

Chapter 1 Capacitance

1.1 Capacitance Multiplier

1.1.1 Basic Circuits and Principles

This part refers to references [1], [2] and [3]. Below are two basic concepts for capacitance multiplication:

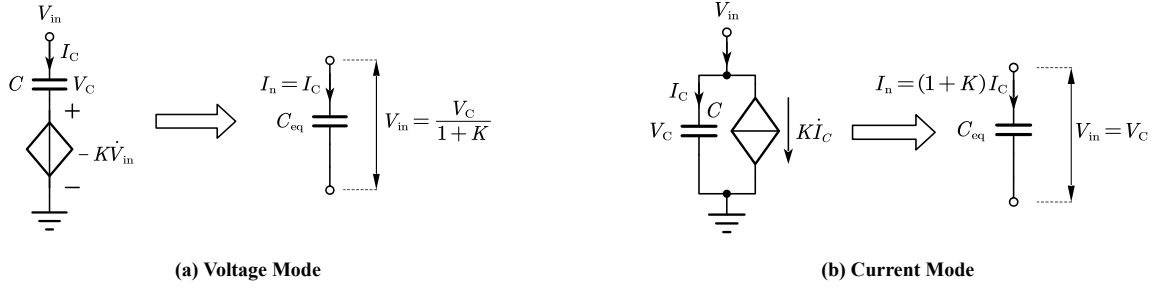


Figure 1.1: Basic Two Concepts for Capacitance Multiplier

Thus, we obtain the equivalent capacitance as:

$$\text{voltage mode: } C_{eq} = \frac{I_n}{SV_n} = \frac{I_C}{S \frac{V_C}{1+K}} = (1+K)C, \quad K > 0 \quad (1.1)$$

$$\text{current mode: } C_{eq} = \frac{I_n}{SV_n} = \frac{(1+K)I_C}{SV_C} = (1+K)C, \quad K > 0 \quad (1.2)$$

A simple implementation of cap multiplier, depicted in Fig. 1.2 (a), combining a unit-gain buffer (voltage follower) and an inverting amplifier, uses a voltage mode. yielding the equivalent capacitance:

$$A = -\frac{R_2}{R_1} = -K \implies C_{eq} = \left(1 + \frac{R_2}{R_1}\right)C \quad (1.3)$$

where $A = -\frac{R_2}{R_1}$ is the closed-loop gain of the inverting amplifier. Since inverting amplifier has a low input impedance, the unit-gain buffer is a necessary. To change it into a two-terminal element, just replace GND with the negative terminal of the input voltage, e.g. $V_{in,-}$, as shown in Fig. 1.2 (b).

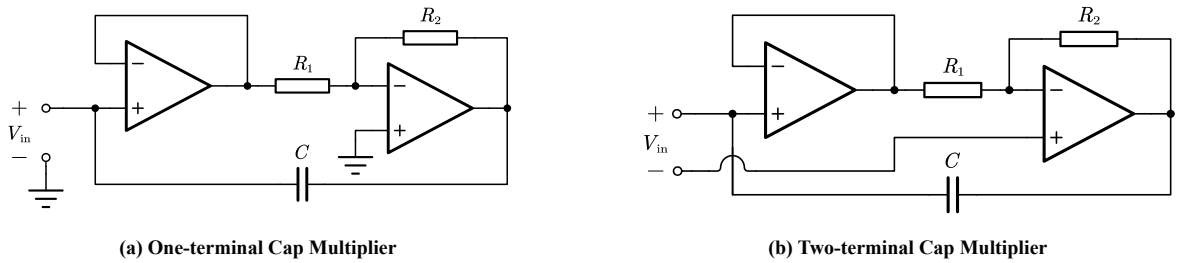


Figure 1.2: A Simple Implementation of Capacitance Multiplier

Refer to reference [1], [2] and [3] for more advanced circuits.

1.1.2 Multisim Simulation

Considering Figure 1.2 (b), set the parameters as Table 1.1. Then we connect it to the RC series circuit to perform a AC sweep to test the capacitance value. The Simulation Circuit is shown in Figure 1.3.

