Laboratory 2: 2024.11.21 - 2024.12.12

14.1 Pre-Lab

(1) Voltage-Voltage Characteristics of Inverting Amplifier

The output voltage V_{out} as a function of input voltage V_{in} is shown in Fig.14.1, where V_{T} is the threshold voltage of teh MOSFET.

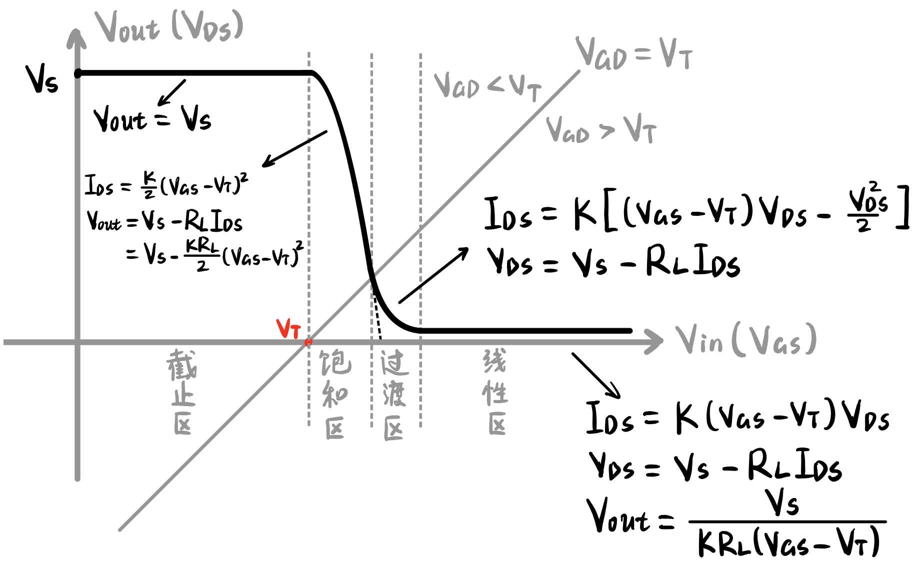


Figure 14.1: Voltage-Voltage Characteristics of Inverting Amplifier

For easy reference, we summarize the formula as:

$$V_{\text{out}} = \begin{cases} V_{\text{S}}, & V_{\text{GS}} \in [0, V_{\text{T}}) & \text{(Cut-Off Region)} \\ V_S - \dfrac{KR_L}{2}\left(V_{\text{GS}} - V_{\text{T}}\right)^2, & V_{\text{GS}} \in [V_{\text{T}}, V_0) & \text{(Saturation Region)} \\ V_{\text{GS}} - V_{\text{T}} + \dfrac{1-\sigma}{KR_L}, & V_{\text{GS}} \in [V_0, V_0 + \Delta V) & \text{(Transition Region)} \\ \dfrac{V_{\text{S}}}{KR_L(V_{\text{GS}} - V_{\text{T}})}, & V_{\text{GS}} \in [V_0 + \Delta V, V_{\max}] & \text{(Linear Region)} \end{cases} \tag{14.1}$$

Where V_0 is the second solution $V_{\text{GS},2}$ of equations:

$$\begin{cases} V_{\text{GS}} = V_S - \frac{KR_L}{2}\left(V_{\text{GS}} - V_{\text{T}}\right)^2 \\ V_{\text{GD}} = V_{\text{T}} \end{cases} \Longrightarrow V_0 = \frac{\sqrt{2KR_LV_S + 1} + KR_LV_T - 1}{KR_L} \tag{14.2}$$

And σ is the discriminant of the quadratic equation:

$$\sigma = \sqrt{K^2R_L^2V_{\text{GS}}^2 + 2KR_L\left(1 - KR_LV_T\right)V_{\text{GS}} + \left[K^2R_L^2V_{\text{T}}^2 - 2KR_L(V_{\text{T}} + V_{\text{S}}) + 1\right]} \tag{14.3}$$

When the input voltage $V_{\text{in}}(V_{\text{GS}}) > V_0$ is large enough, i.e. $V_{\text{in}} \geqslant V_0 + \Delta V$, an approximation can be made:

$$I_{\text{DS}} \approx K\left(V_{\text{GS}} - V_{\text{T}}\right)V_{\text{DS}} \Longrightarrow V_{\text{out}} = \frac{V_S}{KR_L(V_{\text{GS}} - V_{\text{T}})} \tag{14.4}$$

(2) Small Voltage Gain of Inverting Amplifier

We have alread driven the small signal voltage gain during the last homework. Assuming the small AC input voltage is u_{in}, and MOS is biased into saturation region, it follows that:

$$A = \frac{u_{\text{out}}}{u_{\text{in}}} = -g_m R_L = -K\left(V_{\text{GS}} - V_{\text{T}}\right) R_L \tag{14.5}$$

(3) RC Transient Process

By the three-element method, we can obtain:

$$V_{\text{out}} = \frac{R_2 V_{\text{I}}}{R_1 + R_2}\left(1 - e^{-\frac{t}{\tau}}\right), \quad \tau = (R_1 \parallel R_2)\, C = \frac{R_1 R_2}{R_1 + R_2} C \tag{14.6}$$

(4) Transient Time

Given V_{T} in the range $[0, V_{\text{S}}]$ ($V_{\text{S}} = \frac{R_2 V_{\text{I}}}{R_1 + R_2}$), the time where V_{out} reaches V_{T} is:

$$\Delta t = \tau \ln\left(\frac{V_{\text{S}}}{V_{\text{S}} - V_{\text{T}}}\right), \quad \tau = \frac{R_1 R_2}{R_1 + R_2} C, \ \ V_{\text{S}} = \frac{R_2 V_{\text{I}}}{R_1 + R_2} \tag{14.7}$$

14.2 In-Lab

(1) Static Input-Output Relationship of Inverting Amplifier

Measure In-Out Voltage Relationship

Construct the circuit in Fig.14.2, then we can obtain the voltage relationship shown in Fig.14.3.

