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T4 ________________	**B**	F4 ________________

2020
MCM/ICM
Summary Sheet

Build A Long Lasting Sandcastle According to Scientific Model

Summary

Making a nice sandcastle on the beach is exciting for people of all ages. It's not hard to make a good sandcastle while the difficulty is to make it last longer. The foundation of the sandcastle plays an important role in it.

First, we need to study the basic shape of the foundation. According to the drag equation in fluid mechanics, we know that the drag coefficient plays a decisive role. Then, we select some common foundation shapes and compare their drag coefficients one by one to get the two-dimensional structure of the foundation. Then, through the mechanical knowledge, we analyze the stress of the column and the platform generated by the two-dimensional structure, and finally establish the **three-dimensional model of the foundation**. **Secondly**, we analyze the optimum mixing ratio of sand water in sand castle. By idealizing sand particles, we finally establish a **liquid bridge model** to analyze the cohesion between sand particles. According to cubic crystal system, the final sand-water ratio is obtained.**Thirdly**, on the basis of the above results, we consider the influence of water content, slope Angle of 3d platform and rainwater erosion.

In addition, in the sensitivity analysis, we pay attention to a number of factors, such as the proportion of two-dimensional structures and the size of raindrops, in order to analyze their influence on the stability of the model. **Finally**, as far as possible, we write an interesting article to the magazine *Fun in the sun* in a language which is easy to understand, so that more people know how to keep their sand castles longer.

key words : Drag coefficient; Liquid bridge; Cubic crystal system;

Contents

1 Introduction

Sandcastles are the most beautiful landscapes on the beach. They are interesting, but fragile, and they can be destroyed by the erosion of the waves. How to build a solid sand castle requires thinking from multiple factors such as the shape of the foundation and the sand material. In addition to the waves, the erosion of rain also affects sandcastles. It requires further adjustment of the overall structure of the foundation. In addition, there are also many other methods to extend the life of the sandcastle.

Our work is to solve these problems using mathematical methods, including but not limited to micro particle models, macro abstract physical models, etc. Finally came to a complete conclusion.

2 Assumptions and Notations

2.1 Assumptions

Due to the lack of necessary data, we make the following assumptions to help us perform modeling.

1. Fluid density does not change
2. Fluid velocity is constant
3. Uniform force on the force surface of the 3d model
4. The sand particles are thought to be spherical with the same radius
5. Raindrops fall on the model in a direction perpendicular to the ground
6. The probability of raindrops falling at every point on the model is the same

7. Raindrops have the same speed and mass
8. The quantity of the rainfall is contant

2.2 Notations

Here are all the notations and their meanings in this paper.

Symbol	Meaning
F_α	Impact force of water
ρ	Fluid density(constant is ρ = 1g / cm³ (density of water))
u	Fluid velocity
C_d	Drag coefficient
A	Projected area of the force surface of the 3D model
C_e	The perimeter of geometric shape e
L_s	The length of the line segment s
$W^{'}$	Effective underwater gravity
u_{rx}	The relative velocity of flow and sediment in the x direction
$u_x^{'}$	The velocity of water ripple in the x direction
$u_y^{'}$	The velocity of water ripple in the y direction
$\alpha_1, \alpha_2, \alpha_3$	The shape factor of sediment particles(particles are simplified to a sphere, the factor is $\pi/4$ here)
γ_s	The volumetric weight of sediment
γ	The volumetric weight of water
ρ	The density of water
d	The grain size of sediment
θ	The angle of slope
f	Coefficient(s) of friction
C_x	Horizontal drag coefficient
$C_x^{'}$	Water ripple's drag coefficient
C_y	Vertical uplift force coefficient
$C_y^{'}$	Water ripple's uplift force coefficient
Δl	The thickness of the surface sand removed by a single raindrop
S_{Rain}	Areas where raindrops do not coincide with each other
S_α	Areas of α
V_β	Volume of β

3 The best shape of sandcastles

3.1 Model Construction

As we all know, without considering the material of the model, the impact resistance of the model has a greater correlation with the shape of its force surface. However, considering the complexity of the 3d model, we assume that the Fluid velocity u and the Fluid density ρ is constant to simplify the construction of the model.

According to Batchelor, G.K.'s book "An Introduction to Fluid Dynamics"[1], we can get the following formula Eq. (1):

$$F_\alpha = \frac{1}{2}\rho u^2 C_d A \tag{1}$$

In this way, we will find that under our assumption(ρ, u are constants), F_α is only positively correlated with $C_d A$, so we can get Eq. (2).

$$F_\alpha \propto C_d A \tag{2}$$

The full name of C_d is the Drag coefficient. Without considering the material and A, C_d is only related to the shape of the model. In Frank M. White's book "Fluid Mechanics seventh edition"[2], we can learn the C_d of most common models, so we can narrow the range of 3D models with great uncertainty to these common models to choose(shown in Fig. 1).

