

电路原理课程作业、仿真、实验与设计
Homework, Simulation, Laboratory and Design to Accompany
Principles of Electric Circuits

丁毅

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序言

本文为笔者本科时的“电路原理”课程作业、仿真、实验与设计合集(Homework, Simulation, Laboratory and Design to Accompany Principles of Electric Circuits, 2024.9-2025.1)。所有作业课件(包括Homework, Sim, Lab and Design)已上传到网址<https://www.123865.com/s/0y0pTd-R8Kj3>。读者可在笔者的个人网站上找到课程信息、教材、教辅和作业答案等相关资料YiDingg's Website。

由于个人学识浅陋,认识有限,文中难免有不妥甚至错误之处,望读者不吝指正。读者可以将错误发送到我的邮箱dingyi233@mails.ucas.ac.cn,也可以到笔者的GitHub(<https://github.com/YiDingg/LatexNotes>)上提issue,衷心感谢。

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PART I

HOMEWORK, SIMULATION AND

LABORATORY

Homework 01: 2024.08.27 - 2024.09.02

1.1 习题集 1-2: 求题图各电路中的电压 U 和电流 I

- (a) 短路, 因此 $U = 0$, $I = \frac{U_S}{R_i}$
 (b) 开路, 因此 $U = U_s$, $I = 0$
 (c) 构成回路, 因此 $U = \frac{U_S R}{R + R_i}$, $I = \frac{U_S}{R + R_i}$

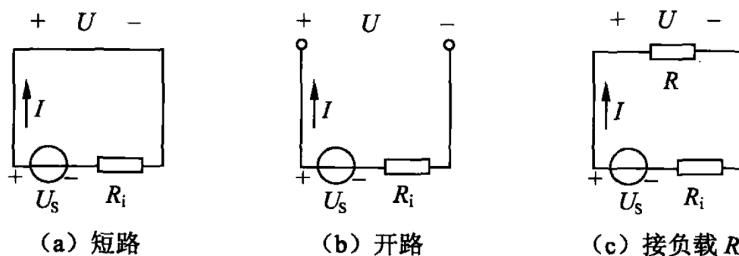


图 1.1: 1.1 习题集 1-2

1.2 习题集 1-9: 求题图 (a) 中的电压 U_{ab} , 图 (b) 中的电阻 R , 图 (c) 中的电压 U_S 和图 (d) 中的电流 I

- (a) $\varphi_a - 3 \text{ V} + 2 \text{ V} = \varphi_b \implies U_{ab} = 1 \text{ V}$
 (b) $I = 1 \text{ A}, 3 - IR = -4 \implies R = 7 \Omega$
 (c) $-3 + U_S = 1 \implies U_S = 4 \text{ V}$
 (d) $R = 2 \Omega, -IR + 2 = 3 \implies I = -0.5 \text{ A}$

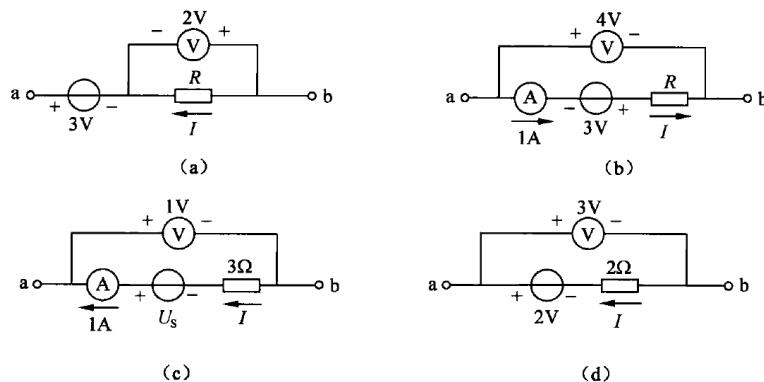


图 1.2: 1.2 习题集 1-9

1.3 习题集 1-10: 求题图电路中的电压 U_{ab}

- (a) 记参考点 a 的电势 $\varphi_a = 0$, 则 $\varphi_c = 2 \text{ V}$, $\varphi_b = -2 \text{ V}$, 因此 $U_{ab} = 2 \text{ V}$
 (b) 记参考点 d 的电势 $\varphi_d = \varphi_b = 0$, 则 $\varphi_c = 6 \text{ V}$, $\varphi_a = -2 \text{ V}$, 因此 $U_{ab} = -2 \text{ V}$

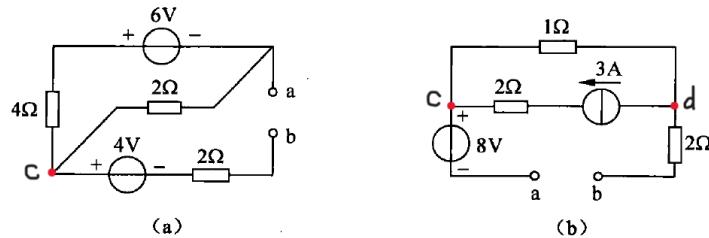


图 1.3: 1.3 习题集 1-10

后补: (b) 中电流源两端仍有电势差, $\varphi_c \neq 6\text{ V}$ 而是 $\varphi_c = -3\text{ V}$, 最终得 $U_{ab} = -5\text{ V}$ 。

1.4 习题集 1-15: 求题图各电路汇总所标出的电压和电流

- (a) $I = -\frac{U}{R} + 4\text{ A} = -2\text{ A}$
- (b) $U = 12\text{ V} + 3\Omega \times 4\text{ A} = 0$
- (c) $I = 8\text{ A} - 6\text{ A} = 2\text{ A}$, $U = 12\text{ V} + 3 \times 8\text{ V} = 36\text{ V}$
- (d) 取点 d 为参考点, 则 $\varphi_d = \varphi_c = 0$, $\varphi_b = \varphi_a = 9\text{ V}$, 于是 $U_1 = 9 + 2 \times 3 = 15\text{ V}$, $U_2 = 9 + 2 \times 2 = 13\text{ V}$, $I = 2 - (9 - 3) = -4\text{ A}$

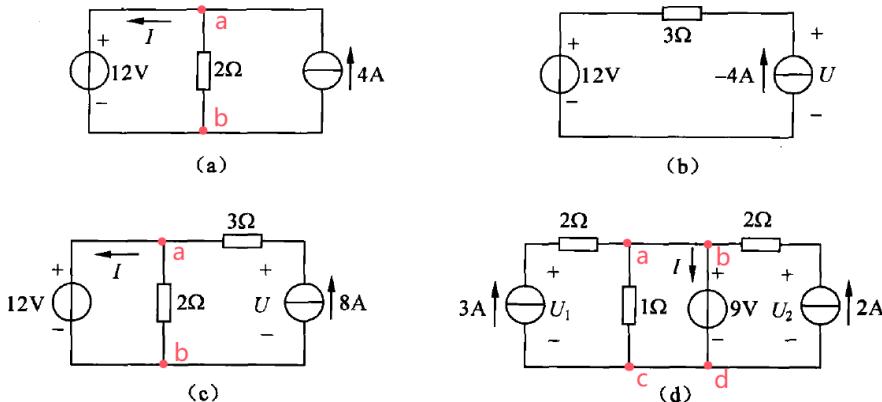


图 1.4: 1.4 习题集 1-15

1.5 习题集 1-29: 题图电路中流过 40Ω 电阻的电流为 2A, 求电流源的电流值 I_S

取点 a 为参考点 $\varphi_a = 0$, 可得 $\varphi_b = 100U_1 - 80$, 于是在结点 a 有电流:

$$I_S + \frac{100U_1 - 80}{5} = 2$$

0.2Ω 电阻处又有 $U_1 = 0.2I_S$, 联立解得 $I_S = 3.6\text{ A}$, $U_1 = 7.2\text{ V}$ 。

1.6 习题集 1-30: 求题图电路中独立电源的功率

这里要注意左二元器件是受控电流源, 因此 $0.5U$ 是指电流大小而非电压。 I_1 处可列出方程:

$$\frac{U}{2} + 12 - \frac{U}{3} = 0.5U \implies U = 36\text{ V} \implies P = UI = 432\text{ W}$$

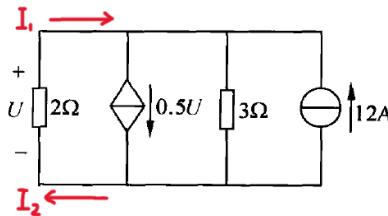
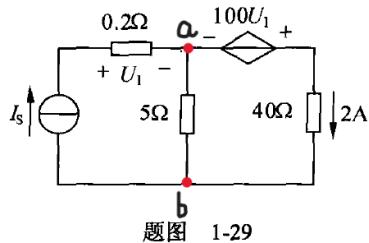


图 1.5: 1.4 习题集 1-29 和 1.5 习题集 1-30

后补: 上面的方程列错了, 错将 I_1 的方向标为由左向右, 应该是由右向左。最后得到 $P = 108 \text{ W}$ 。另外, 也可以直接将受控电流源看作是 2Ω 的电阻, 这样左侧三个电阻并联, 也可求出正确答案 108 W .

1.7 讲义题 1-6: 关联参考方向下, 电阻的 $\alpha > 90^\circ$ 代表什么物理意义

$\alpha > 90^\circ$ 时, 电阻为“负电阻”。

1.8 讲义题 1-7: 充电电池的 1 C 是什么意思, 涓流充电是多少 C, 快速充电是多少 C

充放电倍率 C 的含义:

C (充放电倍率) 表示电池充放电时电流相对电池容量的大小数值, $C = \frac{\text{电池容量}}{\text{充放电所需时间}}$ 。例如, 1 C 电流充电表示电池需要 1 小时充满, 5 C 充电表示电池需要 0.2 小时充满。放电也是类似的, 一个 10 Ah 的电池以 2 C 放电, 表示以 20 A 的电流放电 0.5 h。

若倍率上升, 总时间就会下降, 若倍率下降, 总时间就会上升。通俗来讲, C 代表了电池的爆发力大小, 高倍率的动力电池瞬间放电电流大, 特别适合大电流放电产品使用, 如航模。

涓流充电:

涓流充电是指在电池接近完全充满电后, 采用非常小的电流进行充电, 以弥补电池自放电造成的容量损失。理论倍率 C 约为最大倍率 C_{\max} 的 $\frac{1}{100}$ 至 $\frac{1}{1000}$, 但由于倍率太小, 常常根本无法充电, 一个比较好的方法是脉冲式充电, 例如以 $\frac{C_{\max}}{10}$ 充电 6 s, 然后停止充电 54 s。

快速充电:

快速充电至少要求 1 C, 现阶段的快速充电多在 1.5 C 至 2 C 之间。

1.9 讲义题 1-8 (Multisim 仿真): 用 Multisim 实现课堂仿真的 MOSFET。画出 U_{GS} 固定为 5 V, U_{DS} (横轴) 从 0 V 到 12 V 变化时 I_{DS} (纵轴) 的曲线; 以及 U_{DS} 固定为 10 V, U_{GS} (横轴) 从 0 至 10 V 变化时 I_{DS} (纵轴) 的曲线

仿真电路如图 1.6 所示,

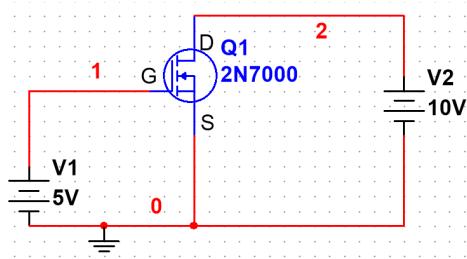


图 1.6: 仿真电路图

先固定 $U_{GS} = 5 \text{ V}$ 不变 (即 $V_1 = 5 \text{ V}$), 横坐标 $U_{DS} \in [0 \text{ V}, 12 \text{ V}]$, 画出 I_{DS} (即 I_2) 的变化曲线, 如图 1.7 所示。再固定 $U_{DS} = 10 \text{ V}$ 不变 (即 $V_2 = 10 \text{ V}$), 横坐标 $U_{GS} \in [0 \text{ V}, 10 \text{ V}]$, 画出 I_{DS} (即 I_2) 的变化曲线, 如图 1.8 所示。

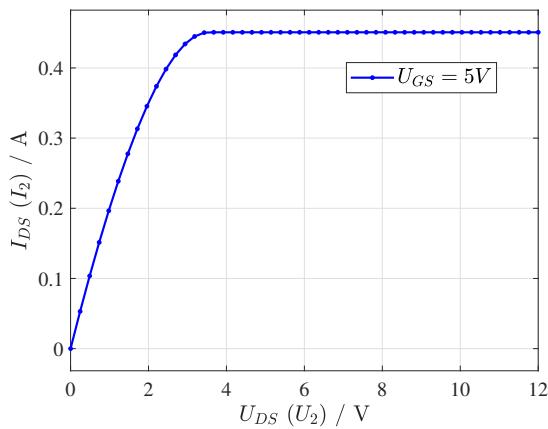


图 1.7: 仿真结果 1

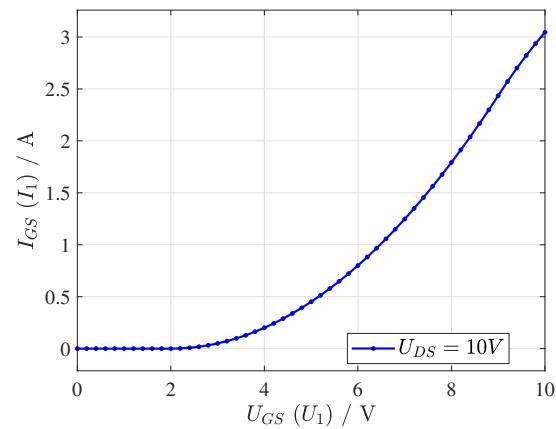


图 1.8: 仿真结果 2

Homework 02: 2024.09.03 - 2024.09.09

2.1 习题集 1-33: 题图电路中 $R_1 = 40$, $R_e = 27$, $R_b = 150$, $R_L = 1500$, 单位都为 Ω , $\alpha = 0.98$, 求电压增益 $\frac{u_2}{u_1}$ 和功率增益 $\frac{p_2}{p_1}$, 其中 p_1 是 u_1 输出的功率, p_2 是 R_L 吸收的功率

左半边回路有:

$$u_1 - 67i_e - (1 - \alpha)i_e \cdot 150 = 0 \implies \frac{u_2}{u_1} = \frac{\alpha i_e R_L}{70 i_e} = \frac{0.98 \times 1500}{70} = 21$$

$$p_2 = (\alpha i_e)^2 R_L, \quad p_1 = u_1 i_e \implies \frac{p_2}{p_1} = \frac{(\alpha i_e)^2 R_L}{70 i_e^2} = 20.58$$

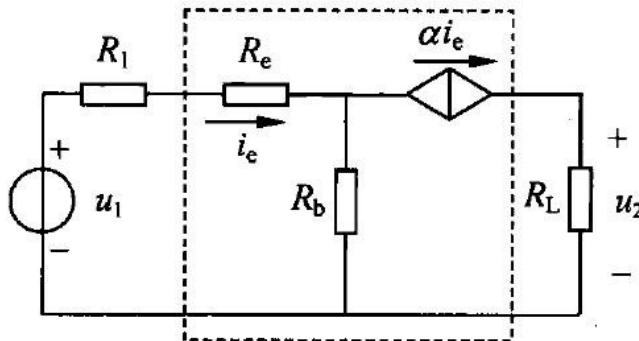


图 2.1: 习题集 1-33

2.2 习题集 2-2: 求题图各电路的入端电阻 R

对图 (a), 化简并联后电桥平衡, 可以得到

$$\frac{1}{R} = \frac{1}{20} + \frac{1}{40} + \frac{1}{40} \implies R = 10 \Omega$$

对图 (b), 经过多次并联化简, 可以得到:

$$R = 8 + \frac{3 \times 6}{3 + 6} = 10 \Omega \quad (2.1)$$

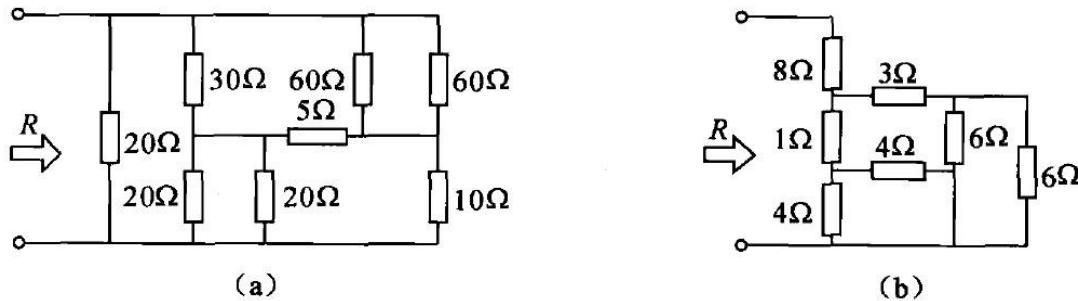


图 2.2: 习题集 2-2

2.3 习题集 2-6: 将题图中各电路化为最简电路

各电路的最简电路图如图 2.4 所示:

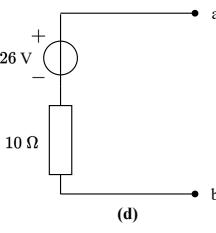
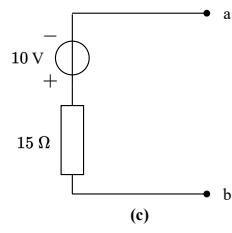
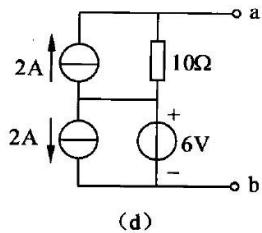
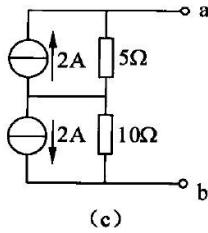
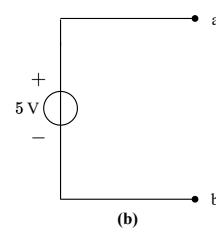
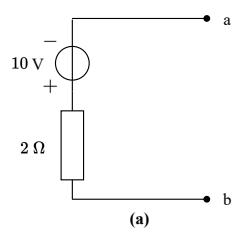
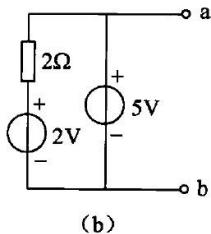
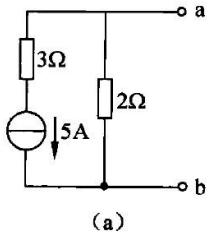


图 2.3: 习题集 2-6

图 2.4: 习题集 2-6 解答

2.4 习题集 2-8: 用电源等效方法求题图中的电流 i

对原电路进行多次等效转换, 得到最简电路如图所示, 进而有:

$$I = \frac{3}{2 + 3 + 5} = 0.3 \text{ A}$$

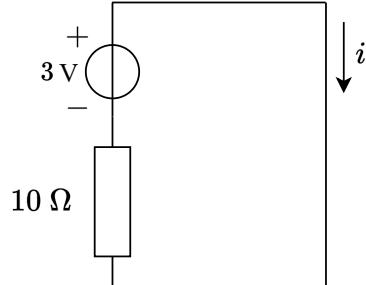
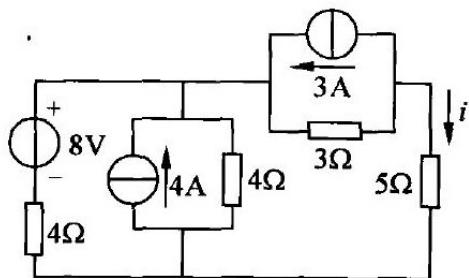


图 2.5: 习题集 2-8

图 2.6: 习题集 2-8 等效电路

2.5 习题集 2-11: 求题图中的电流 I

等效电路图如图 2.8 所示, 由 KVL 得:

$$28 = 4I' + 4(I' - I), \quad 25 = -8I + 4(I' - I) \implies I' = 2.95 \text{ A}, I = -1.1 \text{ A}$$

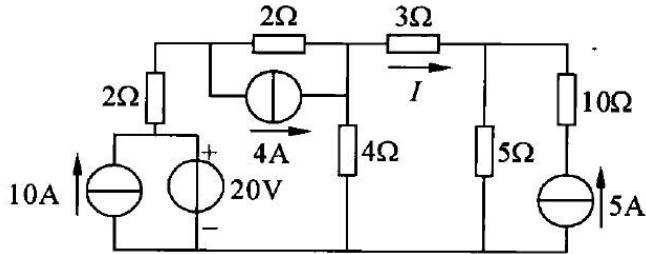


图 2.7: 习题集 2-11

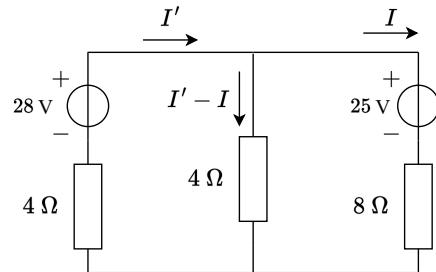


图 2.8: 习题集 2-11 等效电路

2.6 习题集 2-17: 求题图中的电压 U_{ab}

等效电路图如图 2.10 所示, 可以求得:

$$4I - 8 = 12(I - 1) \implies I = 0.5 \text{ A} \implies U_{ab} = 8 - 8I = 4 \text{ V}$$

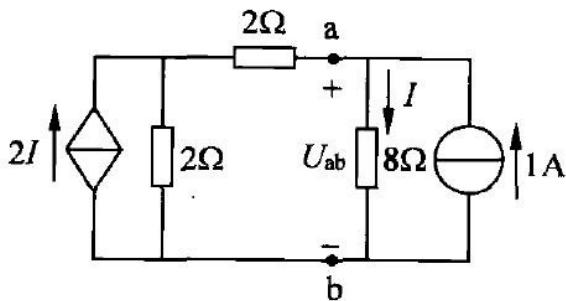


图 2.9: 习题集 2-17

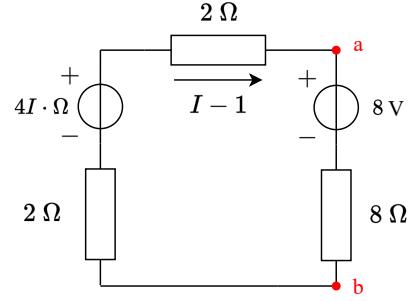


图 2.10: 习题集 2-17 等效电路

2.7 习题集 2-22: 求题图中的电压 U_1 , U_2 和电流源发出的功率

经过电源等效和 Δ -Y 变换, 等效电路图如图 2.12 所示, 回路总电阻 $R = 3 + \frac{4}{9} + \frac{14}{9} = 5 \Omega$, $I_1 = \frac{U}{R} = 1.2 \text{ A}$, 则有:

$$U_1 = 6 - 3 \times 1.2 = 2.4 \text{ V}, U_2 = 2 \times \frac{I}{2} = 1.2 \text{ V}, P = 2U_1 = 4.8 \text{ W}$$

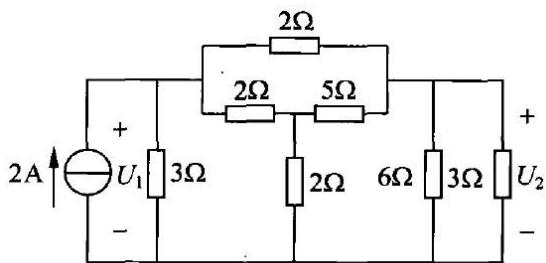


图 2.11: 习题集 2-22

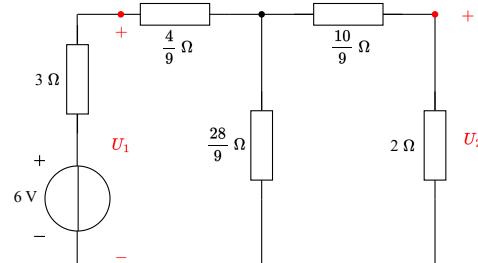


图 2.12: 习题集 2-22

Homework 03: 2024.09.10 - 2024.09.18

3.1 习题集 3-40 (书上答案不正确): 题图电路中, $u_s(t) = \sin 4t \text{ V}$, 电阻 $R_2 = 2R_1 = 1 \text{ k}\Omega$, 求电流 $i(t)$

由虚短和虚断, 可以得到 R_1 处电流为 $i_1 = \frac{u_s}{R_1}$ (从上至下), 于是输出电压 $u_o = 3u_s$, 右侧负载由三个电阻构成, 并联电阻分压 $2u_s$, 最后得电流 $i(t)$:

$$i(t) = \frac{2u_s}{6 \text{ k}\Omega} = \frac{u_s}{3} \text{ mA} \quad (3.1)$$

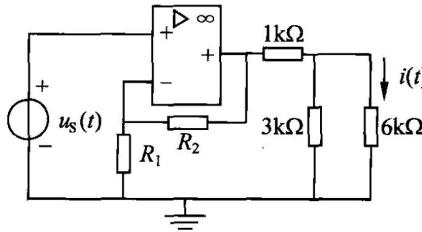


图 3.1: 习题集 3-40

3.2 习题集 3-45 (注意题目单位是 S): 对题图电路, 求电压增益 $\frac{U_o}{U_i}$ 和入端电阻 R_i

如图所示, 将电导全部转换为电阻。由虚断、虚短, 流经 $\frac{1}{10} \Omega$ 电阻的电流为 $i_1 = \frac{u_s}{0.1 \Omega} = 10u_s$ 。右下角两电阻分压, 再由虚短可得 $i_2 = 2U_o$, 于是 $i_3 = i_1 + i_2 = 10U_s + 2U_o$, 由 KVL:

$$0 - \frac{1}{3}(10U_s + 2U_o) = U_o \Rightarrow \frac{U_o}{U_s} = -2 \quad (3.2)$$

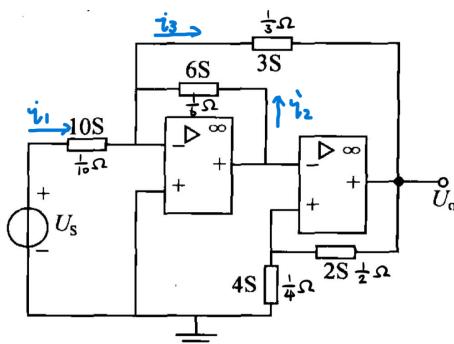
入端电阻 R_i :

$$i_1 = 10U_s \Rightarrow R_i = \frac{1}{10} \Omega \quad (3.3)$$

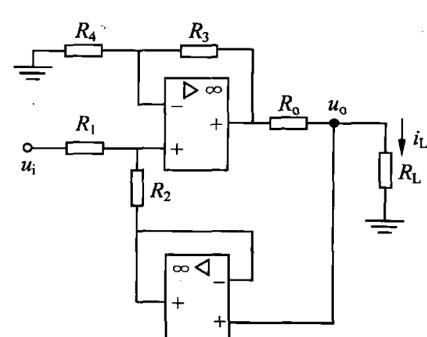
3.3 习题集 3-46: 题图电路中, $R_1 = R_2 = R_3 = R_4 = R_o = R_L$, 求在输入电压 u_i 作用下的负载电流 i_L

依据 KVL、KCL、虚短、虚断, 标出各节点电势, 如图所示。则有:

$$(u_i + u_o) - u_o = i_L R \Rightarrow i_L = \frac{u_o}{R} = \frac{u_i}{R} \quad (3.4)$$



(a) 习题集 3-45



(b) 习题集 3-46

图 3.2: 习题集 3-45 和习题集 3-46

3.4 讲义题 2-19: 求同相比例放大器和反向比例放大器的输入电阻和输出电阻, 放大器均理想, 根据求解结果讨论两种放大器的优劣

(1) 反相比例放大器

对输入电阻, $i_1 = \frac{u_i}{R_1} \Rightarrow R_i = R_1$ 。对输出电阻, 将输入电压源短路, 采用加流求压法, 在输出端接入电流源, 由 $u = iR$ 且 $u = 0$, 得 $R_o = 0$ 。也即:

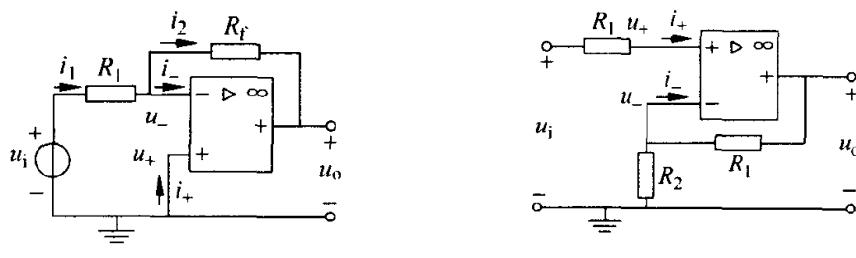
$$R_i = R_1, R_o = 0 \quad (3.5)$$

(2) 同相比例放大器

对输入电阻, R_1 右端断路, 因此 $R_i = \infty$ 。对输出电阻, 将输入电压源短路, 采用加流求压法, 在输出端接入电流源, 由 $u = iR$ 且 $u = 0$, 得 $R_o = 0$ 。也即:

$$R_i = \infty, R_o = 0 \quad (3.6)$$

从输入输出电阻特性来看, 同相比例放大器电气特性更优秀。



(a) 同相比例放大器

(b) 反相比例放大器

图 3.3: 讲义题 2-19

3.5 讲义题 2-20: 求题图各网络的 G 参数

(a) 由 KVL 有:

$$\begin{cases} u_1 + u_2 = i_2 R_1 \\ u_1 = (i_1 - i_2) R_2 \end{cases} \Rightarrow \begin{cases} i_1 = \frac{R_1 + R_2}{R_1 R_2} u_1 + \frac{1}{R_1} u_2 \\ i_2 = \frac{1}{R_1} u_1 + \frac{1}{R_1} u_2 \end{cases}, G = \begin{bmatrix} \frac{R_1 + R_2}{R_1 R_2} & \frac{1}{R_1} \\ \frac{1}{R_1} & \frac{1}{R_1} \end{bmatrix} \quad (3.7)$$

(b) 设 2 号端口的低电位为 u , 也即 $u_{2,-} = u$, 由 KCL:

$$\begin{cases} i_1 + \frac{u+u_2-u_1}{R_2} = \frac{u_1-u}{R_1} \\ \frac{u_1-u}{R_1} = i_2 + \frac{u}{R_2} \\ \frac{u+u_2}{R_1} + \frac{u}{R_2} = i_1 \end{cases} \Rightarrow \begin{cases} i_1 = \frac{(R_1+R_2)u_1+(R_2-R_1)u_2}{2R_1R_2} \\ i_2 = \frac{(R_2-R_1)u_1+(R_1+R_2)u_2}{2R_1R_2} \end{cases}, G = \frac{1}{R_1R_2} \begin{bmatrix} R_1+R_2 & R_2-R_1 \\ R_2-R_1 & R_1+R_2 \end{bmatrix} \quad (3.8)$$

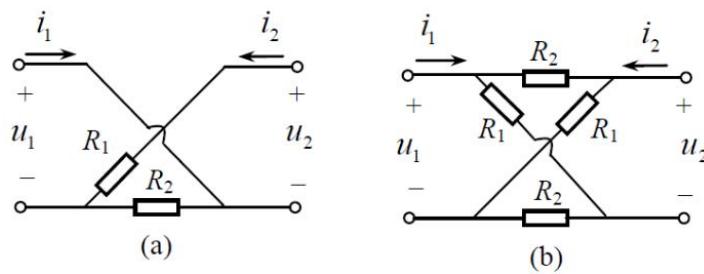


图 3.4: 讲义题 2-20

Homework 04: 2024.09.19 - 2024.09.24

4.1 讲义题 2-20: 求图示各网络的和 R 参数

如图 4.1 (a), 对 (a) 电路有:

$$\begin{cases} u_1 + u_2 = i_2 R_1 \\ u_1 = (i_1 - i_2) R_2 \end{cases} \implies \begin{cases} u_1 = R_2 i_1 + (-R_2) i_2 \\ u_2 = (-R_2) i_1 + (R_1 + R_2) i_2 \end{cases}, \quad \mathbf{R} = \begin{bmatrix} R_2 & -R_2 \\ -R_2 & R_1 + R_2 \end{bmatrix} \quad (4.1)$$

对 (b) 电路, 设 2 号端口的低电位为 u , 也即 $u_{2,-} = u$, 由 KCL:

$$\begin{cases} i_1 + \frac{u+u_2-u_1}{R_2} = \frac{u_1-u}{R_1} \\ \frac{u_1-u}{R_1} = i_2 + \frac{u}{R_2} \\ \frac{u+u_2}{R_1} + \frac{u}{R_2} = i_1 \end{cases} \implies \begin{cases} u_1 + u_2 = R_1 i_1 + R_1 i_2 \\ u_1 - u_2 = R_2 i_1 - R_2 i_2 \end{cases} \quad (4.2)$$

$$\implies \begin{cases} u_1 = \frac{R_1+R_2}{2} i_1 + \frac{R_1-R_2}{2} i_2 \\ u_2 = \frac{R_1-R_2}{2} i_1 + \frac{R_1+R_2}{2} i_2 \end{cases}, \quad \mathbf{R} = \begin{bmatrix} \frac{R_1+R_2}{2} & \frac{R_1-R_2}{2} \\ \frac{R_1-R_2}{2} & \frac{R_1+R_2}{2} \end{bmatrix} \quad (4.3)$$

4.2 讲义题 2-21: 图示电路中 $R_1 = 10 \Omega$, $R_2 = 40 \Omega$, 求

(1) 此二端口网络的 T 参数:

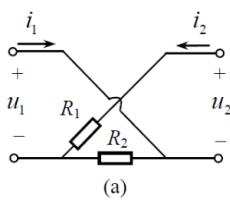
$i_1 = \frac{u_2}{R_2} + (-i_2)$, $u_1 = u_2 - R_1(i_2 - \frac{u_2}{R_2})$, 得到此二端口的 T 参数:

$$\mathbf{T} = \begin{bmatrix} 1 + \frac{R_1}{R_2} & R_1 \\ \frac{1}{R_2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & 10 \Omega \\ \frac{1}{40} \text{ S} & 1 \end{bmatrix} \quad (4.4)$$

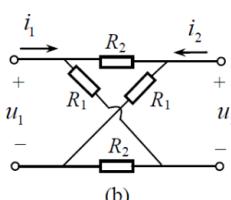
(1) 求 U_{S1} 和 I_1

$(-i_2) = I_2 = 2 \text{ A}$, $u_2 = I_2 R_3 = 40 \text{ V}$, 代入即得:

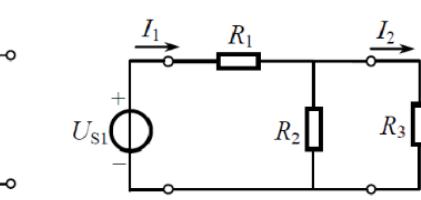
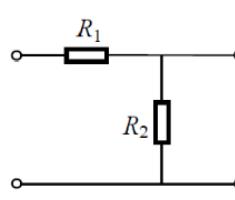
$$\begin{bmatrix} U_{S1} \\ I_1 \end{bmatrix} = \mathbf{T} \cdot \begin{bmatrix} 40 \text{ V} \\ 2 \text{ A} \end{bmatrix} = \begin{bmatrix} 70 \text{ V} \\ 3 \text{ A} \end{bmatrix} \quad (4.5)$$



(a) 讲义题 2-20 图



(b) 讲义题 2-20 图



(b) 讲义题 2-21 图

图 4.1: 讲义题 2-20、讲义题 2-21

4.3 讲义题 2-22: 图示电路中二端口网络的 T 参数为 $T = \begin{bmatrix} 2 & 8 \Omega \\ 0.5 \text{ S} & 2.5 \end{bmatrix}$

(1) 求此二端口的等效电路

T 参数满足 $\det T = 5 - 4 = 1$, 也即满足互易条件, 因此可以等效为 T 型三电阻电路, 如图 4.2 所示。此时的电阻阻值为:

$$R_T = \frac{1}{T_{21}} = 2 \Omega, \quad R_a = R_T(T_{11} - 1) = 2 \Omega, \quad R_b = R_T(T_{22} - 1) = 3 \Omega \quad (4.6)$$

(1) R_2 为何值时其获得最大功率

R_2 吸收的功率为 $p = \frac{u_2^2}{R_2}$, 回路总电阻为 $2 + 2 + 2 \parallel (3 + R_2) = 4 + \frac{2(3+R_2)}{5+R_2}$, 由分压原理得到 u_2 :

$$u_2 = 6 \cdot \frac{\frac{2(3+R_2)}{5+R_2}}{4 + \frac{2(3+R_2)}{5+R_2}} \cdot \frac{R_2}{3 + R_2} = \frac{6}{3 + \frac{13}{R_2}} \quad (4.7)$$

于是 R_2 上的功率 p 为:

$$p = \frac{u_2^2}{R_2} = \frac{36}{\frac{13^2}{R_2} + 78 + 9R_2} \leq \frac{36}{2 \cdot 13 \cdot 3 + 78} \text{ W} = \frac{9}{39} \text{ W} = 0.2308 \text{ W} \quad (4.8)$$

当且仅当 $\frac{13^2}{R_2} = 9R_2$ 取等, 此时 $R_2 = \frac{13}{3} \Omega$ 。

事实上, 视 R_2 为负载, 视电路的剩余部分为电源, 可求得电源的内阻 (也即输出电阻) 为 $R_s = \frac{13}{3} \Omega$, 因此当 $R_2 = R_s = \frac{13}{3} \Omega$ 时, 外部电路 (也即负载 R_2) 有最大功率。

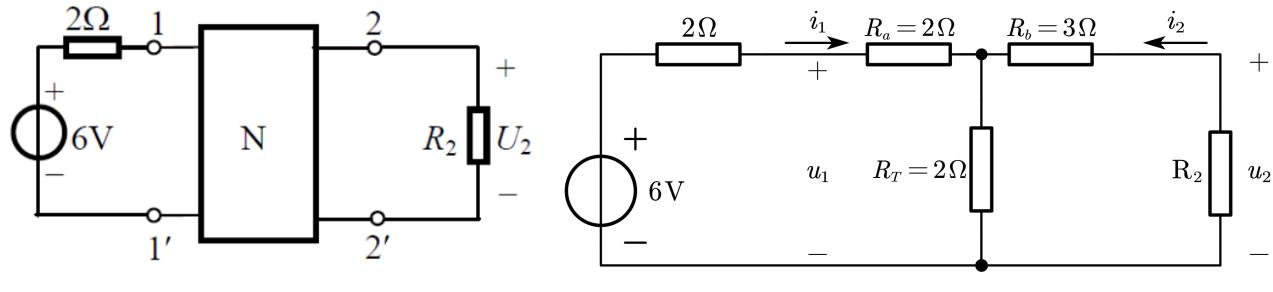


图 4.2: 讲义题 2-22

Homework 05: 2024.09.25 - 2024.10.08

5.1 教材 2-41: 求由 N-E MOS 构成的两输入 NAND 和两输入 NOR 的最大功率, 并指出何时有最大功率

对于 NAND, 仅当两输入都为 1 时有静态功率, 也即最大功率, 设 N-E MOS 的导通电阻为 R_{ON} , 外接电阻 R_L , 电源电压 U_S , 则功率为:

$$P_{\text{NAND}, \max} = \frac{U_S^2}{R_L + 2R_{ON}} \quad (5.1)$$

对于 NOR, 任一输入为 1 时都具有静态功率, 两输入都为 1 时有最大功率:

$$P_{\text{NOR}, \max} = \frac{U_S^2}{R_L + \frac{R_{ON}}{2}} \quad (5.2)$$

5.2 用两个 N-E MOS、两个 P-E MOS 和电源构成静态功率为零的 NAND

题意也即 C-MOS NAND, 我们不妨直接用 C-MOS 构成三种基本逻辑门 (反相器 NOT、或非门 NOR、与非门 NAND), 如图 5.1 所示, 其中红色表示 P-MOS, 蓝色表示 N-MOS。

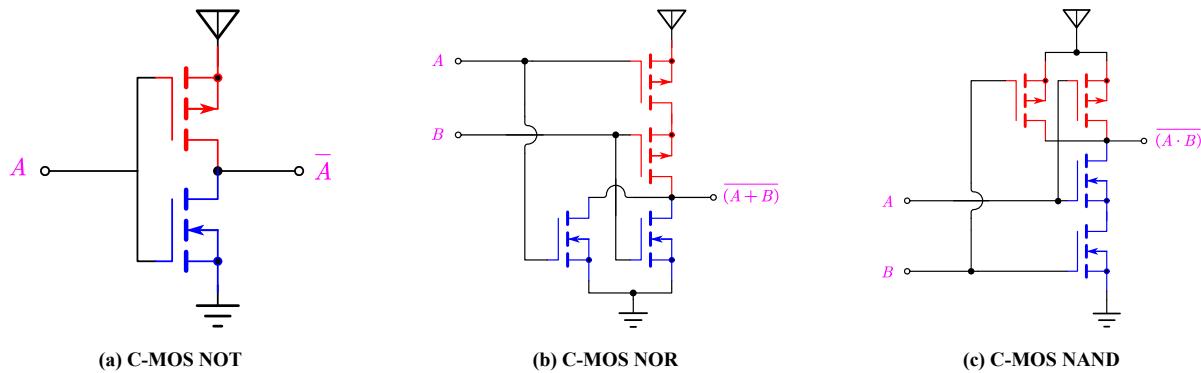


图 5.1: 由 C-MOS 构成三种基本逻辑门

5.3 半加器、全加器、四位加法器

(1) 教材 2-40: 用电源、N-E MOS (最多九个) 和电阻器构成一个半加器 HA

半加器是一种基本的逻辑电路, 用于将两个二进制数相加, 输出一个两位的二进制数, 表示相加的结果。输出的高位和低位分别称为“进位 C ”、“和位 S ”。也就是说, 半加器实际上是“一位加法器”, 能够处理两个一位二进制数的相加, 并输出一个两位二进制结果。设输入为 A 和 B , 则有:

$$Y = (CS)_{(2)} = A_{(2)} + B_{(2)} \quad (5.3)$$

$$C = A \cdot B, \quad S = A \oplus B \quad (5.4)$$

由于要求使用的 MOS 尽量少 (仅使用 N-MOS), 对半加器的逻辑表达式作处理, 利用下面式子可得最简半加器 (7 个 N-MOS), 其数字电路见图 5.2 (a), 实际电路见图 5.3 (a)。

$$C = \overline{(A \cdot B)}, \quad S = \overline{[(A \cdot B) + (A + B)]} \quad (5.5)$$

当然, 考虑到半加器的逻辑表达式, 也可以用异或门 XOR 和与门 AND 直接构成半加器, 我们采用经过优化的 6 MOS 异或门 XOR (三个 N-MOS 和三个 P-MOS), 它的静态功率为 0。由此构成的半加器数字电路见图 5.2 (b), 实际电路见图 5.3 (b)。

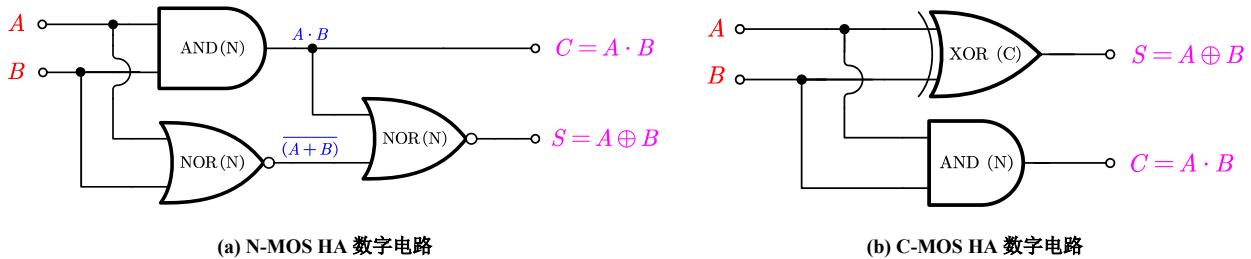


图 5.2: 半加器 HA 数字电路

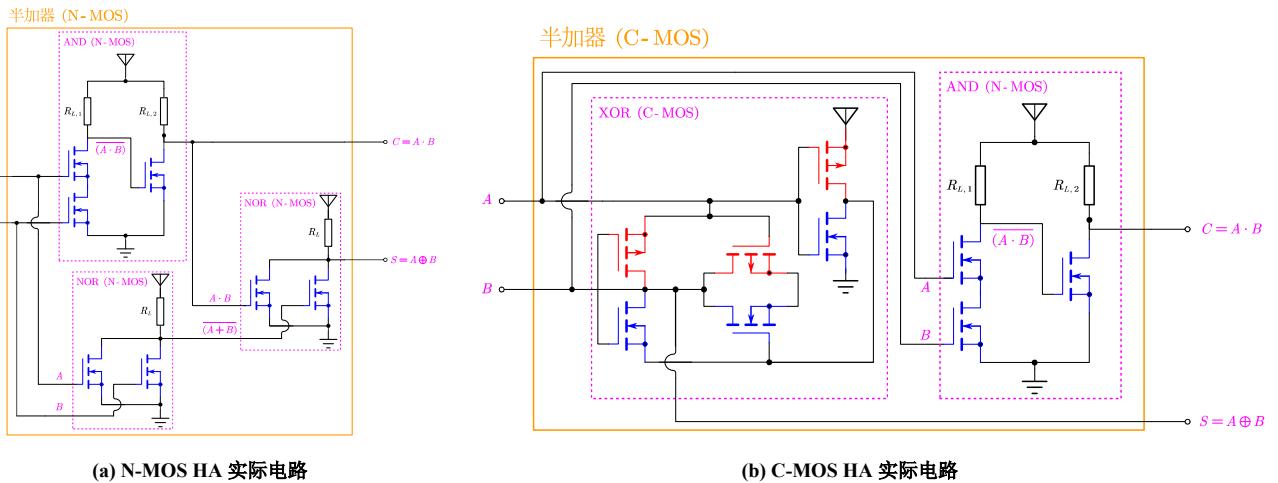


图 5.3: 半加器 HA 实际电路

(2) 用两个半加器 HA 和一个逻辑门构成一个全加器 FA

由半加器可以进一步构造全加器，全加器是一种能够处理三个一位二进制数相加的逻辑电路，输出一个两位的二进制数 $Y = (C_o S)_{(2)}$ ，表示相加的结果。输出的高位称为进位 C_o ，低位称为和位 S 。也就是说，全加器可以理解为“三输入一位加法器”。记全加器的三个输入为 A, B, C ，它们都是一位的二进制数，则全加器可写为：

$$Y = (C_o S)_{(2)} = A_{(2)} + B_{(2)} + C_{(2)} \quad (5.6)$$

$$C = A \cdot B + A \cdot C + B \cdot C, \quad S = A \oplus B \oplus C \quad (5.7)$$

角标 (2) 表示上式为二进制运算。由两个 HA 和一个 OR 即可构成全加器，数字电路如图 5.4。

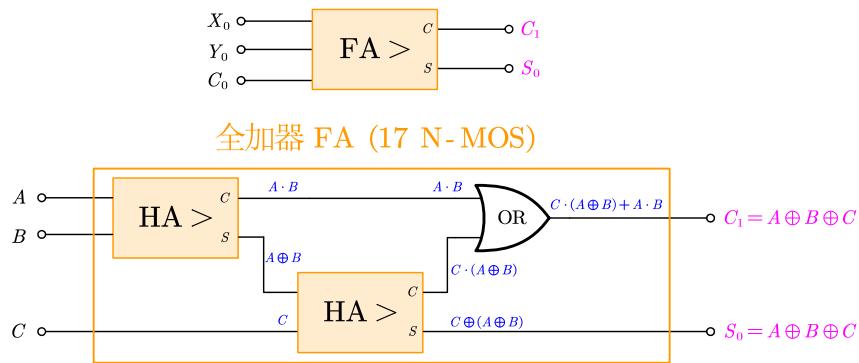


图 5.4: 全加器 FA

(3) 用四个全加器 FA 构成一个四位加法器

四位加法器, 输入两个四位二进制数 X 和 Y , 分别记作 $X = (X_3 X_2 X_1 X_0)_{(2)}$, $Y = (Y_3 Y_2 Y_1 Y_0)_{(2)}$, 输出一个五位二进制数 $Z = (CS_3 S_2 S_1 S_0)_{(2)}$, 代表两数相加的结果。原理及数字电路如图 5.5 所示。

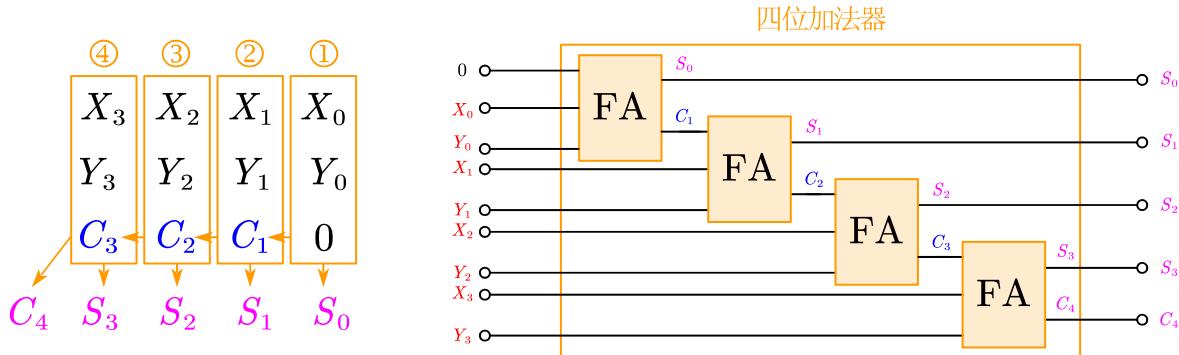


图 5.5: 四位加法器

5.4 习题集 3-20: 用节点法求图中电路的电流 I 和电流源两端电压 U

对电路作等效处理, 得到等效电路如图 5.6 (b) 所示, 则有节点电压方程组如下式左半边。再任意选取一个节点作为参考节点, 这里选择节点 3, 即 $U_3 = 0$, 可以解得:

$$\begin{cases} \text{节点 1: } \left(\frac{5}{8} + 1\right)U_1 - \frac{5}{8}U_2 - U_3 = -10.75 \\ \text{节点 2: } -\frac{5}{8}U_1 + \left(\frac{5}{8} + \frac{3}{5}\right)U_2 - \frac{3}{5}U_3 = +10.75 \\ \text{节点 3: } -U_1 - \frac{3}{5}U_2 + \left(1 + \frac{3}{5}\right)U_3 = 0 \end{cases} \Rightarrow \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} -\frac{129}{32} \text{ V} \\ \frac{215}{32} \text{ V} \\ 0 \end{bmatrix} \quad (5.8)$$

返回到原电路, 可得电流 I 和电流源两端电压 U :

$$I = \frac{U_2 - 8 - U_3}{2} = -\frac{41}{64} \text{ A} = -0.640625 \text{ A}, \quad U = U_2 - U_1 + 6 = \frac{67}{4} \text{ V} = 16.75 \text{ V} \quad (5.9)$$

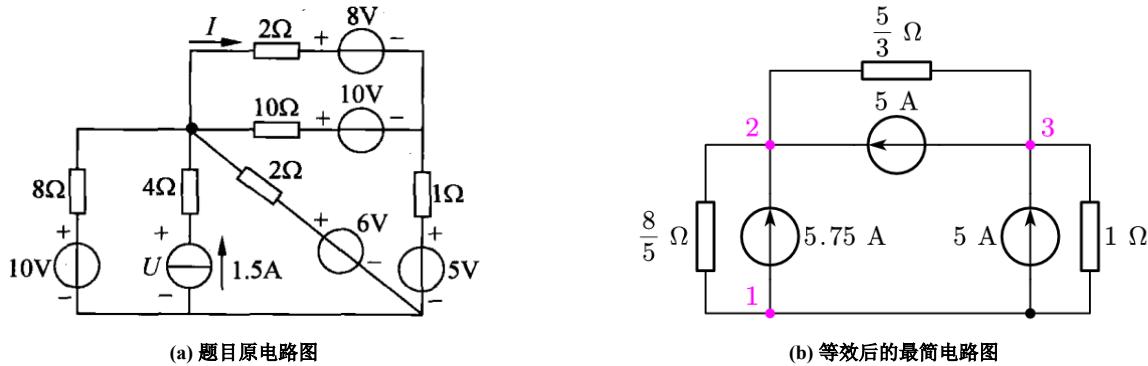


图 5.6: 5.4 习题集 3-20

5.5 习题集 3-21: 用节点法求电流源两端电压 U 和各支路电流

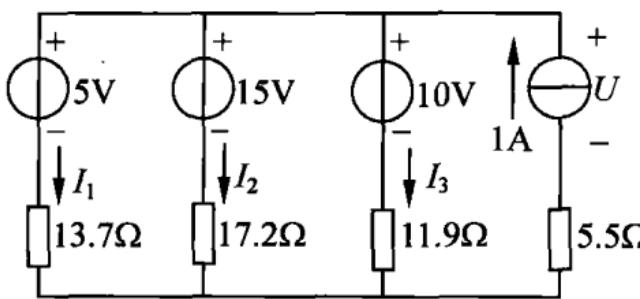
还是对电路作等效处理, 得到等效电路如图 5.7 (b) 所示。这次先选取参考节点再列方程, 以节点 0 为参考点, 得到:

$$\left(\frac{1}{13.7} + \frac{1}{17.2} + \frac{1}{11.9}\right)U_1 = \frac{5}{13.7} + \frac{15}{17.2} + \frac{10}{11.9} + 1 \Rightarrow U_1 = 14.3024 \text{ V} \quad (5.10)$$

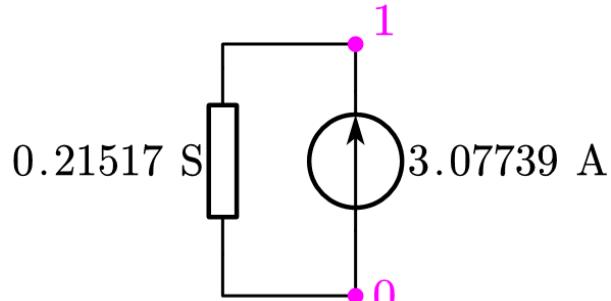
于是得到电压 U 和各支路电流:

$$U = U_1 + 5.5 = 19.8024 \text{ V}, \quad I_1 = \frac{U_1 - 5}{13.7} = 0.6790 \text{ A} \quad (5.11)$$

$$I_2 = \frac{U_1 - 15}{17.2} = -0.0406 \text{ A}, \quad I_3 = \frac{U_1 - 10}{11.9} = 0.3615 \text{ A} \quad (5.12)$$



(a) 题目原电路图



(b) 等效后的最简电路图

图 5.7: 5.5 习题集 3-21

5.6 (选做) 讲义题 3-4: 按指定的参考节点, 列出电路的节点电压方程

原题图和等效电路如图 5.8 所示, 电路中有 4 个节点 (不含无伴电压源) 和 2 个受控源控制变量, 需要列出 3 个节点方程和 2 个控制方程, 如下:

$$\begin{aligned} \text{节点 1: } & (1+2)U_1 - U_2 - 0 = 2u - 1 \\ \text{节点 2: } & -\frac{1}{2}U_1 + \left(\frac{1}{2} + \frac{1}{4}\right)U_2 - \frac{1}{4}U_3 = 3i \\ \text{节点 3: } & 0 - \frac{1}{4}U_2 + \left(1 + \frac{1}{4}\right)U_3 = 1 \\ \text{控制方程: } & i = \frac{U_1 - U_2}{2}, \quad u = U_3 \end{aligned} \quad (5.13)$$

代入化简并求解:

$$\begin{cases} 3U_1 - U_2 - 2U_3 = -1 \\ -8U_1 + 9U_2 - U_3 = 0 \\ 0 - U_2 + 5U_3 = 1 \end{cases} \implies \begin{cases} U_1 = -\frac{25}{76} = -0.3289 \\ U_2 = -\frac{21}{76} = -0.2763 \\ U_3 = \frac{11}{76} = 0.1447 \end{cases} \quad i = -\frac{1}{38} \text{ A} = -0.0263 \text{ A}, \quad u = \frac{11}{76} \text{ V} = 0.1447 \text{ V} \quad (5.14)$$

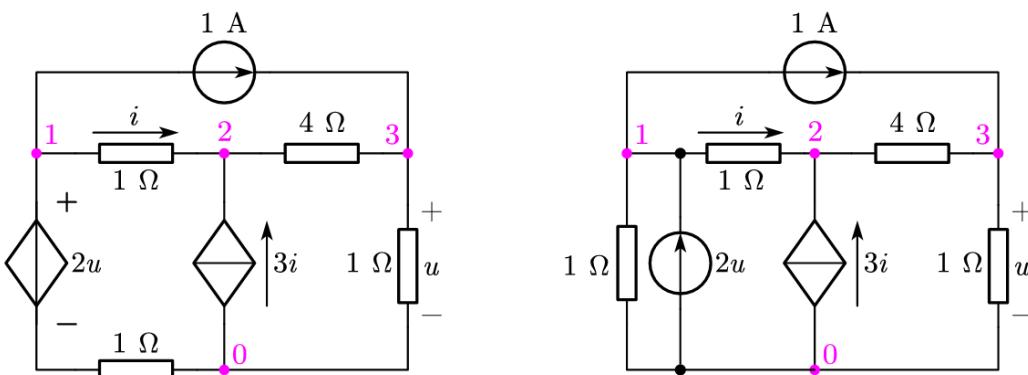


图 5.8: 5.6 讲义题 3-4

5.7 习题集 3-15: 用回路电流法求图中电路的独立电流源功率

原电路已无法继续化简, 电路中有 3 个网格 (含 2 个无伴电流源), 1 个受控源变量, 共需列出 $3+1=2$ 个方程。如图 5.9 (a), 先列出三个网格电流方程, 如下:

$$\text{网格 1: } 100i_1 - 20i_2 - 30i_3 = 0 \quad (5.15)$$

$$\text{网格 2: } i_2 = 1 \quad (5.16)$$

$$\text{网格 3: } i_3 = -0.02U \quad (5.17)$$

又有控制变量 U :

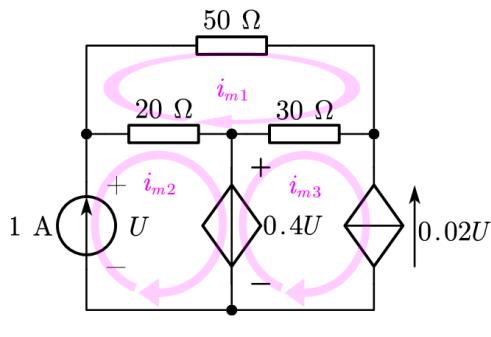
$$0 + U - 20(i_2 - i_1) - 0.4U = 0 \implies U = \frac{100}{3}(i_2 - i_1) \quad (5.18)$$

联立上面四个方程, 可以得到:

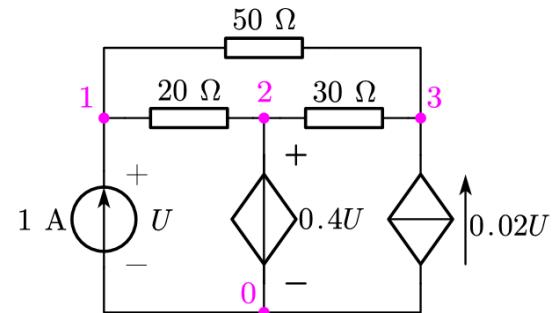
$$\begin{cases} \text{网格 1: } 100i_1 - 20i_2 - 30i_3 = 0 \\ \text{网格 2: } i_2 = 1 \\ \text{网格 3: } -2i_1 + 2i_2 + 3i_3 = 0 \end{cases} \implies \begin{cases} i_1 = 0 \text{ A} \\ i_2 = 1 \text{ A} \\ i_3 = -\frac{2}{3} \text{ A} \end{cases} \quad (5.19)$$

得到功率 P :

$$P = 1 \text{ A} \cdot U = \frac{100}{3}(i_2 - i_1) = \frac{100}{3} \text{ W} = 33.3333 \text{ W} \quad (5.20)$$



(a) 回路电流法



(b) 节点电压法

图 5.9: 5.7 习题集 3-15

不妨也用节点电压法求解一下此题。如图 5.9 (b), 电路中有 4 个节点 (含 1 个无伴电压源) 和 1 个 (受控源) 控制变量, 取节点 0 为参考节点, 共需列出 $(4-1-1)+0+2=4$ 个方程, 如下:

$$\text{节点 1: } \left(\frac{1}{50} + \frac{1}{20} \right) U_1 - \frac{1}{20} U_2 - \frac{1}{50} U_3 = 1 \quad (5.21)$$

$$\text{节点 2: } U_2 = 0.4U \quad (5.22)$$

$$\text{节点 3: } -\frac{1}{50} U_1 - \frac{1}{30} U_2 + \left(\frac{1}{50} + \frac{1}{30} \right) U_3 = 0.02U \quad (5.23)$$

$$\text{控制方程: } U = U_1 \quad (5.24)$$

联立上述四个方程, 可以得到:

$$\begin{cases} \text{节点 1: } \left(\frac{1}{50} + \frac{1}{20} \right) U_1 - \frac{1}{20} U_2 - \frac{1}{50} U_3 = 1 \\ \text{节点 2: } -0.4U_1 + U_2 = 0 \\ \text{节点 3: } -\frac{2}{50} U_1 - \frac{1}{30} U_2 + \left(\frac{1}{50} + \frac{1}{30} \right) U_3 = 0 \end{cases} \implies \begin{cases} U_1 = \frac{100}{3} \text{ V} \\ U_2 = \frac{40}{3} \text{ V} \\ U_3 = \frac{100}{3} \text{ V} \end{cases} \implies P = 1 \text{ A} \cdot U = \frac{100}{3} \text{ W} \quad (5.25)$$

