Simulation 3: 2024.11.26 - 2024.12.17

12.1 Capacitance Multiplier

Relevant resources:

(1) A Review of Capacitance Multiplication Techniques (https://ieeexplore.ieee.org/document/8678969)

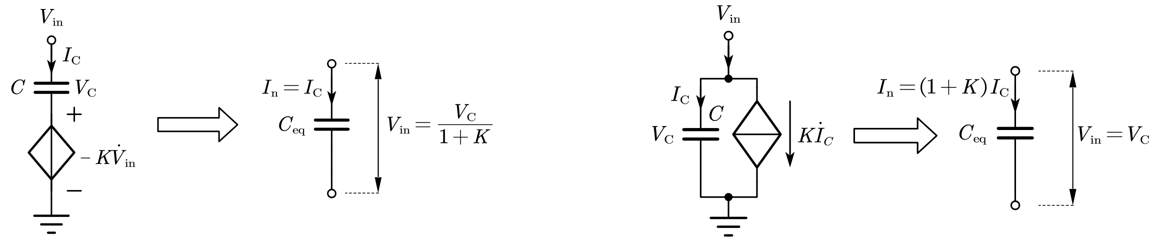
(2) Capacitance Multiplier with Large Multiplication Factor and High Accuracy (https://www.jstage.jst.go.jp/article/elex/15/3/15_15.20171191/_article)

(3) Active Capacitor Multiplier in Miller-compensated Circuits (https://ieeexplore.ieee.org/document/818917)

(4) The Capacitance Multiplier (https://audioxpress.com/article/the-capacitance-multiplier)

(1) Basic Circuits and Principles

Below are two basic concepts for capacitance multiplication:



(a) Voltage Mode **(b) Current Mode**

Figure 12.1: Basic Two Concepts for Capacitance Multiplier

Thus, we obtain the equivalent capacitance as:

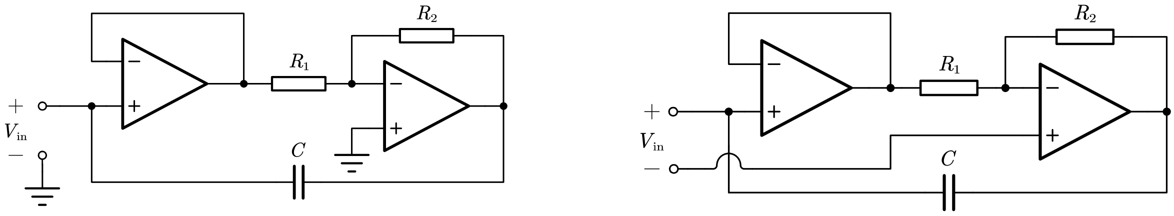
$$\text{voltage mode:}\quad C_{eq} = \frac{I_n}{SV_n} = \frac{I_C}{S\frac{V_C}{1+K}} = (1+K)\,C, \quad K > 0 \tag{12.1}$$

$$\text{current mode:}\quad C_{eq} = \frac{I_n}{SV_n} = \frac{(1+K)I_C}{SV_C} = (1+K)\,C, \quad K > 0 \tag{12.2}$$

A simple implementation of cap multiplier, depicted in Fig.1.2 (a), combining a unit-gain buffer (voltage fllower) and an inverting amplifier, uses a voltage mode. yielding the equivalent capacitance:

$$A = -\frac{R_2}{R_1} = -K \Longrightarrow C_{eq} = \left(1 + \frac{R_2}{R_1}\right) C \tag{12.3}$$

where $A = -\frac{R_2}{R_1}$ is the closed-loop gain of the inverting amplifier. Since inverting amplifier has a low input impedance, the unit-gain buffer is a necessary. To change it into a two-terminal element, just replace GND with the negtive terminal of the input voltage, e.g. $V_{in,-}$, as shown in Fig.1.2 (b).



(a) One-terminal Cap Multiplier **(b) Two-terminal Cap Multiplier**

Figure 12.2: A Simple Implementation of Capacitance Multiplier

(2) Multisim Simulation

Considering Figure 1.2 (b), set the parameters as Table 1.1. Then we connect it to the RC series circuit to perform an AC sweep to test the capacitance value. The Simulation Circuit is shown in Figure 1.3.