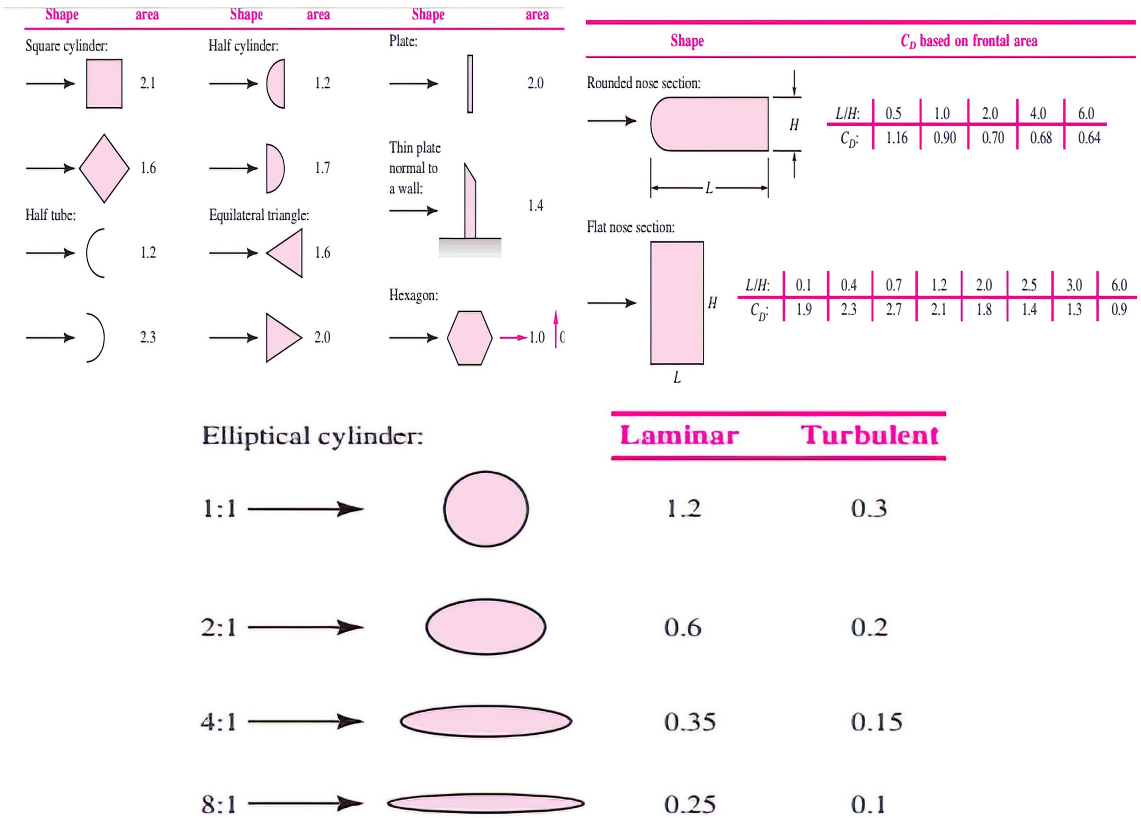


Figure 1: Common Fluid Impact Model

In all the common 2D models in Fig. 1, we can see that the C_d of the ellipse model is small, so we can first determine that the horizontal section of our 3D model is an ellipse. Then we chose an elliptical cylinder as the model(H_0 in height).

Under the premise of using this model, we can guarantee that each horizontal section of the model front view is an ellipse and consider its forces as a whole. The force surface of the model is the surface ACFDBEA(shown in Fig. 2),and its area is as follows(We define $S_{ACFDBEA}$ as the area of surface ACFDBEA, define C_e as the perimeter of ellipse bottom):

$$S_{ACFDBEA} = \frac{1}{2} C_e H_0 \tag{3}$$

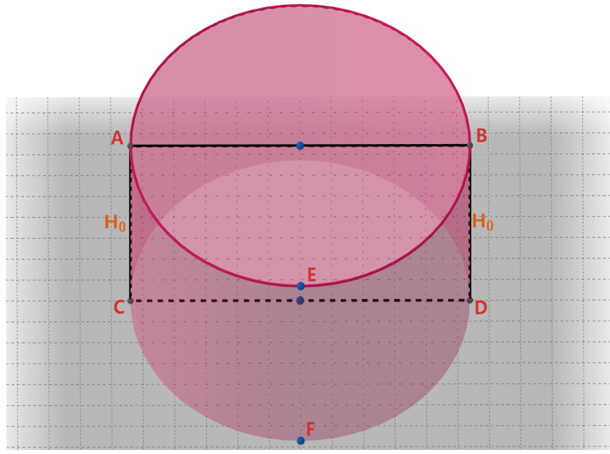


Figure 2: Top view of elliptical cylinder

But according to the decomposition rules of force and knowledge of calculus, we can get that the impact force of water will do the same work on surface ACFDBEA and surface ABCD within unit time.

So when F is working on surface ACFDBEA, we can calculate the work done on surface ACFDBEA by calculating the work done on surface ABCD.