5.8 (选做) 讲义题 3-8: 列出图中电路的网孔电流方程, 并计算受控源的吸收功率

如图, 电路共有 3 个网格 (内含 2 个无伴电流源) 和 1 个控制变量, 将 i_2 和 i_3 合并为超网格后, 共需要列出 2 个网格方程、1 个超网格内部方程和 1 个控制方程, 如下:

$$\left\{ \begin{array}{l} \text{网格 1: } i_1 = 2 \\ \text{网格 2,3: } -3i_1 + (2i_2 + 4i_3) = 4 \\ \text{网格 2,3 内部: } 2i = i_2 - i_3 \\ \text{控制方程: } i = i_3 - i_1 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} i_1 = 2 \\ -3i_1 + 2i_2 + 4i_3 = 4 \\ 2i_1 + i_2 - 3i_3 = 0 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} i_1 = 2 \text{ A} \\ i_2 = 1.4 \text{ A} \\ i_3 = 1.8 \text{ A} \\ i = -0.2 \text{ A} \end{array} \right. \quad (5.26)$$

于是得到吸收功率:

$$P_i = 2i \cdot [0 - (4 - 2(i_2 - i_1))] = -2.08 \text{ W} \quad (5.27)$$

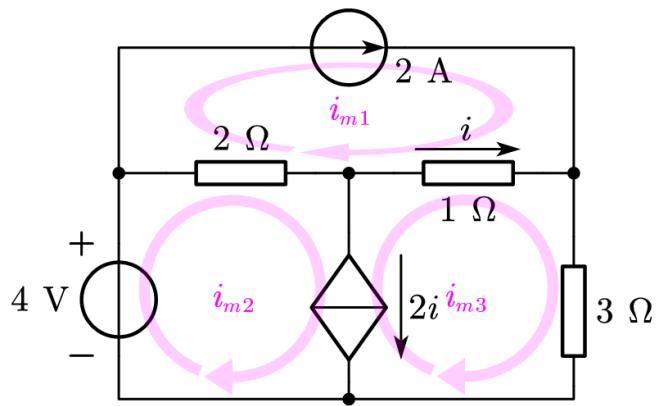


图 5.10: 5.8 讲义题 3-8

Homework 06: 2024.10.09 - 2024.10.15

6.1 习题集 4-10: 题图电路中电流源 $I_{S1} = 2 \text{ A}$, $I_{S2} = 3 \text{ A}$ 。断开 3 A 电流源, 则 2 A 电流源输出 28 W , $U = 8 \text{ V}$ 。断开 2 A 电流源, 则 3 A 的电流源输出 54 W , $U = 12 \text{ V}$ 。求两个电流源同时作用时, 每个电流源输出的功率。

仅有 2 A 电流源时, $U_1 = 14 \text{ V}$, $U_2 = 8 \text{ V}$; 仅有 3 A 电流源时, $U_1 = 12 \text{ V}$, $U_2 = 18 \text{ V}$ 。由叠加定理, 两电流源同时作用时:

$$\begin{cases} U_1 = 14 + 12 = 26 \text{ V} \\ U_2 = 8 + 18 = 26 \text{ V} \end{cases} \Rightarrow \begin{cases} P_{2\text{A}} = 2 \cdot 26 = 52 \text{ W} \\ P_{3\text{A}} = 3 \cdot 26 = 78 \text{ W} \end{cases} \quad (6.1)$$

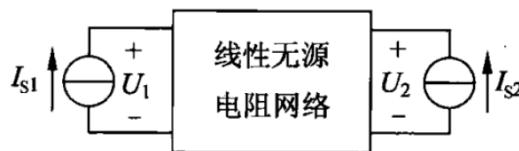


图 6.1: 6.1 习题集 4-10

6.2 习题集 4-24: 题图电路中, 已知 $U_{S1} = 24 \text{ V}$, $U_{S2} = 18 \text{ V}$, $R_1 = 2 \Omega$, $R_2 = 1 \Omega$, $R_3 = 3 \Omega$ 。

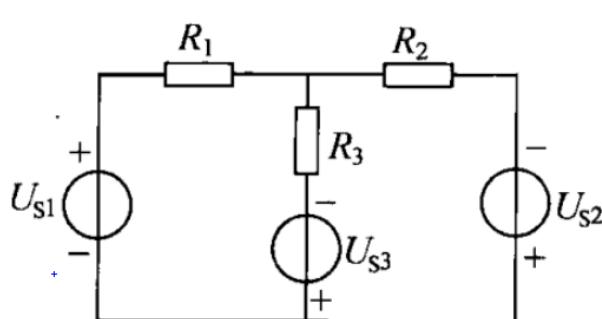
(1) 求 $U_{S3} = 15 \text{ V}$ 时, 通过 R_3 的电流

先求出激励为单个电压源时, 通过 R_3 的电流 (参考方向标在图 6.2 中):

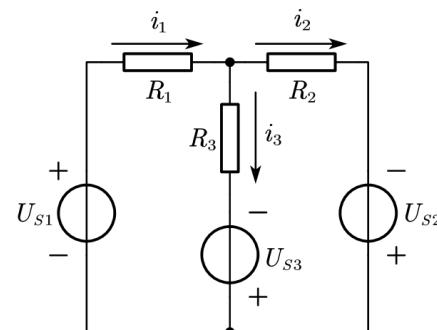
$$U_{S1} : I_{R3} = \frac{24}{11} \text{ A}, \quad U_{S2} : I_{R3} = -\frac{36}{11} \text{ A}, \quad U_{S3} : I_{R3} = \frac{3}{11} U_{S3} \quad (6.2)$$

由叠加定理, 通过 R_3 的电流为:

$$I_{R3} = \frac{24}{11} - \frac{36}{11} + \frac{3}{11} \cdot 15 = 3 \text{ A} \quad (6.3)$$



(a) 原题图



(b) 电流参考方向

图 6.2: 6.2 习题集 4-24

(2) R_3 为多大时可获得最大功率, 值是多少

此题表述有些问题, 既没有说明固定 U_{S3} 的值为多少, 也没有强调“功率”的主语, 是指“ R_3 的功率”还是“电路总功率”, 有些模糊不清。不论功率是前者还是后者, 必须先给定 U_{S3} 才可计算。所以, 我们在这里沿用第一问的条件, 假设 $U_{S1} = 15 \text{ V}$, 则各电源(激励)对电阻的电流为:

$$U_{S1}: I_{R_1} = \frac{U_{S1}}{R_1 + R_2 \parallel R_3}, \quad I_{R_2} = I_{R_1} \cdot \frac{R_3}{R_2 + R_3}, \quad I_{R_3} = I_{R_1} \cdot \frac{R_2}{R_2 + R_3} \quad (6.4)$$

$$U_{S2}: I_{R_1} = I_{R_2} \cdot \frac{R_3}{R_1 + R_3}, \quad I_{R_2} = \frac{U_{S2}}{R_2 + R_1 \parallel R_3}, \quad I_{R_3} = -I_{R_2} \cdot \frac{R_1}{R_1 + R_3} \quad (6.5)$$

$$U_{S3}: I_{R_1} = I_{R_3} \cdot \frac{R_2}{R_1 + R_2}, \quad I_{R_2} = -I_{R_3} \cdot \frac{R_1}{R_1 + R_2}, \quad I_{R_3} = \frac{U_{S3}}{R_3 + R_1 \parallel R_2} \quad (6.6)$$

由叠加定理计算 R_3 的功率和电路总功率:

$$P_{R_3} = I_{R_3}^2 R_3, \quad P = I_{R_1}^2 R_1 + I_{R_2}^2 R_2 + I_{R_3}^2 R_3 \quad (6.7)$$

代入数据, 做数学上的化简和整理, 可得:

$$P_{R_3} = \frac{1089R_3}{(3R_3 + 2)^2} = \frac{1089}{9R_3 + \frac{4}{R_3} + 12}, \quad P = \frac{363}{3R_3 + 2} + 588 \quad (6.8)$$

于是:

当 $R_3 = \frac{2}{3} \Omega$ 时, R_3 有最大功率 $P_{R_3,\max} = \frac{363}{8} \text{ W} = 45.375 \text{ W}$

当 $R_3 = 0$ 时, 有最大电路总功率 $P_{\max} = \frac{1539}{2} \text{ W} = 769.5 \text{ W}$

(3) 求使 R_3 中电流为 0 的 U_{S3}

由叠加定理:

$$\frac{24}{11} - \frac{36}{11} + \frac{3}{11} \cdot U_{S3} = 0 \implies U_{S3} = 4 \text{ V} \quad (6.9)$$

6.3 习题集 4-36: 题图电路中网络 A 内含有独立电压源、电流源和线性电阻。在 (a) 图中测得 $U_{ab} = 10 \text{ V}$, (b) 图中测得 $U_{a'b'} = 4 \text{ V}$, 求 (c) 图中的电压 $U_{a''b''}$ 。

由戴维南定理, 可将网络 A 等效为一个电压源 U 串联一个电阻 R , 如图 6.4 (a) 所示。列出方程:

$$U = 10 - 0.5R, \quad U - 0.4R = 4 \implies R = \frac{20}{3} \Omega, \quad U = \frac{20}{3} \text{ V} \quad (6.10)$$

作电源等效如图 6.4 (b), 求得题图 (c) 中的 $U_{a''b''}$:

$$R_0 = R \parallel 10 \Omega \parallel 8 \Omega = \frac{8}{3} \Omega, \quad I_0 = \frac{U}{R} + 0.5 + 1 = 2.5 \text{ A} \implies U_{a''b''} = I_0 R_0 = \frac{20}{3} \text{ V} = 6.67 \text{ V} \quad (6.11)$$

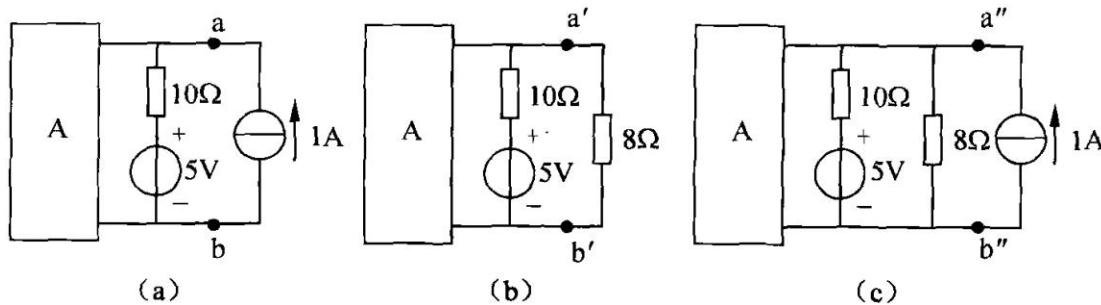


图 6.3: 6.3 习题集 4-36

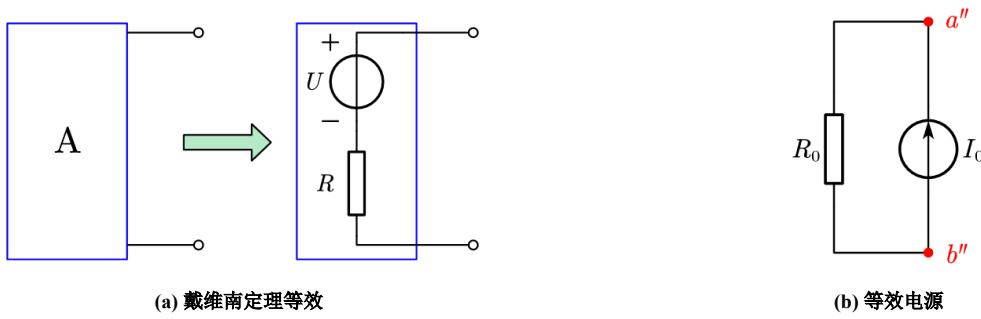


图 6.4: 6.3 习题集 4-36 的等效处理

6.4 习题集 4-37: 题图电路方框内为线性电阻网络, 测得 $U_S = 8 \text{ V}$, $R = 3 \Omega$ 时 $I = 0.5 \text{ A}$; $U_S = 18 \text{ V}$, $R = 4 \Omega$ 时 $I = 1 \text{ A}$ 。求 $U_S = 30 \text{ V}$, $R = 5 \Omega$ 时的电流 I 。

由戴维南定理, 将方框内的线性电阻网络与电压源 U_S 等效为一个电压源 U_0 串联一个电阻 R_0 , 如图 6.5 所示。当 U_S 从 8 V 变为 $18 = \frac{9}{4} \cdot 8 \text{ V}$ 时, 由齐性定理, 等效后的电压源 U_0 应变为 $\frac{9}{4}U_0$, 于是可求得 U_0 和 R_0 :

$$\begin{cases} U_0 = 0.5(R_0 + 3) \\ \frac{9}{4}U_0 = R_0 + 4 \end{cases} \Rightarrow \begin{cases} R_0 = 5\Omega \\ U_0 = 4 \text{ V} \end{cases} \Rightarrow I|_{R=5\Omega} = \left[\frac{\frac{30}{8}U_0}{R_0 + R} \right]_{R=5\Omega} = 1.5 \text{ A} \quad (6.12)$$

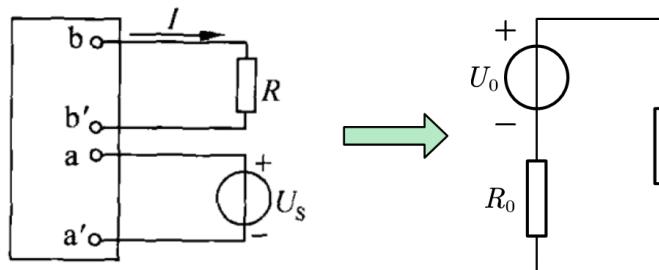


图 6.5: 6.4 习题集 4-37

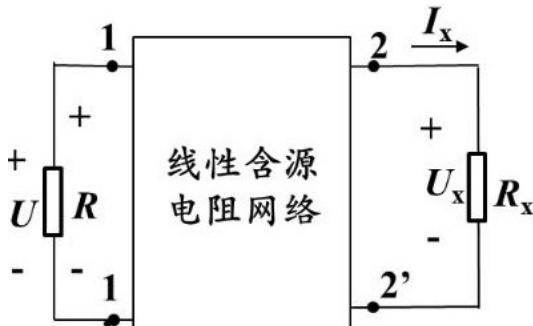
6.5 (选做) 讲义题 3-13: 题图电路中, $R_x = 0$ 时测得 $I_x = 8 \text{ A}$, $U = 12 \text{ V}$; $R_x = \infty$ 时测得 $U_x = 36 \text{ V}$, $U = 6 \text{ V}$ 。求出 $R_x = 9 \Omega$ 时的 U_x 和 U 。

先将除 R_x 外的电路等效为戴维南电路, 设戴维南电路的电压源为 U_1 , 电阻为 R_1 , 由题意可得:

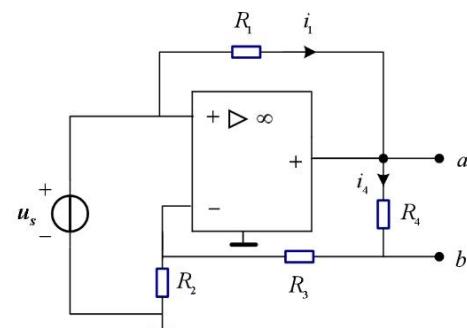
$$\begin{cases} U_1 = 36 \text{ V} \\ R_1 = \frac{36}{8} \Omega = 4.5 \Omega \end{cases} \Rightarrow U_x|_{R_x=9\Omega} = U_1 \cdot \frac{R_x}{R_1 + R_x} = 24 \text{ V} \quad (6.13)$$

然后用替代定理, 将 R_x 替换为电压源 U_x 。由叠加定理和齐性定理, 可得:

$$U = U' + kU_x \Rightarrow \begin{cases} U' = 12 \text{ V} \\ 6 = 12 + k \cdot 36 \end{cases} \Rightarrow \begin{cases} U' = 12 \text{ V} \\ k = -\frac{1}{6} \end{cases} \Rightarrow U|_{R_x=9\Omega} = 12 - \frac{1}{6} \cdot 24 = 8 \text{ V} \quad (6.14)$$



(a) 讲义 3-13 题图



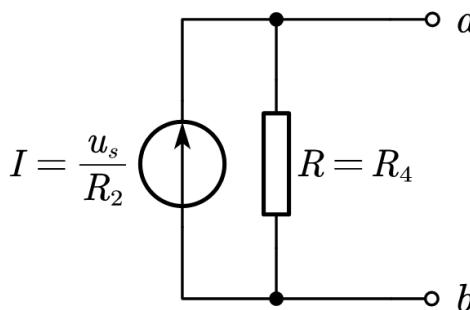
(b) 讲义 3-18 题图

图 6.6: 讲义题 3-13、讲义题 3-18

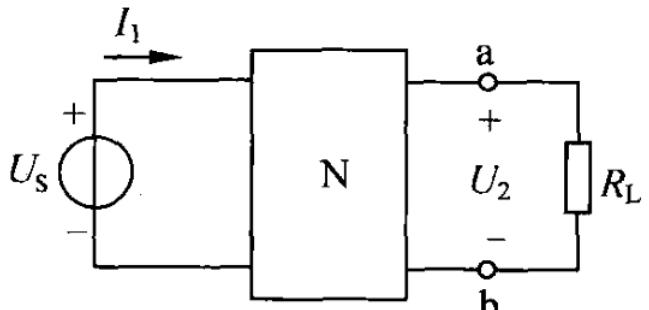
6.6 (选做) 讲义题 3-18: 题图电路中 OPA 理想, 求 ab 端口的诺顿等效电路, 以及电流之比 $\frac{i_1}{i_4}$ (设 ab 端开路)。

由虚短和虚断, $i_4 = \frac{u_s}{R_2} \Rightarrow U_{ab} = \frac{R_4}{R_2} u_s$, 将电压源置零, 可得戴维南等效电阻为 $R = R_4$, 于是诺顿等效电流为 $i = \frac{U}{R} = \frac{u_s}{R_2}$, 如图 6.7 (a) 所示。并且电流之比:

$$u_s - i_1 R_1 = u_s \left(1 + \frac{R_3 + R_4}{R_2}\right) \Rightarrow i_1 = -\frac{R_3 + R_4}{R_1 R_2} \cdot u_s, \quad \boxed{\frac{i_1}{i_4} = -\frac{R_3 + R_4}{R_1}} \quad (6.15)$$



(a) 6.6 讲义题 3-18 诺顿等效电路



(b) 6.7 习题集 4-47

图 6.7: 6.6 等效电路、6.7 题图

6.7 习题集 4-47: 图 6.7 (b) 中, N 为纯电阻网络, 已知 ab 端开路电压 $U_o = 8 \text{ V}$, ab 端纽左端的戴维南等效电阻 $R = 3 \Omega$, 电压源 $U_s = 10 \text{ V}$ 。若 ab 两端接上 $R_L = 2 \Omega$ 电阻时, 电压源 U_s 供出电流 I_1 。求当把 R_L 移走之后, 电流 I_1 变化多少?

由特勒根定理, 设电路总支路数为 b , 由特勒根定理:

$$\sum_{k=1}^2 u_k \hat{i}_k + \sum_{k=3}^b u_k \hat{i}_k = 0 = \sum_{k=1}^2 \hat{u}_k i_k + \sum_{k=3}^b \hat{u}_k i_k \quad (6.16)$$

其中支路 $3 \sim b$ 是网络 N 内部的支路, 由于 N 为纯电阻网络, 有:

$$\sum_{k=3}^b u_k \hat{i}_k = \sum_{k=3}^b i_k R_k \hat{i}_k = \sum_{k=3}^b \hat{u}_k i_k \quad (6.17)$$

于是内部项可以消去, 得到:

$$u_1 \hat{i}_1 + u_2 \hat{i}_2 = \hat{u}_1 i_1 + \hat{u}_2 i_2 \implies U_S I'_1 + 0 = U_S I_1 + U_o \frac{U_o}{R_L + R} \implies \quad (6.18)$$

$$\Delta I_1 = \frac{U_o}{U_S} \cdot \frac{U_o}{R_L + R} = 1.28 \text{ A}$$

(6.19)

6.8 (选做) 讲义题 3-20: 题图电路中, N_R 为只含线性电阻的网络, 试求解电压 U_R .

N_R 为线性电阻网络, 由特勒根定理:

$$u_1 \hat{i}_1 + u_2 \hat{i}_2 = \hat{u}_1 i_1 + \hat{u}_2 i_2 \implies (-5) \cdot U_R + 1 \cdot 10 = 10 \cdot \frac{U_R}{2} + 0 \implies [U_R = 1 \text{ V}] \quad (6.20)$$

无论有没有电源, 特勒根定理中对应支路的参考方向相同, 且都为关联参考方向。比如说, 无论有没有电源, 我们都令电压上正下负, 电流向下为正。

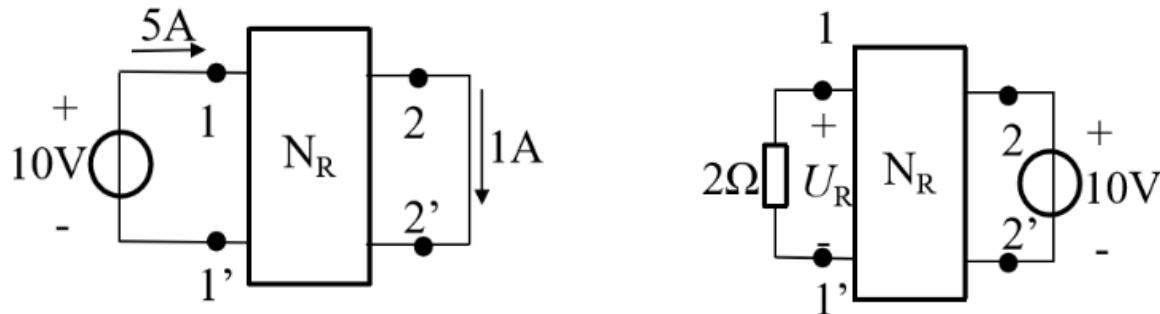


图 6.8: 6.8 讲义题 3-20

Homework 07: 2024.10.15 - 2024.10.22

7.1 习题集 5-1: 已知非线性电阻的 $u-i$ 关系为 $u = 2i + i^3$ (式中 u 的单位是 V, i 的单位是 A), 分别求 $i = 1 \text{ A}$ 、 $i = 2 \text{ A}$ 时的静态电阻和动态电阻

$i = 1 \text{ A}$ 时:

$$R_s = \frac{u}{i} = 2 + i^2 = 3 \Omega, \quad R_d = \frac{du}{di} = 2 + 3i^2 = 5 \Omega \quad (7.1)$$

$i = 2 \text{ A}$ 时:

$$R_s = \frac{u}{i} = 2 + i^2 = 6 \Omega, \quad R_d = \frac{du}{di} = 2 + 3i^2 = 14 \Omega \quad (7.2)$$

7.2 习题集 5-6: 题图 (a) 中非线性电阻的伏安特性如图 (b) 所示, 分别求 $i_s = 1 \text{ A}$ 和 $i_s = 2 \text{ A}$ 时的电压 u 。

两个非线性电阻两端电压 u 相同, 且电流满足 $i_1 + i_2 = i_s$ 。于是在同一坐标系中画出 $i_1 = i_1(u)$ 和 $i_2 = i_2(u)$ 的图像 ($u > 0$), 满足 $i_1 + i_2 = i_s$ 的点即为电路的解。由此得到:

$$i_s = 1 \text{ A} \Rightarrow u = 2 \text{ V}, \quad i_s = 2 \text{ A} \Rightarrow u = 3 \text{ V} \quad (7.3)$$

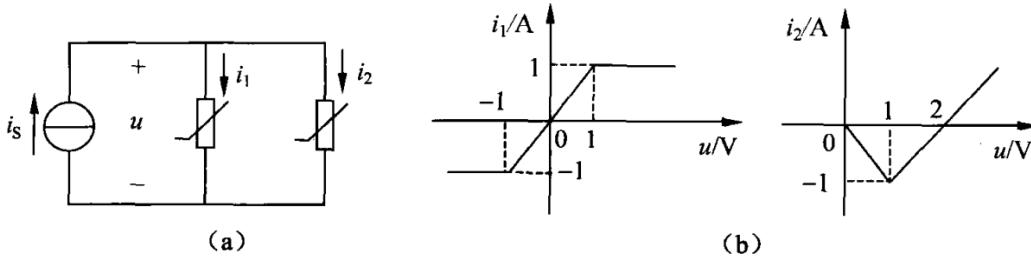


图 7.1: 7.3 习题集 5-11 原题图

7.3 习题集 5-11: 求题图电路中理想二极管 D 所在支路的电路 i

假设二极管导通, 即短路, 由 KCL:

$$\begin{cases} \frac{u-u_1}{40 \text{ K}} + \frac{0-u_1}{60 \text{ K}} - I = 0 \\ \frac{u-u_2}{60 \text{ K}} + \frac{0-u_2}{30 \text{ K}} + I = 0 \\ I = \frac{u_1-u_2}{2 \text{ K}} \end{cases} \Rightarrow \begin{cases} u_1 = \frac{954}{23} \text{ V} = 41.4783 \text{ V} \\ u_2 = \frac{930}{23} \text{ V} = 40.4348 \text{ V} \\ I = \frac{12}{23} \text{ mA} = 0.52174 \text{ mA} \end{cases} \quad (7.4)$$

与假设相符, 因此上式即为所求。

7.4 讲义题 4-4: 用理想二极管模型分析题图电路, 要求安装题号顺序进行

(1) 每个二极管有两种状态 (开路、短路), 整个电路共有几种可能的状态?

不考虑电路是否导通的情况下, 4 个二极管, 每个二极管有 2 种状态, 因此共有 $2^4 = 16$ 种可能的状态。

(2) 电流 i 的方向是怎样的?

电流 i 始终从上至下流经负载电阻 R 。

(3) 沿 (2) 的思路, D1 ~ D4 是怎样的状态是 (可能不止一种) 才能实现 (2) 中的电流? 画出此时的等效电路图。

注意到此电路为全桥整流电路, 借助电流方向可以判断出电路共有 2 种可行状态。在激励的正半周期, D1、D4 导通而 D2、D3 截止; 在激励的负半周期, D2、D3 导通而 D1、D4 截止。

(4) 根据 (1) ~ (3) 的结果, 在坐标系中画出 u_s 和 u 随时间变化的曲线图

二极管理想, 因此有如下公式, 式中电压的单位是 V, 它们的图像如图 7.2 所示。

$$u_s = 12\sqrt{2} \sin(376.8t), \quad u = 12\sqrt{2} |\sin(376.8t)| \quad (7.5)$$

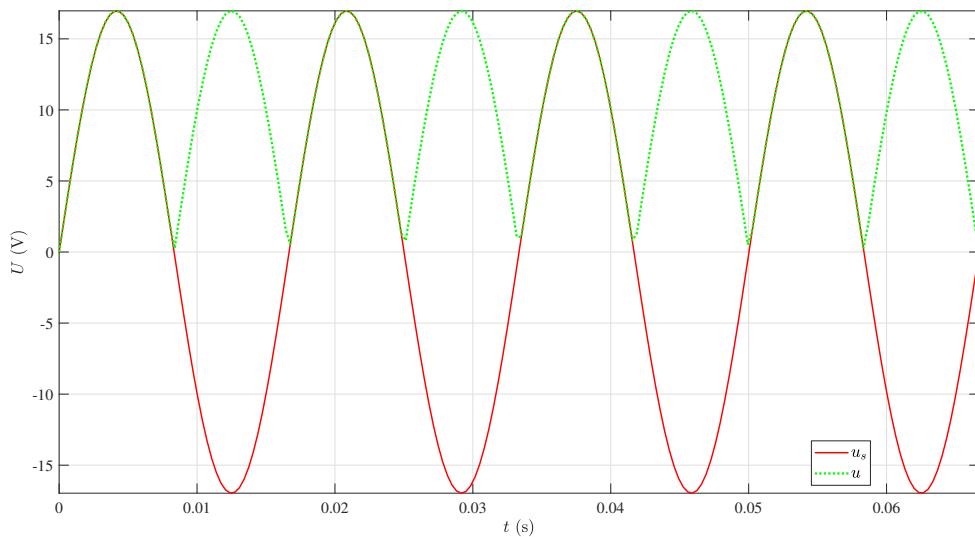


图 7.2: $u_s = u_s(t)$ 和 $u = u(t)$ 的图像

(5) 用 Multisim 仿真验证前面的分析, 要求: (a) 给出仿真电路; (b) 用示波器的两个通道分别显示 u_s 和 u ; (c) 分析示波器波形与第 (4) 问图像的区别。

仿真电路图和示波器波形见图 7.4。由图可知, u 的幅值比理想幅值要低, 这是因为我们在电路中选用了具有实际参数的二极管 SURS8105T3G, 电流通过实际二极管是有压降的, 这消耗了一部分功率, 因此 u 的幅值比理想值稍低。

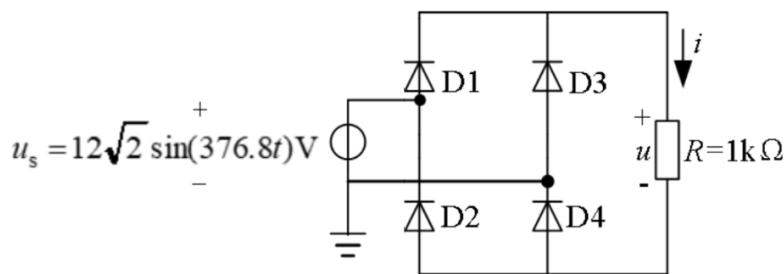
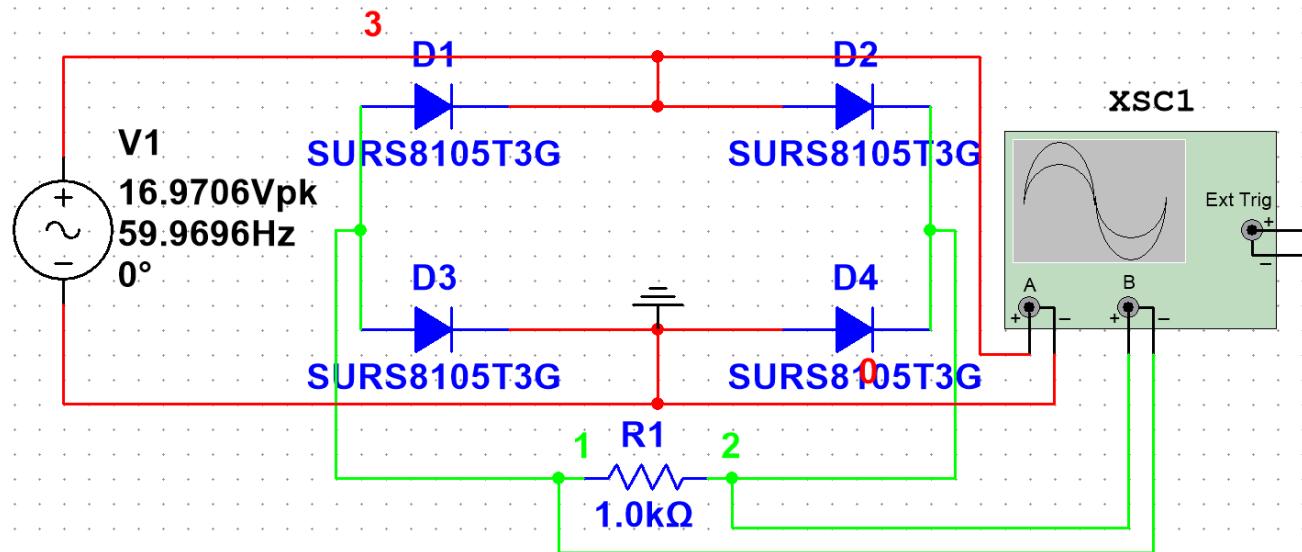
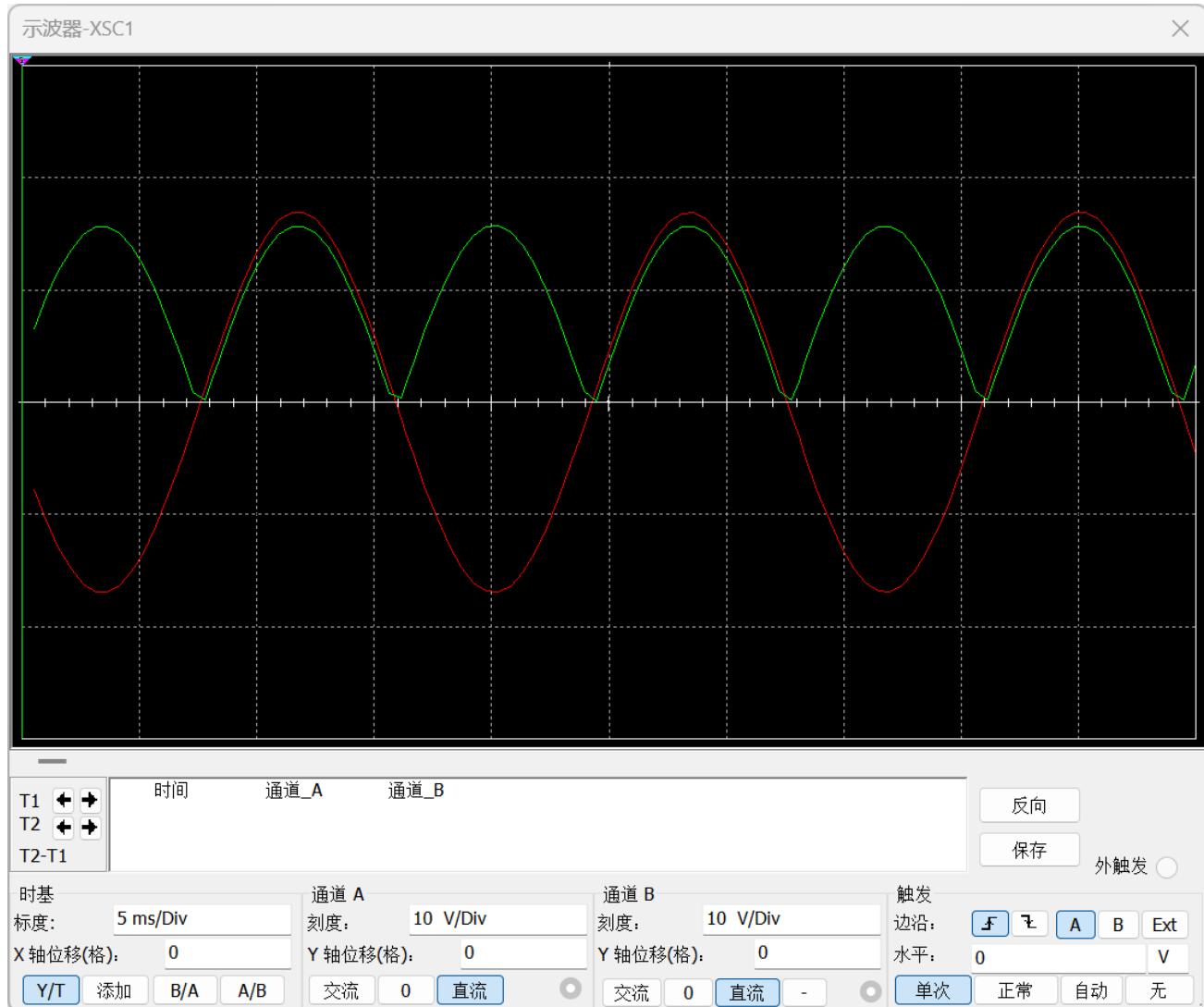


图 7.3: 7.4 讲义题 4-4 原题图



(a) 仿真电路



(b) 示波器波形

图 7.4: Multisim 仿真

(6) 习题集 5-15

电路原理习题集与所有作业课件已上传到网址 <https://www.123865.com/s/0y0pTd-L8Kj3>, 后文不再重复叙述题目。

先求电路的工作点 (U_0, I_0) , 当 $u_s(t) = 0$ 时, 由 KVL:

$$10 = I_0 \cdot 1 + \left(I_0 + \frac{2}{3}I_0^3 \right) \Rightarrow I_0 = 2.0650 \text{ A} \quad (7.6)$$

再考虑小信号带来的变化量, 在小信号电路中, 由 KVL:

$$\begin{cases} R_d = \left. \frac{du}{di} \right|_{i=I_0} = 1 + 2I_0^2 = 9.5281 \Omega \\ u_s(t) = (R + R_d)\Delta i \end{cases} \Rightarrow \Delta i = \frac{1}{10.5281} u_s(t) = 0.0095 \sin(10^3 t) \text{ A} \quad (7.7)$$

因此总电流为:

$$i = I_0 + \Delta i = 2.0650 \text{ A} + 0.0095 \sin(10^3 t) \text{ A} \quad (7.8)$$

7.5 讲义题 4-6

设二极管工作于截止区, 这等价于 $u_{GS} < U_T$ 。而此时 $u_o = 0 \Rightarrow u_{GS} = 2 \text{ V} > U_T = 1 \text{ V}$, 矛盾!

假设二极管工作于线性区, 这等价于 $u_{GS} > U_T$ 且 $u_{DS} < u_{GS} - U_T$ 。由题意, 二极管导通电阻为 $R_{ON} = 1 \text{ K}\Omega$, 因此 $u_o = 2.5 \text{ V} \Rightarrow u_{GS} = -0.5 \text{ V} < U_T$, 矛盾! 因此二极管只能工作在恒流区。

假设二极管工作于恒流区, 这等价于 $u_{GS} > U_T$ 且 $u_{DS} > u_{GS} - U_T$ 。则导通电流为 $i = \frac{K}{2} (u_{GS} - U_T)^2 = \frac{K}{2} (1 - u_o)^2$, 由 KVL:

$$u_o = iR_L = \frac{K}{2} (1 - u_o)^2 \cdot R_L \Rightarrow u_o = 0.6417 \text{ V}, 1.5583 \text{ V} \quad (7.9)$$

$u_o = 1.5583 \text{ V}$ 时 $u_{GS} = 0.4417 \text{ V} < U_T = 1 \text{ V}$, $u_{DS} = 3.4417 \text{ V}$, 不符合假设, 舍去; $u_o = 0.6417 \text{ V}$ 时 $u_{GS} = 1.3583 \text{ V}$, $u_{DS} = 4.3583 \text{ V}$, 符合假设。因此工作区为恒流区, 此时 $u_o = 0.6417 \text{ V}$ 。

7.6 讲义题 4-7

(1) 画出电路, 标明各参数

小信号电路如图 7.5 所示。

(2) 求小信号放大倍数

在 $\max |\Delta u_{GS}| = 2 \text{ mV}$ 的条件下, 容易验证二极管仍工作于恒流区。由 7.5 讲义题 4-6 可知二极管的工作点为 $u_{GS} = 1.3583 \text{ V}$, 则有:

$$i = \frac{K}{2} (u_{GS} - U_T)^2, \quad g_m = \left. \frac{di}{du_{GS}} \right|_{u_{GS}=1.3583 \text{ V}} = K \cdot (u_{GS} - U_T) = 3.583 \times 10^{-3} \text{ A} \cdot \text{V}^{-1} \quad (7.10)$$

$$\Rightarrow \Delta u_o = R_L \Delta i = R_L g_m \Delta u_{GS} \quad (7.11)$$

又 $\Delta u_{GS} = \Delta u_{in} - \Delta u_{out}$, 代入消去 Δu_{GS} , 得到:

$$\Delta u_{out} = R_L g_m (\Delta u_{in} - \Delta u_{out}) \Rightarrow \frac{\Delta u_{out}}{\Delta u_{in}} = \frac{R_L g_m}{1 + R_L g_m} = \frac{3.583}{4.583} = 0.7818 \quad (7.12)$$

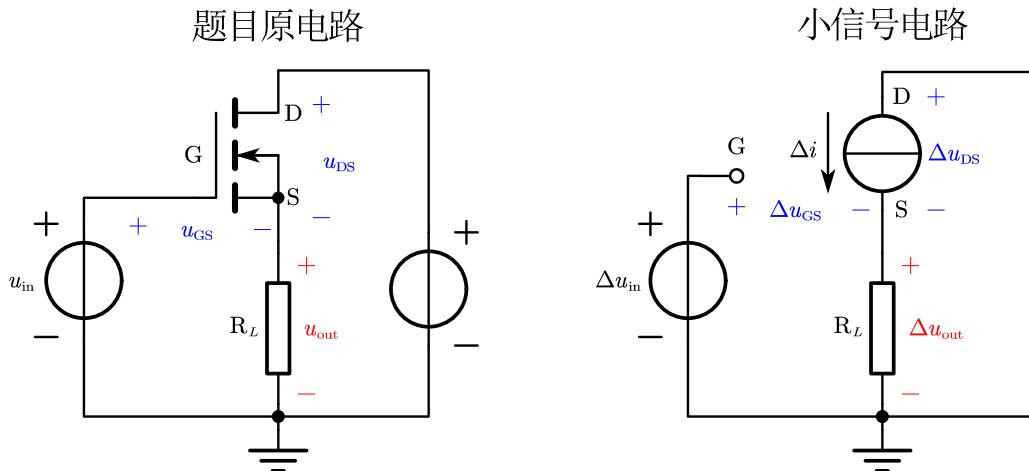


图 7.5: 7.6 讲义题 4-7 电路图

(3) 求小信号电路的输入电阻和输出电阻, 此 MOSFET 的作用是什么?

在小信号电路中, 求输入电阻时无需对输出端口作处理 (即保持开路), 由于 MOS 管 G 端流入的电流近似为 0, 而输入电压 Δu_{in} 非零, 因此输入电阻为 $R_i = \infty$ 无穷大。

求输出电阻时, 将 Δu_{in} 短路, 并在输出端口加极小的电压 u (以保证 MOSFET 仍在工作点附近), 于是 $\Delta u_{GS} = -u$, 有:

$$\begin{cases} \Delta u_{GS} = -u \\ i = \frac{u}{R_L} - \Delta i_{DS} = \frac{u}{R_L} - g_m \Delta u_{GS} \end{cases} \implies R_o = \frac{u}{i} = \frac{1}{\frac{1}{R_L} + g_m} = 218.1977 \Omega \quad (7.13)$$

综上有:

$$R_i = \infty, \quad R_o = \frac{1}{\frac{1}{R_L} + g_m} = 218.1977 \Omega \quad (7.14)$$

由 $\frac{\Delta u_{out}}{\Delta u_{in}} = 0.7818$ 可以看出, MOSFET 降低了激励的微小变化 (例如噪声) 对响应信号的影响, 使得电路更加稳定, 可用于构建差分放大器。

查阅资料知道, 这是一个共漏放大器, 其输出电阻比共源放大器的输出电阻要小。这样的一个放大倍数小于 1, 但输出电阻比较小的信号处理单元适合放在整个信号处理电路的最后一级, 以增强整个电路的带载能力。此外, 由于共漏放大器的输出在 MOSFET 的 S 极, 且在 g_m 较大的情况下输入和输出基本相同 (输出能够跟随输入), 因此该电路也称为源级跟随器。

Homework 08: 2024.10.23 - 2024.10.29

8.1 习题集 1-3

电容有 $q = Cu \Rightarrow i = C \frac{du}{dt}$, 于是:

$$i = i(t) = C \frac{du}{dt} = \begin{cases} 50 \mu\text{A} & , t \in [0, 1 \text{ s}) \\ -250 \mu\text{A} & , t \in [1 \text{ s}, 1.2 \text{ s}) \\ 0 & , t \in [1.2 \text{ s}, 2 \text{ s}) \\ 50 \mu\text{A} & , t \in [2 \text{ s}, 3 \text{ s}) \\ -250 \mu\text{A} & , t \in [3 \text{ s}, 3.2 \text{ s}) \\ 0 & , t \in [3.2 \text{ s}, +\infty) \end{cases} \quad (8.1)$$

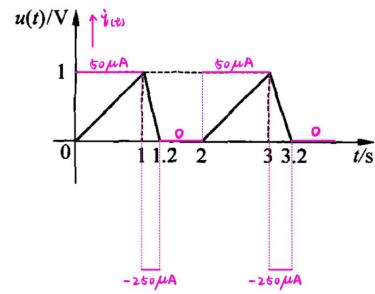


图 8.1: $i = i(t)$ 的图像

8.2 习题集 1-4

电感有 $\Psi = Li \Rightarrow u = L \frac{di}{dt} \Rightarrow i = i_0 \frac{1}{L} \int_0^t u dt$ 。又初始磁能为零, 即 $i_0 = 0$, 于是有:

$$i = i(t) = \begin{cases} 5t^2 \text{ A} & , t \in [0, 1 \text{ s}) \\ (-5t^2 + 20t - 10) \text{ A} & , t \in [1 \text{ s}, 2 \text{ s}) \\ 10 \text{ A} & , t \in [2 \text{ s}, +\infty) \end{cases} \quad (8.2)$$

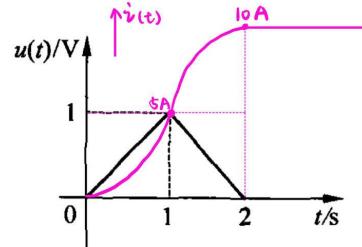


图 8.2: $i = i(t)$ 的图像

Homework 09: 2024.10.30 - 2024.11.05

9.1 习题集 7-6-a

为求 i_1 和 i_C 的初始值, 考虑 $t = 0^+$ 时的等效电路。 $u_{C,0+} = u_{C,0-} = 10 \text{ V}$, 因此 $t = 0^+$ 时电容 C 相当于一个 10 V 电压源, 有:

$$10 + ri_1 - 10 \text{ K}\Omega \cdot i_1 - 20 \text{ K}\Omega \cdot (i_1 + i_S) = 0 \quad (9.1)$$

$$\Rightarrow i_1(0^+) = \frac{10 - 1000}{30 \text{ K} - 50} \text{ A} = -33.0551 \text{ mA}, \quad i_C(0^+) = -(i_S + i_1) = -16.9449 \text{ mA} \quad (9.2)$$

不妨顺手求解一下 $u_C(t)$ 和 $i_C(t)$ 。对电路作戴维南等效以求解 u_C , 如图 9.1 (b) 所示, 有:

$$U_{\text{oc}} = 0 - (10 \text{ K}\Omega - r) \cdot 50 \text{ mA} = (2.5 - 500) \text{ V} = -497.5 \text{ V}, \quad R_{\text{eq}} = 30 \text{ K}\Omega - r = (30 \text{ K} - 50) \Omega \quad (9.3)$$

依据三要素方法, $u_{C,0+} = 10 \text{ V}$, $\tau = R_{\text{eq}}C$, $u_{C,\infty} = U_{\text{oc}} = -497.5 \text{ V}$, 可以得到:

$$u_C(t) = u_{C,0+} \cdot e^{-\frac{t}{\tau}} + u_{C,\infty} \cdot \left[1 - e^{-\frac{t}{\tau}} \right] = 10 e^{-\frac{t}{\tau}} + U_{\text{oc}} \cdot \left[1 - e^{-\frac{t}{\tau}} \right] \quad (9.4)$$

$$i_C(t) = C \frac{du_C}{dt} = -\frac{10}{R_{\text{eq}}} e^{-\frac{t}{\tau}} + \frac{U_{\text{oc}}}{R_{\text{eq}}} e^{-\frac{t}{\tau}}, \quad t \geq 0^+ \quad (9.5)$$

$$\Rightarrow i_C(0^+) = \frac{U_{\text{oc}} - 10}{R_{\text{eq}}} = -16.9449 \text{ mA} \quad (9.6)$$

可以得到同样的结果。

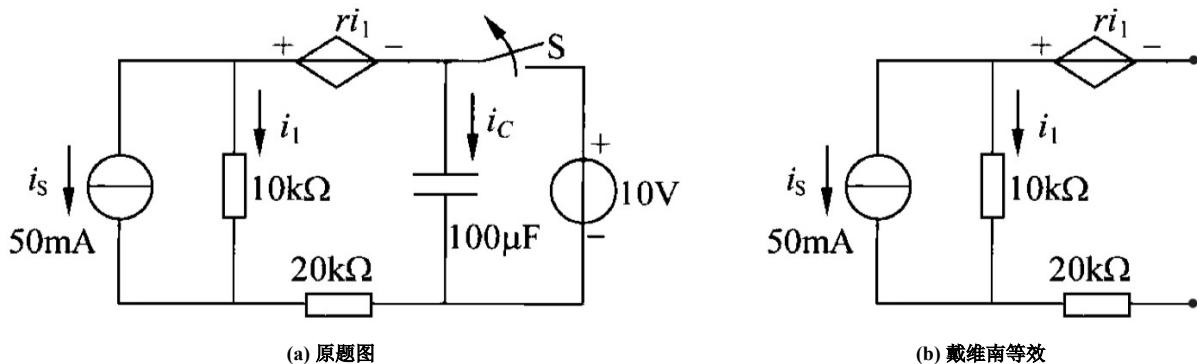


图 9.1: 9.1 习题集 7-6-a

9.2 习题集 7-10

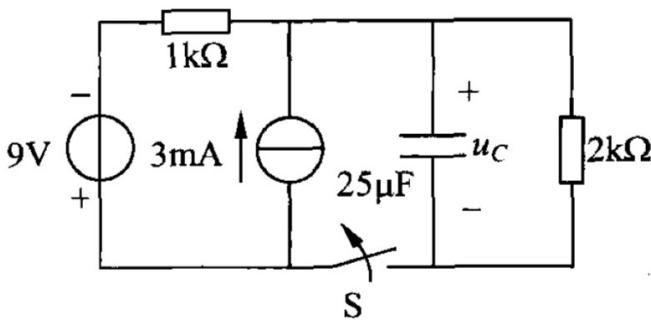
$t = 0$ 时开关断开, 于是:

$$u_{C,0+} = -4 \text{ V}, \quad u_{C,\infty} = 0, \quad \tau = 2 \text{ K}\Omega \cdot C = 50 \text{ ms} \quad (9.7)$$

$$\Rightarrow u_C(t) = u_{C,0+} \cdot e^{-\frac{t}{\tau}} + u_{C,\infty} \cdot \left[1 - e^{-\frac{t}{\tau}} \right] = -4 e^{-\frac{1000}{50}t} \text{ V} \quad (9.8)$$

$$\boxed{u_C(t) = -4 e^{-20t} \text{ V}, \quad t \geq 0^+} \quad (9.9)$$

式中 t 的单位是秒 (s), 作出 $u_C(t)$ 的图像, 如图 9.2 (b) 所示。



(a) 原题图

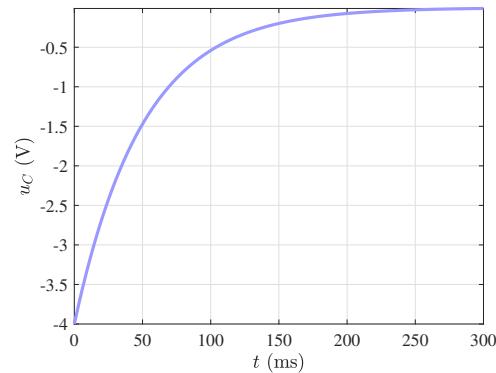
(b) $u_C = u_C(t)$ 的图像

图 9.2: 9.2 习题集 7-10

9.3 习题集 7-12

开关闭合前 ($t = 0^-$)，由电阻分压 $u_{C,0^-} = 18 \cdot \frac{2}{2+3} \cdot \frac{1}{2} = \frac{18}{5}$ V， $t = 0^+$ 后开关闭合，两个 $3\text{ k}\Omega$ 电阻被短路，且电压源置零（零输入响应），只剩下两个 $3\text{ k}\Omega$ 电阻与电容并联，于是：

$$u_{C,0^+} = u_{C,0^-} = \frac{18}{5} \text{ V}, \quad u_{C,\infty} = 0, \quad \tau = R_{\text{eq}}C = 3 \text{ ms} \quad (9.10)$$

$$\Rightarrow u_C(t) = u_{C,0^+} \cdot e^{-\frac{t}{\tau}} + u_{C,\infty} \cdot \left[1 - e^{-\frac{t}{\tau}} \right] = \frac{18}{5} e^{-\left(\frac{t}{3} \times 10^3\right)} \text{ V} \quad (9.11)$$

$$u_C(t) = 3.6 e^{-333.3t} \text{ V}, \quad i_1(t) = -\frac{1}{3\text{ K}} u_C(t) = -1.2 e^{-333.3t} \text{ mA}, \quad t \geq 0^+ \quad (9.12)$$

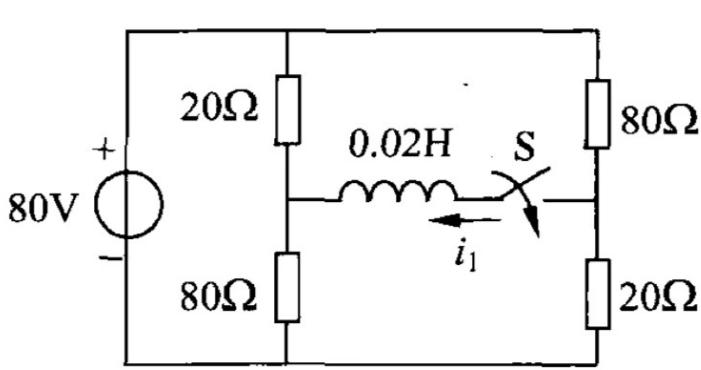
9.4 习题集 7-17

作戴维南等效可得 $U_{\text{oc}} = 80 \cdot \left(\frac{1}{5} - \frac{4}{5} \right) = -48 \text{ V}$ ， $R_{\text{eq}} = (20 \parallel 80) + (20 \parallel 80) = 32 \Omega$ 。因为是零状态响应， $i_{L,0} = 0$ ，于是有：

$$i_{L,0} = 0, \quad i_{L,\infty} = \frac{U_{\text{oc}}}{R_{\text{eq}}} = -1.5 \text{ A}, \quad \tau = \frac{L}{R} = \frac{5}{8} \text{ ms} = 0.625 \text{ ms} \quad (9.13)$$

$$\Rightarrow i_L = -1.5 \cdot \left[1 - e^{-\left(\frac{8t}{5} \times 10^3\right)} \text{ A} \right] = -1.5 + 1.5 e^{-1600t} \text{ A}, \quad t \geq 0^+ \quad (9.14)$$

作出图像，如图 9.3 (b) 所示。



(a) 原题图

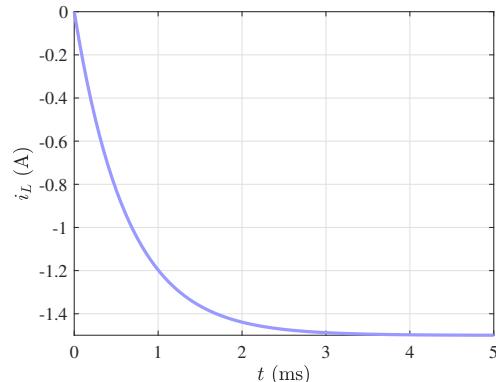
(b) $i_L = i_L(t)$ 的图像

图 9.3: 9.4 习题集 7-17

9.5 习题集 7-28

由电压关系可以发现, 对电感而言, 2 H、5 Ω 和 5 V 三个元件构成一个 RL 电路, 因此有:

$$i_{L,0^+} = \frac{10 \text{ V}}{20 \Omega} = 0.5 \text{ A}, \quad i_{L,\infty} = \frac{5 \text{ V}}{5 \Omega} = 1 \text{ A}, \quad \tau = \frac{L}{R} = \frac{2}{5} \text{ s} = 0.4 \text{ s} \quad (9.15)$$

$$\Rightarrow i_L(t) = 0.5 e^{-2.5t} + 1 \cdot [1 - e^{-2.5t}] \text{ A} = 1 - 0.5 e^{-2.5t} \text{ A}, \quad t \geq 0^+ \quad (9.16)$$

对电容而言, 两个有伴电压源与其并联 (注意有一个 5 Ω 电阻不起作用), 戴维南等效得到 $U_{\text{oc}} = \frac{20}{3} \text{ V}$, $R_{\text{eq}} = \frac{10}{3} \Omega$, 由于 $u_{C,0^+} = 5 \text{ V}$, 可以得到:

$$i_{C,0^+} = \frac{\frac{20}{3} - 5}{\frac{10}{3}} = 0.5 \text{ A}, \quad i_{L,\infty} = 0, \quad \tau = R_{\text{eq}} C = \frac{10}{3} \text{ s} = 0.333 \text{ s} \quad (9.17)$$

$$\Rightarrow i_C(t) = 0.5 e^{-0.3t} \text{ A}, \quad t \geq 0^+ \quad (9.18)$$

设流经 10 V 电压源的电流为 I , 由 KVL 可得 $10 - 10I - 5(I - i_C) = 5 \Rightarrow I = \frac{1}{3}i_C + \frac{1}{3}$, 于是:

$$i(t) = I - i_C - i_L = -\frac{2}{3}i_C - i_L + \frac{1}{3} = -\frac{2}{3} - \frac{1}{3}e^{-0.3t} + 0.5e^{-2.5t} \text{ A} \quad (9.19)$$

$$\Rightarrow i(t) = -0.6667 - 0.3333e^{-0.3t} + 0.5e^{-2.5t} \text{ A}, \quad t \geq 0^+ \quad (9.20)$$

9.6 习题集 7-35

(1) S_1 闭合前瞬间 ($t = 0^-$): $u_C = 1 \text{ V}$

(2) S_1 闭合后瞬间 ($t = 0^+$): $u_C = 1 \text{ V}$, $i_C = 1 \text{ A}$ 。对电路作戴维南等效, 得 $U_{\text{oc}} = 5 \text{ V}$, $R_{\text{eq}} = 4 \Omega$ 。

(3) $t \in (0, 1 \text{ s})$ 的三要素:

$$\begin{cases} \frac{1}{\tau} = \frac{1}{R_{\text{eq}} C} = 1 \text{ Hz} \\ u_{C,0^+} = 1 \text{ V}, u_{C,\infty} = 5 \text{ V} \\ i_{C,0^+} = 1 \text{ A}, i_{C,\infty} = 0 \end{cases} \Rightarrow \begin{cases} u_C(t) = 5 + (1 - 5) \cdot e^{-t} = 5 - 4e^{-t} \text{ V} \\ i_C(t) = 1 \cdot e^{-t} \text{ A} = e^{-t} \text{ A} \end{cases} \quad (9.21)$$

(4) S_2 闭合前瞬间 ($t = 1^-$): $u_C = 5 - \frac{4}{e} \text{ V}$, 记其为 $U_1 = 5 - \frac{4}{e} \text{ V} = 3.5285 \text{ V}$ 。

(5) S_2 闭合后瞬间 ($t = 1^+$): $u_C = U_1$, 对电路作戴维南等效, 得 $U_{\text{oc}} = 4.5 \text{ V}$, $R_{\text{eq}} = 2 \Omega$ 。于是 $i_C = \frac{4.5 - U_1}{2}$, 记作 $I_1 = \frac{4.5 - U_1}{2} = 0.4858 \text{ A}$ 。

(6) $t \in (1 \text{ s}, +\infty)$ 的三要素:

$$\begin{cases} \frac{1}{\tau} = \frac{1}{R_{\text{eq}} C} = 2 \text{ Hz} \\ u_{C,0^+} = U_1 = 3.5285 \text{ V}, u_{C,\infty} = 4.5 \text{ V} \\ i_{C,0^+} = I_1 = 0.4858 \text{ A}, i_{C,\infty} = 0 \end{cases} \Rightarrow \begin{cases} u_C(t) = 4.5 - 0.9715e^{-t} \text{ V} \\ i_C(t) = i_1 \cdot e^{-2(t-1)} \text{ A} = 0.4858e^{-t} \text{ A} \end{cases} \quad (9.22)$$

综上, 我们有:

$$u_C(t) = \begin{cases} 5 - 4e^{-t} \text{ V} & , t \in [0^+, 1 \text{ s}) \\ 4.5 - 0.9715e^{-2(t-1)} \text{ V} & , t \in [1 \text{ s}, +\infty) \end{cases}, \quad i_C(t) = \begin{cases} e^{-t} \text{ A} & , t \in [0^+, 1 \text{ s}) \\ 0.4858e^{-2(t-1)} \text{ A} & , t \in (1 \text{ s}, +\infty) \end{cases} \quad (9.23)$$

作出图像, 如图 9.4 所示。

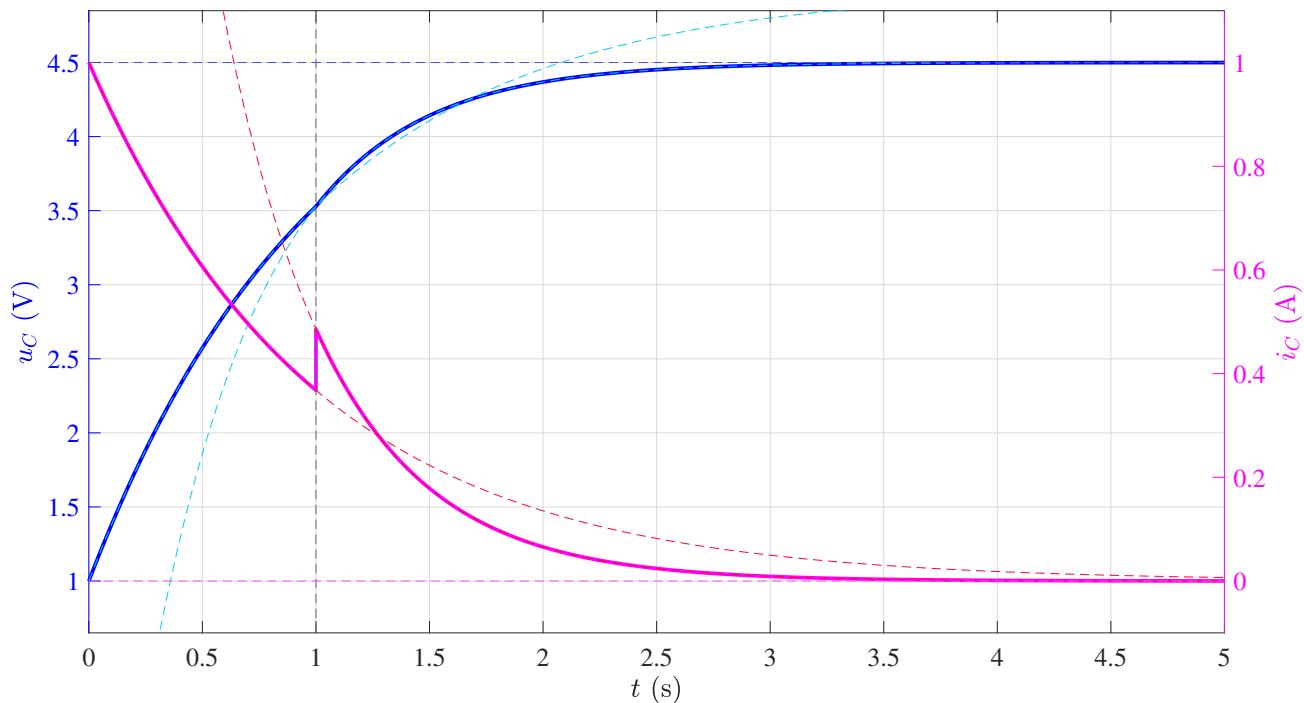


图 9.4: $u_C = u_C(t)$ 和 $i_C = i_C(t)$ 的图像

由于时间安排和 L^AT_EX 计划调整, 后续的几次作业都将在 Notability 上手写, 导出为 PDF 后插入到这里。插入前会对 PDF 进行极致压缩, 以尽量减小文件体积。

由于时间安排和 L^AT_EX 计划调整, 后续的几次作业都将在 Notability 上手写, 导出为 PDF 后插入到这里。插入前会对 PDF 进行极致压缩, 以尽量减小文件体积。

由于时间安排和 L^AT_EX 计划调整, 后续的几次作业都将在 Notability 上手写, 导出为 PDF 后插入到这里。插入前会对 PDF 进行极致压缩, 以尽量减小文件体积。

Homework 10 : 2024.11.06 - 2024.11.12

1. 习题集 8-9

(a) 两电容并联得 $C = 1.5 \text{ F}$, 如图:

以 U_c 为变量, 将激励置零后列方程.