Table 12.1: Simulation Parameters of Capacitance Multiplier

C	R_1	R_2	Operation Amplifier
10 nF	1 KΩ	11 KΩ	LM258P

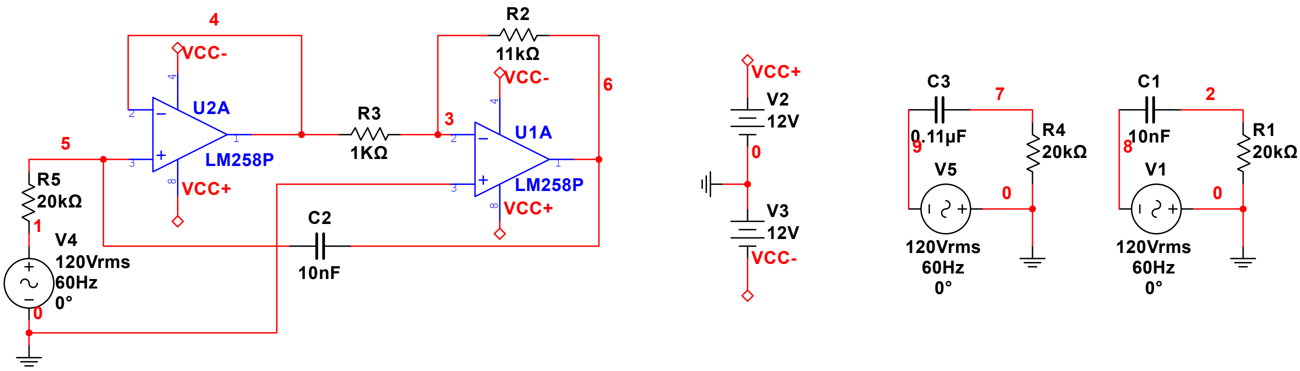


Figure 12.3: Simulation Circuit of Cap Multiplier

Export the simulation data and plot the frequency response (bode plot) of the series RC circuit, as shown in Figure 1.4. The theoretical value of the capacitance of the cap multiplier is 110 nF, and the simulation result confirmed this point.

