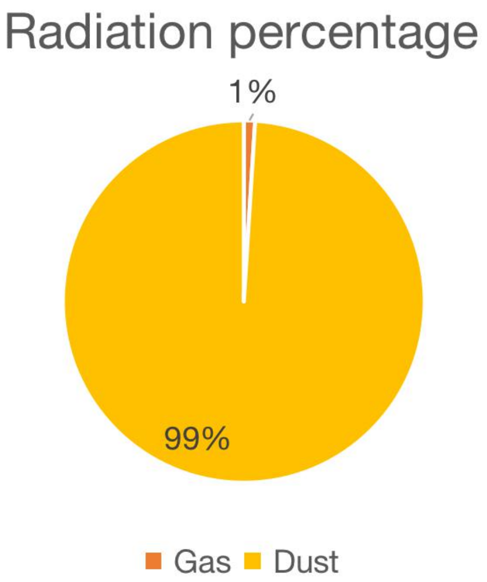


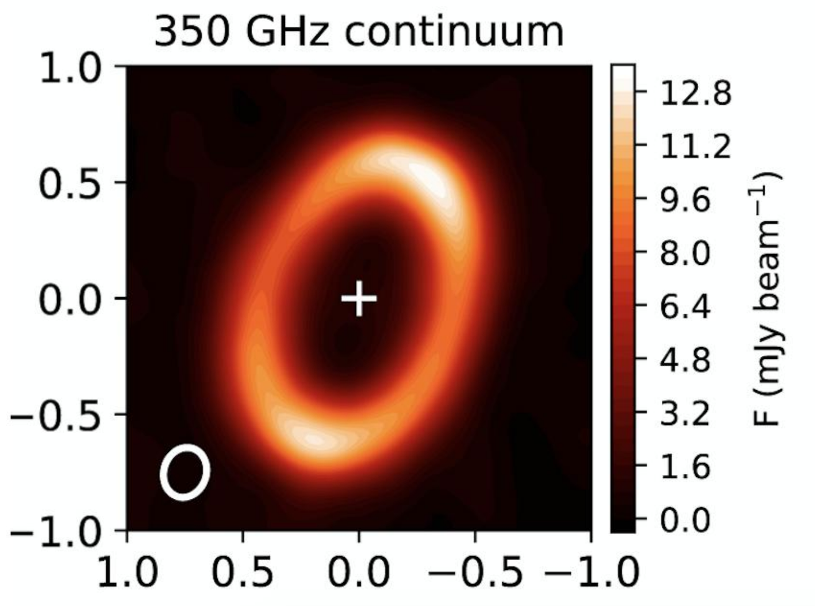


Pebble Isolation Observed

Transition Discs







Muley et al 2019

Dust Profile Gapped/Truncated:

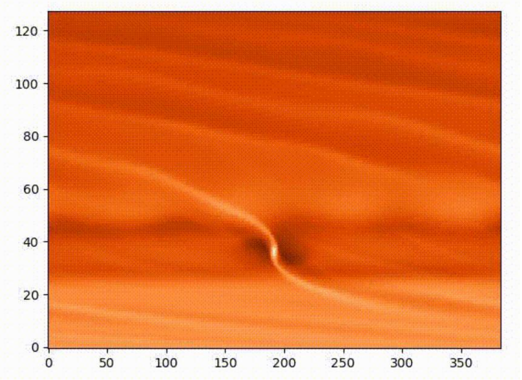
Some of them appears to be entirely devoid of circumstellar material within a certain radius of the star, arguably due to planet formation (or binary companion etc).

Pebble Isolation: General Picture

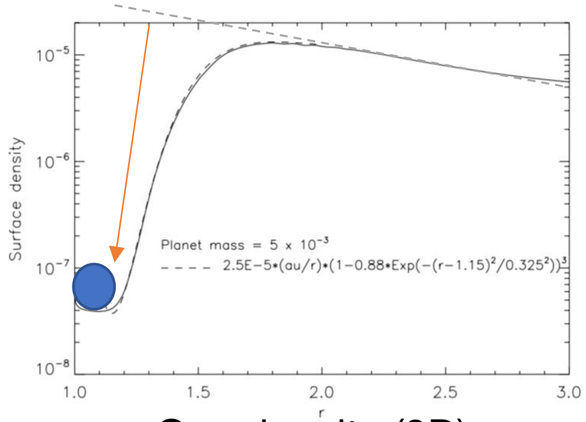
Gap Opening in Gas (Rice et al 2006, Bitsch et al 2018)

Momentum Eqn of Gas: $$\frac{v_\phi^2}{r} = \frac{V_K^2}{r} + \frac{1}{\rho}\frac{dP}{dr}$$

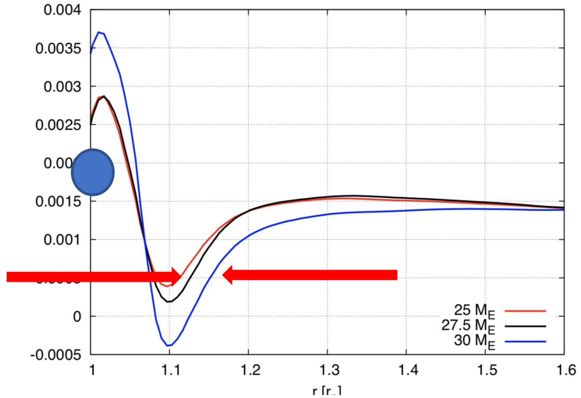
The gas velocity is not strictly Keplerian!