由图, 有:

$$U_L + R i_L + U_C = 0$$

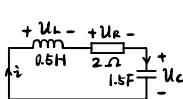
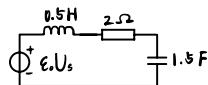
$$LC \ddot{U}_L + RC \dot{U}_L + \dot{U}_L = 0$$

$$\ddot{U}_L + \frac{R}{L} \dot{U}_L + \frac{1}{LC} U_L = 0$$

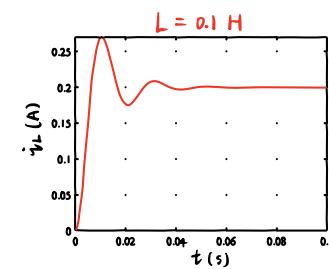
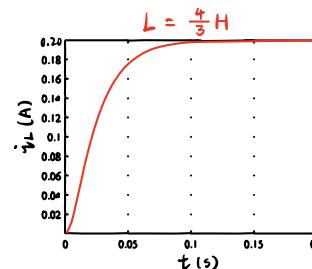
$$\text{衰减系数 } \beta = \frac{R}{2L} = 2 \text{ Hz}, \text{ 固有频率 } \omega_0 = \frac{1}{\sqrt{LC}} = 1.155 \text{ Hz}$$

$\beta > \omega_0$, 因此为过阻尼.

典型 RLC 串联



分别作出 (1) 和 (2) 的图像:



(b) 如图, 仍以 U_C 为变量, 有:

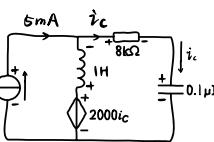
$$LC \ddot{U}_C + RC \dot{U}_C + U_C - 2000i_C = 0$$

$$LC \ddot{U}_C + (R-2000)C \dot{U}_C + U_C = 0$$

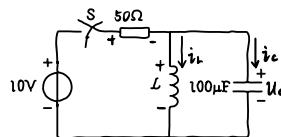
$$\text{衰减系数 } \beta = \frac{R-2000}{2L} = 3000 \text{ Hz}, \text{ 固有角频率 } \omega_0 = \frac{1}{\sqrt{LC}} = 3162.28$$

$\beta < \omega_0$, 为欠阻尼, 振荡角频率 $\omega = \sqrt{\omega_0^2 - \beta^2}$

$$\Rightarrow \omega = \sqrt{10^7 - 9 \times 10^6} = 1000 \text{ rad/s}$$



典型 RLC 并联



2. 习题集 8-10

先列零输入方程:

$$U_C + R(i_L + i_C) = 0$$

$$\ddot{U}_C + \frac{1}{RC} \dot{U}_C + \frac{1}{LC} U_C = 0$$

$$\text{衰减系数 } \beta = \frac{1}{2RC} = 100, \text{ 固有角频率 } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\text{求变量初值: } U_C(0^+) = 0, \dot{U}_C(0^+) = \frac{1}{C} i_C(0^+) = \frac{1}{C} \cdot \frac{U_s}{R} = 2000 \text{ V/s}$$

$$(1) L = \frac{4}{3} \text{ H}, \beta > \omega_0 = \frac{1}{\sqrt{LC}} = 86.6025, \text{ 为过阻尼.}$$

$$\omega = \sqrt{\beta^2 - \omega_0^2} = 100 \cdot \sqrt{1 - \frac{3}{4}} = 50, \text{ 过阻尼时齐次解为:}$$

$$y = e^{-\beta t} (A e^{\omega t} + B e^{-\omega t})$$

$$\text{其中 } A = \frac{1}{2\omega} [(\beta + \omega) y_{(0)} + y'_{(0)}], B = \frac{1}{2\omega} [(-\beta + \omega) y_{(0)} - y'_{(0)}]$$

$$\text{于是 } A = \frac{y_{(0)}}{2\omega} = 20, B = -A = -20$$

$$\Rightarrow U_C(t) = 20 [e^{-50t} - e^{-150t}] \text{ V}$$

$$i_L(t) = \frac{1}{L} [i_L(0^+) + \int_0^t U_C(t) dt] = 0.1 e^{-150t} - 0.3 e^{-50t} + 0.2 \text{ A}$$

$$(2) L = 0.1 \text{ H}, \beta < \omega_0 = \frac{1}{\sqrt{LC}} = 316.23, \text{ 为欠阻尼.}$$

$$\omega = \sqrt{\omega_0^2 - \beta^2} = 100 \cdot \sqrt{1 - \frac{1}{316.23^2}} = 300, \text{ 欠阻尼时齐次解为:}$$

$$y = A e^{-\beta t} \sin(\omega t + \phi)$$

$$\text{其中 } A = y_{(0)} \cdot \sqrt{1 + \frac{1}{\beta^2}}, \phi = \arctan(\frac{\omega}{\beta}), t = \frac{\omega}{\beta + \frac{y'_{(0)}}{\omega}}$$

$$\text{于是 } A = \frac{y_{(0)}}{\omega} = \frac{20}{3}, \phi = 0$$

$$\Rightarrow U_C(t) = \frac{20}{3} e^{-100t} \sin(300t) \text{ V}$$

$$i_L(t) = \frac{1}{L} [i_L(0^+) + \int_0^t U_C(t) dt] = -\frac{1}{15} e^{-100t} \cdot (300 \cos 300t + 60 \sin 300t) + \frac{1}{5}$$

$$= -\frac{4}{15} e^{-100t} \sin(300t + \arctan 3) + \frac{1}{5}$$

写成数值形式, $i_L(t) = -0.2108 e^{-100t} \cdot \sin(300t + 1.2490) + 0.2 \text{ A}$

其中 $\phi = \arctan 3 = 1.2490 = 71.5651^\circ$

3. 习题集 8-17

将电源置零, 为串联 RLC 电路, 有:

$$LC \ddot{U}_L + RC \dot{U}_L + U_L = 0$$

$$\beta = \frac{R}{2L} = 5, \omega_0 = \frac{1}{\sqrt{LC}} = 3$$

$$\beta > \omega_0, \text{ 为过阻尼, 角频率 } \omega = \sqrt{\omega_0^2 - \beta^2} = 4$$

先求 $t=0^+$ 初值:

$$U_C(0^+) = 150 \text{ V}, \dot{U}_C(0^+) = \frac{1}{C} i_C(0^+) = 108 \text{ V/s}$$

齐次解 $y = e^{-\beta t} (A e^{\omega t} + B e^{-\omega t})$, 其中 $\beta=5, \omega=4$

$$\text{全解 } U_C(t) = A e^{-5t} + B e^{-4t} + 150 \text{ V}$$

$$\begin{cases} U_C(0^+) = A + B + 150 = 150 \\ U_C'(0^+) = -A - 9B = 108 \end{cases} \Rightarrow \begin{cases} A = 13.5 \\ B = -13.5 \end{cases}$$

$$\text{故 } U_C(t) = 150 + 13.5 (e^{-5t} - e^{-4t}) \text{ V}$$

4. 习题集 8-18

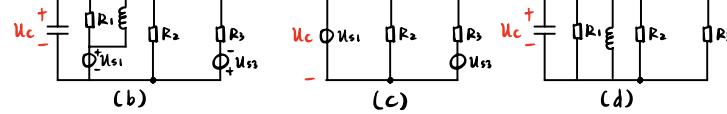
先求初始值, $U_C(0^+) = U_C(0^-) = 1 \text{ V}$,

$$U_C'(0^+) = \frac{1}{C} i_C(0^+) = -\frac{15}{4} \text{ V/s}.$$

考虑稳态解, 等效电路如图 (c), 则:

$$U_C(\infty) = U_{S1} = 1 \text{ V}.$$

将电源置零, 如图 (d), 可知是 RLC 并联电路.



等效电阻 $R = R_1 // R_2 // R_3 = 1 \Omega$, $C = \frac{1}{5} \text{ F}$, $L = \frac{5}{6} \text{ H}$, 列出齐次方程:

$$\ddot{U}_C + \frac{1}{RC} \dot{U}_C + \frac{1}{LC} U_C = 0, \beta = \frac{1}{2\sqrt{LC}} = 2.5, \omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{6}.$$

$\beta > \omega_0$, 因此为过阻尼, $\omega = \sqrt{\beta^2 - \omega_0^2} = 0.5$, 全解为:

$$U_C(t) = e^{-2.5t} (A e^{0.5t} + B e^{-0.5t}) + 1 \text{ (V)}$$

$$= A e^{-2t} + B e^{-3t} + 1 \text{ (V)}$$

$$U_C(0^+) = A + B + 1 = 1, U_C'(0^+) = -2A - 3B = -\frac{15}{4}$$

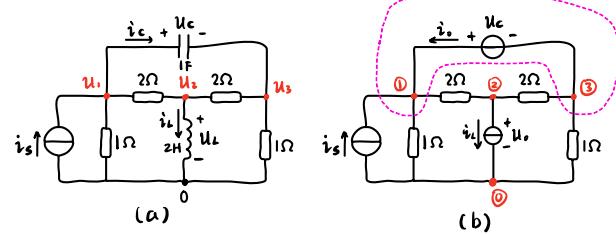
$$\Rightarrow A = -\frac{15}{4}, B = \frac{15}{4}, \text{ 因此:}$$

$$U_C(t) = 1 - 3.75 e^{-2t} + 3.75 e^{-3t} \text{ (V)}$$

5. 习题集 9-5

参考方向如图(a), 用替代法, 将C视为电压源 U_C , L视为电流源 i_L

求解 i_s, U_o 即可.



选取参考节点如图(b), 由节点电压法:

$$\begin{cases} U_1 - U_3 = U_C \\ -\frac{1}{2}U_1 + (\frac{1}{2} + \frac{1}{2})U_2 - \frac{1}{2}U_3 = -i_L \\ (1 + \frac{1}{2})U_1 - (\frac{1}{2} + \frac{1}{2})U_2 + (1 + \frac{1}{2})U_3 = i_s \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ \frac{1}{2} & -1 & \frac{1}{2} \\ \frac{3}{2} & -1 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} U_C \\ i_L \\ i_s \end{bmatrix},$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ \frac{1}{2} & -1 & \frac{1}{2} \\ \frac{3}{2} & -1 & \frac{3}{2} \end{bmatrix}^{-1} \begin{bmatrix} U_C \\ i_L \\ i_s \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{3}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} U_C \\ i_L \\ i_s \end{bmatrix}$$

$$\text{而 } U_L = U_2 = -\frac{3}{2}i_L + \frac{1}{2}i_s$$

$$\text{由节点①的KCL: } i_s = U_1 + \frac{1}{2}(U_1 - U_2) + i_C$$

$$\Rightarrow i_C = i_s - \frac{3}{2}U_1 + \frac{1}{2}U_2$$

$$= i_s - \frac{3}{2} \cdot \frac{1}{2}(U_C - i_L + i_s) + \frac{1}{4} \cdot (-3i_L + i_s)$$

$$= -\frac{3}{4}U_C + \frac{1}{2}i_s$$

写成标准形式 $\frac{dx}{dt} = AX + BS$, 为:

$$\begin{cases} L\dot{i}_L = \frac{3}{2}i_L + \frac{1}{2}i_s \\ C\dot{U}_C = -\frac{3}{4}U_C + \frac{1}{2}i_s \end{cases} \Rightarrow \begin{cases} 2\dot{i}_L = \frac{3}{2}i_L + \frac{1}{2}i_s \\ U_C = -\frac{3}{4}U_C + \frac{1}{2}i_s \end{cases}$$

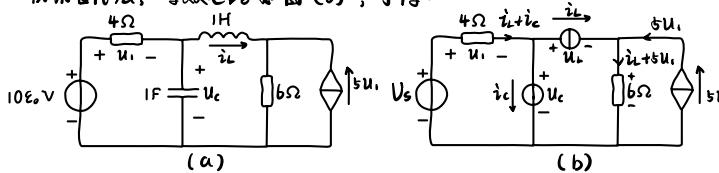
$$\Rightarrow \begin{bmatrix} U_C \\ i_L \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} & 0 \\ 0 & -\frac{3}{4} \end{bmatrix} \begin{bmatrix} U_C \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ \frac{1}{4} \end{bmatrix} i_s, \text{ 对输出变量有:}$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ 0 & -3 & 1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} U_C \\ i_L \\ i_s \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 0 & -3 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} U_C \\ i_L \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} i_s$$

$$\Rightarrow \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 \\ 0 & -1.5 \\ -0.5 & -0.5 \end{bmatrix} \begin{bmatrix} U_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} i_s$$

6. 习题集 9-13

仍用替代法, 等效电路如图(b), 可得:



$$\begin{cases} U_C - U_L - 6[i_L + 5 \cdot 4(i_C + i_L)] = 0 \\ U_s - 4(i_C + i_L) - U_C = 0 \\ i_C = -\frac{1}{4}U_C - i_L + \frac{1}{4}U_s \\ U_L = 31U_C - 6i_L - 30U_s \end{cases}$$

$$\Rightarrow \begin{bmatrix} U_C \\ i_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{4} & -1 \\ 31 & -6 \end{bmatrix} \begin{bmatrix} U_C \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{4} \\ -30 \end{bmatrix} U_s$$

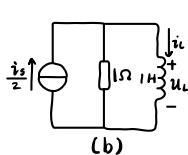
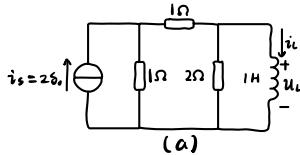
特别地, $t \rightarrow 0$ 时 $U_s = 10V$, 写成数值形式为:

$$\begin{bmatrix} U_C \\ i_L \end{bmatrix} = \begin{bmatrix} -0.25 & -1 \\ 31 & -6 \end{bmatrix} \begin{bmatrix} U_C \\ i_L \end{bmatrix} + \begin{bmatrix} 2.5 \\ -300 \end{bmatrix}$$

Homework 11 : 2024.11.13 - 2024.11.19

1. 习题集 7-41 (书上答案不正确)

对原电路(a)作 Norton 等效, 如(b).



$$\begin{cases} F(t) - F(0) = 8(1 - e^{-t}) & (V), t \in (0, 2) \\ F(t) - F(t-2) = 8e^{-t}(e^2 - 1) & (V), t \in (2, 3) \\ F(t) - F(t-2) = \frac{8(e^{-(t-2)} - e^{-t})}{0} & (V), t \in (3, 5) \\ \quad \quad \quad = 0 & (V), t \in (5, +\infty) \end{cases}$$

这里答案上写的
是错误的

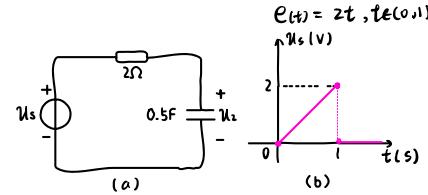
5. 习题集 7-51

先求单位冲激响应:

$$\Delta u_c = \frac{1}{RC} = 1V, \tau = RC = 1s \\ \Rightarrow h_{u_c}(t) = \epsilon(t) e^{-t} (V)$$

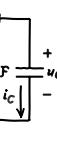
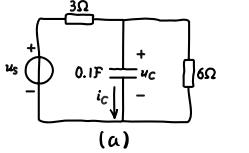
现在输入 $E(t)$ 为

$$E(t) = \begin{cases} 2t, t \in [0, 1] \\ 0, \text{else} \end{cases}, \text{则响应 } f_{u_c}(t) \text{ 为:} \\ f_{u_c}(t) = E(t) * h_{u_c}(t) = \int_{-\infty}^{+\infty} E(t-\tau) h(\tau) d\tau \xrightarrow{\text{卷积}} 2(t-1) e^{-t}, t > 0 \\ \text{令 } F(t) = \int 2(t-1) e^{-t} dt = 2e^{-t}(t+1-t) + C, \text{不妨令 } C=0, \text{则:} \\ F(t) = 2e^{-t}, F(0) = 2 - 2t, F(t-1) = 0 \\ f_{u_c}(t) = \begin{cases} F(t) - F(0) = 2e^{-t} + 2t - 2 & (V), t \in (0, 1) \\ F(t) - F(t-1) = 2e^{-2t} & (V), t \in [1, +\infty) \end{cases}$$



2. 习题集 7-42

等效电路图中 (b), 先求 step response:



本题是典型的冲激电路,
单位冲激带来的突变 $\Delta u_c = \frac{1}{RC}$
因此这里 $\Delta u_c = 10V$. 可直接
得到结果.

是典型的RC串联电路, 由三要素法:

$$S_{u_c}(t) = \left(\frac{2u_c}{3}\right) \cdot (1 - e^{-\frac{t}{6}}) = 2\epsilon(t)(1 - e^{-\frac{t}{6}}) \\ \Rightarrow h_{u_c}(t) = \frac{ds}{dt} = \frac{2\epsilon(t)}{\tau} e^{-\frac{t}{6}} = 10\epsilon(t)e^{-\frac{t}{6}} (V)$$

再求零输入响应, 由三要素法: $U_c(t) = 2 \cdot e^{-\frac{t}{6}} (V), t > 0$

$$\text{得到全响应: } U_c = 2 \cdot e^{-\frac{t}{6}} + 10e^{-\frac{t}{6}} = 12e^{-\frac{t}{6}} (V), t > 0 \\ i_c = C \frac{du_c}{dt} = -6e^{-\frac{t}{6}} (A), t > 0$$

3. 习题集 7-47

先求 $i_{c1}(t)$, S 闭合前等效电容 $C = \frac{1}{3}F$.

$U_c(0^-) = 4V, i_{c1}(0^-) = 0$. S 闭合后等效

电容 $C' = \frac{2}{5}F$, 由于 $q = CU$ 得:

$$U_c(0^+) = \frac{5}{6} U_c(0^-) = \frac{10}{3}V, \text{由三要素法: } \tau = RC' = \frac{4}{5}s, \frac{1}{\tau} = 1.25 \text{ Hz}.$$

$$U_c(t) = \frac{10}{3} \cdot e^{-1.25t} + 4(1 - e^{-1.25t}) (V), t > 0$$

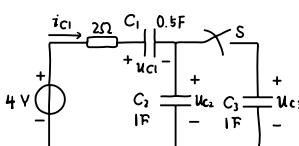
$$\Rightarrow i_{c1}(t) = C' \frac{du_c}{dt} = \frac{2}{5} \cdot \frac{5}{6} e^{-1.25t}, t > 0$$

U_{c3} 与 U_{c2} 恒相等, 它俩并联后 $C'' = C_2 + C_3 = 2F$, 由电容分压:

$$U_{c3}(t) = \frac{0.5}{2+0.5} U_c(t) = \frac{4}{5} - \frac{2}{15} e^{-1.25t} (V), t > 0. \text{ 写成数值}$$

形式: $U_{c3}(t) = 0.8 - 0.1333 e^{-1.25t} (V), t > 0$

$$i_{c1}(t) = 0.3333 e^{-1.25t} (A)$$



7. 习题集 11-2

(1) 规定 $\dot{U} = \frac{U_m}{\sqrt{2}} \angle \phi_u, \dot{I} = \frac{I_m}{\sqrt{2}} \angle \phi_i$. 且统一用正弦(I_m), 则:

$$\dot{U} = 220 \angle 45^\circ V, \dot{I} = \frac{10}{\sqrt{2}} \angle -30^\circ A = 7.0711 \angle -30^\circ A$$

$$(2) \Delta\phi = 45^\circ - (-30^\circ) = 75^\circ$$

(3) 如右图:



4. 习题集 7-49

由卷积定理, 任意激励下的响应 $f(t)$ 为:

$$f(t) = E(t) * h(t) = \int_{-\infty}^{+\infty} 4[\epsilon(t-\tau) - \epsilon(t-\tau-2)] h(\tau) d\tau = \int_{-\infty}^{+\infty} 4[\epsilon_t - \epsilon_{t-2}] h(\tau) d\tau$$

$$\text{令 } F(t) = \int 4 \cdot 2e^{-\tau} d\tau = -8e^{-\tau} + C, \text{不妨令 } C=0, \text{则 } f(t) \text{ 为:}$$

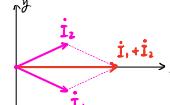
8. 习题集 11-3

$$I_1 = 2\sqrt{2} \angle -\frac{\pi}{6} A, I_2 = 2\sqrt{2} \angle \frac{\pi}{6} A$$

$$\Rightarrow \dot{I}_1 + \dot{I}_2 = 2\sqrt{2} \left(\frac{\sqrt{3}}{2} - \frac{1}{2}i \right) + 2\sqrt{2} \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = 2\sqrt{6} A = 2\sqrt{6} \angle 0 A$$

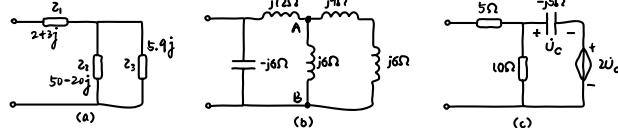
$$\text{于是 } \dot{i}_1 + \dot{i}_2 = 4\sqrt{3} \sin(314t) A = 6.9282 \sin(314t) A.$$

用作图法:



Homework 12 : 2024.11.20 - 2024.11.26

1. 习题集 11-7



$$(1) \quad Z_{in} = Z_1 + Z_2 // Z_3 = (2+3j) + (50-20j) // (5.9j) \approx (2.645 + 9.082j) \Omega$$

$$(2) \quad Z_{AB} = (6j) // (15j) = \frac{30}{7}j$$

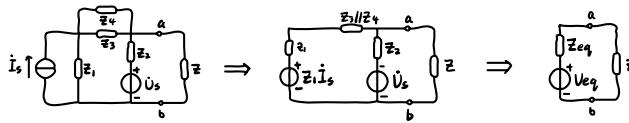
$$Z_{in} = (-6j) // (12j + \frac{30}{7}j) = -9.5j \Omega$$

(3) 叠加电压源可等效为 $Z = 2Z_c = -10j$, 则:

$$Z_{in} = 5 + 10 // (-15j) \approx (11.923 - 4.615j) \Omega$$

2. 习题集 11-38

先求 ab 端口的 Thevenin equivalent circuit.



$$\text{其中 } I_{eq} = \frac{U_s}{Z_2} + \frac{Z_1 \cdot I_s}{Z_1 + Z_2 // Z_4} = 0.8728 - 0.1749j$$

$$Z_{eq} = Z_2 // (Z_1 + Z_2 // Z_4) = 24.2050 + 3.7473j$$

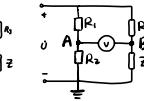
$$U_{eq} = I_{eq} \times Z_{eq} = 21.7821 - 0.9619j$$

$\Re\{Z\} \geq 0$, 因此当且仅当 $Z = -I_m\{Z_{eq}\} = -3.7473j \Omega$ 时, 电流 I 最大. 此时:

$$|I|_{max} = \frac{|U_{eq}|}{|Z_{eq} + Z|} = \frac{|U_{eq}|}{\Re\{Z_{eq}\}} = 0.9008 A$$

答案上写的 3.42 A 是在习题集第一版 $Z_2 = (15 - 10j) \Omega$ 的条件下得到的, 但第二版的条件是 $Z_2 = (111 - 10j) \Omega$.

3. 习题集 11-34

$\dot{U}_{AB} = \dot{U} \left[\frac{R_2}{R} - \frac{Z}{R_2 + Z} \right]$, 先设 $\frac{Z}{R_2 + Z} = x + jy$. 

$\frac{R_2}{R}$ 只能调节 $\frac{Z}{R_2 + Z}$ 的实部, $|\dot{U}_{AB}|$ 最小当且仅当

$$\Re\left\{\frac{R_2}{R} - \frac{Z}{R_2 + Z}\right\} = 0, \text{ 于是 } x = \frac{R_2}{R} = 0.8, \text{ 此时:}$$

$$|\dot{U}_{AB}| = |\dot{U}| \times |I_m\left(\frac{Z}{R_2 + Z}\right)| = 100|j| \Rightarrow y = \pm 0.3. \text{ 因此 } \frac{1}{\frac{R_2}{R} + 1} = x + jy$$

$$\Rightarrow Z = \frac{R_2}{\frac{1}{x+iy} - 1} = (3.5 \pm 15j) \Omega$$

4. 习题集 11-52

不妨以 \dot{I}_1 的相位为参考相位, 设 $\dot{I}_1 = x_1, \dot{I}_2 = x_2 + jy_2$, 则有:

$$\begin{cases} |\dot{I}| = 6 \\ |\dot{I}_1| = 3 \\ |\dot{I}_2| = 4.5 \end{cases} \Rightarrow \begin{cases} (x_1 + x_2)^2 + y_2^2 = 6^2 \\ x_1^2 = 3^2 \\ x_2^2 + y_2^2 = 4.5^2 \end{cases} \Rightarrow \begin{cases} x_1 = 3 \\ x_2 = 1.125 \\ y_2 \approx \pm 4.357 \end{cases}$$

$$\text{于是 } R_2 + jx_2 = \frac{R_1 \dot{I}_1}{\dot{I}_2} \approx 3.333 \mp 12.910j \Omega$$

由于 $x_2 > 0$, 应取 $y_2 = -4.357$, 此时 $R_2 = 3.333 \Omega, x_2 = 12.910 \Omega$

5. 习题集 7-32

先由相量法求稳态解 $\dot{i}_s(t)$. 对于 \dot{i}_2 , 利用叠加定理, 稳态时由激励 \dot{U}_s 产生的响应是恒定电流 $\frac{R_1}{R_1+R_2} \cdot \dot{i}_s = 1 A$.

在激励 $\dot{U}_s = 50\sqrt{2} \angle 90^\circ V$ 下的响应应为

$$\omega = 1000 \text{ rad/s} \quad L = 0.1 \text{ H}$$

图 (b), 可有:

$$\dot{i} = \frac{\dot{U}_s}{R_1 + R_2 + j\omega L} = 0.5 \angle 45^\circ A$$

因此稳态时 $\dot{i}_{2,\infty}(t) = 1 + \frac{\sqrt{2}}{2} \sin(1000t + 45^\circ) A, \dot{i}_{2,\infty}(0^+) = 1.5 A$

而 $\dot{i}_{1,\infty}(t) = \dot{i}_{2,\infty}(t) - \dot{i}_s = -1 + \frac{\sqrt{2}}{2} \sin(1000t + 45^\circ) A, \dot{i}_{1,\infty}(0^+) = -0.5 A$

再求原电路的初始值 (解题要输入解): $\dot{i}_{1,0^+} = -\dot{i}_s = -2 A, \dot{i}_{2,0^+} = 0$.

$$\text{又时间常量 } \tau = \frac{L}{R_1+R_2}, \tau^{-1} = 1000 \text{ Hz, 则:}$$

$$\dot{i}_1(t) = \dot{i}_{1,0^+} \cdot \exp(-\frac{t}{\tau}) + [\dot{i}_{1,\infty}(t) - \dot{i}_{1,\infty}(0^+) \cdot \exp(-\frac{t}{\tau})]$$

$$= (\dot{i}_{1,0^+} - \dot{i}_{1,\infty}(0^+)) \cdot \exp(-\frac{t}{\tau}) + \dot{i}_{1,\infty}(t)$$

$$= -1.5 e^{-1000t} + 0.707 \sin(1000t + 45^\circ) - 1 A, t \geq 0^+$$

$$\dot{i}_2(t) = (\dot{i}_{2,0^+} - \dot{i}_{2,\infty}(0^+)) \cdot \exp(-\frac{t}{\tau}) + \dot{i}_{2,\infty}(t)$$

$$= -1.5 e^{-1000t} + 0.707 \sin(1000t + 45^\circ) + 1 A, t \geq 0^+$$

6. 习题题 6-28

从电容两端向外看去, 有 Thevenin equivalent :

$$\dot{U}_{oc} = \frac{j\omega C}{R + j\omega C} \cdot \frac{U_m}{\sqrt{2}}, \quad U_{oc} = |\dot{U}_{oc}| = \frac{U_m}{\sqrt{2}} \cdot \frac{1}{\sqrt{C^2 R^2 \omega^2 + 1}}$$

$$Z_{eq} = R // \left(\frac{1}{j\omega C} \right) = \frac{1}{C^2 R^2 \omega^2 + 1} (R - \omega C R^2 j)$$

负载阻抗 $Z = R + j\omega L$, $\Im\{L\}$ 可调. 则当 $\omega L = -I_m\{Z_{eq}\}$ 时有最大功率.

$$\text{也即 } \omega L = \frac{\omega C R^2}{C^2 R^2 \omega^2 + 1} \stackrel{\omega^2 C^2 R^2 \gg 1}{\approx} \frac{\omega C R^2}{C^2 R^2 \omega^2} = \frac{1}{\omega C} \Rightarrow L = \frac{1}{\omega^2 C}$$

$$P_{max} = \frac{\Re\{Z\}}{(\Re\{Z\} + \Re\{Z_{eq}\})^2} \cdot U_{oc}^2 \approx \frac{R}{(R+R)^2} \cdot \frac{U_m^2}{2} = \frac{U_m^2}{8R}. \text{ 证毕.}$$

7. 习题题 6-25

$$\dot{U}_s = 100 \angle 30^\circ V, \dot{I}_s = 4 \angle 60^\circ A, Z_1 = Z_3 = 50 \angle 30^\circ \Omega, Z_2 = 50 \angle -30^\circ \Omega.$$

由叠加定理,

$$\dot{U} = \dot{I}_s (Z_1 + Z_2 + Z_3) + \dot{U}_s \frac{Z_2}{Z_2 + Z_3} = 96.603 + 350j V$$

$$\dot{I} = -\dot{I}_s \frac{Z_2}{Z_2 + Z_3} + \frac{\dot{U}_s}{Z_2 + Z_3} = 1 - 1.732j A$$

$$\text{等效电压源复功率为 } \tilde{S}_v = \dot{U}_s \dot{I}^* = 0 + 200j V \cdot A$$

$$\text{电源复功率为 } \tilde{S}_c = \dot{U} \dot{I}^* = 1385.6 + 400j V \cdot A$$

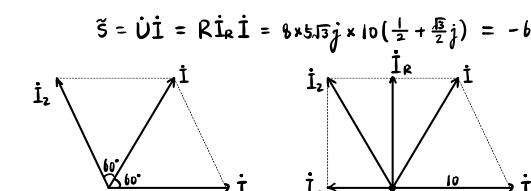
8. 习题题 6-26

$$\text{即 } \dot{I}_1 = 10 + 0j$$

以 \dot{I}_1 的相位为参考相位, $\dot{I} = \dot{I}_1 + \dot{I}_2$ 构成等边三角形, $\dot{I}_2 R = \dot{I}_1 j\omega L$

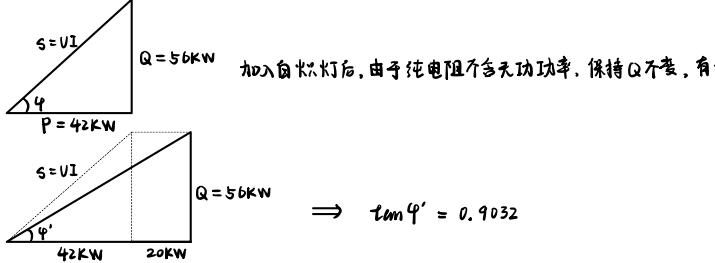
$\Rightarrow \dot{I}_2$ 相位超过 $\dot{I}_1 90^\circ$, 同理 \dot{I}_c 与 \dot{I}_1 反相, 又 $\dot{I}_c = \dot{I}_1 + \dot{I}_2$, 由相量图所示, 由几何关系得 $\dot{I}_c = -\frac{1}{2}\dot{I}_1 = -5$, $\dot{I}_2 = j\frac{\sqrt{3}}{2}|\dot{I}_1| = 5\sqrt{3}j$. 于是:

$$\tilde{S} = \dot{U} \dot{I} = R \dot{I}_2 \dot{I} = 6 \times 5\sqrt{3}j \times 10 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}j \right) = -600 + 200\sqrt{3}j$$



9. 习题集 11-49

$$\cos \varphi = \frac{P}{UI} = 0.6003 \quad (\text{感性}, \varphi > 0) \Rightarrow \tan \varphi = 1.33, \text{ 则各功率关系如图:}$$



$$\text{要使 } I = 325 \text{ A, 需要因数 } \cos \varphi'' = \frac{62 \text{ kW}}{220 \text{ V} \cdot 325 \text{ A}} = 0.8671 \Rightarrow \tan \varphi'' = 0.5745$$

于是需要并联的电容为:

$$C = \frac{P}{\omega U^2} (\tan \varphi' - \tan \varphi'') = \frac{62 \text{ k}}{2\pi \cdot 50 \cdot 220^2} (0.9032 - 0.5745) = 1.34 \text{ mF}$$

$$\text{此时 C 上的无功功率为 } Q_C = P(\tan \varphi'' - \tan \varphi') = -20.38 \text{ kVar}$$

Homework 13 : 2024.11.26 - 2024.12.02

1. 讲义题 6-9

$$(1) \text{传递函数 } H = \frac{1}{2} \cdot \frac{j\omega RC - 1}{j\omega RC + 1}$$

$$(2) |H| = \frac{1}{2}, \theta(\omega) = 180^\circ - 2 \arctan(\omega RC)$$

2. 讲义题

幅频特性 $|H(\omega)| = \begin{cases} 2+\omega, & \omega \in [-2, 0] \\ 2-\omega, & \omega \in [0, 2], \text{ else} \end{cases}$, 相频特性 $\theta(\omega) = -\frac{\pi}{4}\omega$.

该输入是角频率 ω 的 $U_s = U_s \angle \theta_0$, 则输出为 $U_{out} = |H(\omega)| U_s \angle (\theta_0 + \theta)$.

令输入记号 (ω, U_m, θ) , 则输入可写为:

$$U_s = (0, 10, 0) - (1, 6.4, 0) - (2, 3.2, 0) - (3, 2.1, 0)$$

$$\Rightarrow U_{out} = (0, 20, 0) - (1, 6.4, -\frac{\pi}{4}) - (2, 0, -\frac{\pi}{2}) - (3, 0, -\frac{3\pi}{4})$$

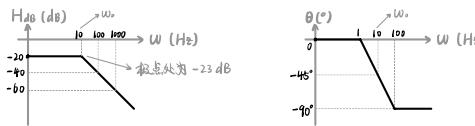
$$U_o = 20 + 6.4 \sin(t + 45^\circ)$$

3. 作出波特图

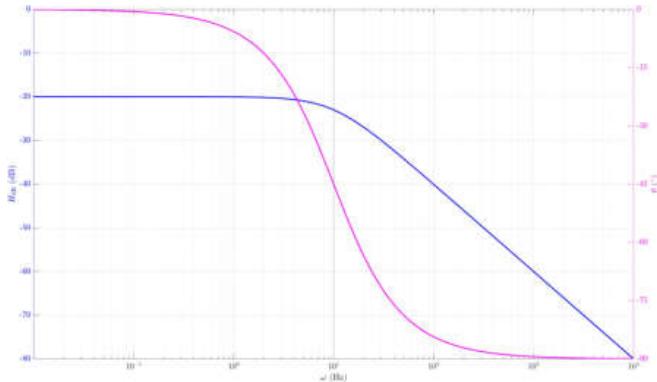
$$(1) H(\omega) = \frac{1}{10 + j\omega} = \frac{1}{10} \cdot \frac{1}{1 + j\frac{\omega}{10}}$$

无零点, 有唯一极点 $\omega_p = 10$, 初始分贝 -20 dB , 初始相位 0° , 终相位 -90° .

波特图如下:



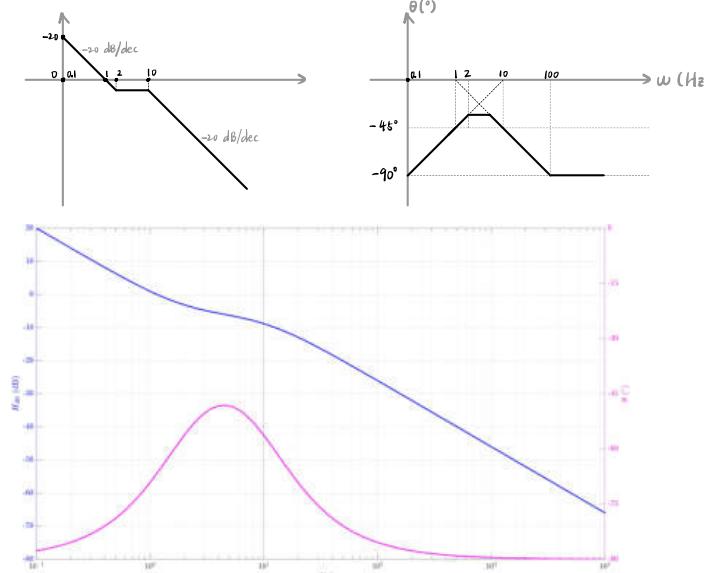
用 Matlab 作图验证:



$$(2) H(\omega) = \frac{5(j\omega+2)}{j\omega(10+j\omega)} = \frac{1+j\frac{\omega}{2}}{j\omega(1+j\frac{\omega}{10})}$$

有极点 $\omega_1 = 0$, 零点 $\omega_2 = 2$, 极点 $\omega_3 = 10$. 初始分贝 $+10 \text{ dB}$, $\omega = 1$ 时约 1 dB (视作 0)

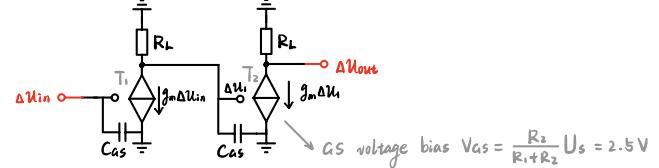
初始相位 -90° , 终相位 -90° . 波特图如下:



4. 讲义题 6-10

将电容与电阻的搭配视为截止频率

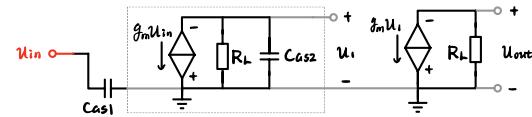
极低 ($\ll 0.1 \text{ Hz}$) 的高通滤波器, 因而带来直流偏置. 则小信号的等效电路如下:



先讨论直流偏置情况: $V_{DS} = 2.5 \text{ V} > V_T = 1 \text{ V}$, MOS导通. 要使 MOS 工作在恒流区(饱和区), 需满足 $V_{DS} < V_{GS} - V_T \Rightarrow V_{DS} > V_{GS} - V_T = 1.5 \text{ V}$, 此时 $I_{DS} = \frac{V_{DD} - V_{DS}}{R_L}$, $\Rightarrow I_{DS} < 0.85 \text{ mA}$. 按饱和区的电流特性 $I_{DS} = \frac{k}{2} (V_{GS} - V_T)^2 = 0.5625 \text{ mA}$, 符合要求, 因此 MOS 工作在恒流区. 有跨导 g_m : $\Delta I_{DS} = g_m \Delta V_{GS}$ $V_{DS} = 4.374 \text{ V}$

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = K(V_{GS} - V_T) = 0.75 \text{ mS}$$

为求输出电阻, 重画等效电路图如下, 其中灰色线路表示电流恒为零(虚断+虚短)



考虑 T_2 的电容 C_{as2} 对 T_1 MOS 的影响, T_1 输出阻抗 $Z_1 = R_L // Z_c = \frac{R_L}{1 + j\omega R_L C_{as}}$.

为方便书写, 将交流小信号 Δu 简记为 u , 作戴维南等效后, 可得 u_{in} 到 u_1 的电压关系:

$$u_1 = -(g_m u_{in}) Z_1$$

而 T_2 的输出阻抗仍为 $Z_2 = R_L$, 因此:

$$u_{out} = -(g_m u_1) Z_2 = Z_1 Z_2 g_m u_{in}$$

$$\Rightarrow H = \frac{u_{out}}{u_{in}} = Z_1 Z_2 g_m^2 = \frac{g_m^2 R_L^2}{1 + j\omega C_{as} R_L} = \frac{56.25}{1 + j\omega 10^3}$$

(1) $\omega = 10^4 \ll 10^7$, 可忽略介计, 于是 $H = 56.25 \Rightarrow u_{out} = 56.25 \sin(10^4 t) \text{ mV}$.

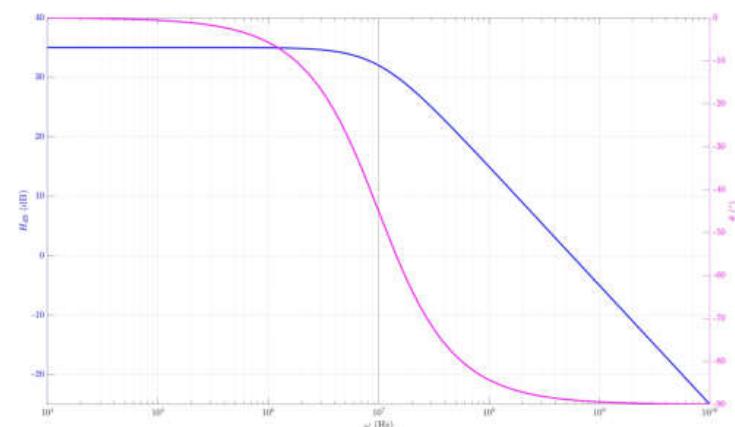
$$V_{out, bias} = V_{DD} - I_{DS} R_L = 4.374 \text{ V}$$

$$\Rightarrow V_{out} = V_{bias} + u_{out} = 4.374 + 0.05625 \sin(10^4 t) \text{ V}$$

(2) 幅值 $|H|$ 下降到初始值的 $\frac{1}{\sqrt{2}}$ 时, 达到半功率频率 ω_c , 有:

$$\omega_c = 10^7 \text{ Hz} = 10 \text{ MHz}, f_c = 1.6 \text{ MHz}$$

不妨作出波特图, 如下:



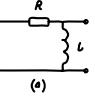
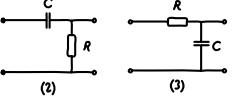
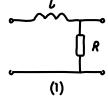
4. 讲义题 6-11

(1) RL 低通滤波器, $H = \frac{1}{1+j\frac{\omega}{\omega_c}}$, 截止频率 $\omega_c = \frac{1}{L} = \frac{R}{L}$

(2) RC 高通滤波器, $H = \frac{j\frac{\omega}{\omega_c}}{1+j\frac{\omega}{\omega_c}}$, 截止频率 $\omega_c = \frac{1}{C} = \frac{1}{RC}$

(3) RC 低通滤波器, $H = \frac{1}{1+j\frac{\omega}{\omega_c}}$, 截止频率 $\omega_c = \frac{1}{C} = \frac{1}{RC}$

(4) RL 高通滤波器, $H = \frac{j\frac{\omega}{\omega_c}}{1+j\frac{\omega}{\omega_c}}$, 截止频率 $\omega_c = \frac{1}{L} = \frac{R}{L}$

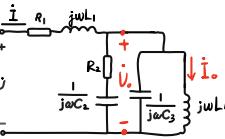
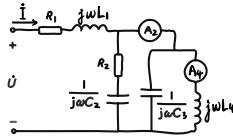


Homework 14: 2024.12.03 - 2024.12.09

1. 习题集 13-9

C_3 和 L_4 发生并联谐振，即 $Z = \frac{1}{\frac{1}{j\omega C_3} + j\omega L_4} = \frac{L_4}{C_3} \cdot \frac{j\omega C_3}{1 - \omega^2 C_3 L_4} \Rightarrow 1 - \omega^2 C_3 L_4 = 0$
 $\Rightarrow \omega = \frac{1}{\sqrt{C_3 L_4}} = 1 \text{ K rad/s}$. 视为断路，则有
 $\dot{U}_o = \frac{R_2 + \frac{1}{j\omega C_2}}{R_1 + \frac{1}{j\omega C_1} + R_2 + j\omega L_1} \dot{U} = \frac{50 - 200j}{100} = (0.5 - 2j) \dot{U}$
 $\dot{I}_o = \frac{\dot{U}_o}{j\omega L_4} = \frac{1 - 4j}{200} \dot{U}, I_o = |\dot{I}_o| = \sqrt{\frac{1+16}{400}} = \sqrt{5} \text{ A} = 4.123 \text{ A}$

即为电流表 A_4 的读数。



2. 习题集 13-12

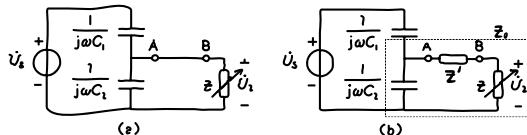
设接入元件的阻抗为 $Z' = R + jX$ ，则 $Z_o = (Z' + Z) // \frac{1}{j\omega C_2} = \frac{(Z' + Z)}{Z' + Z + \frac{1}{j\omega C_2}}$
 $\dot{U}_o = \frac{Z_o}{j\omega C_1 + Z_o} \dot{U} = \frac{Z}{Z' + Z + \frac{1}{j\omega C_2}}$
 $= \dot{U} \frac{1}{\frac{1}{j\omega C_1} + Z_o} \cdot \frac{(Z' + Z) \frac{1}{j\omega C_2}}{Z' + Z + \frac{1}{j\omega C_2}} \cdot \frac{Z}{Z' + Z}$
 $= \dot{U} \frac{1}{1 + j\omega C_2 (Z' + Z)}$

要使上式与 Z 无关，分子的分母应与 Z 成正比例关系，以与分子共同消去变量 Z 。而分母为：

$$\text{分母} = Z \left(1 + \frac{C_2}{C_1}\right) + Z' \left(1 + \frac{C_2}{C_1}\right) + \frac{1}{j\omega C_2}$$

只能有 $Z \left(1 + \frac{C_2}{C_1}\right) + \frac{1}{j\omega C_2} = 0 \Rightarrow Z' = j \frac{1}{\omega(C_1 + C_2)} = j \omega \left[\frac{1}{\omega(C_1 + C_2)} \right]$

显然可以用电感替代，感值 $L = \frac{1}{\omega(C_1 + C_2)} = \frac{1}{(2\pi f)^2 (C_1 + C_2)}$



3. 讲义题 6-24

$$\dot{U}_{out} = \frac{j\omega L_o + \frac{1}{j\omega C}}{j\omega L_o + \frac{1}{j\omega C} + \frac{L/C_o}{j\omega L + \frac{1}{j\omega C}}} \dot{U}_{in}$$

$$\Rightarrow H(j\omega) = \frac{j\omega L_o + \frac{1}{j\omega C}}{j\omega L_o + \frac{1}{j\omega C} + \frac{L/C_o}{j\omega L + \frac{1}{j\omega C}}} = \frac{CL\omega^4 - (C+L)\omega^2 + 1}{CL\omega^4 - (C+L+CL)\omega^2 + 1}$$

要使 $\omega_1 = 3$ 和 $\omega_2 = 7$ 被完全滤除，它们应是 $H(j\omega) = 0$ 的两根，于是

$$\begin{aligned} \omega^2 \omega_1^2 &= 9 \times 49 = 441 = \frac{1}{CL}, \quad \omega_1^2 + \omega_2^2 = 58 = \frac{C+L}{CL}, \quad \text{联立解得:} \\ \begin{cases} C = \frac{1}{49} F \\ L = \frac{1}{441} H \end{cases} \quad \text{或} \quad \begin{cases} C = \frac{1}{49} F \\ L = \frac{1}{9} H \end{cases}. \quad \text{此时 } 3 \text{ rad/s} \text{ 和 } 7 \text{ rad/s} \text{ 的谐波被完全消除.} \\ H &= \frac{(\omega^2 - 1)(\omega^2 - 49)}{441 - \frac{58\omega^2}{441} + 1}, \quad \text{但是, 由于 } H \text{ 分母的根为} \\ &\omega = \sqrt{\frac{59}{2} - \frac{11717}{2}} \quad \text{or} \quad \sqrt{\frac{59}{2} + \frac{11717}{2}} \end{aligned}$$

也即 $\omega = 2.9634$ or 7.0365 ，此时分母为 0， $H = \infty$ ，会产生新的谐波。因此一般情况下，上面的“滤波”并无太大价值。

4. 课件题

等效归纳 $Y = G + j\omega C + \frac{1}{j\omega H} = G + j(\omega C - \frac{1}{\omega H}) \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} = 1 \text{ K}$
 $Q = 2\pi \frac{\text{stored energy in one cycle}}{\text{lost energy in one cycle}} = 2\pi \frac{CV_0^2}{\frac{V_0^2}{R} \cdot T_0} = 2\pi f_0 \cdot CR$
 $= \omega_0 \cdot CR = R \sqrt{\frac{C}{L}} = 10$
 $BW = \frac{\omega_0}{Q} = \frac{1}{RC} = 100 \text{ rad/s}, \quad Z|_{\omega=\omega_0} = R = 10 \text{ k}\Omega$

5. 课件题

(1) $\omega_0 = \frac{1}{\sqrt{LC}} = 10^5 \text{ rad/s} = 100 \text{ K rad/s.}$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{\omega_0 L}{R} = 25$$

$$BW = \frac{\omega_0}{Q} = 4 \text{ K rad/s}$$

$$V = 5\sqrt{2}$$

(2) $\omega = 98 \text{ K} \approx 30 \text{ rad/s}$ (半功率点)，谐振时电流有效值为 $\frac{V}{R}$ A.

$$\text{于是 } I = \frac{V}{R} \text{ A.}$$

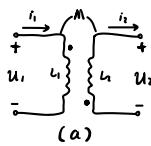
(3) ω_0 时，电容电压 $\dot{U}_c = jQ\dot{U} \Rightarrow U_c = QU = 125\sqrt{2} \text{ V} = 176.75 \text{ V}$

Homework 15: 2024.12.10 - 2024.12.16

1. 习题集 12-4

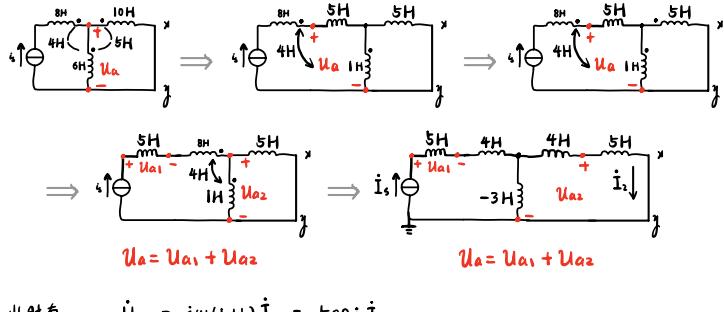
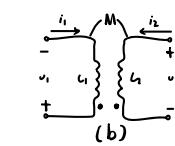
对(a), 同名端异侧:

$$\begin{cases} u_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \\ u_2 = -M \frac{di_1}{dt} - L_2 \frac{di_2}{dt} \end{cases}$$



对(b), 同名端同侧:

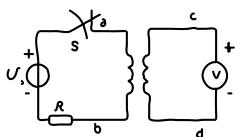
$$\begin{cases} u_1 = -L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ u_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \end{cases}$$



$$\begin{aligned} \text{此时有: } & \dot{U}_{a1} = j\omega(5H)\dot{I}_s = 500j\dot{I}_s \\ & \dot{U}_{a2} = j\omega(5H) \cdot \frac{-3H}{9H+(-3H)} \cdot \dot{I}_s = -250j\dot{I}_s \\ \Rightarrow & \dot{U}_a = \dot{U}_{a1} + \dot{U}_{a2} = 250j\dot{I}_s = 500 \sin(100t+90^\circ) V = 500 \cos(100t) V \end{aligned}$$

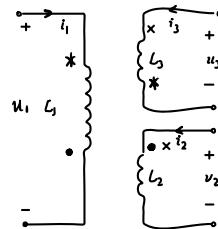
2. 讲义题 6-30

开关断开瞬间电压表正偏，则同名端异侧，即 a, d 是同名端

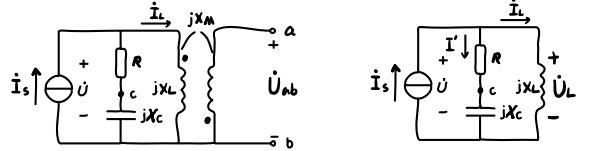


3. 讲义题 6-31

$$\begin{aligned} u_1 &= L_1 \frac{di_1}{dt} - M_{12} \frac{di_2}{dt} - M_{13} \frac{di_3}{dt} \\ u_2 &= -M_{12} \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + M_{23} \frac{di_3}{dt} \\ u_3 &= -M_{13} \frac{di_1}{dt} + M_{23} \frac{di_2}{dt} + L_3 \frac{di_3}{dt} \end{aligned}$$



6. 习题集 12-20



$$X_L = 5 \Omega, X_M = 2 \Omega, X_C = -2 \Omega, R = 4 \Omega, U_{ab} = 4 V.$$

$$Z_{22} \rightarrow \infty, \text{引入阻抗为 } D, \text{ 并且 } \dot{U}_{ab} = -jX_M \dot{I}_L \Rightarrow I_L = 2 A.$$

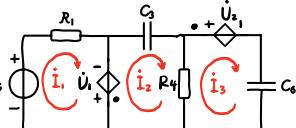
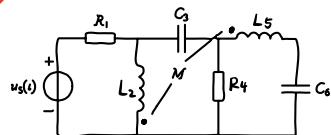
$$\dot{U}_{ab} = -jX_M \dot{I}_L = -\frac{X_M}{X_L} \dot{U}_L = -\frac{2}{5} \dot{U}_L. \text{ 以 } \dot{U}_{ab} \text{ 为参考相位.}$$

$$\text{若 } \dot{U}_{ab} = 4 < 0 V, \text{ 则:}$$

$$\begin{aligned} \dot{U}_s &= \dot{U}_L = -\frac{5}{2} \dot{U}_{ab} = 10 < 180^\circ V \\ \dot{I}_L &= 2 < 90^\circ A, \dot{I}' = \frac{\dot{U}_s}{R+jX_C} = \frac{-10}{4-2j} = 2.236 < -153.4^\circ, \text{ 于是:} \\ \dot{I}_s &= \dot{I}' + \dot{I}_L = 2.236 < 153.4^\circ A \end{aligned}$$

$$\dot{U}_{cb} = jX_C \dot{I}' = 4.472 < 116.6^\circ \Rightarrow \dot{U}_{ac} = \dot{U}_{ab} - \dot{U}_{cb} = 7.211 < -33.7^\circ V$$

4. 习题集 12-13



如图作等效, 其中 $\dot{U}_1 = -jwL_2(\dot{i}_1 - \dot{i}_2) + jwM\dot{i}_3, \dot{U}_2 = -jwM(\dot{i}_1 - \dot{i}_2) + jwL_5\dot{i}_3$, 由

网孔电流法:

$$\begin{cases} R_1 \dot{i}_1 + 0 + 0 = \dot{U}_s + \dot{U}_1 \\ 0 + (\frac{1}{jwC_3} + R_4) \dot{i}_2 + (-R_4) \dot{i}_3 = -\dot{U}_1 \\ 0 + (-R_4) \dot{i}_2 + (\frac{1}{jwC_5} + R_4) \dot{i}_3 = -\dot{U}_2 \end{cases}$$

代入 \dot{U}_1 和 \dot{U}_2 , 得到:

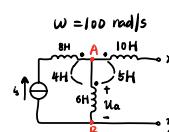
$$\begin{cases} (jwL_2 + R_1) \dot{i}_1 + (-jwL_2) \dot{i}_2 + (-jwM) \dot{i}_3 = \dot{U}_s \\ -jwL_2 \dot{i}_1 + (\frac{1}{jwC_3} + R_4 + jwL_5) \dot{i}_2 + (jwM - R_4) \dot{i}_3 = 0 \\ -jwM \dot{i}_1 + (-R_4 + jwM) \dot{i}_2 + (\frac{1}{jwC_5} + R_4 + jwL_5) \dot{i}_3 = 0 \end{cases}$$

5. 习题集 12-7

① xy 断路时, 节点 AB 是外部节点, 可作去耦等效, 得到:

$$\dot{U}_a = j\omega(6-4)\dot{I}_s = 200j\dot{I}_s$$

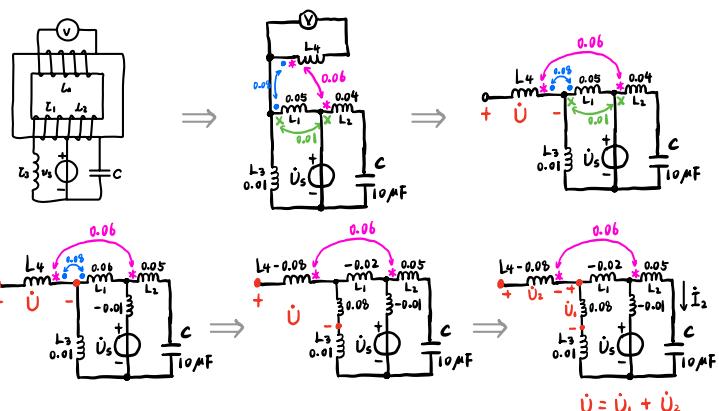
$$\Rightarrow \dot{U}_a = 400 \sin(100t + 90^\circ) V = 400 \cos(100t) V$$



② xy 短路, 作等效时需注意 U_a 对应节点的变化, 具体而言:

7. 习题集 12-21

$\omega = 1000 \text{ rad/s}, U_s = 120 V$, 等效电路如图:



$$\begin{aligned} \dot{U}_1 &= \frac{8}{1+8-2-1} \dot{U}_s = \frac{4}{3} \dot{U}_s, \quad \dot{U}_2 = j\omega(0.06H) \dot{I}_1 = 60j \cdot \frac{\dot{U}_s}{-60j} = -\dot{U}_s \\ \Rightarrow \dot{U} &= \frac{1}{3} \dot{U}_s, \quad U = 40 V \end{aligned}$$

8. 习题集 12-16

阻抗放大 $Z = n^2 R = 6n^2$, $Z = 150 \Omega$ 时阻抗匹配, 获得最大功率:

$$n=5, P_{max} = \frac{U_s^2}{4R} = \frac{1}{6} W = 0.167 W$$

Homework 16 : 2024.12.17 - 2024.12.23

1. 习题集 14-4

(1) 假设电源是△形，转为Y型后 $U_s = \frac{3\dot{U}_A}{\sqrt{3}} = 220V$.

