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Many-objective evolutionary algorithm based on relative non-dominance matrix

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ABSTRACT

Various evolutionary algorithms have been proposed for tackling many-objective optimization problems over the past three decades. However, these algorithms still suffer from the loss of selection pressures due to the existence of dominance resistance. To tackle this issue, this paper proposes a relative non-dominance matrix, based on which a fitness formula is defined. Empirical analyses show that solutions with smaller fitness values are likely to dominate more other solutions in the future evolutionary process, and play a vital role in enhancing the convergence toward to the true Pareto fronts. Additionally, to further ensure the diversity, *k*-means clustering strategy is combined with the relative non-dominance matrix for a new design of the environmental selection, where parameter *k* in the clustering strategy is adjusted adaptively. The proposed algorithm is extensively tested with four state-of-art algorithms on WFG, MaF and DTLZ test suites. Empirical comparisons demonstrate the competitiveness of the proposed algorithm regarding to the convergence, diversity and spread.

1. Introduction

Optimization is ubiquitous in the real-life disciplines, such as solid transportation problem with budget constraint [28] and cloud service composition [44]. Multi-objective Optimization Problems(MOPs) [21] are referred to as problems with two or three conflicting objectives, which can be formulated with the following expression:

$$\begin{aligned} &Minimize F(\mathrm{X}) = (f_1(\mathrm{X}), f_2(\mathrm{X}), \ldots, f_M(\mathrm{X})) \\ &s.t. \mathrm{X} \in \Omega \end{aligned} \tag{1}$$

where $\mathrm{X} = (x_1, x_2, x_3, \ldots, x_D)$ indicates one decision vector of D variables. $f_M(\mathrm{X})$ is the M-th objective function. $\Omega \subseteq R^D$ represents the set of feasible decision vectors. Unlike single objective optimization problems, there does not exist the best solution in MOPs, but a set of trade-off solutions. X_1 strictly Pareto dominates X_2, expressed as $\mathrm{X}_1 \prec \mathrm{X}_2$, if and only if $\forall i \in \{1, 2, 3, \ldots, M\} : f_i(\mathrm{X}_1) \leqslant f_i(\mathrm{X}_2)$ and $\exists j \in \{1, 2, 3, \ldots, M\} : f_j(\mathrm{X}_1) < f_j(\mathrm{X}_2)$. X^* is Pareto optimal if no solution dominates it. Pareto Set (PS) is a collection of Pareto optimal solutions, whose images in the objective space correspond to the Pareto Front (PF).

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Over the past three decades, various methods have been developed to tackle MOPs, such as NSGA-II [10], MOEA/D [42] and SPEA2 [46], just to name a few, and they have shown outstanding performance on a variety of MOPs. However, when these methods are applied to optimization problems with more than three conflicting objectives, which are termed as Many-objective Optimization Problems (MaOPs), their performance is less satisfying. The reason for that phenomenon lies in three aspects. 1) With the increasing number of objectives, more and more solutions become non-dominated, causing insufficient evolutionary pressure and stagnant search behaviors [30]. 2) To approximate the true Pareto front, the population size should be increased with the increasing objective space. However, a larger population size inevitably causes higher computational costs [23]. Moreover, it becomes more and more difficult to identify the extreme or boundary solutions with the increasing objective space. 3) Some feasible methods, such as decomposition-based methods, heavily rely on the high-quality weight vectors. Without any prior knowledge of PF, it is nearly impossible to generate satisfying weight vectors [19]. In addition, the emergence of some problems with irregular Pareto fronts may further aggravate this issue.

In spite of these issues in tackling MaOPs, various methods have been proposed, such as KnEA [43], RVEA [5], NSGA-III [9], MOEA/DD [24], and the like. Generally, these methods follow the same framework and are able to obtain a set of trade-off solutions in a single run. The evident difference between these algorithms mostly lies in the selection strategies, which can be used to divide the current methods into the following categories. 1) Enhanced dominance-based methods, which mainly strengthen the selection pressure toward the Pareto front. Typical variants include ε-dominance [22], L-optimality [47], k-optimality [14], fuzzy dominance [38], and preference order ranking [29]. These improved dominance relations have shown promising performance on MaOPs, although they are likely to converge into the local regions of PFs. 2) Indicator-based methods, such as HypE [1], SMS-EMOA [12] and IBEA [45]. The first two methods are based on the hypervolume (HV) indicator, while the last one defines an indicator to measure each solution's contribution. However, the calculation of HV is very time-consuming, inevitably hindering the scalability of HV-based methods. To alleviate this issue, some methods, such as the computational geometry [3] and Monte Carlo sampling [1], are proposed and have demonstrated promising performance. 3) Decomposition-based methods, which are widely used for tackling MaOPs because they can strengthen the selection pressure to some extent [40]. MOEA/D [42] and its variants [20] are typical decomposition-based methods, and they are featured with weight vectors. NSGA-III [9] also falls into this category, which employs a set of reference points to guide the search directions. However, MOEA/D [42] still suffers from its decomposition strategy because the generated sub-problems trend to be taken over by some superior solutions [40]. The phenomenon becomes more serious with the increasing number of objectives. Regarding to this issue, Li et al. [25] propose to update the sub-problems only by their related solutions. Based on the idea in [40], Yuan et al. [41] restrict a solution to update one of its K closest sub-problems. There are also some methods which adopt ideas different from those discussed above. For example, KnEA [43] combines the traditional non-dominated sorting strategy with a knee-point strategy. Li et al. [24] incorporate decomposition- and Pareto-based methods into the same framework, and propose to employ the Pareto domination to prescreen the population and estimate the local density with the solution associated with predefined weight vectors.

As discussed above, although various methods have been proposed for tackling MaOPs, they still suffer from the loss of selection pressure [43]. Specifically, the Pareto dominance relation plays a significant role in facilitating the convergence toward PFs. However, for MaOPs, the performance of Pareto dominance deteriorates rapidly with the increasing number of objectives. As discussed in paper [16], for DTLZ2 with more than five objectives, the proportion of non-domination in an initial population is more than 90%, significantly diminishing the selection pressure during the evolutionary process. From the definition of Pareto dominance, we can observe that the Pareto dominance relation neither provides a complete ordering of all solutions in the objective space, nor quantifies how much one solution is better than others. On the basis of this observation, Many-Objective Evolutionary Algorithm Based on Relative Non-dominance Matrix(MaOEA-RNM) is proposed. Firstly, the relative non-dominance distance is proposed for measuring any two solutions. Based on the relative non-dominance distance, the relative non-dominance matrix and fitness formula are further defined to distinguish non-dominated solutions. The contributions of this paper can be summarized as follows.

1) The relative non-dominance matrix is formulated for distinguishing non-dominated solutions and enhancing the selection pressure. Based on the analysis of non-dominated solutions as discussed in later section, this paper firstly defines the relative non-dominance distance to distinguish any two solutions. After that, the relative non-dominance matrix and fitness formula are formulated to compute the fitness of each solution. Empirical analysis proves that solutions with smaller fitness values are likely to dominate more other solutions in the further evolutionary process, thus enhancing the convergence of the final solution set.
2) Many-Objective Evolutionary Algorithm based on Relative Non-dominance Matrix (MaOEA-RNM) is proposed. Based on the proposed relative non-dominance matrix and fitness formula, this paper redesigns the mating selection and environmental selection. In addition, in terms of the environmental selection, the k-means clustering strategy is combined with the fitness formula to enhance the convergence and diversity of final non-dominated solutions. Note that, parameter k in k-means clustering strategy is adjusted adaptively.
3) To verify the effectiveness of MaOEA-RNM, this paper employs four state-of-art algorithms for comparisons, including RVEA [5], KnEA [43], NSGA-III [9], and MOEA/D [42]. Experimental results on three popular test suites show that MaOEA-RNM is superior to other optimizers in terms of the convergence, diversity and spread.

The remaining sections detail this paper. In Section 2, existing work related to alleviating the inefficiency of Pareto dominance is analyzed. In Section 3, our motivation is articulated and empirical analyses are presented. After that, MaOEA-RNM is proposed in Section 4, including the mating selection and environmental selection. In Section 5, extensive experiments are conducted to verify the effectiveness of MaOEA-RNM, as well as the analyses of experimental results. Conclusions and future work are presented in Section 6.

2. Related work

This section is to introduce the related work on how to determine superior solutions in a solution set. As discussed above, the inefficiency of Pareto dominance is largely attributed to the increasing number of objectives. Taking this key issue into account, many researchers have contributed their efforts, and various strategies are proposed to remedy this issue, which can be roughly grouped into the following four categories.

1) Objective reduction. For example, Brokhoff and Zitzler [4] investigate how omitting objectives affects problem characteristics, and propose to merge non-conflict objectives into one objective. Deb and Saxena [11] employ the principle component analysis to find the lower-dimensional interactions of each objective. Singh et al. [34] construct an approximated non-dominated front and then use it to identify whether or not each objective can be omitted. These methods above indeed work on some special problems, but the objectives of most real-world problems are irreducible. For this case, sorting the importance of each objective is a feasible method [11]. In addition to that, Said et al. [31] set an error constant using the preference information from decision makers, and further incorporate it into the Pareto dominance relation. However, without prior knowledge and preference information, this method still suffers from the loss of information.
2) Scalar methods. Fabre *et al.* [15] propose to employ weighted sum to guide the search direction. Similar to weighted sum, maximum ranking(MR), global detriment(GD) and profit also fall into this category [15]. Different from simple weighted sum method, Zhou et al. [47] treat all the objectives equally, and take into account both the number of improved objective values and the values of improved objective functions. Farina and Amato [14] employ the simple preference information from decision makers and weighted values to converge to sub-regions of Pareto optimal solutions. Additionally, transforming certain objectives into constraints, such goal attainment [13], is a popular method.
3) Improved Pareto dominance. The core idea is to relax the Pareto dominance relation to enable one solution to dominate others more easily. For instance, Pareto α-dominance [18] defines solutions with a small improvement on some objectives as dominated solutions by setting lower and upper bounds of trade-off rate, although they should be considered as non-dominated solutions according to the Pareto dominance relation. Pareto ε-dominance [22] defines a hypergrid with $((K-1)/\varepsilon)^M$ boxes in the objective space, where M indicates the number of objectives and K is the upper boundary of objectives. Solutions falling into these boxes are considered to be non-dominated. However, as paper [2] points out, ε-dominance has difficulties in determining the most appropriate value. Therefore, Batista et al. [2] propose a cone ε-dominance, which combines the ε-dominance with the Pareto dominance relation together. Control of dominance area [32] is proposed by contracting and expanding the dominance area of solutions. Fuzzy dominance [16] is designed based on fuzzy logic to continuously differentiate solutions into different degrees of optimality beyond the classification of the original Pareto dominance. Strengthened dominance [37] is a newly proposed indicator, which measures the convergence degree of each solution by the sum of each objective. On the contrary, angle dominance [27] is defined based on the angles composed of solutions and each axis node.
4) Indicator-based methods. IBEA[45] is the representative of this kind, which uses a predefined goal to measure the contribution of each solution. SMS-EMOA [12] proposes a metric based on HV and Pareto dominance. However, SMS-EMOA still suffers from the high computational cost. To address this issue, Bader et al. [1] propose to use Monte Carlo simulation to approximate the exact hypervolume values. Similar to HV-based methods for MaOPs, AR-MOEA [35] proposes to employ the enhanced inverted generational distance indicator to distinguish solutions, which is able to accelerate the evolution towards PFs.

As can be seen, various methods have been proposed to remedy the inefficiency of Pareto dominance in maintaining the selection pressure. To tackle this issue, in this paper, for a pair of non-dominated solutions, the relative non-dominance distance is defined to quantify how much one solution is superior to its competitor. Based on that, the relative non-dominance matrix and fitness formula are defined to differentiate different solutions in one Pareto set.

3. Motivation

As can be seen from the discussions above, Pareto dominance can only generate various Pareto fronts. For the non-dominated solutions in the same Pareto fronts, Pareto dominance can do nothing with them, resulting in the loss of selection pressure. Concerning this issue, this section employs Fig. 1 to show the main idea of our motivation. As can be seen, Fig. 1 is a two-dimensional objective space. Solution $A(2,4)$ and $B(4,1.5)$ respectively correspond to two non-dominated solutions, and solution $C(2,1.5)$ is the crosspoint of solution A and B.