Gas density (2D)



Gas density (2D)
(Rice et al 2006)



Pressure gradient parameter $\eta = -\frac{h^2\Omega_k r}{2}\frac{ln\rho}{\ln r} \propto -\frac{\ln P}{\ln r}$
(Bitsch et al 2018)

$$\begin{cases} \dfrac{d\ln\rho}{d\ln r} < 0, \eta > 0, v_\phi < v_K \\ \dfrac{d\ln\rho}{d\ln r} > 0, \eta < 0, v_\phi > v_K \end{cases}$$

Acting through drag force

Drags down the velocity of dust, which loses angular momentum and spirals inwards

Speeds up dust, which is expelled outwards TOWARDS THE MAXIMA!

Quantify: Contaminant Diffusion (Clarke & Pringle 1988)

Diffusion Equation

1-D: everything is a function of R

$$\frac{\partial \Sigma}{\partial t} + \mathrm{div}(\Sigma \mathbf{u}) = 0$$

$$\frac{\partial \sigma}{\partial t} + \mathrm{div}\left(\sigma \mathbf{u} - \kappa \Sigma \nabla \frac{\sigma}{\Sigma}\right) = 0$$

$$\Sigma \left(\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C \right) = \mathrm{div}(\kappa \Sigma \nabla C)$$

Dust to density ratio/concentration

$$C := \frac{\sigma}{\Sigma}$$

Ratio of diffusion coef over viscosity

$$\zeta = \frac{\kappa(R)}{\nu(R)} = constant$$

$$\Sigma \frac{\partial C}{\partial t} + \Sigma v_{\mathrm{R}} \frac{\partial C}{\partial R} = \frac{1}{R} \frac{\partial}{\partial R} \left(R \zeta \nu \Sigma \frac{\partial C}{\partial R} \right)$$

Analytical results with NO PLANET PERTURBATION

Clarke & Pringle 1988

Steady Accretion

$$\Sigma = \Sigma_0 R^{-a}$$

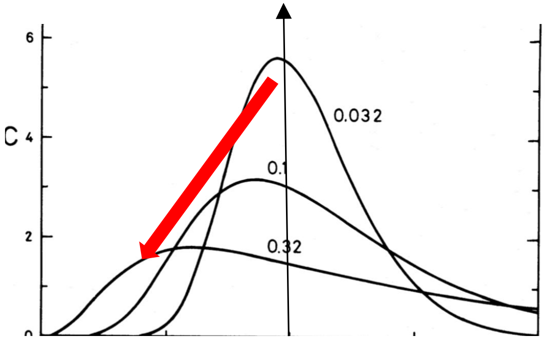
$$v_{\mathrm{R}} = -\frac{3\nu}{2R} = -\frac{\dot{M}}{2\pi \Sigma_0 R^{1-a}}.$$

$$\frac{1}{\nu_0} \frac{\partial C}{\partial t} - \frac{3}{2} R^{a-1} \frac{\partial C}{\partial R} = R^{a-1} \frac{\partial}{\partial R} \left(\zeta R \frac{\partial C}{\partial R} \right)$$

For given condition:

$$C(R,t)|_{t=0} = C_0 \delta(R - R_0)$$

$$C(R,t)|_{R=R_{min}, R=R_{max}} = 0$$



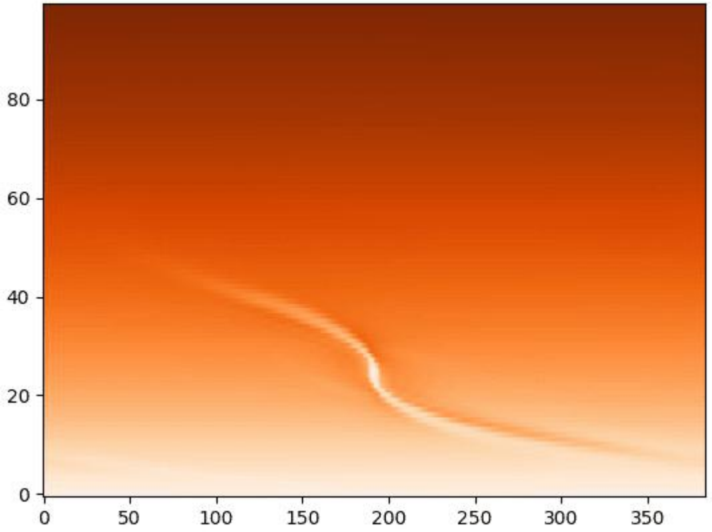
Planet → Gas(2D)

