FRE 7251 Course Project

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March 4, 2025

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1 Introduction

This report documents the design, implementation, and evaluation of a technical trading strategy applied to TQQQ, a leveraged ETF tracking the NASDAQ-100. Daily price data from January 1, 2020, to December 31, 2024, was obtained via the yfinance Python API from Yahoo Finance. The focus of this project is to assess the performance of a moving average crossover strategy under both historical conditions and through an AR(1)-based bootstrap simulation.

In this report, the following aspects are analyzed:

- The trading strategy and rationale behind the chosen moving average parameters.
- Historical performance of the strategy applied to TQQQ.
- The implementation of an AR(1) bootstrap to simulate alternative price paths.
- Performance evaluation of the bootstrapped simulations, including profit/loss distribution analysis.
- Statistical testing of whether the mean total return differs significantly from zero.

2 Trading Strategy and Parameter Choice

Strategy: A simple moving average crossover strategy is employed with the following rules:

- **Entry Signal:** Buy when the 20-day moving average (MA20) of the adjusted closing price crosses above the 50-day moving average (MA50).
- **Exit Signal:** Sell when the MA20 crosses below the MA50.

Rationale: TQQQ is known for its high volatility due to leverage. A moving average crossover is straightforward to implement and can help capture trending periods while avoiding choppy, sideways markets. The 20-day MA serves as a short-term indicator, while the 50-day MA provides a longer-term trend filter. This combination is intended to generate a sufficient number of signals while reducing noise from short-term fluctuations.

3 Historical Performance of the Trading Method

The moving average crossover strategy was applied to TQQQ's historical data from January 1, 2020, to December 31, 2024. The key performance metrics computed on the historical data were as follows:

- **Number of Trades:** 11
- **Total Strategy Return:** 46.22%
- **Buy-and-Hold (B&H) Return:** 492.81%

- **Strategy Sharpe Ratio:** 0.41
- **Buy-and-Hold Sharpe Ratio:** 0.88
- **Winning Trades (%):** 34.96%
- **Maximum Drawdown:** -76.82%

Although the strategy underperformed a pure buy-and-hold approach in terms of absolute return, it was designed to provide risk control and signal exits during adverse market conditions.

4 Bootstrap Implementation and Assumptions

To assess the robustness of the moving average crossover strategy, an AR(1)-based bootstrap simulation was performed with the following steps:

Modeling Returns

The daily returns were modeled using an AR(1) process:

$$r_t = \alpha + \phi\, r_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2)$$

The parameters α, ϕ, and σ were estimated from the historical TQQQ returns.

Simulation

Using the fitted AR(1) model, 100 synthetic return series (each the same length as the historical sample) were generated. For each simulation, the synthetic returns were converted into a price path starting from the historical initial price.

Strategy Application

The moving average crossover strategy was applied to each simulated price series to compute performance metrics identical to the historical evaluation.

Assumptions: The bootstrap assumes that the AR(1) model adequately captures the autocorrelation in TQQQ returns and that the residuals are normally distributed. Although these assumptions may not perfectly hold in real markets, they allow us to explore the potential variability of the strategy's performance under alternative market scenarios.

5 Performance of the Bootstrap

Across the 100 bootstrap simulations, the following average performance metrics were observed:

