Designing of a Co-axial Co-planar Rotor

Yichi Ma

Abstract

This work presents a co-axial and co-planar rotor design that can be equipped on small aerial vehicle. By employing rotor dynamics, blade element theory, vortex panel method, method of Thwaites, and numerical computation, our design can achieve a net torque of zero with two co-planar rotors spinning in opposite direction and minimal required power. The design is able to generate 100 N of thrust by requiring a total power of 1302.5 Watts. Computational results show that the design satisfies all requirements while it meets all constraints. By presenting our design process and providing details of our design, we hope to advance the further research in co-axial and co-planar rotor designs.

1 Introduction

The helicopter is a machine consisted of engineering sophistication and refinement. Its origin has been less clear comparing to the origin of fixed-wing plane which can be traced back to Lilienthal and Wright Brothers. The helicopter is usually defined by using rotating wings to achieve lifting and propulsion. The vertical thrust is achieved by aerodynamic lift forces created by the moving flow with the rotating speed of the blade itself. To make a more efficient design, it requires the bladed to accelerate a large mass of air at relatively low velocity [6].

There are other considerations for the helicopter design: the helicopter must use its rotors to perform vertical lift, hover, descending, and cruising, which increase the complexity of mechanical system and control. Additionally, vibrations are generated by the motors and transmitted to the airframe, resulting in reduced lifespan of the airframe [6].

The common configuration for the helicopter consists of one main rotor and a tail rotor. The tail rotor is used to compensate the torque created by the main rotor. Other configuration for the helicopter can be tandem, transverse, or coaxial rotor configuration. The coaxial rotor is often used to make the net torque of the helicopter vanishes, abandoning the use of tail rotor. However, due to the non linearity of the moment um and energy fluxes, the lower rotor is required to do more work than the upper rotor for the same thrust.

In our work, we want to take the advantage of counter rotating rotor to make the net torque of the system vanishes. We also attempts the co-planar design to avoid the disadvantages like more power requirement of the coaxial configuration.

2 Methods

2.1 Requirements

Constraints and fixed parameters are considered to initialize our design process. The dimensional restrictions are given by [2]:

1) $R_o = 0.3$ m
2) $d_i/D_o = 0.25$ for root cut-out
3) $D_i - d_o = 0.05D_o$ for clearance
3) $\sigma = (NA_bo + nA_ib)/A <= 0.10$ for overall rotor solidity

By combining requirements number 1, 2, 3, we determined that $r_i = 0.075$, and $R_i - r_o = R_o/20 = 0.3/20$. In summary, r_o can vary from 0.075 to 2.785.

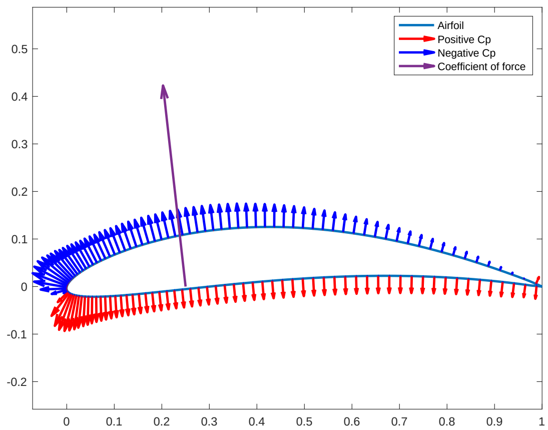

Figure 1: NACA 7512 Boundary, C_p distribution, and the coefficient of force.