Table 1.1: Simulation Parameters of Capacitance Multiplier

C	R_1	R_2	Operation Amplifier
10 nF	1 K Ω	11 K Ω	LM258P

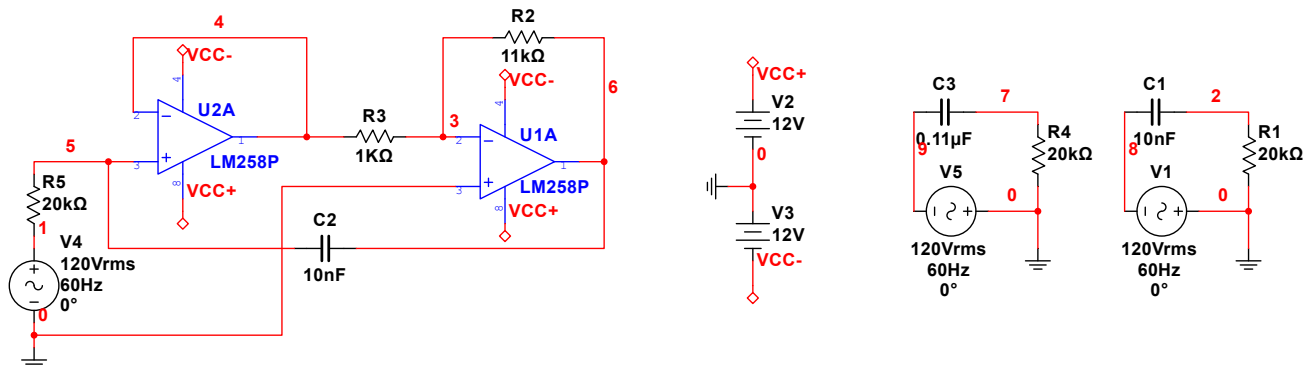


Figure 1.3: Simulation Circuit of Cap Multiplier

Export the simulation data and plot the frequency response (bode plot) of the series RC circuit, as shown in Figure 1.4. The theoretical value of the capacitance of the cap multiplier is 110 nF, and the simulation result confirmed this point.

