P.190 例6.4

$$\hat{\theta}_2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$$

$\Rightarrow$ 母体變異數 σ^2 的不偏估計量

$E(x_i)=\mu$

$V(x_i)=\sigma^2=E(x_i^2)-\mu^2$

$E(\bar{x})=\mu$

$V(\bar{x})=\frac{\sigma^2}{n}=E(\bar{x})-\mu^2$

$$E(\hat{\theta}_1)=E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}\right)=\frac{1}{n}E\left(\sum_{i=1}^{n}x_i^2-n\bar{x}^2\right)$$

$$=\frac{1}{n}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\frac{n-1}{n}\sigma^2\neq\sigma^2$$

$$E(\hat{\theta}_2)=E\left(\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}\right)=\frac{1}{n-1}E\left(\sum_{i=1}^{n}x_i^2-n\bar{x}^2\right)$$

$$=\frac{1}{n-1}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)=\sigma^2$$