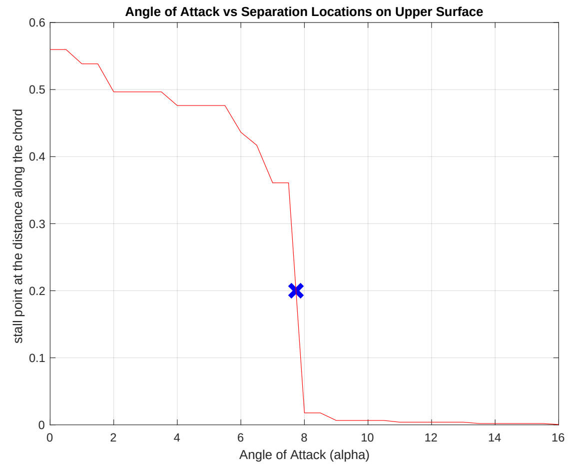

Figure 2: Angle of attack (α) and the separation locations on the upper surface. The blue cross represents the stall point.

Another constraint is the output thrust of the inner and outer rotors, which is: $T_i + T_o = 100$ N. T_i is the thrust provided by the inner rotor, while T_o is the thrust provided by the outer rotor. We will consider the T_i varying from 0 to 100. The number of inner blade is fixed at n = 4 and the outer blade as N = 3 to avoid possible resonances. There should be no flow separation since we wish to achieve the maximum lift. We also neglect the viscous drag and vortex induced drag to simplify our computation.

In summary, we need to choose the value of r_o and T_i for our design while satisfying other constraints.

2.2 Theory

Consider the cross section of the rotor blade as the two-dimensional airfoil sections. The lift coefficient and drag coefficient can be determined by using vortex panel method of an airfoil. In our design, we employed NACA 7512 airfoil as the profile for rotor blade cross section. In vortex panel method, we calculate the vortice strength at each discretized control points on the airfoil to ensure a continous streamline over the airfoil boundary [3]. Next, the velocity of the flow at the boundary can be determined. Ultimately, the coefficients of pressure C_p are computed based on the local velocity. Using the coefficient of pressure, we obtained the coefficient of lift and coefficient of drag of the airfoil.

To designate the angle of attack for our rotor blade, we concern about the flow separation and stall angle, the point where the maximum lift can be obtained. The separation points for a particular angle of attack are determined by using Thwaite's method. The criteria is given by [1]:

$$K(x) = \frac{0.45}{V(x)^6}\frac{dV}{dx}\int_0^x V(x)^5 dx = -0.09 \tag{1}$$

where V is the dimensionless local velocity obtained from the vortex panel method and x is the distance from the stagnation point at the leading edge of the airfoil. Next, the separation distance is plotted at each angle of attack as shown in Fig. 2. We establish that the stall angle is the angle of attack when the separation point is at 0.2 of the whole chord length. Thus, from Fig. 2, we determine that the stall angle is at 7.73 degrees. To avoid flow separation, we set our angle of attack at 6.50 degrees for our airfoil profile. Revoking the vortex panel method given the angle of attack at 6.50 degrees, we acquired the lift coefficient (C_l) as 1.7024 and a drag coefficient (C_d) close to zero: 0.0028. The result is shown graphically in Fig. 1. We will than employ blade element theory by La Glauert.

As the rotor blades generate a downward flow when in operation,the airfoil sections experience reduced airflow passing in the direction of the chord. Hence, the relative velocity effect must be considered into the design process. The extra inclination angle is given by:

$$tan(\phi) = \frac{V_h}{r\omega} \tag{2}$$

where V_h is the hover induced airflow velocity, r is the distance from the rotational axis, and ω is the angular velocity. The hover induced flow velocity is given by: $V_h = \sqrt{T/(2\rho A_i)}$ from rotor dynamics [1]. The blade twist angle ($\beta(r)$) can be determined by adding $\phi(r)$ and angle of attack (α) together: $\beta(r) = \phi(r) + \alpha$. Now, the coefficient of normal force (C_n) and the coefficient of tangential force (C_t) can be determined by applying rotational matrix with an angle of ϕ of C_d and C_l. This relation can be expressed as:

$$C_n = C_l cos(\phi) - C_d sin(\phi) \tag{3}$$

$$C_t = C_l sin(\phi) + C_d cos(\phi) \tag{4}$$

The coefficient of normal force can be used to determine the section thrust along the blade. The section thrust can be directly computed by using:

$$t = C_n(r)\frac{1}{2}\rho V_r^2 l(r) \tag{5}$$

where t is the section thrust force and l(r) is the chord length which varies with distance from the rotation axis (r). We also know that the blade is elliptically loaded, which means the section thrust force t_i also has the following relation:

$$t = t_{max}\sqrt{1 - \frac{y^2}{(b/2)^2}} \tag{6}$$

For the inner blade, $y = r - (\frac{r_o - r_i}{2} + r_i)$ and $b = r - o - r_i$. The total thrust force for each blade is the sum of all section thrust force on that blade: $T = \int tdr$. The total thrust force is given by the requirement, so t_{max} can be obtained by: $T = \frac{1}{2}\pi t_{max}(r_o - r_i)$. After t_{max} is determined, we relate Eq. 5 and Eq. 6. They become:

$$t = t_{max}\sqrt{1 - \frac{y^2}{(b/2)^2}} = C_n(r)\frac{1}{2}\rho V_r^2 l(r) \tag{7}$$

V_r is also dependent on r. By Pythagorean theorem, $V_r = \sqrt{V_h^2 + \omega^2 r^2}$. With all known parameters, we can solve for $l(r)$ at all locations of r.

The section torque (q_i) is given by: $q_i(r) = Ct(r)\frac{1}{2}\rho V_r^2 l(r) r$. The total torque of the rotor with n blades is determined by: $Q = n\int q_i(r)dr$. The total torque for the inner rotor must cancel exactly with the total torque of the outer rotor to enable a stable operation of the co-axial rotors: $Q_{inner} + Q_{outer} = 0$. This requirement eliminates many choices for r_o and T_i. To help us to find the design space of r_o and T_i while $Q_{inner} + Q_{outer} = 0$. As shown in Fig. 3, a contour plot is generated by plotting $|Q_{inner} - Q_{outer}| = 0$

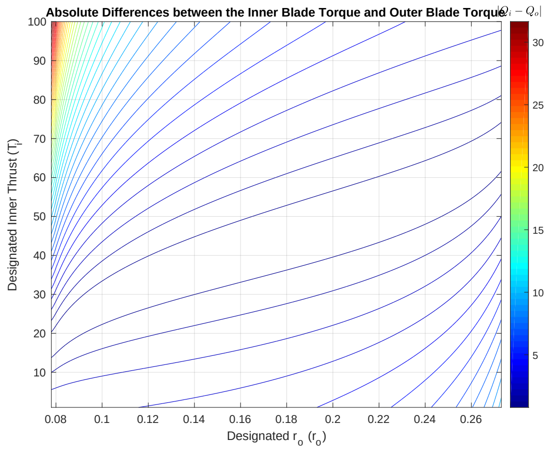

Figure 3: Contour plot of the absolute difference of inner and outer rotor torque.

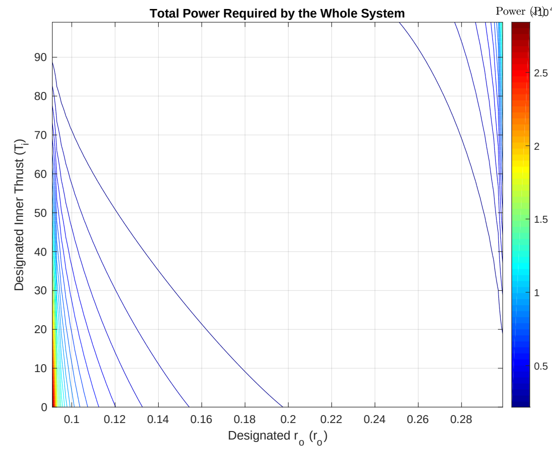

Figure 4: Contour plot of the total power required.