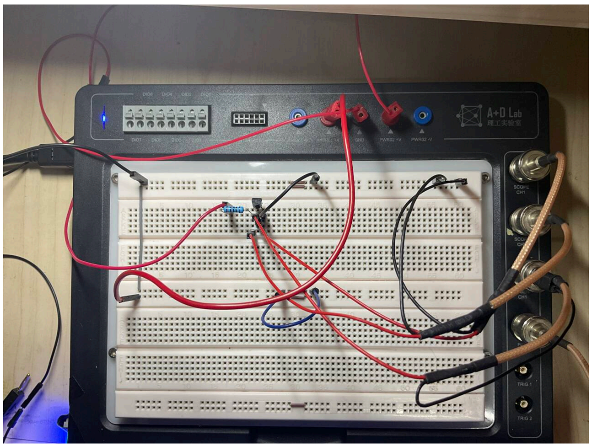


Figure 14.2: Measure In-Out Voltage Relationship

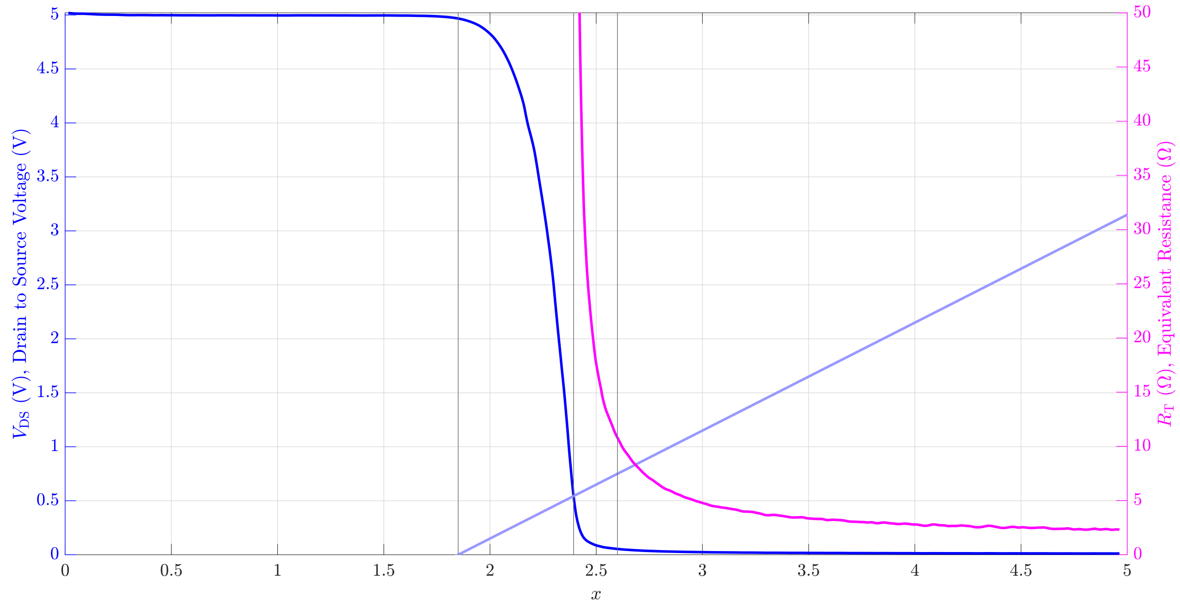


Figure 14.3: Operational Characteristics of Inverting Amplifier

The Threshold of The MOSFET

With $V_S = 5.0185$ V, $R_L = 1$ KΩ (998 Ω) and the data obtained in the last section, we can get the threshold voltage of the MOSFET:

$$V_T = 1.85 \text{ V}, \quad V_0 = 2.3934 \text{ V}, \quad \Delta V = 0.2066 \text{ V} \tag{14.8}$$

Tables of Input-Output Voltage

Table 14.1: Input-Output Voltage Relationship

Output (V)	5	4	3	2	1	0.0116
Input (V)	0.2765	2.1767	2.2686	2.3238	2.3712	4.9599

(2) Small Signal Voltage Gain

Voltage Gain of Inverting Amplifier

With the data measured in section (1), we can derived the voltage gain $A_v = \frac{\mathrm{d}V_{DS}}{\mathrm{d}V_{GS}}$ via matrix difference (see Fig.14.4).