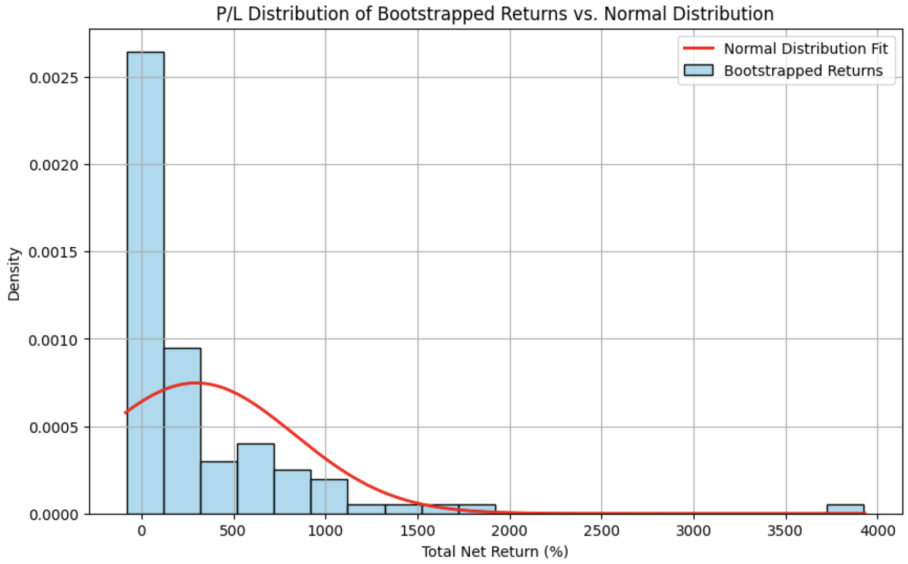
- **Average Number of Trades:** 12.22

- **Average Total Strategy Return:** 565.09%
- **Average Buy-and-Hold Return:** 1973.12%
- **Average Strategy Sharpe Ratio:** 0.78
- **Average Buy-and-Hold Sharpe Ratio:** 1.01
- **Average Winning Trades (%):** 31.72%
- **Average Maximum Drawdown:** -57.37%

These results suggest that while the moving average crossover strategy tends to generate a positive total return, its performance exhibits a wide range of outcomes. Notably, the bootstrap simulations reveal that the strategy's return distribution is right-skewed, indicating the possibility of occasional very high gains, but also a non-negligible risk of substantial drawdowns.

6 Profit/Loss Distribution Analysis

The profit/loss (P/L) distribution of the bootstrapped total returns was analyzed by:




```
Mean Total Return: 297.30%
t-statistic: 5.57
p-value: 0.0000
The mean total return differs significantly from zero (p < 0.05).
```

1. **Plotting the Histogram:** The histogram of total returns was overlaid with a fitted normal distribution (using the sample mean and standard deviation). This comparison revealed that the bootstrapped returns are not well-described by a normal distribution—the histogram is right-skewed with heavier tails, indicating a higher probability of extreme outcomes.

2. **Statistical Testing:** A one-sample t-test was conducted to assess whether the mean total return differs significantly from zero. The test produced a t-statistic of approximately 5.57 with a p-value less than 0.0001, thereby rejecting the null hypothesis. This result confirms that the average total return from the bootstrap simulations is statistically significantly positive.

7 Conclusion

The moving average crossover strategy applied to TQQQ from 2020 to 2024 demonstrates a moderately positive performance under historical conditions, although it does not outperform a buy-and-hold strategy in absolute terms. However, the AR(1)-based bootstrap analysis suggests that under simulated market conditions, the strategy can achieve higher average returns with a similar risk profile, albeit with a wide distribution of outcomes.

The profit/loss distribution analysis highlights a right-skewed profile with fat tails—indicating that while there is the potential for substantial gains, there remains a risk of significant drawdowns. The statistically significant positive mean total return across simulations supports the strategy's viability, but additional risk management enhancements (e.g., stop-loss orders) could further improve the downside risk profile.

Future work may involve exploring alternative moving average windows, incorporating additional technical indicators, or implementing dynamic risk controls to further refine the strategy's performance.

Appendix

A complete table of performance metrics for each of the 100 bootstrap simulations is provided in the accompanying Excel file, and detailed plots of the P/L distribution compared to a normal distribution are included in the supplementary materials.

