

Supervised Learning

Overview/Linear Regression Models

Sources: A. Mueller, ESLR, and R. Tibshirani

Lecture Overview

- Overview with key terms and basic model
- Least Squares walk through
- Ridge and Lasso Regression
- Code introduced along the way

Supervised Learning

$$(x_i, y_i) \propto p(x, y) \text{ i.i.d.}$$

$$x_i \in \mathbb{R}^p$$

$$y_i \in \mathbb{R}$$

$$f(x_i) \approx y_i$$

$$f(x) \approx y$$

K nearest neighbors

- A simple algorithm:
 1. Define a measurement of similarity between observations in dataset
e.g.- Euclidean Distance
 2. Define “k” parameter (i.e.- number of nearest neighbors to evaluate in training data)

K nearest neighbors

- A simple algorithm:

Model makes prediction with new test observation by....

3. Calculate observation's similarity to all observations in training data using distance measurement
4. Find k nearest neighbors in training data.
5. Assign category from majority of these neighbors to observation
(e.g. – take majority vote)

K nearest neighbors

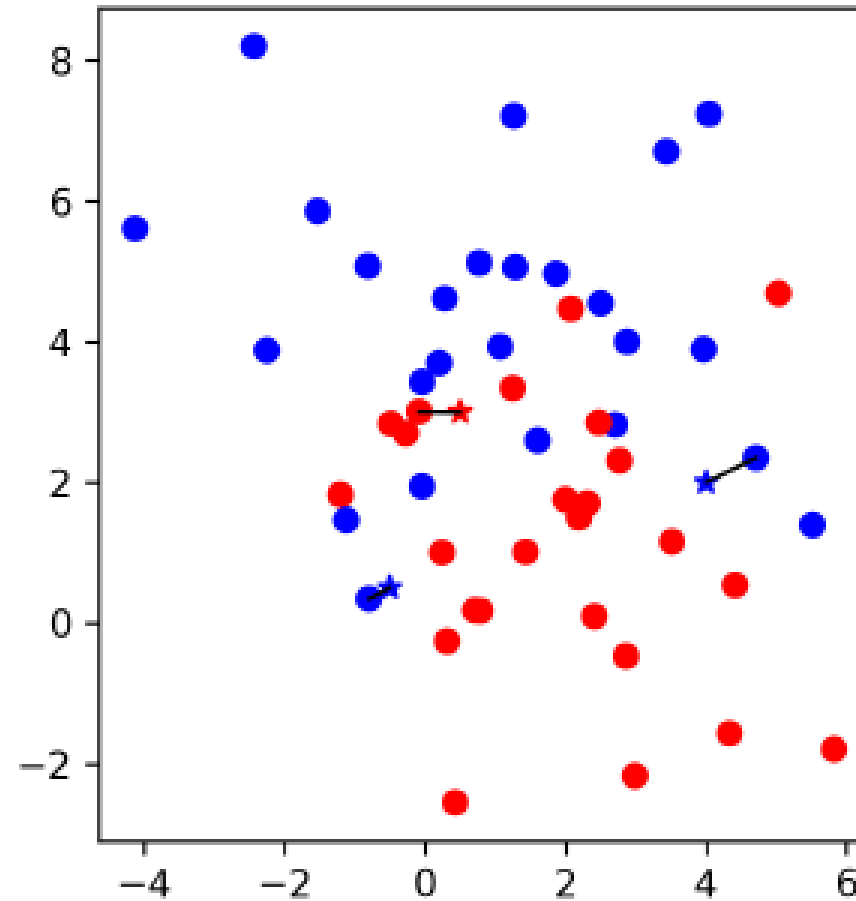
Euclidean distance:

$$d(x, x') = \sqrt{(x_1 - x'_1)^2 + (x_2 - x'_2)^2 + \dots + (x_n - x'_n)^2}$$

For any two observations in data, subtract one row from the other, square result, add together, then take square root

Similar observations have smaller distances

Nearest Neighbors



Nearest Neighbors

training set

$$X = \begin{pmatrix} 1.1 & 2.2 \\ 6.7 & 0.5 \\ 2.4 & 9.3 \\ 1.5 & 0.0 \\ 0.5 & 3.5 \\ 5.1 & 9.7 \\ 3.7 & 7.8 \end{pmatrix}$$
$$y = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

test set

Training Set X	Training Set y	Test Set X	Test Set y
1.1	2.2	0	1
6.7	0.5	1	1
2.4	9.3	1	0
1.5	0.0	0	1
0.5	3.5	1	0
5.1	9.7	0	0
3.7	7.8	0	0