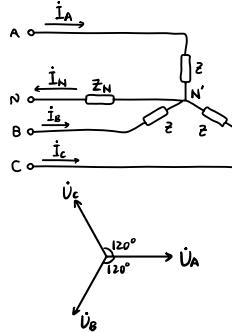
由于负载对称，无论 Z_N 是多少，均有：

$$\dot{I}_A = \frac{\dot{U}_A}{Z} = 4.4 \angle -36.9^\circ A$$

$$\dot{I}_B = \dot{I}_A \angle -120^\circ = 4.4 \angle -156.9^\circ A$$

$$\dot{I}_C = \dot{I}_A \angle -240^\circ = 4.4 \angle -276.9^\circ A$$

(2) 如图：



2. 习题集 14-7

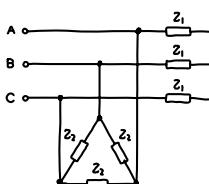
设电源△型，转为Y型后 $U_s = \frac{3\dot{U}_A}{\sqrt{3}} = 220V$.

将△型转为Y型， $Z' = \frac{Z_2}{3} = \frac{50-180j}{3} \Omega$ ，则

$$Z_1/Z' = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z'}} = (22.11 - 20.60j) \Omega$$

$$\text{各线电流为: } \dot{I}_A = \frac{220V}{Z_1/Z'} = 7.28 \angle 43.0^\circ A,$$

$$\dot{I}_B = 7.28 \angle -77^\circ A, \quad \dot{I}_C = 7.28 \angle -197.0^\circ A$$



3. 习题集 14-12

类似地，转为Y型电源 $U_s = 220V$

$$(1) S 打开时为对称负载: \dot{I}_A = \frac{220V}{Z_A} = 1.97 \angle -63.4^\circ A$$

$$\dot{I}_B = 1.97 \angle -183.4^\circ A, \quad \dot{I}_C = 1.97 \angle -303.4^\circ A$$

$$(2) S 闭合时，作 $\Delta \rightarrow Y$ 变换得: $Z_o = Z_1 + Z_A + Z_B = (150 + 250j) \Omega$$$

$$Z'_A = \frac{Z_1 Z_A}{Z_o} = Z'_B = (17.65 + 20.59j) \Omega, \quad Z'_C = \frac{Z_1 Z_B}{Z_o} + Z_C = (66.18 + 139.71j) \Omega$$

非对称负载，公共端 \dot{U}_N 为:

$$\dot{U}_{N'} = \frac{\dot{U}_A + \dot{U}_B + \dot{U}_C}{Z_o} = \frac{-3.26 - 11.19j}{0.0508 - 0.0618j} = (47.49 - 69.46j) V$$

于是各线电流为:

$$\dot{I}_A = \frac{\dot{U}_A - \dot{U}_{N'}}{Z_A} = 6.84 \angle -27.4^\circ A$$

$$\dot{I}_B = \frac{\dot{U}_B - \dot{U}_{N'}}{Z'_B} = 7.30 \angle 168.1^\circ A$$

$$\dot{I}_C = \frac{\dot{U}_C - \dot{U}_{N'}}{Z'_C} = 1.97 \angle 56.6^\circ A$$

$$\text{总功率 } P = 3 I_A^2 \frac{R}{3} = 10890 W$$

4. 习题集 14-14 (讲义题 6-43)

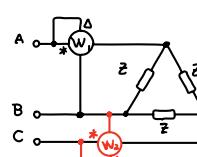
$$(1) \Delta \text{型负载和电源, 作 } \Delta \rightarrow Y \text{ 变换, 则 A 相电流 } \dot{I}_A = \frac{220V \cdot \sqrt{3}}{\sqrt{3} \cdot Z} = 13.47 \angle -45^\circ A$$

$$\text{总功率 } P = 3 \frac{220V}{\sqrt{3}} I_A \cos \varphi_A = 3629.4 W$$

(2) 采用共B相接法，如图：

$$\text{则 } P_1 = 220 \times 13.47 \times \cos(-45^\circ - 30^\circ) = 767.0 W$$

$$P_2 = 220 \times 13.47 \times \cos(-45^\circ + 30^\circ) = 2862.4 W$$



5. 习题集 14-19 (习题集例 14-4)

$$(1) 转为 220V Y型电源, A 相向电动机提供电流 $I_{A1} = \frac{P}{3 \times 220V \times \cos \varphi} = 3.22 A$$$

$$\cos \varphi = 0.8 (\varphi > 0) \Rightarrow \varphi = 36.9^\circ, \text{ 以 } \dot{U}_A \text{ 为参考相量, 则有 } \dot{I}_{A1} = 3.22 \angle -36.9^\circ A$$

再考虑△型负载 $Z: \dot{I}_{A2} = \frac{3 \dot{U}_A}{Z} = 7.00 \angle -59.0^\circ A$

$$\Rightarrow \dot{I}_A = \dot{I}_{A1} + \dot{I}_{A2} = 10.07 \angle -51.39^\circ A$$

$$\text{同理 } \dot{I}_B = 10.07 \angle -171.39^\circ A, \quad \dot{I}_C = 10.07 \angle -291.39^\circ A$$

(2) 负载总功率为 $P = 3 I_A I_B \cos \varphi_A = 4147.3 W$, 其中 $\varphi_A = 51.39^\circ, \tan \varphi_A = 1.2522$

要求 $\cos \varphi = 0.9, \beta \tan \varphi = 0.4843$, 保持有功功率不变, 则所需Y型电容为(每个):

$$f = 50 Hz \Rightarrow C = \frac{P / (\tan \varphi_A - \tan \varphi)}{3 \omega U^2} = 69.82 \mu F$$

6. 习题集 15-21

可视为理想变压器, 即:

$$Z = R_L + j\omega L + n^2 R_2 = 30 + j\omega L$$

$$U_s = 70 + 100 \sin 1000t + 50 \sin 3000t V$$

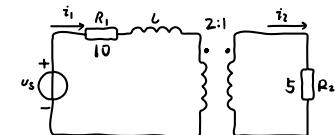
$$\textcircled{1} \omega = 0, \quad \omega L = 0, \quad i_1 = \frac{70}{10} = 7 A$$

$$\textcircled{2} \omega = 1000, \quad \omega L = 40, \quad \dot{i}_1 = \frac{100 \angle 0^\circ}{30+40j} = 2 \angle -53.1^\circ A, \quad i_1 = 2 \sin(1000t - 53.1^\circ) A$$

$$\textcircled{3} \omega = 3000, \quad \omega L = 120, \quad \dot{i}_1 = \frac{50 \angle 0^\circ}{30+120j} = 0.40 \angle -76.0^\circ A, \quad i_1 = 0.816 \sin(3000t - 76.0^\circ) A$$

$$\Rightarrow \begin{cases} i_1 = 7 + 2 \sin(1000t - 53.1^\circ) + 0.406 \sin(3000t - 76.0^\circ) A \\ i_2 = 0 + 4 \sin(1000t - 53.1^\circ) + 0.816 \sin(3000t - 76.0^\circ) \end{cases}$$

$$I_1 = \sqrt{7^2 + (\frac{2}{10})^2 + (\frac{0.406}{5})^2} = 7.147 A, \quad I_2 = \sqrt{\frac{4^2 + 0.816^2}{2}} = 2.886 A$$



7. 讲义题 6-47

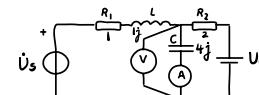
$$(1) 仅有 \dot{U}_S 作用时: $R_2/Z_C = \frac{2+4j}{2+4j} = 1.6 + 0.8j \Omega$$$

$$\dot{I} = \frac{\dot{U}_S}{1+1j+(1.6+0.8j)} \cdot \frac{2}{2+4j} = 0.1414 \dot{U}_S \angle -98.1^\circ$$

$$\dot{U} = \frac{1}{j\omega C} \dot{I} = 0.5657 \angle 171.9^\circ$$

$$i = 0.1414 U_S \sin(\omega t - 98.1^\circ) A,$$

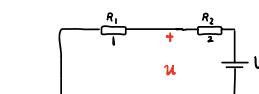
$$u = 0.5657 U_S \sin(\omega t + 171.9^\circ) V$$



仅有 U_o 作用时: $i = 0, u = \frac{1}{3} U_o$

$$\Rightarrow \begin{cases} U^2 = \frac{U^2}{9} + \left(\frac{0.5657 U_S}{12}\right)^2 = 100 \\ I^2 = 0 + \left(\frac{0.1414 U_S}{\sqrt{2}}\right)^2 = 4 \end{cases}$$

$$\Rightarrow U_S = 20.0 V, \quad U_o = 16.0 V$$



(2) 注意不能简单将(总响应)有效值相乘得到功率.

$$\text{对 } U_o: P = \frac{16^2}{3} = 108 W$$

$$\text{对 } U_S: \dot{I}_S = \frac{\dot{U}_S}{1+1j+(1.6+0.8j)} = 6.32 \angle -34.7^\circ A$$

$$P = 20 \times 6.32 \times \cos 34.7^\circ = 104 W$$

Simulation 1: 2024.09.10 - 2024.09.24

10.1 仿真 2-1: 题目详见图 10.4 (b)

(1) 单 OPA 实现电压运算

电路如图 10.1 (a) 所示, 接线端示意图见图 10.1 (b)。

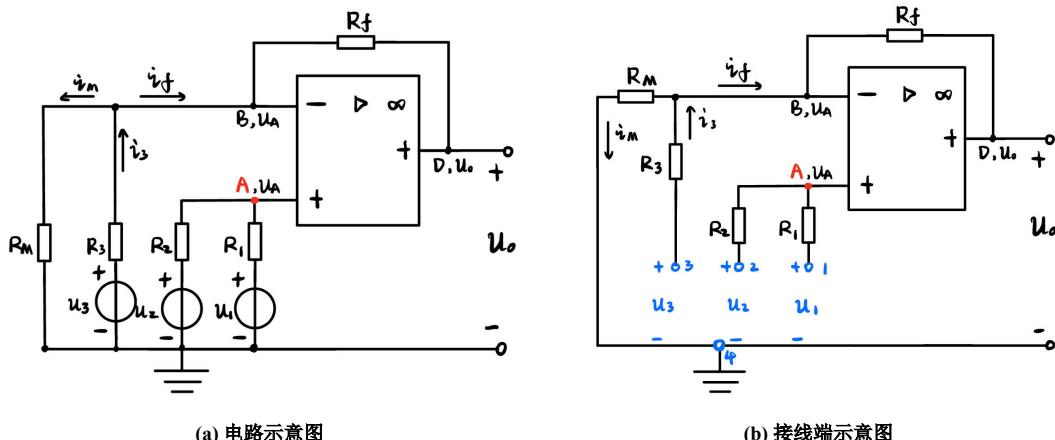


图 10.1: 单 OPA 实现电压运算

下面分析其输出特性。由虚断，在 u_1 和 u_2 构成的回路中，设正向流经 u_2 的电流为 i_2 ，则有：

$$i_2 = \frac{u_2 - u_1}{R_1 + R_2} \implies u_A = u_2 - i_2 R_2 = \frac{R_2 u_1 + R_1 u_2}{R_1 + R_2} \quad (10.1)$$

由虚短, B 点的电势也为 u_A , 于是:

$$i_3 = \frac{u_3 - u_A}{R_3}, \quad i_M = \frac{u_A}{R_M} \implies i_f = i_3 - i_M = \frac{u_3 - u_A}{R_3} - \frac{u_A}{R_M} = \frac{u_3}{R_3} - \left(\frac{1}{R_3} + \frac{1}{R_M} \right) u_A \quad (10.2)$$

由虚断和 KVL:

$$u_o = u_A - i_f R_f = u_A - \frac{R_f}{R_3} u_3 + \left(\frac{R_f}{R_3} + \frac{R_f}{R_M} \right) u_A = \left(1 + \frac{R_f}{R_3} + \frac{R_f}{R_M} \right) u_A - \frac{R_f}{R_3} u_3 \quad (10.3)$$

将 y_4 的表达式代入，最终得到：

$$u_o = \left(1 + \frac{R_f}{R_3} + \frac{R_f}{R_M}\right) \frac{1}{\frac{R_1}{R_2} + 1} u_1 + \left(1 + \frac{R_f}{R_3} + \frac{R_f}{R_M}\right) \frac{\frac{R_1}{R_2}}{\frac{R_1}{R_2} + 1} u_2 - \frac{R_f}{R_3} u_3 \quad (10.4)$$

我们需要 u_1, u_2, u_3 前的系数分别为 3, 2, -0.5, 于是有:

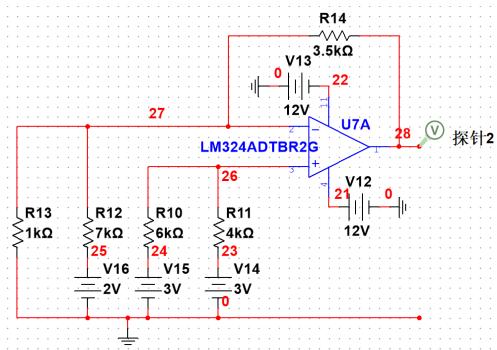
$$\begin{cases} \left(1 + \frac{R_f}{R_3} + \frac{R_f}{R_M}\right) \frac{1}{\frac{R_1}{R_2} + 1} = 3 \\ \left(1 + \frac{R_f}{R_3} + \frac{R_f}{R_M}\right) \frac{\frac{R_1}{R_2}}{\frac{R_1}{R_2} + 1} = 2 \\ -\frac{R_f}{R_3} = -0.5 \end{cases} \implies \begin{cases} R_1 = \frac{2}{3}R_2 & , R_2 > 0 \\ R_3 = 2R_f, R_M = \frac{2}{7}R_f & , R_f > 0 \end{cases} \quad (10.5)$$

为了保持 OPA 的理想性，我们应选择 $K\Omega$ 量级的电阻，同时，为了降低电路的整体功率，减少消耗，电阻阻值应该尽量大。综合下来，不妨选取 $R_2 = 6 K\Omega$, $R_f = 3.5 K\Omega$ ，此时所有电阻阻值为：

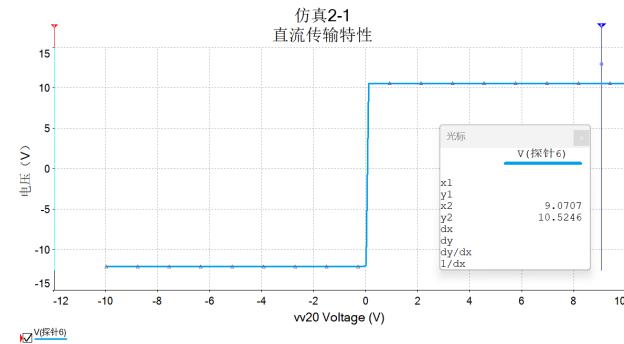
$$R_1 = 4 \text{ K}\Omega, R_2 = 6 \text{ K}\Omega, R_3 = 7 \text{ K}\Omega, R_M = 1 \text{ K}\Omega, R_f = 3.5 \text{ K}\Omega \quad (10.6)$$

如图 10.2 (a), 在 Multisim 中进行仿真, 得到的结果如下表所示:

项目	1			2			3			4		
	x, u_1	y, u_2	z, u_3	x, u_1	y, u_2	z, u_3	x, u_1	y, u_2	z, u_3	x, u_1	y, u_2	z, u_3
	1	1	1	1	3	2	-2	2	0	3	3	2
理论输出 (V)	$3 + 2 - 0.5 = 4.5$			$3 + 6 - 1 = 8$			$-6 + 4 - 0 = -2$			$9 + 6 - 1 = 14$		
仿真输出 (V)	4.50			8.00			-2.00			10.494		



(a) 单 OPA 实现电压运算



(b) OPA 饱和电压

图 10.2: 仿真电路图与 OPA 饱和电压

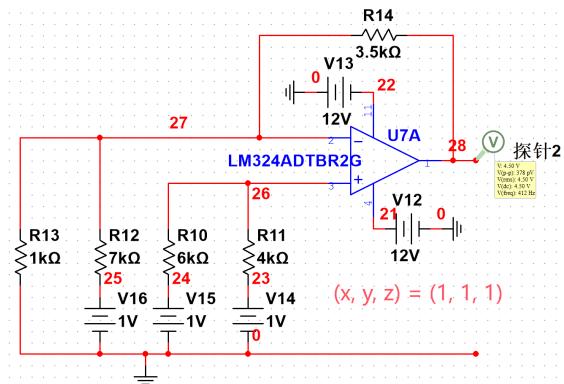
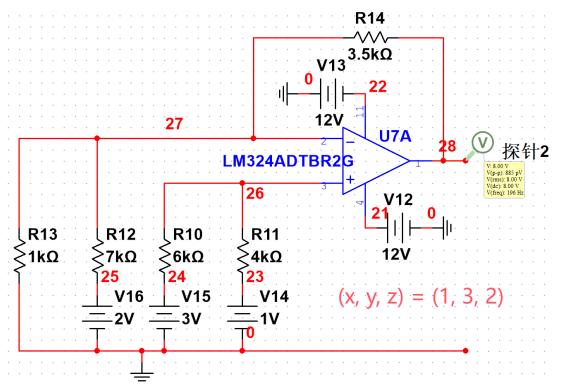
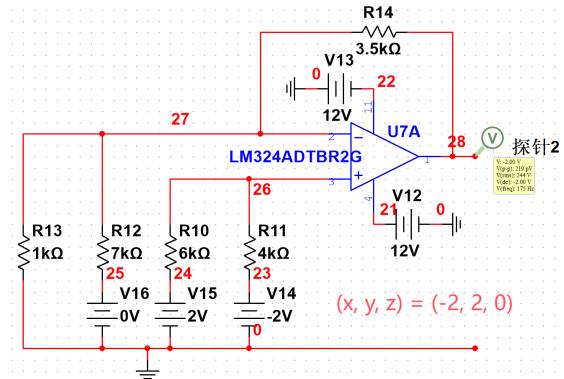
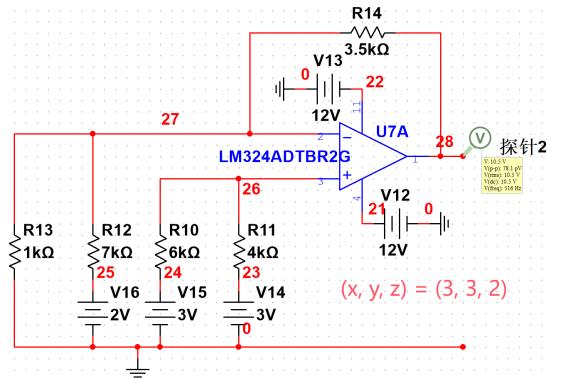
(a) $(x, y, z) = (1, 1, 1)$ (b) $(x, y, z) = (1, 3, 2)$ (c) $(x, y, z) = (-2, 2, 0)$ (d) $(x, y, z) = (3, 3, 2)$

图 10.3: 仿真具体结果图

由表可见, 除了最后一组数据, 仿真结果与理论结果完全一致。最后一组之所以不同, 是因为输出电

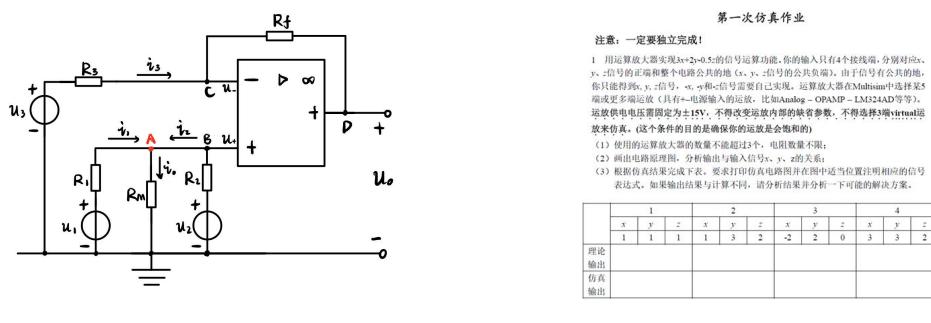
压 u_o 超出了此 OPA 的饱和电压 U_{sat} , 导致输出电压 $u_o = U_{\text{sat}} = 10.494\text{V}$ 。如图 10.2 (b) 所示, 此 OPA (LM324ADTBR2G) 的饱和电压为 10.525V , 与解释相符。具体仿真时的结果见图 10.3。

(2) 一些失败的例子

注意到, 减法器是在反相加法器的基础上, 串联入电压源(和电阻)改变了 u_+ 端的电压。这样, 在最终的输出电压 u_o 中, u_- 端的电源电压会带负号, u_+ 端的电源电压带正号。用类似的思想, 我们可以对减法器进行改造, 最终仅用一个 OPA 便实现 $3x + 2y - 0.5z$ 的电压运算。

一种方法是向 u_+ 端再串联一个电压源, 使得输出 u_o 中两正一负, 然后通过电阻值来调整系数, 但是, 这样不满足接线端的要求(三正一地)。另一种方法是向 u_- 端再并联一个电压源, 使得输出 u_o 中两负一正($-u_1, -u_2, +u_3$), 最后通过电阻值来调整系数, 但是, 这样得到的是两负一正而不是两正一负, 虽满足了接线端要求, 却不是我们需要的结果。

其实, 我们只需要向 u_+ 端的电压源再并联一个电压源即可, 如图所示。下面分析其输出特性。



(a) 失败的例子

(b) 仿真作业 2-1

图 10.4: 示意图

在 u_1, R_1, u_2, R_2 和 R_M 构成的局部电路中, 由 KVC 得点 A 处的电势 u_A :

$$\begin{cases} u_1 - R_1 i_1 - R_M(i_1 + i_2) = 0 \\ u_2 - R_2 i_2 - R_M(i_1 + i_2) = 0 \end{cases} \Rightarrow \begin{cases} i_1 = \frac{(R_2 + R_M)u_1 - R_M u_2}{R_1 R_2 + R_1 R_M + R_2 R_M} \\ i_2 = \frac{(R_1 + R_M)u_2 - R_M u_1}{R_1 R_2 + R_1 R_M + R_2 R_M} \end{cases} \Rightarrow u_A = \frac{R_2 R_M u_1 + R_1 R_M u_2}{R_1 R_2 + R_1 R_M + R_2 R_M}$$

也即点 B 和非反相输入端的电势 $u_+ = u_B = u_A$ 。由虚短, $u_- = u_+$, 可得电流 i_3 。再由虚断, 经过电阻 R_f 求得 D 点电势, 也即输出电压 u_o 。

$$i_3 = \frac{u_3 - u_-}{R_3} = \frac{1}{R_3}(u_3 - \frac{R_2 R_M u_1 + R_1 R_M u_2}{R_1 R_2 + R_1 R_M + R_2 R_M}) \quad (10.7)$$

$$u_o = u_A - i_3 R_f = (1 + \frac{R_f}{R_3})u_A - \frac{R_f}{R_3}u_3 \quad (10.8)$$

$$= (1 + \frac{R_f}{R_3}) \cdot \frac{\frac{R_M}{R_1}u_1 + \frac{R_M}{R_2}u_2}{1 + \frac{R_M}{R_1} + \frac{R_M}{R_2}} - \frac{R_f}{R_3}u_3 \quad (10.9)$$

$$= \frac{1 + \frac{R_f}{R_3}}{1 + \frac{R_M}{R_1} + \frac{R_M}{R_2}} \left(\frac{R_M}{R_1}u_1 + \frac{R_M}{R_2}u_2 \right) - \frac{R_f}{R_3}u_3 \quad (10.10)$$

最后调整电阻阻值。为了保持 OPA 的理想性, 电阻需要在 $\text{K}\Omega$ 量级, 令电阻比例例如下:

$$\begin{cases} \frac{R_f}{R_3} = 0.5 \\ \frac{1 + \frac{R_f}{R_3}}{1 + \frac{R_M}{R_1} + \frac{R_M}{R_2}} \cdot \frac{R_M}{R_1} = 3 \\ \frac{1 + \frac{R_f}{R_3}}{1 + \frac{R_M}{R_1} + \frac{R_M}{R_2}} \cdot \frac{R_M}{R_2} = 2 \end{cases} \Rightarrow \begin{cases} R_f = \frac{1}{2}R_3 \\ R_1 = -2R_M \\ R_2 = -\frac{4}{3}R_M \end{cases} \quad (10.11)$$

显然这不可能实现, 舍弃。

10.2 仿真 2-2

仿真电路如图 10.5 (a) 所示, 对输入电压进行参数扫描, 输出通过电压源的电流, 得到图 10.5 (b)。这里需要注意, 在 Multisim 中, 电流的参考方向始终是高电势指向低电势 (包括电压源), 因此, 仿真输出中的 $I(V11)$ 是从上往下通过 V11 的电流 (而不是从下至上), 电压源 V11 的实际电流为 $i = -I(V11)$ 。

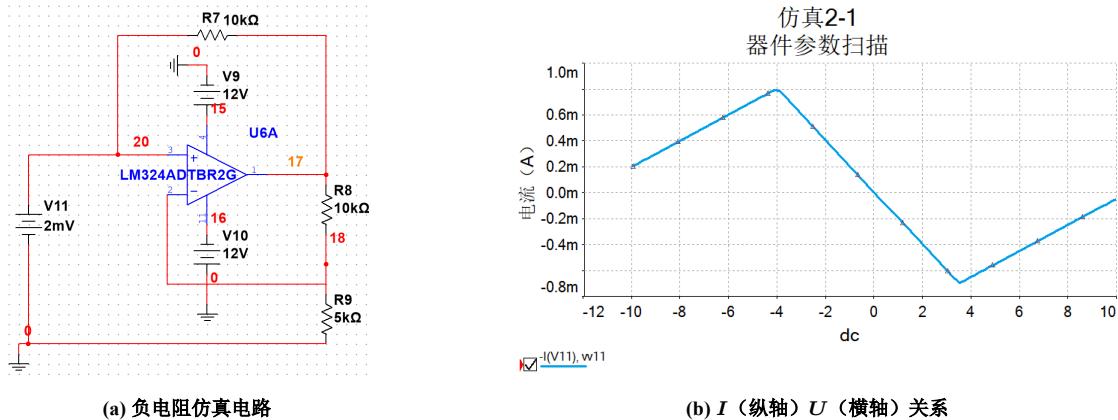


图 10.5: 负电阻仿真

简记电压源 V11 的电压为 u , 继续仿真输出电压 u_o 关于输入电压 u 的变化, 将数据导出后在 Matlab 中绘制曲线, 如图 10.6 (a)。再将 $I-U$ 关系转化为 $U-I$ 关系, 如图 10.6 (b)。可以发现, 在线性区内, 电路表现为负阻。而线性区外的两段折线位于 OPA 的饱和区, 此时 u_o 始终为饱和电压, 电路呈现正电阻, 且阻值为:

$$\begin{cases} i = \frac{u - U_{\text{sat}}}{R_1} & , u > 3.54 \text{ V} \\ i = \frac{u + U_{\text{sat}}}{R_1} & , u < -4.05 \text{ V} \end{cases} \implies R_{\text{sat}} = R_1 = 10 \text{ k}\Omega \quad (10.12)$$

这与图 10.6 (b) 中曲线的斜率是相符的。而在线性区, 负电阻 $R = -\frac{10 \text{ k}\Omega}{10 \text{ k}\Omega} \cdot 5 \text{ k}\Omega = -5 \text{ k}\Omega$, 这也是符合的。

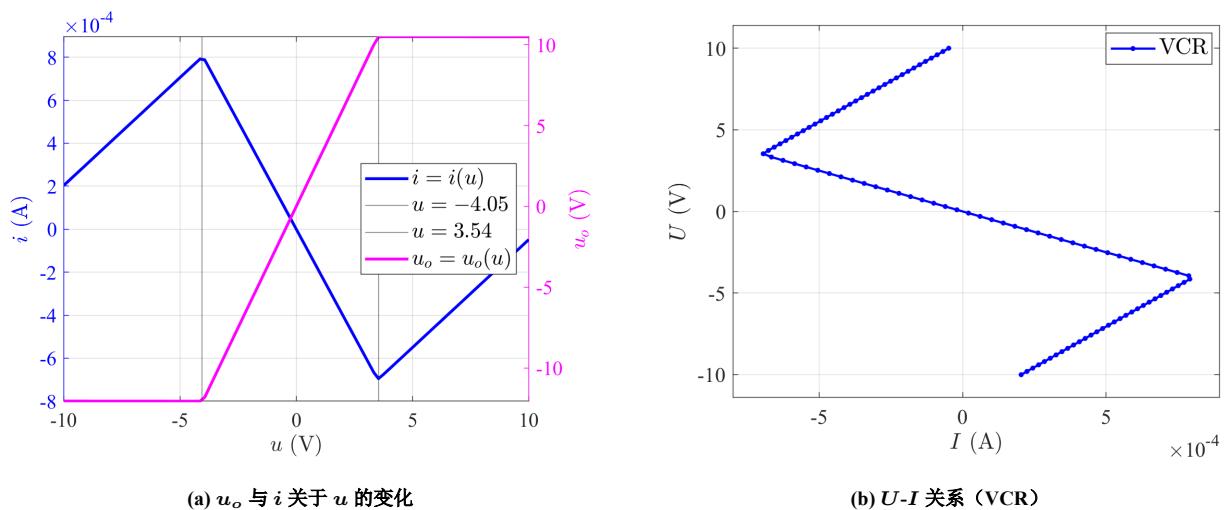


图 10.6: 仿真结果分析

Simulation 2: 2024.10.22 - 2024.11.07

11.1 放大器设计

考虑如图 11.1 (a) 所示的共源放大器, 图 11.1 (b) 是其等效电路图和小信号电路 (已默认 MOSFET 工作在恒流区)。我们需要 MOSFET 工作在恒流区, 因此有第一个限制条件:

$$u_{DS} > u_{GS} - U_T > 0 \quad (11.1)$$

为了确保 MOSFET 不在截止区, 不妨将上面的限制改为 $u_{DS} > u_{GS} - 1.3 U_T > 0$ 。设输入信号的直流偏置为 U_0 (GS 端的直流电压), 即输入信号 $u_{in} = U_0 + U_m \sin(\omega t + \varphi)$, 小信号放大倍数为 A , 则有:

$$\begin{cases} g_m = \left[\frac{di_{DS}}{du_{GS}} \right]_{u_{GS}=U_0} = K(U_0 - U_T) \\ \Delta u_{DS} = -g_m R_L \Delta u_{GS} \end{cases} \implies A = \frac{\Delta u_{DS}}{\Delta u_{GS}} = -KR_L(U_0 - U_T) \quad (11.2)$$

这是第二个限制条件。对图 11.1 (a), 设右侧电压源的电压为 U_S (在本题中是 12 V), 则有:

$$u_{DS} = U_S - i_{DS} R_L \quad (11.3)$$

三个限制条件联立, 可以得到:

$$R_L = \frac{|A|}{K(U_0 - U_T)}, \quad U_0 \in \left(1.3 U_T, U_T + \frac{2}{2+A} U_S \right) \quad (11.4)$$

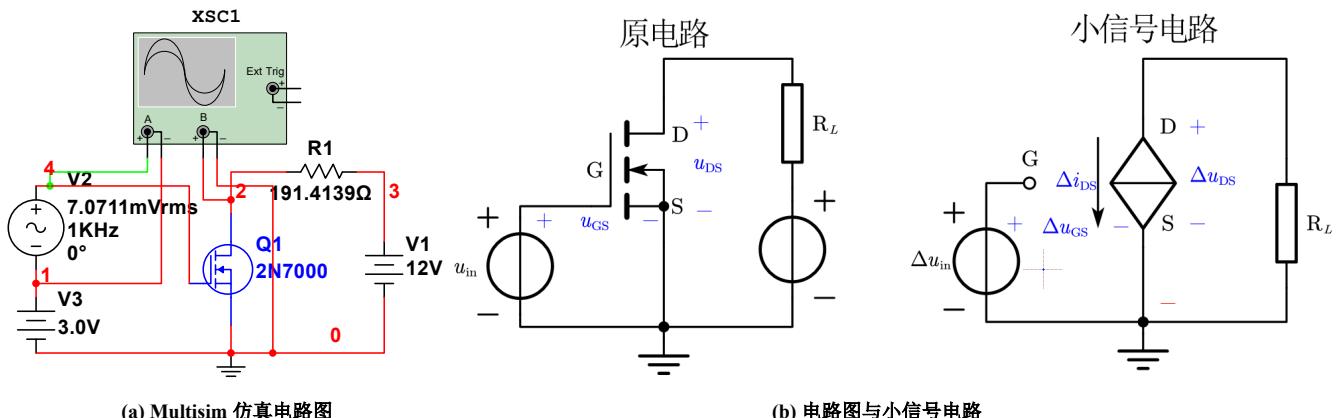


图 11.1: 共源放大器

基于上面的理论, 可以为电路中各参数选择合适的值。在两周前的 Lab 1 实验中, 我们已经得到了 N 沟道增强型 MOSFET 2N7000 的阈值电压 $U_T = 2.003$ V 以及 $K = 0.1048 \text{ A} \cdot \text{V}^{-2}$ 。本次实验的放大倍数为 $A = 20$, 还需要确定电阻 R_L 和输入信号的直流偏置 U_0 的值。代入已知数据, 可以得到:

$$R_L = \frac{190.8397}{U_0 - 2.003}, \quad U_0 \in (2.6039 \text{ V}, 3.0939 \text{ V}) \quad (11.5)$$

不妨分别令 $U_0 = 2.8 \text{ V}, 2.9 \text{ V}, 3.0 \text{ V}$, 计算对应的 R_L 电阻值, 并在 Multisim 中进行三次仿真, 得到的示波器结果如图 11.2 所示。其中绿色是输入信号, 红色是放大后的输出信号。需要注意, Multisim 中的 AC Source 的“电压”参数是指有效值而非幅度值, 因此, 若要使输入小信号幅度为 10 mV, 应设置“电压”参数为 $\frac{10}{\sqrt{2}} \text{ mV} = 7.0711 \text{ mV}$ 。

由表和图可以知道, 理论与实际符合得较好。

表 11.1: 不同参数下的放大倍数

U_0 (V)	R_L (Ω)	Δu_{in} (mV)	Δu_{out} (mV)	$ A $
2.8	239.4475	- 9.9700	184.0440	18.4598
2.9	212.7533	- 9.9560	202.3830	20.3277
3.0	191.4139	- 9.9700	213.9680	21.4612

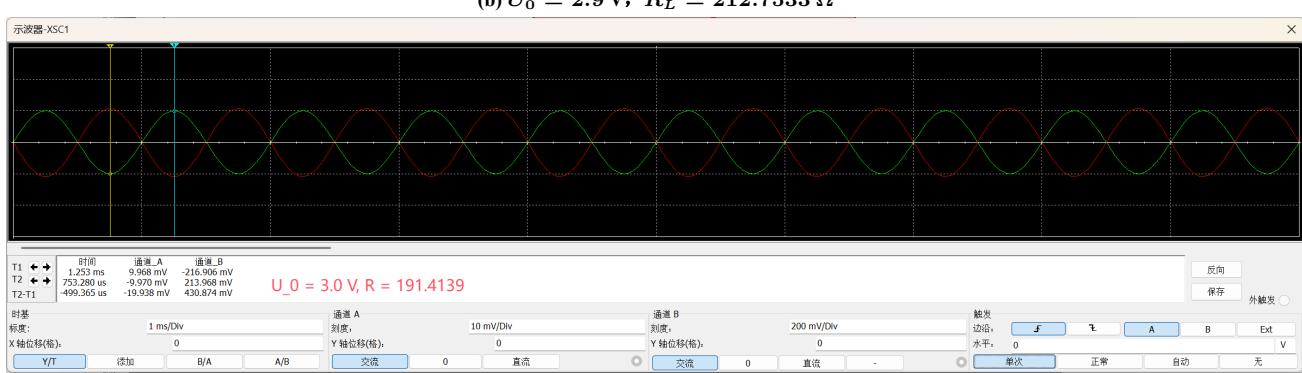
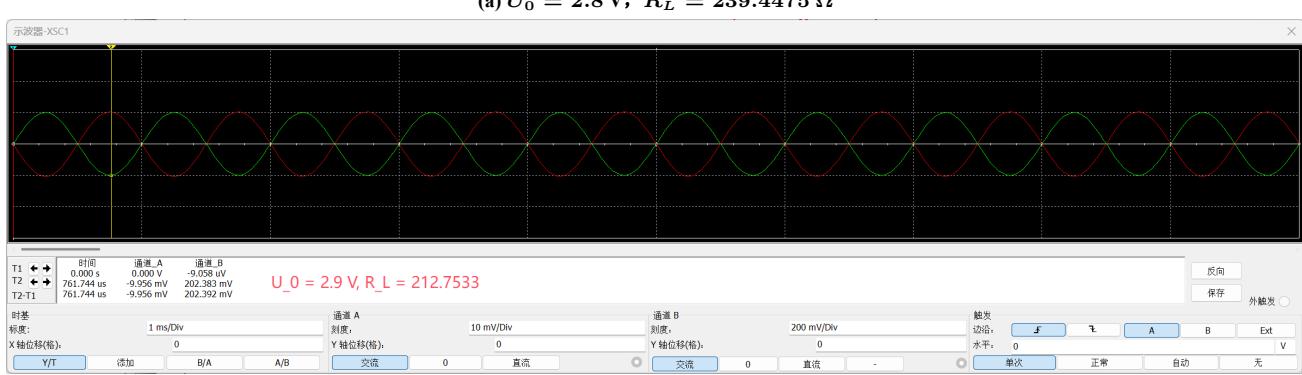
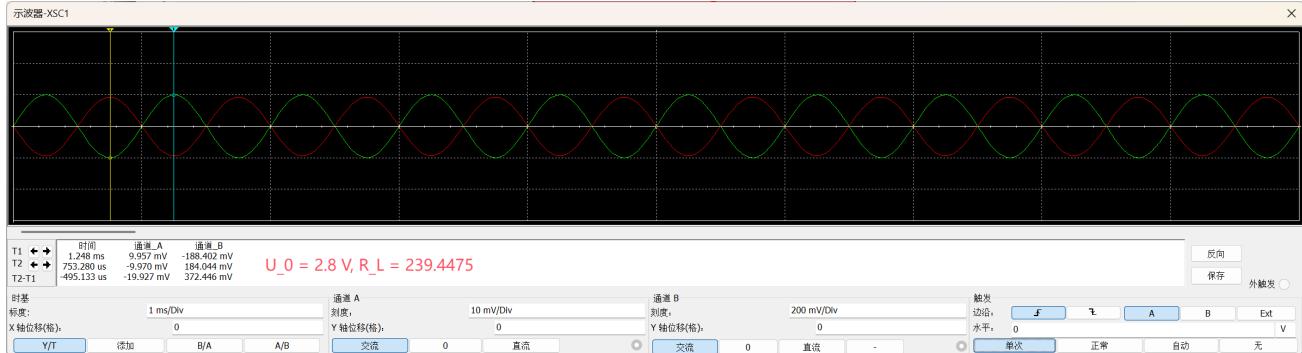


图 11.2: 不同参数下的小信号放大仿真结果

经过多次尝试, 发现使理论值和仿真结果符合得极好的一组参数值为 $U_T = 2.35 \text{ V}$ 和 $K = 0.17 \text{ A} \cdot \text{V}^{-2}$, Multisim 中的 2N7000 模型的参数如图 11.3 所示, 我似乎并没有找到与之关联的参数。具体原因有待进一步的探究。



图 11.3: Multisim 中 2N7000 的仿真参数

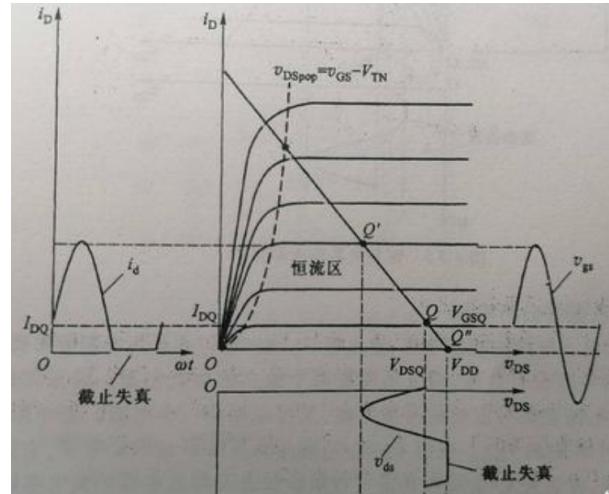


图 11.4: 输出信号发生截止失真的原因

11.2 放大波形失真的原因

题目中的波形发生了截止失真，这是信号幅度过大和 u_{DS} 直流偏置过大共同造成的，如图 11.4 所示。当小信号幅度较大，且 u_{DS} 的直流偏置也较大时，工作点的振动范围内可能部分包含截止区，此时输出信号呈水平直线，称为“截止失真”，最大不失真输出电压为：

$$U_{\text{out,max}} = U_S - \left(U_S - \frac{K}{2} (U_0 - U_T)^2 \cdot R_L \right) = \frac{K}{2} (U_0 - U_T)^2 \cdot R_L \quad (11.6)$$

其中 U_0 是输入信号的直流偏置电压。在我们的设计中， $R_L = \frac{|A|}{K(U_0 - U_T)}$ ，代入即得 $U_{\text{out,max}} = 10 \text{ V}$ ，而 $U_0 = 3.0 \text{ V}$ 时的输出电压范围在 5.5 V 到 6.0 V 之间（由 i_{DS} 的范围即可算出），因此不会出现截止失真。

对本题而言，不存在输入输出信号幅度过大的说法，因此题目中的问题电路可以通过适当调小 u_{DS} 的直流偏置来改善或避免信号失真。由于直流工作点的 u_{DS} 满足：

$$U_{DS} = U_S - i_{DS} R_L = U_S - \frac{K R_L}{2} (u_{GS} - U_T)^2 \quad (11.7)$$

所以调小 u_{DS} 等价于调大 u_{GS} ，也即调大输入信号的直流电压值 U_0 。

Simulation 3: 2024.11.26 - 2024.12.17

12.1 Capacitance Multiplier

Relevant resources:

- (1) A Review of Capacitance Multiplication Techniques (<https://ieeexplore.ieee.org/document/8678969>)
- (2) Capacitance Multiplier with Large Multiplication Factor and High Accuracy
(https://www.jstage.jst.go.jp/article/elex/15/3/15_15.20171191/_article)
- (3) Active Capacitor Multiplier in Miller-compensated Circuits (<https://ieeexplore.ieee.org/document/818917>)
- (4) The Capacitance Multiplier (<https://audiopress.com/article/the-capacitance-multiplier>)

(1) Basic Circuits and Principles

Below are two basic concepts for capacitance multiplication:

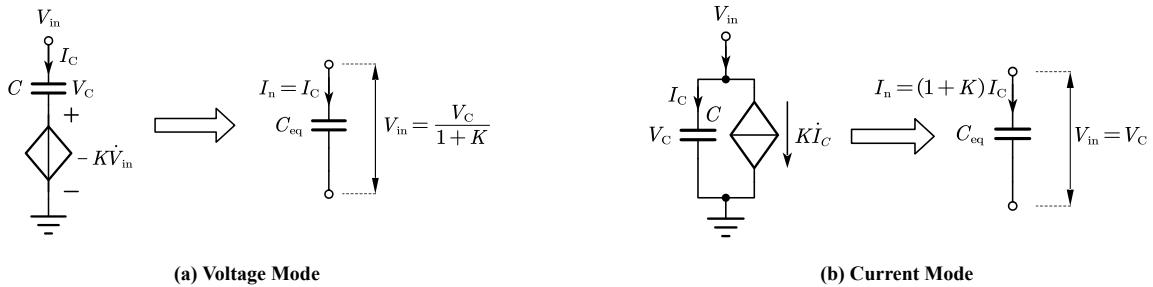


Figure 12.1: Basic Two Concepts for Capacitance Multiplier

Thus, we obtain the equivalent capacitance as:

$$\text{voltage mode: } C_{\text{eq}} = \frac{I_n}{S V_n} = \frac{I_C}{S \frac{V_C}{1+K}} = (1+K) C, \quad K > 0 \quad (12.1)$$

$$\text{current mode: } C_{\text{eq}} = \frac{I_n}{S V_n} = \frac{(1+K) I_C}{S V_C} = (1+K) C, \quad K > 0 \quad (12.2)$$

A simple implementation of cap multiplier, depicted in Fig.1.2 (a), combining a unit-gain buffer (voltage follower) and an inverting amplifier, uses a voltage mode. yielding the equivalent capacitance:

$$A = -\frac{R_2}{R_1} = -K \implies C_{\text{eq}} = \left(1 + \frac{R_2}{R_1}\right) C \quad (12.3)$$

where $A = -\frac{R_2}{R_1}$ is the closed-loop gain of the inverting amplifier. Since inverting amplifier has a low input impedance, the unit-gain buffer is a necessary. To change it into a two-terminal element, just replace GND with the negative terminal of the input voltage, e.g. $V_{\text{in},-}$, as shown in Fig.1.2 (b).

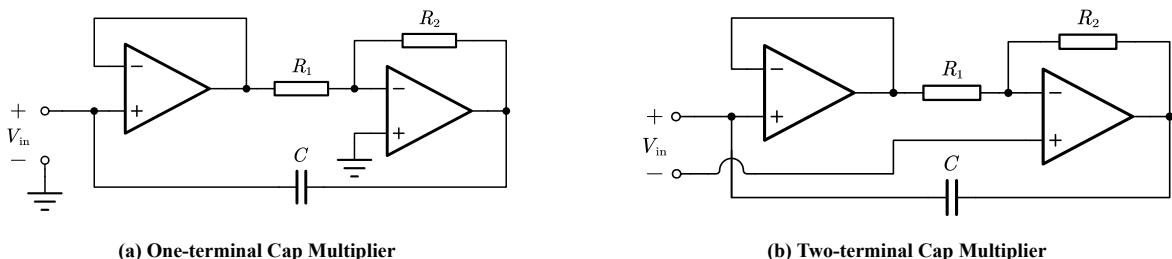


Figure 12.2: A Simple Implementation of Capacitance Multiplier

(2) Multisim Simulation

Considering Figure 1.2 (b), set the parameters as Table 1.1. Then we connect it to the RC series circuit to perform an AC sweep to test the capacitance value. The Simulation Circuit is shown in Figure 1.3.

Table 12.1: Simulation Parameters of Capacitance Multiplier

C	R_1	R_2	Operation Amplifier
10 nF	1 K Ω	11 K Ω	LM258P

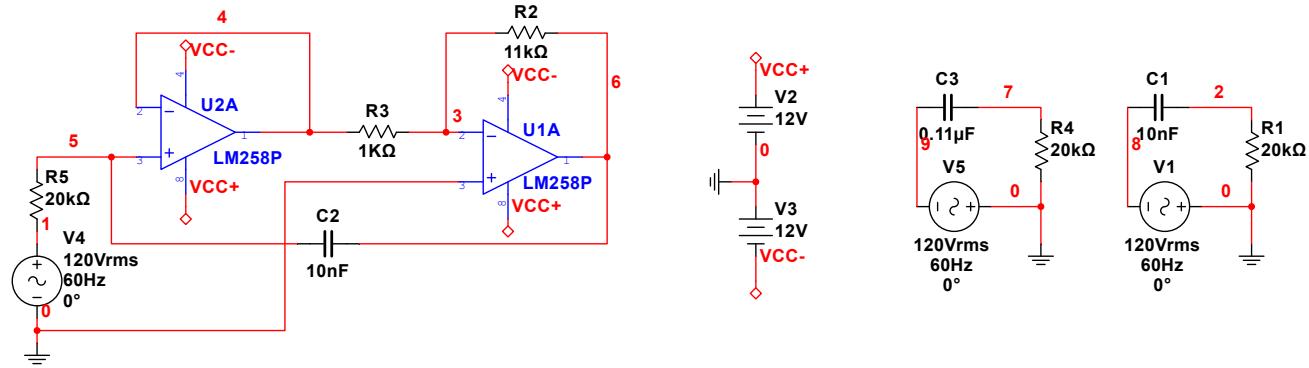


Figure 12.3: Simulation Circuit of Cap Multiplier

Export the simulation data and plot the frequency response (bode plot) of the series RC circuit, as shown in Figure 1.4. The theoretical value of the capacitance of the cap multiplier is 110 nF, and the simulation result confirmed this point.

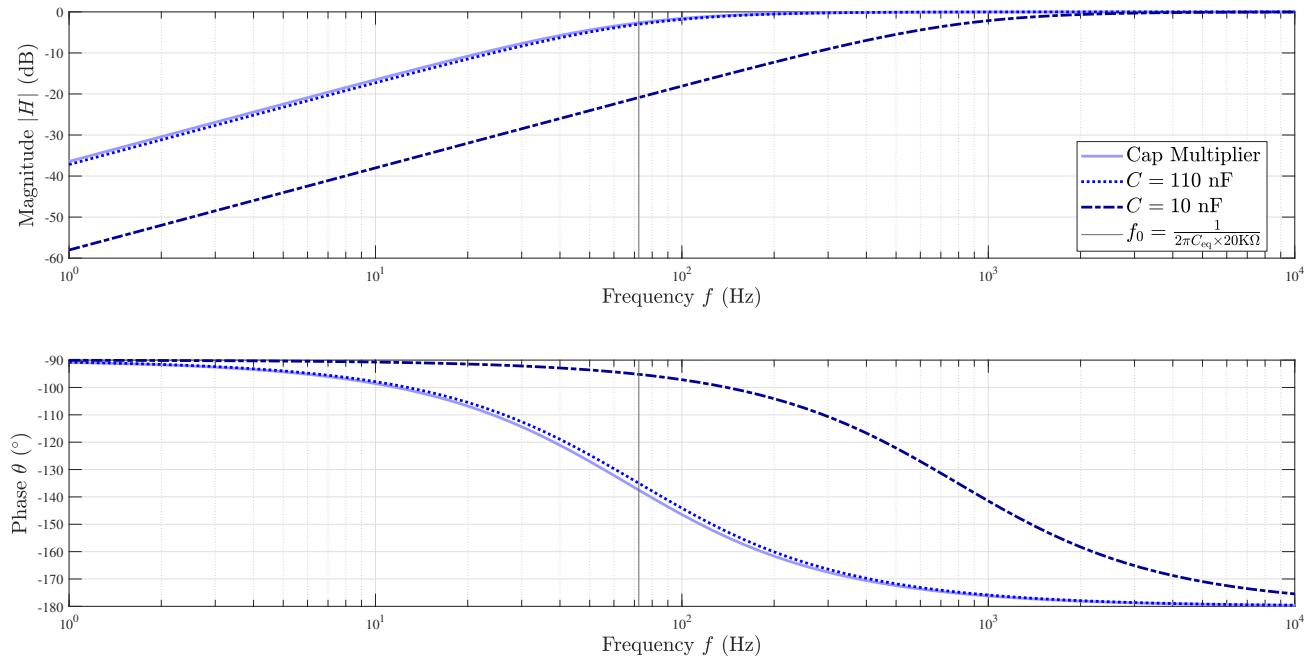


Figure 12.4: AC Sweep of the Cap Multiplier

12.2 The Wien Bridge Oscillator

You must have seen that a number of resistors and capacitors can be connected together with an inverting amplifier to produce an oscillating circuit. Wien bridge oscillator is one of the simplest sine wave oscillators which uses an RC network in place of the conventional LC tuned tank circuit to produce a sinusoidal output waveform.

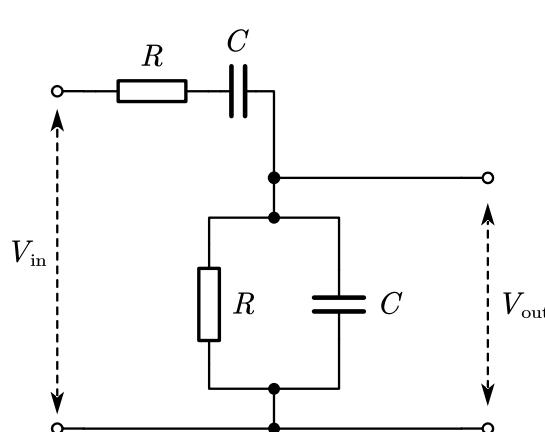
The Wien Bridge Oscillator is based on a noninverting amplifier, using a series RC circuit connected with a parallel RC of the same component values as a feedback circuit.

(1) Basic Circuit and Principles

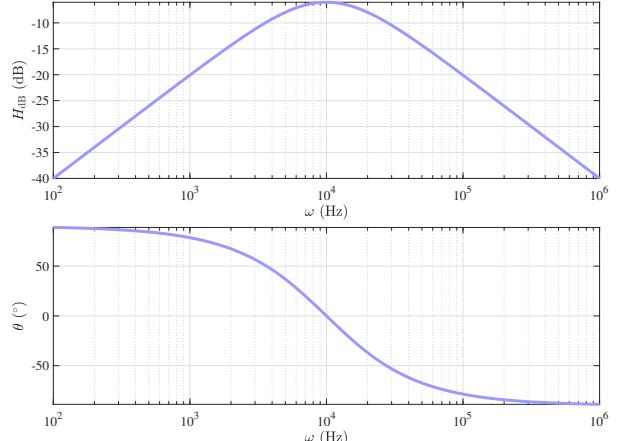
Consider the RC circuit in Figure 1.5 (a), the voltage gain H of the series RC circuit is:

$$H(j\omega) = \frac{R \parallel (\frac{1}{j\omega C})}{R + \frac{1}{j\omega C} + R \parallel (\frac{1}{j\omega C})} = \frac{1}{1 + \frac{(R^2 - \frac{1}{\omega^2 C^2}) + \frac{2R}{j\omega C}}{\frac{R}{j\omega C}}} , \quad H|_{\omega=\frac{1}{RC}} = \frac{1}{1 + \frac{\frac{2R}{j\omega C}}{\frac{R}{j\omega C}}} = \frac{1}{3} \quad (12.4)$$

Defined to obtain a 0° phase shift, the resonant frequency f_0 is the key to Wien bridge oscillator. And at the point we have $H = \frac{1}{3}$. Let's set $R = 10 \text{ K}\Omega$, $C = 10 \text{ nF}$ and sketch the bode plot of this RC circuit in Figure 1.5 (b).

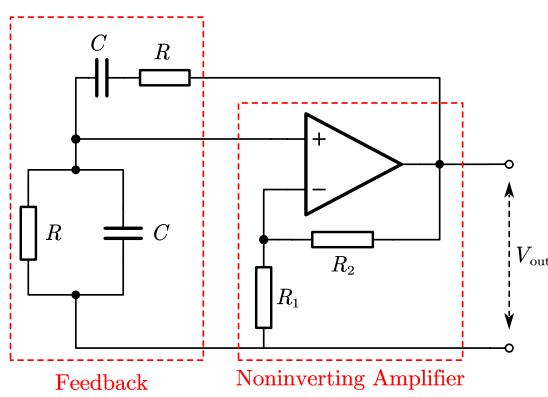


(a) RC Feedback Circuit

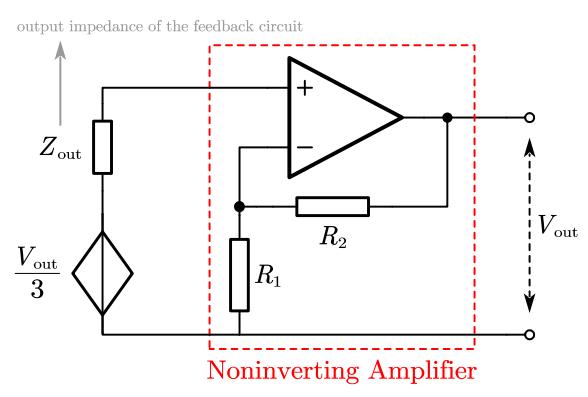


(b) Bode Plot of the RC Circuit

Figure 12.5: Wien Bridge Oscillator's Feedback Circuit



(a) Wien Bridge Oscillator



(b) Equivalent Circuit of Wien Bridge Oscillator

Figure 12.6: Wien Bridge Oscillator and Its Equivalent Circuit

Since a noninverting amplifier has extreme high input impedance and low output impedance, the coupling effect of the two circuits is negligible. In other words, the output impedance of noninverting amplifier (combing with the

input impedance of feedback circuit), and the output impedance of feedback circuit (combing with input impedance of noninverting amplifier) can be ignored. Thus, the Wien bridge oscillator, depicted in Figure 1.6 (a), has the equivalent circuit shown in Figure 1.6 (b).

The oscillation frequency f_0 of the Wien Bridge Oscillator is given by:

$$\omega_0 = \frac{1}{RC}, \quad f_0 = \frac{1}{2\pi RC} \quad (12.5)$$

As the voltage gain of noninverting amplifier is:

$$A_v = 1 + \frac{R_2}{R_1} \quad (12.6)$$

yielding the start-oscillation condition:

$$A_v > 3 \iff R_2 > 2R_1 \quad (12.7)$$

Assuming R_2 is slightly greater than $2R_1$, and there is a noise signal consists of a series of frequency, including $f_0 = \frac{1}{2\pi RC}$. Then at the selected resonant frequency f_0 , $\frac{1}{3}A_v > 1$, so the positive feedback will cancel out the negative feedback signal, causing the circuit to oscillate, until it reaches a voltage saturation (dependent on power supply). However, at the other frequency, $\frac{1}{3}A_v < 1$ so the negative feedback will cancel out the positive, resulting other frequency signal fading away.

The closer the ratio $\frac{R_2}{R_1}$ is to 2^+ , the better the waveform, but the longer the start-up time. Define $a = \frac{1}{3} \left(1 + \frac{R_2}{R_1} \right)$ as the periodic gain, a not-bad approximation for the start-up time is:

$$t_{\text{start}} = \frac{1}{f} \log_{a^3} \left(\frac{V_{\text{limit}}}{V_{\text{noise}}} \right) - 0.02 \quad (12.8)$$

where the unit of t is seconds, V_{limit} is the limit amplitude of output voltage, V_{noise} is the amplitude of noise.

By the way, if R_2 exceeds $2R_1$ too much, for example $R_2 = 3R_1$, the output waveform will be seriously distorted. Also, due to the slew rate limitations of operational amplifiers, frequencies above 1 MHz are unachievable without the use of special high frequency op-amps.

(2) Multisim Simulation of the Basic Circuit

Set the parameters in Figure 1.6 (a) as below, and run the simulation.

Table 12.2: Simulation Parameters of Wien Bridge Oscillator

R	C	R_1	R_2	Operation Amplifier	VCC
10 K Ω	10 nF	10 K Ω	20.1 K Ω	LM258P	± 12 V

The start-up time is about 570 ms, shown in Figure 1.7.

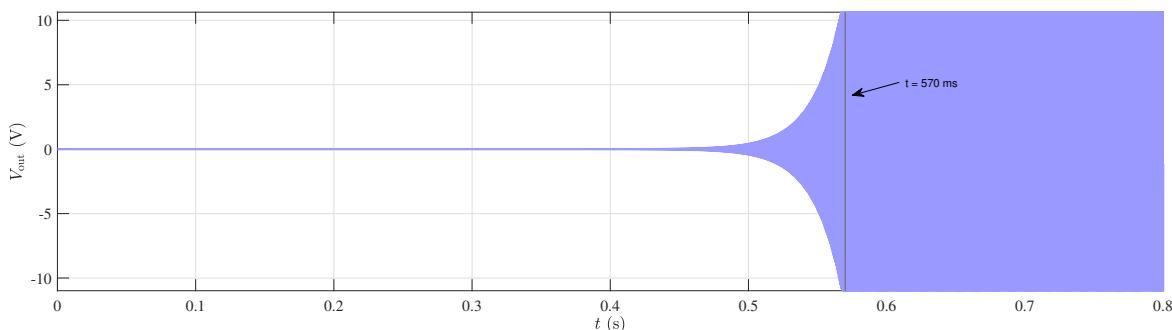


Figure 12.7: Start-up Time of Wien Bridge Oscillator

Export the simulation data, and perform a spectrum and distortion analysis in Matlab. Then we obtain the waveform and spectrum shown in Figure 1.8.