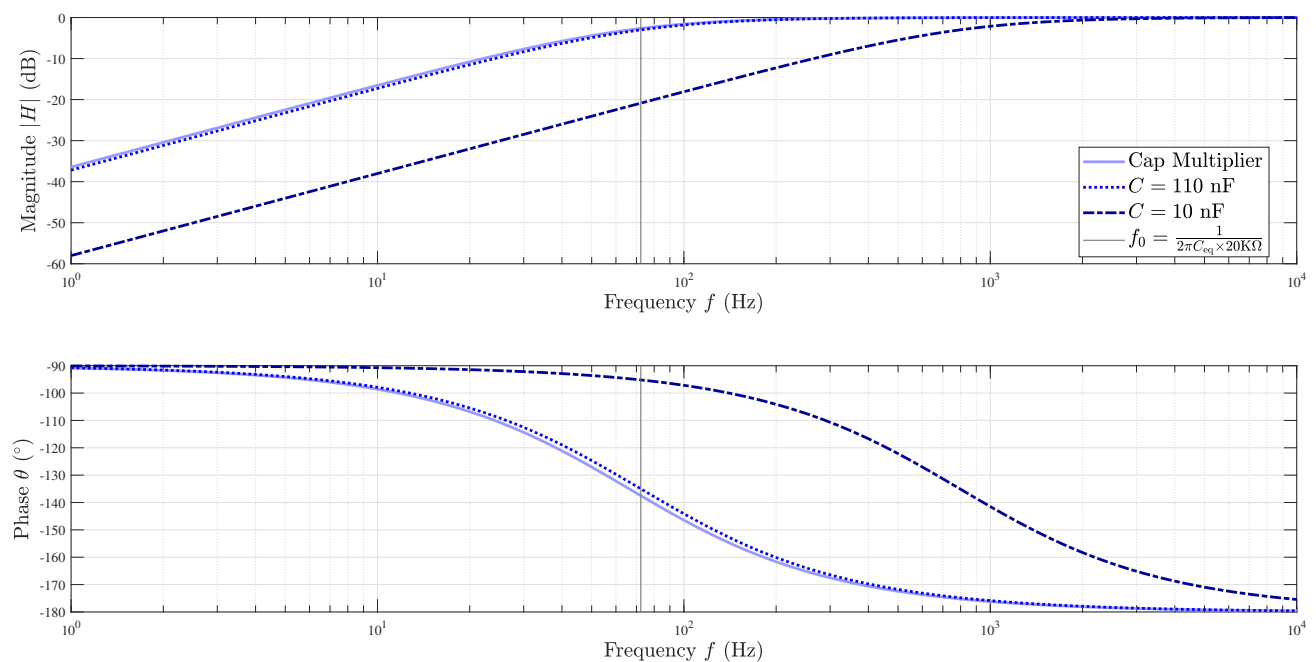


Figure 1.4: AC Sweep of the Cap Multiplier

Chapter 2 Osillators

2.1 The Wien Bridge Oscillator

You must have seen that a number of resistors and capacitors can be connected together with an inverting amplifier to produce an oscillating circuit. One of the simplest sine wave oscillators which uses a RC network in place of the conventional LC tuned tank circuit to produce a sinusoidal output waveform, is called the **Wien Bridge Oscillator**.

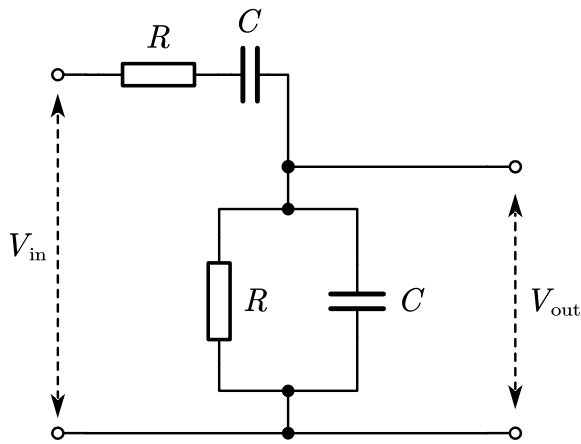
The Wien Bridge Oscillator is based on a noinverting amplifier, using a series RC circuit connected with a parallel RC of the same component values as a feedback circuit.

2.1.1 Basic Circuit and Principles

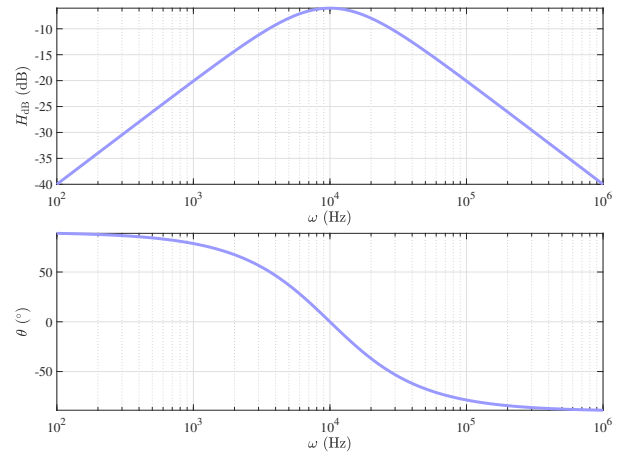
Consider the RC circuit in Figure 2.1 (a), the voltage gain H of the series RC circuit is:

$$H(j\omega) = \frac{R \parallel (\frac{1}{j\omega C})}{R + \frac{1}{j\omega C} + R \parallel (\frac{1}{j\omega C})} = \frac{1}{1 + \frac{(R^2 - \frac{1}{\omega^2 C^2}) + \frac{2R}{j\omega C}}{\frac{R}{j\omega C}}}, \quad H|_{\omega=\frac{1}{RC}} = \frac{1}{1 + \frac{2R}{j\omega C}} = \frac{1}{3} \quad (2.1)$$

Defined to obtain a 0° phase shift, the resonant frequency f_0 is the key to Wien bridge oscillator. And at the point we have $H = \frac{1}{3}$. Let's set $R = 10 \text{ K}\Omega$, $C = 10 \text{ nF}$ and sketch the bode plot of this RC circuit in Figure 2.1 (b).