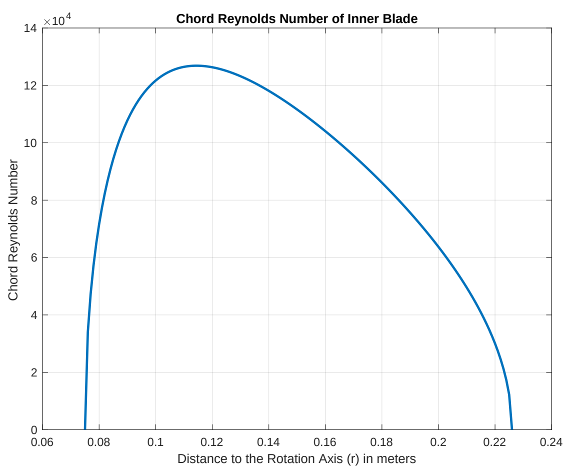

Figure 5: Chord Reynolds Number of inner blade.

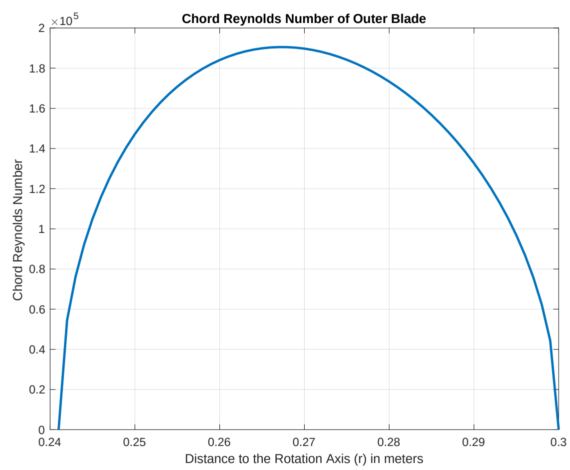

Figure 6: Chord Reynolds Number of outer blade.

as the z-axis, r_o as the x-axis, and T_i as the y-axis. The center diagonal space is where the total torque becomes zero. However, we also consider the required power to be minimized for our system. The power of the rotor required is given by: $P = \sqrt{\frac{T^3}{2\rho A_d}}$, where T is the thrust of the rotor and the A_d as the disk area. As presented in Fig. 4, a contour plot of total power ($P_{total} = P_{inner} + P_{outer}$) required is generated.

The power contour graph further limits our choice of r_o and T_i. We aimed at a balanced design, so we designated r_o as 0.226 [m] and T_i as 53 [N] as our final decision. R_i is than to be determined as 0.241 [m], and outer rotor thrust (T_o) is fixed to 47 [N].

We also checked the chord Reynolds number as demonstrated in Fig. 5 and Fig. 6. The chord Reynolds Number should be less than 200000 to ensure laminar flow. By inspecting the graph, our design satisfied the requirement of Reynolds Number less than 200000.

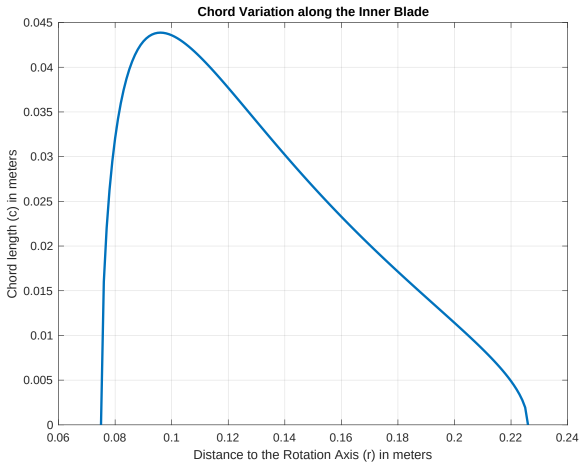

Figure 7: Chord length along the inner blade.