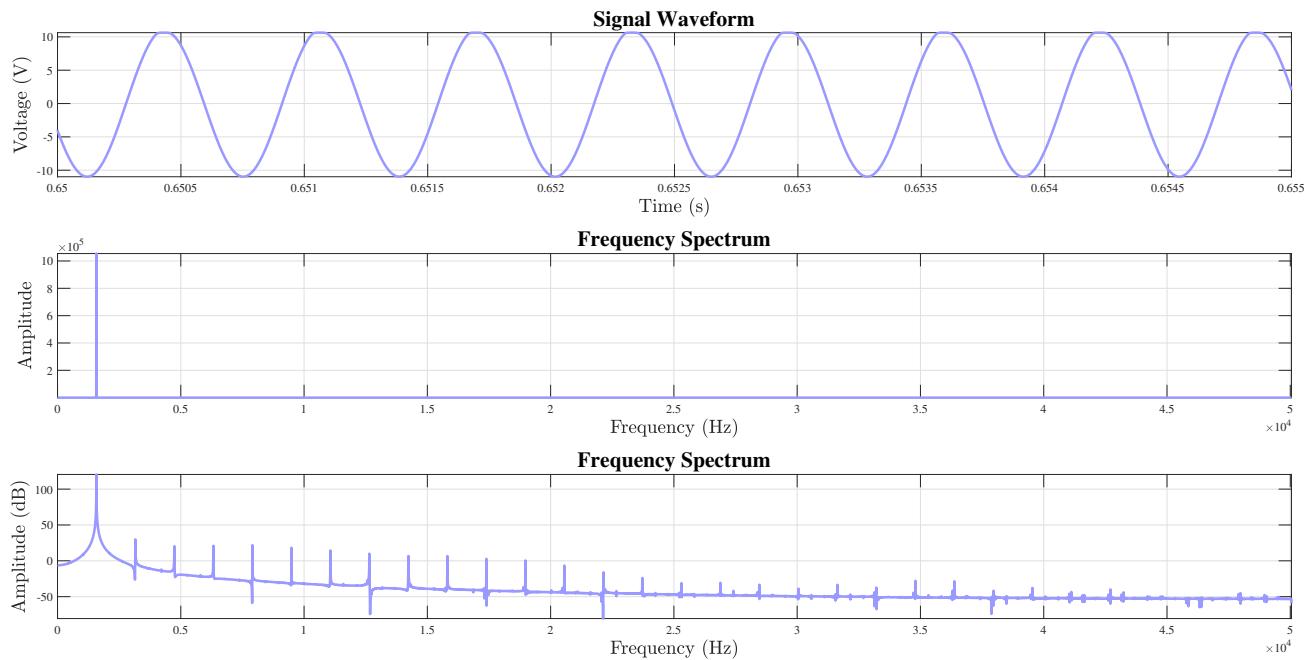


Figure 12.8: Spectrum Analysis of the Simulation Circuit

As we can see, the main output waveform is a sine wave at the resonant frequency f_0 , the simulated oscillation frequency is:

$$\begin{aligned} f_{\text{simu}} &= 1.5758 \text{ KHz} \\ f_{\text{theo}} &= 1.5915 \text{ KHz} \\ \eta &= \frac{f_{\text{simu}} - f_{\text{theo}}}{f_{\text{theo}}} = -0.98 \% \end{aligned} \quad (12.9)$$

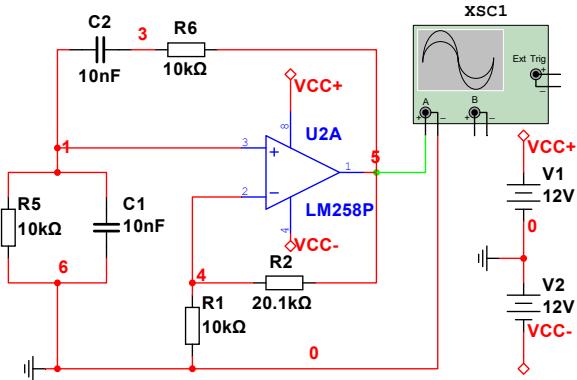
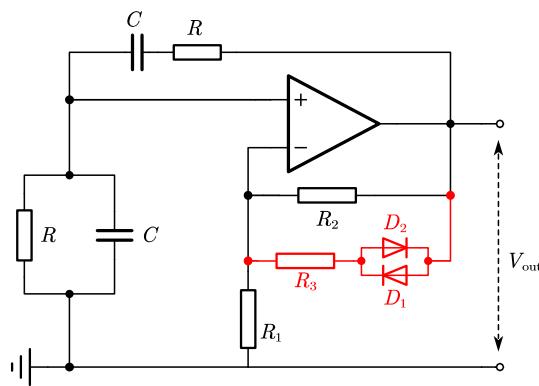


Figure 12.9: Simulation Circuit of Wien Bridge Oscillator

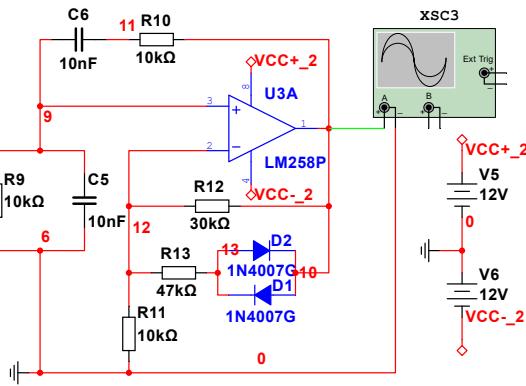
(3) Decrease the Start-up Time

We have noted that the output waveform is distorted when R_2 exceeds $2R_1$ too much, but the start-up time is too long when R_2 is too close to $2R_1$. Therefore, we need to optimize the circuit to achieve a better waveform and shorter start-up time, exemplified in Figure 1.10.

In Figure 1.10 (a), we added a resistance R_3 and two diodes D_1 and D_2 . When the output voltage amplitude is less than the threshold voltage of diodes V_D , the diodes are off, and the circuit is the same as the basic circuit. When the output is greater than V_D , the diodes are on, and the resistance of R_3 is added to the circuit (parallel with R_2), which reduces the gain of amplifier.



(a) Optimized Circuit



(b) Simulation Circuit

Figure 12.10: Optimize the Start-up Time of Wien Bridge Oscillator

Simulation circuit is shown in Figure 12.10 (b), the start-up time is reduced to about 10 ms (see Figure 1.11), and the output waveform is shown in Figure 12.12.

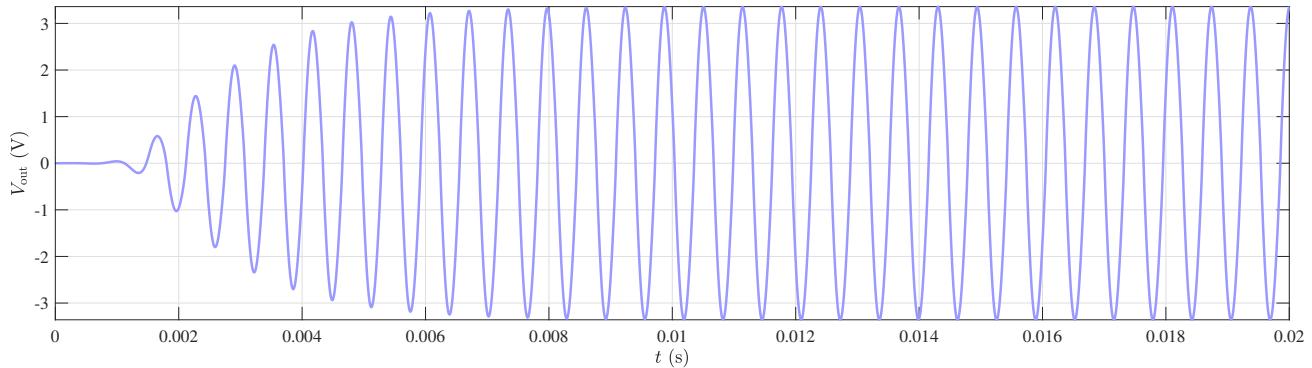


Figure 12.11: Optimized Start-up Time

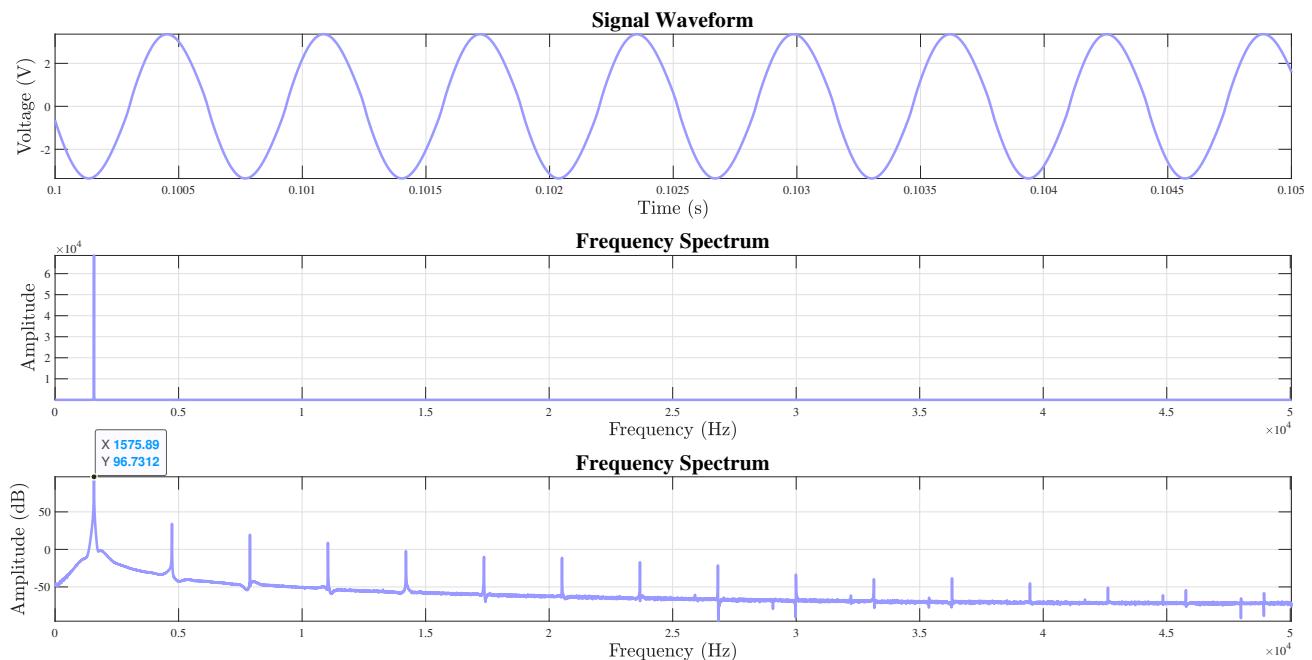


Figure 12.12: Spectrum Analysis of Optimized Circuit

Although the output frequency still focuses on f_0 and the distortion completely disappears, we have to note that

the amplitude of the output waveform is significantly reduced. If larger amplitudes are desired, a resistor can be added to divide V_{out} to a suitable level, leading to a larger output amplitude. Refer to *Fundamentals of Microelectronics* (3rd edition) (Behzad Razavi) page 661 for more details. For more alternative methods to optimize the start-up time, see https://blog.csdn.net/qq_29356039/article/details/132611987.

(4) Generate a Square Wave

An R_2 greater than $2R_1$ will result in the output waveform being clipped at the output voltage limitations. In other words, if we let $R_2 \gg 2R_1$, the waveform becomes a square wave. To prove our surmise, reset $R_1 = 1\text{ K}\Omega$, $R_2 = 30\text{ K}\Omega$ in Table 1.2, without changing the other parameters. We obtain the output waveform shown in Figure 1.13.

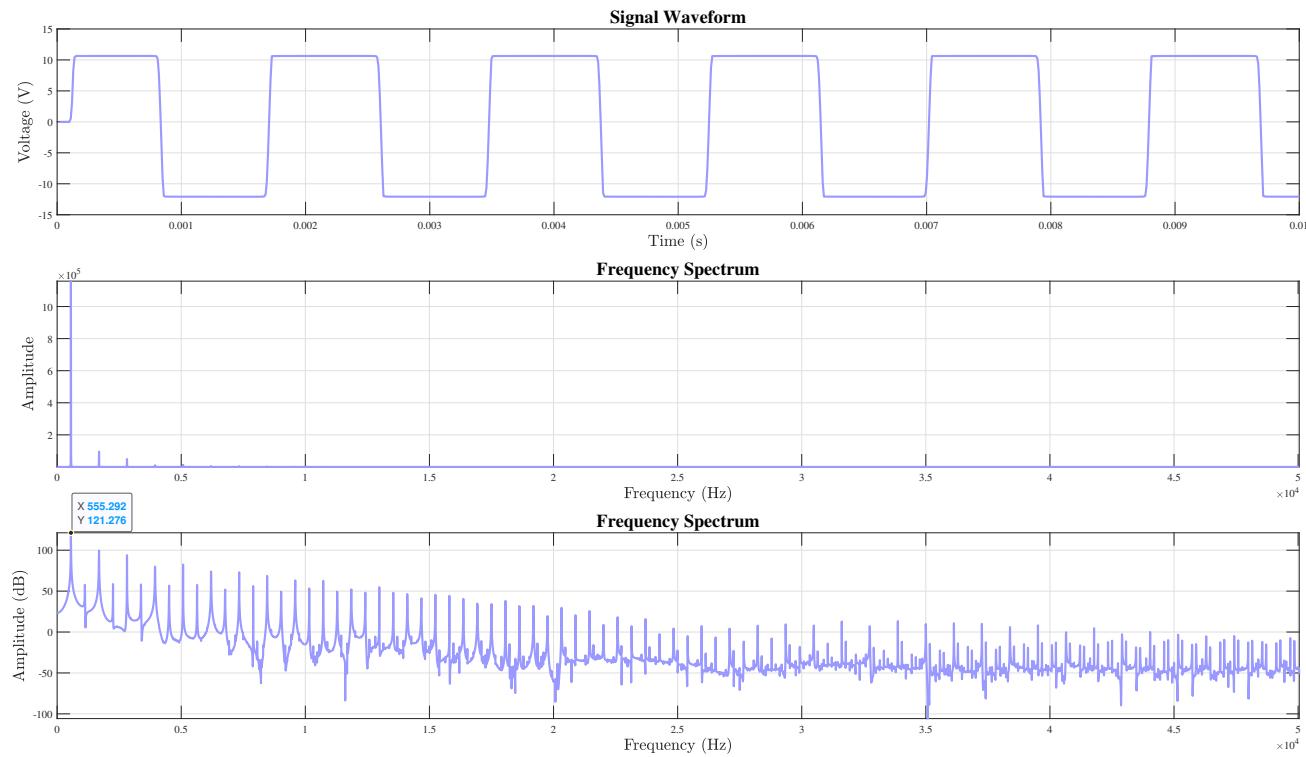


Figure 12.13: Square Wave Output of Wien Bridge Oscillator

Since the frequency of the square wave is difficult to calculate and control, the Wien Bridge Oscillator is not suitable for generating square waves in practical applications. To obtain an output signal with DC offset, see https://blog.csdn.net/qq_29356039/article/details/132611987 for more details.

Laboratory 1: 2024.10.11 - 2024.10.31

所有 Lab 实验课件（以及作业、仿真课件）见网址 <https://www.123865.com/s/0y0pTd-L8Kj3>, 后文不再重复实验要求和实验题目。

12.1 Pre-Lab

(1) The Thevenin and Norton equivalents of the network:

$$U_{\text{The}} = \frac{R_3 U}{R_1 + R_3}, \quad R = R_2 + R_1 \parallel R_3 \quad (12.1)$$

$$I_{\text{Nor}} = \frac{U}{R_1 \left(1 + \frac{R_2}{R_1 \parallel R_3} \right)}, \quad R = R_2 + R_1 \parallel R_3 \quad (12.2)$$

(2) Evaluate the results via $R_1 = 50 \Omega$, $R_2 = 1.5 \text{ K}\Omega$, $R_3 = 2.2 \text{ K}\Omega$:

$$U_{\text{The}} = 4.8889 \text{ V}, \quad I_{\text{Nor}} = 0.003156 \text{ A}, \quad R = 1548.8889 \Omega \quad (12.3)$$

(3) With $U_s = 5 \text{ V}$, $R_{\text{ON}} = 4 \Omega$ and $R_D = 1 \text{ K}\Omega$, the output voltage of all combinations of input voltage are shown as below :

Table 12.1: The output voltage u_o of In-Lab (3)

Gate	Input voltage u_i	Logic input	Output voltage u_o (V)	Logic output
NOT	$< U_T$	0	$U_s = 5$	1
NOT	$> U_T$	1	$\frac{R_{\text{ON}} U_s}{R_{\text{ON}} + R_D} = 0.0199$	0
NOR	$u_1 < U_T, u_2 < U_T$	(0, 0)	$U_s = 5$	1
NOR	$u_1 > U_T, u_2 < U_T$	(1, 0)	$\frac{R_{\text{ON}} U_s}{R_{\text{ON}} + R_D} = 0.0199$	0
NOR	$u_1 < U_T, u_2 > U_T$	(0, 1)	$\frac{R_{\text{ON}} U_s}{R_{\text{ON}} + R_D} = 0.0199$	0
NOR	$u_1 > U_T, u_2 > U_T$	(1, 1)	$\frac{R_{\text{ON}} U_s}{R_{\text{ON}} + 2R_D} = 0.0010$	0
NAND	$u_1 < U_T, u_2 < U_T$	(0, 0)	$U_s = 5$	1
NAND	$u_1 > U_T, u_2 < U_T$	(1, 0)	$U_s = 5$	1
NAND	$u_1 < U_T, u_2 > U_T$	(0, 1)	$U_s = 5$	1
NAND	$u_1 > U_T, u_2 > U_T$	(1, 1)	$\frac{2R_{\text{ON}} U_s}{2R_{\text{ON}} + R_D} = 0.0397$	0

(4, 5) The truth tabel and the circuit diagram are as below:

Table 12.2: Input-output truth table

Logic input (A, B, C)	Logic output
(0, 0, 0)	1
(0, 0, 1)	1
(0, 1, 0)	1
(0, 1, 1)	1
(1, 0, 0)	0
(1, 0, 1)	1
(1, 1, 0)	1
(1, 1, 1)	1

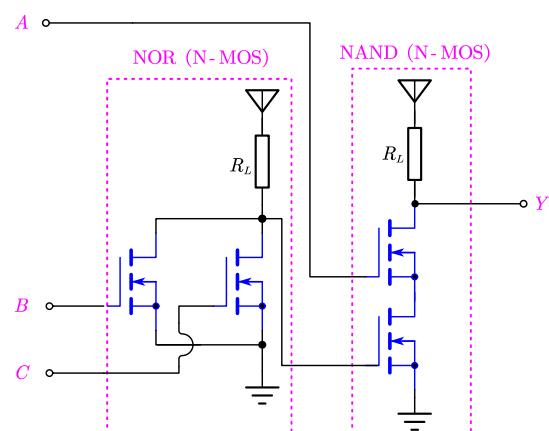
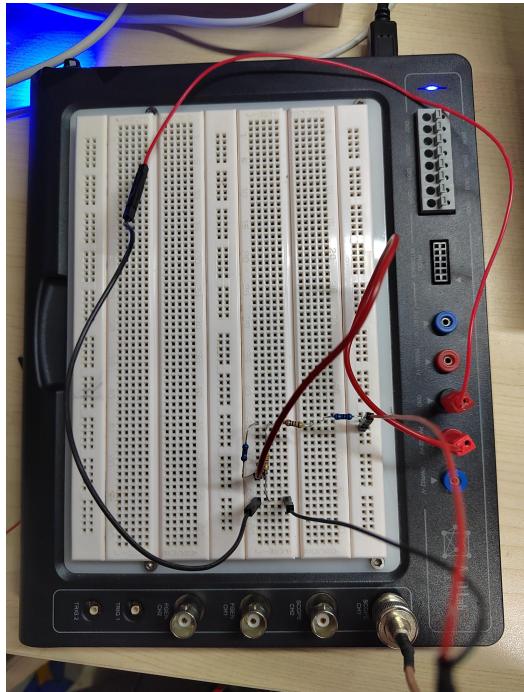


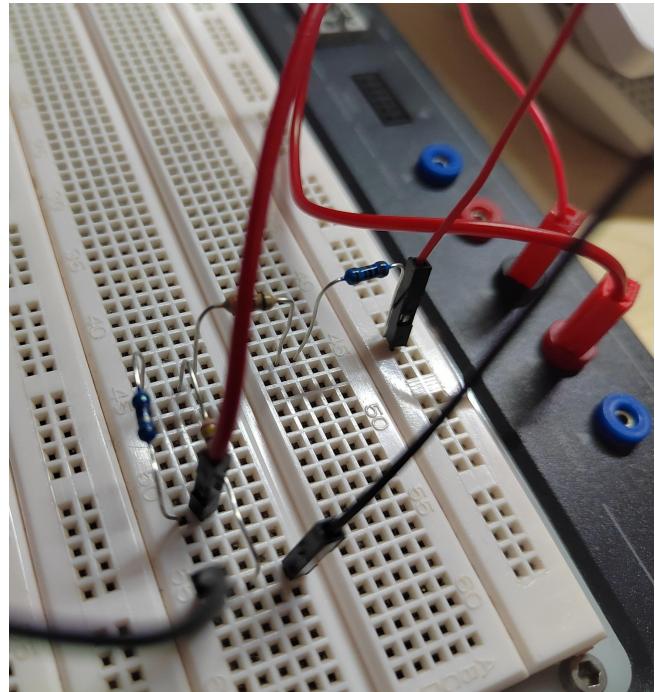
Figure 12.1: The circuit diagram

12.2 In-Lab

- (1) The wire connection is shown in the Figure 12.2, with $R_1 = 51.0 \Omega$ (51.0 Ω), $R_2 = 1.50 \text{ K}\Omega$ (1.47 $\text{K}\Omega$) and $R_3 = 2.40 \text{ K}\Omega$ (2.36 $\text{K}\Omega$). Outside parentheses is the original value and the actual measured value is in parentheses.



(a) Global view



(b) Partial view

Figure 12.2: The actual circuit connection of In-Lab (1)

- (2) Use the multi-meter and the open-circuit voltage is 4.89 V and the short-circuit current is 3.17 mA.



(a) Open-circuit voltage



(b) Short-circuit current

Figure 12.3: Measure the short-circuit current and open-circuit voltage

- (3) Connect the resistor and measure the port voltage, the results are as below:

Table 12.3: The port voltage

Resistance (Ω)	561 (560)	1.00 K (0.99 K)	2.20 K (2.19 K)
Voltage (V)	1.30	1.92	2.89

(4) We use the signal generator to provide the voltage u_{GS} , and the oscilloscope to measure the output voltage u_{DS} .

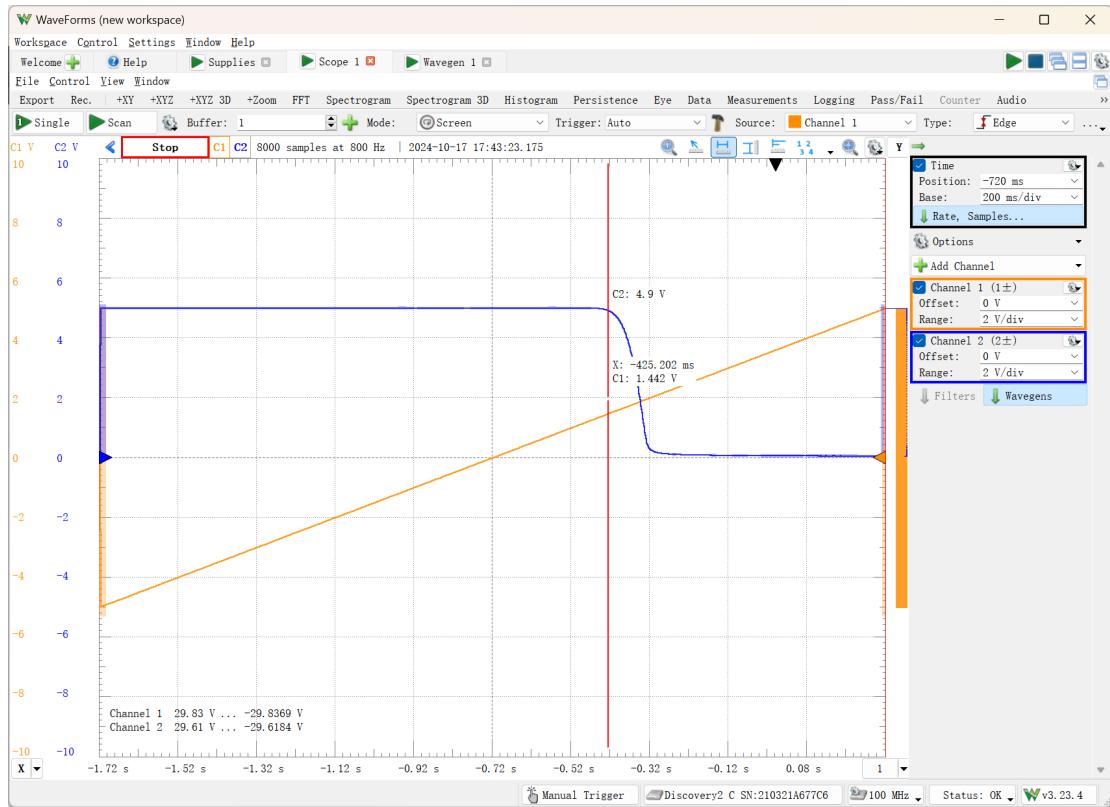


Figure 12.4: u_{DS} starts to fall

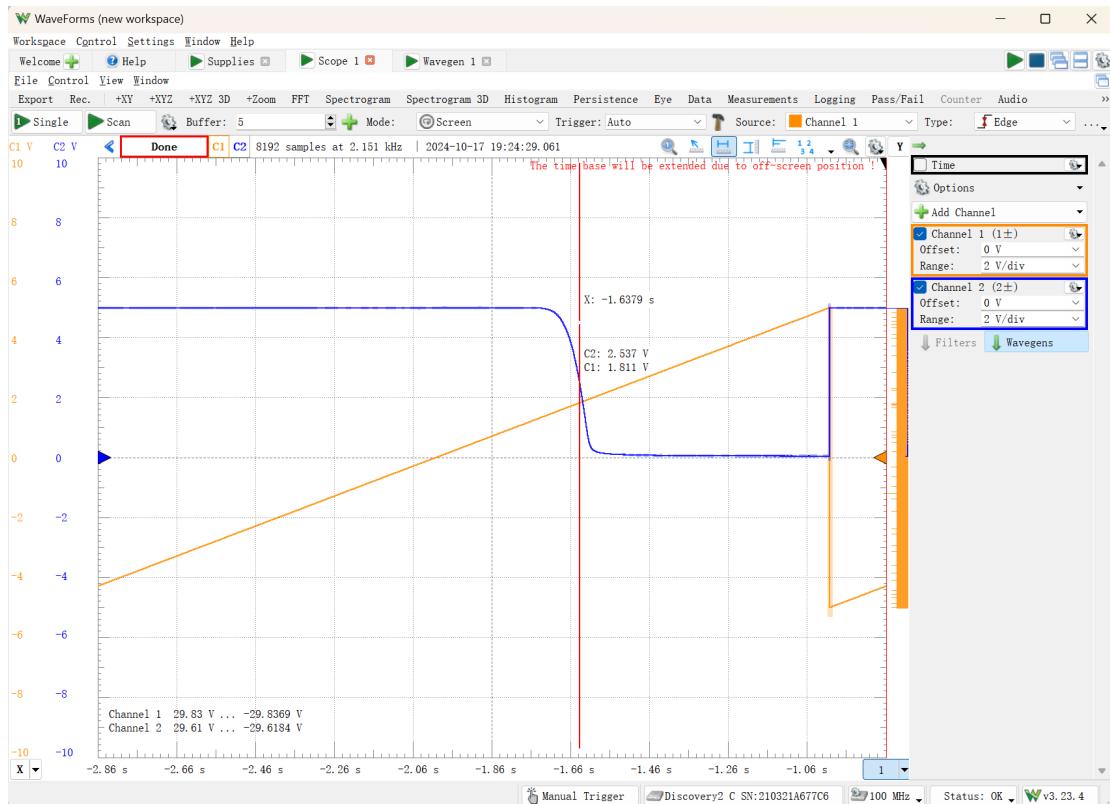


Figure 12.5: u_{DS} is reduced to half its original value

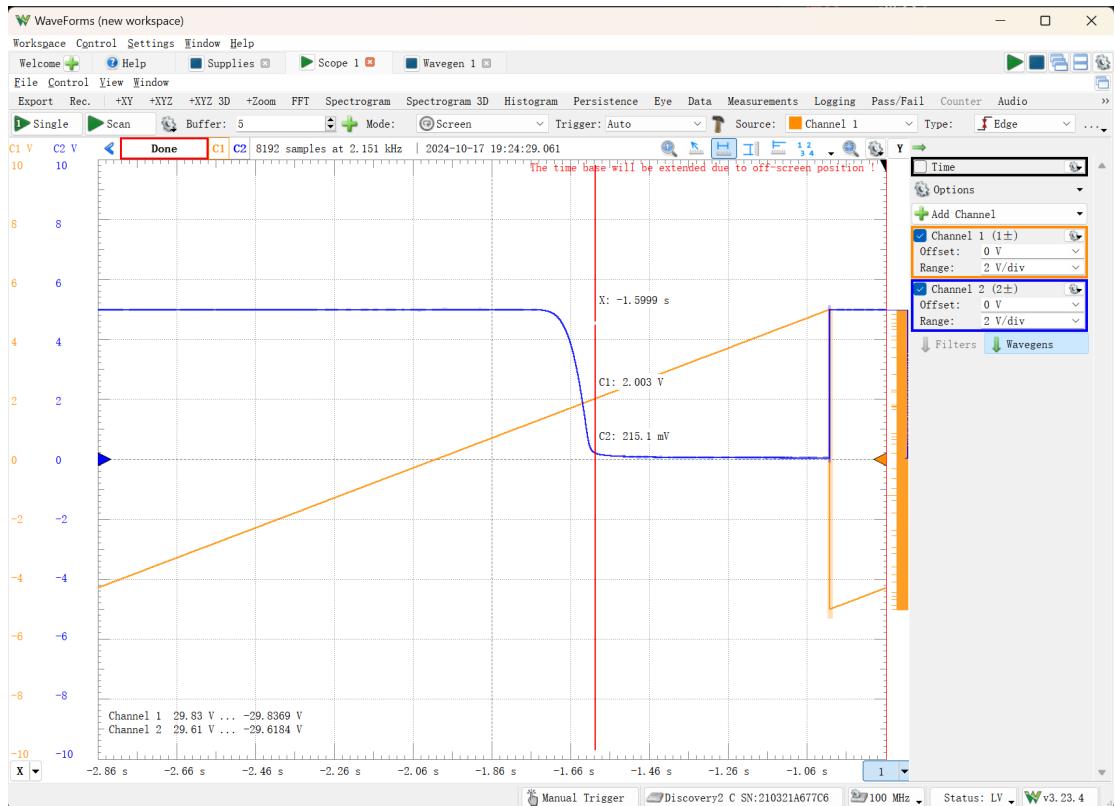
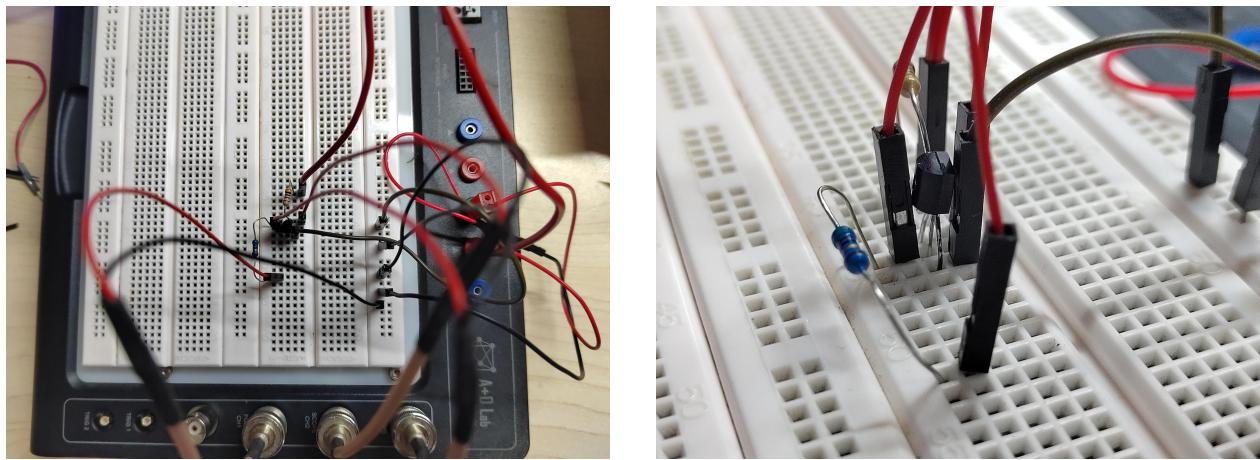


Figure 12.6: u_{DS} is almost approaching zero

The results are shown in the three figures above. It is $u_{GS} = 1.442$ V when u_{DS} starts to fall, $u_{GS} = 1.811$ V when the u_{DS} is reduced to half its original value, and $u_{GS} = 2.003$ V when u_{DS} is almost approaching zero. To ensure MOSFET is fully on, the threshold voltage should be $U_T = 2.003$ V.

(5) The actual wire connection is shown as below. $R_{DS} = 2.8 \Omega$ when $U_{GS} = 5$ V and $R_{DS} = 8.7 \Omega$ when $U_{GS} = 2$ V.



(a) The actual wire connection of In-Lab (5)

(b) The actual wire connection of In-Lab (5)

Figure 12.7

(6) The actual wire connection is shown as below. $u_o = 4.989$ V when the switch is closed (logic input 0) and $u_o = 0.018$ V when the switch is opened (logic input 1).

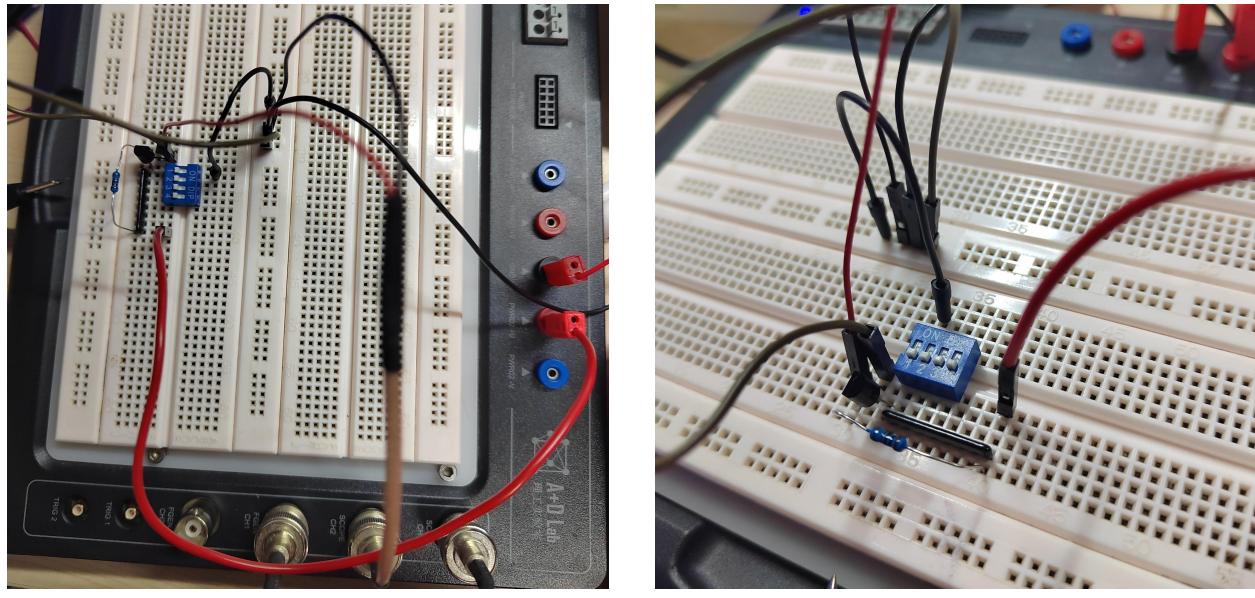


Figure 12.8: The actual wire connection of In-Lab (6)

(7, 8) Construct the NOR Gate and NAND, measure their output voltage u_o of all combinations of switch positions respectively. The results are as below:

Table 12.4: The output voltage u_o results of In-Lab (7, 8)

Gate	Switch positon	Logic Input	Output voltage (V)
NOR	(closed, closed)	(0, 0)	4.989
NOR	(opened, closed)	(0, 1)	0.018
NOR	(closed, opened)	(1, 0)	0.018
NOR	(opened, opened)	(1, 1)	0.00874
NAND	(closed, closed)	(0, 0)	4.989
NAND	(opened, closed)	(0, 1)	4.989
NAND	(closed, opened)	(1, 0)	4.989
NAND	(opened, opened)	(1, 1)	0.03212

(9) The actual wire connection is shown as below.

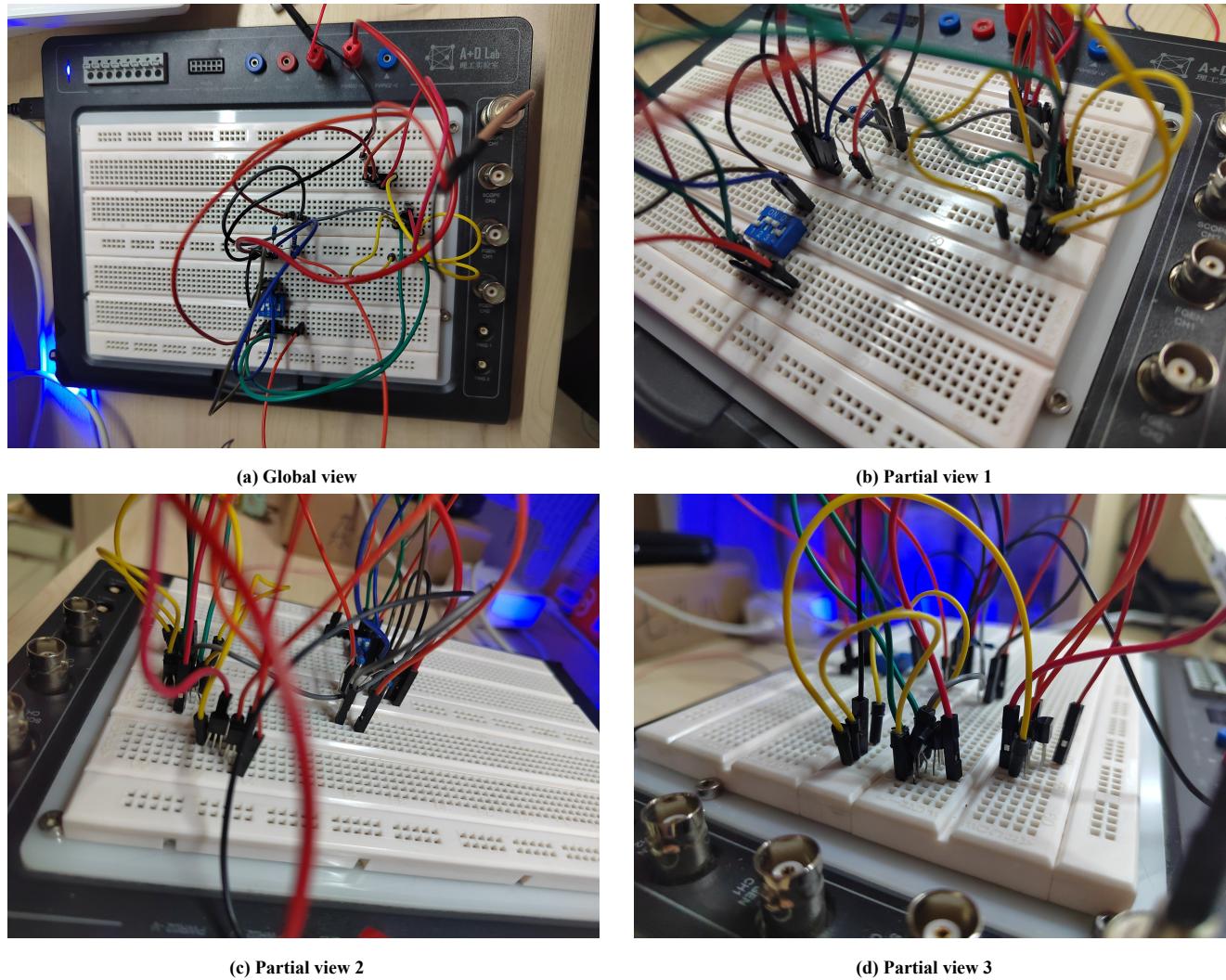
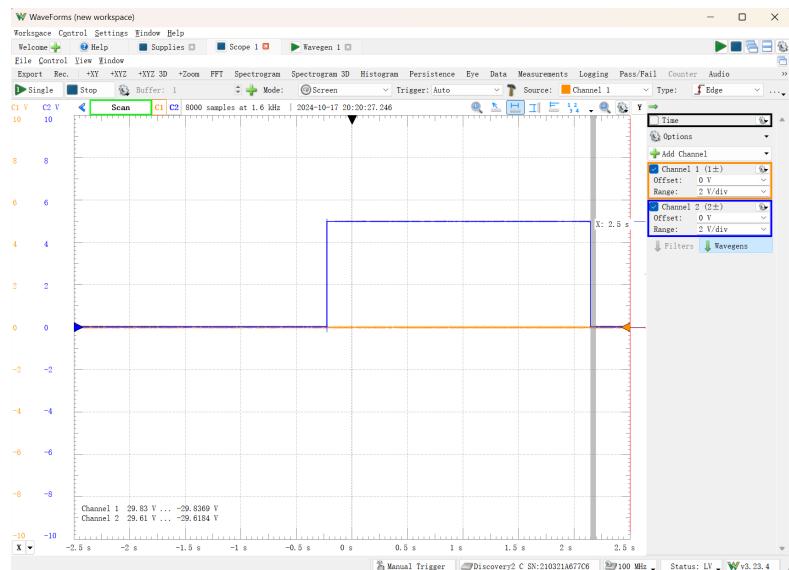


Figure 12.9: The actual wire connection of In-Lab (9)

And the measurement results are:

Table 12.5: u_o results of In-Lab (9)

(A, B, C)	u_o (V)
(0, 0, 0)	4.991
(0, 0, 1)	4.987
(0, 1, 0)	4.987
(0, 1, 1)	4.987
(1, 0, 0)	0.01962
(1, 0, 1)	4.987
(1, 1, 0)	4.987
(1, 1, 1)	4.987

Figure 12.10: u_o becomes 19.62 mV only when (A, B, C) = (1, 0, 0)

12.3 Post-Lab

(1) The Experimental parameters and their actual values is:

Table 12.6: Comparison of the values bewteen Pre-Lab (2) and In-Lab (2)

Parameter	Theoretical	Experimental	Percentage error
$U_{\text{The}} (\text{V})$	4.8889	4.89	+ 0.02 %
$I_{\text{Nor}} (\text{mA})$	3.156	3.17	+ 0.44 %
R	1548.8889	1542.5868	- 0.41 %

Percentage error = $\frac{|y - \hat{y}|}{y}$ standing for "Absolute Percentage Error", where y is the experimental value and \hat{y} is the theoretical value.

In Pre-Lab (2), the given theoretical resistance is $R_1 = 50 \Omega$, $R_2 = 2.2 \text{ K}\Omega$, $R_3 = 1.5 \text{ K}\Omega$. However, the actual resistance we used in In-Lab (2) was $R_1 = 51.0 \Omega$, $R_2 = 1.47 \text{ K}\Omega$ and $R_3 = 2.40 \text{ K}\Omega$. Reevaluate the related parameters as below and these should be more realistic results.

Table 12.7: Comparison under the actual values

Parameter	Actual theoretical	Experimental	Percentage error
$U_{\text{The}} (\text{V})$	4.8942	4.89	- 0.09 %
$I_{\text{Nor}} (\text{mA})$	3.220	3.17	- 1.58 %
R	1519.921	1542.5868	+ 1.47 %

When using multi-meter as a voltage-meter, as it is not ideal, the experimental voltage should be slightly lower the theoretical value. Also, the experimental current should be higher than the theoretical. These exactly matches the results in Table 12.7.

(2) According to Table 12.3, the corresponding port current (with the actual resistance) is:

Table 12.8: The port current

Resistance (Ω)	561 (560)	1.00 K (0.99 K)	2.20 K (2.19 K)
Voltage (V)	1.30	1.92	2.89
current (mA)	2.3214	1.9394	1.3196

(3) The port's $i-u$ relationship and the data obtained during In-Lab (3) are shown in the Figure 12.11 on the top of the next page.

(4) According to the data sheet of N-MOSFET 2N7000, R_{DS-ON} is typically 1.7Ω and not exceeding 7.5Ω when $U_{GS} = 5 \text{ V}$. Our experimental data is 2.8Ω , which is within the range of the data sheet.

We can know the threshold voltage U_T of 2N7000 is about 2.0 V according to the Figure 12.12 from the data sheet. The comparison of MOSFET data is shown in Table 12.9.

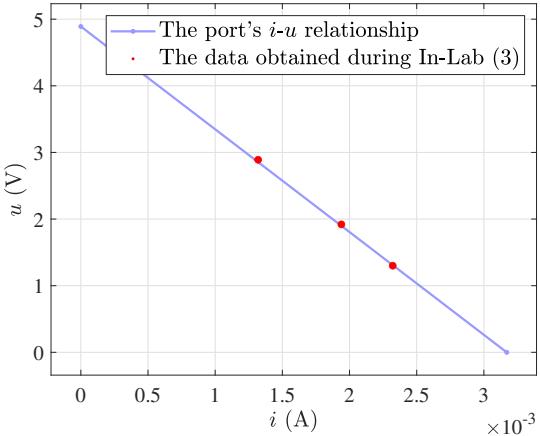
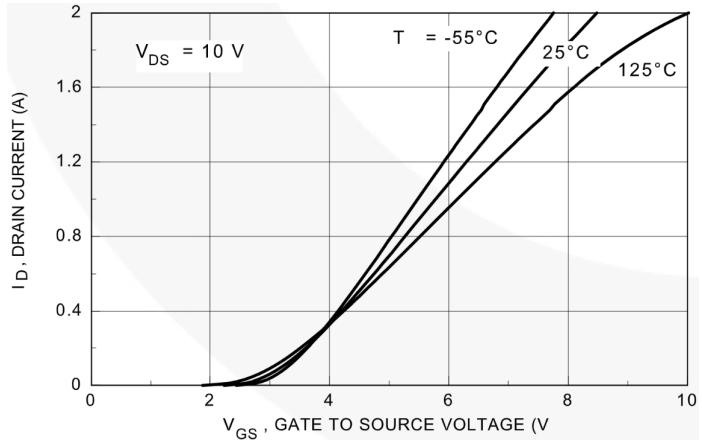
Figure 12.11: The port's i - u relationship

Figure 12.12: The threshold voltage of 2N7000

Table 12.9: Comparison of MOSFET data

Type	R_{DS-ON} (Ω)	U_T (V)
Data sheet	typical 1.7 and max 7.2	between 1.85 and 2.5
Experiment	2.8	2.003

(5) With $R_{ON} = 2.8 \Omega$ measured during In-Lab (5), we can get the theoretical output voltage of different switch positions, comparing with the experimental values in Table 12.10.

Table 12.10: The output voltage u_o using R_{ON} measured during In-Lab (5)

Gate	Input voltage u_i	Logic input	Theoretical output u_o (V)	Experimental output (V)
NOT	$< U_T$	0	$U_s = 5$	4.989
NOT	$> U_T$	1	$\frac{R_{ON}U_s}{R_{On}+R_D} = 0.0140$	0.018
NOR	$u_1 < U_T, u_2 < U_T$	(0, 0)	$U_s = 5$	4.989
NOR	$u_1 > U_T, u_2 < U_T$	(1, 0)	$\frac{R_{ON}U_s}{R_{On}+R_D} = 0.014$	0.018
NOR	$u_1 < U_T, u_2 > U_T$	(0, 1)	$\frac{R_{ON}U_s}{R_{On}+R_D} = 0.014$	0.018
NOR	$u_1 > U_T, u_2 > U_T$	(1, 1)	$\frac{R_{ON}U_s}{R_{On}+2R_D} = 0.0070$	0.009
NAND	$u_1 < U_T, u_2 < U_T$	(0, 0)	$U_s = 5$	4.989
NAND	$u_1 > U_T, u_2 < U_T$	(1, 0)	$U_s = 5$	4.989
NAND	$u_1 < U_T, u_2 > U_T$	(0, 1)	$U_s = 5$	4.989
NAND	$u_1 > U_T, u_2 > U_T$	(1, 1)	$\frac{2R_{ON}U_s}{2R_{On}+R_D} = 0.0278$	0.032

(6) Suppose the supply voltage is 5 V, to achieve the best symmetric noise margins, we specify the voltage parameters of MOSFET as below:

Table 12.11: Voltage parameters of MOSFET

	Input	Output
Low	$U_{IL} = 1.442 \text{ V}$	$U_{OL} = 5 \text{ V}$
High	$U_{IH} = 2.003 \text{ V}$	$U_{OH} = 0 \text{ V}$

When the input voltage is lower than $U_{LI} = 1.442$ V (logic 0), the MOSFET is off so that the inverter produce a high voltage closed to $U_{OH} = 5$ V (logic 1). On the contrary, when the input voltage is higher than $U_{HI} = 2.003$ V (logic 1), the MOSFET is on so that the inverter produce a low voltage closed to $U_{OL} = 0$ V (logic 0).

Laboratory 2: 2024.11.21 - 2024.12.12

14.1 Pre-Lab

(1) Voltage-Voltage Characteristics of Inverting Amplifier

The output voltage V_{out} as a function of input voltage V_{in} is shown in Fig. 14.1, where V_T is the threshold voltage of the MOSFET.

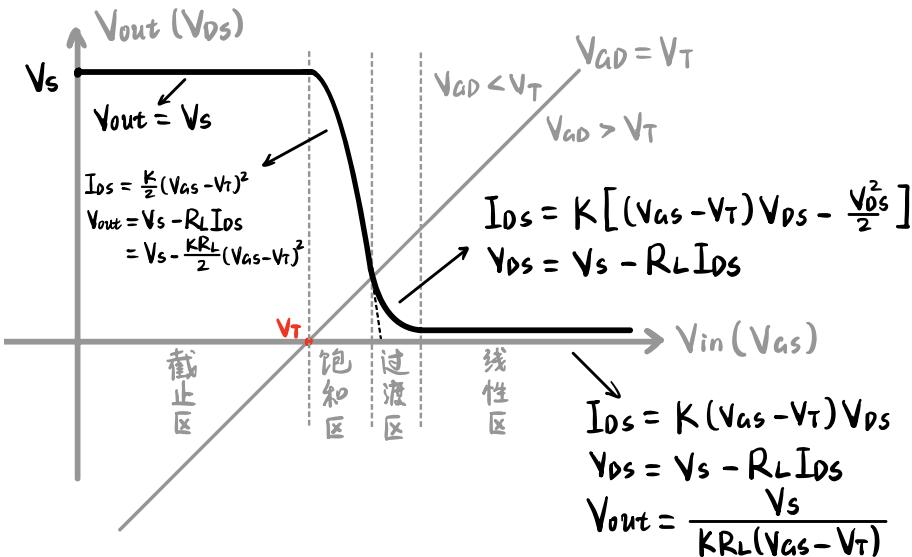


Figure 14.1: Voltage-Voltage Characteristics of Inverting Amplifier

For easy reference, we summarize the formula as:

$$V_{\text{out}} = \begin{cases} V_s, & V_{\text{gs}} \in [0, V_T] \\ V_s - \frac{KR_L}{2}(V_{\text{gs}} - V_T)^2, & V_{\text{gs}} \in [V_T, V_0] \\ V_{\text{gs}} - V_T + \frac{1-\sigma}{KR_L}, & V_{\text{gs}} \in [V_0, V_0 + \Delta V] \\ \frac{V_s}{KR_L(V_{\text{gs}} - V_T)}, & V_{\text{gs}} \in [V_0 + \Delta V, V_{\text{max}}] \end{cases} \quad (14.1)$$

Where V_0 is the second solution $V_{\text{gs},2}$ of equations:

$$\begin{cases} V_{\text{gs}} = V_s - \frac{KR_L}{2}(V_{\text{gs}} - V_T)^2 \\ V_{\text{gd}} = V_T \end{cases} \implies V_0 = \frac{\sqrt{2KR_LV_s + 1} + KR_LV_T - 1}{KR_L} \quad (14.2)$$

And σ is the discriminant of the quadratic equation:

$$\sigma = \sqrt{K^2R_L^2V_{\text{gs}}^2 + 2KR_L(1 - KR_LV_T)V_{\text{gs}} + [K^2R_L^2V_T^2 - 2KR_L(V_T + V_s) + 1]} \quad (14.3)$$

When the input voltage $V_{\text{in}}(V_{\text{gs}}) > V_0$ is large enough, i.e. $V_{\text{in}} \geq V_0 + \Delta V$, an approximation can be made:

$$I_{\text{ds}} \approx K(V_{\text{gs}} - V_T)V_{\text{ds}} \implies V_{\text{out}} = \frac{V_s}{KR_L(V_{\text{gs}} - V_T)} \quad (14.4)$$

(2) Small Voltage Gain of Inverting Amplifier

We have already driven the small signal voltage gain during the last homework. Assuming the small AC input voltage is u_{in} , and MOS is biased into saturation region, it follows that:

$$A = \frac{u_{\text{out}}}{u_{\text{in}}} = -g_m R_L = -K(V_{\text{GS}} - V_{\text{T}}) R_L \quad (14.5)$$

(3) RC Transient Process

By the three-element method, we can obtain:

$$V_{\text{out}} = \frac{R_2 V_{\text{I}}}{R_1 + R_2} \left(1 - e^{-\frac{t}{\tau}}\right), \quad \tau = (R_1 \parallel R_2) C = \frac{R_1 R_2}{R_1 + R_2} C \quad (14.6)$$

(4) Transient Time

Given V_{T} in the range $[0, V_{\text{S}}]$ ($V_{\text{S}} = \frac{R_2 V_{\text{I}}}{R_1 + R_2}$), the time where V_{out} reaches V_{T} is:

$$\Delta t = \tau \ln \left(\frac{V_{\text{S}}}{V_{\text{S}} - V_{\text{T}}} \right), \quad \tau = \frac{R_1 R_2}{R_1 + R_2} C, \quad V_{\text{S}} = \frac{R_2 V_{\text{I}}}{R_1 + R_2} \quad (14.7)$$

14.2 In-Lab

(1) Static Input-Output Relationship of Inverting Amplifier

Measure In-Out Voltage Relationship

Construct the circuit in Fig.14.2, then we can obtain the voltage relationship shown in Fig.14.3.

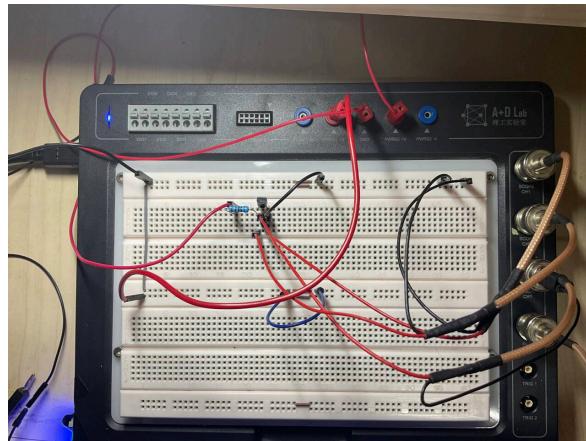


Figure 14.2: Measure In-Out Voltage Relationship

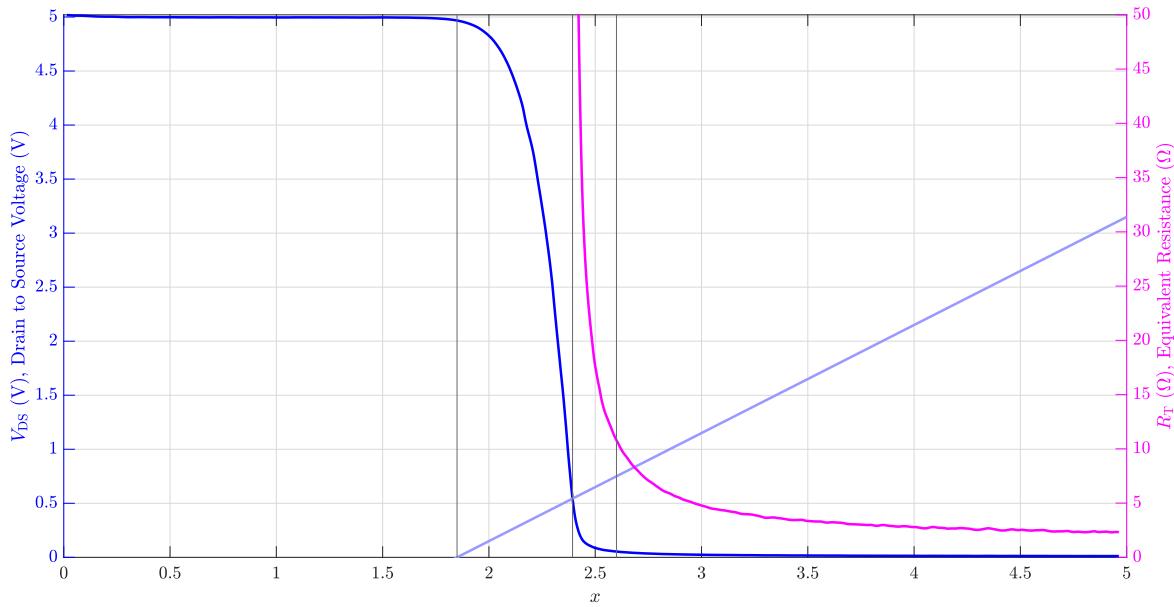


Figure 14.3: Operational Characteristics of Inverting Amplifier

The Threshold of The MOSFET

With $V_S = 5.0185$ V, $R_L = 1$ KΩ (998 Ω) and the data obtained in the last section, we can get the threshold voltage of the MOSFET:

$$V_T = 1.85 \text{ V}, \quad V_0 = 2.3934 \text{ V}, \quad \Delta V = 0.2066 \text{ V} \quad (14.8)$$

Tables of Input-Output Voltage

Table 14.1: Input-Output Voltage Relationship

Output (V)	5	4	3	2	1	0.0116
Input (V)	0.2765	2.1767	2.2686	2.3238	2.3712	4.9599

(2) Small Signal Voltage Gain

Voltage Gain of Inverting Amplifier

With the data measured in section (1), we can derived the voltage gain $A_v = \frac{dV_{DS}}{dV_{GS}}$ via matrix difference (see Fig.14.4).

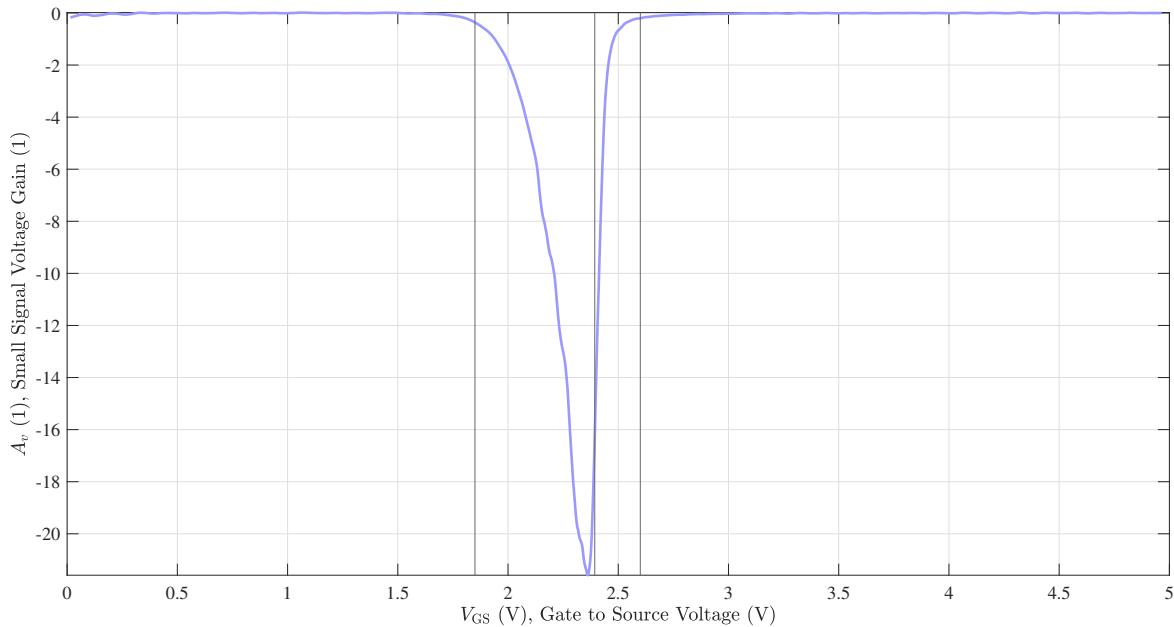


Figure 14.4: Small Signal Voltage Gain of Inverting Amplifier

Since transconductance satisfies $A = -g_m R_L$, we can also get the transconductance by dividing the limiting resistance $R_L = 1 \text{ K}\Omega$ (998Ω).

Construct the circuit to measure the voltage gain where the output voltage is 2 V and the sine wave has 50 mV amplitude (from -50 mV to +50 mV). The measured result and the data in Fig.14.4 is:

$$(A_v)_{\text{meas}} = \frac{-1.0056 \text{ V}}{50 \text{ mV}} = -20.1120, \quad (A_v)_{\text{fig}} = -20.1513 \quad (14.9)$$

As we can see, almost no deviation.