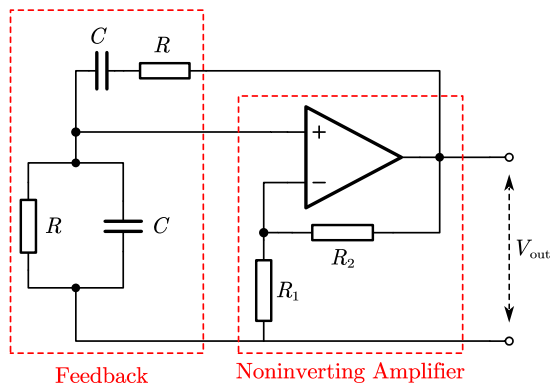


(a) RC Feedback Circuit

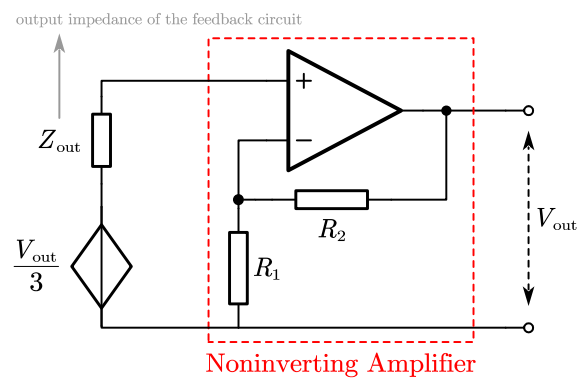


(b) Bode Plot of the RC Circuit

Figure 2.1: Wien Bridge Osillator's Feedback Circuit



(a) Wien Bridge Osillator



(b) Equivalent Circuit of Wien Bridge Osillator

Figure 2.2: Wien Bridge Osillator and Its Equivalent Circuit

Since a noninverting amplifier has extreme high input impedance and low output impedance, the coupling effect of the two circuits is negligible. In other words, the output impedance of noninverting amplifier (combing with the input impedance of feedback circuit), and the output impedance of feedback circuit (combing with input impedance of noninverting amplifier) can be ignored. Thus, the Wien bridge oscillator, depicted in Figure 2.2 (a), has the equivalent circuit shown in Figure 2.2 (b).

The frequency of oscillations for a Wien Bridge Oscillator is given as:

$$\omega_0 = \frac{1}{RC}, \quad f_0 = \frac{1}{2\pi RC} \quad (2.2)$$

As the voltage gain of noninverting amplifier is:

$$A_v = 1 + \frac{R_2}{R_1} \quad (2.3)$$

yielding the **start-oscillation condition**:

$$A_v > 3 \iff R_2 > 2R_1 \quad (2.4)$$

Assuming R_2 is slightly greater than $2R_1$, and there is a noise signal consists of a series of frequency, including $f_0 = \frac{1}{2\pi RC}$. Then at the selected resonant frequency f_0 , $\frac{1}{3}A_v > 1$, so the positive feedback will cancel out the negative feedback signal, causing the circuit to oscillate, until it reaches a voltage saturation (dependent on power supply). However, at the other frequency, $\frac{1}{3}A_v < 1$ so the negative feedback will cancel out the positive, resulting other frequency signal fading away.

The closer the ratio $\frac{R_2}{R_1}$ is to 2^+ , the better the waveform, but the longer the start-up time. Define $a = \frac{1}{3} \left(1 + \frac{R_2}{R_1}\right)$ as the periodic gain, a not-bad approximation for the start-up time is:

$$t_{\text{start}} = \frac{1}{f} \log_{a^3} \left(\frac{V_{\text{limit}}}{V_{\text{noise}}} \right) - 0.02 \quad (2.5)$$

where the unit of t is seconds, V_{limit} is the limit amplitude of output voltage, V_{noise} is the amplitude of noise.

By the way, if R_2 exceeds $2R_1$ too much, for example $R_2 = 3R_1$, the output waveform will be seriously distorted. Also, due to the slew rate limitations of operational amplifiers, frequencies above 1 MHz are unachievable without the use of special high frequency op-amps.

2.1.2 Multisim Simulation of the Basic Circuit

Set the parameters in Figure 2.2 (a) as below, and run the simulation.

Table 2.1: Simulation Parameters of Wien Bridge Oscillator

R	C	R_1	R_2	Operation Amplifier	VCC
10 K Ω	10 nF	10 K Ω	20.1 K Ω	LM258P	± 12 V

The start-up time is about 570 ms, shown in Figure 2.3.