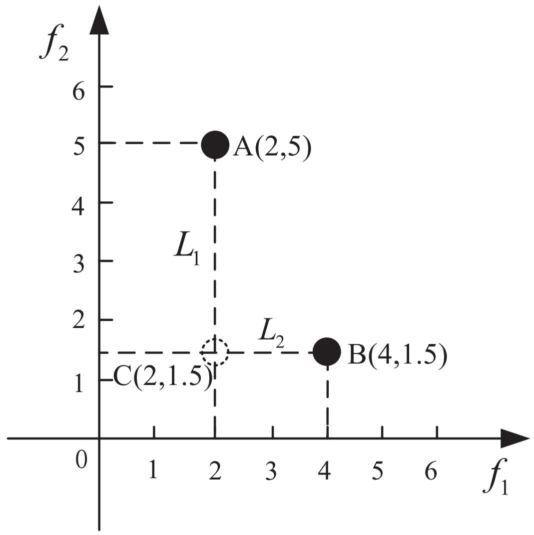


Fig. 1. Illustration of relative non-dominance distance.

According to the definition of Pareto dominance, solution A and B are non-dominated with each other due to the following two conditions: 1) The second objective of solution A is greater than that of solution B; 2) The first objective of solution B is greater than that of solution A. Regarding to the first condition, let solution A move down to solution C along $L_1 = 3.5$. Then, it can be said that solution A after moving along L_1, namely solution C, dominates solution B; On the contrary, for the second condition, solution B will dominate solution A if it moves to solution C along $L_2 = 2$. In terms of the moving length, it is obvious that solution B is expected to reach solution C faster than solution A in the future evolutionary process, indicating solution B is relatively better than solution A although they are non-dominated with each other.

Therefore, according to the observations above, the relative non-dominance distance from solution X_i to X_j, corresponding to $L_1 = 3.5$ and $L_2 = 2$ in Fig. 1, can be formulated with the following formula:

$$R^{i,j} = \sqrt{\sum_{m=1}^{M}((f_m(X_i) - f_m(X_j))^2 \times c_m)}$$

where M is the number of objectives, $f_m(X_i)$ indicates the m-th objective of solution X_i and parameter c_m is defined as follows:

$$\begin{aligned} &C = (c_1, c_2, \ldots, c_m, \ldots, c_M) \\ &s.t. c_m = \begin{cases} 1, f_m(X_i) > f_m(X_j) \\ 0, f_m(X_i) \leqslant f_m(X_j) \end{cases} \end{aligned} \tag{3}$$

In formula (2), $R^{i,j}$ is the relative non-dominance distance from solution X_i to X_j. Strictly speaking, $R^{i,j}$ is the shortest distance from solution X_i to the dominating area where any one solution definitely Pareto dominates solution X_j. It should be specially noted that $R^{i,j}$ and $R^{j,i}$ are essentially different from each other. Only when $R^{i,j} = 0$ and $R^{j,i} = 0$, solution X_i is equal to X_j.

Note that formula (2) is only limited to two solutions. In spite of that, formula (2) is not only designed for two non-dominated solutions, but also for solutions with domination relations. In other words, although the relative non-dominance distance is initially proposed to measure the differences between two non-dominated solutions, the relative non-dominance distance between any two solutions can obtained without knowing the relations of them. Moreover, for any two solutions with domination relations, the relative non-dominance distance from the dominated solution to the dominating solution is actually the Euclidean distance between them, which can be easily understood according to formula (2).

Assuming there are N members in a population, then based on the relative non-dominance distance, there exists a relative non-dominance matrix R of $N \times N$ elements as follows:

$$R = \begin{bmatrix} 0 & R^{1,2} & R^{1,3} & \ldots & R^{1,N} \\ R^{2,1} & 0 & R^{2,3} & \ldots & R^{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R^{N-1,1} & R^{N-1,2} & R^{N-1,3} & \ldots & R^{N-1,N} \\ R^{N,1} & R^{N,2} & R^{N,3} & \ldots & 0 \end{bmatrix} \tag{4}$$

where $R^{N-1,N}$ means the relative non-dominance distance from solution X_{N-1} to X_N . Note that matrix R is an asymmetric matrix because $R^{i,j}$ is generally not equal to $R^{j,i}$. In addition, matrix R has some special characteristics as follows.

Theorem 1. *If* $\forall i,j \in \{1,2,\ldots,N\} : R^{i,j} = 0, R^{j,i} \neq 0$*, then solution* X_i *dominates solution* X_j*.*

Proof. According to formula (2), because $R^{i,j} = 0$, each objective of solution X_i in the objective space is smaller than or at least equal to the corresponding objective of solution X_j . Moreover, $R^{j,i} \neq 0$ indicates that at least one objective of solution X_j is more than that of solution X_i. According to the definition of Pareto dominance, it is evident that solution X_i dominates solution X_j.

Theorem 2. *If* $\forall i,j \in \{1,2,\ldots,N\} : R^{i,j} \neq 0, R^{j,i} \neq 0$*, then solution* X_i *and* X_j *are non-dominated with each other.*

Proof. Because $R^{i,j} \neq 0$, at least one objective of solution X_i is more than that of solution X_j. On the contrary, $R^{j,i} \neq 0$ indicates that at least one objective of solution X_j is more than that of solution X_i . Therefore, solution X_i and X_j are non-dominated with each other according to the analysis above.

Theorem 3. *If* $\forall i,j \in \{1,2,\ldots,N\}$*, solution* X_i *is not equal to solution* X_j *, then there does not exist the case that* $R^{i,j} = 0, R^{j,i} = 0$*.*

Proof. According formula (2), $R^{i,j} = 0$ indicates that each objective of solution X_i is less than or equal to that of solution X_j . On the contrary, $R^{j,i} = 0$ implies each objective of solution X_j is less than or equal to that of solution X_i. Taking into account the two cases above, solution X_i should be equal to solution X_j, which is obviously contradictory with the precondition that solution X_i is not equal to solution X_j. Therefore, Theorem 3 is true.

Corollary 1. *If* $\exists i \in \{1,2,\ldots,N\}, \forall j \in \{1,2,\ldots,i-1,i+1,\ldots,N\} : \sum_{j=1,j\neq i}^{N} R^{i,j} = 0 \wedge \sum_{j=1,j\neq i}^{N} R^{j,i} \neq 0$*, then solution* X_i *dominates all the other solutions.*

Proof. According to formula (2), the relative non-dominance distance between any two solutions is always equal to or more than zero. Therefore, $\sum_{j=1,j\neq i}^{N} R^{i,j} = 0$ indicates that $R^{i,j} = 0$ is always true for solution X_i and any one solution X_j . In addition, according to Theorem 3, $R^{j,i} \neq 0$ is always true for any one solution X_j. Based on the two aspects above, according to Theorem 1, solution X_i evidently dominates all the solutions except for itself.

Based on the relative non-dominance matrix R and the theorems above, the fitness value of solution X_i can be defined as follows:

$$F_i = \sum_{j=1,j\neq i}^{N} R^{i,j} \tag{5}$$

where $R^{i,j}$ is the relative non-dominance distance from solution X_i to X_j. The smaller the fitness value $R^{i,j}$ is, the better solution X_i is in terms of the convergence. To illustrate our idea more clearly, we take Fig. 2 as an illustrative example.

Assume there are five solutions in the objective space, A(2,12), B(4,7), C(6,5.5), D(8,4) and E(12,2). The fitness value of solution C can be obtained using formula (5), namely $F_C = 11$. Next, move solution $C(6,5.5)$ to $C^1(4,4)$. Corresponding fitness value is $F_{C^1} = 4$. Obviously, it can be seen that solution $C^1(4,4)$ dominates $B(4,7)$ and $D(8,4)$. Keep moving $C^1(4,4)$ to $C^2(2,2)$. The fitness value of $C^2(2,2)$ is $F_{C^2} = 0$. It is evident that solution $C^2(2,2)$ completely dominates $A(2,12), B(4,7), D(8,4)$ and $E(12,2)$. By comparing $F_C = 11, F_{C^1} = 4$ and $F_{C^2} = 0$, we can see that solution $C^2(2,2)$ is better than $C^1(4,4)$, and $C^1(4,4)$ is superior to $C(6,5.5)$ in the terms of the number of dominated solutions. Additionally, as solution $C(6,5.5)$ moves, its fitness value decreases accordingly.

To have a dynamic observation of the discussion above, this paper considers the following bi-objective SCH benchmarking problem [33]:

$$\begin{cases} \min f_1 = & x^2 \\ \min f_2 = & (x-2)^2 \end{cases} \tag{6}$$

The Pareto optimal set of SCH is $x \in [0,2]$. Firstly, sample 100 non-dominated solutions on the true Pareto front by varying x from 0 to 2 with interval being 0.02, in which $S_a(0.0025, 3.8025)$ and $S_b(0.0529, 3.1329)$ are two randomly selected solutions. Next, move $S_a(0.0025, 3.8025)$ to (0, 0) along the two paths: Path1 $S_a(0.0025, 3.8025) \rightarrow (0,1) \rightarrow (0,0)$, and Path2:

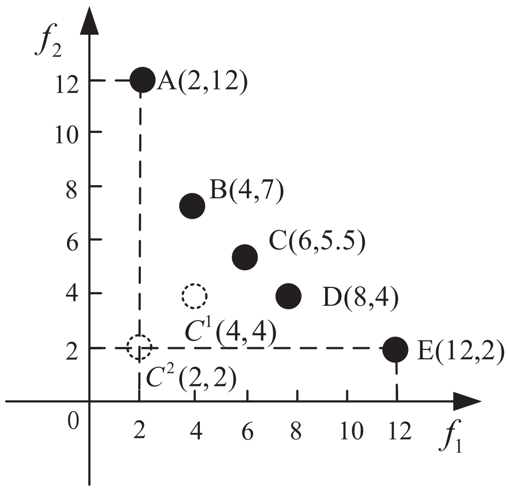
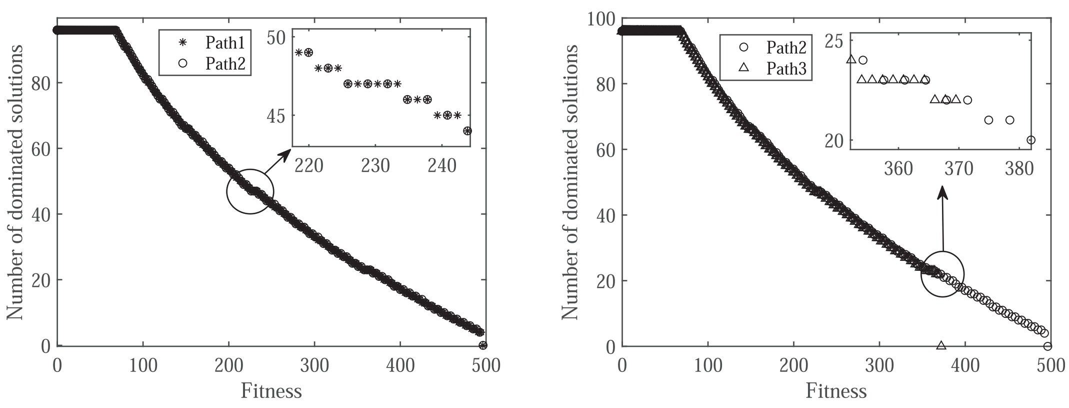


Fig. 2. Illustration of the motivation for MaOEA-RNM.

$S_a(0.0025, 3.8025) \rightarrow (0, 0)$. After that, move $S_b(0.0529, 3.1329)$ to (0,0) along the following Path3: $S_b(0.0529, 3.1329) \rightarrow (0, 1) \rightarrow (0, 0)$. As Fig. 3(a) shows, plot the fitness values and the numbers of solutions dominated by solution S_a during the moving processes along Path1 and Path2. In the same way, the moving process of solution S_b along Path3 is obtained. Fig. 3(b) shows the comparison curves of Path2 and Path3.

Fig. 3(a) shows the differences of solution S_a moving to (0, 0) along various paths. As can be seen, the curves of two different moving processes are nearly the same. A smaller fitness value generally corresponds to a larger number of dominated solutions. From the enlarged region in Fig. 3(a), it can be seen that there indeed exist some nuances between the two moving processes. However, from Fig. 3(a), it still can be roughly said that no matter how solution S_a reaches (0, 0), solution S_a always follows the similar rule. Fig. 3(b) compares the curves of different starting points moving to (0, 0) along different paths. It is evident that their outlines are similar except for the down-right bifurcation, which is caused by the positions of starting points. In summary, although different solutions have various moving paths, they approximately follow the same rule that solutions with smaller fitness values are likely to dominate other solutions with larger fitness values in the future evolutionary process.