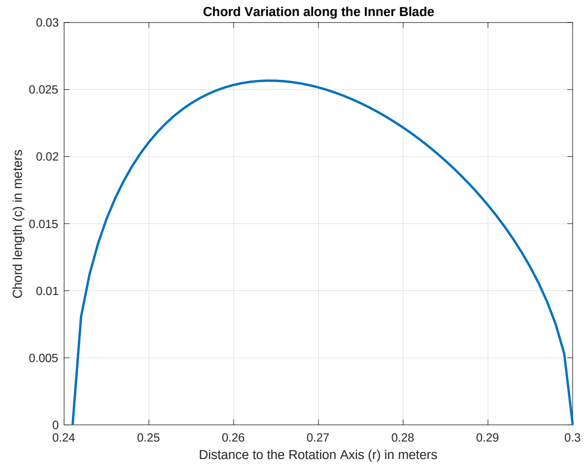

Figure 8: Chord length along the outer blade.

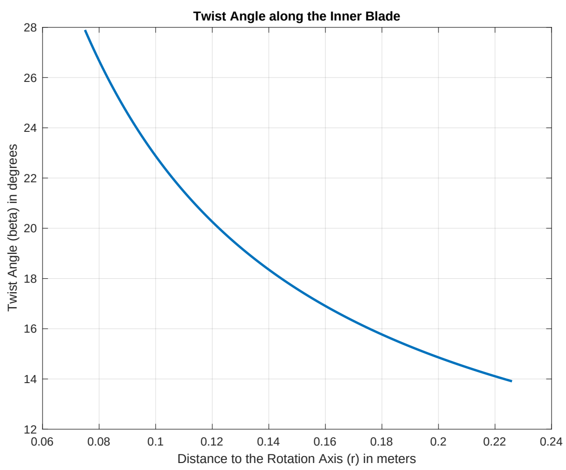

Figure 9: Twist angle along the inner blade.

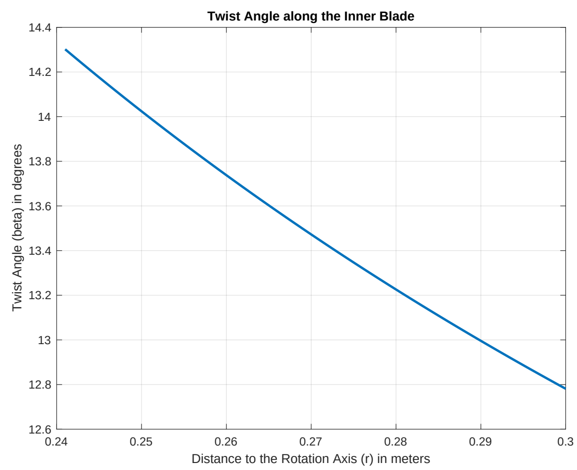

Figure 10: Twist angle along the outer blade.

3 Design

The stall angle for NACA 7512 airfoil is determined to be 7.7 degrees. As a result, we choose the angle of attack (α) to be 6.5 degrees to avoid flow separation. The varying chord length of the inner and outer blade are shown in Fig. 7 and Fig. 8. It is clear that the chord length converges to zero at the root and the tip. The maximum chord length for the inner blade can be found as 0.044 [m] and the maximum chord length for the outer blade can be determined as 0.026 [m]. The overall solidity is 0.9, which is below the 0.1 limit.

The twist angle of the inner blade is also presented in Fig. 9 and Fig. 10. We can clearly see that the twist angle decreases as the distance to the rotation axis increases. The reason is that ωr term in $tan(\phi) = V_h/(\omega r)$ gets larger at larger distance, making ϕ become smaller. With chord length and twist angle determined, we now have our final design. First we scale the NACA 7512 airfoil boundary points [4] by a factor of chord length at the distance from the rotation axis: $X_{scaled}(r) = Xl(r)$ and $Y_{scaled}(r) = Yl(r)$.

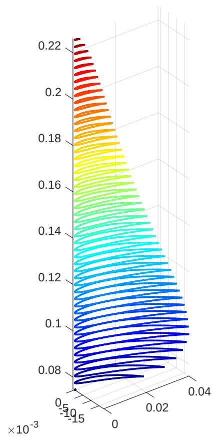

Figure 11: An isometric view of the inner blade.