Sample	# round-trip trades	Total net return (%)	Sharpe ratio	B&H return (%)	B&H Sharpe ratio	Win %	Max Drawdown (%)
1	9	301.1269895	0.851201116	168.9248831	0.660007153	24.87046632	-55.90510771
2	14	22.37783174	0.338453995	97.80456331	0.561977081	27.89291883	-48.77423715
3	15	-50.73631723	-0.052183339	292.8467105	0.776343251	27.89291883	-70.39706999
4	9	110.6583516	0.573492079	109.0088111	0.577652344	25.90673575	-68.85519874
5	11	6761.83867	1.800094538	26583.92103	2.062381312	40.41450777	-60.32935033
6	16	420.344933	0.922563707	2085.241491	1.315547096	33.67875648	-44.43507148
7	14	77.90194546	0.503403736	834.4419756	1.039345396	32.12435233	-62.41278557
8	12	360.2942777	0.860785657	1570.953789	1.216696854	32.21070812	-63.34652198
9	10	429.6506562	0.97045713	108.5219591	0.575778835	28.58376511	-58.65028499
10	15	7.171487553	0.290376059	561.4568248	0.937288247	29.2746114	-79.37879351
11	10	1794.816093	1.360503612	6743.250515	1.653644199	38.51468048	-43.2789964
12	14	68.15083801	0.476121383	139.597783	0.622930194	26.33851468	-67.79099972
13	12	-34.48201115	0.012845676	-96.59443596	-0.659539645	18.65284974	-68.21981386
14	13	-43.76039829	0.022021709	91.37581104	0.554654081	28.49740933	-69.09324442
15	14	344.5277006	0.850931195	8702.663373	1.734037418	36.96027634	-47.81646412
16	14	74.51526117	0.493262918	58.92032375	0.496532699	24.78411054	-73.82082103
17	11	129.126361	0.622364942	12.69170292	0.378054887	24.35233161	-48.30373167
18	14	71.92405348	0.490305083	471.1242502	0.886031803	30.22452504	-66.53941185
19	15	242.2709627	0.744005342	2690.149276	1.354532601	36.78756477	-80.34881211
20	11	426.1032106	0.957175974	449.4894072	0.87908146	28.75647668	-71.73984541
21	14	65.87816007	0.475253016	888.1814846	1.061712078	32.2970639	-56.41393824
22	12	367.642156	0.87811217	2203.781714	1.321350616	33.85146805	-38.75582201
23	9	1895.177035	1.401063165	4205.545026	1.513867012	36.09671848	-48.10205922
24	14	184.6078891	0.68375299	2016.062025	1.2980683	36.52849741	-53.86511955
25	11	2596.81728	1.476576011	2393.889285	1.329097499	37.56476684	-38.22122052
26	10	171.665679	0.675854943	143.0494904	0.628635848	29.44732297	-49.77527293
27	11	16.88633368	0.324711602	229.6859659	0.722296431	28.84283247	-53.14396384
28	11	270.1713061	0.803472856	66.7270534	0.510178193	28.23834197	-51.01069765
29	8	2071.33393	1.406900621	2269.968361	1.296789939	35.83765112	-56.53063327
30	13	350.6396565	0.855003779	771.1608584	1.020225062	36.87392055	-44.95502385
31	11	406.2885161	0.934799641	233.5749155	0.725115338	28.23834197	-52.94083905
32	13	178.3376348	0.686989744	93.03982188	0.554656019	27.28842832	-38.2830757
33	12	-12.64370068	0.157905031	-43.00415841	0.158270848	24.26597582	-52.23103289
34	10	356.8472945	0.87314381	658.3551356	0.974575094	30.39723661	-76.72235624
35	12	233.9758683	0.751085915	2766.404382	1.413222966	36.70120898	-50.35497796
36	13	122.2786433	0.589360988	746.2605903	1.000067106	29.70639033	-63.4702132
37	14	-3.496951382	0.213487942	172.1372054	0.660143534	23.83419689	-68.1527619
38	13	19.80384636	0.342218748	24.36787913	0.41908338	29.8791019	-66.10809787
39	8	360.4944439	0.871979156	400.9406677	0.850117376	32.9015544	-49.65843324
40	11	1667.66258	1.307188101	9014.275622	1.725667682	41.19170984	-49.26110585
41	12	149.5760104	0.63394802	449.0019164	0.878478058	33.07426598	-56.20316981
42	10	2559.419251	1.537586261	2050.159293	1.311788847	38.51468048	-40.28414005
43	11	558.7570789	0.987470162	2525.34277	1.35989718	37.39205527	-54.2835218