Evidently, with the decrease of the fitness value, corresponding solution will dominate more other solutions, until the fitness value is equal to zero, which means all the other solutions are dominated as Corollary 1 indicates. However, one may think that the proposed method performs a similar role with indicator-based methods in terms of choosing one key solution, such as KnEA [43]. Actually, there are two essentially different aspects between them. 1) The knee-point strategy is focused on the maximum HV value of the current population, which has been stated in the original paper [43]. On the contrary, formula (5) emphasizes the potential of one solution in the future evolutionary process. 2) In some cases, the knee-point strategy and formula (5) may result in completely different solutions. Again, take Fig. 2 as an illustrative example. Let us determine one key solution among the five solutions. According to the knee-point strategy, solution $B(4, 7)$ has the



(a) Comparison of Path1 and Path2 (b) Comparison of Path2 and Path3

Fig. 3. Dynamic observation of different moving processes.

largest distance to the line determined by solution $A(2,12)$ and $E(12,2)$. However, according to formula (5), $F_B = 11.5$ and $F_C = 11$. It is evident that formula (5) prefers solution $C(6,5.5)$. By comparing the two results, it can be seen that the two methods have their own preferences. In summary, HV-based methods prefer solutions which maximize the HV values, while the fitness formula proposed in this paper focuses on the one which trends to dominate more others in the future evolutionary process. Most importantly, extensive comparisons in later experiments further demonstrate the differences between MaOEA-RNM and KnEA.

4. Proposed method

This section details the proposed MaOEA-RNM. Firstly, the general framework of MaOEA-RNM is presented. In the sequential, the selection strategy based on formula (5) is introduced. After that, the environmental selection strategy is elaborated.

4.1. General framework of MaOEA-RNM

The main framework of MaOEA-RNM is similar to that of NSGA-II, which has the following components. One population of N members is firstly initialized after inputting population size N. In the sequential, the relative non-dominance matrix is constructed using formula (4) as introduced above. Third, the tournament selection strategy is performed to select N individuals from the parent population. In the tournament selection strategy, the Pareto dominance relation and fitness formula are used as the selection criterions. The offspring population is generated using the variation method, and it is then combined with the parent population. Following that, the combined population is sorted to generate different non-dominated levels. In the end, an environmental selection strategy is performed to select N members from the combined population as the next population. Note that, to strengthen the convergence and diversity of the next population, formula (5) and k-means strategy are employed in the environmental selection. Repeat the procedures above until the termination criterion is reached. The descriptions above correspond to the general framework of MaOEA-RNM presented in Algorithm 1.

Algorithm 1: General Framework of MaOEA-RNM

Input: N (population size)
Output: P (population)
$P \leftarrow Initialize(N)$
while *termination criterion not reached* **do**
  $R \leftarrow$ Relative non-dominance matrix(P, N)
  $P' \leftarrow$ Mating selection(P, R, N)
  $P \leftarrow P \cup Variation(P', N)$
  $F \leftarrow$ Non-dominated sorting(P) % Various non-dominated levels are generated; F_i is the first Pareto front such that $|F_1 \cup F_2 ... \cup F_i| \geq N$
  $P \leftarrow$ Environmental selection(F, N)
end

In the following subsections, we will present the details of the main components above, including the relative non-dominance matrix, mating selection and environmental selection, which are essentially the core features of MaOEA-RNM and obviously different from common methods for MaOPs.

4.2. Relative non-dominance matrix

The function of Relative non-dominance matrix is to calculate the relative non-dominance distances from one solution to other solutions based on formula (2), and simultaneously construct the relative non-dominance matrix as formula (4) defines, seeing line 2–5 in Algorithm 2. Repeat the procedures above until all the solutions are visited. The resultant matrix is used for the comparison of any two solutions.

Algorithm 2: Relative Non-dominance Matrix (P, N)

Input: P (population), N (population size)
Output: R (Relative Non-dominance matrix)
1 $R \leftarrow zeros(N, N)$
2 **for** *all* $P_i \in P$ **do**
3   **for** *all* $P_j \in P$ **do**
4     $R^{i,j} \leftarrow$ Relative non-dominance distance(P_i, P_j) % compute the relative non-dominance distance $R^{i,j}$ from P_i to P_j using formula (2); and simultaneously construct the relative non-dominance matrix as formula (4) indicates.
5   **end**
6 **end**

4.3. Mating selection

Mating selection is to select N high-quality solutions for reproduction. As Algorithm 3 exhibits, solution P_a and P_b are randomly picked up from the parent population. If $R^{a,b} = 0, R^{b,a} \neq 0$, indicating P_a dominates P_b as Theorem 1 shows, then P_a is included into P'. On the contrary, if $R^{b,a} = 0, R^{a,b} \neq 0$, then P_b is included into P', referring to line 4–7. If $R^{a,b} \neq 0, R^{b,a} \neq 0$, meaning P_a and P_b are non-dominated with each other as Theorem 2 indicates, compare their relative non-dominance distances, seeing line 9–12. The solution with a smaller relative non-dominance distance is chosen as the offspring member. In the case that both of them share the same value, randomly choose either of them.

Algorithm 3: Mating Selection (P, R, N)

```
   Input: P (population), R (relative non-dominance matrix), N (population
          size)
   Output: P'
 1 P' ← ∅ % next population
 2 while |P'| ≤ N do
 3     randomly choose P_a and P_b from P
 4     if R^{a,b} = 0, R^{b,a} ≠ 0 then
 5         P' ← P' ∪ {P_a}
 6     else if R^{b,a} = 0, R^{a,b} ≠ 0 then
 7         P' ← P' ∪ {P_b}
 8     else if R^{b,a} ≠ 0, R^{a,b} ≠ 0 then
 9         if R^{a,b} < R^{b,a} then
10             P' ← P' ∪ {P_a}
11         else if R^{a,b} > R^{b,a} then
12             P' ← P' ∪ {P_b}
13         else
14             if rand() < 0.5 then
15                 P' ← P' ∪ {P_a}
16             else
17                 P' ← P' ∪ {P_b}
18             end
19         end
20     end
21 end
```

4.4. Environmental selection

Environmental selection is to select N solutions from the combination of parent and offspring populations. The detailed procedure of environmental selection is presented in Algorithm 4. The input parameters are F and the population size N. Note that F is a set of different non-dominated levels. The last level F_i in F is the first level so that $|F_1 \cup F_2 \ldots \cup F_i| \geqslant N$.

Algorithm 4: Environmental selection (F, N)

```
   Input: F (various non-dominated levels), N (population size)
   Output: P
 1 P_temp ← ∪_{j=1}^{i-1} F_j
 2 k ← N − |P_temp|
 3 P ← P ∪ P_temp
 4 S ← cluster F_i into k sets using k-means clustering strategy
 5 for all S_i ∈ S do
 6     F_{s_i} ← Fitness computation(S_i, |S_i|)
 7     I ← choose the solution with minimum fitness value
 8     P ← P ∪ {I}
 9 end
```

As shown in Algorithm 4, the first $i-1$ levels are firstly excluded from F to determine the number of solutions chosen from the last non-dominated level, seeing line 1–3. After that, cluster solutions in F_i into k sets by the k-means clustering strategy based on the Euclidean distance, referring line 4. Then, compute the fitness values of all the solutions in S_i. The solution with the minimum fitness value is included into the next population. Repeat the procedure above until k solutions are determined.

As shown in Algorithm 4, the k-means clustering strategy is used to partition level F_i into k sets, where parameter k is determined by the number of solutions to be selected from level F_i. As discussed in Section 3, solutions with small fitness values are likely to dominate other solutions in the future evolutionary process, definitely improving the convergence towards the true Pareto front. However, the diversity is not taken into account as formula (5) indicates. Thus, the diversity should be guaranteed by other methods, such as the k-means clustering strategy. Such a design can be illustrated with Fig. 4.

Similar to Fig. 2, there are five solutions in Fig. 4. According to formula (5), the fitness values of A, B, C, D, and E are 29.5, 11.5, 11, 14 and 28, respectively. Assume the current task is to select three solutions from the five solutions. If the k-means strategy is not employed, according to formula (5), solution B, C and D should be chosen. Obviously, the resultant solutions are not satisfying. However, on the contrary, if the k-means clustering strategy is performed before the selection operation, the chosen solutions are A, C and E. By comparing the two resultant sets, it can be seen that the latter strategy is more beneficial than the former one in terms of keeping the diversity. It should be specially noted that parameter k in the k-means clustering strategy is determined by the number of solutions to be selected from the last level. That is to say, parameter k is adjusted adaptively.

4.5. Computational complexity analysis

As introduced above, MaOEA-RNM mainly consists of five components, including the relative non-dominance matrix, mating selection, non-dominated sorting and environmental selection. For an optimization problem with M objectives and D decision variables, one population of N members participates in the optimization process. According to formula (2) and (4), the computational complexity of the relative non-dominance matrix is $O(MN^2)$. The mating selection has a computational complexity of $O(N^2)$ to build up a mating pool according to formula (5). Genetic variations employ the simulated binary crossover and polynomial mutation to optimize each solution, and its computational complexity is $O(DN)$. In the worst case, the non-dominated sorting needs a computational complexity of $O(MN^2)$. The environmental selection is to select N members from the combined population. In the worst case that the first non-dominated level has $N-1$ solutions and the second one has $N+1$ solutions, the computational complexity is $O(MN^2)$. As for the k-means clustering strategy, in the worst case, the upper boundary of its computational complexity is $O(MN^2)$. In summary, considering all the computations above, one generation of MaOEA-RNM needs a maximum computational complexity of $O(MN^2)$. Compared to most MOEA for MaOPs, such as KnEA and NSGA-III, both of which have a computational complexity of $O(MN^2)$ in the worst case, MaOEA-RNM is computationally comparable to them.

5. Experimental results and analyses

To assess MaOEA-RNM, this paper employs RVEA [5], KnEA [43], NSGA-III [9] and MOEA/D [42] as comparison algorithms. The main reason for selecting RVEA and KnEA is that both of them are devoted to recognize points in the preferred regions, and they are two contrasts to the idea of MaORA-RNM. NSGA-III and MOEA/D, as representative methods for MaOPs, are widely used as comparison algorithms in many papers [5,8]. The four comparison algorithms are briefly described as follows:

RVEA is featured with decomposing the original MOPs into a number of single-objective problems using a set of reference vectors. In addition, RVEA elucidates user preferences to target a preferred subset of the whole Pareto front. In RVEA, a penal-

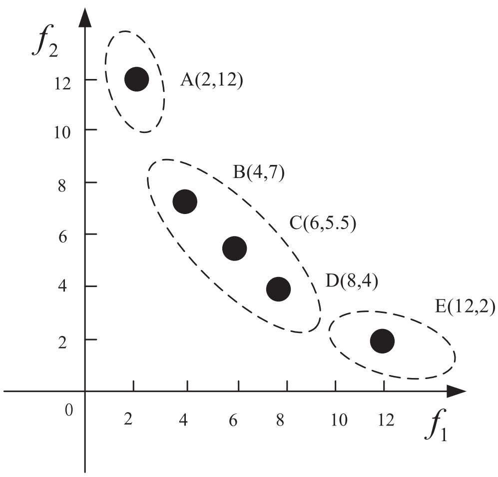


Fig. 4. Illustration of the use of k-means clustering strategy.

ized distance is adopted to balance the convergence and diversity, and an adaptation strategy is designed to dynamically adjust the reference vectors according to the scales of objective functions.

KnEA is proposed based on the idea that knee points are naturally preferred among non-dominated solutions without explicit user preference. In KnEA, the knee points are shown to be an approximation of a bias toward a large hypervolume. No additional diversity maintenance strategy is designed to keep the diversity.

NSGA-III, as the enhanced version of NSGA-II, emphasizes the population members that are non-dominated, yet close to a set of supplied reference points. The diversity is enhanced by defining and adaptively updating a set of well-spread reference points in the objective space.

MOEA/D is a decomposition-based method. It decomposes a multi-objective problem into a set of scalar optimization sub-problems, which are optimized by only using information from their several neighboring sub-problems.