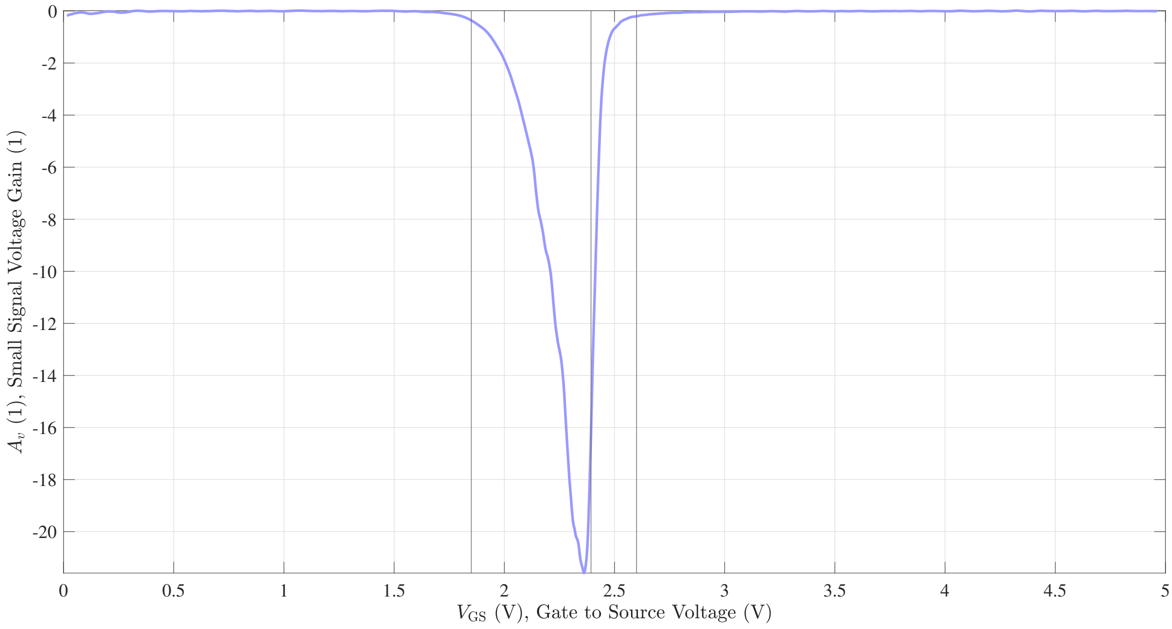


Figure 14.4: Small Signal Voltage Gain of Inverting Amplifier

Since transconductance satisfies $A = -g_m R_L$, we can also get the transconductance by dividing the limiting resistance $R_L = 1$ KΩ (998 Ω).

Construct the circuit to measure the voltage gain where the output voltage is 2 V and the sine wave has 50 mV amplitude (from -50 mV to + 50 mV). The measured result and the data in Fig.14.4 is:

$$(A_v)_{\text{meas}} = \frac{-1.0056\ \text{V}}{50\ \text{mV}} = -20.1120, \quad (A_v)_{\text{fig}} = -20.1513 \tag{14.9}$$

As we can see, almost no deviation.

Clipping Distortion

Set the amplitude of sine wave to 50 mV ($[-50$ mV, $+50$ mV$]$), the lower and upper bias limits are (see Fig.14.5 and Fig.14.6):

$$V_{\text{bias,min}} = 2.00\ \text{V}, \quad V_{\text{bias,max}} = 2.39\ \text{V} \tag{14.10}$$