Nearest Neighbors

KNN with scikit-learn

```
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(X, y)

from sklearn.neighbors import KNeighborsClassifier
knn = KNeighborsClassifier(n_neighbors=1)
knn.fit(X_train, y_train)
print("accuracy: {:.2f}".format(knn.score(X_test, y_test)))
y_pred = knn.predict(X_test)
```

accuracy: 0.77

Nearest Neighbors

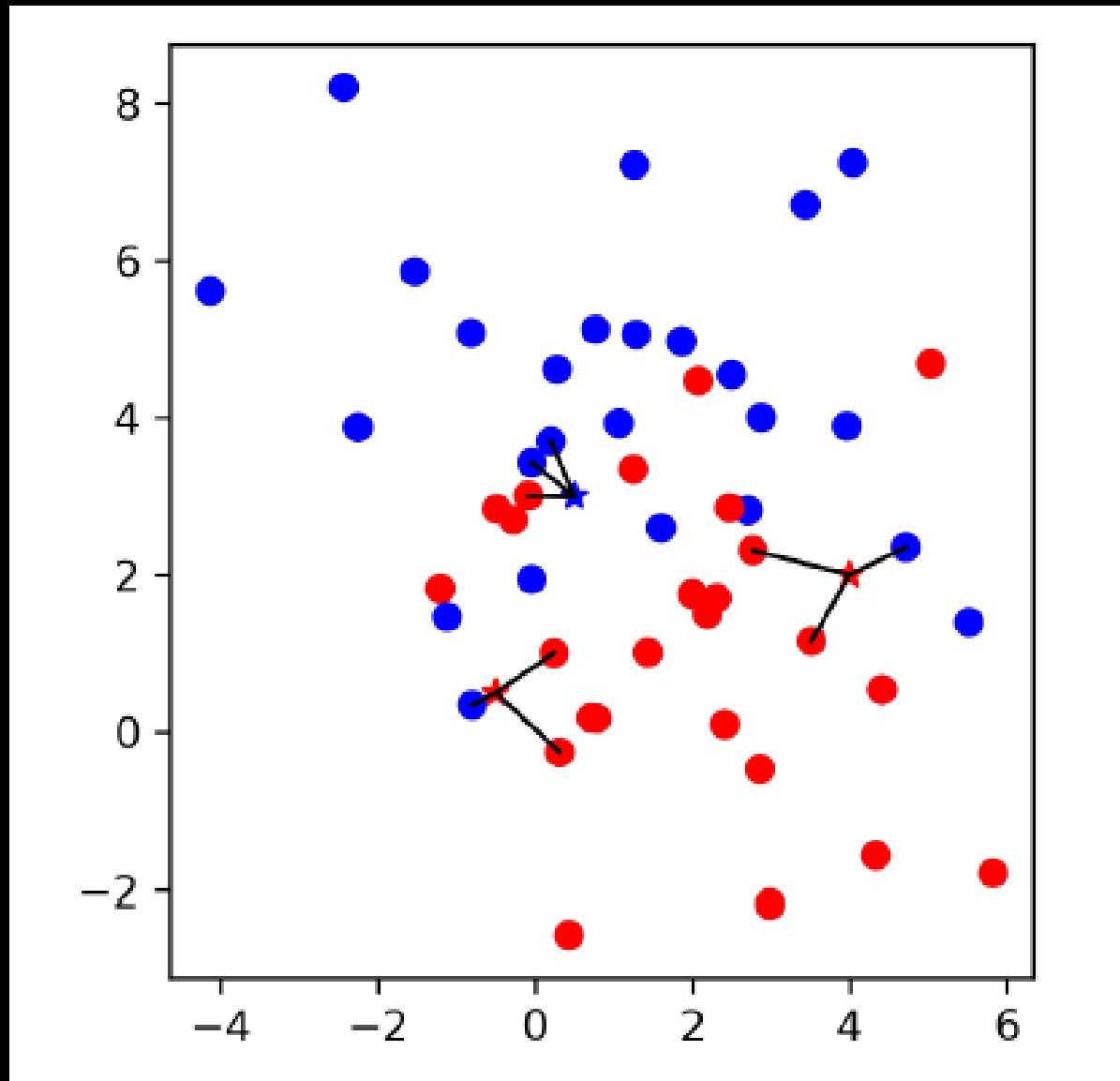
KNN with scikit-learn

```
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(X, y)

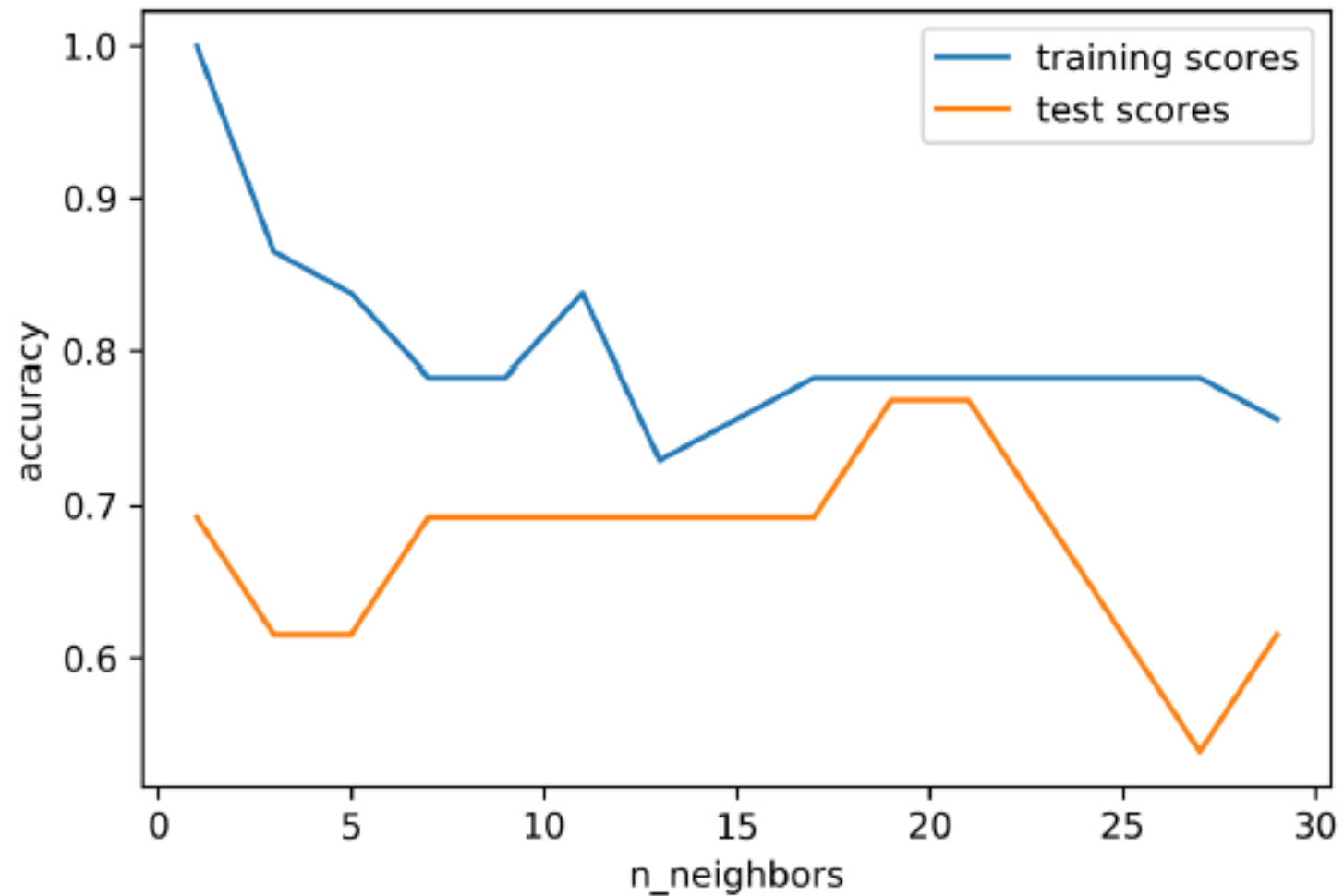
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print("accuracy: {:.2f}".format(knn.score(X_test, y_test)))
y_pred = knn.predict(X_test)