5.1. Experimental settings

5.1.1. Benchmarking instances

To comprehensively test MaOEA-RNM, three popular test suites are employed, including WFG [17], MaF [6] and DTLZ [37]. In WFG test suite, WFG1-WFG9 with 8, 10, 12 and 15 objectives are employed, and the number of decision variables are 17, 19, 21 and 24, respectively. The maximum number of evaluations is 250000 for WFG1, 150000 for WFG2 and 8000 for other WFG problems. MaF test suite is specially proposed for many-objective optimization by suggesting a set of test problems with a good representation of various real-world scenarios. In MaF test suite, MaF1-MaF9 with 8, 10, 12 and 15 objectives are used for comparison purpose. The numbers of decision variables(D) for MaF1-MaF9 are listed in Table 1, where M indicates the number of objectives. The maximum number of function evaluations is set to 800000 for MaF3 and MaF4, and 400000 for other test problems. For both of the two test suites above, the population size is set to 200, 220, 240 and 260 for problems with 8, 10, 12 and 15, respectively. For DTLZ test suite, 3-objective DTLZ1-DTLZ6 and two varients, including IDTLZ2 and C2_DTLZ2, are used for verifying the effectiveness of MaOEA-RNM on MOPs. The decision number for DTLZ test suite is set to $k + M - 1$, where k is 5 for DTLZ1 and 10 for others. The population size is set to 100, and the maximum number of evaluations is set to 30000 for DTLZ3 and 10000 for others.

5.1.2. Performance indicators

Generally, one algorithm for MaOPs can be assessed from various perspectives, such as the convergence, diversity and extent of the final non-dominated solutions. The convergence indicates the closeness of the obtained solutions to the true Pareto front, while the diversity implies how diverse the obtained solutions are. Both of them can be calculated with IGD [7] as follows:

$$IGD = \frac{\sum_{v \in P_*} d(v, P)}{|P^*|} \tag{7}$$

where P is the set of solutions obtained by any methods. P^* is a set of sampled Pareto optimal solutions on the true Pareto front, and $d(v, P)$ indicates the minimal distance between point v and all the points in P. In this experiment, P^* is a set of around 8000 points for each test problem, which are generated by PlatEMO [36].

The second performance indicator is Spread(Δ) [39], which is used to measure the extent of spread of the final non-dominated solutions. The definition of Spread is described as follows:

$$\Delta = \frac{\sum_{i=1}^{M} d(E_i, P) + \sum_{X \in P} |d(X, P) - \bar{d}|}{\sum_{i=1}^{M} d(E_i, P) + (|P| - M) \times \bar{d}} \tag{8}$$

where $d(X, P) = \min_{Y \in P, Y \neq X} ||F(X) - F(Y)||, \bar{d} = \frac{1}{|P|} \sum_{X \in P} d(X, P), P$ is the final non-dominated set. $\{E_1, \ldots, E_M\}$ are M extreme solutions in the set of Pareto optimal solutions, and M is the number of objectives.

Table 1
Parameter settings of MaF test suite.

Instance	M	D
MaF1-MaF6	8	17
	10	19
	12	21
	15	24
MaF7	8	27
	10	29
	12	31
	15	34
MaF8-MaF9	8,10,12,15	2

The last performance indicator is hypervolume (HV) [1], which is widely used in most papers to measure the convergence and diversity of the final non-dominated solutions. Different from IGD and Spread, a larger HV value implies a better performance of one algorithm. For computing HV, the reference point is set to (1,1,...,1). The corresponding objective values are normalized by $1.1 \times z^{nad}$ before calculation, where z^{nad} indicates the nadir point of the Pareto front.

5.1.3. General settings

For fair comparison, the parameters of all the employed algorithms are set according to the original papers. For example, for both NSGA-III and MOEA/D, the SBX probability is set to 1, and the mutation probability is set to $1/D$, where D is the dimension of decision variables. In addition, in both NSGA-III and MOEA/D, the two-layered reference point strategy is also employed to generate the uniformly distributed weight vectors. Parameter T in KnEA follows the original setting in paper [43]. In RVEA, the parameter controlling the rate of change of penalty is set to 2, and the frequency of employing reference vector adaptation is 0.1. To further show the differences of them, Friedman test [8], which is a non-parametric statistical test, is employed for statistical comparisons. All the algorithms run 100 times on the same machine with Matlab 7.9.

5.2. Experimental results on WFG test suite

WFG test suite is a set of widely used benchmarking problems for evaluating the methods for MaOPs, which is characterized with various true Pareto fronts, such as nonseparability, multimodality and biased parameters. WFG test suite poses great challenges to the current methods for MaOPs.

As Table 2 shows, this paper compares the averages and standard deviations of IGDs over 100 runs obtained by the employed algorithms, where the best values are highlighted with gray background. The last row indicates the Friedman test results. From Table 2, it can be seen that MaOEA-RNM performs the best on WFG5, WFG6 and WFG9 with 8, 10, 12 and 15 objectives in comparison with RVEA, KnEA, NSGA-III and MOEA/D. In addition, on WFG4, WFG7 and WFG8 with 8, 10, and 12 objectives, MaOEA-RNM still outperforms other compared algorithms. However, the proposed method is slightly inferior to NSGA-III on WFG1 with 12 and 15 objectives and WFG3 with 8, 10 and 15 objectives, which are featured with flat bias and linear shapes, respectively. The reason may be that the reference point strategy in NSGA-III is able to guide the population toward certain promising regions. Moreover, on WFG3 with 12 objectives and 15-objective WFG4, WFG7 anf WFG8, MaOEA-RNM shows no superiority over KnEA, which is famous for the knee point strategy. However, according to the Friedman test results in the last row, it is evident the overall performance of MaOEA-RNM is outstanding compared with other methods.

Table 3 exhibits the extent of spread of non-dominated solutions obtained by all the algorithms. The smaller the indicator value is, the better corresponding algorithm performs. As can be seen, MaOEA-RNM outperforms all the compared algorithms on all test instances except for WFG5 with 12 objectives and WFG6 with 15 objectives. WFG5 and WFG6 have the same Pareto front shapes, but the decision spaces are essentially different with each other. WFG5 is famous for its deceptiveness, and WFG6 is a non-separable instance. On 12-objective WFG5 and 15-objective WFG6, NSGA-III is superior to MaOEA-RNM partly because of its unique reference point strategy. Overall, MaOEA-RNM wins 34 out of 36 instances, exhibiting significant advantages over compared algorithms. Besides, the Friedman test results further exhibit the statistical superiority of MaOEA-RNM over other comparison methods.

Fig. 5 visually presents the parallel coordinates of non-dominated solutions obtained by all the five algorithms on 10-objective WFG2, WFG4, WFG6 and WFG8, along with the parallel coordinates of true Pareto fronts sampled by PlatEMO [36]. As can be seen, compared with the true Pareto front of WFG2, although MaOEA-RNM fails to converge to the first three objectives, it still performs promising on the other objectives. Also, it is obvious that the convergence of MaOEA-RNM outperforms the compared algorithms. On WFG4, MaOEA-RNM nearly obtains the same shape as the true Pareto front of WFG4. RVEA, KnEA and NSGA-III perform very well in terms of the convergence, but is slightly inferior to MaOEA-RNM regarding to the diversity. MOEA/D performs the worst in both the convergence and diversity. On both of WFG6 and WFG8, the overall performance of MaOEA-RNM is comparable with KnEA and NSGA-III, but evidently outperforms RVEA and MOEA/D.

5.3. Experimental results on MaF test suite

Table 4 shows the averaged IGD values obtained by the five algorithms. As can be seen from Table 4, MaOEA-RNM performs the best on both MaF1 and MaF8. On MaF2 with 10, 12, and 15 objectives, MaF4 with 12 and 15 objectives, MaF5 with 10, 12 and 15 objectives and MaF7 with 8 objectives, KnEA performs better than MaOEA-RNM. As explained above, the reason may be that the knee-point strategy employed in KnEA naturally prefers certain solutions, which are shown to be an approximation of a bias toward a large hypervolume. On MaF3 with 8, 10, 12 and 15 objectives, RVEA exhibits its advantage over MaOEA-RNM due to the dynamical adjustment of reference vectors in the objective space. MOEA/D, famous for its decomposition strategy, also outperforms MaOEA-RNM on MaF6 with 8, 10 and 15 objectives and MaF9 with 8 objectives. In spite of the observations above, MaOEA-RNM still statistically performs the best in comparison with other methods.

Table 5 presents the averaged Spread values on MaF1-MaF9 with 8, 10, 12, and 15 objectives. From Table 5, it can be observed that MaOEA-RNM is the best optimizer on MaF3,MaF5, MaF6, MaF7 and MaF8, compred with other algorithms. On MaF2, KnEA achieves better values in comparison with other algorithms. On MaF9 with 12 and 15 objectives, MaOEA-

Table 2
Averaged IGD values on WFG test suite.

Problem	M	MaOEA-RNM	RVEA	KnEA	NSGA-III	MOEA/D
WFG1	8	1.5103e+0 (9.27e-1)	1.6024e+0 (9.32e-1)	1.6819e+0 (1.11e+0)	1.7649e+0 (1.29e+0)	1.7973e+0 (2.45e-1)
	10	1.3357e+0 (1.38e-1)	1.1105e+0 (1.52e-2)	1.0441e+0 (2.96e-2)	1.1495e+0 (9.22e-2)	2.0631e+0 (1.31e-1)
	12	1.7459e+0 (1.88e-1)	1.1397e+0 (1.27e-2)	1.1425e+0 (2.53e-3)	1.0540e+0 (1.01e-2)	1.6383e+0 (6.74e-2)
	15	2.5881e+0 (1.99e-1)	1.6153e+0 (9.93e-2)	1.6115e+0 (1.57e-2)	1.4227e+0 (3.29e-2)	2.6228e+0 (2.10e-1)
WFG2	8	9.9682e-1 (3.10e-2)	9.9462e-1 (7.50e-3)	1.0356e+0 (2.91e-2)	1.3828e+0 (6.33e-1)	1.7237e+0 (4.97e-2)
	10	1.2119e+0 (1.47e-2)	1.1109e+0 (3.54e-2)	1.2414e+0 (2.94e-2)	1.4929e+0 (6.66e-2)	2.0958e+0 (1.70e-2)
	12	1.2532e+0 (6.97e-2)	1.3937e+0 (6.59e-2)	1.2861e+0 (8.16e-3)	1.3193e+0 (1.76e-1)	1.7339e+0 (1.17e-2)
	15	1.6908e+0 (7.33e-3)	1.7851e+0 (1.27e-1)	1.6991e+0 (5.90e-2)	1.7361e+0 (2.07e-4)	2.4724e+0 (2.39e-2)
WFG3	8	1.5365e+0 (1.22e-1)	2.1951e+0 (8.13e-1)	2.0162e+0 (1.30e-1)	1.4362e+0 (6.26e-1)	3.7957e+0 (9.83e-2)
	10	2.6458e+0 (1.57e-1)	4.3168e+0 (7.11e-1)	2.2726e+0 (8.77e-1)	9.1543e-1 (8.16e-2)	8.7403e+0 (7.92e-1)
	12	3.8379e+0 (2.42e-1)	7.0273e+0 (1.04e+0)	2.9380e+0 (6.61e-1)	3.3165e+0 (1.68e-1)	6.1890e+0 (2.42e-2)
	15	5.6779e+0 (6.96e-1)	6.0569e+0 (1.50e+0)	3.9196e+0 (1.40e-1)	2.0236e+0 (3.40e-1)	8.6383e+0 (4.16e-1)
WFG4	8	2.8297e+0 (1.21e-3)	3.0171e+0 (8.49e-3)	3.1685e+0 (6.90e-2)	2.9642e+0 (5.57e-3)	6.8069e+0 (2.65e-1)
	10	4.2599e+0 (8.36e-2)	4.5539e+0 (5.23e-2)	4.7528e+0 (4.19e-2)	4.8209e+0 (6.19e-3)	9.3045e+0 (2.18e-1)
	12	5.6766e+0 (1.78e-2)	6.8560e+0 (9.16e-2)	6.4218e+0 (4.97e-2)	6.7405e+0 (4.07e-2)	1.2774e+1 (1.16e-1)
	15	7.8055e+0 (1.01e-1)	8.8664e+0 (5.19e-1)	7.4997e+0 (3.09e-1)	8.4773e+0 (3.45e-1)	1.6173e+1 (7.65e-2)
WFG5	8	2.8443e+0 (3.88e-2)	2.9710e+0 (6.21e-3)	3.1847e+0 (1.25e-2)	2.9420e+0 (3.59e-3)	6.4580e+0 (6.72e-2)
	10	4.1379e+0 (4.50e-2)	4.5737e+0 (5.66e-2)	4.7307e+0 (2.15e-2)	4.7451e+0 (1.36e-2)	8.9736e+0 (1.87e-1)
	12	5.5372e+0 (4.09e-2)	6.6931e+0 (2.20e-1)	6.2640e+0 (7.04e-2)	6.7357e+0 (4.54e-2)	1.2204e+1 (2.53e-1)
	15	7.4235e+0 (5.64e-2)	8.8183e+0 (1.67e-1)	7.5662e+0 (4.33e-2)	7.7820e+0 (3.55e-1)	1.6007e+1 (1.62e-1)
WFG6	8	2.9753e+0 (8.34e-2)	3.0542e+0 (2.08e-2)	3.3589e+0 (1.03e-1)	2.9761e+0 (2.96e-3)	6.9768e+0 (6.25e-2)
	10	4.3781e+0 (8.59e-2)	4.5395e+0 (2.17e-1)	4.8888e+0 (2.92e-2)	4.8164e+0 (4.60e-3)	9.5462e+0 (5.54e-2)
	12	5.6795e+0 (1.27e-2)	7.0312e+0 (3.38e-3)	6.6206e+0 (1.02e-1)	6.7931e+0 (7.02e-2)	1.2651e+1 (4.16e-1)
	15	7.6663e+0 (8.41e-2)	8.8390e+0 (5.08e-2)	8.9845e+0 (1.82e-1)	8.2158e+0 (8.98e-2)	1.6192e+1 (1.85e-2)
WFG7	8	2.9058e+0 (4.05e-2)	3.0323e+0 (1.92e-2)	3.1329e+0 (1.35e-2)	2.9742e+0 (8.04e-3)	7.0890e+0 (9.42e-3)
	10	4.1531e+0 (1.14e-2)	4.6088e+0 (2.79e-2)	4.6441e+0 (1.81e-2)	5.0684e+0 (4.29e-1)	9.4066e+0 (7.55e-3)
	12	5.7165e+0 (2.46e-1)	6.0929e+0 (8.20e-2)	6.1782e+0 (1.84e-2)	7.0019e+0 (2.94e-2)	1.3049e+1 (7.30e-3)
	15	7.8339e+0 (3.36e-1)	7.8892e+0 (8.44e-1)	7.4544e+0 (1.02e-1)	8.6196e+0 (1.22e-1)	1.6416e+1 (8.40e-2)
WFG8	8	3.0409e+0 (1.21e-1)	3.1327e+0 (1.25e-2)	3.3700e+0 (4.12e-2)	3.5840e+0 (5.13e-1)	6.1435e+0 (2.31e-1)
	10	4.3366e+0 (4.06e-2)	4.3692e+0 (9.55e-2)	4.8741e+0 (4.79e-2)	4.4707e+0 (5.81e-2)	8.7676e+0 (1.89e-1)
	12	6.1260e+0 (2.96e-1)	6.9171e+0 (1.17e-1)	6.4905e+0 (1.02e-2)	6.7660e+0 (3.72e-1)	1.1122e+1 (2.82e-1)
	15	8.4539e+0 (2.09e-1)	9.0591e+0 (6.56e-1)	7.4264e+0 (1.11e-1)	9.2364e+0 (8.43e-2)	1.4552e+1 (4.74e-1)
WFG9	8	2.8758e+0 (3.89e-2)	3.0343e+0 (3.04e-2)	3.1036e+0 (4.25e-3)	2.9322e+0 (8.42e-3)	6.5955e+0 (1.08e-1)
	10	4.1420e+0 (4.30e-3)	4.4826e+0 (1.12e-1)	4.5871e+0 (2.85e-2)	4.5007e+0 (2.12e-2)	8.7983e+0 (2.91e-1)
	12	5.3051e+0 (2.61e-2)	5.8430e+0 (8.54e-2)	6.0961e+0 (1.01e-1)	6.4501e+0 (2.48e-1)	1.2041e+1 (7.08e-1)
	15	7.1681e+0 (6.95e-2)	7.6540e+0 (3.02e-1)	7.1682e+0 (1.75e-1)	7.9131e+0 (1.75e-1)	1.4541e+1 (1.91e+0)
Ranks		1.72	2.9	2.6	2.83	4.94