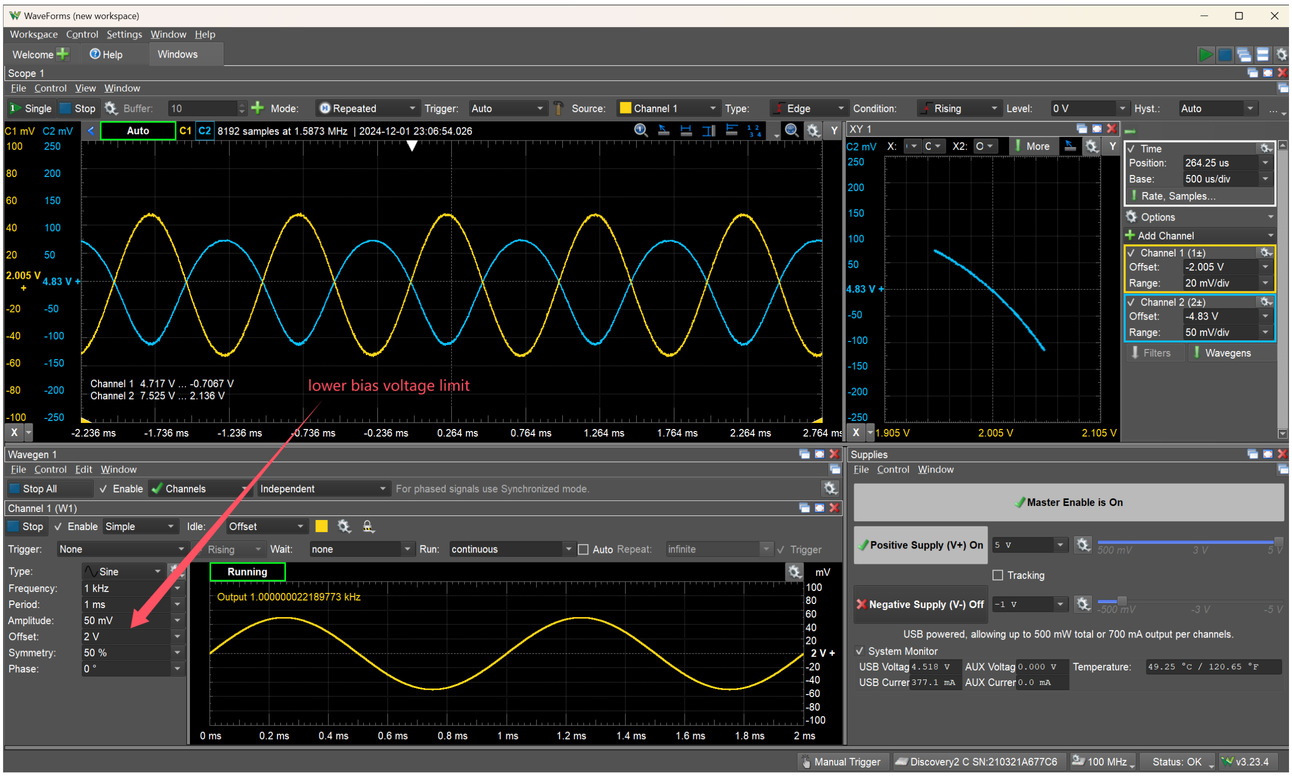


Figure 14.5: The Lower Limit of The Input Bias Voltage

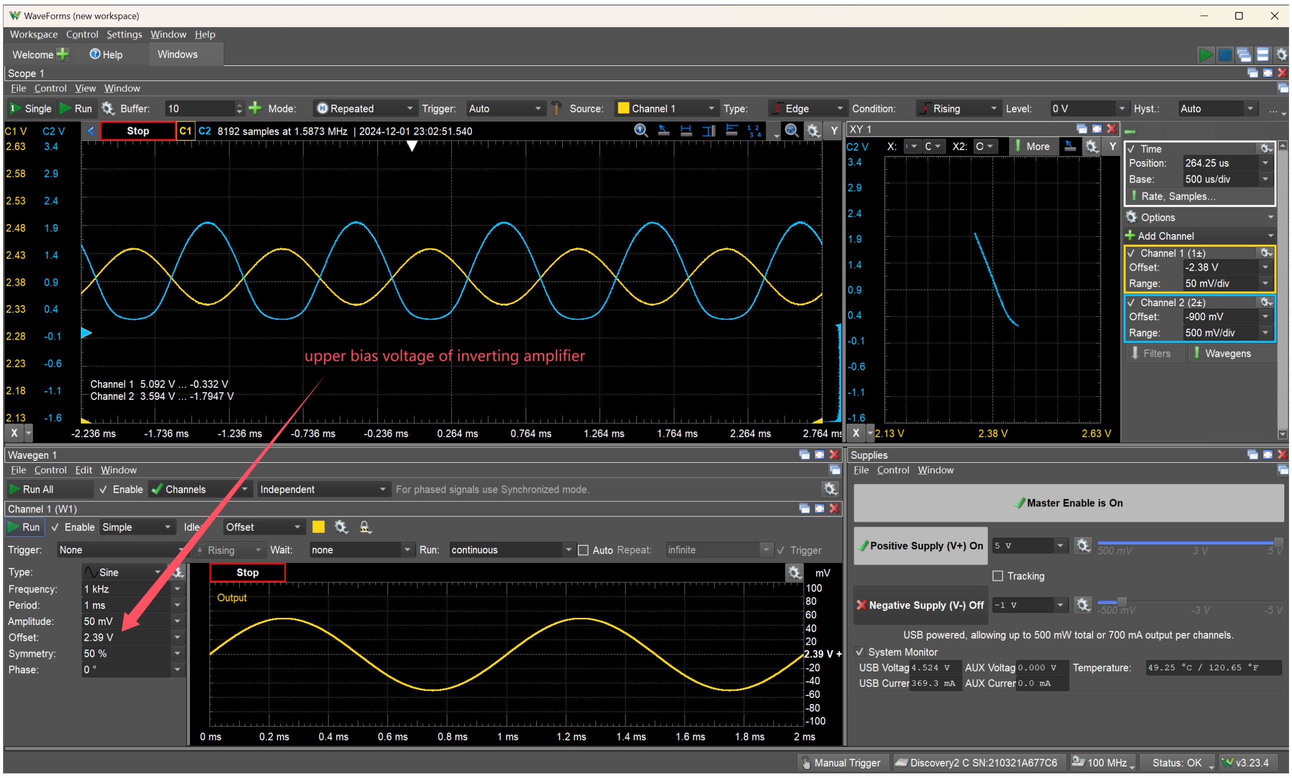


Figure 14.6: The Upper Limit of The Input Bias Voltage

(3) The Delay of Inverting Amplifier as a Digital Inverter

Measure The Gate to Source Capacitance of Inverting Amplifier

Use a 500 KΩ resistor and a 30 KΩ resistor to measure the gate to source capacitance C_{GS} of the MOSFET, respectively. The measured results are:

$$R_L = 500\ \mathrm{K\Omega},\quad \lim_{t\to 0^+}\frac{\mathrm{d}V_{\mathrm{out}}}{\mathrm{d}t} = 57070.3\ \mathrm{V\cdot s^{-1}},\quad V_{\mathrm{steady}} = 3.3198\ \mathrm{V} \Longrightarrow C = \frac{V_0}{R_L k_{0^+}} = 116.3407\ \mu\mathrm{pF} \tag{14.11}$$

$$R_L = 30.0\ \mathrm{K\Omega},\ \lim_{t\to 0^+}\frac{\mathrm{d}V_{\mathrm{out}}}{\mathrm{d}t} = 948889.8\ \mathrm{V\cdot s^{-1}},\quad V_{\mathrm{steady}} = 4.9032\ \mathrm{V} \Longrightarrow C = \frac{V_0}{R_L k_{0^+}} = 172.2434\ \mu\mathrm{pF} \tag{14.12}$$