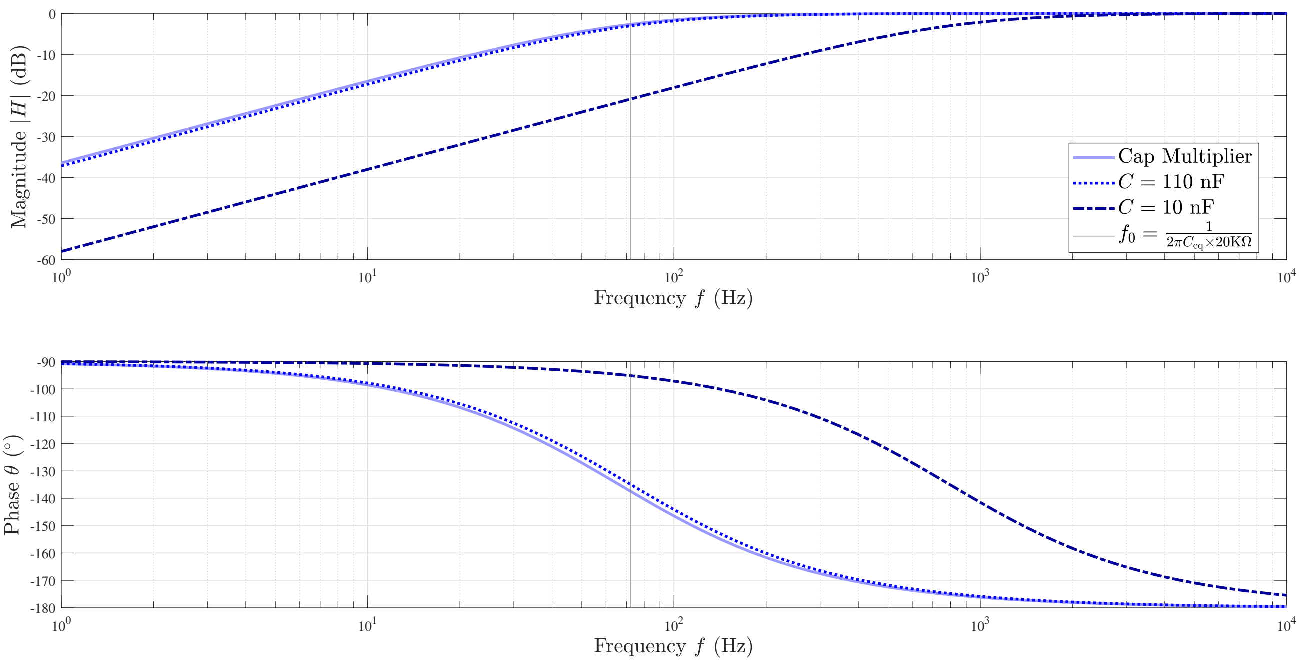


Figure 12.4: AC Sweep of the Cap Multiplier

12.2 The Wien Bridge Oscillator

You must have seen that a number of resistors and capacitors can be connected together with an inverting amplifier to produce an oscillating circuit. Wien bridge oscillator is one of the simplest sine wave oscillators which uses an RC network in place of the conventional LC tuned tank circuit to produce a sinusoidal output waveform.

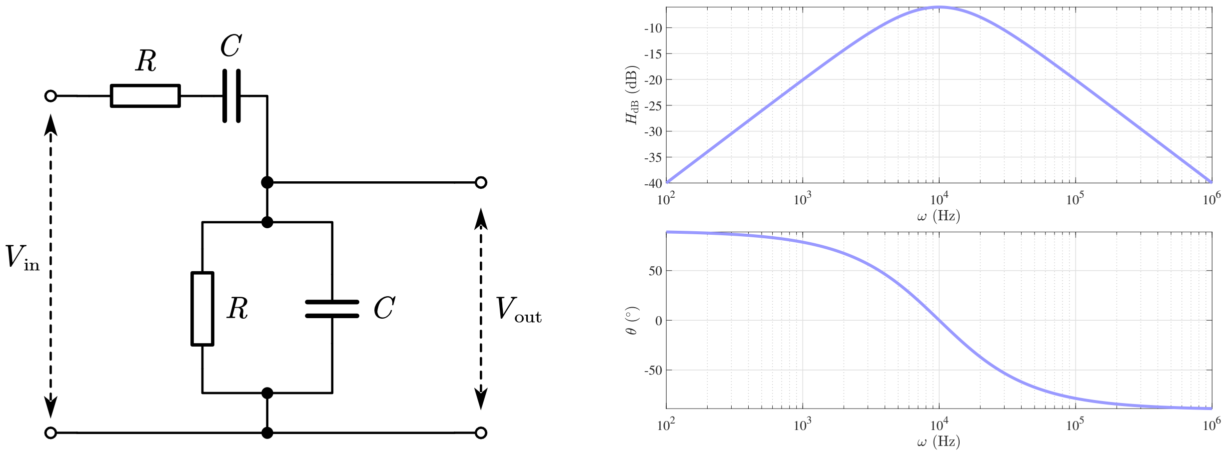
The Wien Bridge Oscillator is based on a noninverting amplifier, using a series RC circuit connected with a parallel RC of the same component values as a feedback circuit.

(1) Basic Circuit and Principles

Consider the RC circuit in Figure 1.5 (a), the voltage gain H of the series RC circuit is:

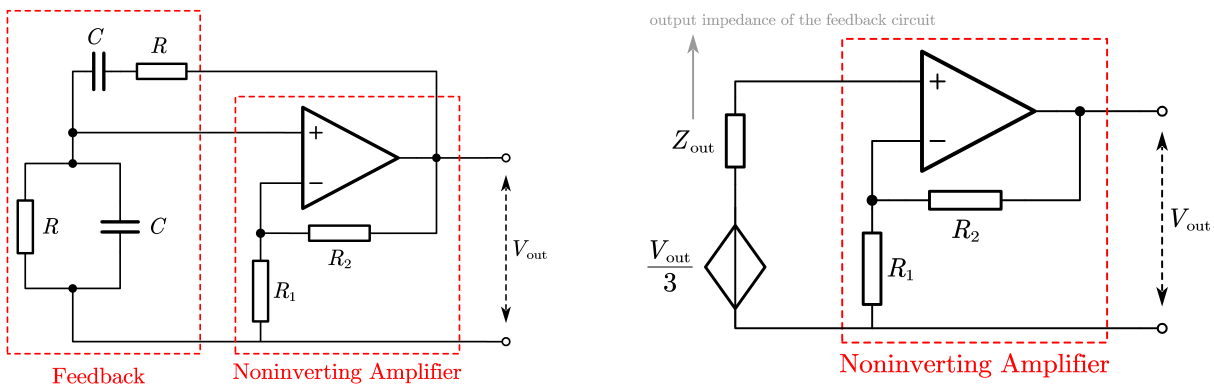
$$H(j\omega) = \frac{R \parallel (\frac{1}{j\omega C})}{R + \frac{1}{j\omega C} + R \parallel (\frac{1}{j\omega C})} = \frac{1}{1 + \frac{(R^2 - \frac{1}{\omega^2 C^2}) + \frac{2R}{j\omega C}}{\frac{R}{j\omega C}}}, \quad H|_{\omega = \frac{1}{RC}} = \frac{1}{1 + \frac{\frac{2R}{j\omega C}}{\frac{R}{j\omega C}}} = \frac{1}{3} \tag{12.4}$$