RNM is less competitive. Still, MaOEA-RNM exhibits great advantages over other algorithms according to the Friedman test results presented in the last row.

Fig. 6 visually compares the non-dominated solutions obtained by all the algorithms on MaF2, MaF4, MaF6 and MaF8 instance with 10 objectives. For MaF2, both the convergence and diversity of MaOEA-RNM perform better compared with RVEA, KnEA, NSGA-III, and MOEA/D. Besides, both NSGA-III and MOEA/D perform poor regarding to the convergence. On MaF4, MaOEA-RNM fits the true Pareto front of MaF4 very well. By contrast, RVEA, KnEA, NSGA-III and MOEA/D perform poor. On MaF6, MaOEA-RNM fails to converge to the true Pareto front, but is still better that other algorithms. On MaF8, the overall performance of MaOEA-RNM is the best compared with other algorithms. Note that, although both MOEA/D and KnEA have the similar shapes as the true Pareto front, the diversity of MOEA/D and the convergence of KnEA are still poor.

Table 3
Averaged Spread values on WFG test suite.

Problem	M	MaOEA-RNM	RVEA	KnEA	NSGA-III	MOEA/D
WFG1	8	4.1237e-1 (4.56e-2)	8.9868e-1 (4.80e-2)	6.4245e-1 (9.07e-2)	7.4314e-1 (6.69e-2)	9.0364e-1 (1.64e-1)
	10	3.8772e-1 (1.40e-2)	1.0595e+0 (2.53e-2)	7.1559e-1 (1.94e-2)	7.5620e-1 (1.65e-2)	8.2995e-1 (3.30e-2)
	12	4.2246e-1 (2.18e-2)	1.2733e+0 (4.89e-2)	7.2181e-1 (4.15e-2)	1.0067e+0 (3.08e-2)	9.8679e-1 (1.12e-2)
	15	4.4384e-1 (1.43e-3)	1.2809e+0 (9.14e-2)	7.3238e-1 (6.82e-2)	1.0415e+0 (2.02e-2)	1.0018e+0 (1.67e-2)
WFG2	8	3.4247e-1 (1.26e-2)	5.3169e-1 (7.13e-2)	5.4955e-1 (1.01e-2)	8.6987e-1 (2.60e-1)	7.2913e-1 (9.01e-3)
	10	3.3723e-1 (2.45e-2)	5.9400e-1 (3.85e-2)	5.5284e-1 (6.38e-2)	7.9479e-1 (2.74e-2)	8.0970e-1 (8.60e-3)
	12	3.5288e-1 (1.01e-3)	8.3860e-1 (3.04e-2)	6.3113e-1 (2.42e-2)	8.8828e-1 (7.42e-2)	1.0126e+0 (3.97e-3)
	15	3.6070e-1 (1.04e-2)	9.0367e-1 (3.85e-2)	5.5939e-1 (2.66e-2)	8.5993e-1 (2.91e-2)	1.0244e+0 (2.37e-3)
WFG3	8	2.6666e-1 (1.68e-2)	2.6799e-1 (1.89e-2)	3.8637e-1 (2.28e-2)	7.2442e-1 (6.71e-2)	5.5059e-1 (2.38e-3)
	10	2.8917e-1 (1.78e-2)	3.2905e-1 (7.54e-2)	3.9190e-1 (4.31e-2)	6.0696e-1 (5.92e-2)	7.4773e-1 (1.23e-2)
	12	3.0317e-1 (1.41e-2)	3.9401e-1 (4.76e-2)	3.8210e-1 (1.05e-2)	6.9874e-1 (5.30e-2)	1.2089e+0 (2.23e-4)
	15	2.7100e-1 (8.44e-4)	4.7389e-1 (9.65e-2)	4.5991e-1 (2.26e-3)	7.4839e-1 (1.69e-1)	1.1698e+0 (1.80e-2)
WFG4	8	2.0858e-1 (1.41e-2)	2.6734e-1 (2.08e-3)	4.5626e-1 (3.75e-2)	2.7406e-1 (8.39e-3)	6.8309e-1 (1.71e-1)
	10	2.1140e-1 (7.29e-3)	3.4621e-1 (1.70e-2)	4.4392e-1 (2.00e-2)	3.6464e-1 (5.40e-3)	9.1763e-1 (3.82e-1)
	12	2.1705e-1 (2.41e-3)	5.3529e-1 (4.19e-2)	4.4664e-1 (1.65e-3)	3.1738e-1 (2.29e-1)	1.1284e+0 (8.00e-2)
	15	2.1882e-1 (1.26e-2)	6.5771e-1 (3.19e-3)	4.5917e-1 (6.93e-2)	3.8453e-1 (2.73e-1)	1.1516e+0 (9.11e-3)
WFG5	8	2.0055e-1 (2.19e-2)	2.7948e-1 (3.72e-4)	4.6056e-1 (4.08e-2)	2.8061e-1 (4.18e-3)	1.1174e+0 (2.94e-2)
	10	1.9521e-1 (2.33e-2)	3.5247e-1 (2.80e-3)	4.3178e-1 (3.35e-2)	3.7118e-1 (1.07e-2)	1.1749e+0 (6.30e-2)
	12	2.2854e-1 (9.47e-3)	5.5005e-1 (1.78e-2)	4.4226e-1 (3.71e-2)	1.6487e-1 (4.15e-3)	1.2092e+0 (2.06e-3)
	15	2.0348e-1 (6.28e-3)	6.6559e-1 (2.43e-2)	4.0984e-1 (3.84e-3)	3.6946e-1 (1.44e-1)	1.1166e+0 (4.47e-3)
WFG6	8	2.0828e-1 (8.94e-3)	2.9232e-1 (1.08e-2)	6.1509e-1 (2.01e-2)	2.7398e-1 (1.56e-3)	7.9114e-1 (9.13e-2)
	10	1.9403e-1 (1.30e-2)	3.4301e-1 (5.53e-2)	5.0385e-1 (5.91e-2)	3.6510e-1 (5.18e-4)	8.1697e-1 (8.38e-2)
	12	1.9019e-1 (5.48e-3)	4.2867e-1 (9.44e-4)	5.0969e-1 (1.91e-3)	1.9930e-1 (7.84e-3)	1.1438e+0 (9.28e-2)
	15	2.0085e-1 (1.48e-3)	6.0624e-1 (1.76e-3)	5.0587e-1 (8.34e-2)	1.9398e-1 (1.08e-2)	1.1069e+0 (1.30e-2)
WFG7	8	2.2940e-1 (4.13e-3)	2.7834e-1 (1.42e-3)	4.5727e-1 (2.74e-2)	2.7140e-1 (1.00e-2)	1.1381e+0 (2.97e-1)
	10	2.1708e-1 (3.60e-3)	3.5260e-1 (1.13e-2)	4.8443e-1 (2.67e-2)	5.1305e-1 (2.64e-1)	7.8641e-1 (7.33e-2)
	12	1.8979e-1 (1.14e-2)	4.2334e-1 (4.47e-2)	4.4105e-1 (1.82e-2)	6.0278e-1 (1.88e-1)	1.1175e+0 (1.23e-3)
	15	2.2068e-1 (2.62e-3)	4.8001e-1 (1.15e-1)	4.2237e-1 (6.44e-3)	7.1758e-1 (3.98e-2)	1.0921e+0 (1.13e-2)
WFG8	8	2.0169e-1 (8.77e-4)	2.9417e-1 (1.88e-2)	6.0802e-1 (1.80e-2)	6.2575e-1 (2.21e-1)	7.4675e-1 (1.41e-1)
	10	2.0047e-1 (8.05e-3)	3.0775e-1 (4.15e-3)	5.9278e-1 (6.88e-2)	3.6060e-1 (1.47e-2)	1.2259e+0 (5.49e-2)
	12	1.9203e-1 (1.12e-2)	3.8564e-1 (1.23e-1)	4.8333e-1 (5.30e-2)	6.5587e-1 (6.54e-2)	1.1921e+0 (5.62e-3)
	15	1.8491e-1 (1.36e-3)	6.0799e-1 (2.61e-2)	4.7151e-1 (4.26e-2)	6.9340e-1 (5.24e-2)	1.1210e+0 (1.73e-2)
WFG9	8	1.9390e-1 (1.12e-2)	3.0263e-1 (1.75e-2)	3.7122e-1 (2.27e-2)	2.9375e-1 (6.10e-3)	1.2147e+0 (1.81e-2)
	10	2.2806e-1 (8.12e-3)	3.4111e-1 (8.54e-3)	3.9871e-1 (3.01e-2)	3.7097e-1 (1.15e-2)	1.2674e+0 (8.80e-3)
	12	1.9920e-1 (1.38e-2)	3.5034e-1 (8.64e-2)	3.8897e-1 (1.93e-3)	3.8049e-1 (1.26e-1)	1.1578e+0 (6.51e-2)
	15	2.3382e-1 (1.11e-2)	5.5639e-1 (7.17e-3)	3.8170e-1 (4.76e-2)	4.9548e-1 (8.34e-2)	1.2044e+0 (1.68e-1)
Ranks		1.06	2.97	3.00	3.17	4.81

5.4. Experimental results on DTLZ test suite

The subsection is to verify the effectiveness of MaOEA-RNM on DTLZ test suite with 3 objectives. The final non-dominated solutions are evaluated with HV and IGD indicators. The averaged HV and IGD values are presented in Tables 6 and 7, respectively. The best results are highlighted with gray background.