44	10	1419.516999	1.29278215	4220.251999	1.496998588	38.68739206	-48.86285079
45	12	24.94350816	0.374601442	892.4109854	1.057808943	32.98791019	-74.10581012
46	12	437.3590031	0.912021438	2827.365856	1.37698445	33.85146805	-43.87515896
47	14	479.8573546	0.928520887	5122.426207	1.54299315	36.6148532	-63.78722174
48	15	-24.19134423	0.154116985	392.8288445	0.846152592	27.54749568	-83.18581717
49	12	428.6130314	0.897183846	1755.278665	1.251194454	37.8238342	-46.14010016
50	11	13.73915847	0.29828896	-56.94141133	0.099290747	21.76165803	-70.39505535
51	12	128.0224296	0.613822205	2.278734377	0.360650227	25.56131261	-51.80486538
52	10	1003.849467	1.20865485	1332.246561	1.187349184	36.01036269	-51.15407199
53	10	762.5931658	1.131547674	747.9104238	1.005408087	29.79274611	-60.27101347
54	13	175.2101924	0.669165531	1727.267211	1.221198315	32.38341969	-57.20741415
55	13	23.433749	0.340656092	182.5054342	0.67218016	28.0656304	-65.48441816
56	9	1152.353301	1.254630388	2742.800514	1.399153104	36.44214162	-38.83003497
57	11	25.61079345	0.355650661	79.22332102	0.526557513	29.44732297	-74.48042199
58	13	89.4900802	0.529042984	333.5872611	0.809891436	28.0656304	-62.38099493
59	12	929.5936778	1.162238133	891.9840559	1.051396574	31.86528497	-50.06493919
60	11	859.2311579	1.169747116	530.7631783	0.925096919	33.93782383	-45.66368907
61	11	677.1148806	1.021075193	6096.324182	1.591848532	39.72366149	-65.1170402
62	12	773.9484782	1.115436482	1700.417945	1.246356901	34.62867012	-69.42479939
63	15	-15.44654644	0.189672287	240.9598011	0.730502661	26.68393782	-61.61212697
64	15	11.61042222	0.310418511	756.7416259	1.015379012	31.60621762	-77.78515082
65	12	687.5826872	1.067971403	3003.645865	1.407248548	34.97409326	-53.4456027
66	15	57.20064569	0.45268582	182.839568	0.676918435	28.49740933	-72.84040028
67	12	116.2547366	0.58176428	303.5190066	0.785300699	29.36096718	-43.99997604
68	13	64.58331858	0.471989832	1433.444177	1.194057848	32.46977547	-74.02115498
69	13	224.2227911	0.738581189	871.8506055	1.056679813	31.6925734	-58.64068722
70	10	1427.239954	1.269761419	2147.22846	1.28757498	36.6148532	-51.37438069
71	12	41.67486214	0.409644401	797.6781078	1.027704927	28.75647668	-71.25393421
72	13	-11.82690924	0.228250685	856.2824578	1.050789072	33.24697755	-47.69014715
73	12	1599.430782	1.39983116	1319.58613	1.164888717	35.06044905	-48.37750706
74	13	55.31024846	0.452690537	709.2513726	0.99572811	34.11053541	-69.13426469
75	11	1049.536609	1.220283506	3461.455301	1.46903196	35.31951641	-36.15803512
76	14	55.10933782	0.444628191	103.6917575	0.575670843	27.02936097	-77.55107242
77	12	287.219849	0.806665864	1077.964257	1.113854144	32.98791019	-68.57911365
78	16	241.8999539	0.752450452	3379.641516	1.438341604	32.03799655	-60.84333629
79	10	527.1186883	1.02494236	817.1495049	1.028010823	28.6701209	-49.68649938
80	13	549.5789821	1.009022843	859.87213	1.046654267	32.55613126	-41.56540051
81	13	4716.361404	1.712676638	27512.59641	2.09849035	39.11917098	-35.66898448
82	11	1270.657734	1.269776754	2429.00487	1.360002214	36.78756477	-41.3123526
83	12	279.0871904	0.791991899	2265.933573	1.319449548	34.19689119	-53.91336049
84	13	249.8481586	0.787420941	246.546804	0.740464294	31.0880829	-52.55975355

85	14	42.3205242	0.400068354	-25.65701245	0.254597064	24.17962003	-56.95715633
86	11	98.7272428	0.546103521	445.6870777	0.873777615	28.32469775	-69.16374488
87	9	253.8509932	0.787858105	139.5704958	0.62226225	26.59758204	-44.40640487
88	12	459.3534543	0.955676584	2689.388015	1.389772751	32.38341969	-49.64461128
89	12	462.5733124	0.947483607	2462.072942	1.363627171	35.14680484	-56.48552155
90	14	60.45819001	0.457044732	146.5029077	0.631214887	28.32469775	-63.03923766
91	17	89.79740781	0.528085585	1390.64017	1.18242237	29.53367876	-72.62526948
92	13	99.05878482	0.546985052	298.5182313	0.778315249	31.51986183	-57.19490123
93	11	343.0349769	0.848878456	1269.068925	1.151549443	34.54231434	-60.5901726
94	14	388.2855152	0.876885552	3873.821557	1.483929747	38.08290155	-62.77675557
95	14	127.8823736	0.609009609	67.50700659	0.509658021	28.0656304	-56.26109888
96	11	57.79934073	0.448910516	23.34915712	0.419977997	25.90673575	-57.64994701
97	13	818.2634949	1.158203509	2288.090985	1.344500893	32.2970639	-41.53813286
98	11	661.4375519	1.077026275	1952.722831	1.28503209	33.41968912	-62.6877284
99	12	3131.494433	1.591349598	7126.516349	1.709812733	41.10535406	-42.62554191
100	15	448.6712177	0.964785688	912.4042947	1.067143538	29.18825561	-55.37561429
Average	12.22	565.09	0.78	1973.12	1.01	31.72	-57.37