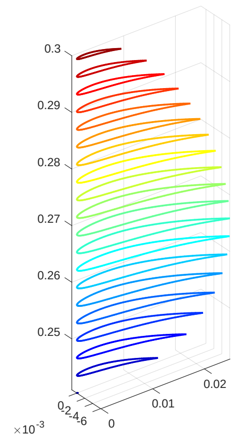

Figure 12: An isometric view of the outer blade.

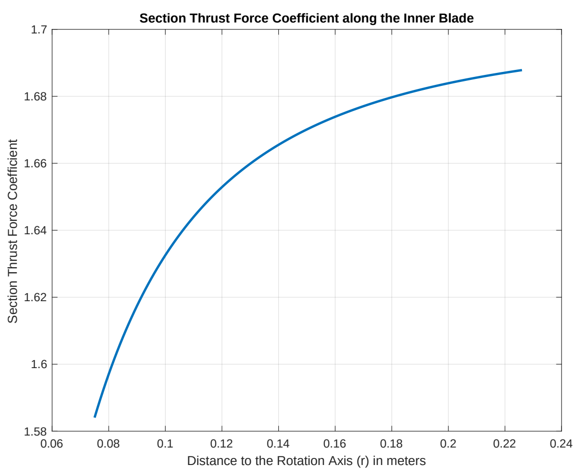

Figure 13: Section thrust force coefficient along the inner blade.

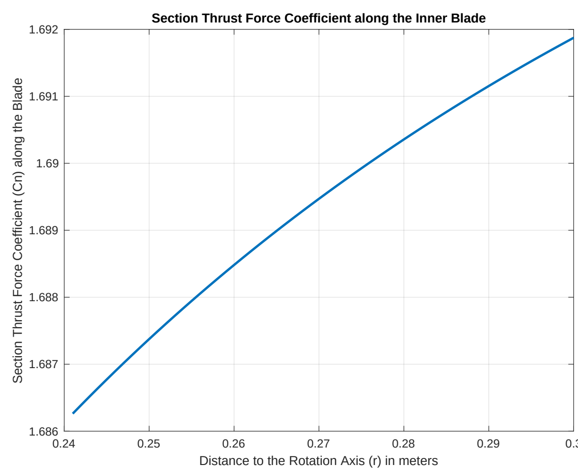

Figure 14: Section thrust force coefficient along the outer blade.

Then, we rotate the airfoil boundary points by $\beta(r)$:

$$X_{final} = X_{scaled}cos(\beta(r)) + Y_{scaled}sin(\beta(r)) \tag{8}$$

$$Y_{final} = -X_{scaled}sin(\beta(r)) + Y_{scaled}cos(\beta(r)) \tag{9}$$

X_{final} and Y_{final} are the profile boundary points for each cross section of the blade. These points are is plotted in three-dimensions for better visualization, as shown in Fig. 11 and Fig. 12.

The section thrust (which is normal to the disk area) force coefficient C_n is also presented in Fig. 13 and Fig. 14. The coefficient of section thrust is the maximum at the tip since ϕ is at the minimum at the tip. The section tangential force coefficient C_t is shown in Fig. 15 and Fig. 16. The graphs explain well that the maximum value of the coefficient occurs at the root of the blade as ϕ is the largest at the

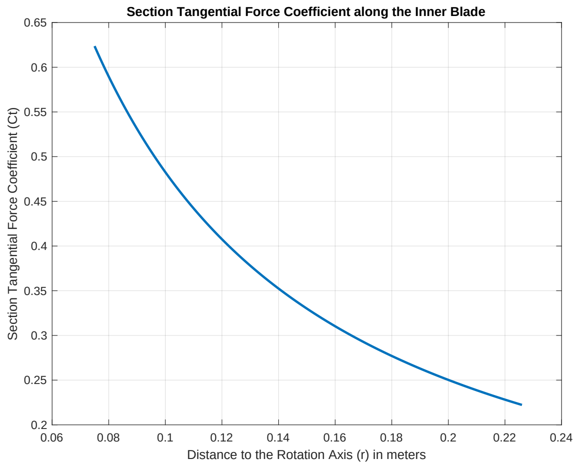

Figure 15: Section tangential force coefficient along the inner blade.