We can easily get the size of ABCD(Here we define S_{ABCD} as the area of surface ABCD, define L_{Sa} as the short axis length at the bottom of the elliptical cylinder):

$$S_{ABCD} = L_{Sa} H_0 \tag{4}$$

Therefore, according to Eq. (1), the impact force on the elliptical cylinder model is(We define F_{model1} as the impact force on the elliptical cylinder model):

$$F_{model1} = \frac{1}{2} \rho u^2 C_d L_{Sa} H_0 \tag{5}$$

Under the premise that the horizontal section of the new model is an ellipse, we chose an elliptical platform with the same height(H_0) as the previous elliptical cylinder, and made the upper and lower bottom of the model tangent in the top view at the endpoints of the long axis which does not contact the water.

The diagram is as follows:

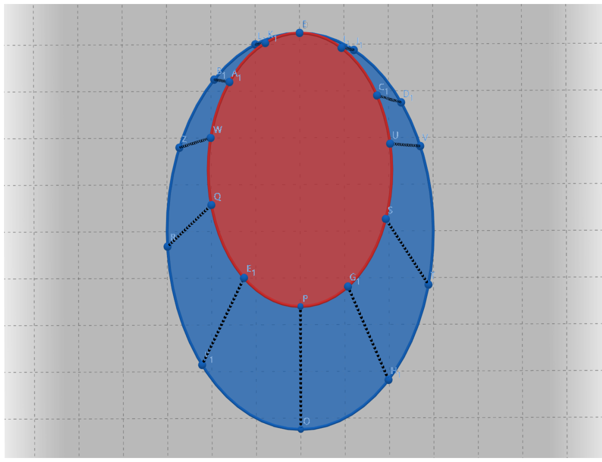


Figure 3: Top view of elliptical platform

Then, we can also guarantee that each horizontal section of the model front view is an ellipse. But the size of each horizontal section is different, so we analyze a single horizontal section from the perspective of differential.

The horizontal section of the model front view is shown in the figure below:

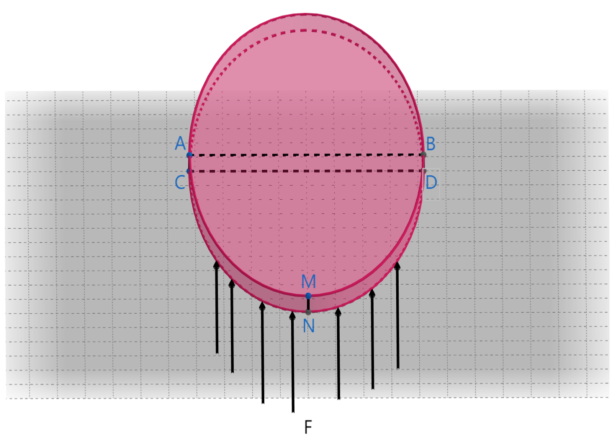


Figure 4: overlook section diagram

As shown in Fig. 4, in a cylinder with a height of Δh formed by pulling up the horizontal section of the model, the force surface is the surface ACNDBMA(Here we define ΔS as the area of surface ACNDBMA, define ${C_e}'$ as perimeter of any ellipse), so the area of surface ACNDBMA can be obtained:

$$\Delta S = \frac{1}{2}{C_e}'\Delta h \tag{6}$$

Like we did before, we can replace ΔS equivalent to ΔS′(we define L_{RT} as the minor axis length of the horizontal section ellipse):

$$\Delta S' = L_{RT}\Delta h \tag{7}$$

In this way, according to Eq. (1), we can quickly get the expression of the force acting on the surface ABCD(We define it as ΔF) as below:

$$\Delta F = \frac{1}{2}\rho u^2 C_d L_{RT}\Delta h \tag{8}$$

And in any horizontal section(h from the bottom of the model), L_{RT} can be expressed as(We define L_{Bsa} as the short axis length at the bottom of the elliptical platform(AB in Fig .5), L_{Tsa} as the short axis length at the top of the elliptical platform(CD in Fig .5)):

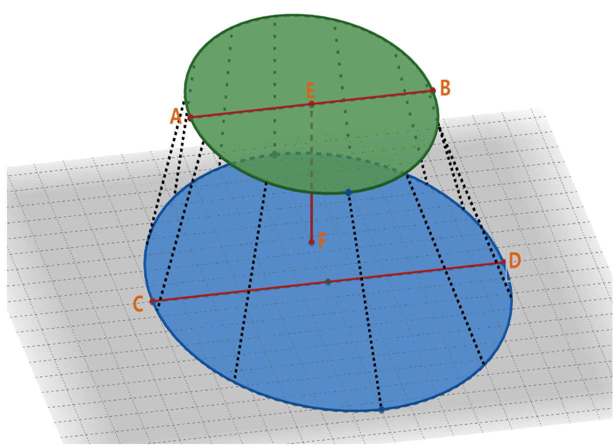


Figure 5: Side view of model

$$L_{RT} = L_{Tsa} + \frac{H_0 - h}{H_0}(L_{Bsa} - L_{Tsa}) \tag{9}$$

So from the overall model, the impact force on the elliptical platform can be obtained by integration(We define F_{model2} as the impact force on the elliptical platform):

$$\begin{aligned}
F_{model2} &= \int_0^{H_0} \Delta F \\
&= \frac{1}{2} \cdot \rho \cdot u^2 \cdot C_d \cdot \int_0^{H_0} L_{RT} \cdot dh \\
&= \frac{1}{2} \cdot \rho \cdot u^2 \cdot C_d \cdot \int_0^{H_0} [L_{Tsa} + \frac{H_0 - h}{H_0} \cdot (L_{Bsa} - L_{Tsa}) \cdot] dh \\
&= \frac{1}{4} \cdot \rho \cdot u^2 \cdot C_d \cdot (L_{Bsa} + L_{Tsa}) \cdot H_0
\end{aligned} \tag{10}$$