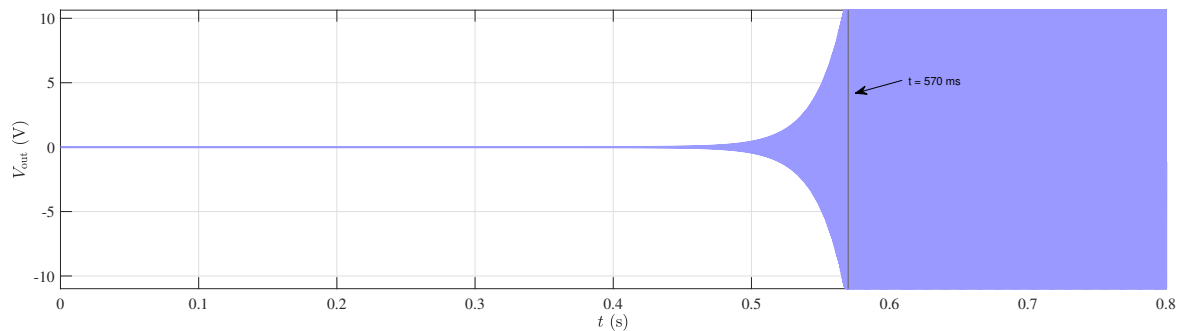


Figure 2.3: Start-up Time of Wien Bridge Oscillator

Export the simulation data, and perform a spectrum and distortion analysis in Matlab. Then we obtain the waveform and spectrum shown in Figure 2.4.

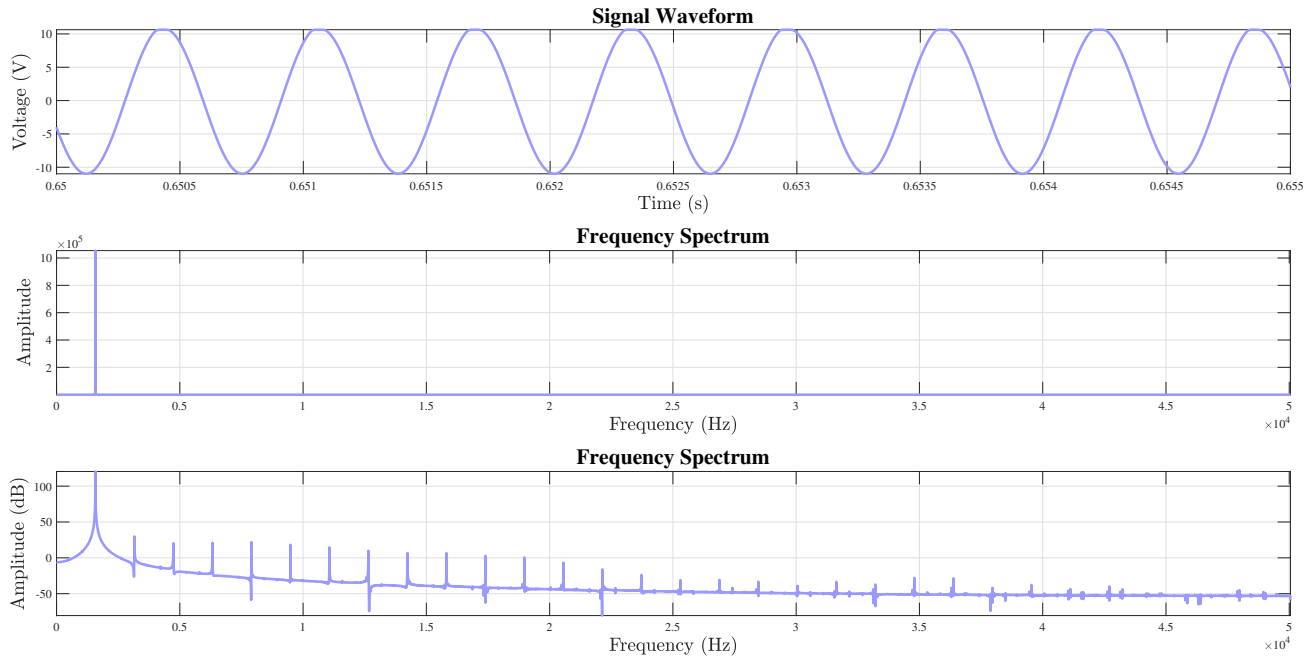


Figure 2.4: Spectrum Analysis of the Simulation Circuit

As we can see, the main output waveform is a sine wave at the resonant frequency f_0 , the simulated oscillation frequency is:

$$\begin{aligned} f_{\text{simu}} &= 1.5758 \text{ KHz} \\ f_{\text{theo}} &= 1.5915 \text{ KHz} \\ \eta &= \frac{f_{\text{simu}} - f_{\text{theo}}}{f_{\text{theo}}} = -0.98 \% \end{aligned} \quad (2.6)$$