FARGO3D Benítezllambay & Masset2016

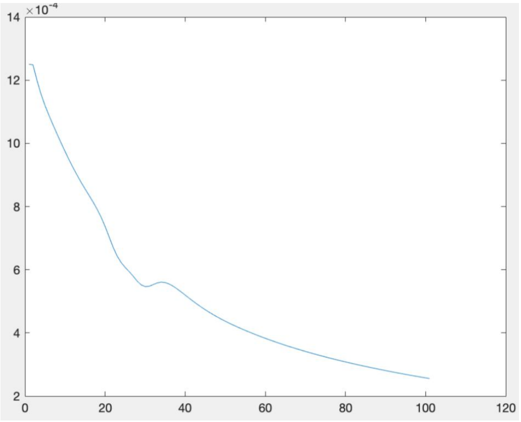
Parameters:

Default unit system:
central star mass=1
orbital radius=1
G=1
Planet=0.001(a planet core)

1. h_0: 0.05
2. Σ_0: 6.3661977237e-4
3. ν_0: 1.0e-5
4. α: 1.0
5. β: 0.25

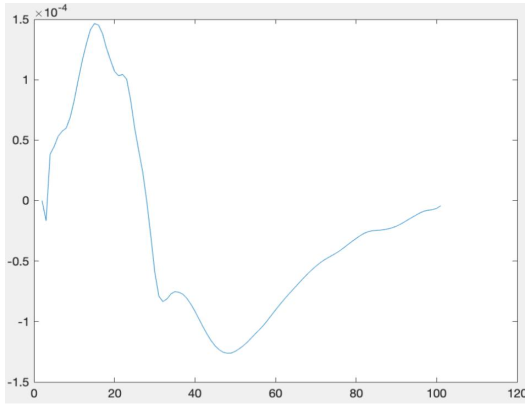


Radial cross-section



Gas Density

Radial Direction



Radial Velocity

Radial Direction

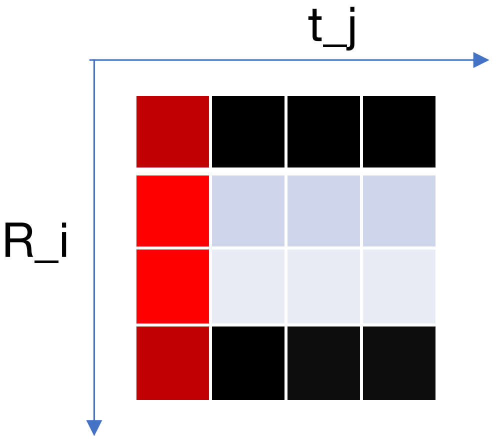
Numerical Method

Jacobi Iteration with MATLAB

$C(R,t) = C(R_i, t_j)$

$$\frac{\partial C}{\partial R}_{i,j} \approx \frac{C(R_{i+1}, t_j) - C(R_{i-1}, t_j)}{\Delta R}$$

$$\frac{\partial C}{\partial t}_{i,j} = \frac{C\left(R_i, t_j\right) - C\left(R_i, t_{j-1}\right)}{\Delta t}$$



Boundary condition

Initial condition

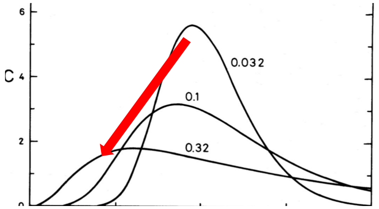
Boundary condition

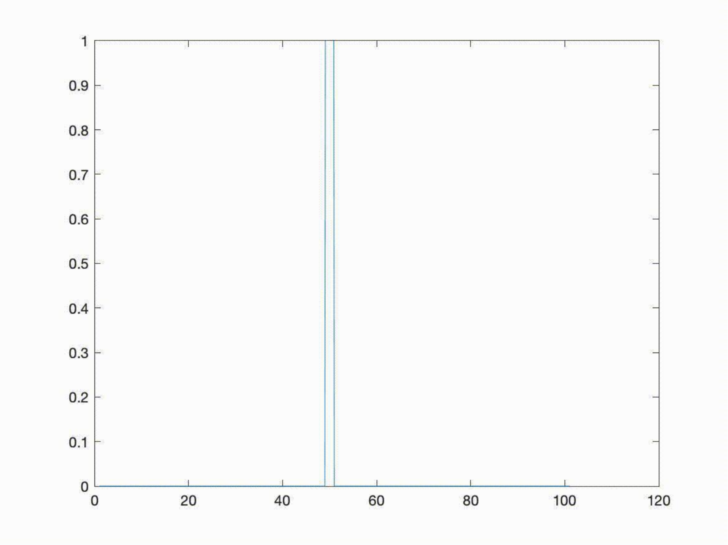
$$C_{i,j} = f(C_{i+1,j}, C_{i-1,j}, C_{i,j-1})$$

Continue to iterate until the C matrix becomes stable!