Defined to obtain a 0° phase shift, the resonant frequency f_0 is the key to Wien bridge oscillator. And at the point we have $H = \frac{1}{3}$. Let's set $R = 10\ \mathrm{K\Omega}$, $C = 10\ \mathrm{nF}$ and sketch the bode plot of this RC circuit in Figure 1.5 (b).



(a) RC Feedback Circuit **(b) Bode Plot of the RC Circuit**

Figure 12.5: Wien Bridge Oscillator's Feedback Circuit



(a) Wien Bridge Oscillator **(b) Equivalent Circuit of Wien Bridge Oscillator**

Figure 12.6: Wien Bridge Oscillator and Its Equivalent Circuit

Since a noninverting amplifier has extreme high input impedance and low output impedance, the coupling effect of the two circuits is negligible. In other words, the output impedance of noninverting amplifier (combing with the input impedance of feedback circuit), and the output impedance of feedback circuit (combing with input impedance of noninverting amplifier) can be ignored. Thus, the Wien bridge oscillator, depicted in Figure 1.6 (a), has the equivalent circuit shown in Figure 1.6 (b).

The oscillation frequency f_0 of the Wien Bridge Oscillator is given by:

$$\omega_0 = \frac{1}{RC}, \quad f_0 = \frac{1}{2\pi RC} \tag{12.5}$$

As the voltage gain of noninverting amplifier is:

$$A_v = 1 + \frac{R_2}{R_1} \tag{12.6}$$

yielding **the start-oscillation condition**:

$$A_v > 3 \Longleftrightarrow R_2 > 2R_1 \tag{12.7}$$

Assuming R_2 is slightly greater than $2R_1$, and there is a noise signal consists of a series of frequency, including $f_0 = \frac{1}{2\pi RC}$. Then at the selected resonant frequency f_0, $\frac{1}{3}A_v > 1$, so the positive feedback will cancel out the negative feedback signal, causing the circuit to oscillate, until it reaches a voltage saturation (dependent on power supply). However, at the other frequency, $\frac{1}{3}A_v < 1$ so the negative feedback will cancel out the positive, resulting other frequency signal fading away.

The closer the ratio $\frac{R_2}{R_1}$ is to 2^+, the better the waveform, but the longer the start-up time. Define $a = \frac{1}{3}\left(1 + \frac{R_2}{R_1}\right)$ as the periodic gain, a not-bad approximation for the start-up time is:

$$t_{\text{start}} = \frac{1}{f} \log_{a^3}\left(\frac{V_{\text{limit}}}{V_{\text{noise}}}\right) - 0.02 \tag{12.8}$$

where the unit of t is seconds, V_{limit} is the limit amplitude of output voltage, V_{noise} is the amplitude of noise.

By the way, if R_2 exceeds $2R_1$ too much, for example $R_2 = 3R_1$, the output waveform will be seriously distorted. Also, due to the slew rate limitations of operational amplifiers, frequencies above 1 MHz are unachievable without the use of special high frequency op-amps.

(2) Multisim Simulation of the Basic Circuit

Set the parameters in Figure 1.6 (a) as below, and run the simulation.

Table 12.2: Simulation Parameters of Wien Bridge Oscillator

R	C	R_1	R_2	Operation Amplifier	VCC
10 KΩ	10 nF	10 KΩ	20.1 KΩ	LM258P	± 12 V

The start-up time is about 570 ms, shown in Figure 1.7.