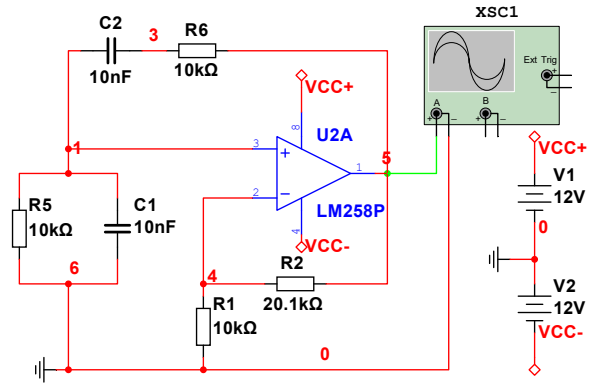


Figure 2.5: Simulation Circuit of Wien Bridge Oscillator

2.1.3 Decrease the Start-up Time

We have noted that the output waveform is distorted when R_2 exceeds $2R_1$ too much, but the start-up time is too long when R_2 is too closed to $2R_1$. Therefore, we need to optimize the circuit to achieve a better waveform and shorter start-up time, examplified in Figure 2.6.

In Figure 2.6 (a), we added a resistance R_3 and two diodes D_1 and D_2 . When the output voltage amplitude is less than the threshold voltage of diodes V_D , the diodes are off, and the circuit is the same as the basic circuit. When the output is greater than V_D , the diodes are on, and the resistance of R_3 is added to the circuit (parallel with R_2), which reduces the gain of amplifier.

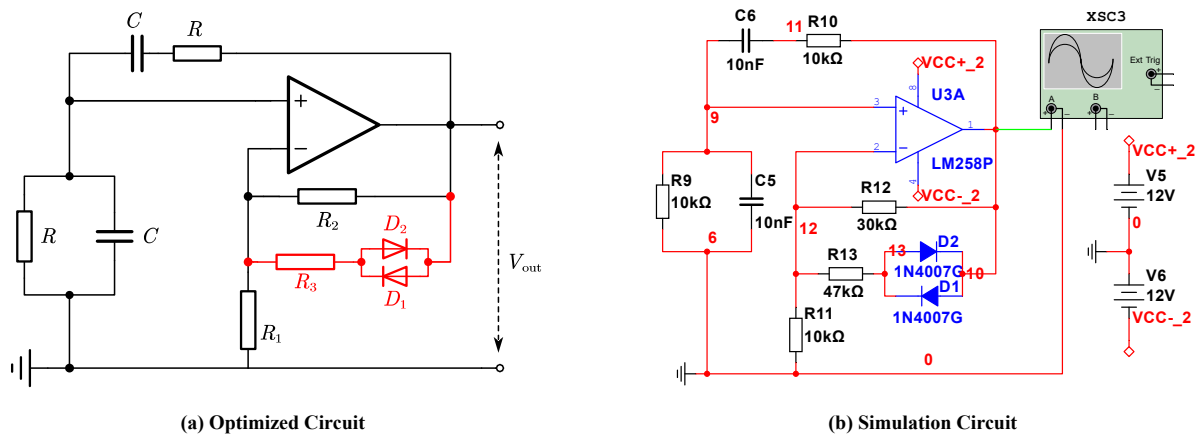


Figure 2.6: Optimize the Start-up Time of Wien Bridge Osillator

Simulation circuit is shown in Figure 2.6 (b), the start-up time is reduced to about 10 ms (see Figure 2.7), and the output waveform is shown in Figure 2.8.