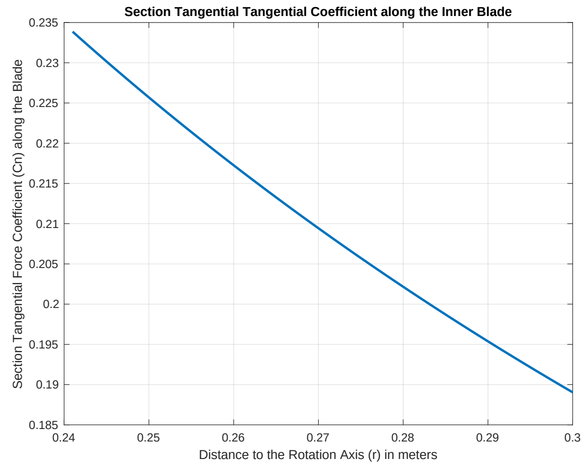

Figure 16: Section tangential force coefficient along the inner blade.

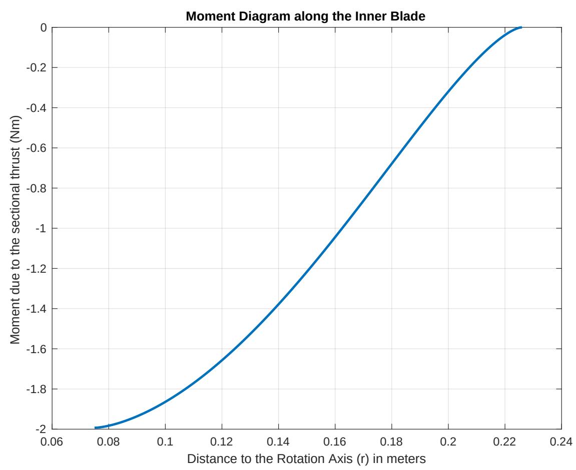

Figure 17: Moment distribution on the inner blade.

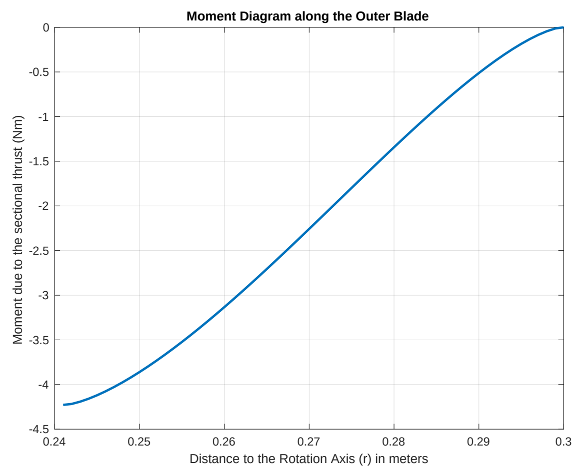

Figure 18: Moment distribution on the outer blade.

root, generating largest drag. By using Eq. **??** and Eq. **??**, we obtained the total torque for inner rotor (including all blades) as 1.5701 [Nm], and the total torque for the outer rotor as 1.5695 [Nm]. The values of both total torque are approximately equal to each other, and while the rotors rotate in opposite direction, the net torque of the system vanishes. For the rotor motor, the total torque required is the sum of inner and outer rotor torque together, which is 3.1396 [Nm].

The power for the inner rotor is calculated as 652.4 [Watts] and for the outer rotor is 650.1 [Watts] by using Eq. **??**. The total power for the system to sustain hover is determined as 1302.5 [Watts].

We also determined the bending due to the sectional thrust at r distance. We assume that the root of the blade is fixed, providing an reaction moment. The result is shown as a moment diagram in Fig. 17 and Fig. 18.