Method Test

Green function initial

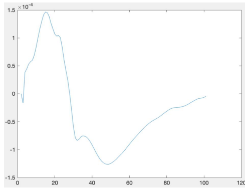




Gas → Grains (1D)

$$\Sigma\frac{\partial C}{\partial t} + \Sigma v_{\rm R}\frac{\partial C}{\partial R} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\zeta\nu\Sigma\frac{\partial C}{\partial R}\right) \longrightarrow \frac{\partial C}{\partial t} + v_{\rm R}(R)\frac{\partial C}{\partial R} = \kappa_0[(2+\frac{\partial \ln \Sigma}{\partial \ln R}(R))\frac{\partial C}{\partial R} + R\frac{\partial^2 C}{\partial R^2}]$$

Modified Gas Velocity



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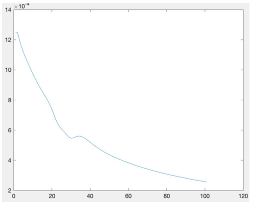

(Extracted azimuthal mean 1-D data from the simulation)

For given condition:

$$C(R,t)|_{t=0} = \frac{0.0001}{\Sigma(R)}$$

$$\partial_R C(R,t)|_{R=R_{min},R=R_{max}} = 0$$

Results:

Gas → Grains (1D)

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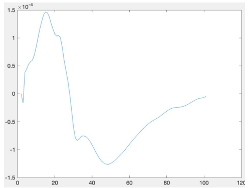

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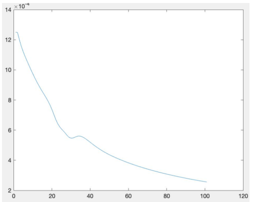

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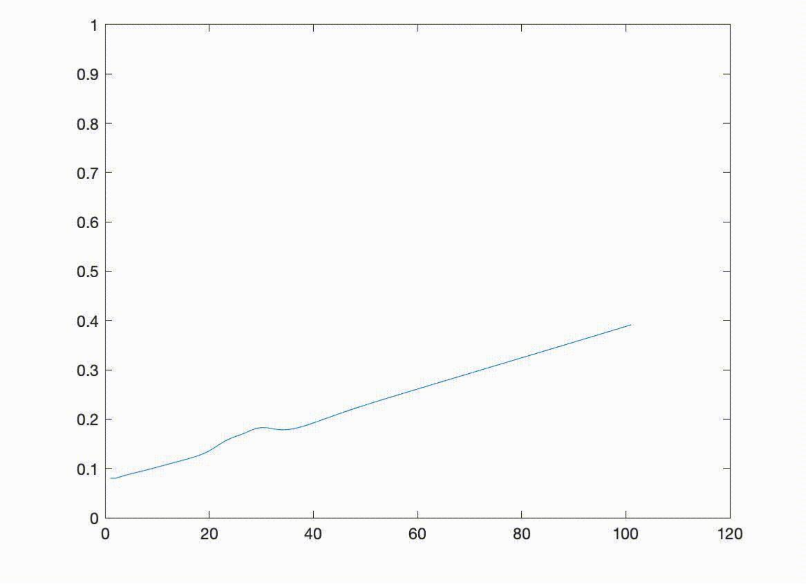
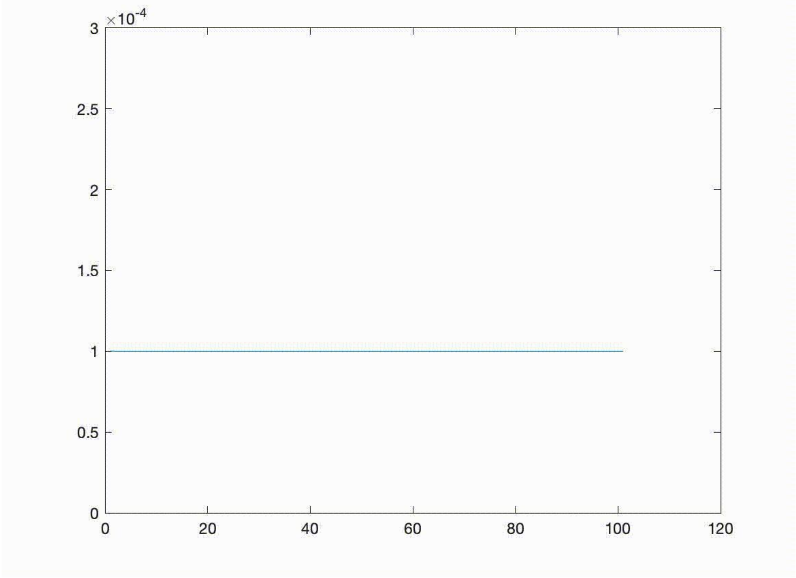
Results:

Concentration Profile C



Density Profile σ



Modification (+ Gas Drag)

v: relative azimuthal velocity; u: radial velocity

$$
\begin{cases}
\frac{1}{2}u\Omega_k = -\frac{\rho_p}{\rho}\frac{v-v_p}{\tau_s} + \frac{1}{2}\xi\Omega_K \\
\frac{1}{2}u_p\Omega_k = \frac{v-v_p}{\tau_s} \\
-2v_p\Omega_k = \frac{u-u_p}{\tau_s} \\
-2v\Omega_k = -\frac{\rho_p}{\rho}\frac{u-u_p}{\tau_s} + 2\eta\Omega_k
\end{cases}
$$

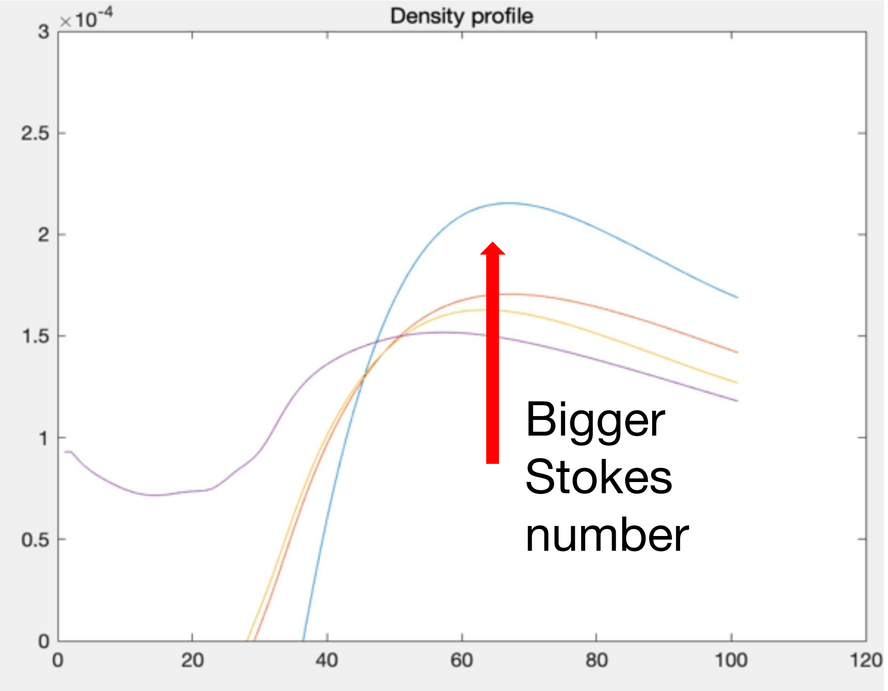
$$\eta = -\frac{h^2\Omega_k r}{2}\frac{ln\rho}{\ln r}$$

$$\frac{1}{2}\xi = \frac{\partial}{pr^2\Omega_K\partial r}\left(\rho\nu r^3\frac{\partial\Omega_k}{\partial r}\right)$$

Affiliated with relaxation time.
Eqn 1,4 ---- Feedback
Eqn 2,3 ----Gas drag

$$
\begin{cases}
\frac{1}{2}u_g\Omega_k = \frac{v_{gas}-v_g}{\tau} \\
-2v_g\Omega_k = \frac{u_{gas}-u_g}{\tau}
\end{cases}
\Rightarrow
u_g = \frac{u_{gas}}{1+(\tau\Omega_k)^2}
$$

Stokes number



Conclusion:
After becoming stable, the bigger dust grains (pebbles) are more likely to be totally blocked.(PEBBLE ISOLATION)

Simulation By Super Computer(Owen 2014)

Two-Fluid Simulation:

$$\frac{\partial \Sigma_g}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}\left(R^{1/2}\nu\Sigma_g\right) - \frac{2\Lambda\Sigma_g R^{3/2}}{\sqrt{GM_*}}\right]$$

$$\frac{\partial \Sigma_d^i}{\partial t} = -\frac{1}{R}\frac{\partial}{\partial R}\left[R\Sigma_d^i v_d^i - \frac{\nu}{Pr}R\Sigma_g\frac{\partial}{\partial R}\left(\frac{\Sigma_d^i}{\Sigma_g}\right)\right]$$

$$\Lambda = \begin{cases} -\frac{q^2 GM_*}{2R}\left(\frac{R}{\max(H,|R-a)}\right)^4 & \text{if } R < a \\ \frac{q^2 GM_*}{2R}\left(\frac{a}{\max(H,|R-a|)}\right)^4 & \text{if } R > a \end{cases}$$

q : planet-star mass ratio.