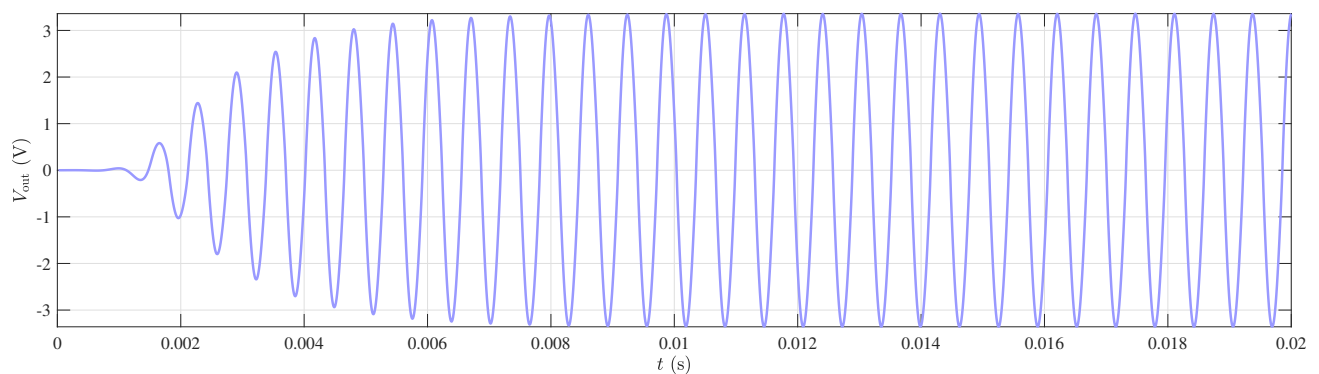


Figure 2.7: Optimized Start-up Time

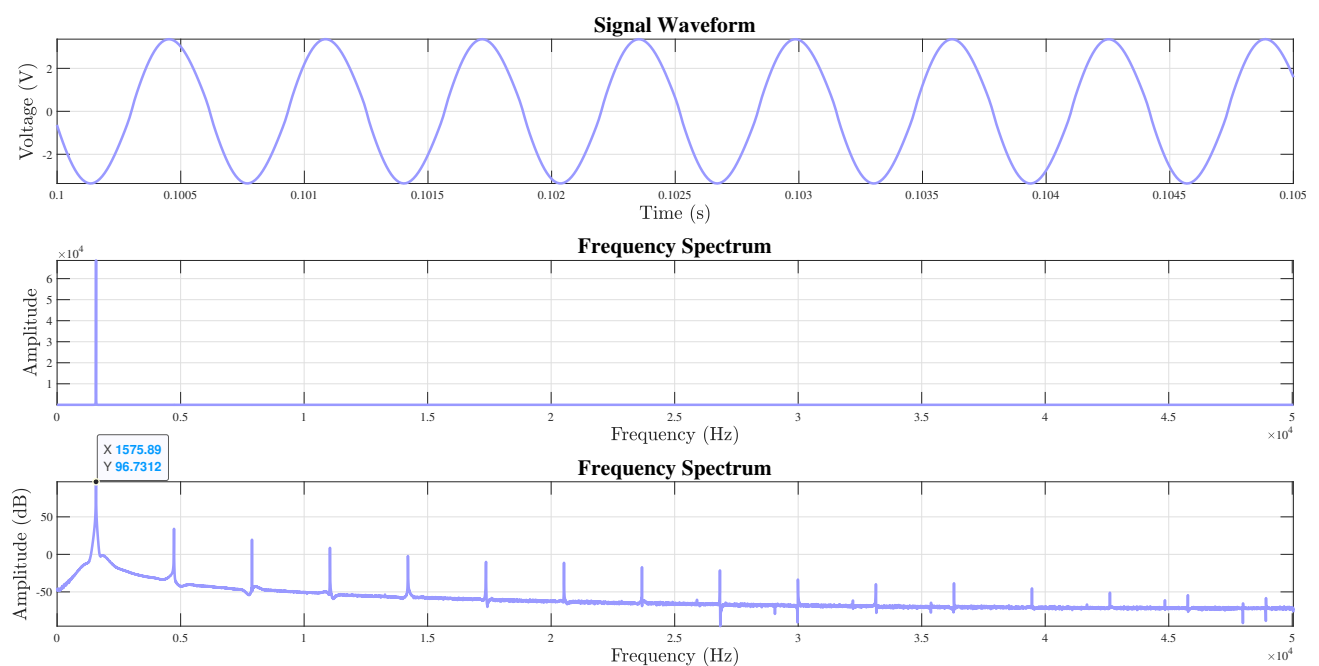


Figure 2.8: Spectrum Analysis of Optimized Circuit

Although output frequency still focus on f_0 and the distortion completely disappears, we have to note that the

amplitude of the output waveform is significantly reduced. For more alternative methods to optimize the start-up time, see https://blog.csdn.net/qj_29356039/article/details/132611987.

2.1.4 Generate a Square Wave

A R_2 greater than $2R_1$ will result the output waveform to be clipped at the output voltage limitations. In other words, if we let $R_2 \gg 2R_1$, the waveform becomes a square wave. To prove our surmise, reset $R_1 = 1\text{ K}\Omega$, $R_2 = 30\text{ K}\Omega$ in Table 2.1, without changing the other parameters. We obtain the output waveform shown in Figure 2.9.