Then by comparing F_{model1} and F_{model2} with Eq. (5) and Eq. (10), it is not difficult to conclude that F_{model2} is smaller than F_{model1}, so an elliptical platform model should be used.

3.2 Model Simulation and Analysis

As for the specific shape of the ellipse, we can get a set of data from the ellipse related table in Fig. 1 and analyze it.

We used interpolation to analyze this set of data, rendered two sets of function images obtained(shown in Fig .6). So we can learn from the function image(we defined R_e as the ratio of the long axis and the short axis of the Elliptical section of the model): as the R_e increases, the rate of C_d decrease gradually reduces, and approaches 0 when the R_e is 3. Combining feasibility and aesthetics, we can set the R_e to 2.5 - 3.

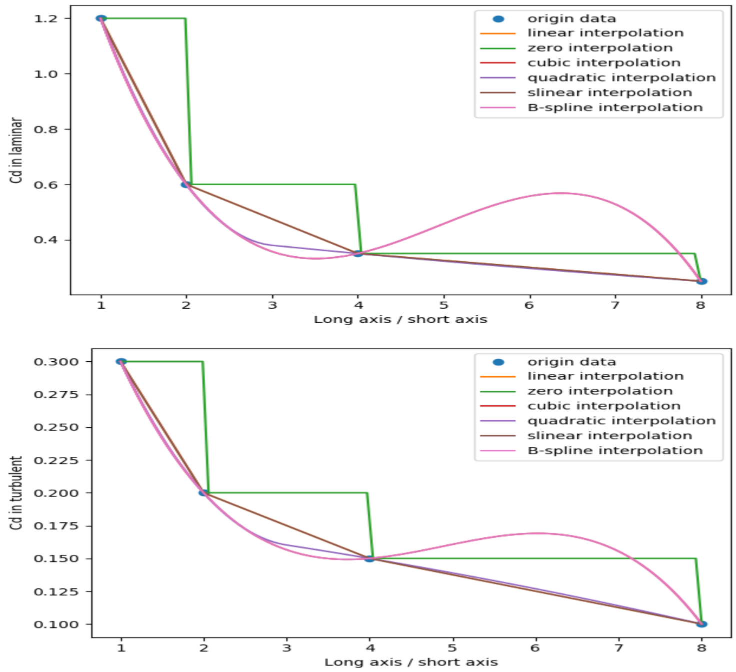


Figure 6: Analyze the relationship between the ratio of long and short axes and C_d

On the other hand, we notice in Fig. 1 that the C_d of half cylinder is lower than Equilateral triangle, and consider the construction problem on the side surface of the model (straight shapes may not be optimal). So we combine the force analysis to conclude that a curved surface on the side may reduce the impact. The force analysis diagram is as follows:

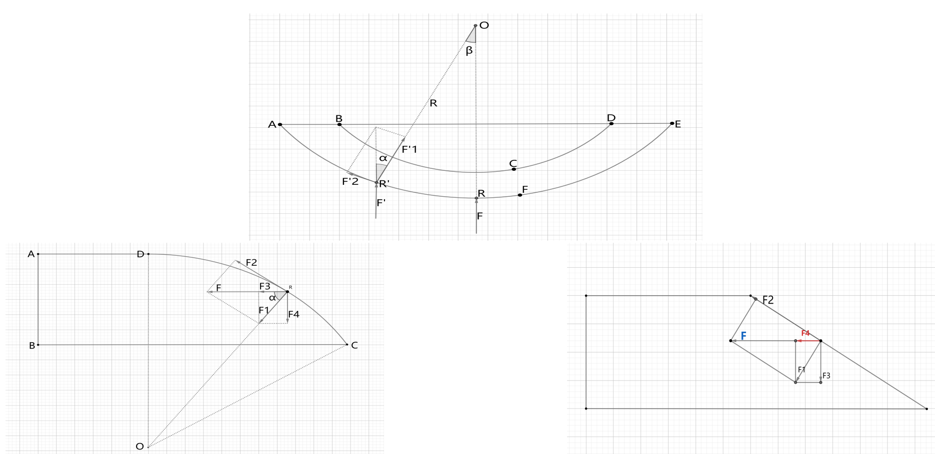


Figure 7: Force analysis of vertical section

4 The best proportion of sand and water

4.1 Cohesion between two spheres

4.1.1 Meniscus formation

Liquid surface tension and capillary action can cause cohesion between wet particles[3]. At the same time, it is considered that liquid having a pressure of P_l forms a meniscus under the action of air having a pressure of P_a. If the radius of curvature between liquid and air is r_1 and r_2 ,then the ΔP can be given by the Young-Laplace equation as

$$\Delta P = P_a - P_l = \gamma[\frac{1}{r_1} + \frac{1}{r_2}] \tag{11}$$

where the γ is the surface tension between air and liquid.