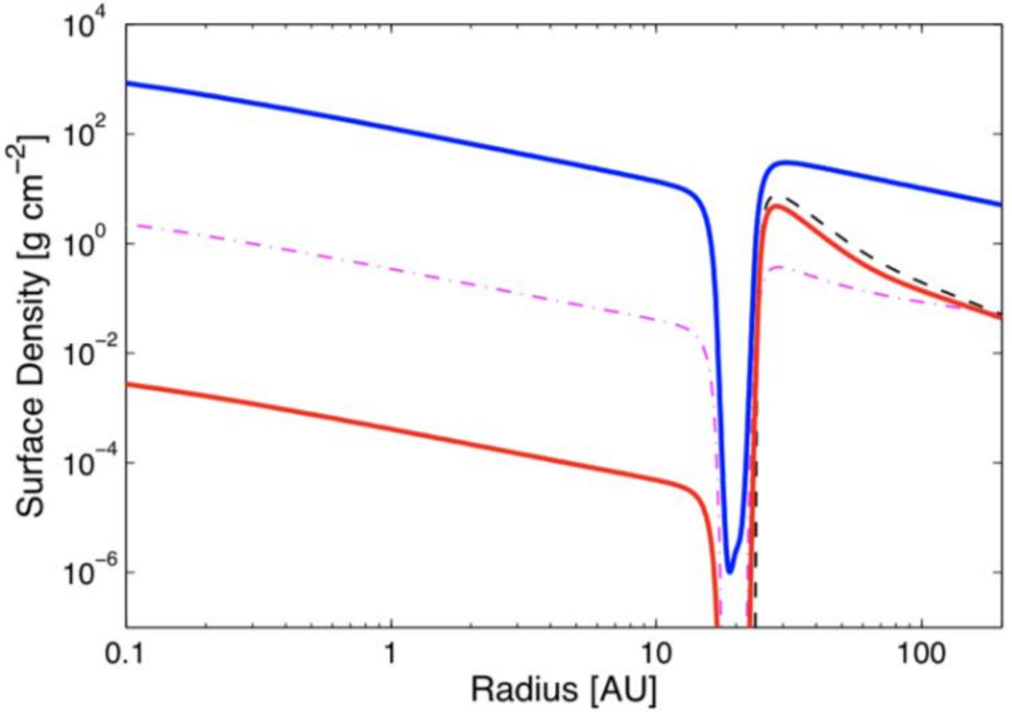


FIG. 1.— The dust and gas distribution for simulation A: a disc with a 4.0M_J planet on a circular orbit at 20AU, without including the effect of radiative feedback on the dust. The thick blue line shows the gas surface density, while the thick red line shows the total dust surface density. The dot-dashed and dashed line shows the surface density of sub-micron (∼0.1μm) and mm-sized (∼ 1 mm) dust particles respectively, note these are not plotted with physical units (e.g. g cm^{-2}), but rather; both dashed and dot-dashed lines are scaled such that their individual dust-to-gas ratios are 0.01 at large radius.