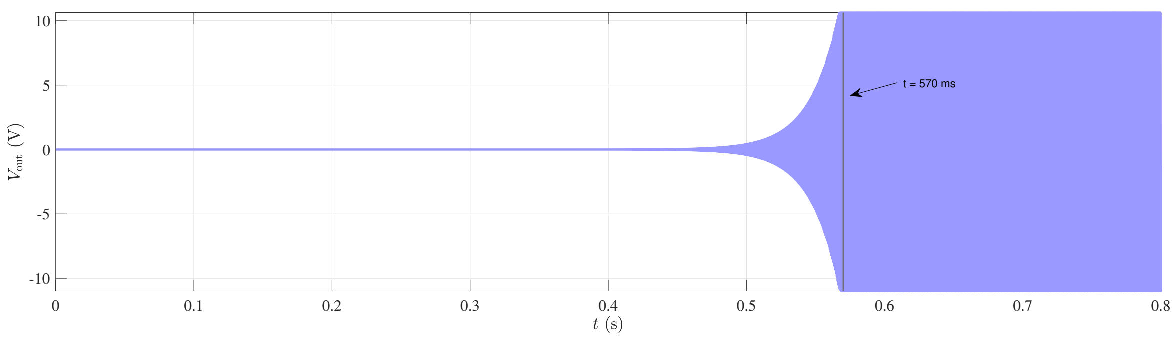


Figure 12.7: Start-up Time of Wien Bridge Oscillator

Export the simulation data, and perform a spectrum and distortion analysis in Matlab. Then we obtain the waveform and spectrum shown in Figure 1.8.

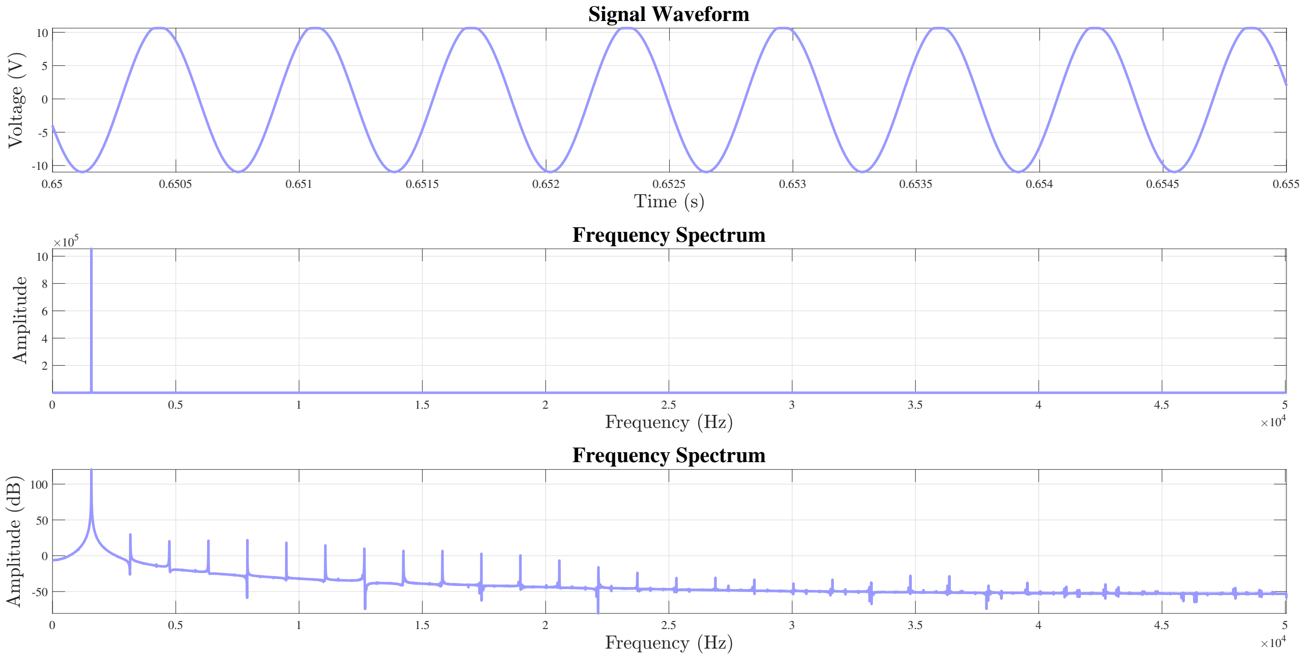


Figure 12.8: Spectrum Analysis of the Simulation Circuit

As we can see, the main output waveform is a sine wave at the resonant frequency f_0, the simulated oscillation frequency is:

$$
\begin{aligned}
f_{\text{simu}} &= 1.5758 \text{ KHz} \\
f_{\text{theo}} &= 1.5915 \text{ KHz} \\
\eta = \frac{f_{\text{simu}} - f_{\text{theo}}}{f_{\text{theo}}} &= -0.98\ \%
\end{aligned}
\tag{12.9}
$$

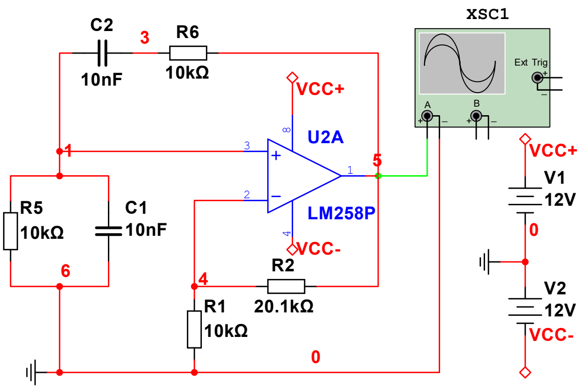
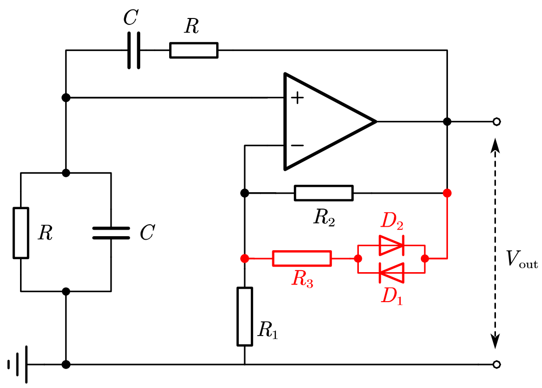


Figure 12.9: Simulation Circuit of Wien Bridge Oscillator

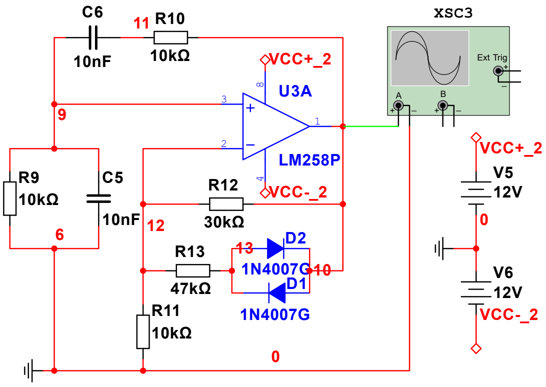
(3) Decrease the Start-up Time

We have noted that the output waveform is distorted when R_2 exceeds $2R_1$ too much, but the start-up time is too long when R_2 is too closed to $2R_1$. Therefore, we need to optimize the circuit to achieve a better waveform and shorter start-up time, exemplified in Figure 1.10.

In Figure 1.10 (a), we added a resistance R_3 and two diodes D_1 and D_2. When the output voltage amplitude is less than the threshold voltage of diodes V_{D}, the diodes are off, and the circuit is the same as the basic circuit. When the output is greater than V_{D}, the diodes are on, and the resistance of R_3 is added to the circuit (parallel with R_2), which reduces the gain of amplifier.



(a) Optimized Circuit



(b) Simulation Circuit

Figure 12.10: Optimize the Start-up Time of Wien Bridge Oscillator

Simulation circuit is shown in Figure 1.10 (b), the start-up time is reduced to about 10 ms (see Figure 1.11), and the output waveform is shown in Figure 1.12.