4.1.2 Liquid bridge model between two particles

Liquid bridge between two particles (shown in Fig. 8) causes cohesion[4]. In order to clearly describe the force of water between sand particles, we assume that sand particles can be regarded as spherical particles with the same radius and ignore the effect of gravity.

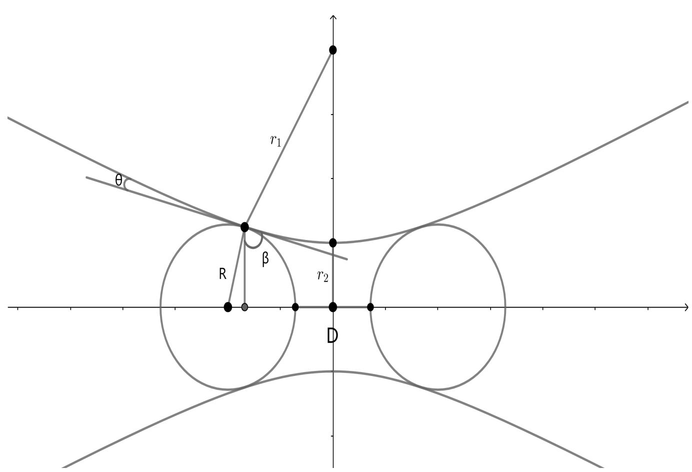


Figure 8: Schematic diagram of a liquid bridge between identical spheres

4.1.3 Liquid bridge force

Because the attractive force between the spheres caused by the liquid meniscus is given by the sum of surface tension and suction[3], the Dorge method is used to solve the problem of liquid bridge force. The force is given by

$$F_{bridge} = 2\pi r_2 \gamma + \pi r_2^2 \Delta P \tag{12}$$

with

$$\Delta P = \gamma[\frac{1}{r_1} - \frac{1}{r_2}] \tag{13}$$

From the geometric relationship[4], we can get

$$r_1 = [D/2 + R(1 - \cos\beta)]/[\cos(\beta + \theta)] \tag{14}$$

and

$$r_2 = Rsin\beta - [1 - sin(\beta + \theta)]r_1 \tag{15}$$

when $R \gg r_1 \gg r_2$ and $D \ll 2r_1 cos\theta$, a simplified expression can be written as

$$F_{bridge} = 2\pi R\gamma \cos\theta[1 - \frac{D}{2r_1 \cos\theta}] \tag{16}$$

Substituting Eq. (14) into Eq. (16),we can get

$$F_{bridge} = 2\pi R\gamma(\cos\theta - \frac{\cos(\beta + \theta)}{1 + \frac{2R(1-\cos\beta)}{D}}) \tag{17}$$

Now we qualitatively analyze Eq. (17): On the one hand, we know that π and R are constants, and γ is a constant under certain conditions. On the other hand, θ and β will change with the change of D. When D becomes smaller and the volume of liquid does not change, the contact angle θ will become smaller, the half-filling angle β will become bigger, and the sum of θ and β will become bigger. So we can draw the first conclusion that when D becomes smaller and the volume of liquid does not change, F_{bridge} will become bigger. Similarly, the second conclusion is that when the volume of liquid becomes bigger and D does not change, β and θ will have the same change as before and F_{bridge} will get bigger, too. We have found similar conclusions in the other literatures and figures to show these conclusions[4].

Through the following figures(Fig. 9, Fig. 10), we can know the sensitivity of the model: D has a linear effect on F; when $0 < V < 2 \cdot 10^{-8} m^3$ the effect on F is great, and when $V > 2 \cdot 10^{-8} m^3$ the effect on F is small.

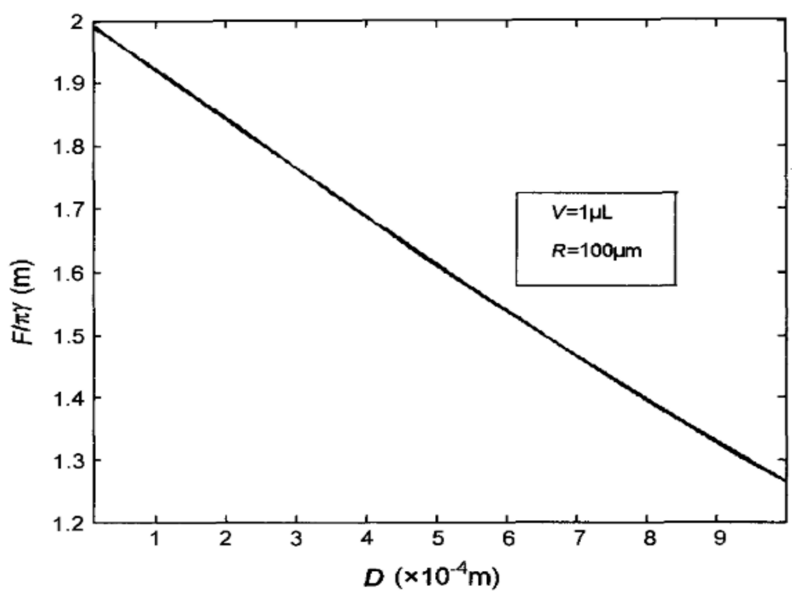


Figure 9: Relationship between liquid bridge force and separation distance

4.2 Optimum stress and sand-water ratio

Based on these two conclusions, we constructed the following model(shown in Fig. 11) to maximize the cohesion between two particles.