Figure 14.7: Gate to Source Capacitance of 2N7000, $R_L = 500\ \mathrm{K\Omega}$

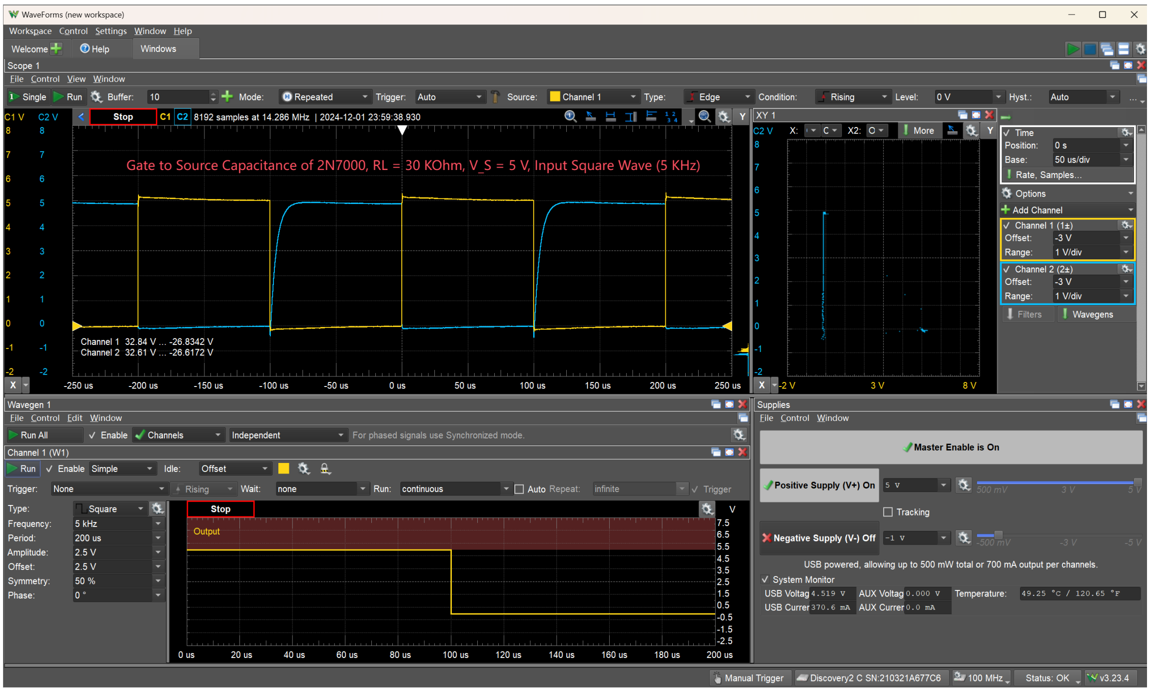
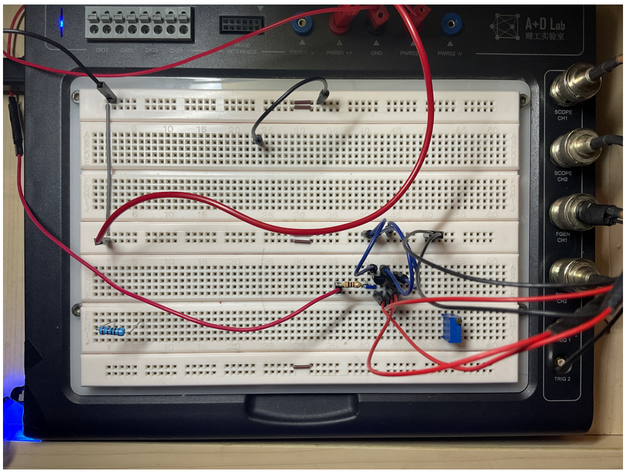
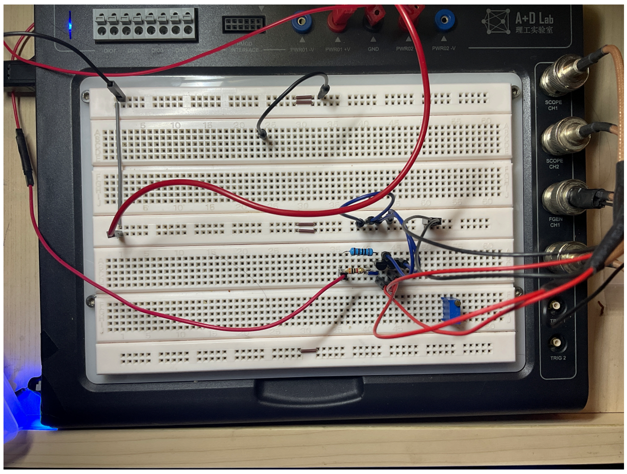


Figure 14.8: Gate to Source Capacitance of 2N7000, R_L = 30 KΩ

It is puzzling that $C =$ with $R_L = 500$ KΩ. Actually, with the resistance $R_L = 500$ KΩ, the steady voltage V_0 is limited to 3.3198 V (see Fig.14.7), which is not the expected value. Why is it like that? Because the oscilloscope input resistance is not large enough. We keep this to the Post-Lab,



(a) The Gate to Source Capacitance of Inverting Amplifier



(b) The Delay of Inverting Amplifier as a Digital Inverter

Figure 14.9: In-Lab 3.1 and In-Lab 3.2

Measure The Delay of Inverting Amplifier

With $R_{L,1} = 30$ KΩ and $R_{L,2} = 1$ KΩ, construct the circuit in Fig.14.9 (b), obtain the delay time of the inverting amplifier as a digital inverter (see Fig.14.10 and Fig.14.11).

$$\text{start to fall: } \Delta t_1 = 1.504\ \mu\text{s}, \quad \text{reach low: } \Delta t_2 = 3.799\ \mu\text{s} \tag{14.13}$$