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accuracy: 0.77

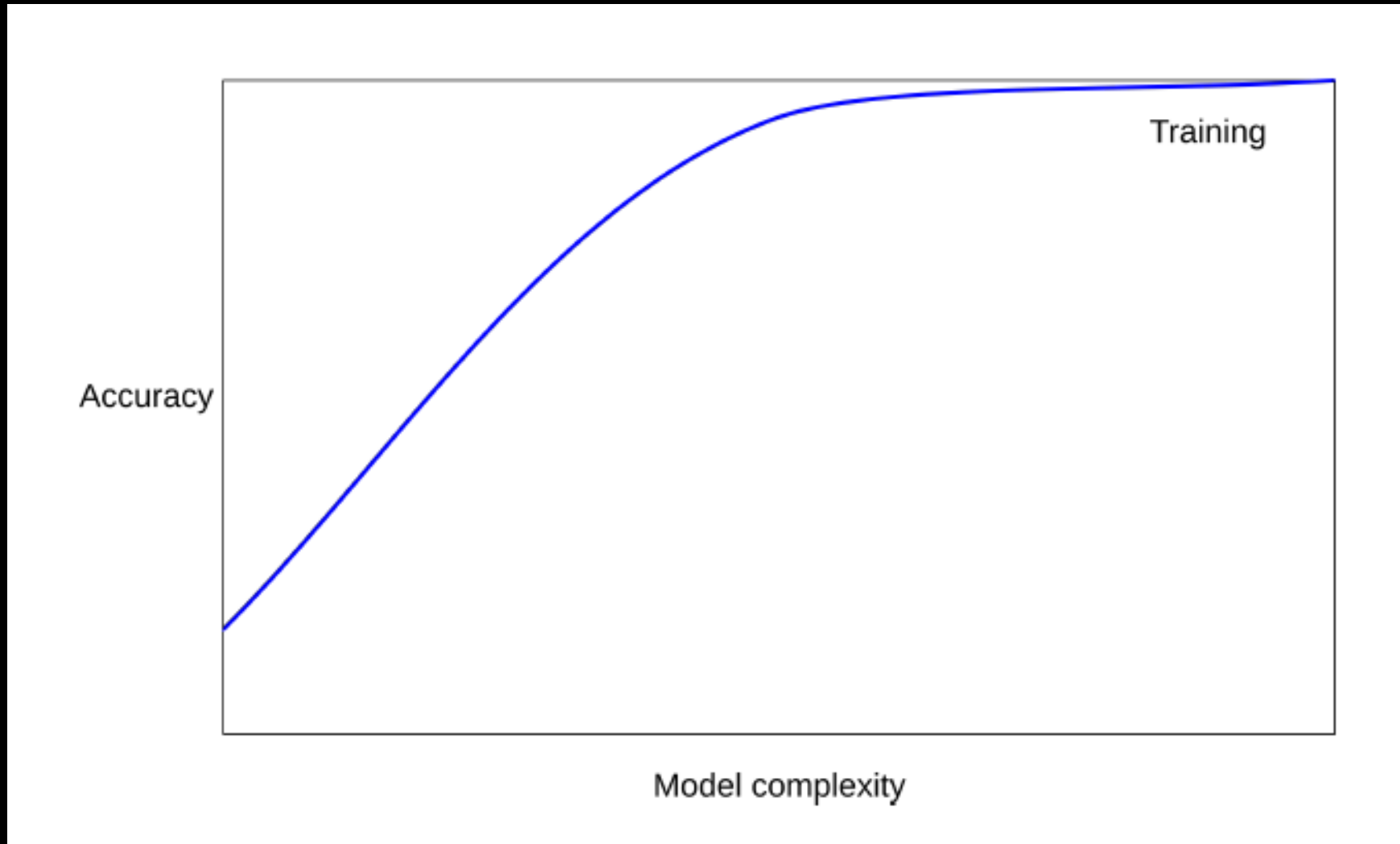
More Neighbors



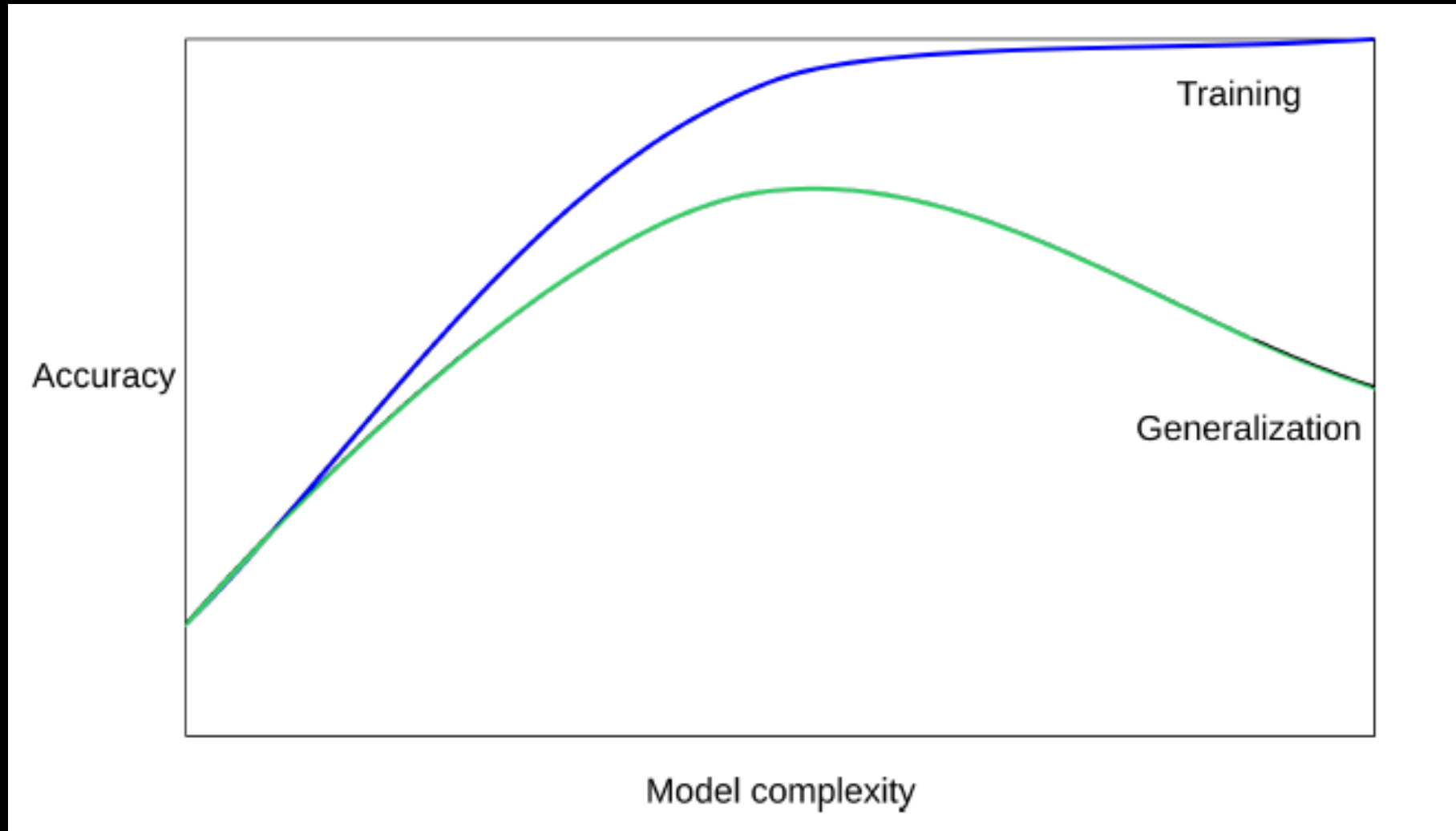
More Neighbors



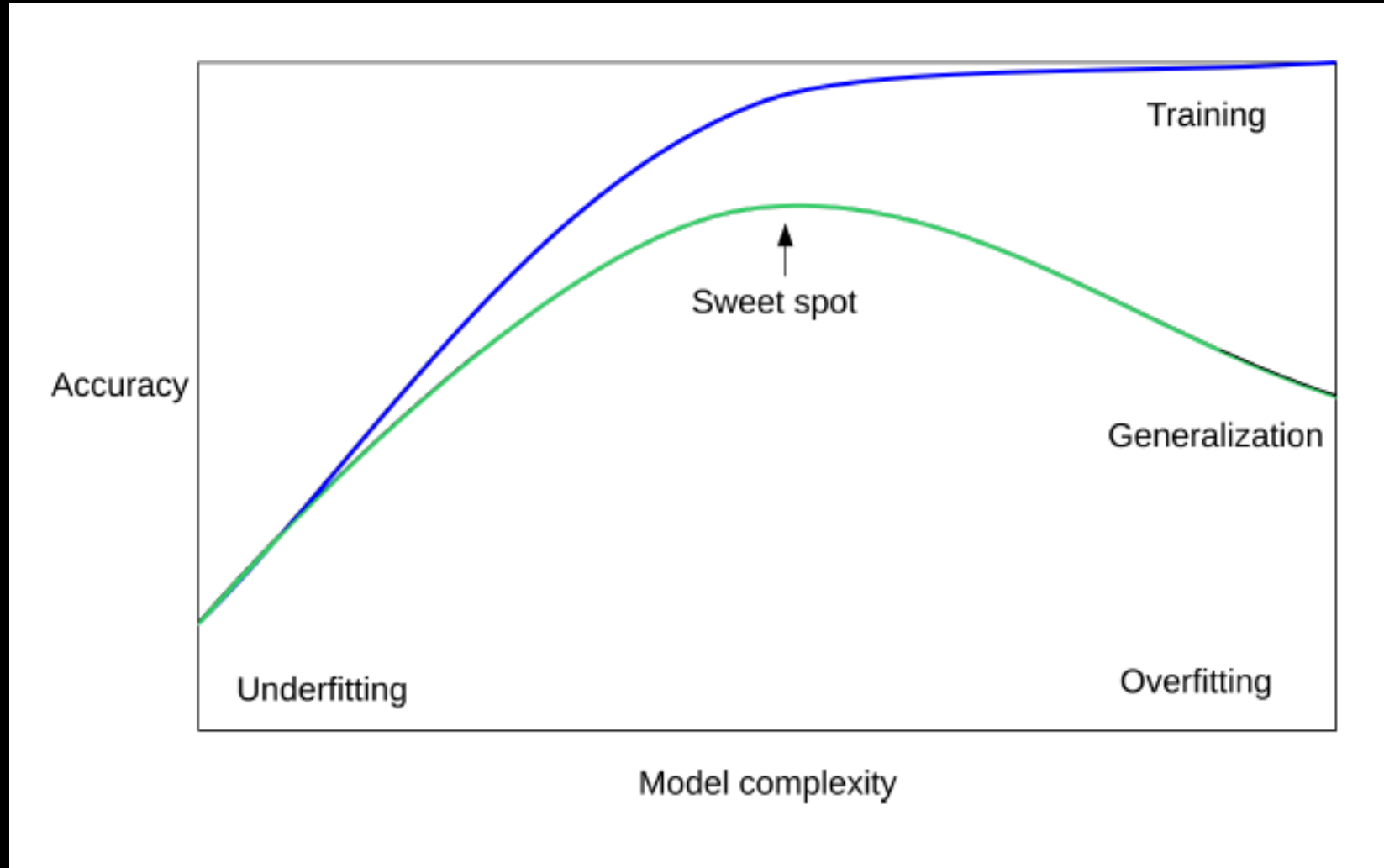
Overfitting and Underfitting



Overfitting and Underfitting



Overfitting and Underfitting



Other considerations for KNN Models

- Practically speaking, it's better to...
 - Scale your IVs before taking distance calculations
 - Use z scores for example
 - Also good to use odd number for k to break any ties in majority votes

Other considerations for KNN Models

- Minimal training but expensive testing.
