1 Introduction

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3 第一章第一节

向后加权隐式格式:

$$\begin{split} \frac{u_j^k - u_j^{k-1}}{h_t} &= a\theta \frac{u_{j+1}^k - 2u_j^k + u_{j-1}^k}{h_x^2} + a(1-\theta) \frac{u_{j+1}^{k-1} - 2u_j^{k-1} + u_{j-1}^{k-1}}{h_x^2} \quad \text{(1)} \\ & \text{其中} \quad \theta \quad \in \quad [0,1] \quad \text{为权重, 其截断误差} \quad R \quad = \\ a\left(\frac{1}{2} - \theta\right) h_t \left[\frac{\partial^3 u}{\partial x^2 \partial t}\right]_j^k + O(h_t^2 + h_x^2), \text{ 因此当} \quad \theta \quad = \quad \frac{1}{2} \text{ 时,} \end{split}$$

方程具有 $O(h_t^2 + h_x^2)$ 精度, 称为 Crank-Nicolson 格式 (CN 格式)。 公式1的增长因子及稳定性条件为:

$$G(h_t, \sigma) = \frac{1 - 4(1 - \theta)ar\sin^2\frac{\sigma h}{2}}{1 + 4\theta ar\sin^2\frac{\sigma h}{2}}, \begin{cases} r \leqslant \frac{1}{2\alpha(1 - 2\theta)}, & \theta \in [0, \frac{1}{2})\\ \frac{1}{2}\% + \frac{1}{2}\% + \frac{1}{2}\% + \frac{1}{2}\% + \frac{1}{2}\% \end{cases}$$
(2)

Theorem. 1 (这是一个 Line Theorem): 你好你好你好

Theorem. 2 (这是一个 Block Theorem): 你好你好你好

定理 2 的证明:

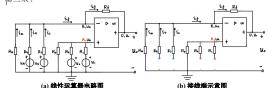
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表 1: 符号含义与约定					
符号	符号含义	单位			
符号 1 符号 2 符号 3 符号 4	含义 1 含义 2 含义 3 含义 4	单位 1 单位 2 单位 3 单位 4			

3.1 线性运算器(自设计)

图 1 是在加法器、减法器的基础上,自己设计的线性运算器,它可以实现 任意数量的输入(电压)信号的任意线性运算。事实上,在此线性运算器中,电阻 R_M 和电阻 R_P 是关键,因为在正相信号间的比例、反相信号间的比 例分别确定时,这两个电阻实现了正信号和负信号之间的比例调整,使得最 终输出的正、负信号可以任意大或任意小(最小即为0,不占任何比例)。

图中,红色端是加法信号,蓝色端是减法信号,绿色端为公共地(可只保



型 1: 自设计线性运算器 Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam

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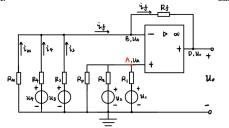


图 2: 这里是图注

我们先研究图 1 (a) 的输出特性,再讨论如果没有电阻 R_M 或 R_P ,输 出电压会受到什么限制。输出电压的推导是简单的,先考虑点 A 的电势 u_A ,

$$\mu_A = \frac{\frac{u_1}{R_1} + \frac{u_2}{R_2}}{\frac{1}{-} + \frac{1}{-} + \frac{1}{-}}$$
(3)

 u_{Δ} 的推导,除了列 KCL, KVL 硬解之外,还可以这样: 先将 R_{n} 断路,这样 u_2, R_2, u_1, R_1 构成并联的两个实际电压源(自带电阻),容易求得此时点 A 的电

$$u_A = \frac{R_2 u_1 + R_1 u_2}{R_1 + R_2} = \frac{\frac{u_1}{R_1} + \frac{u_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$
 于是,我们再并联一个实际电压源 P 后,由数学上直接推广,可以得到 u_A 为;

$$\iota_{A} = \frac{\frac{u_{1}}{R_{1}} + \frac{u_{2}}{R_{2}} + \frac{u_{p}}{R_{p}}}{\frac{1}{p} + \frac{1}{p} + \frac{1}{p}}$$
(5)

再令 $u_p = 0$,即得图 1 中的原始 u_A 。 再考虑左侧的电流组,并利用虚断:

$$\begin{array}{l} (1) & \text{ For Earth District Dis$$

$$\implies u_o = u_A - R_f \left[\frac{u_3}{R_3} + \frac{u_4}{R_4} - u_A \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_m} \right) \right]$$

$$= \left[1 + R_f \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_m} \right) \right] u_A - R_f \left(\frac{u_3}{R_3} + \frac{u_4}{R_4} \right)$$
(8)