As shown in Table 6, MaOEA-RNM performs the best on DTLZ2, DTLZ3, DTLZ5, DTLZ6, IDTLZ2 and C2_DTLZ2 compared with other methods. On DTLZ1 and DTLZ4, MaOEA-RNM is inferor to NSGA-III and RVEA, respectively. The Friedman test results in the last row shows that MaOEA-RNM is the best optimizer. Moreover, from Table 7, the same phenomenon is observed again. That is, MaOEA-RNM consistently performs better on DTLZ2, DTLZ3, DTLZ5, DTLZ6, IDTLZ2 and C2_DTLZ2 and worse on DTLZ1 and DTLZ4. The final statistical tests further confirm the analysis above.

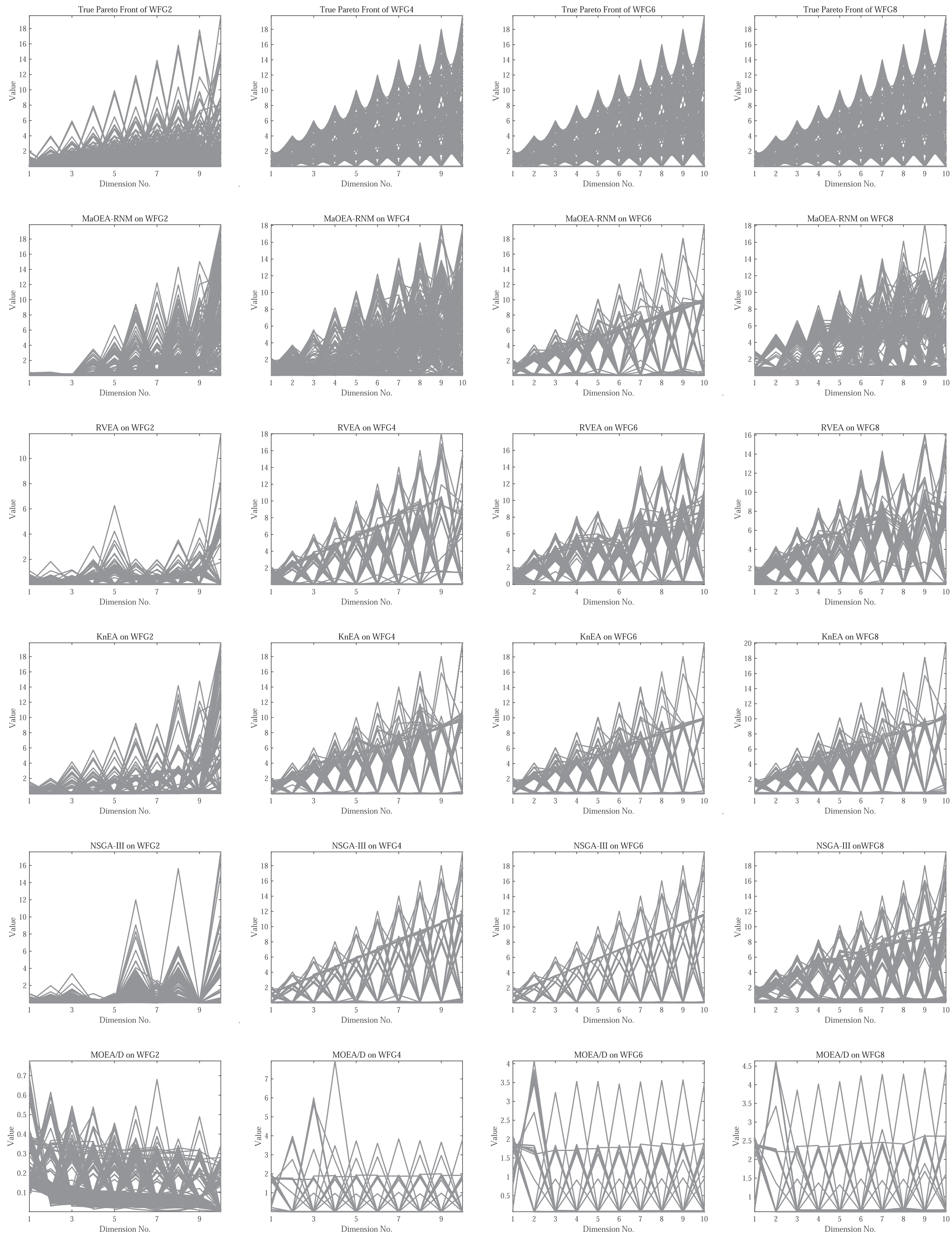


Fig. 5. Parallel coordinates of non-dominated fronts on WFG2, WFG4, WFG6 and WFG8 instance with 10 objectives. The first, second, third and forth column corresponds to WFG2, WFG4, WFG6 and WFG8, respectively.

Table 4
Averaged IGD values on MaF test suite.

Problem	M	MaOEA-RNM	RVEA	KnEA	NSGA-III	MOEA/D
MaF1	8	1.9955e-1 (1.98e-3)	6.1504e-1 (1.36e-2)	1.9974e-1 (4.37e-3)	2.5399e-1 (1.30e-2)	4.3626e-1 (4.12e-5)
	10	2.3383e-1 (4.87e-3)	6.7965e-1 (8.25e-2)	2.3630e-1 (3.43e-3)	2.6285e-1 (5.47e-3)	4.6905e-1 (2.36e-4)
	12	2.6004e-1 (2.17e-3)	6.5458e-1 (6.55e-2)	2.6559e-1 (9.41e-4)	2.9374e-1 (4.40e-3)	5.3823e-1 (1.78e-5)
	15	3.0859e-1 (1.10e-2)	7.4199e-1 (6.32e-2)	3.0878e-1 (7.63e-3)	3.1589e-1 (1.95e-3)	5.7591e-1 (2.03e-4)
MaF2	8	1.6338e-1 (4.03e-3)	2.1186e-1 (5.41e-3)	1.6766e-1 (3.83e-3)	3.3348e-1 (1.76e-1)	2.1689e-1 (1.53e-4)
	10	1.8755e-1 (2.90e-3)	2.3539e-1 (1.52e-2)	1.5720e-1 (1.53e-3)	2.2571e-1 (3.65e-4)	3.3316e-1 (1.02e-3)
	12	2.0013e-1 (1.18e-3)	5.7845e-1 (3.10e-1)	1.7700e-1 (1.13e-2)	2.2472e-1 (1.09e-2)	3.1029e-1 (4.40e-5)
	15	2.8988e-1 (1.18e-3)	8.1348e-1 (4.77e-4)	1.8978e-1 (2.57e-3)	2.1073e-1 (1.23e-4)	3.6381e-1 (7.37e-6)
MaF3	8	7.1819e+3 (5.10e+3)	8.9609e-2 (4.92e-3)	2.4920e+6 (1.42e+6)	3.9701e-1 (1.62e-1)	1.6103e-1 (1.10e-4)
	10	1.4265e+4 (1.03e+4)	8.5964e-2 (3.85e-3)	8.9331e+5 (2.86e+5)	1.5583e-1 (3.73e-2)	1.5372e-1 (1.67e-4)
	12	1.0874e+4 (3.11e+3)	8.2998e-2 (1.47e-3)	2.7806e+7 (3.54e+7)	1.7560e-1 (3.36e-2)	1.1640e-1 (1.05e-4)
	15	1.4568e+4 (2.50e+3)	8.8850e-2 (4.86e-4)	3.0492e+5 (4.09e+5)	1.8548e-1 (1.81e-2)	1.3042e-1 (8.65e-5)
MaF4	8	1.8190e+1 (5.55e-1)	3.5108e+1 (1.94e+0)	2.1080e+1 (8.40e-2)	3.1844e+1 (1.04e+0)	9.9785e+1 (2.12e+0)
	10	7.7035e+1 (1.15e+0)	2.2258e+2 (6.19e+1)	8.6487e+1 (3.02e+0)	9.6399e+1 (2.28e+0)	4.3121e+2 (1.11e+1)
	12	2.9706e+2 (3.29e+0)	9.4127e+2 (3.55e+2)	2.4529e+2 (1.62e+1)	5.2492e+2 (1.00e+1)	1.8095e+3 (8.95e+0)
	15	2.3051e+3 (1.65e+2)	8.3847e+3 (4.43e+2)	1.4026e+3 (1.99e+2)	3.9472e+3 (1.08e+2)	1.7408e+4 (8.86e+2)
MaF5	8	1.4729e+1 (1.20e-1)	2.2968e+1 (3.02e-2)	1.9600e+1 (1.49e+0)	2.1595e+1 (1.56e-2)	8.1864e+1 (2.99e-1)
	10	8.9097e+1 (1.33e+0)	1.0646e+2 (1.59e+0)	8.5991e+1 (4.77e+0)	9.8087e+1 (5.55e-2)	2.9945e+2 (3.80e-1)
	12	3.8925e+2 (2.95e+0)	3.4179e+2 (1.03e+2)	3.0633e+2 (2.50e+1)	4.5981e+2 (7.70e-1)	1.0983e+3 (5.94e-1)
	15	2.7736e+3 (1.19e+2)	3.5523e+3 (5.65e+2)	1.9769e+3 (2.55e+1)	2.5926e+3 (2.08e+1)	7.3238e+3 (6.28e-1)
MaF6	8	7.7376e-1 (2.07e-2)	9.7777e-2 (3.84e-2)	9.1792e-1 (1.29e+0)	1.3342e-1 (1.52e-1)	2.4333e-2 (5.04e-6)
	10	9.0552e-1 (5.72e-3)	1.3391e-1 (5.01e-3)	4.2348e+0 (4.08e-1)	3.1289e-1 (4.43e-3)	6.6156e-2 (5.87e-6)
	12	8.2726e-1 (1.26e-2)	4.0316e-1 (4.31e-1)	1.7510e+1 (9.33e+0)	2.9625e-1 (3.53e-2)	3.4097e-1 (4.26e-1)
	15	9.3001e-1 (2.77e-2)	4.2195e-1 (4.09e-1)	1.5098e+1 (1.49e+0)	2.9973e-1 (7.01e-3)	5.2210e-2 (4.02e-6)
MaF7	8	6.5657e-1 (7.22e-3)	1.9387e+0 (9.03e-2)	6.1173e-1 (6.37e-3)	7.6779e-1 (3.82e-2)	1.8884e+0 (5.68e-3)
	10	8.0364e-1 (9.70e-3)	3.1333e+0 (4.44e-1)	8.9363e-1 (1.37e-2)	9.8550e-1 (6.65e-3)	3.7552e+0 (1.05e+0)
	12	1.1133e+0 (1.02e-2)	3.3036e+0 (3.56e-1)	1.3861e+0 (1.02e-1)	1.5012e+0 (1.14e-1)	3.1635e+0 (1.75e+0)
	15	1.6690e+0 (1.97e-1)	4.4871e+0 (7.03e-1)	2.7232e+0 (8.07e-1)	4.2739e+0 (1.17e+0)	3.5650e+0 (6.82e-1)
MaF8	8	1.1100e-1 (1.85e-5)	7.4857e-1 (2.82e-2)	1.6179e-1 (1.59e-2)	3.6294e-1 (8.93e-2)	6.6252e-1 (2.27e-3)
	10	1.2396e-1 (5.37e-3)	8.6908e-1 (4.96e-2)	1.6670e-1 (8.97e-3)	3.9530e-1 (4.44e-3)	9.2694e-1 (1.21e-3)
	12	1.2589e-1 (1.91e-3)	1.2306e+0 (2.49e-1)	1.6807e-1 (1.60e-2)	4.8381e-1 (1.15e-1)	1.1213e+0 (6.53e-4)
	15	1.3856e-1 (1.40e-4)	1.1878e+0 (1.83e-1)	1.7584e-1 (1.86e-2)	3.3827e-1 (1.40e-4)	1.3567e+0 (3.45e-3)
MaF9	8	1.9241e+0 (2.88e-1)	6.2057e-1 (8.57e-3)	5.6997e+0 (9.95e-1)	5.6056e-1 (2.18e-1)	2.6712e-1 (3.10e-5)
	10	2.3820e+0 (6.00e-2)	6.1286e-1 (4.46e-3)	1.2448e+2 (3.48e+1)	4.1427e-1 (5.24e-2)	2.2467e+0 (2.73e+0)
	12	4.6937e+0 (3.07e+0)	1.4305e+0 (7.73e-2)	1.2205e+1 (9.14e-1)	6.1686e-1 (2.91e-1)	3.1712e+0 (3.51e+0)
	15	2.2956e-1 (9.59e-2)	1.2666e+0 (1.13e-1)	2.3520e-1 (6.46e-2)	3.4385e-1 (5.46e-2)	6.2737e+0 (7.55e+0)
Ranks		2.28	3.58	2.64	2.83	3.67

In summary, from the experimental analyses of MaOEA-RNM on WFG, MaF and DTLZ test suites, it can be seen that MaOEA-RNM exhibits evident superiority over comparison algorithms. The reason for that phenomenon is as follows. NSGA-III, MOEA/D and RVEA are based on the idea of decomposition. That is to say, these algorithms trend to select solutions approaching certain regions or directions. Although these selected solutions are able to ensure the diversity, they may affect the convergence to some extent. Similar conclusions can be found in [26]. As discussed in Section 3, KnEA is featured with the knee-point strategy, which prefers solutions trending to maximize the HV of the current population. On the contrary, MaOEA-RNM is dedicated to select solutions which are likely to dominate other solutions in the future evolutionary process.