PART 2:
Implication on Planet Formation

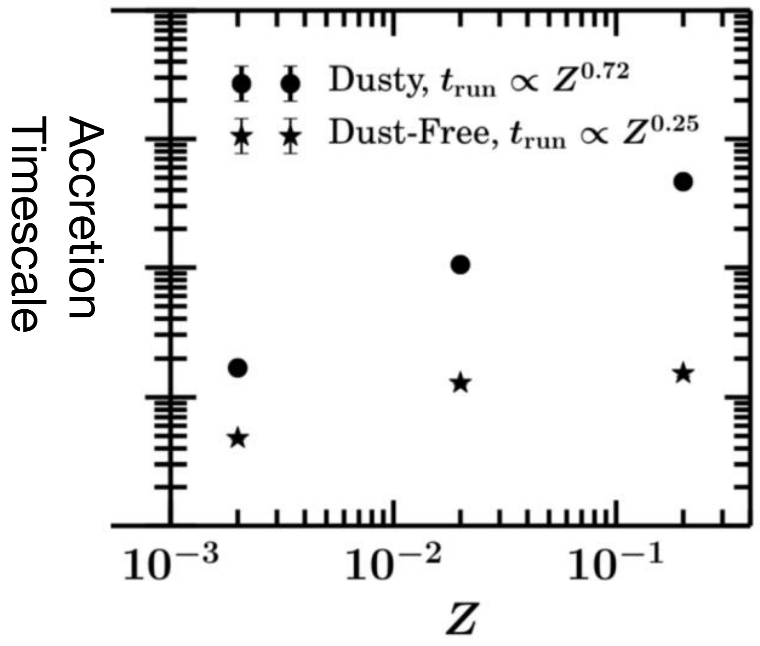
Opacity

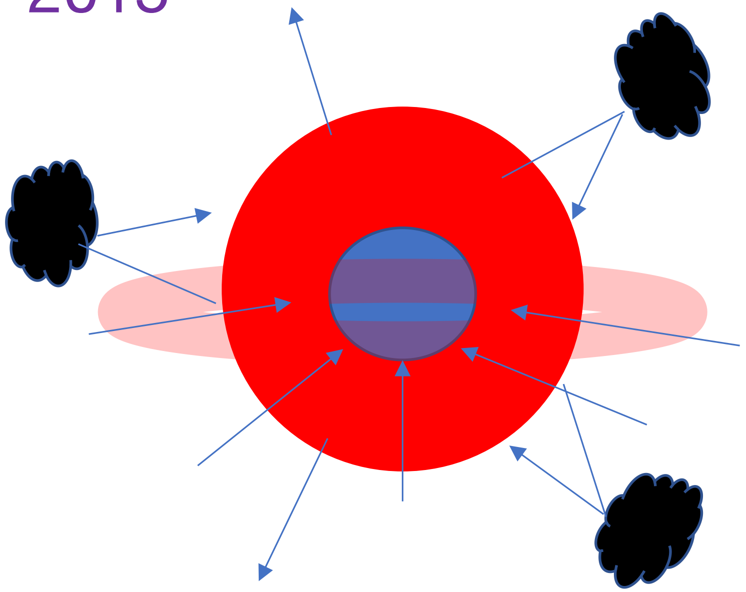
$$\kappa_{\rm rcb} = \kappa_0(\rho_{\rm rcb}/\rho_0)^\alpha(T_{\rm rcb}/T_0)^\beta(Z/Z_0)^\delta$$

Contributed by dust grains and metallicity!

- Grain/Metal Contaminant↓
- Opacity ↓
- Cooling↑
- Accretion↑

"To Cool is to Accrete!"
——Lee &Chiang 2015





Opacity in Pit and Pileup

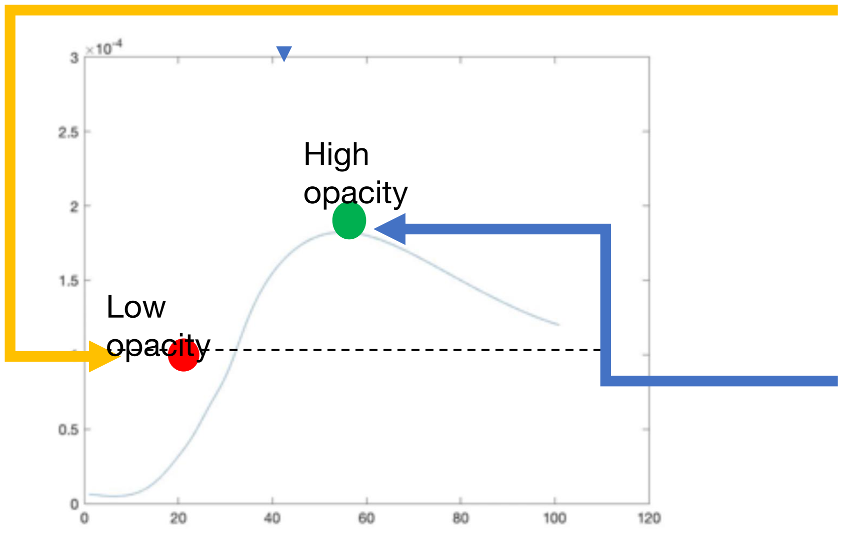
Creationist’s Point of View

Opacity/Metallicity is a pre-set **CONSTANT**! →

Low Metallicity/Opacity -> Quick Accretion, Short Timescale

High Metallicity/Opacity -> Slow Accretion, Long Timescale

Evolutionist’s Point of View



Density profile

PIT:

Accreting planets have the ability to lower the original dust density around the vicinity by generating dust barriers

enhancing its own cooling and accretion

Hard to form a Super-Earth by itself

PILEUP:

A new core’s (once formed) accretion would be hindered by the opacity accumulated under the influence of the first planet

Might remain a Super-Earth

NEXT STEP: QUANTIFY THE EFFECT ON TIMESCALE

Opacity in Pit and Pileup

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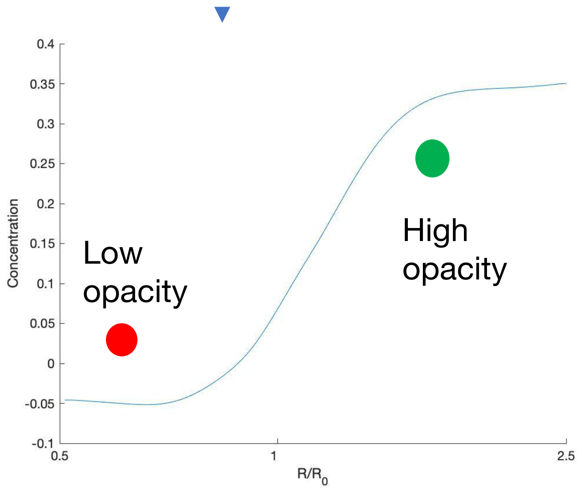

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Might remain a Super-Earth

NEXT STEP: QUANTIFY THE EFFECT ON TIMESCALE

Self-Gravity in Pileup (+Gas Drag +Feedback)