 - Training model is fast/easy
 - Testing model, however, requires large training dataset to draw from each time for model predictions

KNN regression model

- Same algorithm as classification, except..
 - Take average value of y for nearest neighbors
 - **Use diff't code:**
 - `KNeighborsRegressor(n_neighbors=3)`

Parametric and non-parametric models

- Parametric models

- Assume functional form of population parameters model is trying to examine
 - E.g. Gaussian/Poisson/Binomial/etc.
- Prediction relies on parameters learned from training data

- Non-parametric models

- No explicit assumptions about the functional form
- Assume test data has same functional form as training data, however
- Prediction can lean heavily on training data

Model Evaluation

training set

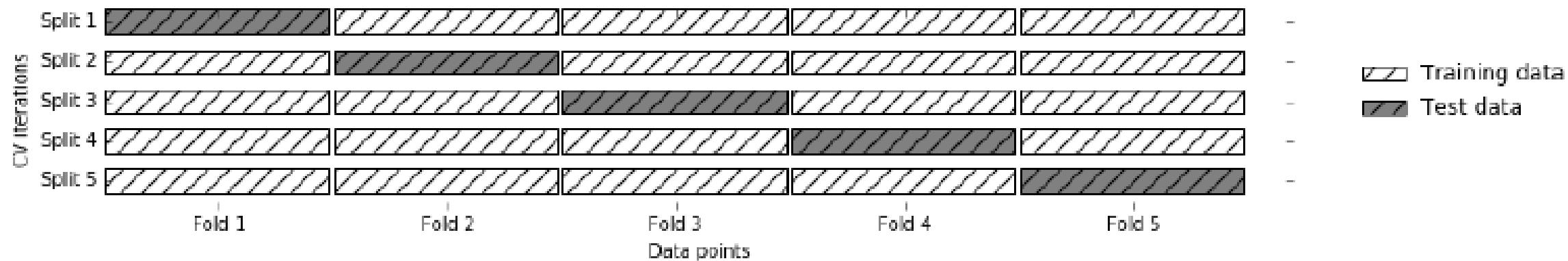
$X =$

1.1	2.2
6.7	0.5
2.4	9.3
1.5	0.0
0.5	3.5
5.1	9.7
3.7	7.8

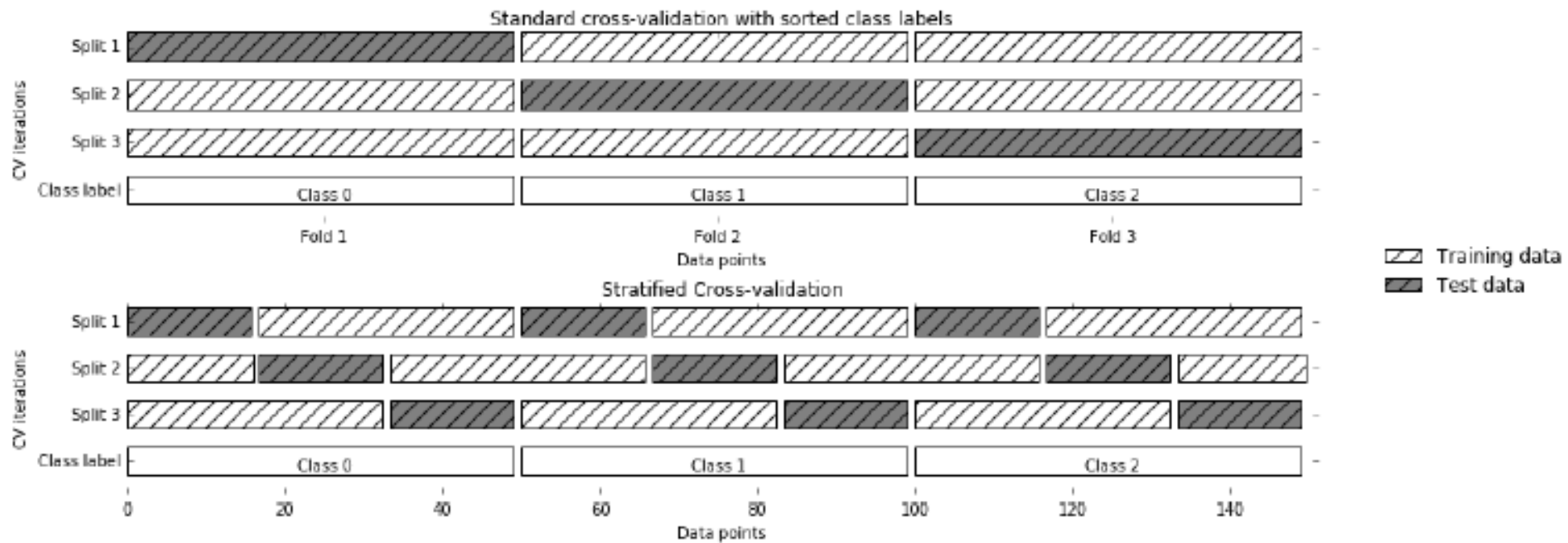
$y =$

0
1
1
0
1
0
0

K-fold Cross Validation



Stratified K-fold Cross Validation



Stratified: Ensure relative class frequencies in each fold reflect relative class frequencies on the whole dataset.

Stratified K-fold Cross Validation

- LeaveOneOut : `KFold(n_folds=n_samples)` High variance, takes a long time
- Better: RepeatedKFold. Apply `KFold` or `StratifiedKFold` multiple times with shuffled data. Reduces variance!
 - E.g. – You do 5 fold CV 10 times with data randomly shuffled before each 5 fold CV

Defaults for CV in scikit-learn

- Three-fold is default number of folds
- For classification cross-validation is stratified
- `train_test_split` has `stratify` option: `train_test_split(X, y,`
- `stratify=y)`
- No shuffle for repeat sampling by default!

Tuning model hyper-parameters with CV

- For KNN we can choose many values for k
- How do we choose a reasonable value?
 - Generate models for lots of different parameters
 - Evaluate them using CV
 - Choose model that predicts new data best

Grid Search with CV using Loop:

```
from sklearn.model_selection import cross_val_score

X_train, X_test, y_train, y_test = train_test_split(X, y)
cross_val_scores = []

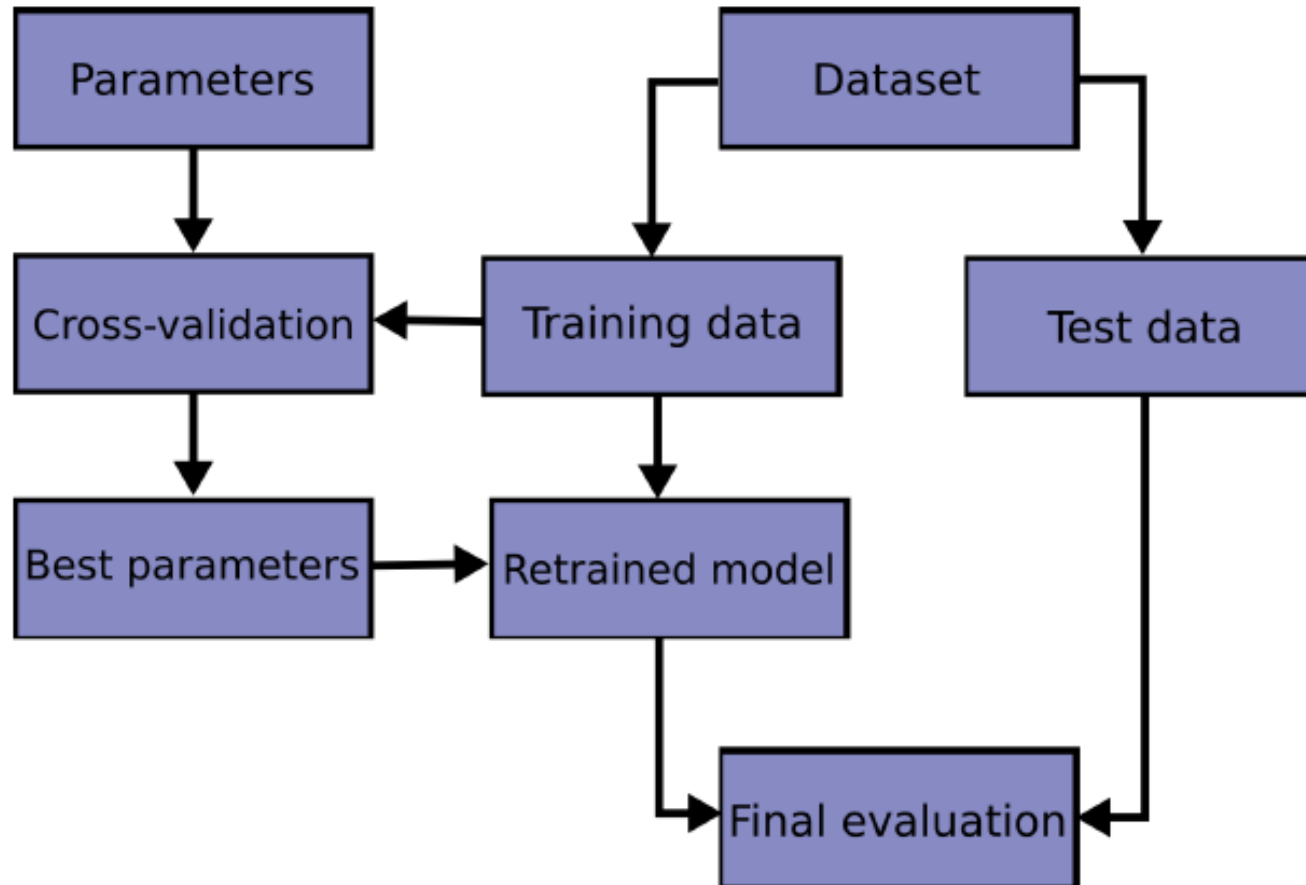
for i in neighbors:
    knn = KNeighborsClassifier(n_neighbors=i)
    scores = cross_val_score(knn, X_train, y_train, cv=10)
    cross_val_scores.append(np.mean(scores))

print("best cross-validation score: {:.3f}".format(np.max(cross_val_scores)))
best_n_neighbors = neighbors[np.argmax(cross_val_scores)]
print("best n_neighbors: {}".format(best_n_neighbors))

knn = KNeighborsClassifier(n_neighbors=best_n_neighbors)
knn.fit(X_train, y_train)
print("test-set score: {:.3f}".format(knn.score(X_test, y_test)))
```