Table 5
Averaged Spread values on MaF test suite.

Problem	M	MaOEA-RNM	RVEA	KnEA	NSGA-III	MOEA/D
MaF1	8	2.3602e-1 (3.83e-2)	1.0683e+0 (4.27e-2)	2.0225e-1 (1.43e-2)	5.8916e-1 (3.42e-2)	1.3580e+0 (2.80e-2)
	10	2.5022e-1 (1.16e-2)	1.2005e+0 (8.53e-2)	2.7481e-1 (7.98e-2)	5.7089e-1 (8.01e-2)	1.3231e+0 (2.71e-2)
	12	2.4520e-1 (1.42e-2)	1.1550e+0 (8.93e-2)	3.5919e-1 (1.27e-1)	6.3676e-1 (1.17e-2)	1.2231e+0 (5.61e-3)
	15	2.7758e-1 (1.32e-2)	1.2426e+0 (1.18e-2)	2.9872e-1 (4.20e-2)	6.0710e-1 (1.39e-2)	1.1884e+0 (4.79e-3)
MaF2	8	2.1906e-1 (1.22e-2)	2.1869e-1 (2.11e-2)	1.3747e-1 (5.51e-3)	8.3275e-1 (9.77e-2)	4.4999e-1 (7.86e-3)
	10	2.4725e-1 (1.42e-2)	2.3157e-1 (5.17e-3)	1.3574e-1 (2.62e-3)	6.8285e-1 (1.02e-1)	7.1244e-1 (5.62e-3)
	12	2.2743e-1 (3.82e-2)	6.7700e-1 (2.66e-1)	1.6457e-1 (1.80e-2)	6.6726e-1 (2.86e-2)	7.1563e-1 (1.46e-3)
	15	2.5571e-1 (2.52e-2)	8.8091e-1 (3.10e-3)	2.2978e-1 (9.04e-3)	6.8169e-1 (2.24e-3)	9.8924e-1 (7.58e-4)
MaF3	8	3.7687e-1 (8.14e-4)	4.7067e-1 (4.10e-2)	9.9632e-1 (7.96e-2)	2.0640e+0 (4.33e-2)	6.1065e-1 (4.66e-4)
	10	3.7556e-1 (8.68e-3)	4.1732e-1 (3.85e-2)	9.2824e-1 (8.07e-2)	2.0077e+0 (9.69e-2)	6.8212e-1 (3.49e-4)
	12	3.5183e-1 (2.90e-2)	8.1567e-1 (9.31e-3)	8.5152e-1 (3.93e-2)	1.7626e+0 (1.29e-1)	9.9634e-1 (6.38e-4)
	15	3.6846e-1 (2.75e-2)	8.7491e-1 (2.13e-2)	8.6830e-1 (1.18e-1)	1.9200e+0 (4.19e-2)	9.5921e-1 (4.69e-3)
MaF4	8	2.7550e-1 (6.31e-2)	7.7590e-1 (2.15e-1)	6.3873e-1 (5.00e-2)	9.0072e-1 (5.91e-3)	1.3006e+0 (1.33e-2)
	10	1.1097e+0 (1.19e+0)	1.5159e+0 (1.35e-1)	7.2463e-1 (1.05e-2)	8.7099e-1 (2.93e-2)	1.3496e+0 (2.23e-2)
	12	2.7742e-1 (2.43e-2)	3.3094e+0 (2.55e+0)	7.2483e-1 (2.46e-2)	9.9231e-1 (1.73e-3)	1.2290e+0 (1.05e-2)
	15	2.8405e-1 (1.64e-2)	1.7600e+0 (2.34e-1)	8.2476e-1 (1.32e-2)	1.0097e+0 (3.62e-3)	1.1437e+0 (4.66e-2)
MaF5	8	2.7487e-1 (1.42e-2)	6.2423e-1 (4.32e-3)	7.2875e-1 (5.32e-2)	8.5616e-1 (1.82e-3)	1.0446e+0 (2.59e-2)
	10	2.6008e-1 (8.03e-3)	8.8672e-1 (1.25e-1)	9.2559e-1 (7.54e-2)	1.1601e+0 (1.17e-3)	1.0242e+0 (1.48e-2)
	12	2.4987e-1 (1.75e-2)	1.5495e+0 (1.37e-1)	1.1142e+0 (3.68e-2)	1.2443e+0 (1.50e-2)	1.0034e+0 (1.27e-3)
	15	2.4975e-1 (2.63e-3)	1.9080e+0 (1.04e-1)	1.2300e+0 (1.78e-2)	1.2910e+0 (1.40e-2)	1.0004e+0 (1.07e-4)
MaF6	8	3.1638e-1 (6.81e-3)	6.1714e+6 (8.73e+6)	8.9840e-1 (9.32e-2)	1.3062e+0 (2.19e-2)	1.6111e+0 (8.15e-4)
	10	3.2955e-1 (1.81e-2)	1.2244e+0 (1.95e+0)	5.4688e-1 (5.32e-2)	8.5849e-1 (1.15e-1)	1.8276e+0 (1.27e-3)
	12	2.8130e-1 (1.86e-2)	3.3139e+0 (3.24e+0)	5.5371e-1 (7.58e-2)	8.9377e-1 (4.14e-2)	1.2472e+0 (3.43e-1)
	15	2.5424e-1 (2.43e-2)	2.9857e+0 (2.77e+0)	3.6402e-1 (1.19e-1)	7.3853e-1 (1.01e-1)	1.5631e+0 (4.49e-2)
MaF7	8	2.1713e-1 (1.64e-2)	8.8233e-1 (1.22e-2)	3.1765e-1 (1.57e-2)	4.1373e-1 (6.23e-2)	9.7960e-1 (3.16e-3)
	10	1.2157e-1 (1.97e-2)	9.9701e-1 (3.50e-3)	1.8909e-1 (1.92e-1)	5.3401e-1 (7.48e-2)	9.7087e-1 (8.81e-3)
	12	1.8288e-1 (2.87e-3)	1.0197e+0 (5.01e-2)	2.3475e-1 (7.12e-2)	5.7264e-1 (3.53e-3)	9.9107e-1 (8.19e-3)
	15	1.8989e-1 (1.71e-2)	8.9670e-1 (2.57e-2)	2.3019e-1 (3.85e-2)	6.7487e-1 (1.36e-2)	9.9064e-1 (1.44e-3)
MaF8	8	2.8359e-1 (2.28e-2)	1.2204e+0 (6.02e-2)	9.1290e-1 (1.36e-2)	1.0972e+0 (1.63e-2)	8.9810e-1 (1.48e-2)
	10	2.8740e-1 (1.32e-2)	1.4798e+0 (3.69e-1)	8.7938e-1 (4.53e-2)	1.0365e+0 (5.04e-3)	9.4223e-1 (8.37e-3)
	12	2.7248e-1 (1.02e-2)	1.2816e+1 (1.49e+1)	8.3660e-1 (1.50e-2)	1.1520e+0 (3.17e-2)	9.5235e-1 (1.14e-2)
	15	2.8308e-1 (8.97e-3)	2.3145e+0 (7.34e-1)	8.7164e-1 (6.42e-2)	1.1458e+0 (2.74e-2)	9.7597e-1 (3.83e-3)
MaF9	8	6.4584e-1 (5.61e-2)	2.3342e+0 (4.64e-1)	1.6678e+0 (5.69e-2)	1.8346e+0 (7.45e-2)	9.2516e-1 (6.20e-4)
	10	8.1585e-1 (7.75e-2)	1.2987e+0 (9.55e-2)	1.0308e+0 (1.24e-1)	1.7897e+0 (5.82e-3)	1.0680e+0 (9.71e-2)
	12	7.2704e-1 (1.04e-1)	6.0726e-1 (5.67e+0)	1.2008e+0 (4.74e-2)	1.9060e+0 (5.57e-3)	9.7749e-1 (3.18e-2)
	15	1.8391e+0 (6.72e-2)	1.1850e-1 (9.22e-1)	1.6890e+0 (4.56e-1)	1.7436e+0 (4.88e-1)	9.7772e-1 (3.51e-2)
Ranks		1.38	3.85	2.22	3.72	3.83

Namely, MaOEA-RNM pays more attention to the convergence in the future, which is the essential difference between MaOEA-RNM and other comparison algorithms.

5.5. Effectiveness of parameter *k*

To have a direct understanding of parameter *k* in the *k*-means clustering technology, this part plots the changing processes of parameter *k* during the optimization processes of WFG2 and MaF5 with 4 and 10 objectives. In addition, the numbers of non-dominated solutions in the first non-dominated level are also presented to highlight the effectiveness of parameter *k*. The population is 100 and the maximum generation is set to 100. Figs. 7 and 8 exhibit the experimental results.

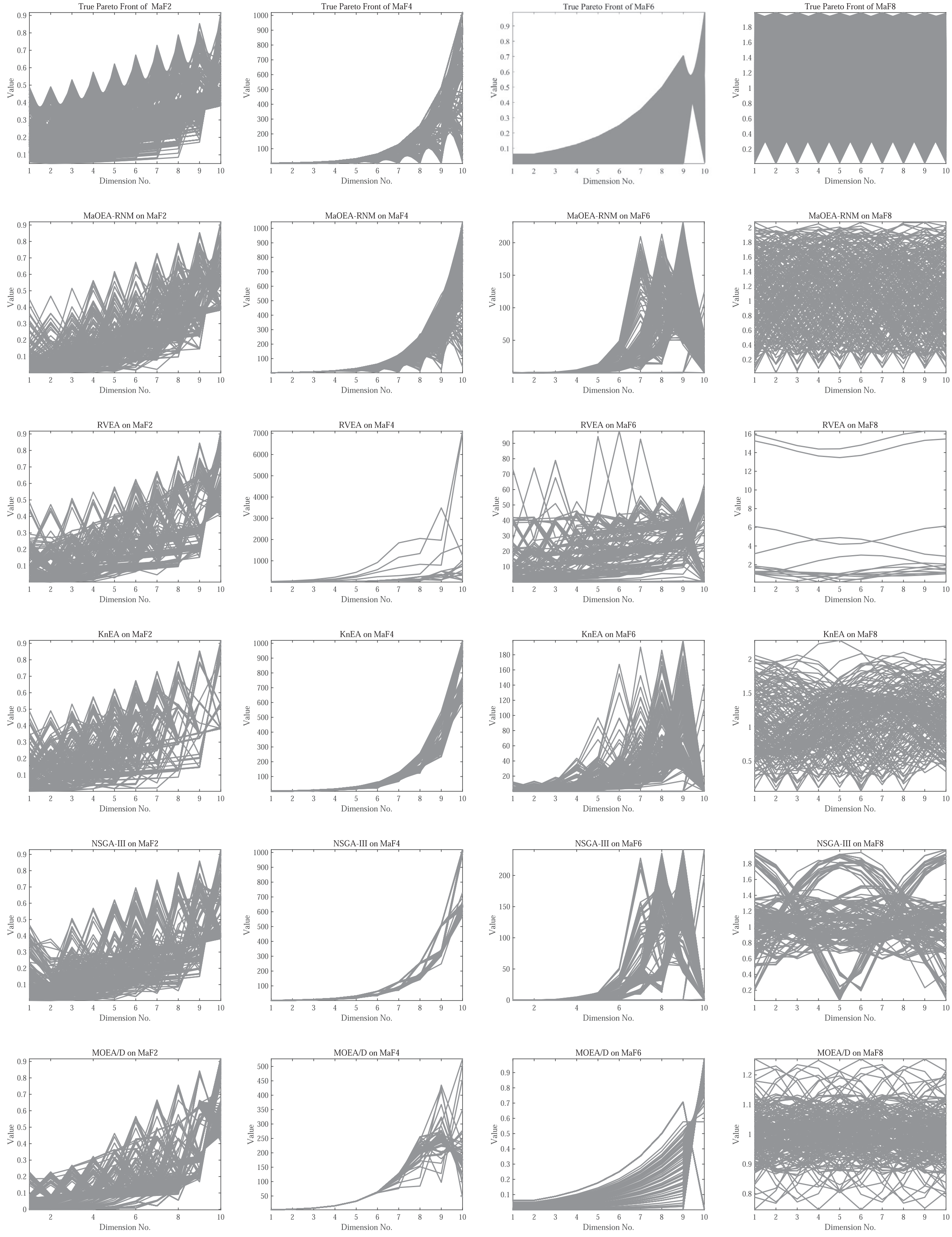


Fig. 6. Parallel coordinates of non-dominated fronts on MaF2, MaF4, MaF6 and MaF8 instance with 10 objectives. The first, second, third and forth column corresponds to MaF2, MaF4, MaF6 and MaF8 instance, respectively.