 $u_{o} = \frac{1 + \frac{R_{f}}{R_{m}} \left(\frac{1}{R_{3}} + \frac{1}{R_{4}}\right)}{\frac{1}{R_{5}} + \left(\frac{1}{R_{1}} + \frac{1}{R_{2}}\right)} \cdot \left(\frac{\mathbf{u}_{1}}{R_{1}} + \frac{\mathbf{u}_{2}}{R_{2}}\right) - R_{f} \cdot \left(\frac{\mathbf{u}_{3}}{R_{3}} + \frac{\mathbf{u}_{4}}{R_{4}}\right)$

$$u_o = \frac{1}{\frac{1}{R_s} + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{R_f}{R_f} \cdot \left(\frac{1}{R_3} + \frac{1}{R_4}\right)$$
 (9) 这样,对于所有加法信号,可以通过 R_1 , R_2 间的比例来调整它们在加法中的输出比例。举创地、滹泞信号通过 R_2 , R_3 间的比例来调整它们在滹泞中

的输出比例,类似地,减法信号通过 R_3 , R_4 间的比例来调整它们在减法中 的输出比例。最后通过 R_f, R_m, R_p 来调整加法、减法之间的输出比例。在 R_f, R_m, R_p 都可变时, 易证(减法占比) $R_f \in [0, \infty)$, (加法占比) $\frac{\frac{1+R_f\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_m}\right)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_p}}\in[0,\,\infty),\, \exists E$ 全部系数都具有任意性,此线

上面的电路容易推广到任意输入信号个数的情形。假设有 m 个加法信号 u_{s_1},\ldots,u_{s_m} ,它们对应的串联电阻分别 R_{s_1},\ldots,R_{s_m} ;以及 n 个减 法信号 u_{r_1},\ldots,u_{r_n} ,它们对应的串联阻值分别 R_{r_1},\ldots,R_{r_n} 。直接 由数学上推广出去,得到输出电压 u。的表达式为:

$$u_{o} = \left(\frac{1 + \frac{R_{i}}{R_{m}} + R_{f} \sum_{i=s_{i}}^{i=s_{m}} \frac{1}{R_{i}}}{\frac{1}{R_{i}} + \sum_{i=r_{i}}^{i=r_{i}} \frac{1}{R_{i}}}\right) \cdot \sum_{i=s_{i}}^{i=s_{m}} \frac{u_{i}}{R_{i}} - R_{f} \cdot \sum_{i=r_{i}}^{i=r_{n}} \frac{u_{i}}{R_{i}}$$
(10)

此线性运算器的具体仿真示例见 Homework 3.

4 对于正入射的情况,写出菲涅尔公式

菲涅尔公式如下: 菲涅尔公式如下: 菲涅尔公式如下: 菲涅尔公式如下: 菲涅尔公式如下: 菲涅尔公式如下: 菲涅尔公式如下: 菲涅尔公式如下: 菲涅 尔公式如下: 菲涅尔公式如下: 菲涅尔公式如下: 菲涅尔公式如下:

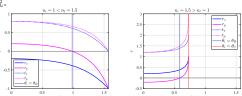
类型	振幅反射系数 r		振幅透射系数 t	
s被	$r_s = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t}$	$-\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$	$t_s = \frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t}$	$+\frac{2 \sin \theta_t \cos \theta_i}{\sin(\theta_i + \theta_t)}$
p被	$r_p = \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_t \cos \theta_i + n_i \cos \theta_t}$	$+\frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$	$t_p = \frac{2n_i \cos \theta_i}{n_i \cos \theta_t + n_t \cos \theta_i}$	$+\frac{2 \sin \theta_t \cos \theta_i}{\sin(\theta_i + \theta_t) \cos(\theta_i - \theta_t)}$

$$r_{p} = (-r_{s}) = \frac{n_{t} - n_{i}}{n_{t} + n_{i}}, \quad t_{p} = t_{s} = \frac{2n_{i}}{n_{i} + n_{t}}$$

$$F = R_{s} = R_{p} = \left(\frac{n_{t} - n_{i}}{n_{t} + n_{t}}\right)^{2}$$
(12)

$$F = R_s = R_p = \left(\frac{n_t - n_i}{n_s + n_s}\right)^2 \tag{12}$$

情况。



(a) 由空气入射玻璃($n_i=1,\;n_t=1.5$)(b) 由玻璃入射空气($n_i=1.5,\;n_t=1$) 图 3: 振幅系数 r 随入射角 θ 。的变化

5 一自然光以 Brewster Angle 入射到空气中的一块玻璃。 已知功率透射率为 0.86。

*事实上,本题题设并不合理,是不符合实际的。我们先给出解题过程, 再说明为何题设不合理。

(1) 求功率的反射率:

T = 0.86, 由能量守恒, 功率反射率 R = 0.14。

(2) 若输入为 1000 W, 求透射光 s 分量上的功率

光束为自然光,因此 s 分量和 p 分量的功率相同,都为 500 W,也即 $\Phi_{e,i,s} = \Phi_{e,i,p} = 500 \text{ W}$ 。文由 Brewster Angle 入射,因此反射光的 p分量为 0,也即 $R_p = 0$,于是:

$$T_p = 1 - R_p = 1, \quad T_s = 2T - T_p = 0.72$$
 (13)

由此可求得透射光 s 分量上的辐射通量(即辐射功率)。 $\Phi_{e,t,s}=T_s\Phi_{e,t,s}=0.72\times500\,\mathrm{W}=360\,\mathrm{W}$ (14)