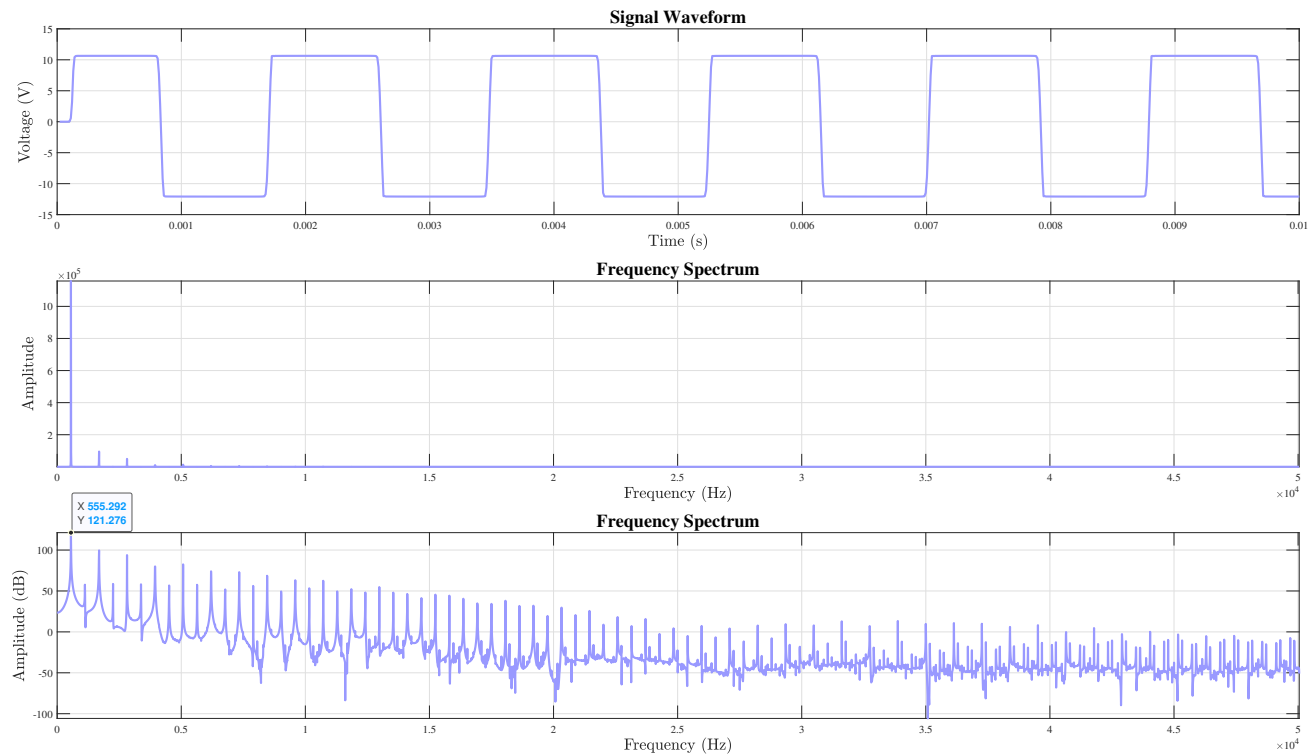


Figure 2.9: Square Wave Output of Wien Bridge Oscillator

Since the frequency of the square wave is difficult to calculate and control, the Wien Bridge Oscillator is not suitable for generating square waves in practical application.

2.1.5 Output With a DC Offset

see https://blog.csdn.net/qj_29356039/article/details/132611987

Reference

- [1] G.A. Rincon-Mora. Active capacitor multiplier in Miller-compensated circuits. *IEEE Journal of Solid-State Circuits*, 35(1):26–32, 2000. <https://ieeexplore.ieee.org/document/818917>.
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- [3] Ivan Padilla-Cantoya, Luis Rizo-Dominguez, and Jesus E. Molinar-Solis. Capacitance multiplier with large multiplication factor, high accuracy, and low power and silicon area for floating applications. *IEICE Electronics Express*, 15(3):20171191–20171191, 2018. <https://doi.org/10.1587/elex.15.20171191>.
- [4] Behzad Razavi. *Fundamentals of Microelectronics*. University of California Press, 2nd edition, 2014.
- [5] Behzad Razavi. *Design of Analog CMOS Integrated Circuits*. McGraw-Hill Education, 2nd edition, 2017.