Clipping Distortion

Set the amplitude of sine wave to 50 mV ($[-50 \text{ mV}, +50 \text{ mV}]$), the lower and upper bias limits are (see Fig.14.5 and Fig.14.6):

$$V_{\text{bias,min}} = 2.00 \text{ V}, \quad V_{\text{bias,max}} = 2.39 \text{ V} \quad (14.10)$$

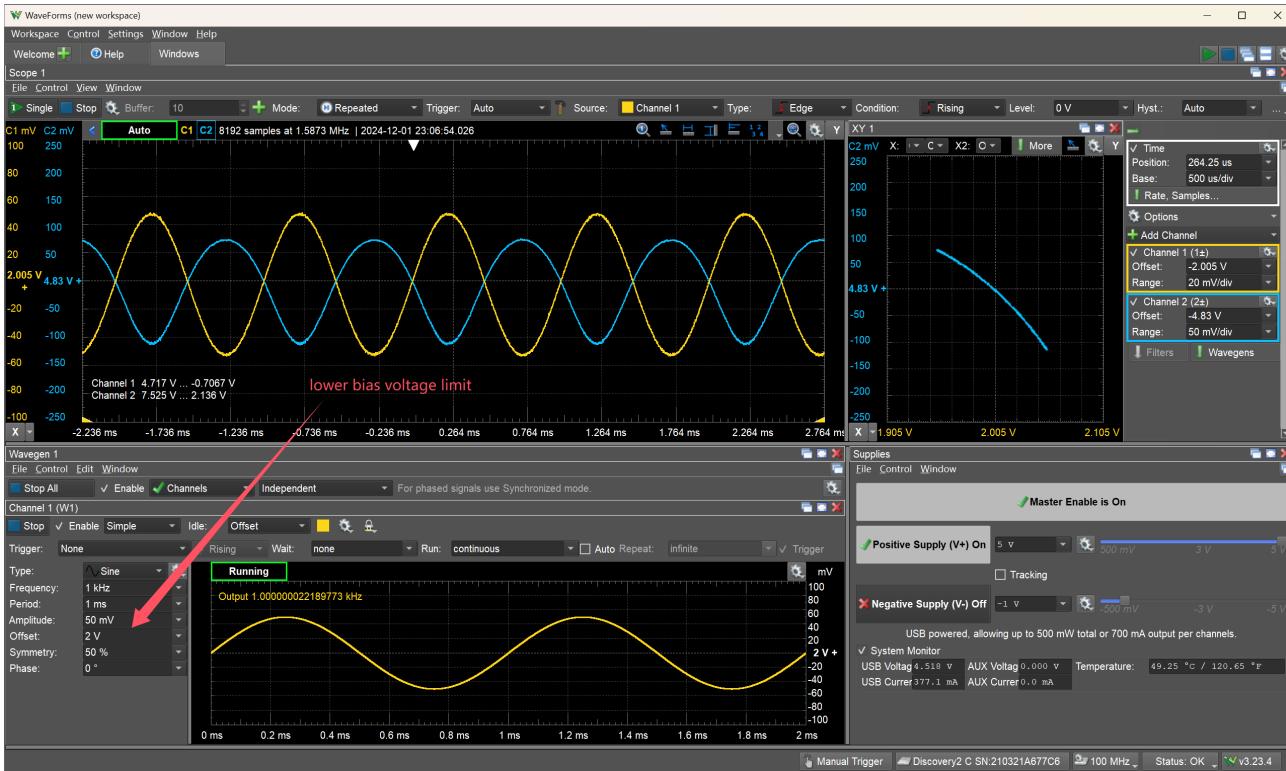


Figure 14.5: The Lower Limit of The Input Bias Voltage

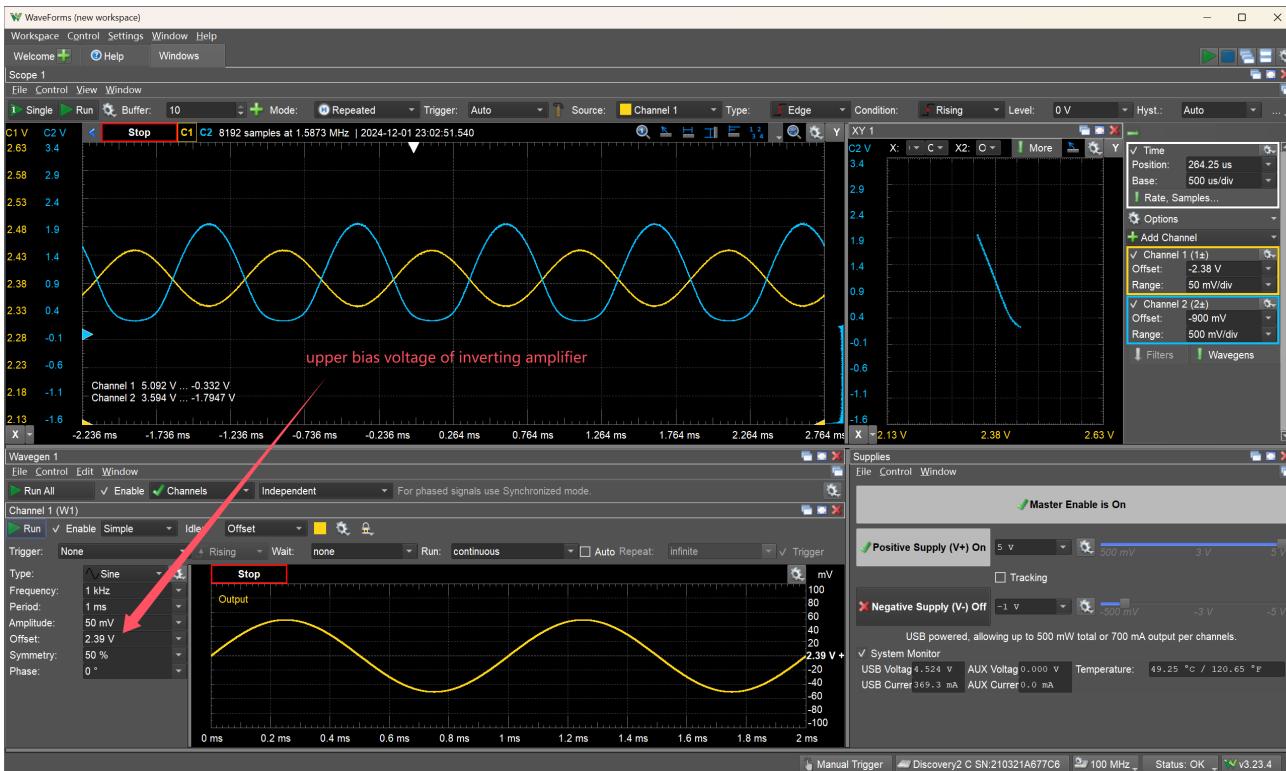


Figure 14.6: The Upper Limit of The Input Bias Voltage

(3) The Delay of Inverting Amplifier as a Digital Inverter

Measure The Gate to Source Capacitance of Inverting Amplifier

Use a $500\text{ K}\Omega$ resistor and a $30\text{ K}\Omega$ resistor to measure the gate to source capacitance C_{GS} of the MOSFET, respectively. The measured results are:

$$R_L = 500\text{ K}\Omega, \lim_{t \rightarrow 0^+} \frac{dV_{out}}{dt} = 57070.3\text{ V} \cdot \text{s}^{-1}, V_{steady} = 3.3198\text{ V} \implies C = \frac{V_0}{R_L k_0^+} = 116.3407\text{ }\mu\text{F} \quad (14.11)$$

$$R_L = 30.0\text{ K}\Omega, \lim_{t \rightarrow 0^+} \frac{dV_{out}}{dt} = 948889.8\text{ V} \cdot \text{s}^{-1}, V_{steady} = 4.9032\text{ V} \implies C = \frac{V_0}{R_L k_0^+} = 172.2434\text{ }\mu\text{F} \quad (14.12)$$

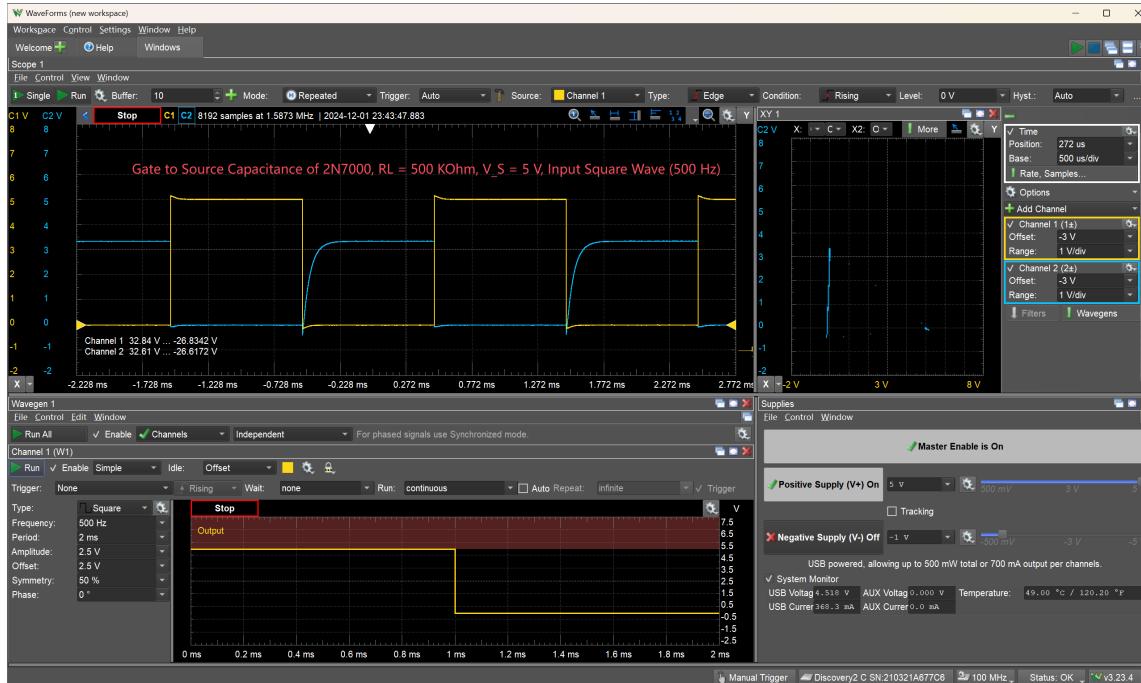
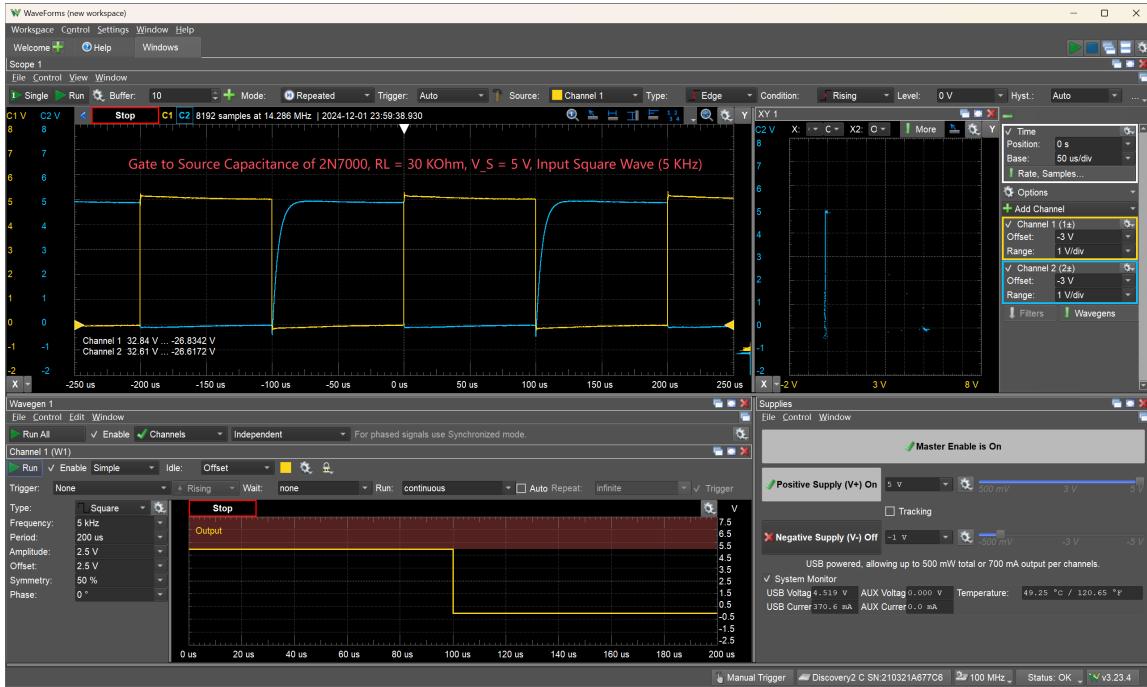
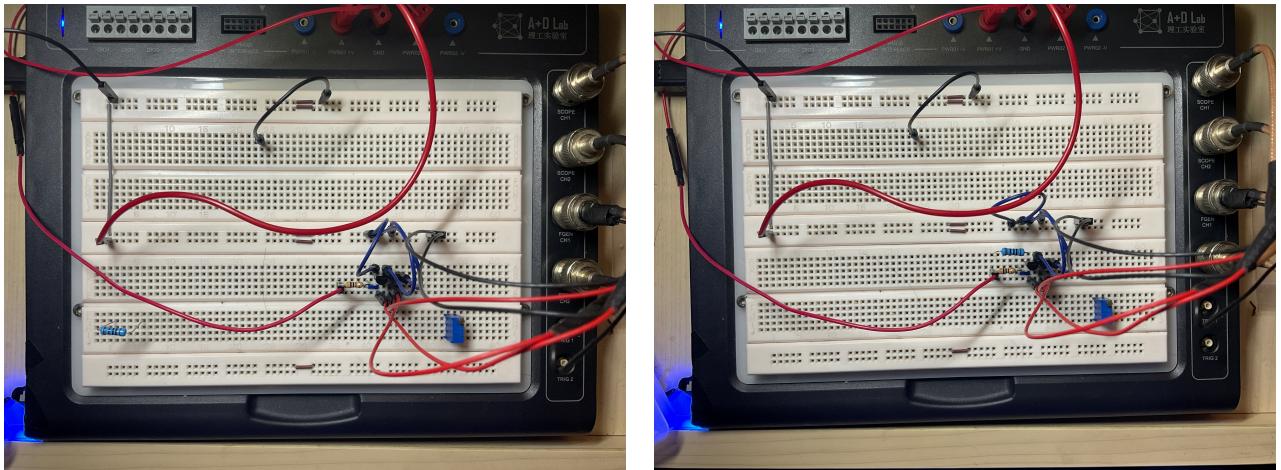


Figure 14.7: Gate to Source Capacitance of 2N7000, $R_L = 500\text{ K}\Omega$

Figure 14.8: Gate to Source Capacitance of 2N7000, $R_L = 30 \text{ k}\Omega$

It is puzzling that $C =$ with $R_L = 500 \text{ k}\Omega$. Actually, with the resistance $R_L = 500 \text{ k}\Omega$, the steady voltage V_0 is limited to 3.3198 V (see Fig.14.7), which is not the expected value. Why is it like that? Because the oscilloscope input resistance is not large enough. We keep this to the Post-Lab,



(a) The Gate to Source Capacitance of Inverting Amplifier

(b) The Delay of Inverting Amplifier as a Digital Inverter

Figure 14.9: In-Lab 3.1 and In-Lab 3.2

Measure The Delay of Inverting Amplifier

With $R_{L,1} = 30 \text{ k}\Omega$ and $R_{L,2} = 1 \text{k}\Omega$, construct the circuit in Fig.14.9 (b), obtain the delay time of the inverting amplifier as a digital inverter (see Fig.14.10 and Fig.14.11).

$$\text{start to fall: } \Delta t_1 = 1.504 \mu\text{s}, \quad \text{reach low: } \Delta t_2 = 3.799 \mu\text{s} \quad (14.13)$$

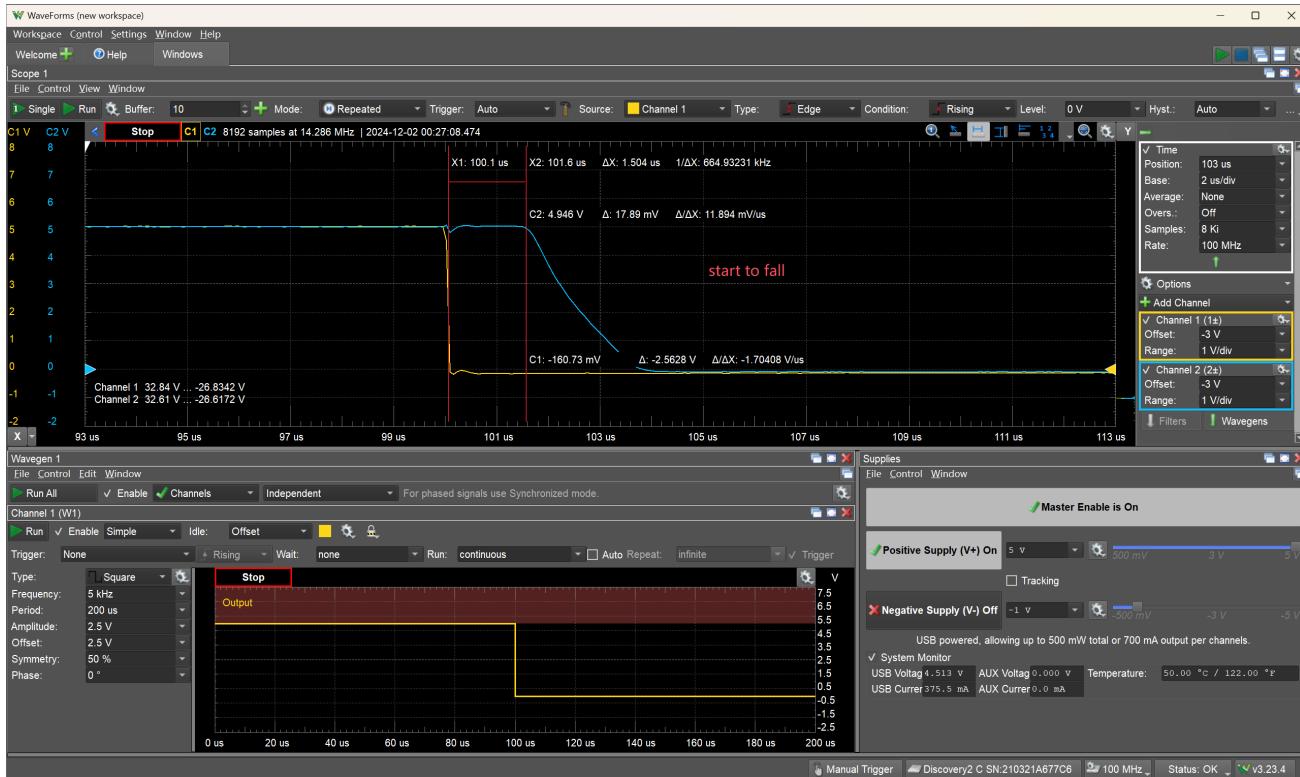


Figure 14.10: The Output Voltage Starts to Fall

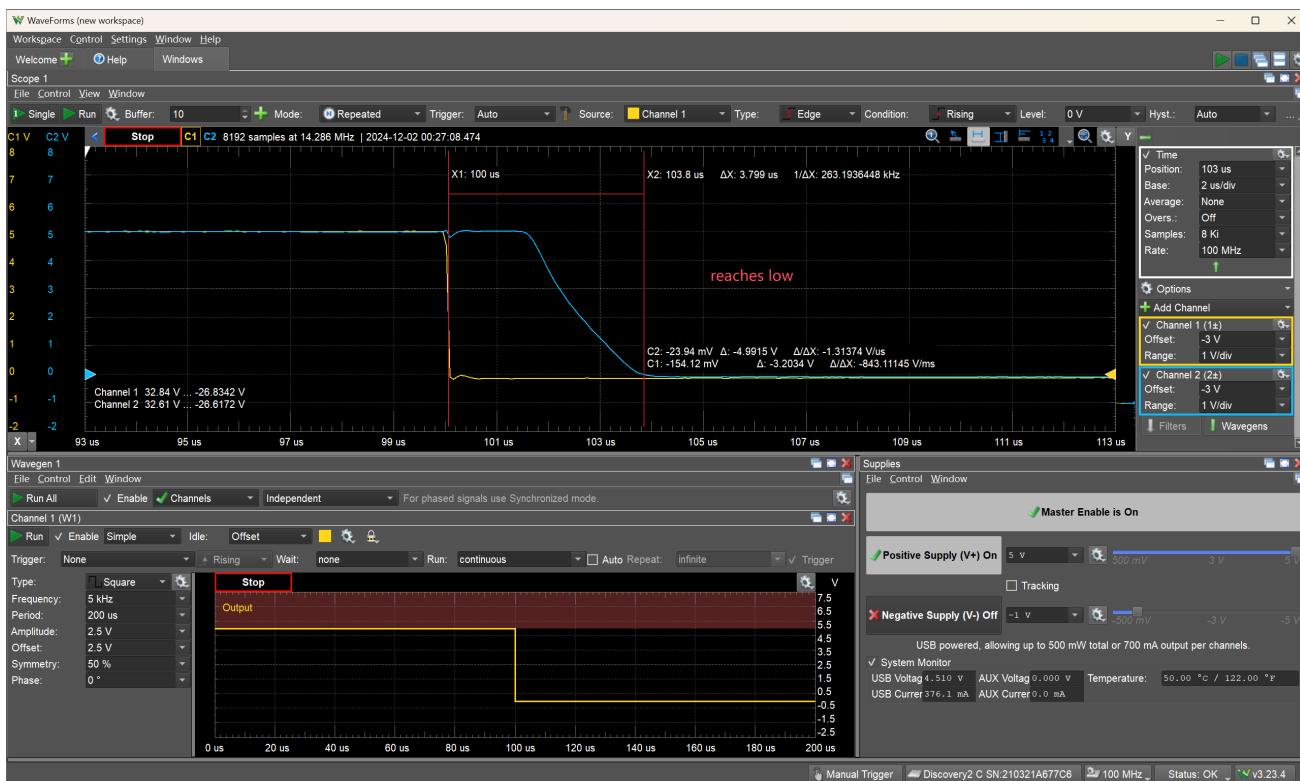


Figure 14.11: The Output Voltage Reaches to Low

It is interesting that resonance phenomena occurs, see Fig.14.12 and Fig.14.13.

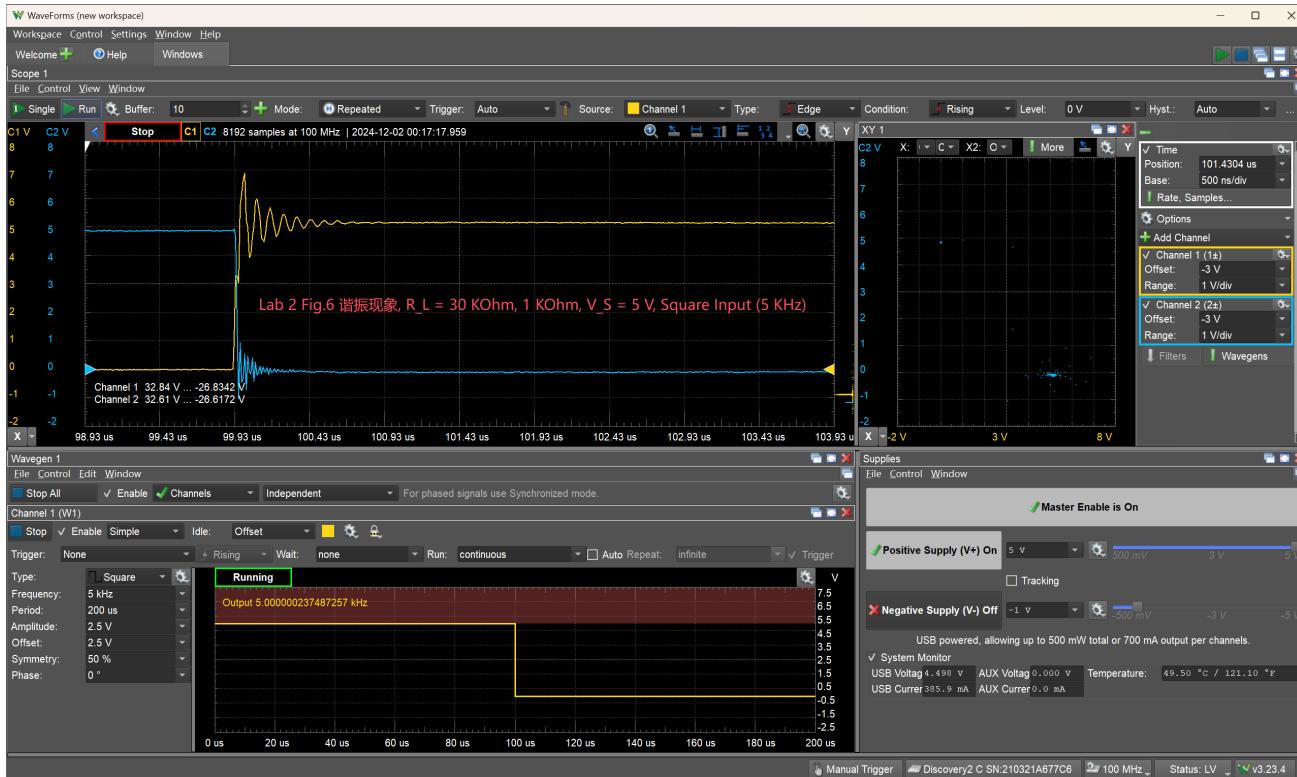


Figure 14.12: Input Voltage (Yellow) and Output Voltage (Blue) of The First MOS

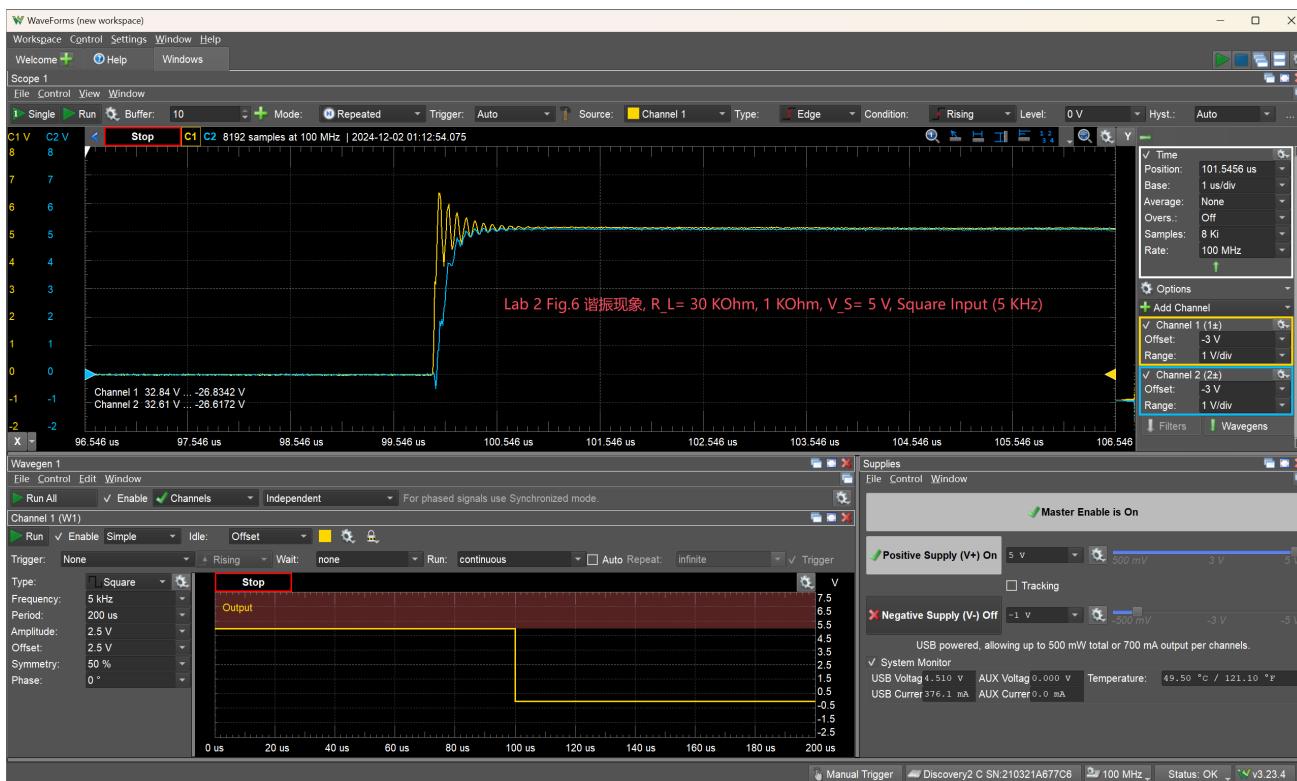


Figure 14.13: Input Voltage (Yellow) and Output Voltage (Blue) of The Second MOS

14.3 Post-Lab

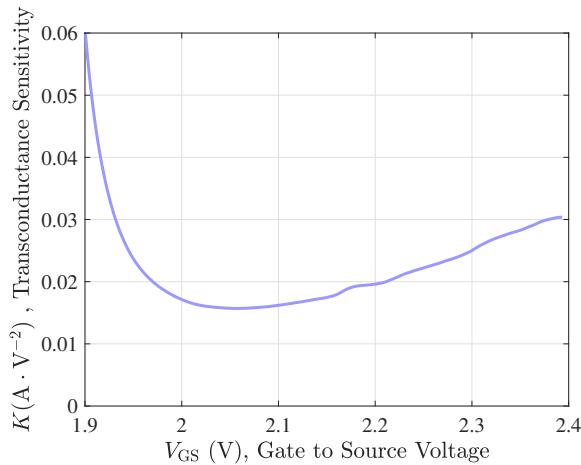
(1) Voltage-Voltage Characteristics Comparison

Transconductance Sensitivity K

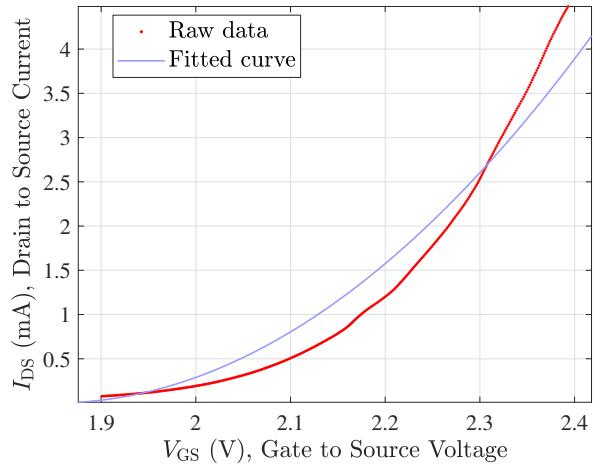
Use the voltage-voltage characteristics data in section (1) to calculate the transconductance sensitivity K of the MOSFET:

$$\begin{cases} I_{DS} = \frac{K}{2} (V_{GS} - V_T)^2 \\ V_{DS} = V_S - I_{DS} R_L \end{cases} \implies K = \frac{2(V_S - V_{DS})}{R_L (V_{GS} - V_T)^2} \quad (14.14)$$

Where $V_T = 1.85$ V, $R_L = 1$ KΩ (998 Ω) and $V_S = 5.0185$ V. Plot the curve $K = K(V_{GS})$, as shown in Fig.14.14 (a):



(a) Transconductance Sensitivity K as a Function of V_{GS}



(b) Drain to Source Current as a Function of V_{GS}

Figure 14.14: Transconductance Sensitivity

As we can see, K is not a ideal constant value. So we try fitting I_{DS} as a function of V_{DS} , and the result is shown in Fig.14.14 (b), $R^2 = 0.9511$.

$$I_{DS} = \frac{K}{2} (V_{GS} - V_T)^2, \quad K = 0.02572 \text{ A} \cdot \text{V}^{-2}, \quad V_T = 1.85 \text{ V} \quad (14.15)$$

It is funny that the fitting result is excellent if we use $I_{DS} = \frac{K}{2} (V_{GS} - V_T)^3$, which has a high $R^2 = 0.9977$.

Comparison of Operational Characteristics

With the four parameters $V_T = 1.85$ V, $K = 0.02572 \text{ A} \cdot \text{V}^{-2}$, $V_S = 5$ V and $R_L = 1$ KΩ (998 Ω), we can compute and plot the theoretical operational characteristics of the inverting amplifier, as shown in Fig.14.15. Below are the other parameters for our theoretical model:

$$V_0 = \frac{\sqrt{2KR_LV_S + 1} + KR_LV_T - 1}{KR_L} = 2.4376 \text{ V}, \quad \Delta V = 1.5(V_0 - V_T) = 0.8814 \text{ V} \quad (14.16)$$

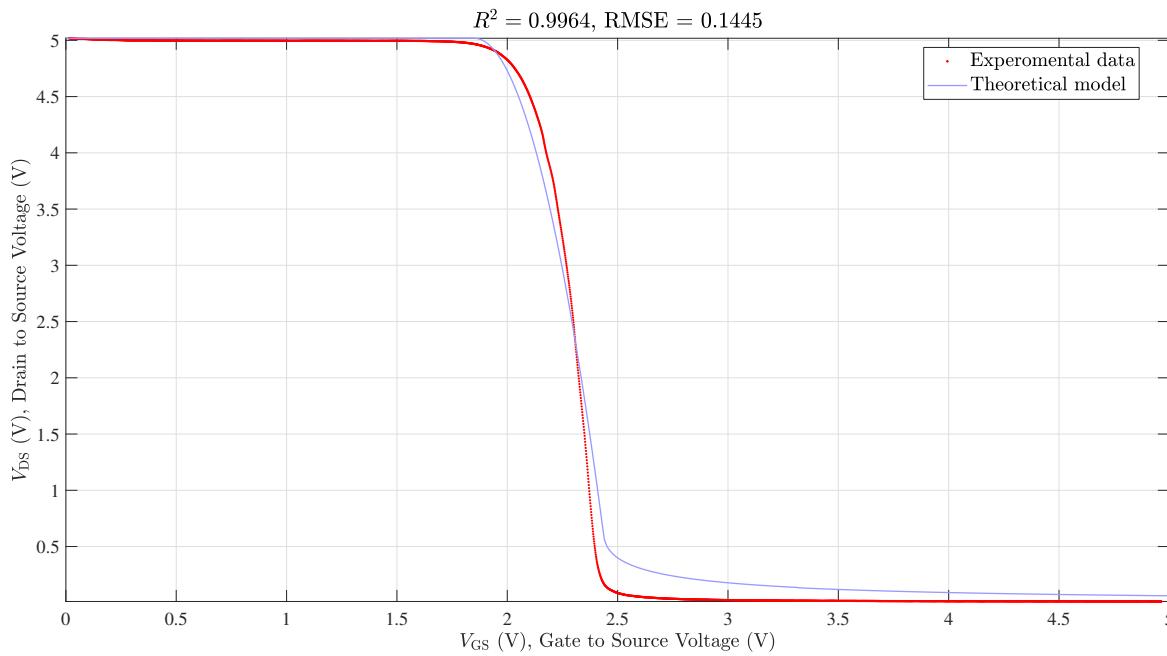


Figure 14.15: Operational Characteristics Comparison

(2) Small Signal Voltage Gain

The voltage gain A_v measured during In-Lab 2-2 (section (2)), by the operational characteristics obtained in In-Lab 2-1 (section (1)) and the theoretical module from Pre-Lab are respectively (at Output $V_{GS} = 2$ V):

Table 14.2: Voltage Gain at Output 2 V by Different Methods

By Experimental Measurement	By Operational Characteristics	By Theoretical Op Characteristics
-20.1120	-20.1513	-12.1670

(3) Capacitance and Delay Analysis

Total Input Capacitance

In Fig.14.7, we have seen that the output voltage drops to 3.3198 V, when $R_L = 500 \text{ k}\Omega$ and $V_S = 5 \text{ V}$. It follows that:

$$V_{\text{out}} = \frac{R_{\text{osci}}}{R_{\text{osci}} + R_L} V_S \implies R_{\text{osci}} = \frac{R_L}{\frac{V_S}{V_{\text{out}}} - 1} = 987.9 \text{ k}\Omega \approx 1 \text{ M}\Omega \quad (14.17)$$

Assuming $V_{\text{source}} = 5 \text{ V}$, we can obtain the total input capacitance, including GS capacitance C_{GS} and oscilloscope input capacitance C_{osci} :

$$\begin{cases} k_{0+} = \frac{V_{\text{steady}}}{\tau} \\ V_{\text{steady}} = \frac{R_{\text{osci}}}{R_{\text{osci}} + R_L} \cdot V_{\text{source}} \\ \tau = (R_{\text{osci}} \parallel R_L)(C_{\text{GS}} + C_{\text{osci}}) \end{cases} \implies C_{\text{GS}} + C_{\text{osci}} = \frac{V_S}{k_{0+} R_L} = 175.2225 \mu\text{F} \quad (14.18)$$

If oscilloscope input capacitance C_{osci} is about 15 pF, then we have $C_{\text{GS}} \approx 160 \mu\text{F}$.

The Delay Used as a Digital Inverter

Let $C_{GS} \approx 160 \mu\text{pF}$, $V_T = 1.85 \text{ V}$, $V_0 + \Delta V = 3.22 \text{ V}$ $V_I = 5 \text{ V}$, yielding:

$$\begin{cases} \Delta t = \tau \ln \left(\frac{V_S}{V_S} - V_T \right) \\ \tau = \frac{R_1 R_2}{R_1 + R_2} C \\ V_S = \frac{R_2}{R_1 + R_2} \cdot V_I \end{cases} \implies \text{start to fall: } \Delta t_1 = 0.1287 \mu\text{s}, \quad \text{reach low: } \Delta t_2 = 1.3040 \mu\text{s} \quad (14.19)$$

PART II

BUCK CONVERTER DESIGN

An Improved Buck Circuit

《电路原理》电路设计报告

2024.11.07 - 2024.12.31

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Buck 电路是一种开关型 DC-DC 变换器，用于将输入的（高）直流电压转换为输出（低）直流电压。在本次，我们将设计一个开关频率和占空比可调的 Buck 电路。其中，开关频率主要影响输出的正弦（三角）纹波大小，占空比可以控制输出电压大小。

需要注意的是，开关频率和占空比都可能影响电路的稳定性（如抗噪性能），实际使用时应做抗干扰测试，或者进一步改进后再使用。

1 设计要求

利用运算放大器 OPA 和 MOSFET 实现 Buck 电路，下面是具体的设计要求：

- (1) 输出电压 3.3 V，电压输出纹波比 $r = \frac{\Delta U_o}{U_{o,\text{ave}}} < 5\%$, $3.3 \text{ V} \times 5\% = 165 \text{ mV}$, 相当于 $\pm 82.2 \text{ mV}$;
- (2) 使用一个 +15 V 电源和一个 -15 V 电源供电；
- (3) MOSFET 型号为 2N7000；
- (4) 运放型号为 LM258N；
- (5) 整流二极管型号为 1N4007；
- (6) 电阻电容电感数量不限；最后的滤波电容最大值不超过 $470 \mu\text{F}$ ，最后的负载电阻不要小于 $2 \text{ k}\Omega$ ；
- (7) 电感非理想，所以需要测试电感的 Q 值和 DCR (Direct Current Resistance)，仿真时将 DCR 考虑进去；
- (8) 电容非理想，所以需要测试电容的 ESR (Equivalent Series Resistance) 和实际容值；
- (9) 实验报告里面包含原理说明、设计的电路图、仿真结果、实际电路照片、实际电路测试结果及分析。

2 脉冲发生器

2.1 脉冲发生器原理

Buck 电路中最关键的无疑是 MOS 的开关作用，为了能产生提供给 MOSFET 的脉冲电压，我们先讨论脉冲发生器 (Pulse Generator)，典型的脉冲发生器如下图所示：

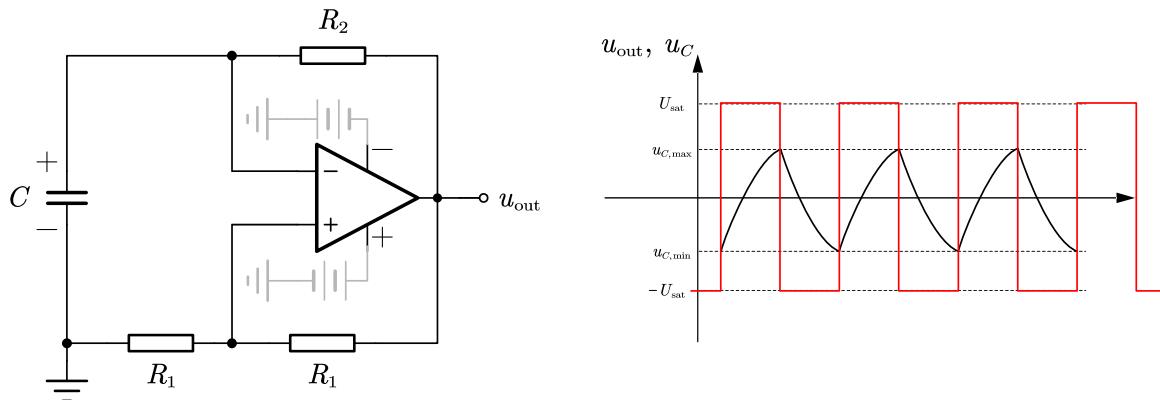


图 1: 典型脉冲发生器 (Typical Pulse Generator)

为 OPA 提供 +15 V 和 -15 V 电压，记 OPA 饱和电压为 $\pm U_{\text{sat}}$ 。设 $t = 0$ 时 $u_C < 0$ 位于最小值，处于上升阶段，则此时 $U_o = +U_{\text{sat}}$ ，当 u_C （也即 u_- ）上升到 $u_+ = \frac{R_1}{R_1+R_1}U_{\text{sat}} = 0.5U_{\text{sat}}$ 并超过它的瞬间，OPA 输出突变为 $-U_{\text{sat}}$ ， u_C 进入下降阶段。

同理，在下降阶段， $U_o = -U_{\text{sat}}$ 。当 u_C 下降到 $u_+ = -\frac{R_1}{R_1+R_1}U_{\text{sat}} = -0.5U_{\text{sat}}$ 并低于它的瞬间，OPA 输出突变为 $+U_{\text{sat}}$ ， u_C 进入上升阶段。如此循环往复，即可得到脉冲信号。由于上升和下降过程完全对称，脉冲占空比恒为 50 % 不变，且周期为：

$$T = 2R_2C \ln \left(1 + 2 \frac{R_1}{R_1} \right) = 2R_2C \ln 3, \quad f = \frac{1}{T} \quad (1)$$

为了方便参考，我们也给出 OPA 正负电源分别接 V_{DD} 和 V_{SS} 时的脉冲周期：

$$T = 2R_2C \ln \left(\frac{1 - \frac{V_{SS}}{V_{DD}} \cdot \frac{R_f}{R_f + R_1}}{1 - \frac{R_f}{R_f + R_1}} \right), \quad f = \frac{1}{T} \quad (2)$$

2.2 脉冲发生器改进方案

显然，占空比恒为 50 % 会使 Buck 电路输出电压恒为 $0.5U_S$ ，这无法满足我们的需求，需要改进脉冲发生器。改进方案如图 3 所示。

在图 3 中，我们将 R_f 换为滑动变阻器（可调电阻），并在 R_2 与电容之间加入二极管与滑动变阻器 R_k 的并联，这样便可以通过 R_f 调节电容电压振荡的幅度，从而调节脉冲频率，同时又能通过 R_k 与二极管的并联，实现上升下降阶段有不同的时间常量 τ ，以此来调节脉冲占空比。

设二极管导通电阻 R_D ，改进的脉冲发生器占空比 k 和周期 T 为（为 OPA 提供 +15 V 和 -15 V 电压）：

$$k = \frac{1}{1 + \frac{R_2 + R_D \| R_k}{R_2 + R_k}}, \quad T = (2R_2 + R_k + R_D \| R_k) C \ln \left(1 + \frac{2R_f}{R_1} \right), \quad f = \frac{1}{T} \quad (3)$$

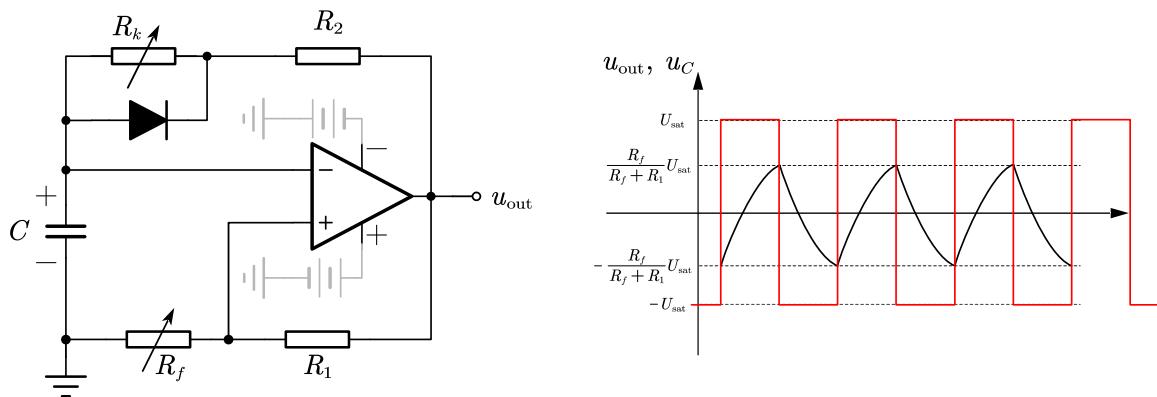
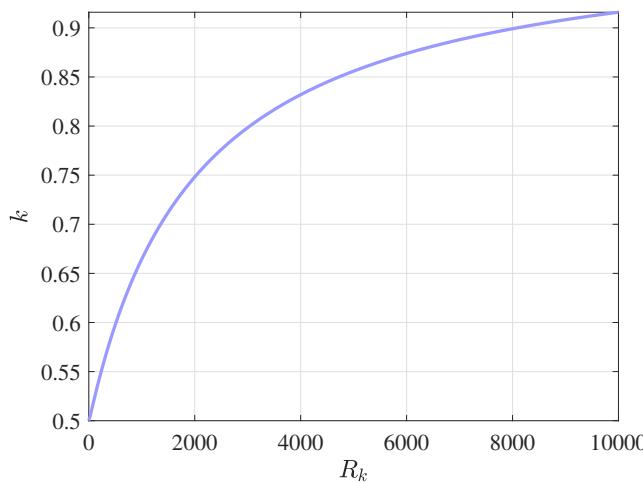
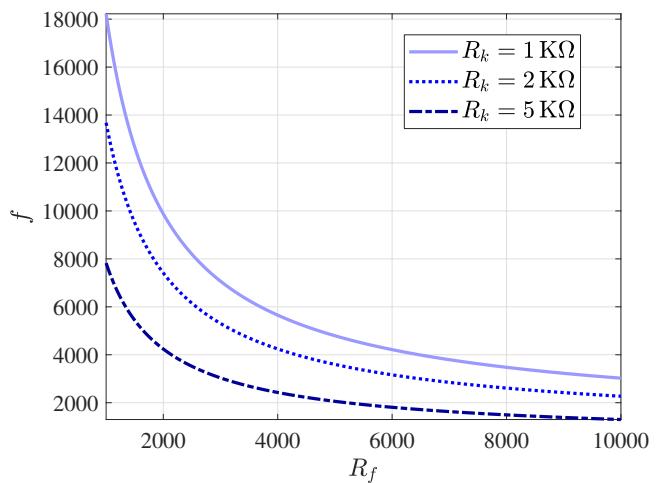


图 2: 改进的脉冲发生器 (Improved Pulse Generator)



(a) 占空比 k 关于 R_k 的变化情况



(b) 频率 f 关于 R_f 的变化情况

图 3: 改进后的脉冲发生器

为了直观感受 R_k 对占空比的调节作用和 R_f 对频率的调节作用，我们作出占空比 k 关于 R_k 变化的图像，以及频率 f 关于 R_f 变化的图像。令 $R_2 = 1 \text{ K}\Omega$, $R_D = 10\Omega$ ，则占空比如图 3 (a) 所示；再令 $C = 100 \text{ nF}$, $R_1 = 10 \text{ K}\Omega$ ，作出频率变化如图 3 (b) 所示。

3 Buck 电路

3.1 Buck 电路原理

图 4 是一个典型的 Buck 电路：

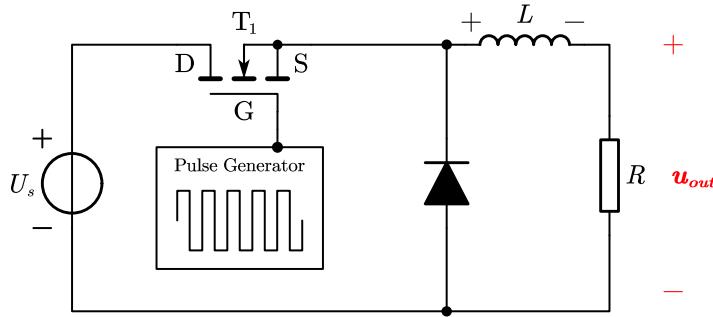


图 4: 典型 Buck 电路 (Typical Buck Circuit)

我们只考虑电路达到稳定工作状态时的情况，即输出电压在平均值附近小范围摆动。为了推导其工作原理，先作出一些必要的假设：

- (1) 经过可接受的启动时间后，电路能够达到稳定输出状态，此时输出电压在均值附近小幅振荡；
- (2) MOSFET 可视为 Switch-Resistor Model，且导通电阻极小（相比于 KΩ 量级）：这意味着电路其它电阻在 KΩ 级别，且 MOSFET 各级电压满足：

$$u_{GS} > U_T \quad \text{and} \quad u_{GD} > U_T \quad (4)$$

- (3) 二极管可视为 Switch-Source-Resistor Model，且导通电阻极小（相比于 KΩ 量级）；
- (4) MOSFET 开关周期远小于电感时间常量 τ ，即 $T \ll \tau$ ，这等价于开关频率 $f \gg \frac{1}{\tau}$ ，此时输出纹波可近似视为三角波（或正弦波）；
- (5) MOSFET 开关频率不高于 500 KHz，以避免高频状态下元件性能异常，此时可不考虑高频效应。

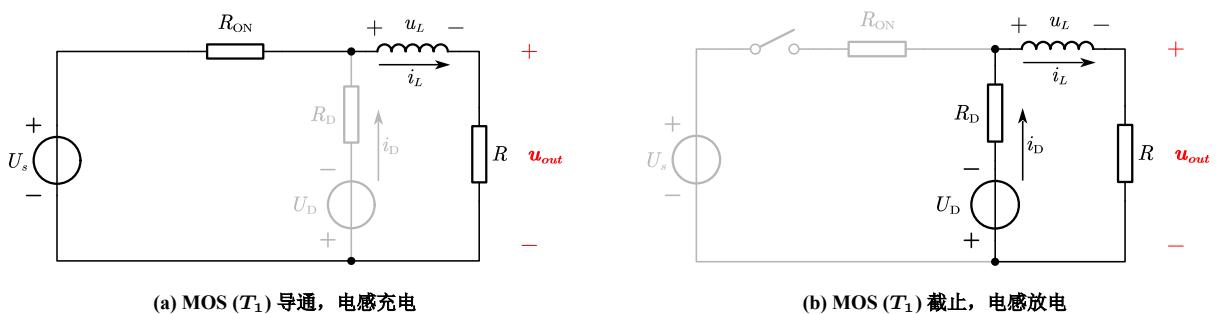


图 5: Buck Circuit 等效电路

设脉冲频率为 f ，占空比为 k ，一个周期 T 分为高电平 T_{ON} 和低电平 T_{OFF} 。下面作具体的推导。设 $t = 0$ 时，MOSFET 的 G 级接收到高电平脉冲信号，MOSFET 导通，等效电路如图 5(a) 所示。此时电感电流应为最小值，设其为 $(i_L)_{min}$ ，稳态值是 $\frac{U_s}{R + R_L + R_{ON}}$ ，由三要素法，有：

$$i_L(t) = (i_L)_{min} \cdot e^{-\frac{t}{\tau_{ON}}} + \frac{U_s}{R + R_L + R_{ON}} \cdot \left(1 - e^{-\frac{t}{\tau_{ON}}}\right) \Rightarrow \quad (5)$$

$$i_L(t) = \left[(i_L)_{min} - \frac{U_s}{R + R_L + R_{ON}} \right] \cdot e^{-\frac{t}{\tau_{ON}}} + \frac{U_s}{R + R_L + R_{ON}}, \quad \tau_{ON} = \frac{L}{R + R_L + R_{ON}} \quad (6)$$

经过 T_{ON} 时间，MOSFET 关断，则 $[0, T_{ON}]$ 时间段内的电感电流增量为：

$$(\Delta i_L)_{ON} = \left[(i_L)_{min} - \frac{U_s}{R + R_L + R_{ON}} \right] \cdot \left(1 - e^{-\frac{T_{ON}}{\tau_{ON}}}\right), \quad \tau_{ON} = \frac{L}{R + R_L + R_{ON}} \quad (7)$$

类似的思路, 设 $t = 0$ 时, MOSFET 的 G 级接收到低电平脉冲信号, MOSFET 关断, 等效电路如图 5 (b)。此时电感电流应为最大值, 设其为 $(i_L)_{\max}$, 由于二极管存在导通压降 U_D , 电流“稳态值”是 $-\frac{U_D}{R+R_L+R_D}$, 由三要素法, 有:

$$i_L(t) = (i_L)_{\max} \cdot e^{-\frac{t}{\tau_{\text{OFF}}}} - \frac{U_D}{R + R_L + R_D} \cdot \left(1 - e^{-\frac{t}{\tau_{\text{OFF}}}}\right) \Rightarrow \quad (8)$$

$$i_L(t) = \left[(i_L)_{\max} + \frac{U_D}{R + R_L + R_D} \right] \cdot e^{-\frac{t}{\tau_{\text{OFF}}}} - \frac{U_D}{R + R_L + R_D}, \quad \tau_{\text{OFF}} = \frac{L}{R + R_L + R_D} \quad (9)$$

经过 T_{OFF} 时间, MOSFET 又导通, 则 $[0, T_{\text{OFF}}]$ 时间段内的电感电流增量为:

$$(\Delta i_L)_{\text{OFF}} = \left[(i_L)_{\max} + \frac{U_D}{R + R_L + R_D} \right] \cdot \left(1 - e^{-\frac{T_{\text{OFF}}}{\tau_{\text{OFF}}}}\right) \quad (10)$$

电路输出达到稳态, 所以应有 $(\Delta i_L)_{\text{ON}} + (\Delta i_L)_{\text{OFF}} = 0$, 简记 $e_{\text{ON}} = e^{-\frac{T_{\text{ON}}}{\tau_{\text{ON}}}}$ 和 $e_{\text{OFF}} = e^{-\frac{T_{\text{OFF}}}{\tau_{\text{OFF}}}}$, 得到:

$$(i_L)_{\max} = \frac{U_s}{R + R_L + R_{\text{ON}}} \cdot \frac{1 - e_{\text{ON}}}{1 - e_{\text{ON}}e_{\text{OFF}}} - \frac{U_D}{R + R_L + R_D} \quad (11)$$

$$(i_L)_{\min} = e_{\text{OFF}} (i_L)_{\max} = \frac{U_s}{R + R_L + R_{\text{ON}}} \cdot \frac{e_{\text{OFF}} - e_{\text{ON}}e_{\text{OFF}}}{1 - e_{\text{ON}}e_{\text{OFF}}} - \frac{e_{\text{OFF}} U_D}{R + R_L + R_D} \quad (12)$$

$$\Delta i_L = (1 - e_{\text{OFF}}) (i_L)_{\max} = \frac{U_s}{R + R_L + R_{\text{ON}}} \cdot \frac{(1 - e_{\text{ON}})(1 - e_{\text{OFF}})}{1 - e_{\text{ON}}e_{\text{OFF}}} - \frac{U_D(1 - e_{\text{OFF}})}{R + R_L + R_D} \quad (13)$$

由于功率损耗发生在二极管导通电阻 R_D 、压降 U_D 和 MOSFET 导通电阻 R_{ON} 上, 我们可以进一步计算功率损耗和转化效率:

$$P_{\text{loss}} = k (i_L)_{\text{average}}^2 R_{\text{ON}} + (1 - k) (i_L)_{\text{average}}^2 R_D + (1 - k) (i_L)_{\text{average}} U_D \quad (14)$$

$$= (i_L)_{\text{average}}^2 [k R_{\text{ON}} + (1 - k) R_D] \quad (15)$$

$$\Rightarrow \eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{loss}}} = \frac{(u_o)_{\text{average}}}{(u_o)_{\text{average}} + (i_o)_{\text{average}} [k R_{\text{ON}} + (1 - k) R_D + (1 - k) U_D]} \quad (16)$$

如果取近似 $(u_o)_{\text{average}} = k U_s$, $(i_o)_{\text{average}} = \frac{k U_s}{R}$, 则转化效率为:

$$\eta = \frac{R}{R + k R_{\text{ON}} + (1 - k) R_D + \frac{(1-k) U_D}{k U_s} R} \quad (17)$$

3.2 Buck 改进方案 1

一方面, 上面的电路, 输出电压纹波比 $r = \frac{\Delta U_o}{U_{o,\text{ave}}}$ 可能较大; 另一方面, 二极管的导通压降 U_D 可能使电路效率明显降低 (尤其在输出低电平时)。改进方案见图 6。

在图 6 中, 一方面, 我们在输出加一个较大的电容 C , 这可以明显降低输出纹波幅度; 另一方面, 我们用另一个 MOSFET 来替代二极管 D, 此 MOSFET 的 G 级由脉冲信号经过反相器得到, 这样相当于将二极管的导通压降 U_D 降低为 0, 同时将 R_D 换为 R_{ON} , 既可以避免压降 U_D 带来的占空比失调, 也可以提高电路的转化效率。

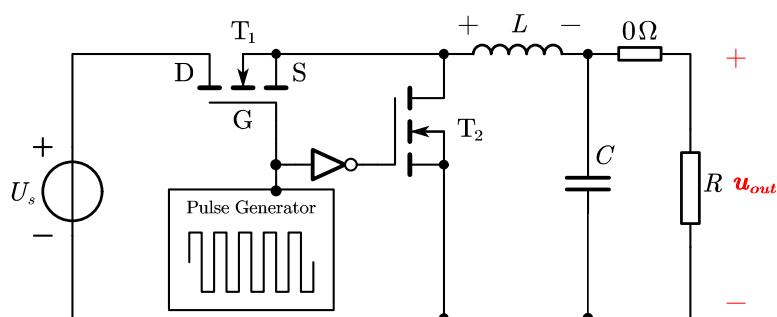


图 6: Buck 改进方案 1

另外，我们在输出端串联了一个 0Ω 电阻，这使得我们可以方便地串入小电阻（示波器测电压）或电流表来测量输出电流。改进之后，开关 MOS 导通与不导通时的等效电路如下图：