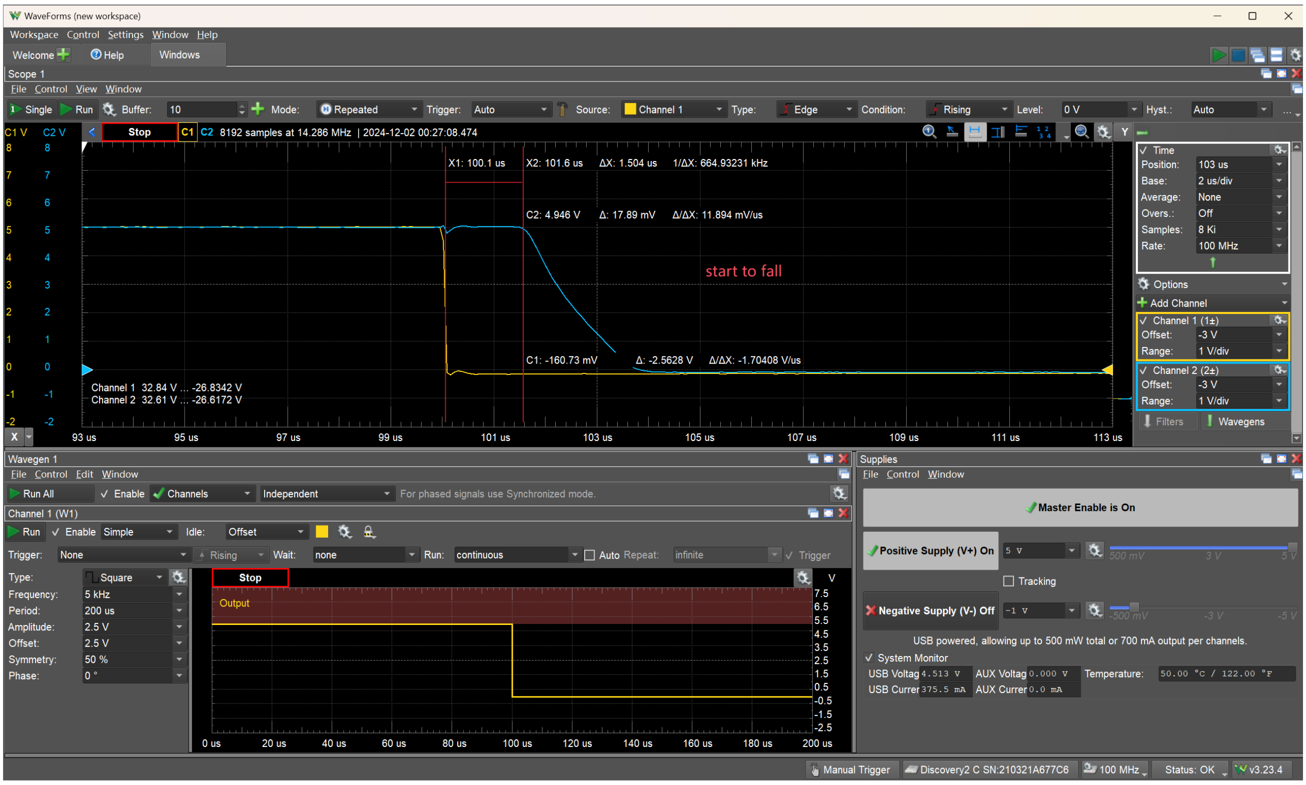


Figure 14.10: The Output Voltage Starts to Fall

Figure 14.11: The Output Voltage Reaches to Low

It is interesting that resonance phenomena occurs, see Fig.14.12 and Fig.14.13.