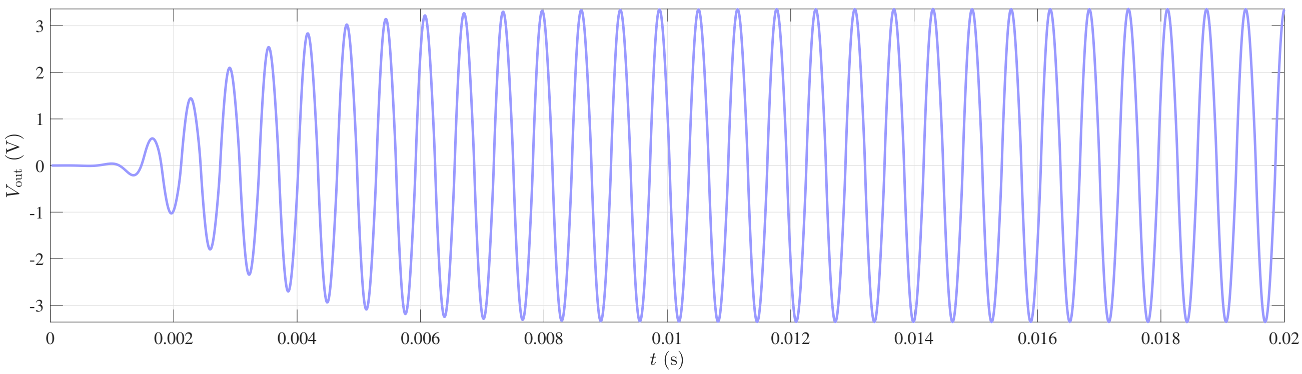


Figure 12.11: Optimized Start-up Time

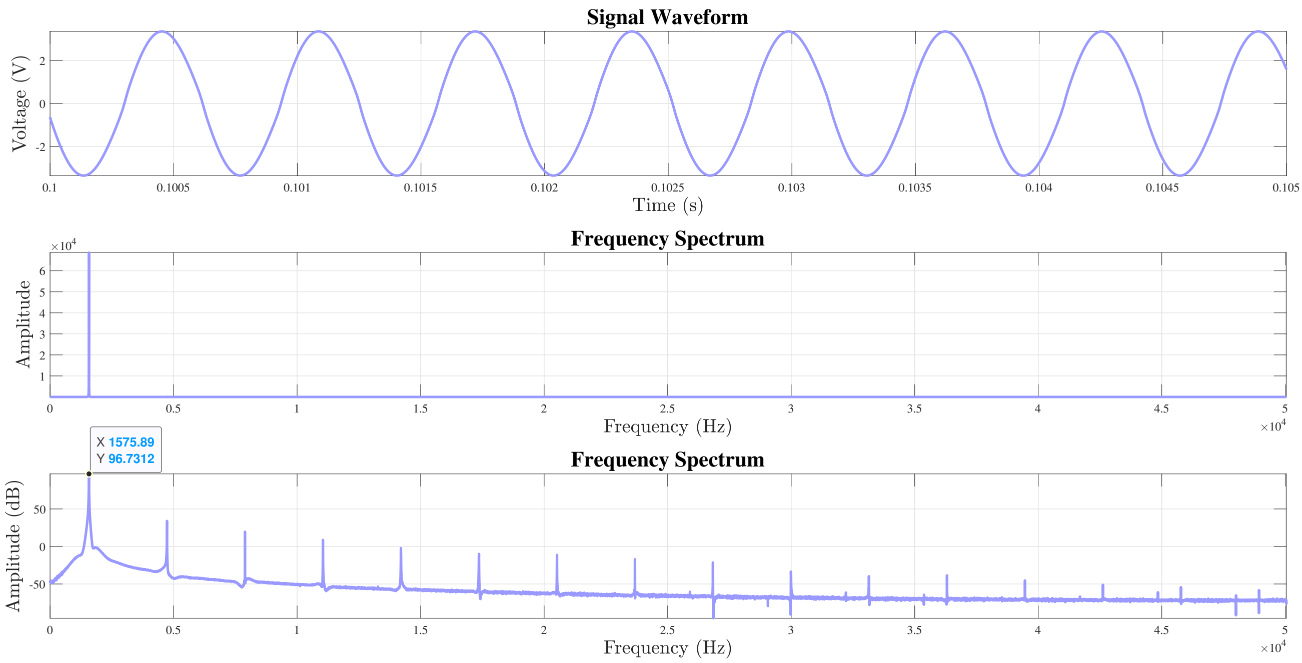


Figure 12.12: Spectrum Analysis of Optimized Circuit

Although the output frequency still focuses on f_0 and the distortion completely disappears, we have to note that the amplitude of the output waveform is significantly reduced. If larger amplitudes are desired, a resistor can be added to divide V_{out} to a suitable level, leading to a larger output amplitude. Refer to *Fundamentals of Microelectornics* (3rd edition) (Behzad Razavi) page 661 for more detials. For more alternative methods to optimize the start-up time, see https://blog.csdn.net/qq_29356039/article/details/132611987.