In this model, the contact angle $\theta = 0$ is for the perfect wet case[4]. When $D \approx 0$, $r_2 \approx R$, $\Delta P \approx 0$, the liquid bridge force will reach the maxi-

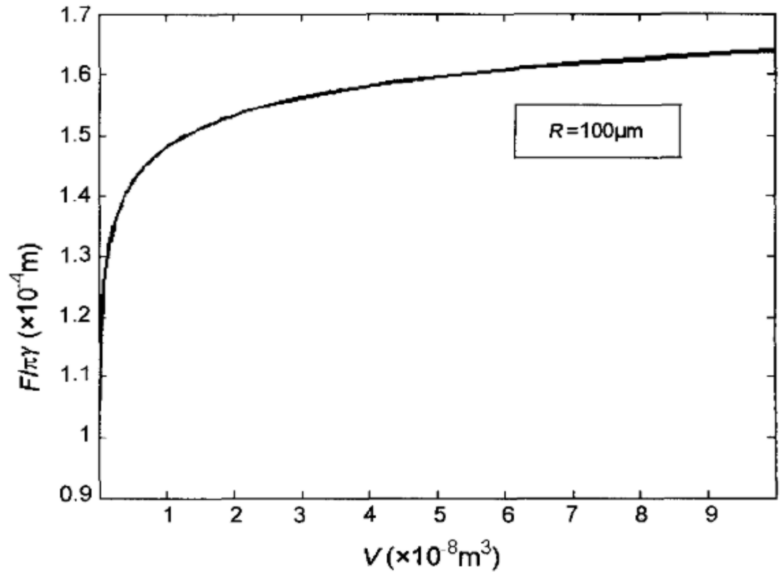


Figure 10: Relationship between liquid bridge force and liquid bridge volume

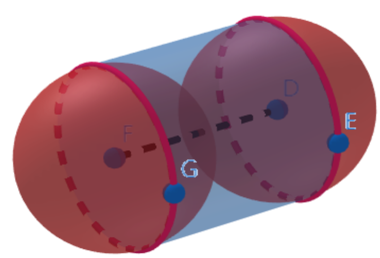


Figure 11: Model that can theoretically form maximum cohesion

mum. It is given that

$$F_{birdge} = 2\pi R\gamma \tag{18}$$

and the volume of liquid is

$$V_{liquid} = \frac{2}{3}\pi R^3 \tag{19}$$

Then, we extend it to a model in three dimensions. In order to get each particle as close as possible to each other, we refer to the Primitive cubic in the cubic crystal system and built a model of anhydrous sand accumulation(shown Fig. 12).

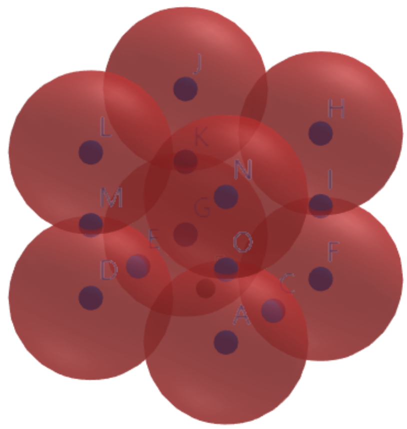


Figure 12: model of anhydrous sand accumulation

Then we add a liquid bridge between two adjacent particles to form the final model in three dimensions(shown Fig. 13).

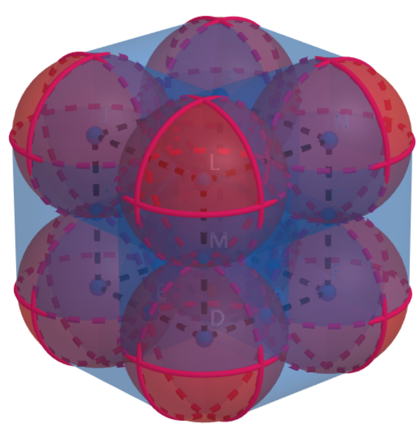


Figure 13: final model in three dimensions

At this time, the volume ratio of sand to water is given that

$$\frac{V_{sand}}{V_{water}} = \frac{\frac{4}{3}\pi R^3}{8R^3 - \frac{4}{3}\pi R^3} = \frac{\pi}{6-\pi} \approx 1.099 \tag{20}$$

The main component of sand is silicon dioxide. So we take the density of sand as 2.65g/cm3. The sea water on the coastline is exposed to direct sunlight so the temperature is high and the salt content is low. According to this reason, we take the density of seawater as 1.02g/cm3. So the mass ratio of sand to water is given that

$$\frac{m_{sand}}{m_{water}} = \frac{\rho_{sand}V_{sand}}{\rho_{water}V_{water}} = \frac{2.65\pi}{1.02(6-\pi)} \approx 2.855 \tag{21}$$

As a conclusion, the volume ratio of sand to water is about 1.099 and the mass ratio of sand to water is about 2.855.