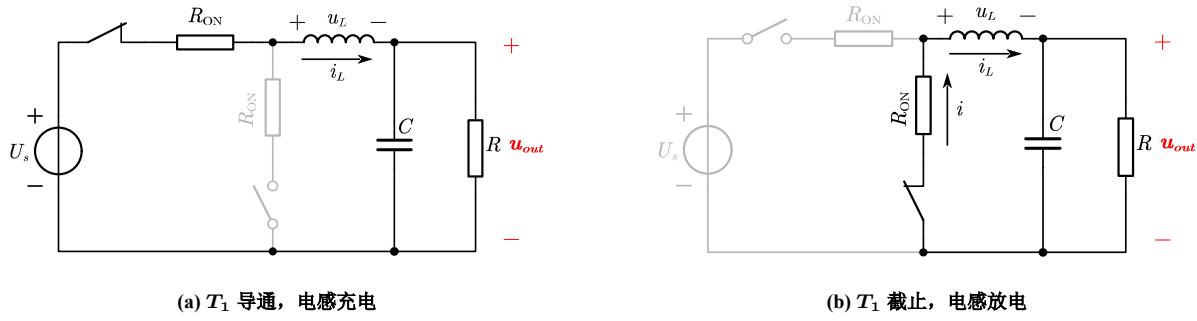


图 7: 改进方案 1 等效电路

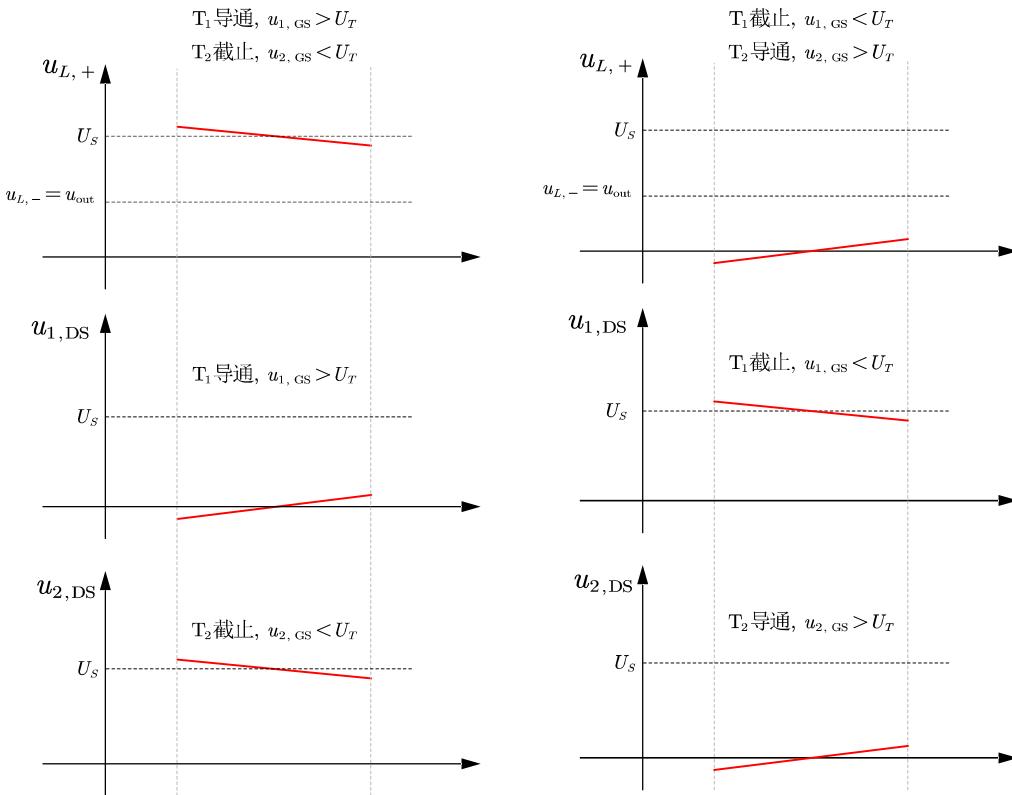


图 8: 改进方案 1 中各元件电压波形

作出两种状态下的各元件电压波形，如图 8 所示。稳定工作时，可分为两种状态。记开关 MOS 为下面以输出负端 $u_{o,-}$ 为电压参考点，定性地分析两种状态下的电路行为：

- (1) 开关 MOS (T_1) 导通：此时二极管 MOS (T_2) 截止，电路如图 7 (a)。 $u_{L,-}$ 维持在 u_{out} 附近，电感正端（即 T_2 D 极） $u_{L,+} = u_{2,DS}$ 在 U_S 附近小幅下降（近似线性），这使得 T_1 的 DS 电压 $u_{1,DS}$ 在 0 附近小幅线性上升（从负到正）；
- (2) 开关 MOS (T_1) 截止：此时二极管 MOS (T_2) 导通，电路如图 7 (b)。 $u_{L,-}$ 仍维持在 u_{out} 附近，电感正端（即 T_2 D 极） $u_{L,+} = u_{2,DS}$ 在 0 附近小幅线性上升（从负到正）， T_1 的 DS 电压 $u_{1,DS}$ 在 U_S 附近小幅线性下降；

3.3 Buck 改进方案 2

如果想进一步降低输出纹波，可考虑下图所示的改进方案：

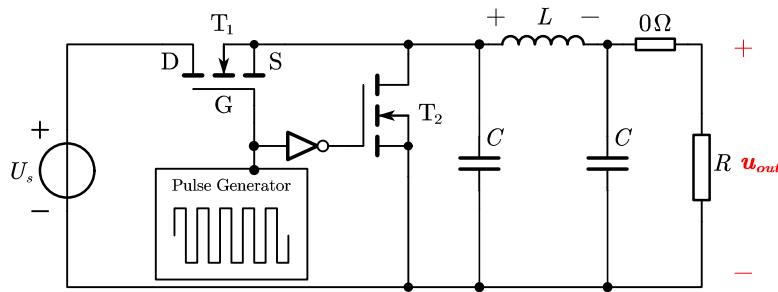


图 9: Buck Circuits 改进方案 2

与第一种方案相比，这种方案只是将 LC 滤波换为了 $CLC \pi$ 型滤波，看似变化不大，实则从原理层面作了改变。

由于 T_2 两端并联了较大的电容 C ，它 DS 间的电压 $u_{2,DS}$ 无法发生突变，经过半定量推导可知 $u_{2,DS}$ 会在 u_{out} 附近以锯齿波形小幅振荡。同理， T_1 的 DS 电压 $u_{1,DS}$ 在 $(U_s - u_{out})$ 附近以锯齿波形小幅振荡。

也就是说，如果想采用此种电路，便需要 MOSFET 在 $\max\{u_{out}, U_s - u_{out}\}$ 能正常工作，对元件的要求比较高，2N7000 不能满足此要求，需要另选其他型号。

从右图中的电压波形可以看到，各元件的工作方式发生了根本性的改变。

虽然对元件要求高，但这样带来的收益也是极高的，在后文的仿真性能测试中，我们将看到， CLC 滤波可以将输出电压纹波降至 1 mV 以下，达到 0.030 % 的纹波率。

此改进方案的等效电路如图 11 所示。

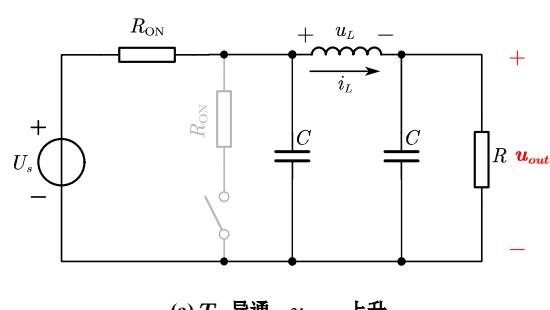
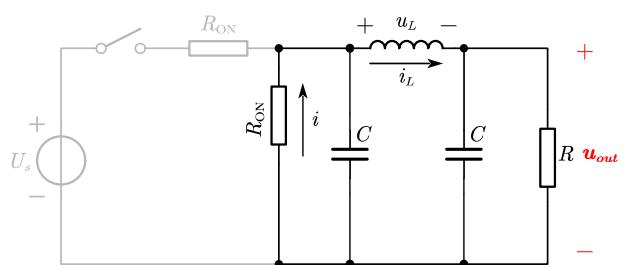
(a) T_1 导通, $u_{L,+}$ 上升(b) T_1 截止, $u_{L,+}$ 下降

图 11: 改进方案 2 等效电路

4 仿真电路性能测试

4.1 改进方案 1 仿真测试

如图 12 搭建仿真电路，选取负载电阻 $R = 5 \text{ K}\Omega$ ，电路各元件的参数见图中标注，需要强调的是 $R_k = 2350 \Omega$ 、 $R_f = 2.2 \text{ K}\Omega \sim 2.6 \text{ K}\Omega$ 。后文进行仿真电路的性能测试。

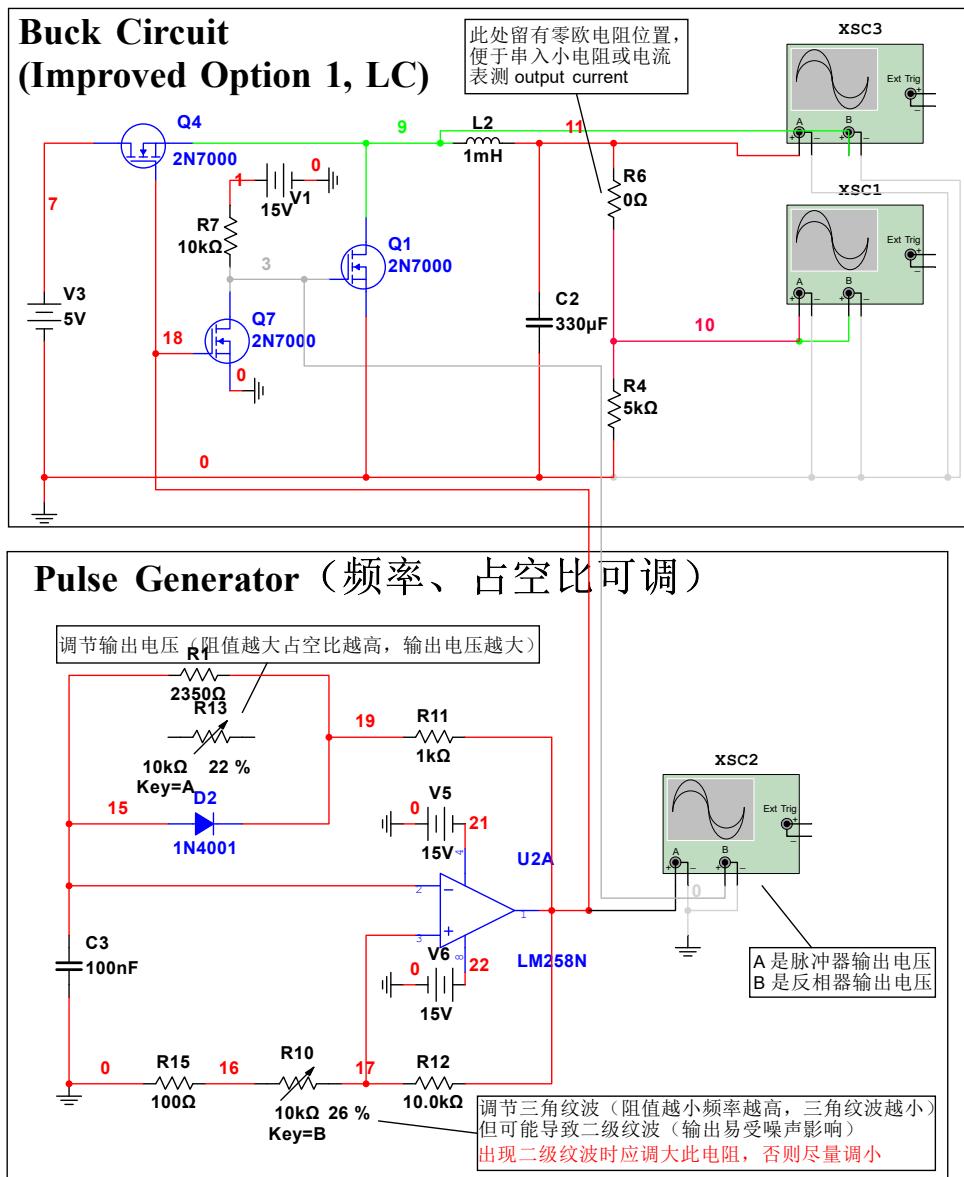


图 12: Buck Circuit 方案 1 仿真电路

4.1.1 方案 1 输出性能

3.3 V 工作点性能:

- (1) 输出电压: 3.320 V ($\pm 17 \text{ mV}$, $r = 0.5120 \%$), 此时 $R_k = 2350 \Omega$ 、 $R_f = 2.6 \text{ k}\Omega$, 详见图 13;
- (2) 启动时间: 13 ms, 20 ms 后完全稳定, 详见图 19。

极限性能测试:

- (1) 最小输出电压: 2.117 V ($\pm 5.0 \text{ mV}$, $r = 0.2362 \%$), 此时 $R_k = 0$ 、 $R_f = 2.2 \text{ k}\Omega$;
- (2) 最大输出电压: 4.26 V ($\pm 62.4 \text{ mV}$, $r = 1.4648 \%$), 此时 $R_k = 10 \text{ k}\Omega$ 、 $R_f = 2.4 \text{ k}\Omega$;

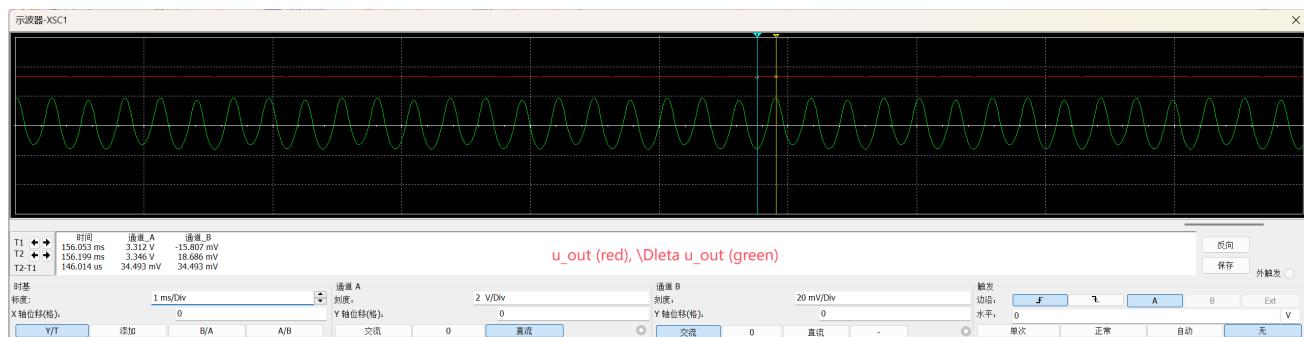
在方案一的基础上, 我们修改参数做了多次仿真测试, 结果汇总在下表:

表 1: 方案 1 不同参数下的仿真结果

OPA	L (mH)	C (μ F)	C_f (nF)	u_{out} (V)	Δu_{out} (mV)	r	R_k (10 K Ω)	R_f (10 K Ω)	输出稳定性
LM258P	2	300	100	3.2975	± 12.5	0.7582 %	20 %	38 %	一般
LM258P	4	300	100	3.3090	± 6.00	0.3626 %	20 %	36 %	较好
LM258P	4.7	300	100	3.3209	± 5.00	0.3011 %	20 %	36 %	极好
LM258P	4.7	330	100	3.3215	± 4.63	0.2788 %	22 %	40 %	极好
LM258P	6	300	100	3.2955	± 1.50	0.0910 %	36 %	8 %	较好
LM318N	6	300	100	3.3445	± 0.50	0.0299 %	14 %	20 %	较好
LM318N	6	300	50	3.3685	± 0.50	0.0297 %	14 %	42 %	较好
LM318N	6	300	10	3.3233	± 0.26	0.0157 %	38 %	20 %	较差

4.1.2 方案 1 工作点各元件电压波形

下面是各元件在工作点的电压波形:

图 13: 输出电压 u_{out} 图 14: MOS T_1 电压图 15: MOS T_2 电压

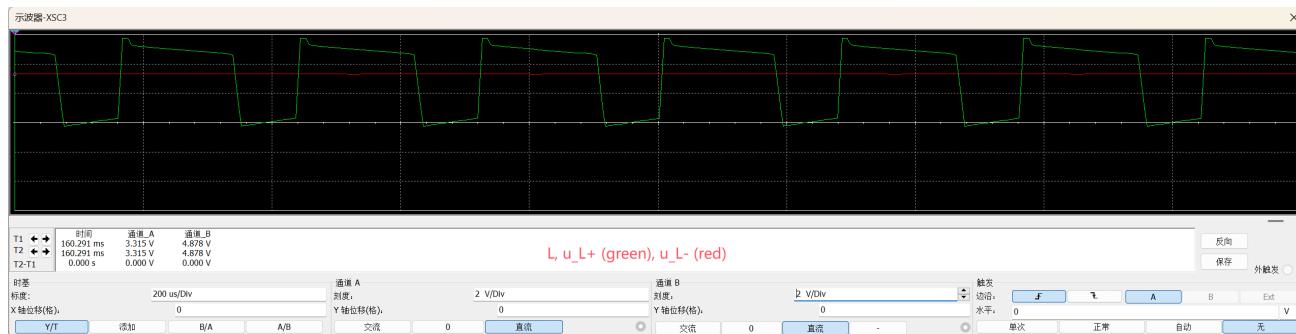
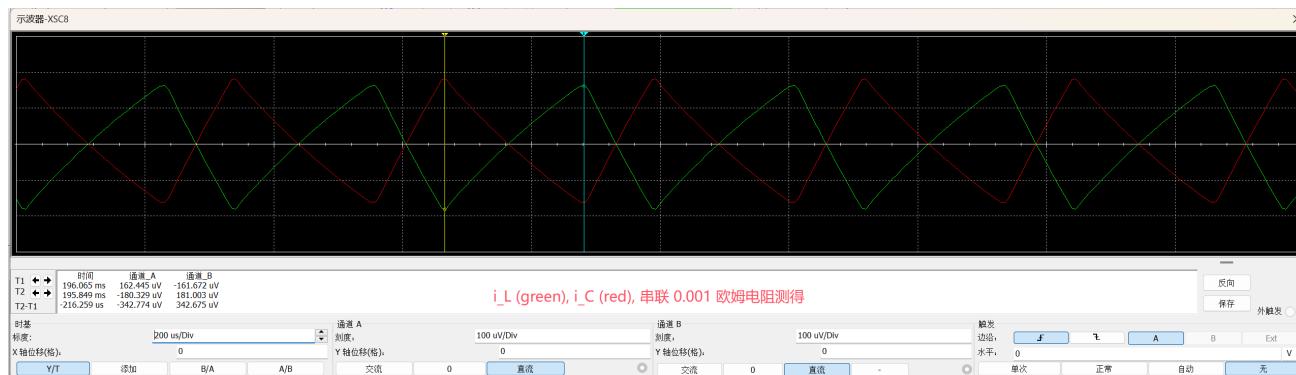
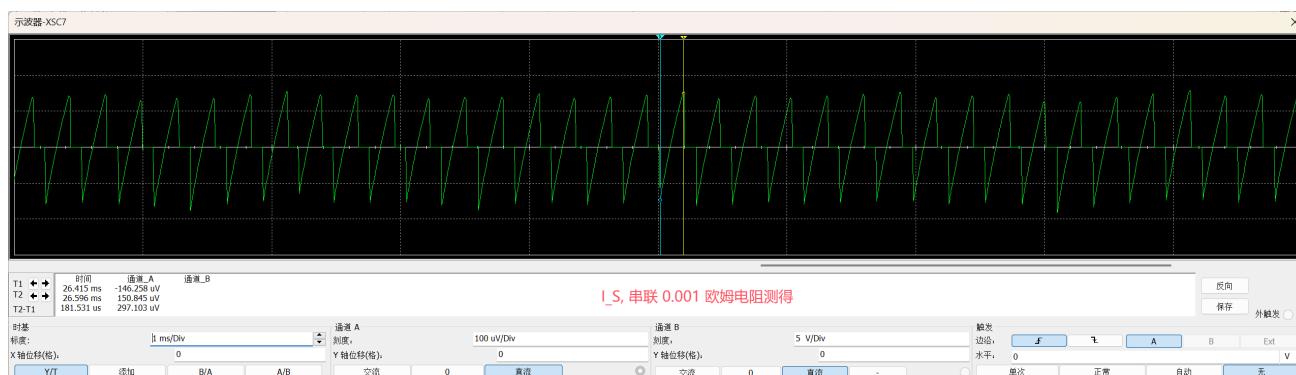
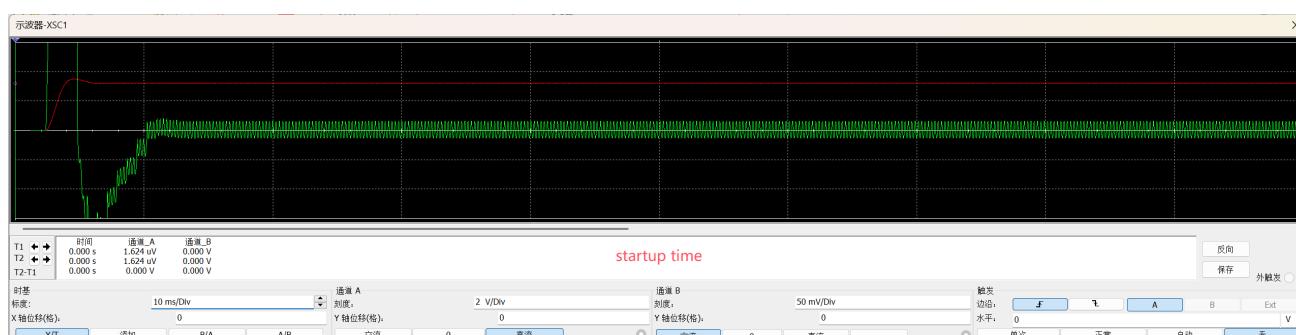
图 16: 电感电压 u_L 图 17: 电感电流 i_L 和电容电流 i_C (从下向上)图 18: 电压源输出电流 I_S 

图 19: 电路启动时间

4.2 改进方案 2 仿真测试

如图 20 搭建仿真电路，同样选取负载电阻 $R = 5 \text{ K}\Omega$ ，电路各元件的参数见图中标注，需要强调的是 $R_k = 2520 \Omega$ 、 $R_f = 3.2 \text{ K}\Omega$ 。后文进行仿真电路的性能测试。

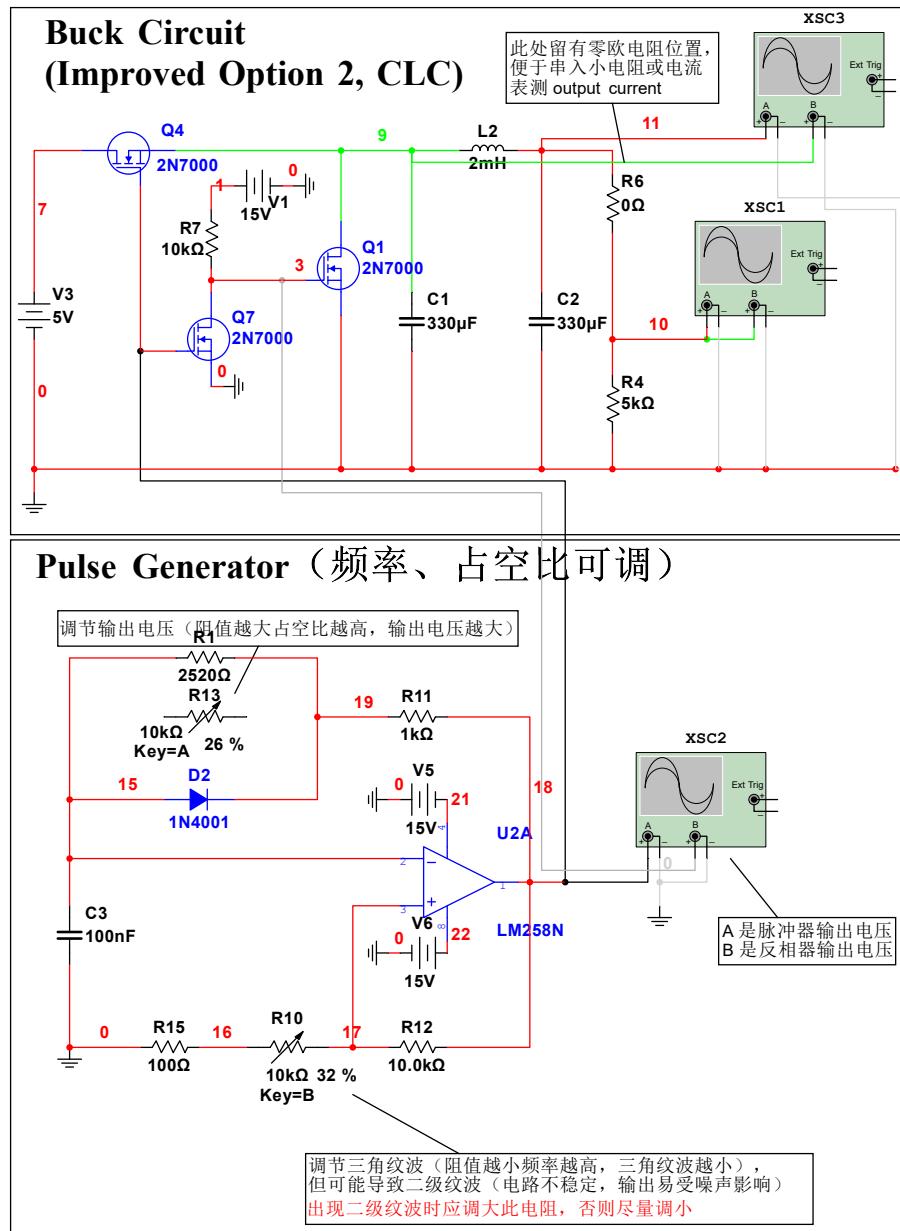


图 20: Buck Circuit 方案 2 仿真电路

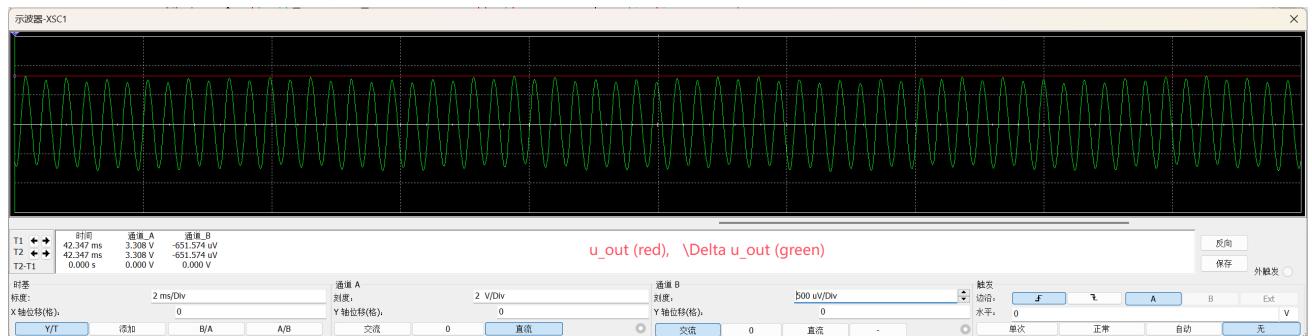
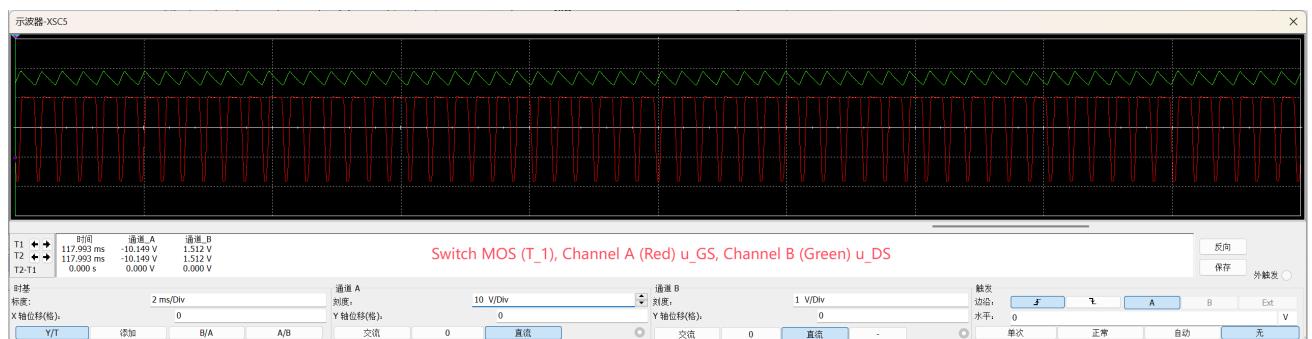
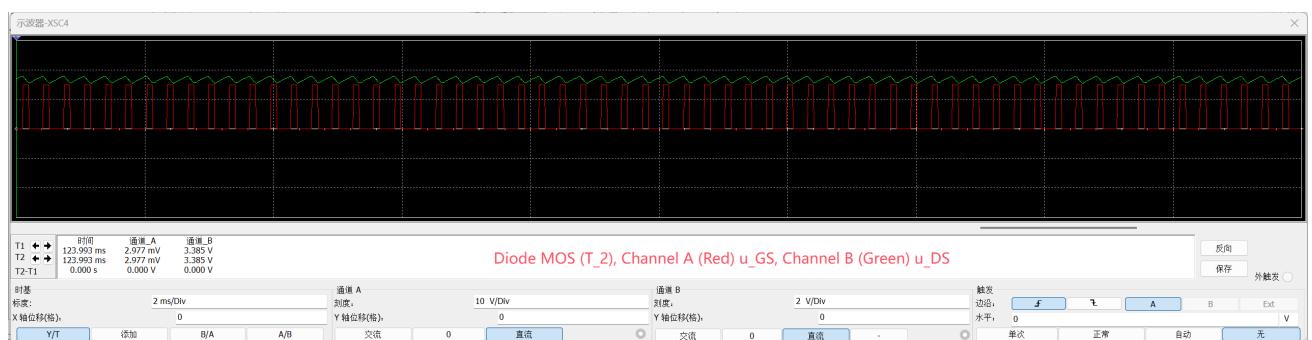
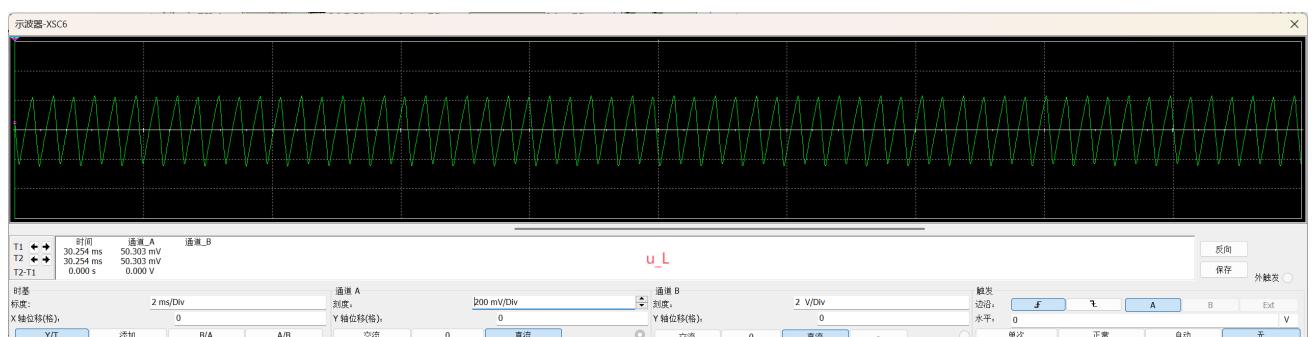
4.2.1 各元件电压波形

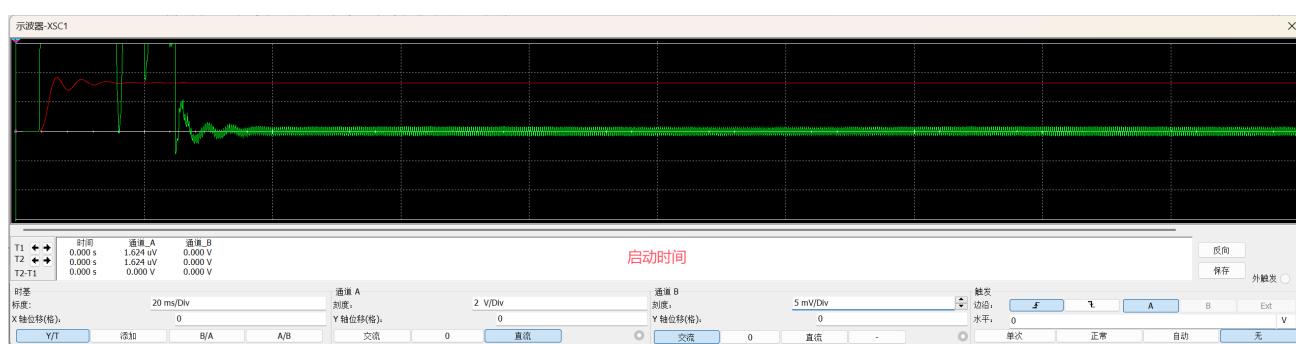
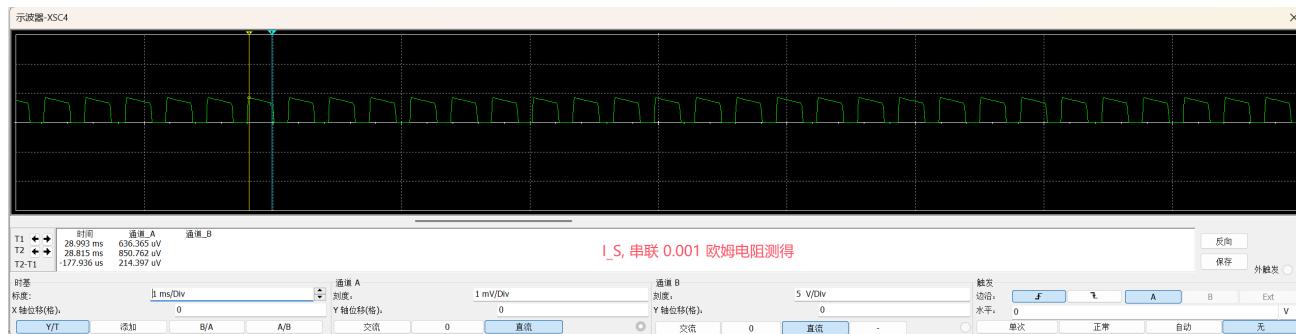
性能测试：

- (1) 输出电压: $3.308 \text{ V} (\pm 0.678 \text{ mV}, r = 0.0205\%)$, 此时 $R_k = 2520\Omega$ 、 $R_f = 3.2 \text{ K}\Omega$, 详见图 21;
- (2) 启动时间: 35 ms, 40 ms 后完全稳定, 详见图 26;
- (3) 最小输出电压: $1.895 \text{ V} (\pm 0.252 \text{ mV}, r = 0.0133\%)$, 此时 $R_k = 0$ 、 $R_f = 5.2 \text{ K}\Omega$;
- (4) 最大输出电压: $4.252 \text{ V} (\pm 8.87 \text{ mV}, r = 0.2086\%)$, 此时 $R_k = 10 \text{ K}\Omega$ 、 $R_f = 2.6 \text{ K}\Omega$;

4.2.2 方案 2 工作点各元件电压波形

下面是各元件在工作点的电压波形：

图 21: 输出电压 u_{out} 图 22: MOS T_1 电压图 23: MOS T_2 电压图 24: 电感电压 u_L



4.3 Typical 方案仿真测试

不妨对典型方案进行仿真测试。选取 T_2 为整流二极管 2N7000, 得到电感和电容的电流波形 (串联 0.001 Ω 电阻测得), 见 27 和图 28, 出现了“振荡情形”, 事实上, 这是因为在此小段, 二极管在导通与截止之间不断跳变。与改进电路相比, Typical 情形下输出稳定性较差, 有难以消除的二级纹波。

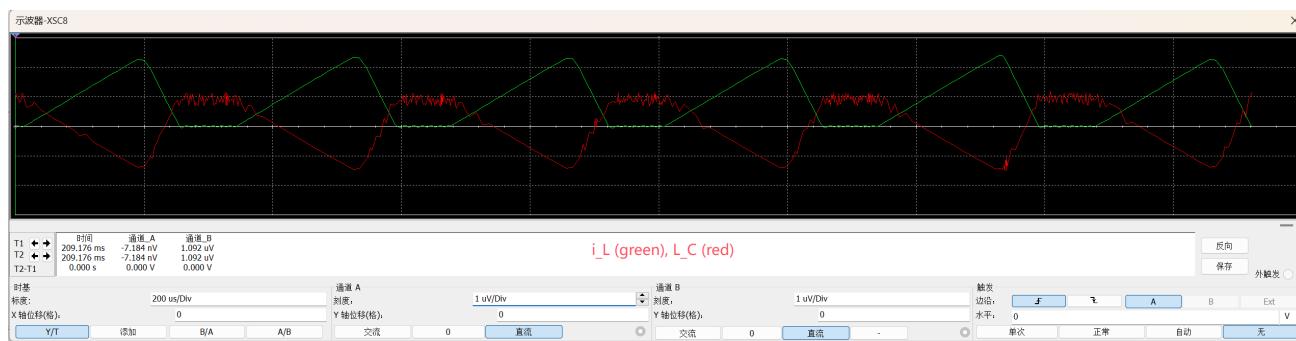


图 27: Typical 方案中的 i_L 和 i_C

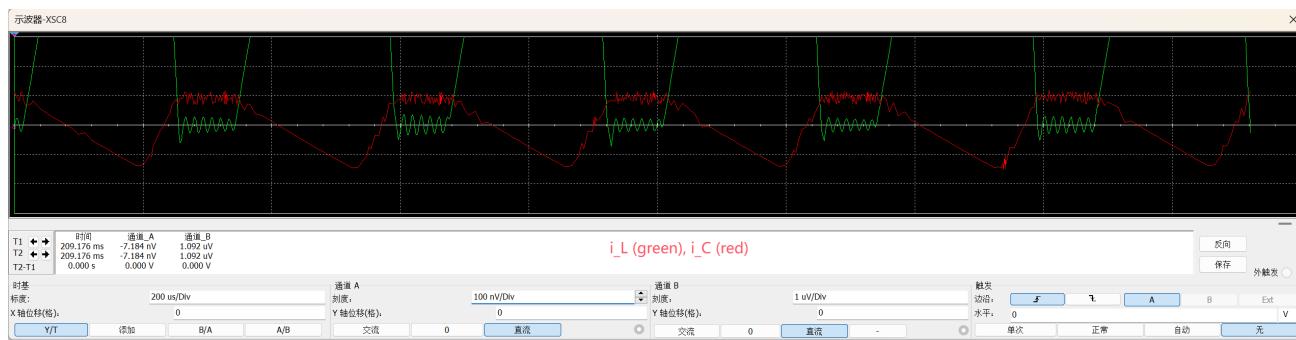


图 28: 电流出现“振荡”

5 设计合理性检查

5.1 设计参数检查

由于设计要求中的元件限制，我们采用上文的方案 1 进行设计检查，并搭建实际电路进行性能测试。下面查找各元件的 Data Sheet^①，依据仿真中的一系列结果，对设计的合理性进行检查，判断各元件是否能正常工作，如果不能，需要修改设计参数以保证元件正常工作。

- (1) MOS T_1 (2N7000) : T_1 截止时， u_{DS} 在 5 V 附近，未超过 60 V，符合标准； T_1 导通时， $u_{GS} = 10$ V， u_{DS} 在 [-200 mV, 200 mV] 之间，依据 Data Sheet 给出的 $V_{SD} - I_{SD}$ 和 $V_{DS} - I_{DS}$ 曲线图， T_1 的电流在 [-1mA, 150 mA] 之间，而 Absolute Maximun Ratings 中规定 Maximum drain current (continuous) = 200 mA, Maximum drain current (pulsed) = 500 mA，符合要求。
- (2) MOS T_2 (2N7000) : T_2 截止时， u_{DS} 在 5 V 附近，符合标准； T_2 导通时， $u_{GS} = 15$ V，电压 u_{DS} 在 [-200 mV, 200 mV] 之间，同上，符合要求。
- (3) 电容、电感的电压电流：依次检查，均符合要求。

5.2 关键元件检查

电路中最关键的无疑是脉冲发生器中的运放，由此产生的脉冲信号是整个电路的枢纽，因此，有必要考察在前文仿真参数下，实际运放是否能正常工作^②。下面分别给出 LM258P、NE5532P 和 LM318 在不同输入下的输出脉冲波形（输入是频率 f 占空比 50 % 的三角波）。

由图可知，为了保证输出方波的理想性，LM258P 的工作频率应在 4 KHz 以下，NE5532P 的工作频率应在 50 KHz 以下，LM318 的工作频率应在 250 KHz 以下。我们前文中的仿真电路工作频率为 3.5 KHz 至 3.8 KHz，因此设计要求中的 LM258P 可以符号要求。

5.3 设计修改

需要注意，由于设计要求中的运放 LM258N 是单电源运放，电源输入为 VCC 和 GND，而我们上面的设计中使用了双电源运放，电源输入为 +VCC 和 -VCC，需要对 Pulse Generator 进行修改。

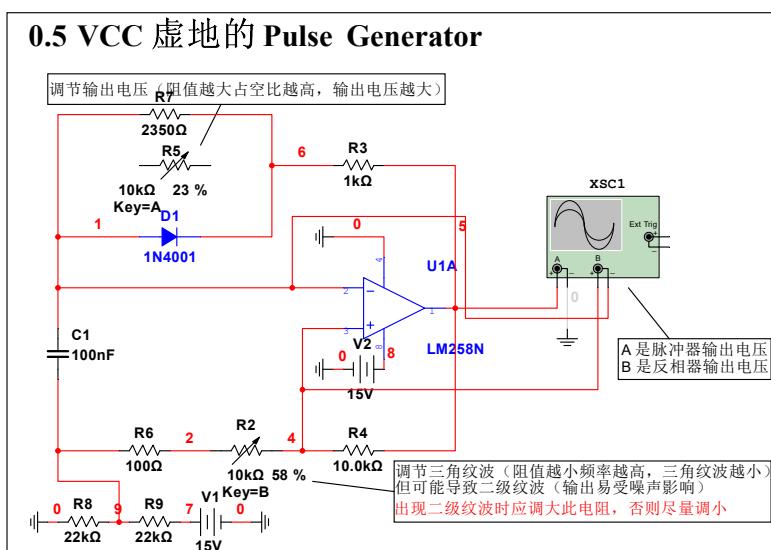


图 29: 利用了虚地的 Pulse Generator (Single Source)

如图 29 所示，创建电压为 0.5 VCC 的“虚地”，这时运放的输出是介于 0 和 VCC 之间的脉冲电压，仍

^① 各元件的 Data Sheet 已整理到网址 <https://www.12365.com/s/0y0pTd-oJkj3>。

^② 事实上，这一步应在仿真设计之前完成，因为我们必须依据已有的元件，确定合适的电路工作点，才能进行仿真。否则，仿真结果可能与实际电路有较大差异，又需要重新设计，这会浪费很多时间精力。

可以直接用于驱动 MOSFET，无需其它修改。

虽然电容的上下突变点发生了变化（幅度变为原来的一半），但其变化速度同样变为了原来的一半。如果运放的正负输出完全对称，正负饱和电压也相等，那么数学上可以知道，修改之后不会对 Buck 电路产生任何影响。但是，实际运放的正负输出并不对称，正负饱和电压也不一定相等，会对 Buck 电路产生一定的影响，所以我们需要重新测定电路工作点参数及性能。

5.4 单电源(虚地)仿真测试

使用上一节所述的虚地 Pulse Generator (Single Source) 进行仿真，得到 Buck 电路工作点输出电压：3.304 V ($\pm 29 \text{ mV}$, $r = 1.7554\%$)，此时 $R_k = 3.0 \text{ K}\Omega$ 、 $R_f = 1.0 \text{ K}\Omega$ ，详见图 30。部分元件的电压波形如上所示，由图可以看到，输出电压的稳定性较差，纹波较大，可能需要进一步的调整。我们在仿真的过程中也发现，参数 R_f 和 R_k 的小变化会对输出电压产生较大的影响，尤其是影响输出的稳定性。

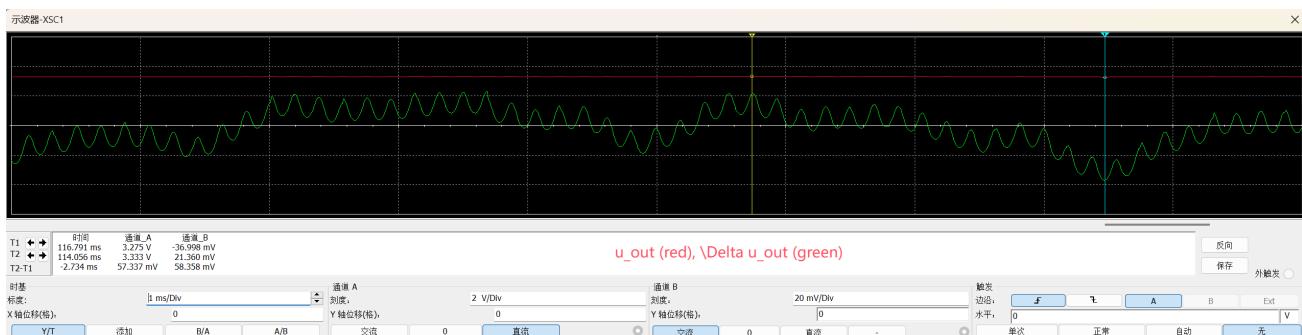


图 30: 输出电压 u_{out}



图 31: MOS T_1 电压

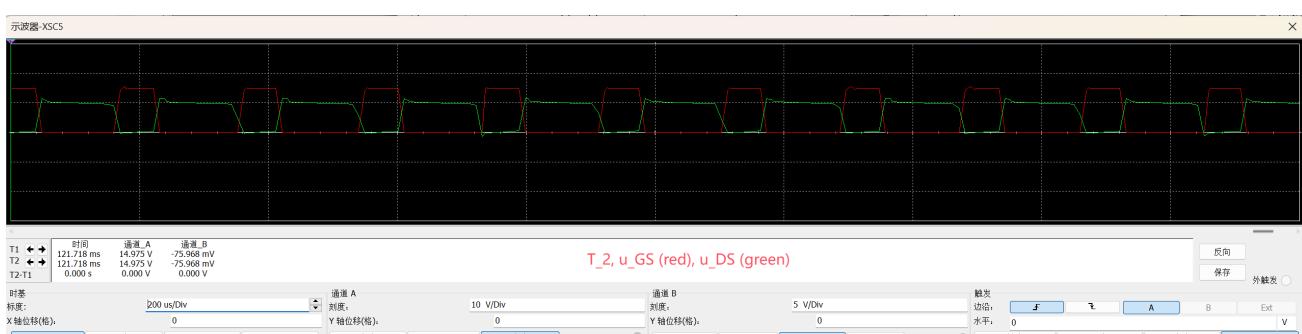


图 32: MOS T_2 电压

6 实际电路性能测试

6.1 脉冲发生器

如图 33 搭建脉冲发生器，运放使用 LM258P。用 RL 电路测得 2 mH 电感的实际值为 2.0795 mH (DCR = 3.4979 Ω)，其它元件的参数见表 2。

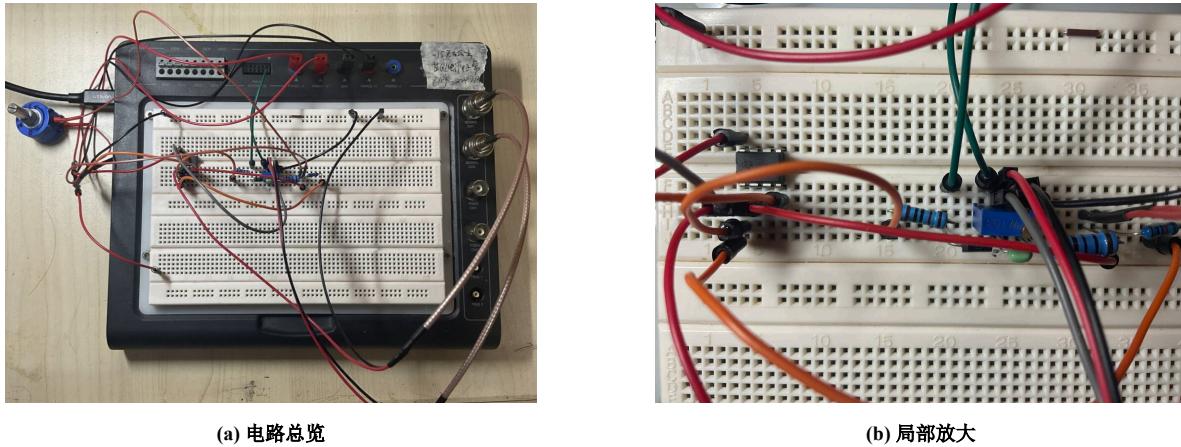


图 33: 脉冲发生器实际电路

为了方便使用，我们将脉冲发生器焊接在洞洞板上，如下图：

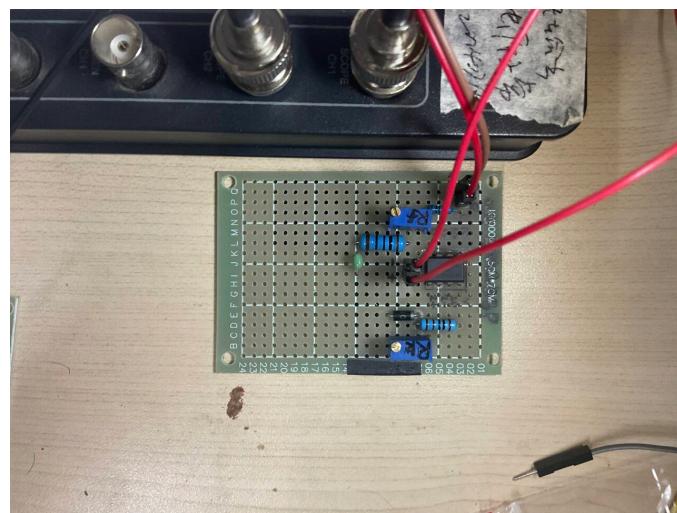
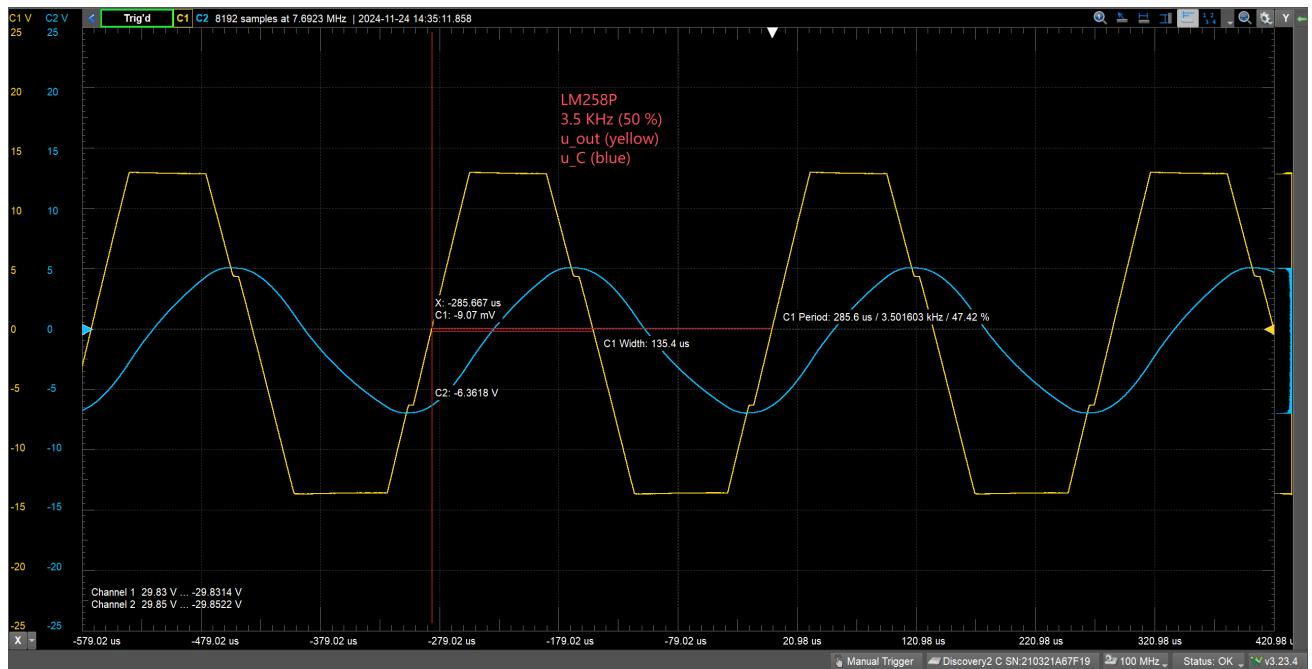
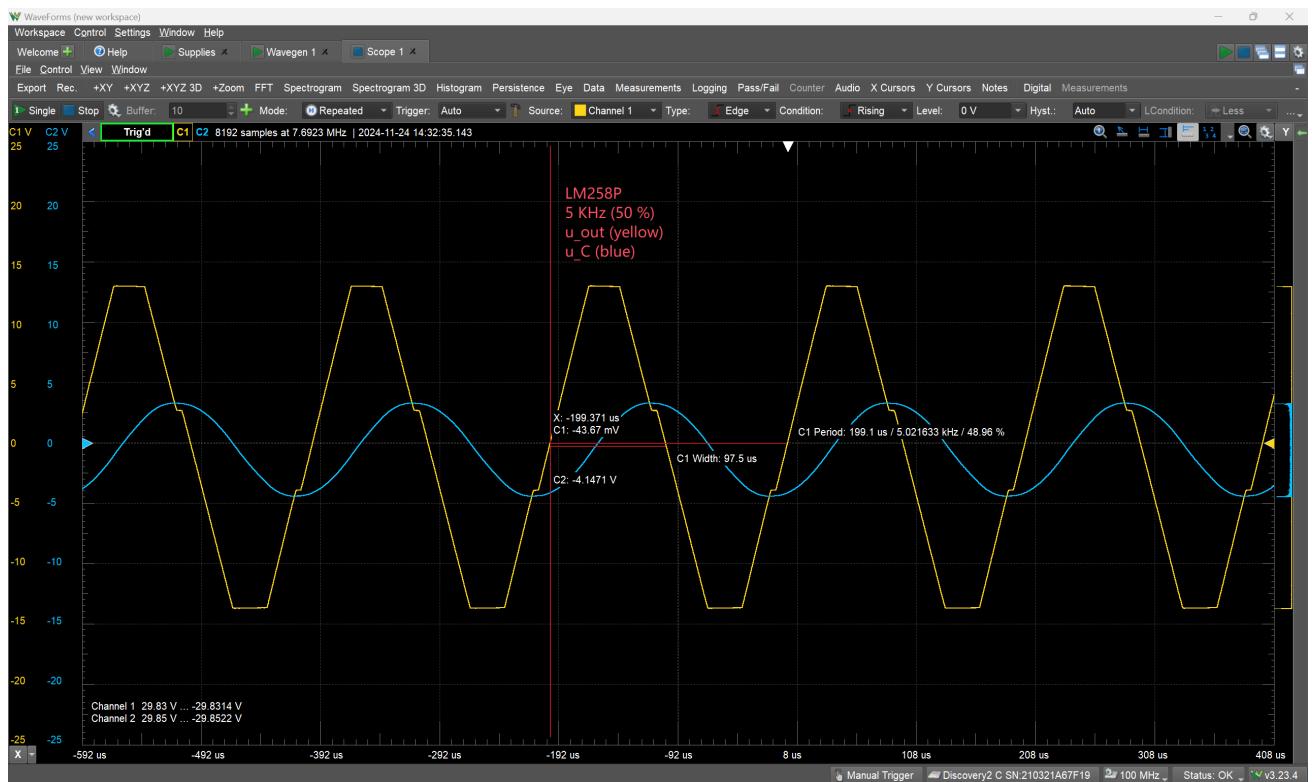


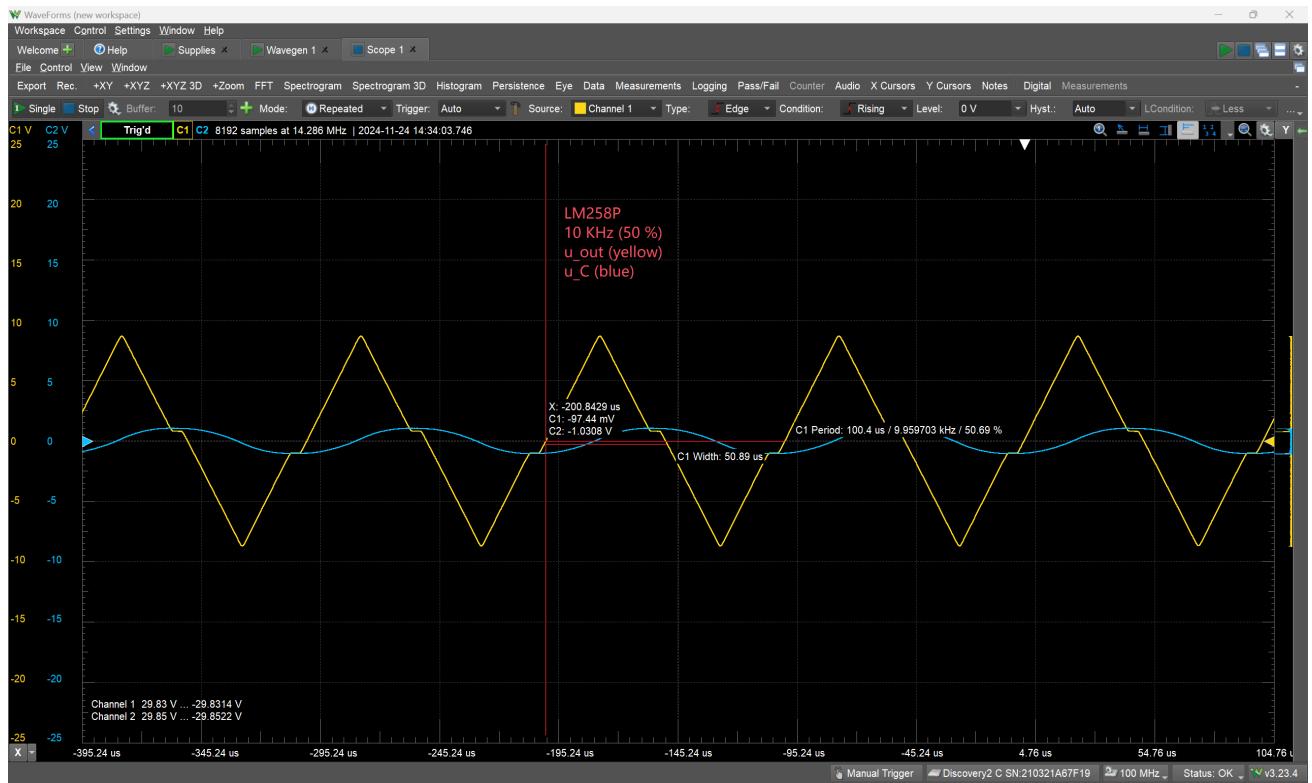
图 34: 将脉冲发生器焊接在洞洞板上

表 2: 实际电路元件参数

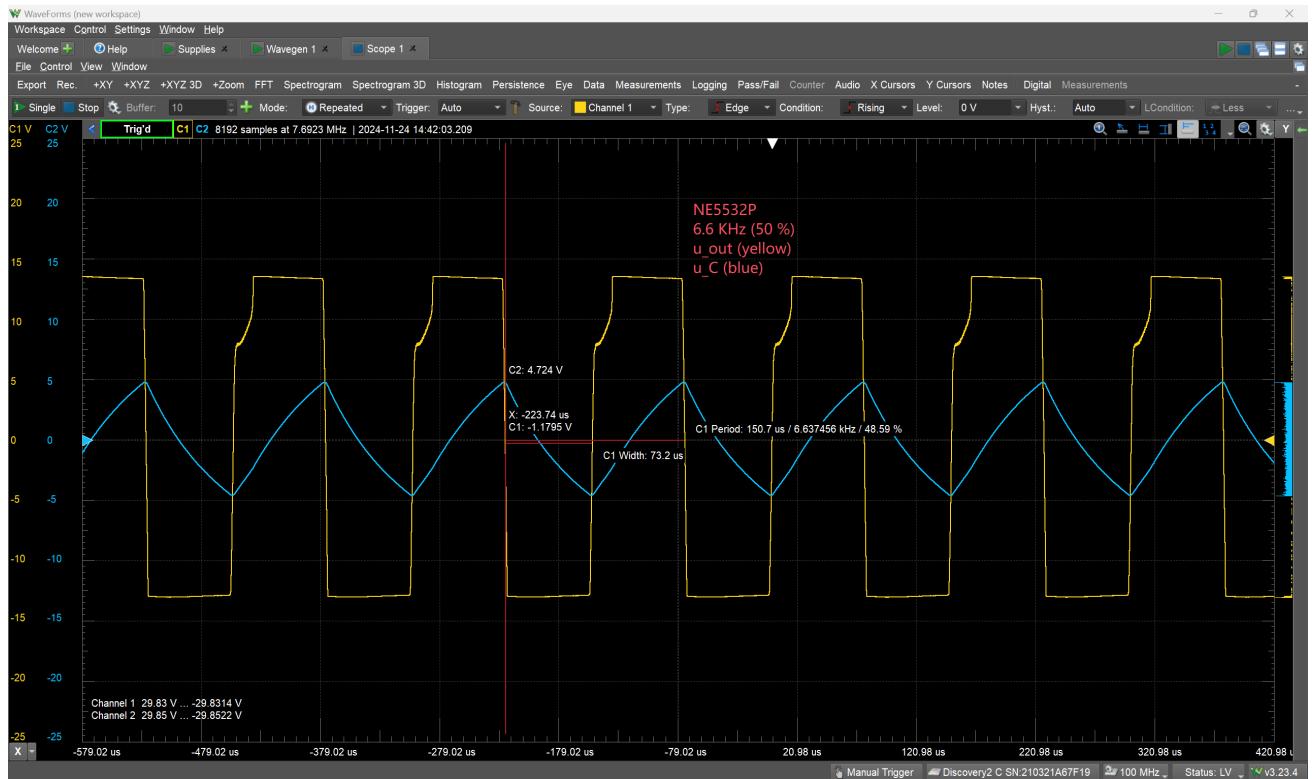
元件	330 μF 电容	100 nF 电容	100 Ω 电阻	1 K Ω 电阻	5 K Ω 负载	10 K Ω 电阻
实测值	300 μF	97.4 nF	98.4 Ω	994 Ω	5.02 K Ω	9.89 K