(4) Generate a Square Wave

An R_2 greater than $2R_1$ will result in the output waveform being clipped at the output voltage limitations. In other words, if we let $R_2 \gg 2R_1$, the waveform becomes a square wave. To prove our surmise, reset $R_1 = 1\ \text{K}\Omega$, $R_2 = 30\ \text{K}\Omega$ in Table 1.2, without changing the other parameters. We obtain the output waveform shown in Figure 1.13.

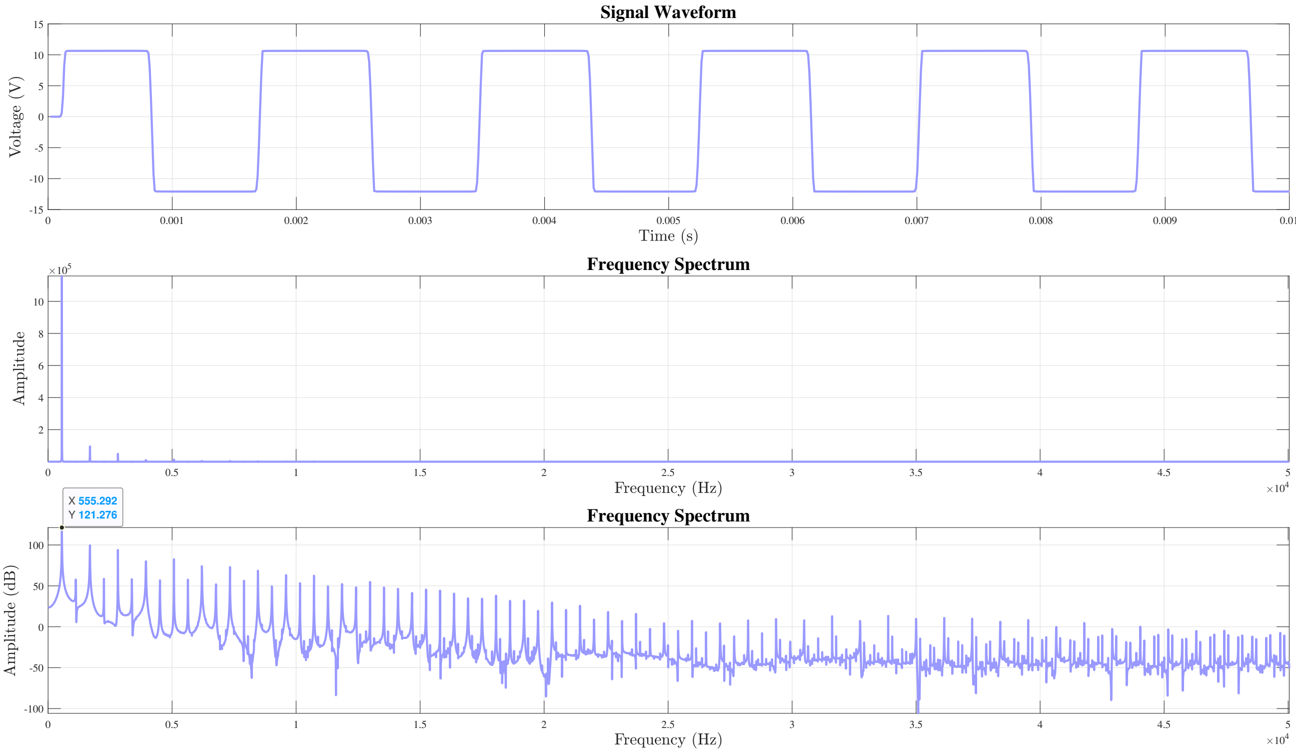


Figure 12.13: Square Wave Output of Wien Bridge Oscillator

Since the frequency of the square wave is difficult to calculate and control, the Wien Bridge Oscillator is not suitable for generating square waves in practical applications. To obtain an output signal with DC offset, see https://blog.csdn.net/qq_29356039/article/details/132611987 for more details.