```
best cross-validation score: 0.967
best n_neighbors: 9
test-set score: 0.965
```

Conceptual Overview



GridSearchCV

```
from sklearn.model_selection import GridSearchCV

X_train, X_test, y_train, y_test = train_test_split(X, y, stratify=y)

param_grid = {'n_neighbors': np.arange(1, 15, 2)}

grid = GridSearchCV(KNeighborsClassifier(), param_grid=param_grid, cv=10)
grid.fit(X_train, y_train)

print("best mean cross-validation score: {:.3f}".format(grid.best_score_))
print("best parameters: {}".format(grid.best_params_))

print("test-set score: {:.3f}".format(grid.score(X_test, y_test)))
```

```
best mean cross-validation score: 0.967
best parameters: {'n_neighbors': 9}
test-set score: 0.993
```

GridSearchCV

```
import pandas as pd
results = pd.DataFrame(grid.cv_results_)
results.columns
```

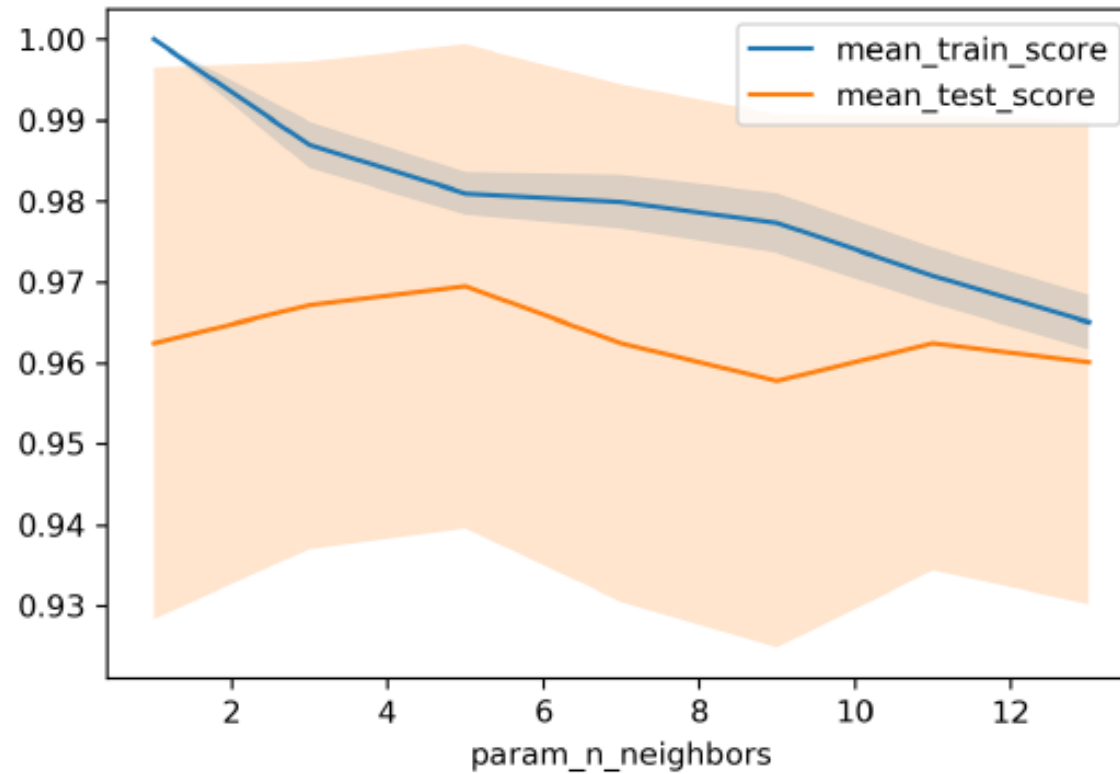
```
Index(['mean_fit_time', 'mean_score_time', 'mean_test_score',
       'mean_train_score', 'param_n_neighbors', 'params', 'rank_test_score',
       'split0_test_score', 'split0_train_score', 'split1_test_score',
       'split1_train_score', 'split2_test_score', 'split2_train_score',
       'split3_test_score', 'split3_train_score', 'split4_test_score',
       'split4_train_score', 'split5_test_score', 'split5_train_score',
       'split6_test_score', 'split6_train_score', 'split7_test_score',
       'split7_train_score', 'split8_test_score', 'split8_train_score',
       'split9_test_score', 'split9_train_score', 'std_fit_time',
       'std_score_time', 'std_test_score', 'std_train_score'],
      dtype='object')
```

```
results.params
```

```
0    {'n_neighbors': 1}
1    {'n_neighbors': 3}
2    {'n_neighbors': 5}
3    {'n_neighbors': 7}
4    {'n_neighbors': 9}
5    {'n_neighbors': 11}
6    {'n_neighbors': 13}
Name: params, dtype: object
```

GridSearchCV

n_neighbors Search Results



Conclusion of Sup. Learning Overview

Questions before we move on?

Part 2 of Lecture: Linear regression models

- Least Squares walk through
- Ridge and Lasso Regression
- Code introduced along the way