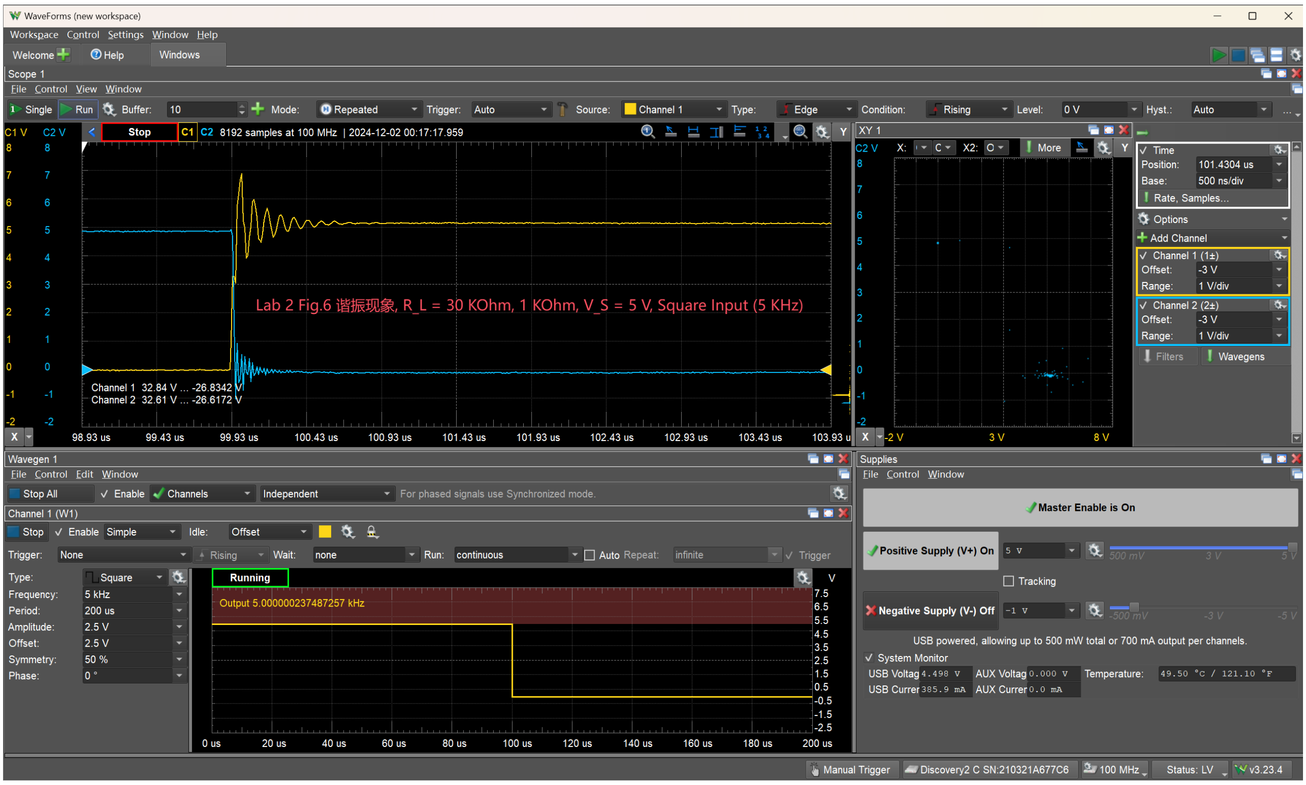


Figure 14.12: Input Voltage (Yellow) and Output Voltage (Blue) of The First MOS

Figure 14.13: Input Voltage (Yellow) and Output Voltage (Blue) of The Second MOS

14.3 Post-Lab

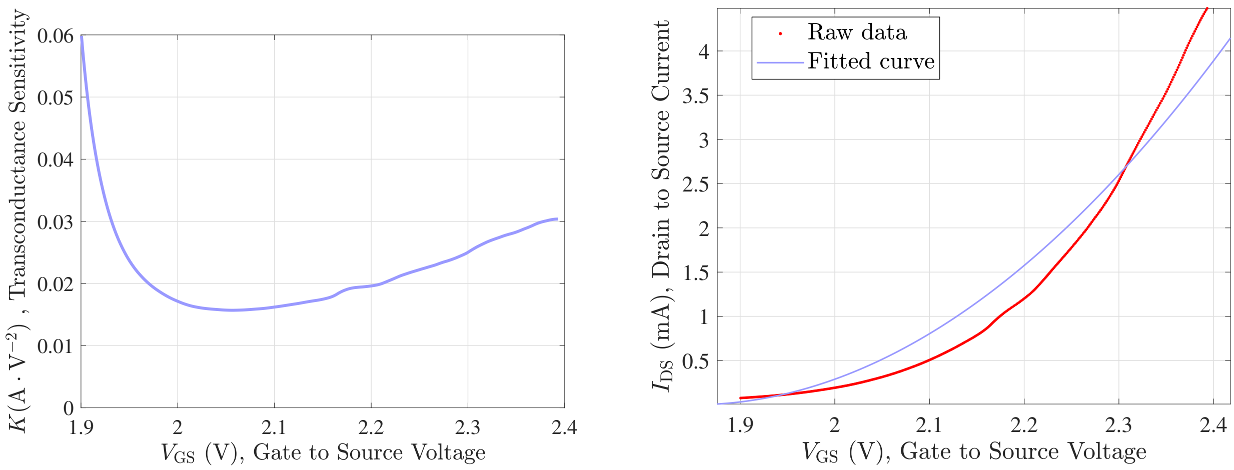
(1) Voltage-Voltage Characteristics Comparison

Transconductance Sensitivity K

Use the voltage-voltage characteristics data in section (1) to calculate the transconductance sensitivity K of the MOSFET:

$$\begin{cases} I_{\mathrm{DS}} = \frac{K}{2}\left(V_{\mathrm{GS}} - V_{\mathrm{T}}\right)^2 \\ V_{\mathrm{DS}} = V_{\mathrm{S}} - I_{\mathrm{DS}} R_L \end{cases} \Longrightarrow K = \frac{2\left(V_{\mathrm{S}} - V_{\mathrm{DS}}\right)}{R_L\left(V_{\mathrm{GS}} - V_{\mathrm{T}}\right)^2} \tag{14.14}$$

Where $V_{\mathrm{T}} = 1.85$ V, $R_L = 1\ \mathrm{K\Omega}\ (998\ \Omega)$ and $V_{\mathrm{S}} = 5.0185$ V. Plot the curve $K = K(V_{\mathrm{GS}})$, as shown in Fig.14.14 (a):



(a) Transconductance Sensitivity K as a Function of V_{GS} **(b) Drain to Source Current as a Function of V_{GS}**

Figure 14.14: Transconductance Sensitivity

As we can see, K is not a ideal constant value. So we try fitting I_{DS} as a function of V_{DS}, and the result is shown in Fig.14.14 (b), $R^2 = 0.9511$.

$$I_{\mathrm{DS}} = \frac{K}{2}\left(V_{\mathrm{GS}} - V_{\mathrm{T}}\right)^2, \quad \mathbf{K = 0.02572\ A \cdot V^{-2}}, \quad V_{\mathrm{T}} = 1.85\ \mathrm{V} \tag{14.15}$$

It is funny that the fitting result is exllecent if we use $I_{\mathrm{DS}} = \frac{K}{2}\left(V_{\mathrm{GS}} - V_{\mathrm{T}}\right)^3$, which has a high $R^2 = 0.9977$.