(3) 不合理的原因: 玻璃折射率为 0.554902

在题设条件下,R=0.14,默认空气折射率为 1,则唯一的未知量是玻璃折射率 n_t ,这是可以求解的,方程如下:

$$R = \frac{1}{2}(R_s + R_p) = 0.14, \quad \theta_i = \theta_B = \arctan\left(\frac{n_t}{n_i}\right), \quad n_i = 1 \Longrightarrow (15)$$

$$s(\arctan n_t) - \sqrt{n_t^2 - \sin^2(\arctan n_t)}\right]^2 + \begin{bmatrix} n_t^2 \cos(\arctan n_t) - \sqrt{n_t^2 - \sin^2(\arctan n_t)} \\ + \frac{n_t^2 \cos(\arctan n_t) - \sqrt{n_t^2 - \sin^2(\arctan n_t)}}{n_t^2 \cos(\arctan n_t)} \end{bmatrix}$$

此方程有唯一未知量 n_t ,用 Matlab 解此非线性方程组,得到玻璃折射率 n_t

$$n_t = 0.554902$$
, $\theta_t = \theta_B = 0.506599$ rad $= 29.025970^{\circ}$
 $\theta_t = 1.064198$ rad $= 60.974030^{\circ}$, $\theta_C = 0.588245$ rad $= 33.703947^{\circ}$
 $R = 0.140000$, $R_s = 0.280000$, $R_p = 0.000000$
 $T = 0.860000$, $T_s = 0.720000$, $T_p = 1.000000$

而一般玻璃的折射率在1.5左右,即使是特殊玻璃(例如高折射率镜片), 也基本在 1.3 至 1.9 之间, 0.5 的玻璃折射率显然是不合理的。即使是考虑介 质折射率关于波长的变化(如 X 射线或 Gamma 射线),也不会达到如此低的

6 上题修正: 一自然光由空气入射玻璃,玻璃折射率为 1.5, 已知功率透射率为 0.86。

(1) 求功率的反射率:

T = 0.86, 由能量守恒, 功率反射率 R = 0.14。

(2) 若输入为 1000 W, 求透射光 s 分量上的功率

光束为自然光,因此 s 分量和 p 分量的功率相同,都为 500 W。先求解入 射角 θ_i , 由菲涅尔定理和能量关系:

$$R = \frac{1}{2}(R_s + R_p) \tag{18}$$

$$R = \frac{1}{2} (R_s + R_p)$$
(18)
$$R_s = \left[\frac{\cos \theta_i - \sqrt{n_{ti}^2 - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{n_{ti}^2 - \sin^2 \theta_i}} \right]^2, R_p = \left[\frac{n_{ti}^2 \cos \theta_i - \sqrt{n_{ti}^2 - \sin^2 \theta_i}}{n_{ti}^2 \cos \theta_i + \sqrt{n_{ti}^2 - \sin^2 \theta_i}} \right]^2$$
(19)

$$\left[\frac{\cos\theta_i - \sqrt{1.5^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{1.5^2 - \sin^2\theta_i}}\right]^2 + \left[\frac{1.5^2 \cos\theta_i - \sqrt{1.5^2 - \sin^2\theta_i}}{1.5^2 \cos\theta_i + \sqrt{1.5^2 - \sin^2\theta_i}}\right]^2 = 2 \times 0.14 (20)$$

 $\begin{array}{c} \theta_i = 1.173220 \ \mathrm{rad} = 67.220559^\circ \\ R = 0.140000, \quad R_s = 0.256933, \ R_p = 0.023067 \\ T = 0.860000, \quad T_s = 0.743067, \ T_p = 0.976933 \end{array}$

于是透射光 s 分量上的辐射通量为: $\Phi_{e,t,s} = T_s \Phi_{e,i,s} = 0.743067 \times 500 \text{ W} = 371.5335 \text{ W}$

7 光束垂直入射到玻璃-空气界面,玻璃折射率 1.5, 求出 能量反射率和透射率

 $\theta_i = 0$ 时,由菲涅尔定律和能量关系,有:

$$R = \frac{1}{2}(R_s + R_p), \quad T = 1 - R$$
 (23)

$$R = \frac{1}{2} (R_s + R_p), \quad T = 1 - R$$

$$R_s = \left[\frac{\cos \theta_i - \sqrt{n_{ti}^2 - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{n_{ti}^2 - \sin^2 \theta_i}} \right]^2 = \left[\frac{1 - n_{ti}}{1 + n_{ti}} \right]^2$$
(22)

$$R_{p} = \left[\frac{n_{ti}^{2} \cos \theta_{i} - \sqrt{n_{ti}^{2} - \sin^{2} \theta_{i}}}{n_{ti}^{2} \cos \theta_{i} + \sqrt{n_{ti}^{2} - \sin^{2} \theta_{i}}} \right]^{2} = \left[\frac{n_{ti}^{2} - n_{ti}}{n_{ti}^{2} + n_{ti}} \right]^{2}$$
(25)

空气入射玻璃: R = 0.04, T = 0.96玻璃入射空气: R = 0.04, T = 0.96

也即无论从哪边入射,能量反射率和诱射率分别为 0.04 和 0.96。