Table 6
Averaged HV values on DTLZ test suite.

Problem	M	MaOEA-RNM	RVEA	KnEA	NSGA-III	MOEA/D
DTLZ1	3	5.0736e-1 (4.04e-1)	1.1834e-2 (2.05e-2)	6.2381e-1 (2.92e-1)	7.5579e-1 (2.58e-2)	3.1371e-1 (4.26e-1)
DTLZ2	3	5.5273e-1 (1.60e-3)	5.5381e-1 (1.49e-3)	5.4297e-1 (4.37e-3)	5.5248e-1 (1.46e-4)	5.5432e-1 (1.74e-4)
DTLZ3	3	5.2688e-1 (4.80e-3)	9.2302e-2 (1.60e-1)	3.1588e-1 (2.74e-1)	3.3632e-1 (2.46e-1)	3.3002e-1 (2.86e-1)
DTLZ4	3	4.8479e-1 (1.20e-1)	5.5333e-1 (9.81e-4)	3.4195e-1 (4.06e-4)	4.0065e-1 (2.68e-1)	1.7455e-1 (1.45e-1)
DTLZ5	3	1.9894e-1 (2.72e-4)	1.4934e-1 (7.49e-3)	1.8518e-1 (9.69e-3)	1.9366e-1 (2.29e-4)	1.8240e-1 (4.75e-4)
DTLZ6	3	1.9845e-1 (2.07e-3)	1.1390e-1 (3.52e-2)	1.9819e-1 (2.72e-4)	1.9180e-1 (1.23e-3)	1.7493e-1 (2.55e-3)
IDTLZ2	3	5.3239e-1 (6.27e-4)	5.0706e-1 (6.73e-4)	5.2950e-1 (2.45e-4)	5.1663e-1 (3.08e-3)	5.2251e-1 (2.51e-3)
C2_DTLZ2	3	5.0529e-1 (2.71e-3)	4.9733e-1 (2.87e-3)	4.8886e-1 (1.31e-2)	4.9653e-1 (5.54e-3)	4.8641e-1 (2.29e-4)
Ranks		4.15	2.00	3.08	3.00	2.77

Table 7
Averaged IGD values on DTLZ test suite.

Problem	M	MaOEA-RNM	RVEA	KnEA	NSGA-III	MOEA/D
DTLZ1	3	1.2559e-1 (1.53e-1)	9.6330e-1 (7.39e-1)	2.3412e-1 (2.67e-1)	5.5354e-2 (7.09e-3)	7.2857e-1 (1.13e+0)
DTLZ2	3	5.9689e-2 (1.07e-3)	5.9967e-2 (2.56e-4)	6.6683e-2 (3.15e-3)	5.9873e-2 (2.72e-4)	5.9728e-2 (2.43e-4)
DTLZ3	3	7.3194e-2 (1.72e-2)	1.0417e+0 (1.06e+0)	3.5928e-1 (4.89e-1)	2.2177e-1 (2.12e-1)	6.6279e-1 (1.17e+0)
DTLZ4	3	2.9996e-1 (2.78e-1)	5.5467e-2 (2.10e-4)	4.2240e-1 (2.38e-1)	2.7774e-1 (4.45e-1)	7.4372e-1 (2.33e-1)
DTLZ5	3	5.1854e-3 (2.64e-4)	8.0433e-2 (7.81e-3)	1.5665e-2 (9.62e-3)	1.1774e-2 (8.27e-4)	3.2446e-2 (4.91e-4)
DTLZ6	3	5.3144e-3 (1.24e-3)	1.0669e-1 (1.67e-2)	5.3378e-3 (1.08e-4)	1.9534e-2 (4.03e-3)	3.2902e-2 (2.27e-3)
IDTLZ2	3	5.9139e-2 (2.02e-3)	8.2595e-2 (1.75e-3)	8.1324e-2 (1.20e-2)	7.4008e-2 (1.01e-3)	7.5169e-2 (3.15e-3)
C2_DTLZ2	3	4.5740e-2 (1.47e-3)	5.2349e-2 (8.87e-4)	6.6053e-2 (9.54e-3)	4.8785e-2 (1.02e-3)	5.2953e-2 (5.14e-4)
Ranks		1.69	3.88	3.63	1.88	3.94

From WFG2 with 4 objectives in Fig. 7, it can be seen that, at the initial stage, the number of non-dominated solutions does not exceed the population size and parameter k is adjusted adaptively to determine the number of clusters. When the number of non-dominated solutions in the first non-dominated level is more than the population size, k is set to 100, implying that 100 clusters are generated to select 100 non-dominated solutions according to the relative non-dominance

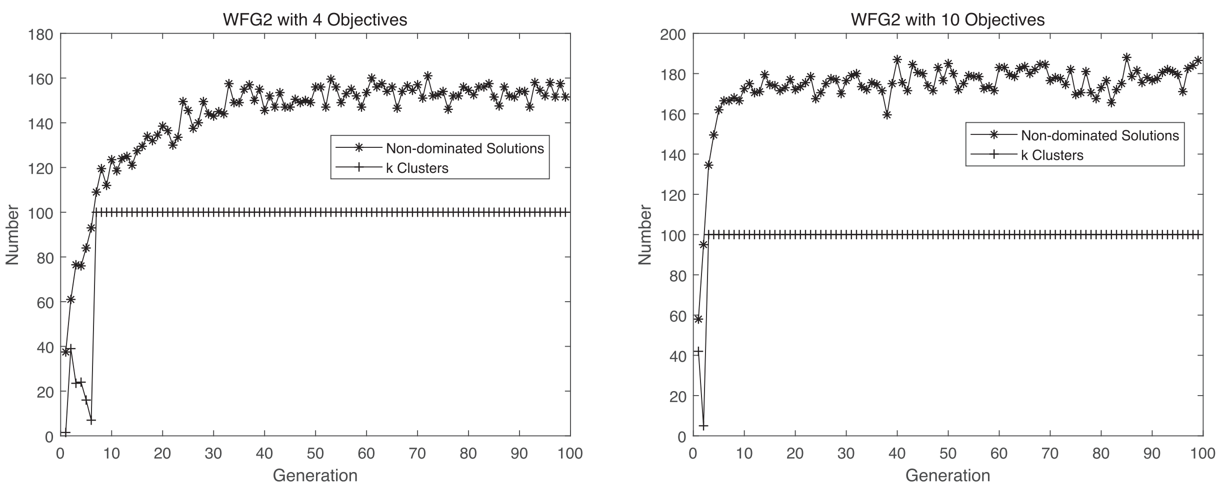


Fig. 7. Illustration of parameter k during the optimization process of WFG2 with 4 and 10 objectives.

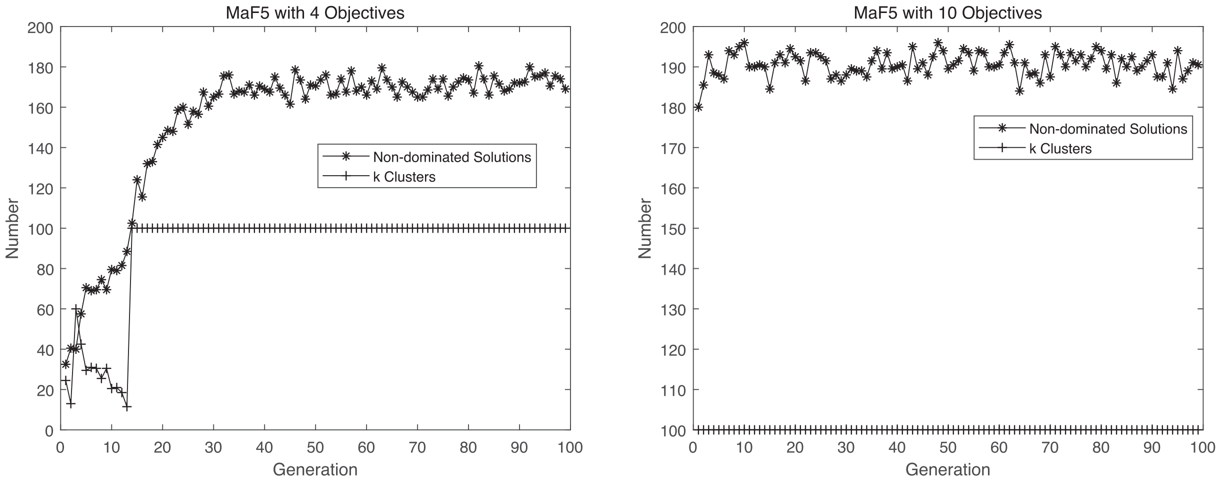


Fig. 8. Illustration of parameter k during the optimization process of MaF5 with 4 and 10 objectives.

matrix proposed in this paper. The same phenomenon can also be observed on MaF5 with 4 objectives in Fig. 8. Moveover, for WFG2 and MaF5 with 10 objectives in Figs. 7 and in 8, the changing processes of parameter k are difficult to be observed because nearly all the members in the initial population are non-dominated.

6. Conclusions

In this paper, to tackle the issue that a large number of non-dominated solutions result in the loss of selection pressure, the relative non-dominance matrix is proposed. Based on the relative non-dominance matrix, the fitness formula is defined. Experimental analysis indicates that solutions with smaller fitness values are considered to be superior to others in terms of the number of dominated solutions. Based on the relative non-dominance matrix, the mating selection strategy is redesigned to further distinguish the promising solutions. In addition, the relative non-dominance matrix is further incorporated into the environmental selection in combination with the k-means clustering strategy to ensure the convergence and diversity. In addition, parameter k in the k-means clustering strategy is adjusted adaptively. The proposed algorithm, namely MaOEA-RNM, is tested against four state-of-art algorithms on three popular test suites with three, eight, ten, twelve and fifteen objectives. The experimental results demonstrate that MaOEA-RNM is more outstanding compared with other algorithms in terms of the convergence and diversity. Moreover, MaOEA-RNM is evidently superior to comparison algorithms regarding to the spread. Further comparisons of the parallel coordinates of final non-dominated solutions demonstrate the effectiveness of MaOEA-RNM.

From the experimental observations, it can be said that the relative non-dominance matrix plays a key role in choosing promising solutions for promoting the population toward the true Pareto fronts. Further work will be focused on the following two aspects. One aspect is to deepen the research on the relative non-dominance matrix to enhance the selection pressure. Although there exist a lot of algorithms for MaOPs, effective methods for MaOPs with large-scale decision variables are still rare. Thus, the second aspect is to further investigate the relative non-dominance matrix on large-scale many-objective problems.

CRediT authorship contribution statement

Maoqing Zhang: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Validation, Visualization, Writing - original draft, Writing - review & editing. **Lei Wang:** Supervision, Funding acquisition, Conceptualization, Investigation, Methodology, Project administration, Resources, Writing - review & editing. **Weian Guo:** Supervision, Funding acquisition, Conceptualization, Data curation, Project administration, Resources, Writing - review & editing. **Wuzhao Li:** Supervision, Funding acquisition, Data curation, Formal analysis, Project administration, Resources, Writing - review & editing. **Dongyang Li:** Data curation, Writing - review & editing. **Bo Hu:** Data curation, Writing - review & editing. **Qidi Wu:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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