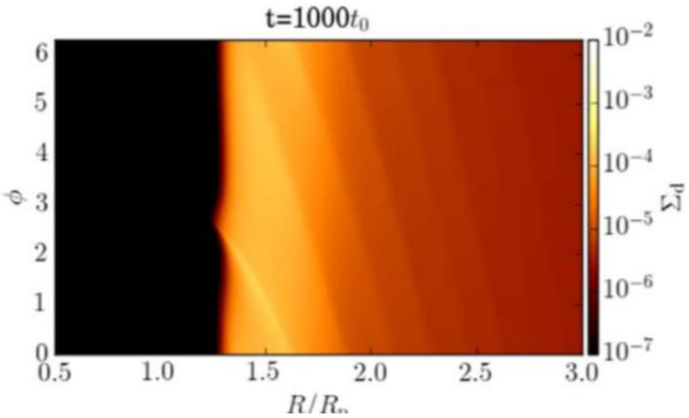
$$\begin{cases} \frac{1}{2}u\Omega_k = -\frac{\rho_p}{\rho}\frac{v-v_p}{\tau_s} + \frac{1}{2}\xi\Omega_K \\ \frac{1}{2}u_p\Omega_k = \frac{v-v_p}{\tau_s} \\ -2v_p\Omega_k = \frac{u-u_p}{\tau_s} \\ -2v\Omega_k = -\frac{\rho_p}{\rho}\frac{u-u_p}{\tau_s} + 2\eta\Omega_k \end{cases}$$

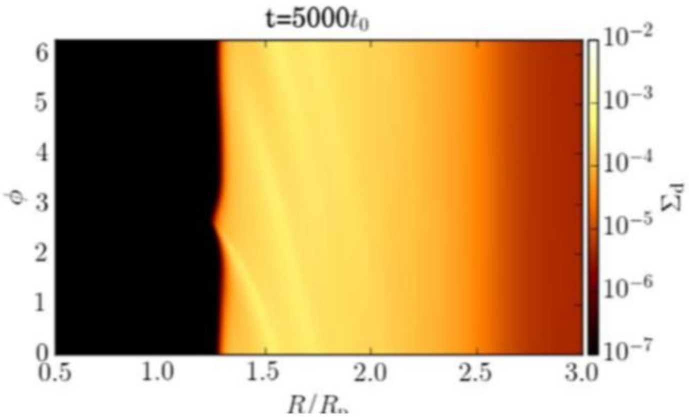
$$\xrightarrow{\Delta v := v - v_p,\ \Delta u := u - u_p}$$

$$\begin{cases} -\Delta v = \eta - \frac{1}{2}(1+C)\frac{\Delta u}{S_t} = 0 \\ \Delta u = \xi - 2(1+C)\frac{\Delta v}{S_t} = u_g \end{cases}$$

Critical Concentration $C = \frac{2\eta S_t}{u_g} - 1$

Flattening:
After reaching critical concentration, the dust begins to move outwards and gain a positive radial velocity, to accumulate elsewhere





Simulation by Kanagawa et al. 2018

Ccrit>>1: the grains accumulates enough to generate self-gravity, might form planetesimals and cores

Ccrit<<1: the grains reach a ceiling and flattens out before generating self gravity

Chain Reaction..?

Summary

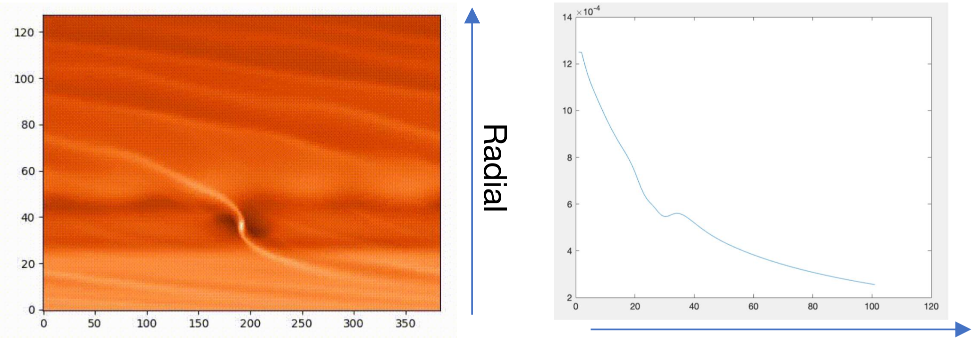
PART 1
Dust Diffusion in Protostellar disks

- Planet formation
- Gap Opening and Pebble Isolation
  (Explained with Diffusion Equation)

Conclusion: Accreting planets have the ability to change the dust density around the vicinity by generating dust barriers

PART 2
Effects on Planet Formation

- Opacity
  - Lower opacity/dust density around it, enhances its own accretion -> gas giant?
  - High opacity/dust density in the pileup, hinders the accretion of a new core ->super earth?
- Gravity: Generates gravity instability/core forming in the pileup if the Critical Concentration before “flattening” reaches ~1



Gas Density distribution 2D->1D

Dust Density distribution