5 Impact of rainwater and model adjustment

5.1 Effect of water content

In section 4, we have built a model that the water almost fills the gap between sand particles to get greater cohesion. However, under the effect of rain, the content of water will increase directly, which will make the gap between the particles to become larger. In terms of what we've concluded before, if the gap becomes larger, the force of liquid bridge will be smaller which makes the looseness of the sand increase.

Therefore, if the area where the beach is located is rainy, we can consider reducing the water content of the wet sand to get more water absorption under the premise of ensuring the stability of the sand castle.

5.2 Effect of water current

When rainwater falls on the slope of the foundation, water flow will be formed. Under certain water flow conditions, the static equilibrium state of the sand will be damaged, and it will change from static to dynamic. So we need to discuss the critical conditions of sand changing from static to dynamic. The movement of water flow on the slope can be decomposed into three directions. We refer to the model of incipient conditions of sediment particle slipping on the slope established by MA Zi-pu et al[5].

In the model, the speed of the water flow is decomposed into speeds in three directions(shown in Fig. 14).

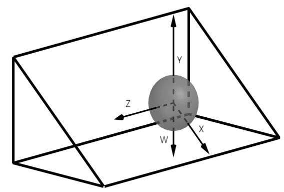


Figure 14: Sketch of 3D Cartesian coordinate system on slope

According to the right-hand rule, the direction of the parallel slope downward is the positive direction of the X-axis, the direction perpendicular to the slope upward is the positive direction of the Y-axis, and the direction perpendicular to the X-axis and Y-axis and pointing to the reader is the positive direction of the z-axis.

In this paper[5], the author gives the following three formulas. All these three equations represent the minimum velocity of water that makes sand to slide:

- When the main flow is in the x direction

$$u_{rx} = \sqrt{\frac{2\alpha_1 d(\gamma_s - \gamma)(f cos\theta - sin\theta)}{\rho(\alpha_3 C_x + f\alpha_2 C_y)}} \tag{22}$$

- When the main flow is in the y direction

$$u_x^{'} = \sqrt{\frac{2\alpha_1 d(\gamma_s - \gamma)(f cos\theta - sin\theta)}{\alpha_3 f \rho C_x'}} \tag{23}$$

- When the main flow is in the z direction

$$u_y^{'} = \frac{2fW^{'}cos\theta - 2\sqrt{W^2 sin^2\theta + (\frac{\rho d^2}{2}\alpha_3 C_x' u_x'^2)^2}}{f\rho d^2 \alpha_2 C_y'} \tag{24}$$

the notations involved are the:

$W^{'}$: effective underwater gravity

u_{rx} : the relative velocity of flow and sediment in the x direction

$u_x^{'}$: the velocity of water ripple in the x direction

$u_y^{'}$: the velocity of water ripple in the y direction

γ_s : the volumetric weight of sediment

γ : the volumetric weight of water

C_x : horizontal drag coefficient

$C_x^{'}$: water ripple's drag coefficient

C_y : vertical uplift force coefficient

$C_y^{'}$: water ripple's uplift force coefficient

$\alpha_1, \alpha_2, \alpha_3$: Constant, the shape factor of sediment particles(particles are simplified to a sphere, the factor is $\pi/4$ here)

ρ : Constant, the density of water

d : Constant, the grain size of sediment

f : Constant, coefficient(s) of friction

θ : the angle of slope

The above three equations indicate the threshold at which the sediment changes from static to dynamic under different flow directions. We can see that if the speed of water is constant, most variables related only to the speed of water are also constant. Then we exclude constants, the only remaining variable in the equation is θ , the angle of slope. Through the analysis of the equations, we can know that when θ becomes smaller, the threshold value will become larger and the difficulty of changing sand particles from static to dynamic will increase. With this conclusion, we can reduce the angle of slope of the model to have a better stability.

5.3 Effect of the impacting force of water

The impact of rain will take away the sand of the sandcastle model, which is something we cannot ignore.

We can get some data from Papers by Ding Jiaming and Wang Yonghe [6]: In the case of heavy rainfall of $50\ mm/h$, we assume that raindrops are water balls with a diameter of $4\ mm$, and have the same speed and mass, and the same time The probability of raindrops falling at every point on the model is the same.

One hour of rainfall in this case, there is $5{\cdot}10^{-2}\ m^3$ rainfall per m^2 , which is $1.49{\cdot}10^6$ Raindrops, so the area where raindrops do not coincide with each other is $18.7\ m^2$(we defined it as S_{Rain}), each time the rain hits the surface of the model, it will take away the surface sand of $5{\cdot}10^{-4}\ m$(we define $_rl$ as the thickness of the surface sand removed by a single raindrop).