The center of the pressure (X_{cp}) is defined as : $X_{cp} = \frac{\int x C_p dx}{\int C_p dx}$. We found that $X_{cp} = 0.19$ numerically. X_{cp} helps us to determine the moment about the leading edge: $M_{LE} = X_{cp} L$, where L is the section lift

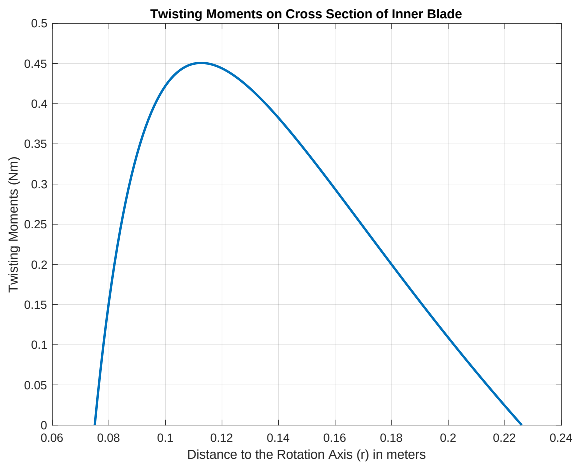

Figure 19: Twisting moment distribution along the inner blade.

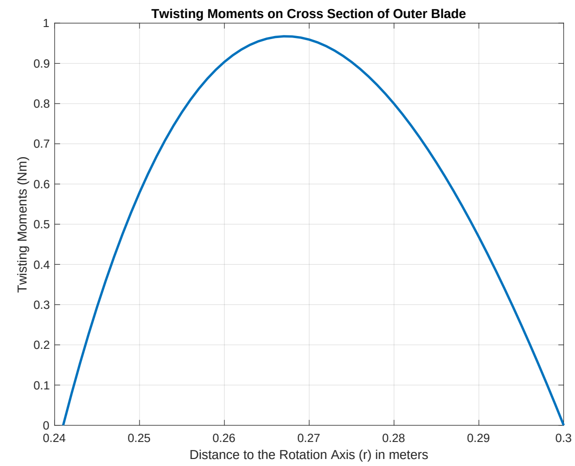

Figure 20: Twisting moment distribution along the outer blade.

for the blade. The moment about the leading edge is computed at each distance from the rotation axis on the blade and presented in Fig. 19 and Fig. 20.

4 Conclusion and Future Work

In this work, a co-axial co-planar rotor design is presented with necessary design parameters and shapes. We illustrate the shape of the the airfoil by scaling the NACA 7512 airfoil with our chord variation and rotate the section airfoil by our blade twist angle. We also verify our total net torque to be zero, no occurrence of flow separation, and below the chord Reynolds Number constraint. The section thrust force coefficient, tangential force coefficient, and required power are also calculated.

In the future, we will optimize our design by further decrease the volume of the blade and power for the rotor. For right now, our blade dimension is picked solely based on satisfying the design requirements and minimal power shown in the contour plot. We have not investigated the parameters such as outer blade radius, rotational speed, and blade numbers. By exploring these parameters, we might find a better design with minimal power and zero net torque. Furthermore, we will calculate our efficiency of the system and also explore its relation with the blade dimensions and thrust distributions on two rotors.

References

[1] Savaş, Ö. *ME 163 Notes*. UC Berkeley 2021.

[2] Ibarra, E. *ME 163 Summer 2021 Final Project Description*. UC Berkeley 2021.

[3] Kuethe, A. M. and Chow, C-Y. *Foundations of aerodynamics*. 5th edition. Wiley 1998.

[4] "NACA 7512 Airfoil" Airfoil Tools. http://www.airfoiltools.com/airfoil/ (accessed Aug. 13, 2021).

[5] I. R. Abbott and A. E. Von Doenhoff. *Theory of Wing Sections*. New York, NY, USA: Dover, 1959.

[6] Leishman G. *Principles of Helicopter Aerodynamics*. Cambridge, Cambridge Univerisity Press, 2000.