Comparison of Operational Characteristics

With the four parameters $V_T = 1.85$ V, $K = 0.02572\ \mathrm{A}\cdot\mathrm{V}^{-2}$, $V_{\mathrm{S}} = 5$ V and $R_L = 1\ \mathrm{K\Omega}\ (998\ \Omega)$, we can compute and plot the theoretical operational characteristics of the inverting amplifier, as shown in Fig.14.15. Below are the other parameters for our theoretical model:

$$V_0 = \frac{\sqrt{2KR_LV_S + 1} + KR_LV_T - 1}{KR_L} = 2.4376\ \mathrm{V}, \quad \Delta V = 1.5\left(V_0 - V_{\mathrm{T}}\right) = 0.8814\mathrm{V} \tag{14.16}$$

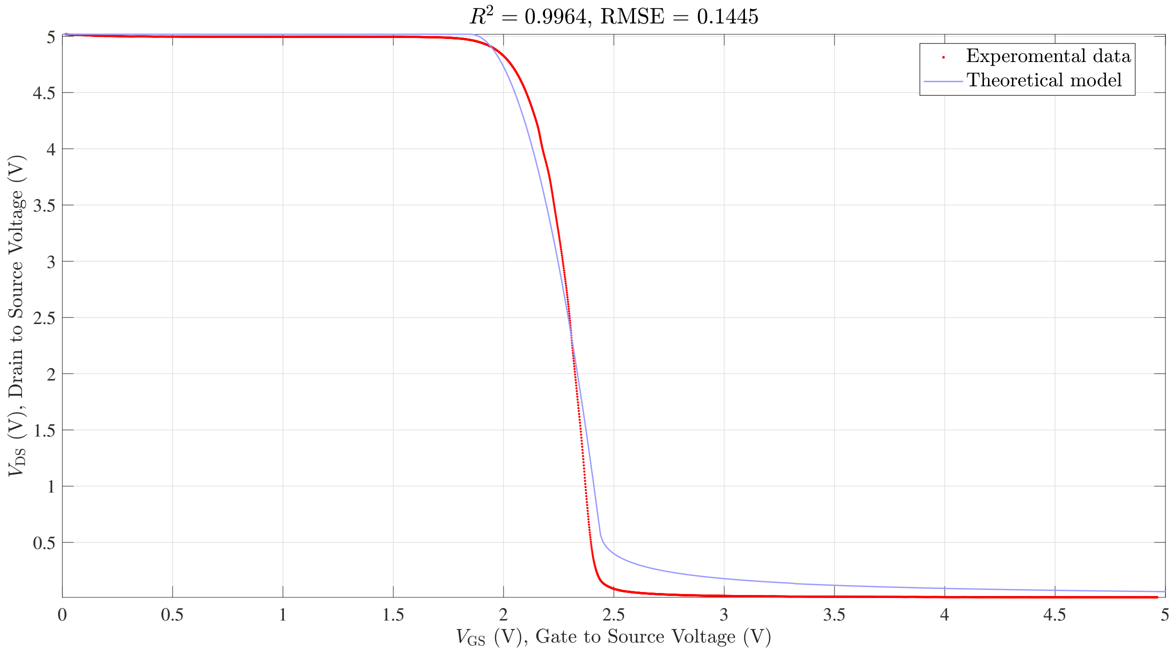


Figure 14.15: Operational Characteristics Comparison

(2) Small Signal Voltage Gain

The voltage gain A_v measured during In-Lab 2-2 (section (2)), by the operational characteristics obtained in In-Lab 2-1 (section (1)) and the theoretical module from Pre-Lab are respectively (at Output $V_{\text{GS}} = 2$ V):

Table 14.2: Voltage Gain at Output 2 V by Different Methods

By Experimental Measurement	By Operational Characteristics	By Theoretical Op Characteristics
-20.1120	-20.1513	-12.1670

(3) Capacitance and Delay Analysis

Total Input Capacitance

In Fig.14.7, we have seen that the output voltage drops to 3.3198 V, when $R_L = 500$ KΩ and $V_{\text{S}} = 5$ V. It follows that:

$$V_{\text{out}} = \frac{R_{\text{osci}}}{R_{\text{osci}} + R_L} V_{\text{S}} \Longrightarrow R_{\text{osci}} = \frac{R_L}{\frac{V_{\text{S}}}{V_{\text{out}}} - 1} = 987.9 \text{ K}\Omega \approx 1\text{M}\Omega \tag{14.17}$$

Assuming $V_{\text{source}} = 5$ V, we can obtain the total input capacitance, including GS capacitance C_{GS} and oscilloscope input capacitance C_{osci}:

$$\begin{cases} k_{0+} = \frac{V_{\text{steady}}}{\tau} \\ V_{\text{steady}} = \frac{R_{\text{osci}}}{R_{\text{osci}}+R_L} \cdot V_{\text{source}} \\ \tau = (R_{\text{osci}} \parallel R_L)(C_{\text{GS}} + C_{\text{osci}}) \end{cases} \Longrightarrow C_{\text{GS}} + C_{\text{osci}} = \frac{V_{\text{S}}}{k_{0+} R_L} = 175.2225\ \mu\text{pF} \tag{14.18}$$

If oscilloscope input capacitance C_{osci} is about 15 pF, then we have $C_{\text{GS}} \approx 160\ \mu$pF.

The Delay Used as a Digital Inverter

Let $C_{\mathrm{GS}} \approx 160\ \mu$pF, $V_{\mathrm{T}} = 1.85$ V, $V_0 + \Delta V = 3.22$ V $V_{\mathrm{I}} = 5$ V, yielding:

$$\begin{cases} \Delta t = \tau\ \ln\left(\frac{V_{\mathrm{S}}}{V_{\mathrm{S}}} - V_{\mathrm{T}}\right) \\ \tau = \frac{R_1 R_2}{R_1+R_2} C \\ V_{\mathrm{S}} = \frac{R_2}{R_1+R_2} \cdot V_{\mathrm{I}} \end{cases} \Longrightarrow \text{start to fall: } \Delta t_1 = 0.1287\mu\text{s}, \quad \text{reach low: } \Delta t_2 = 1.3040\mu\text{s} \tag{14.19}$$