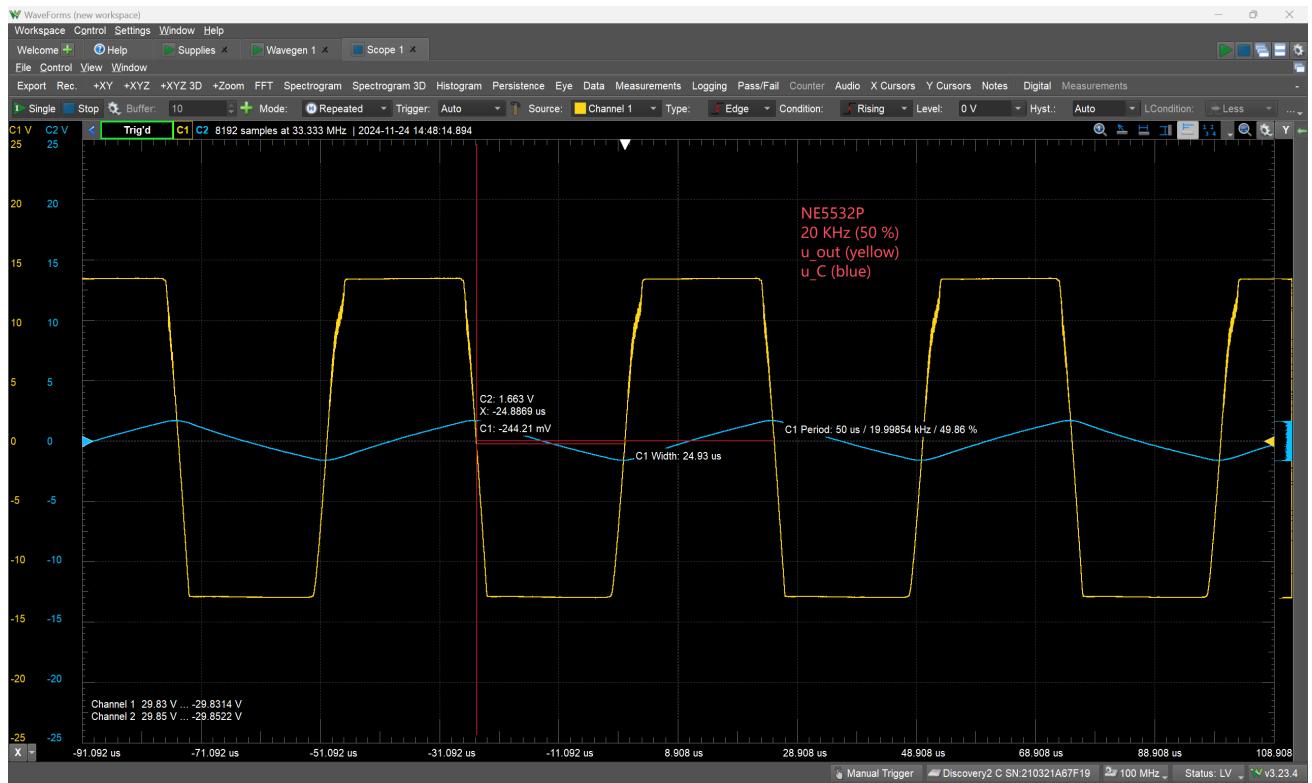
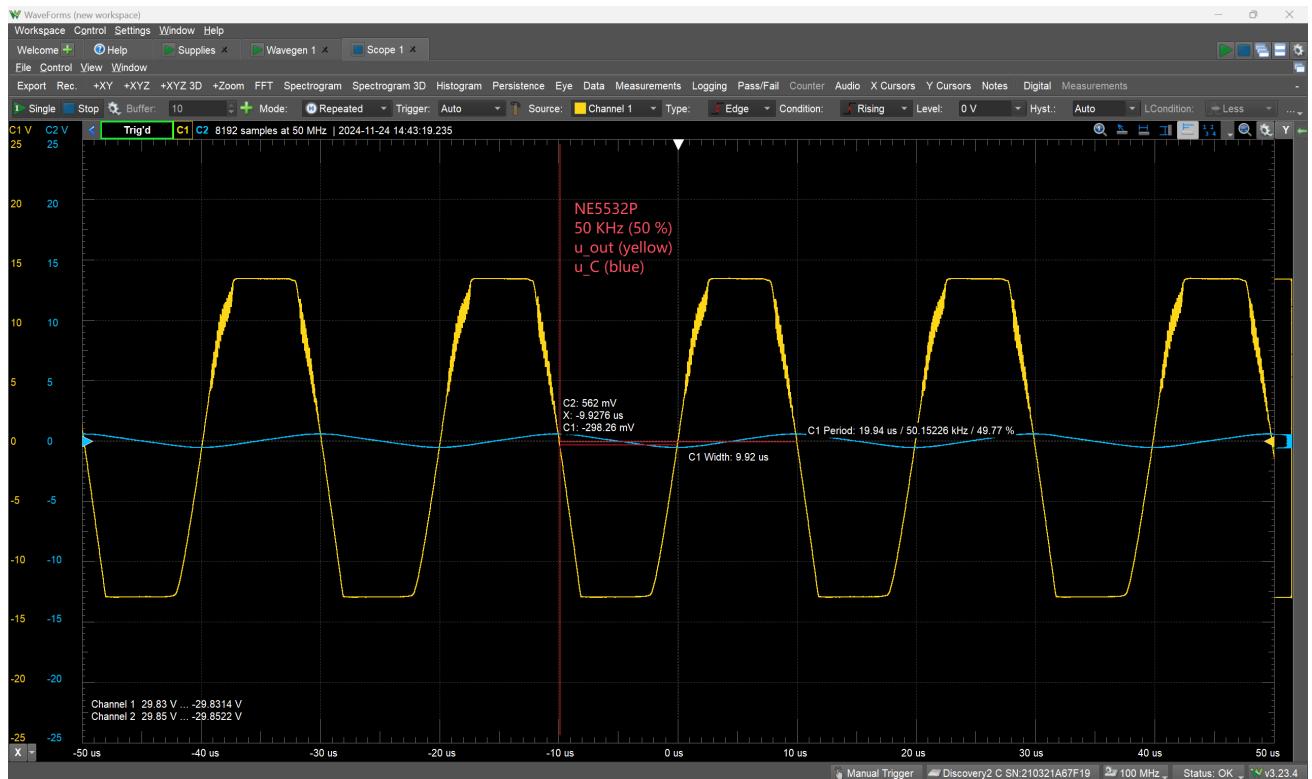
我们分别测试了脉冲发生器使用运放 LM258P 和 NE5532P 的输出情况。LM258P 的输出波形如下：

图 35: LM258P, $f = 3.5$ KHz图 36: LM258P, $f = 5$ KHz

图 37: LM258P, $f = 10 \text{ KHz}$

NE5532P 的输出波形如下:

图 38: NE5532P, $f = 6.6 \text{ KHz}$

图 39: NE5532P, $f = 20$ KHz图 40: NE5532P, $f = 50$ KHz

6.2 完整电路

在脉冲发生器的基础上，搭建完整 Buck 电路，负载电阻 $R = 5 \text{ K}\Omega$ ，如图 41。

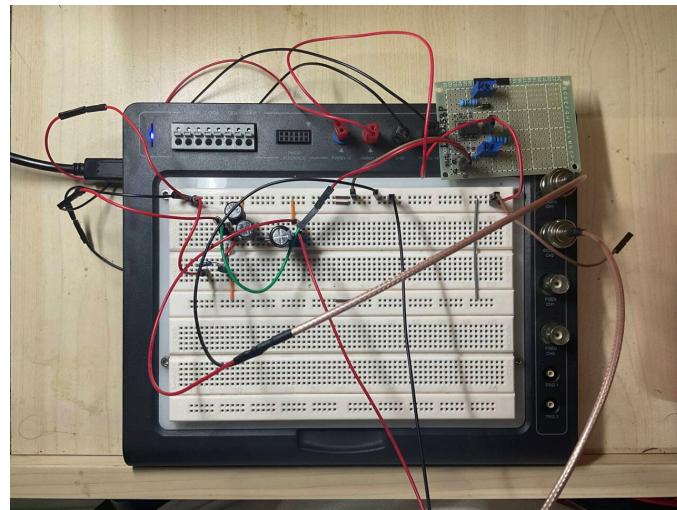


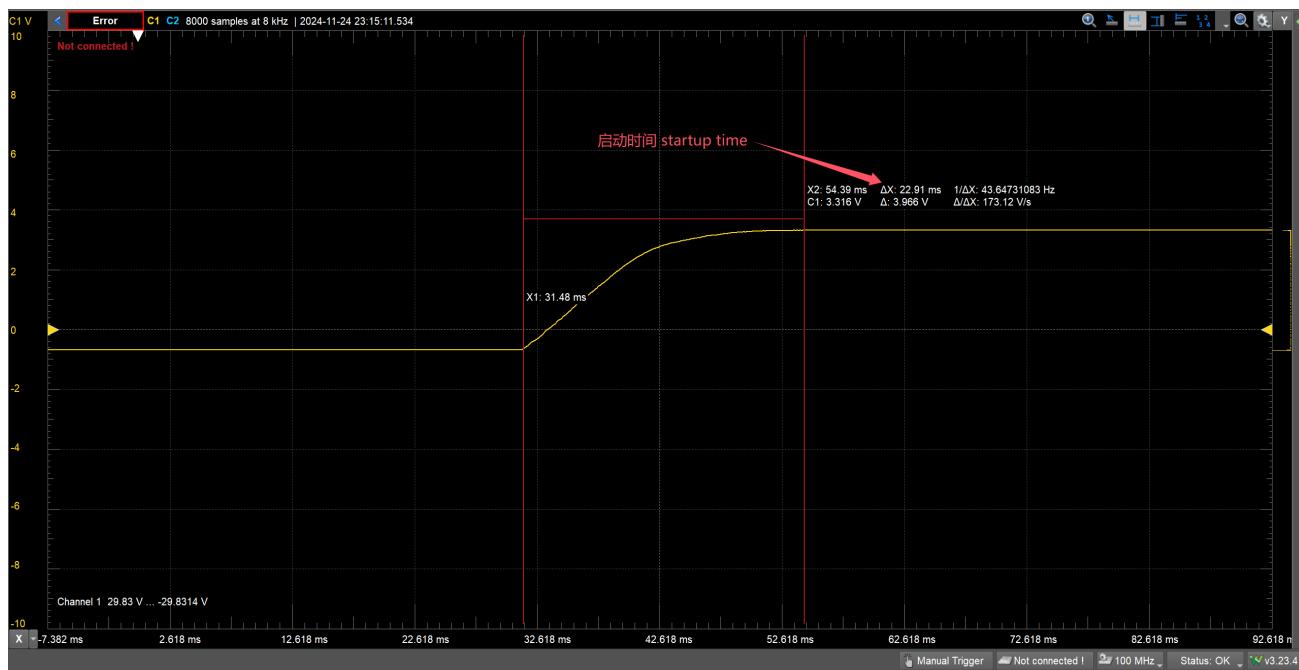
图 41: 完整 Buck 电路

6.3 性能测试

电感 $L = 4 \text{ mH}$ 时的性能如下:

- (1) 输出电压: $3.308 \text{ V} (\pm 7.00 \text{ mV}, r = 0.4232\%)$, 详见图 42;
- (2) 启动时间: $20 \text{ ms} \sim 25 \text{ ms}$, 详见图 43;
- (3) 最小输出电压: $0.822 \text{ V} (\pm 2.30 \text{ mV}, r = 0.5594\%)$;
- (4) 最大输出电压: $4.223 \text{ V} (\pm 12.0 \text{ mV}, r = 0.5683\%)$;

图 42: 实际电路输出电压 ($L = 4 \text{ mH}$)

图 43: 实际电路启动时间 ($L = 4 \text{ mH}$)

另外，我们也分别测试了电感 $L = 2 \text{ mH}$ 和 $L = 6 \text{ mH}$ 时的性能，

- (1) $L = 2 \text{ mH}$ ，输出电压: $3.3415 \text{ V} (\pm 14.50 \text{ mV}, r = 0.8679\%)$ ，详见图 44；
- (2) $L = 6 \text{ mH}$ ，输出电压: $3.306 \text{ V} (\pm 5.00 \text{ mV}, r = 0.3025\%)$ ，详见图 45；

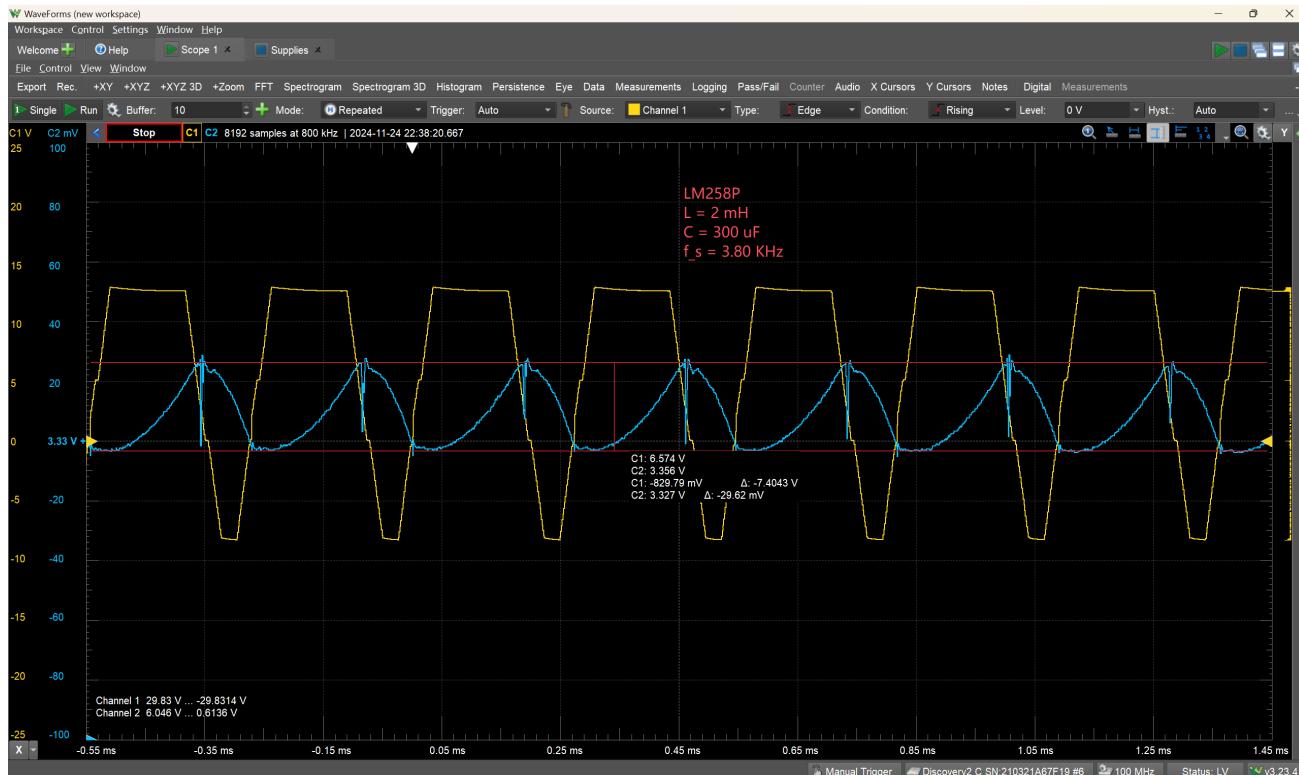
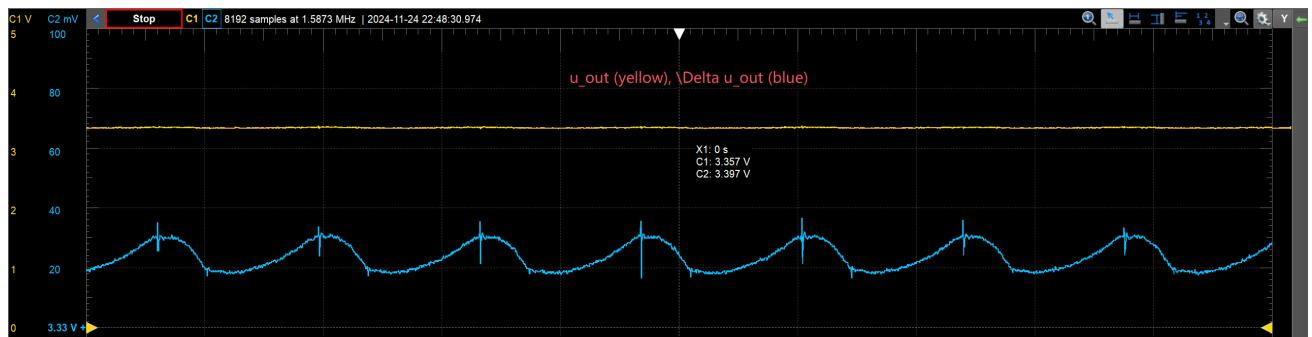
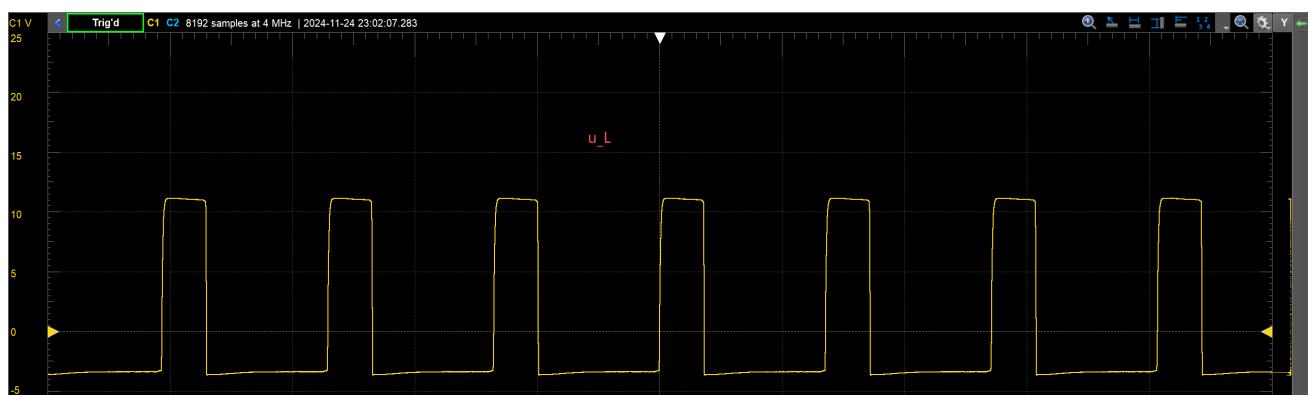
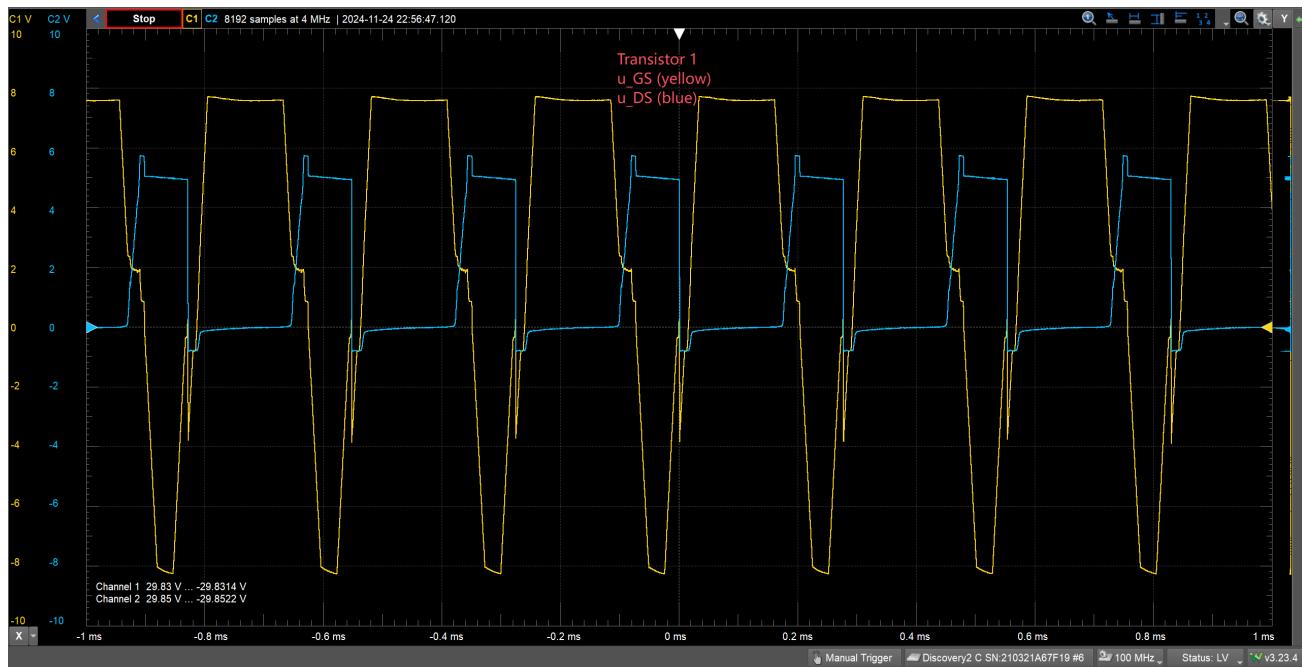
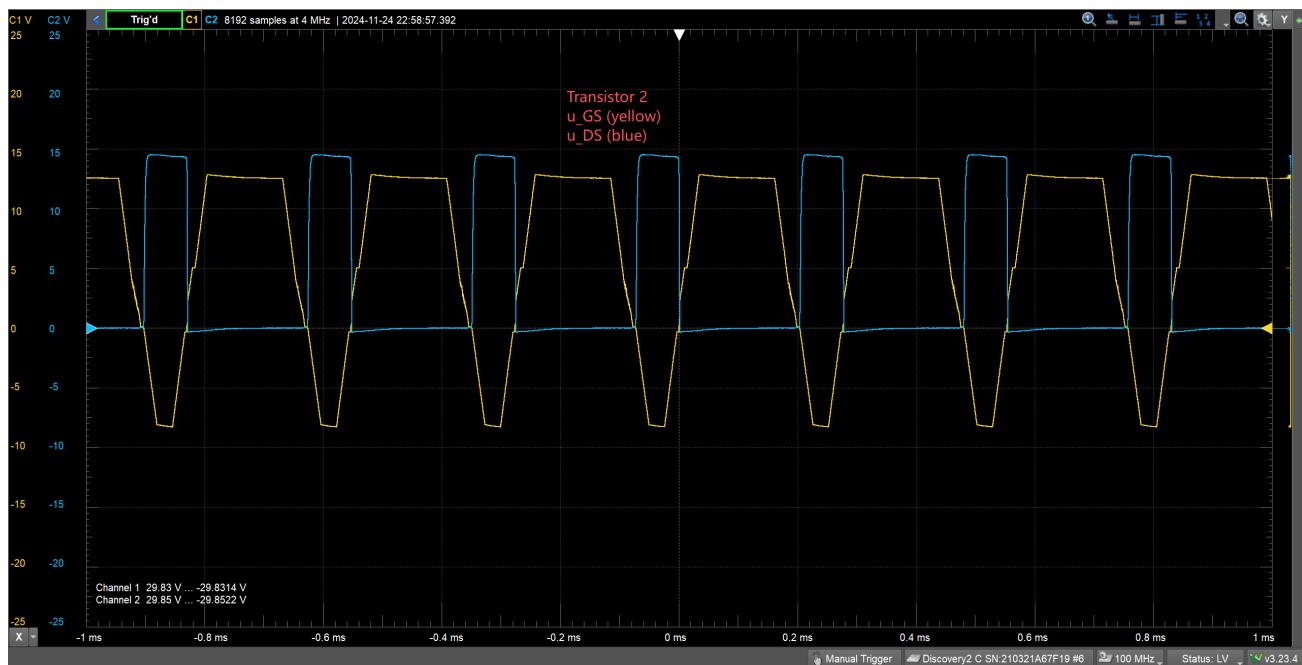
图 44: 实际电路输出电压 ($L = 2 \text{ mH}$)

图 45: 实际电路输出电压 ($L = 6 \text{ mH}$)

6.3.1 工作点各元件电压波形

由上面的输出性能可知, $L = 4 \text{ mH}$ 是比较合适的选择, 下面给出此时各元件的电压波形, 由图可以看出, 实际结果与仿真结果基本一致。

图 46: 输出电压 u_{out} 图 47: 电感电压 u_L

图 48: MOS T_1 电压图 49: MOS T_2 电压

7 设计总结与讨论

总的来看，改进方案 1 中 $L = 4 \text{ mH}$ 、 $C = 300 \mu\text{F}$ 、 $C_f = 100 \text{ nF}$ 是一个比较合适的选择，此时开关频率约为 3.8 KHz，输出电压纹波小，稳定性较好，启动时间也较短。

此次设计让我充分理解了电路的完整设计过程。在进行一个电路设计之前，应当首先确定整体需求，然后根据需求选择合适的元件，再进行理论上的推导。基于理论和所选元件，选择合适的参数进行仿真，最后搭建实际电路进行性能测试。在这个过程中，我们需要不断调整参数，直到满足设计要求。在实际搭建电路时，我们还需要注意元件的实际参数与仿真参数的差异，以及元件的工作范围，保证电路的正常工作。

设计进行到一半的时候，我突然疑惑起来：为了减小纹波，我们进行了这么多改进，那么为何不直接采

用最简单的电阻分压呢？如果只是为了得到稳定的输出，只需要可变电阻分压与一个电压跟随器隔离电路，便能得到可调且精确的输出电压。后来，查阅资料的过程中，我发现许多文献中的 Buck 电路负载都使用了 10Ω 或类似的小电阻，而不是我们设计中的 $K\Omega$ 级别，这才明白原因。在实际应用中，Buck 电路的功率通常很大，等效负载电阻可以很小，而简单的电阻分压带载能力极为有限，功率很小。

设计中指明负载电阻不能小于 $2 K\Omega$ ，也是出于安全考虑。因为我们使用的金属膜电阻，额定功率在 $\frac{1}{8} W$ 到 $1 W$ 不等，过大的电压会烧坏电阻，带来安全隐患。我自己在实验的过程中便发现，在 $1 K\Omega$ ($1 W$) 金属膜电阻两端加 $20 V$ 电压，电阻升温及其明显，且弥漫出一股特殊的“香味”，这显然是不安全的。未能在大功率的条件下进行本次设计，着实有些遗憾。

受理论水平限制，本次设计中我们并没有详细讨论电路的效率与损耗，这其实是任何电源的关键点之一。另外，实际的电源芯片通常都有反馈电路，可以适应不同的负载，自动维持输出电压，我认为这是电源设计的灵魂。有了反馈电路，使用者才可以摆脱繁重的参数调整，电源才能更加智能化。

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附录 A 部分元件实测参数

A.1 通用二极管 1N4007 IV 特性

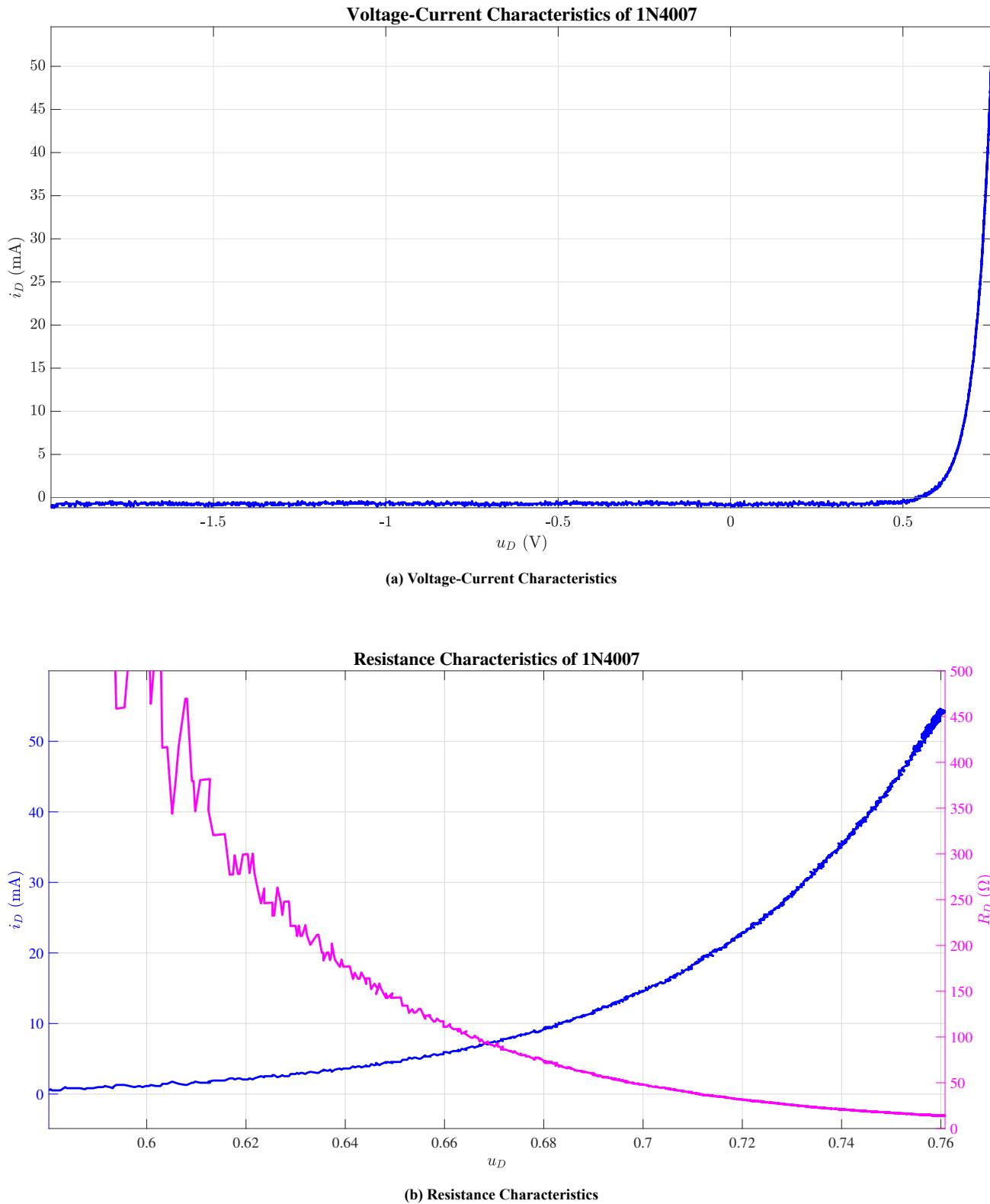


图 50: Characteristics of 1N4007

A.2 N-MOSFET 2N7000 IV 特性

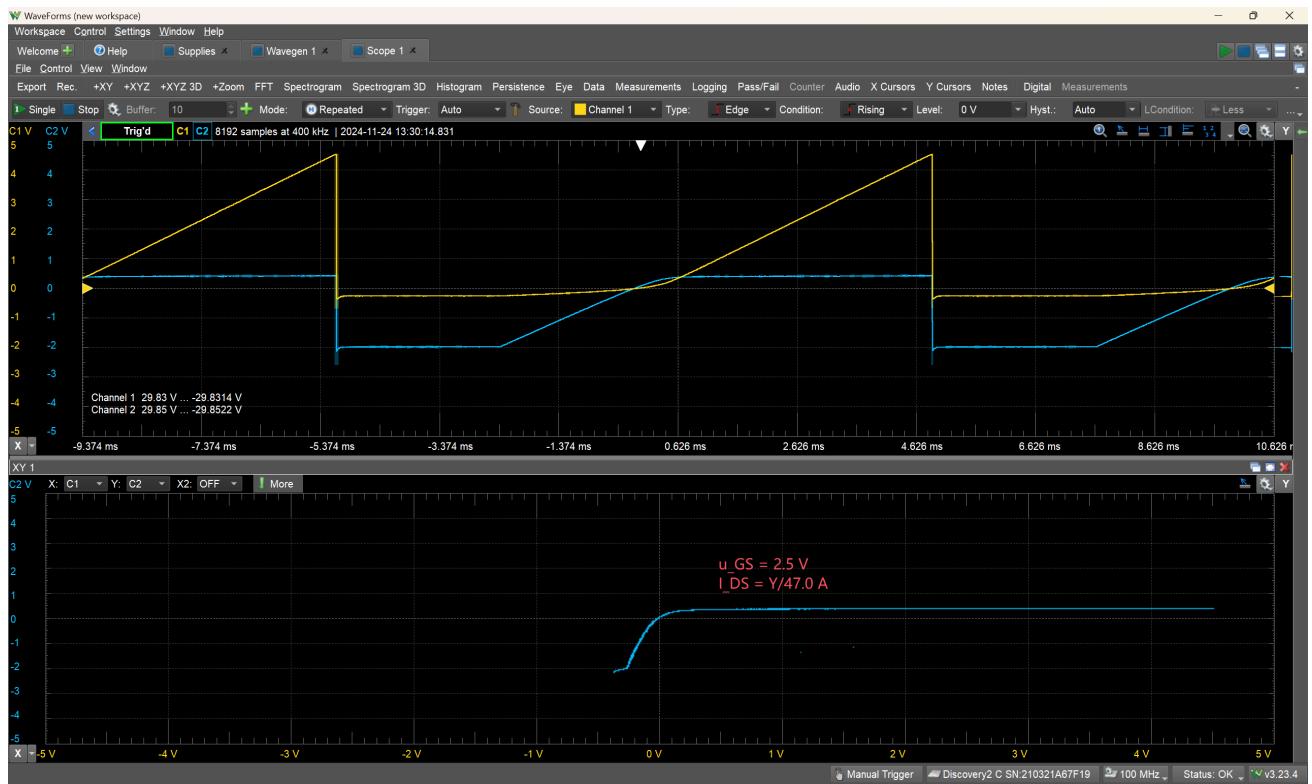


图 51: u_{DS} - i_{DS} ($u_{GS} = 2.5 \text{ V}$, 2 V 时 MOS 不导通, 全输入截止)

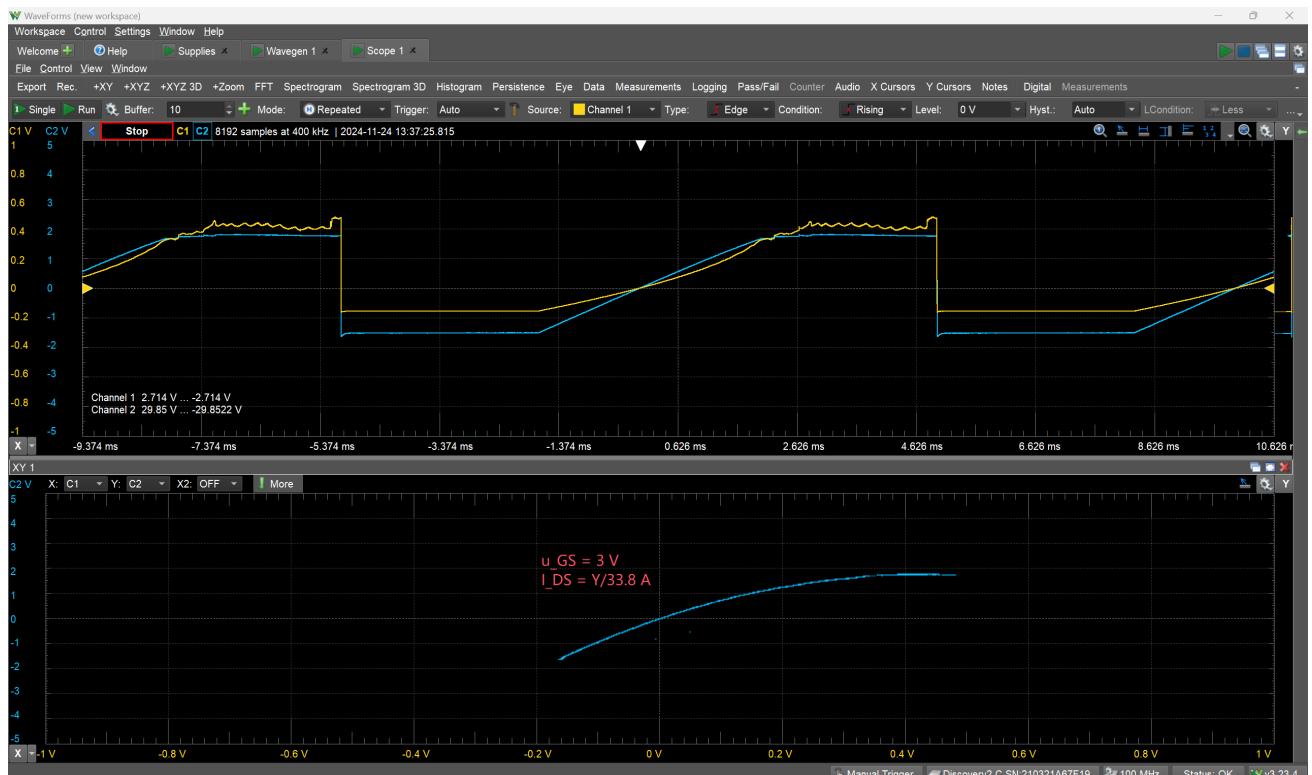


图 52: u_{DS} - i_{DS} ($u_{GS} = 3 \text{ V}$)

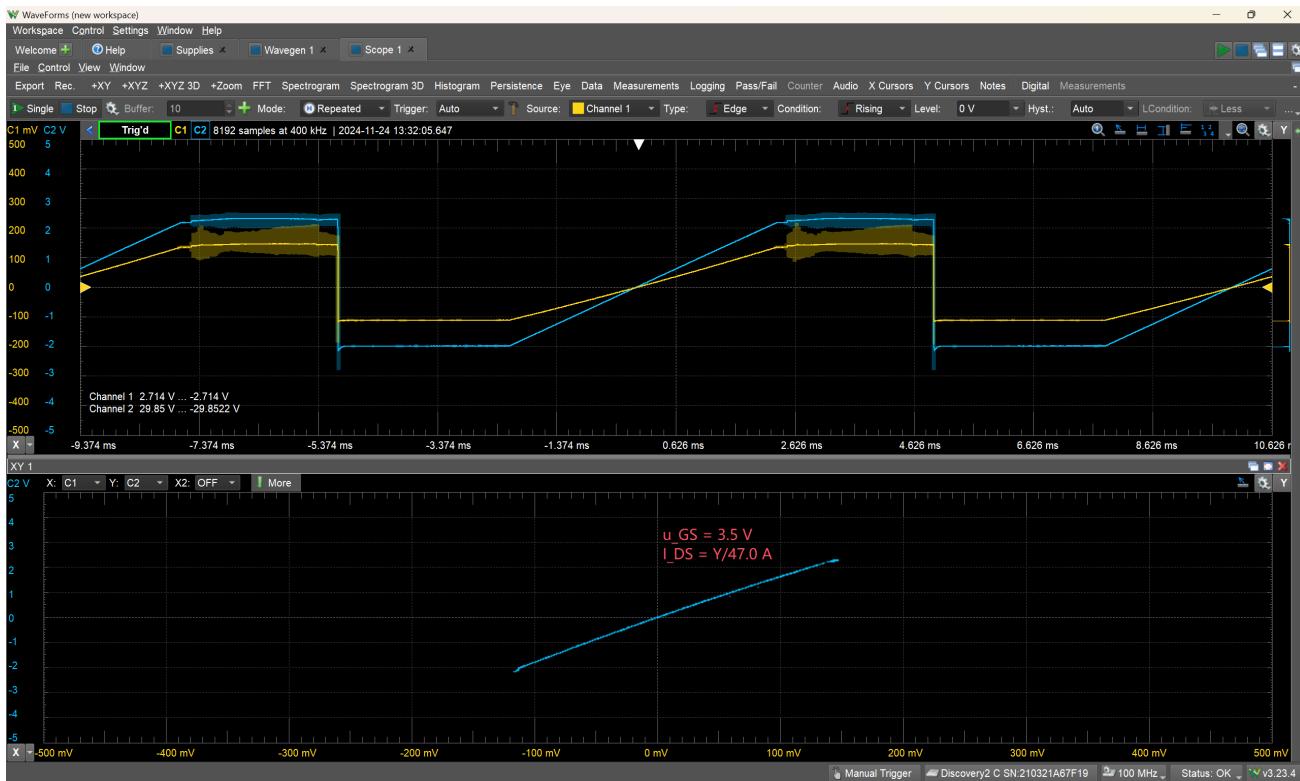


图 53: $u_{DS}-i_{DS}$ ($u_{GS} = 3.5 \text{ V}$)

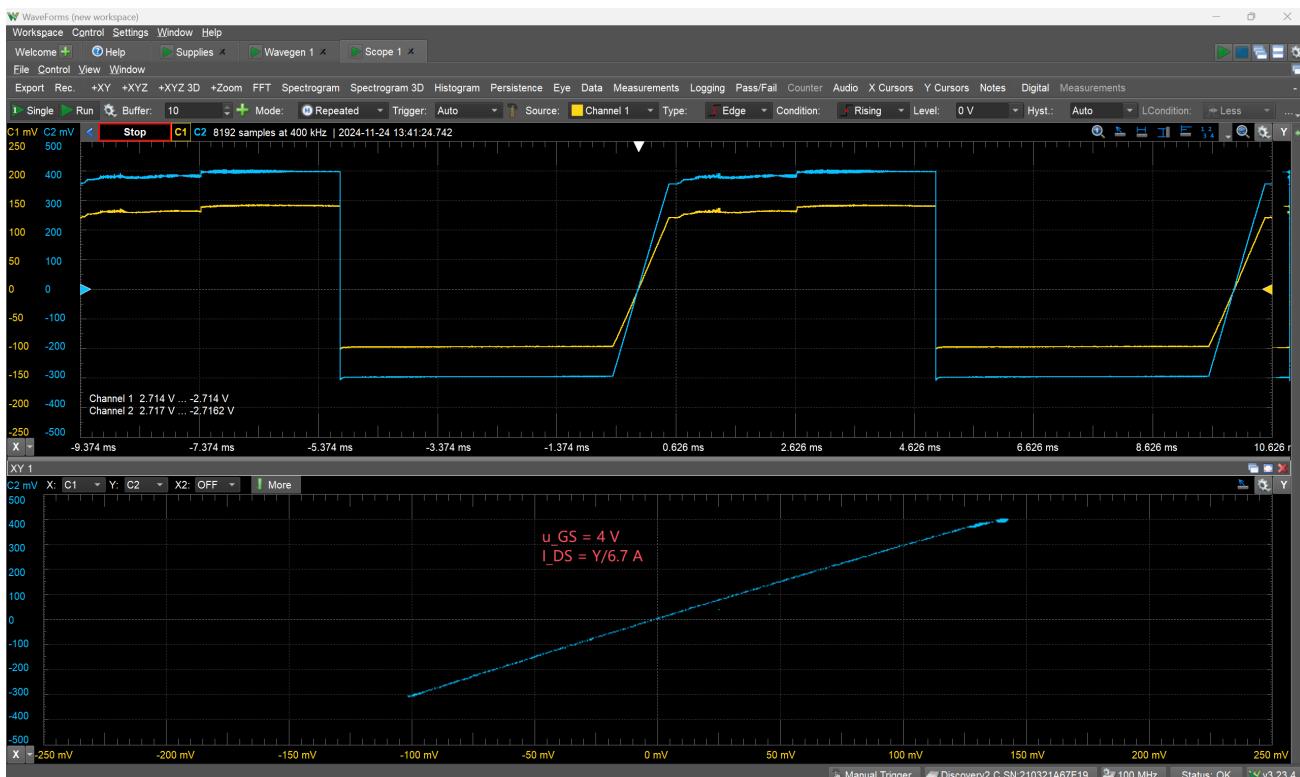
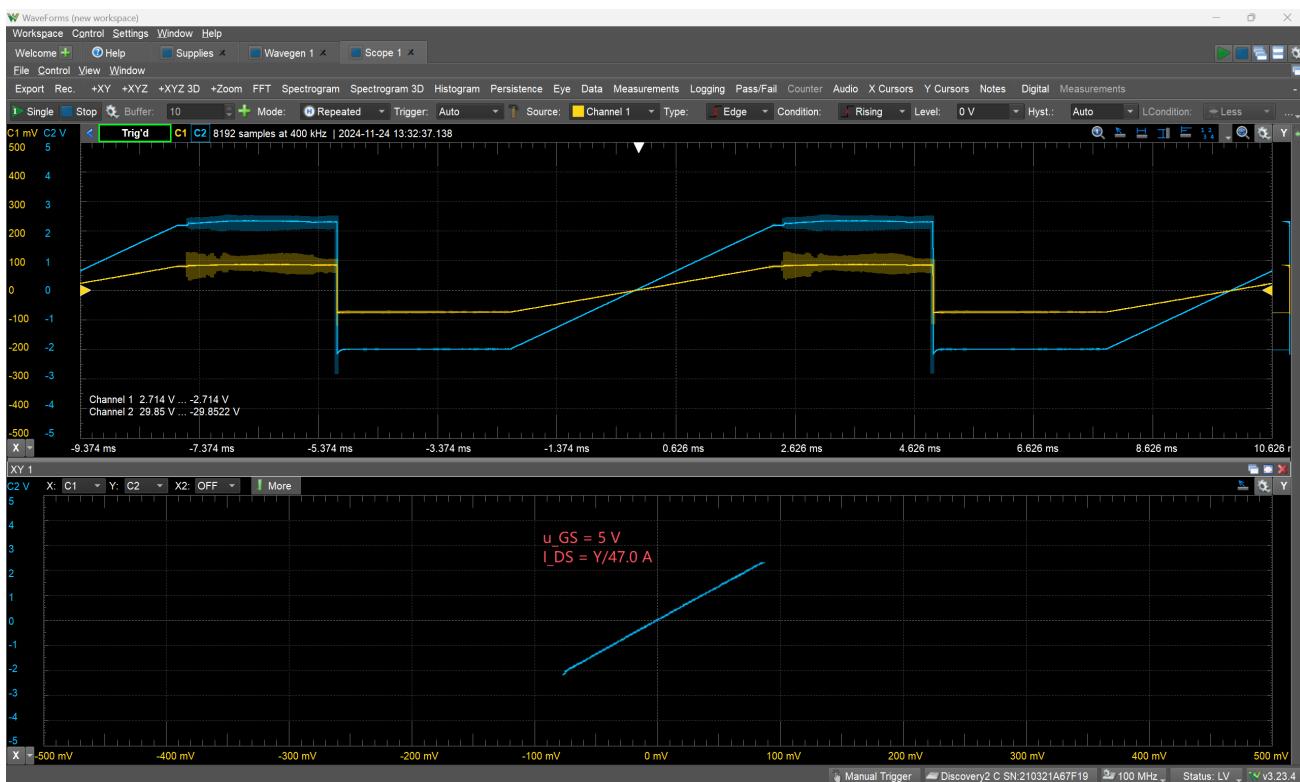


图 54: $u_{DS}-i_{DS}$ ($u_{GS} = 4 \text{ V}$)

图 55: u_{DS} - i_{DS} ($u_{GS} = 5$ V)

附录 B Matlab 源码

```

1  %% Design: Buck Circuits
2
3  %% 脉冲序列发生器
4  % 占空比 k 关于 R_k
5  R = 1e3;
6  R_D = 10;
7
8  k = @(R_p) 1 ./ ( 1 + ( R + (R_D.*R_p./(R_D+R_p)) )./( R + R_p ) );
9  R_p_array = linspace(0, 10e3, 300);
10 stc2 = MyPlot(R_p_array, k(R_p_array));
11 stc2.label.x.String = '$R_k$';
12 stc2.label.y.String = '$k$';
13 stc2.leg.Visible = 'off';
14 %MyExport_pdf
15
16 R_f_array = linspace(1000, 10e3, 300);
17 C = 100*10^3 * 10^(-12);      % 10^3 是 10nF 即 10e-9
18 R_1 = 10e3;
19 f = @(R_k, R_f) 1 ./ ( (2*R + R_k + (R_D.*R_k./(R_D+R_k)))*C*log(1 + 2*R_f/R_1) );
20 stc2 = MyPlot(R_f_array, [f(1000, R_f_array); f(2000, R_f_array); f(5000, R_f_array)]);
21 stc2.label.x.String = '$R_f$';
22 stc2.label.y.String = '$f$';
23 stc2.leg.String = ["$R_k = 1 \backslash \mathrm{K} \Omega $"; "$R_k = 2 \backslash \mathrm{K} \Omega $"; "$R_k = 5 \backslash \mathrm{K} \Omega $"];
24 MyExport_pdf
25
26 u_min = 3.312
27 u_max = 3.346
28 u = 0.5*( u_min + u_max )
29 Delta_u = u_max - u_min

```

```

30 disp(['r = ', num2str(Delta_u/u * 100), ' %'])
31 disp(['+- ', num2str(0.5*Delta_u*1000), ' mV'])
32
33
34
35 Idt = (9.866 + 92.896)*131.771/2 - 78.729*144.677/2 % 单位 mA*us
36 W_S = 5*Idt % 单位 V*mA*s
37 W_out = u*(u/5)*256.906
38 u/5e3*1000
39
40 17e-3/3.32*100
41 5.0e-3/2.117*100
42 62.4e-3/4.26*100
43
44 0.678e-3/3.308*100
45 0.252e-3/1.895*100
46
47 12/(12 + 30 + 30)*15
48
49 T = 274e-6;
50 f = 1/T
51
52 T = 500e-6;
53 f = 1/T
54
55 3.3*0.05*1000
56
57 5e-3/3.3*100
58 disp('-----')
59 disp(['500 uV 百分比: ', num2str(500e-6/3.3*100), ' %'])
60 disp(['1 mV 百分比: ', num2str(1e-3/3.3*100), ' %'])
61 disp(['10 mV 百分比: ', num2str(10e-3/3.3*100), ' %'])
62 disp(['50 mV 百分比: ', num2str(50e-3/3.3*100), ' %'])
63 disp(['100 mV 百分比: ', num2str(100e-3/3.3*100), ' %'])
64 disp('-----')
65
66 2.12/5
67 4.35/5
68 [2.5/5, 4/5]
69 5*0.43
70
71 1.895/5
72 2.115/5
73 4.42/5
74 [2.5/5, 4/5]
75 5*0.43
76
77 0.27e-3/1.895*100
78 8.87e-3/4.252*100
79 0.766e-3/3.308*100
80
81 1/320e-6
82
83 %% 双 MOS Buck 电路理论值计算
84 U_p = 15; % 脉冲高电压 V
85 f = 200e3; % 开关频率 Hz
86 k = 0.661; % 占空比
87
88 L = 100e-3; % 电感 H
89 U_S = 5; % 电源电压
90 R = 5e3; % 负载电阻
91

```

```

92 % MOS 管参数:
93 R_On = 2;
94
95 % 参数准备
96 tau = L/(R + R_On);
97 T = 1./f;
98 T_On = T*k;
99 T_Off = T*(1-k);
100 e_On = exp(-T_On/tau);
101 e_Off = exp(-T_Off/tau);
102 I_S = U_S/(R+R_On);
103
104 % 理论结果
105 I_o_max = I_S*(1 - e_On)/(1-e_On*e_Off);
106 I_o_min = I_S*(1 - e_On)/(1-e_On*e_Off)*e_Off;
107 Delta_I_o = I_S* (1 + e_On^2 - 2*e_On)/(1-e_On^2);
108 I_o_average = 0.5*(I_o_max + I_o_min);
109
110 U_o_max = I_o_max*R;
111 U_o_min = I_o_min*R;
112 Delta_U_o = Delta_I_o*R;
113 U_o_average = I_o_average*R;
114
115 U_persent = Delta_U_o/U_o_average;
116
117 % 输出结果
118 %disp(['I_o_average = ', num2str(I_o_average*1000), ' mA']);
119 %disp(['Delta_I_o = ', num2str(Delta_I_o*1000), ' mA']);
120 disp(['U_o_average = ', num2str(U_o_average), ' V']);
121 disp(['Delta_U_o = ', num2str(Delta_U_o*1000), ' mV']);
122 disp(['U_persent = ', num2str(U_persent*100), ' %']);
123
124 %% 反相器 2N7000 171 导通电阻 (U_DS = 5V 固定, U_GS 0 to 5V)
125 clc, clear, close all
126 U_o = readmatrix("C:\Users\13081\Downloads\U_DS = 5V, U_GS 0 to 5V.txt");
127 U_o = U_o';
128 U_GS = linspace(0, 5, size(U_o, 2));
129 %MyPlot(U_GS, U_o);
130 % 计算导通电阻
131 U_S = 5;
132 R_limit = 993;
133 R_T = R_limit./(U_S./U_o - 1);
134 stc2 = MyYYPlot([U_GS; U_GS], [U_o; R_T]);
135 yyaxis(stc2.axes, "right")
136 ylim([0 100])
137 xline(1.9)
138 yline(10)
139
140 xline(1.73)
141 yline(40)
142 ylim([0 50])
143
144 stc2.label.x.String = "$u_{\mathsf{GS}}$";
145 stc2.label.y_left.String = "$u_{\mathsf{DS}}$";
146 stc2.label.y_right.String = "$R_{\mathsf{T}}$";
147 stc2.leg.String = ["$u_{\mathsf{DS}}$"; "$R_{\mathsf{T}}$"];
148 stc2.axes.Title.String = "Resistance of 2N7000 (U_S = 5V, U_GS form 0 to 5V)";
149
150
151
152 % 10 欧姆时 u_GS = 1.8, 40 欧姆时 u_GS = 1.73
153

```

```

154 %% 反相器 2N7000 171 导通电阻 (U_DS = 15V 固定, U_GS 0 to 5V)
155 clc, clear, close all
156 U_o = readmatrix("C:\Users\13081\Downloads\U_DS = 15V, U_GS 0 to 5V.txt");
157 U_o = U_o';
158 U_o(U_o >= 15) = 14.999
159 U_GS = linspace(0, 5, size(U_o, 2));
160 %MyPlot(U_GS, U_o);
161 % 计算导通电阻
162 U_S = 15;
163 R_limit = 993;
164 R_T = R_limit./(U_S./U_o - 1);
165 stc2 = MyYYPlot([U_GS; U_GS], [U_o; R_T]);
166 yyaxis(stc2.axes, "right");
167 ylim([0 100])
168 xline(2.05)
169 yline(10)
170
171 xline(1.89)
172 yline(40)
173 ylim([0 50])
174
175 stc2.label.x.String = "$u_{\mathrm{GS}}$";
176 stc2.label.y_left.String = "$u_{\mathrm{DS}}$";
177 stc2.label.y_right.String = "$R_{\mathrm{T}}$";
178 stc2.leg.String = ["$u_{\mathrm{DS}}$"; "$R_{\mathrm{T}}$"];
179 stc2.axes.Title.String = "Resistance of 2N7000 (U_S = 15V, U_GS form 0 to 5V)";
180
181
182 % 10 欧姆时 u_GS = 2.05, 40 欧姆时 u_GS = 1.89
183
184 %% 电感感值测量 (L, 100 mH)
185 U_o = readmatrix("C:\Users\13081\Downloads\电感感值测量 (L_1, 第一次).txt");
186 U_o = U_o';
187 % 计算等效电阻
188 U_o_max = U_o(2, 1)
189 R_limit = 993;
190 U_S = 5;
191 R_L = R_limit*(U_S./U_o_max(1) - 1) % 直流等效电阻
192 % 计算感值
193 R_e = R_limit + R_L;
194 MyPlot(U_o(1, :), U_o(2, :));
195 yline(U_o_max/exp(1))
196 tau = (-0.000185126) - (-0.000271056) % 人工读数得到 tau
197 L = tau*R_e;
198 disp(['L = ', num2str(L*1000), ' mH']);
199 disp(['DCR = ', num2str(R_L), ' Ohm']);
200
201 %% 电感感值测量 (L, 2 mH)
202 U_o = readmatrix("C:\Users\13081\Downloads\电感测量 (2 mH, U_s = 2V).txt");
203 U_o = U_o';
204 % 计算等效电阻
205 U_o_max = U_o(2, 1)
206 R_limit = 46.9;
207 U_S = 2;
208 R_L = R_limit*(U_S./U_o_max(1) - 1) % 直流等效电阻
209 % 计算感值
210 R_e = R_limit + R_L;
211 MyPlot(U_o(1, :), U_o(2, :));
212 yline(U_o_max/exp(1))
213 tau = (0.04015908) - (0.0401179) % 人工读数得到 tau
214 L = tau*R_e;
215

```

```

216 disp(['L = ', num2str(L*1000), ' mH']);
217 disp(['DCR = ', num2str(R_L), ' Ohm']);
218
219 %% 电容测量 (JH, 214 uF)
220 U_o = readmatrix("C:\Users\13081\Downloads\电容测量 (JH, 214 uF).txt");
221 U_o = U_o';
222 % 计算等效电阻
223 U_o_max = U_o(2, 1);
224 R_limit = 993;
225 U_S = 5;
226 R_L = R_limit*(U_S./U_o_max(1) - 1) % 直流等效电阻
227 % 计算感值
228 R_e = R_limit + R_L;
229 MyPlot(U_o(1, :), U_o(2, :));
230 yline(U_o_max/exp(1));
231 tau = (-0.000185126) - (-0.000271056) % 人工读数得到 tau
232 L = tau*R_e;
233 %disp(['L = ', num2str(L*1000), ' mH']);
234
235
236 %% 1N4007 IV Characteristics
237 Data = readmatrix("C:\Users\13081\Downloads\1N7007 IV Characteristics (23.5 欧电阻作电流表).txt");
238 %Data = ProcessData(Data);
239 Data = Data(830:6236, :);
240 R_L = parallel(47.1, 47.0); % 用作电流表的电阻 (Channel 2)
241
242 u_D = Data(1, :);
243 i_D = Data(2, :)/R_L*1000;
244
245 stc1 = MyPlot(u_D, i_D);
246 stc1.axes.TickLabelInterpreter = 'latex';
247 stc1.axes.Title.String = "Voltage-Current Characteristics of 1N4007";
248 stc1.label.x.String = '$u_D$ (V)';
249 stc1.label.y.String = '$i_D$ (mA)';
250 stc1.leg.Visible = 'off';
251 c = GetMyColors;
252 stc1.plot_1.Color = c{2};
253 yline(0)
254 %xlim([0.4 max(Data(1, :))])
255 MyExport_pdf_docked
256
257 yyp = MyYYPlot([u_D(u_D > 0.58); u_D(u_D > 0.58)], [i_D(u_D > 0.58); R_D(u_D > 0.58)]);
258 yyaxis(yyp.axes, "right");
259 ylim([0 500])
260 yyp.leg.Visible = 'off';
261 yyp.axes.Title.String = "Resistance Characteristics of 1N4007";
262 yyp.label.x.String = '$u_D$';
263 yyp.label.y_right.String = '$R_D$ (\Omega)';
264 yyp.label.y_left.String = '$i_D$ (mA)';
265 MyExport_pdf_docked
266
267 syms x
268 eq = (200-x)*(100-x) == 10000;
269 re = solve(eq)
270 vpa(re)
271
272 TtoF(297e-6)
273 ShowOutput(3.323523, 3.323)
274
275 %% 4 mH

```

```

278 ShowOutput(3.301, 3.315)
279 ShowOutput(0.820, 0.8246)
280 ShowOutput(4.211, 4.235)
281
282 % 6 mH
283 ShowOutput(3.311, 3.301)
284 % 2 mH
285 ShowOutput(3.327, 3.356)
286
287 function R = parallel(R_1, R_2)
288     R = 1./(1./R_1 + 1./R_2);
289 end
290
291 function R = parallel_n(R_array)
292     R = 1./sum(1./R_array);
293 end
294
295 function f = TtoF(T)
296     f = 1./T;
297 end
298
299 function ShowOutput(u_min, u_max)
300     u = 0.5*( u_min + u_max);
301     Delta_u = u_max - u_min;
302     disp(['u = ', num2str(u, '%.4f'), ' V (+- ', num2str(0.5*Delta_u*1000, '%.2f'), ' mV']]);
303     disp(['r = ', num2str(Delta_u/u * 100, '%.4f'), ' %']);
304 end
305
306 function GetOutput(u_min, Delta_u)
307     u = u_min + 0.5*Delta_u;
308     Delta_u = u_max - u_min;
309     disp(['u = ', num2str(u, '%.4f'), ' V (+- ', num2str(0.5*Delta_u*1000, '%.2f'), ' mV']]);
310     disp(['r = ', num2str(Delta_u/u * 100, '%.4f'), ' %']);
311 end
312
313 function Data = ProcessData(Data)
314     % 转为行向量或行矩阵
315     if size(Data, 1) > size(Data, 2)
316         Data = Data';
317     end
318     i = 1;
319     % 获取一个周期的起始点
320     while sum(Data(1, i) < Data(1, i+ (1:8))) > 0
321         i = i+1;
322     end
323     begin = i+1;
324     i = begin;
325     % 获取一个周期的结束点
326     while sum(Data(1, i) < Data(1, i+ (1:8))) > 0
327         i = i+1;
328     end
329     End = i;
330     Data = Data(:, begin:1:End);
331 end

```