Falling on the ground of $1\ m^2$ in this case, raindrops will overlap 18 times at the same position; so on the surface of $S_e\ m^2$(we define S_e as the actual area affected by raindrops), raindrops will overlap $S_e{\cdot}S_{Rain}$ times at the same position, in order to simplify the model, we assume that the length of the long and short axes of the ellipse's upper ellipses and the height of the elliptical platform are reduce at a speed of Δl m/s, so Δl can be expressed as:

$$\Delta l = L_r{\cdot}\frac{S_e{\cdot}S_{Rain}}{3600} \tag{25}$$

Because the raindrop force is perpendicular to the ground(shown in Fig .15), it can be easily obtained according to the knowledge of force decomposition and calculus that the surface of the raindrop can be equivalent to the bottom ellipse of the elliptical platform, so S_e can be expressed as(we define l_{bx} and l_{by} as half of the long and short axes of the bottom ellipse of the elliptical platform):

$$S_e = \pi \cdot l_{bx} \cdot l_{by} \tag{26}$$

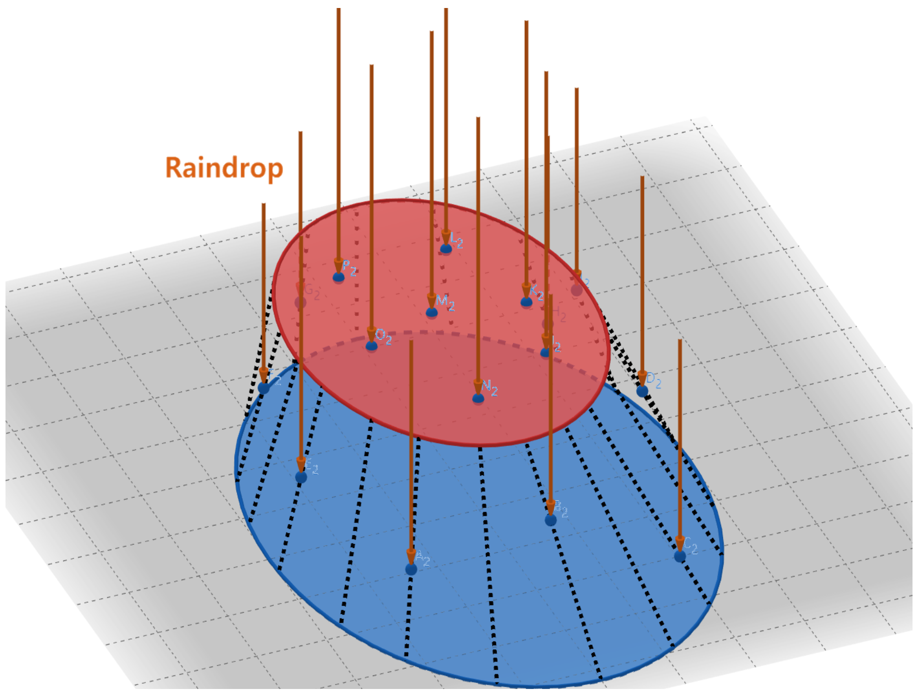


Figure 15: Schematic diagram of rain impact

In this case, Δl is a constant, from this we can get the expressions of the height of the elliptical mesa and the short axis of the top surface of the ellipse mesa:(we define H_0, H, l_{tx0}, l_{tx} as the origin height of the elliptical platform, the real height of the elliptical platform, half of the origin short axis of the top surface of the ellipse mesa, half of the real short axis of the top surface of the ellipse mesa, and define t as the duration of rain):

$$H = H_0 - \Delta l \cdot t \tag{27}$$

$$l_{tx} = l_{tx0} - \Delta l \cdot t \tag{28}$$

We guarantee the ratio of the long and short axes of the top ellipse to 2.5, then we can get the real-time volume of the entire model by integrating(we define V_{mir}, ΔS, Δh, h as the real-time volume of the model, the area of the horizontal section ellipse and the height of horizontal section ellipse, distance from top):

$$
\begin{aligned}
V_{mir} &= \int_0^H \Delta S \\
&= \int_0^H \pi \cdot [l_{tx} + \frac{h(l_{bx} - l_{tx})}{H}] \cdot [l_{ty} + \frac{h(l_{by} - l_{ty})}{H}] \cdot dh \\
&= \pi \cdot l_{tx} l_{ty} H + \frac{H}{6} \cdot (2 l_{bx} l_{by} + l_{tx} l_{by} + l_{ty} l_{bx} - 4 l_{tx} l_{ty}) \\
&= 2.5\pi \cdot l_{tx}^2 H + \frac{H}{6} \cdot (5 l_{bx}^2 + 5 l_{tx} l_{bx} - 10 l_{tx}^2) \qquad (29)
\end{aligned}
$$

In order to test the sensitivity of the model, we set multiple sets of data for H_0, l_{tx0} while keeping the model volume constant, and used matplotlib package to draw the function image in python(shown in Fig .16):

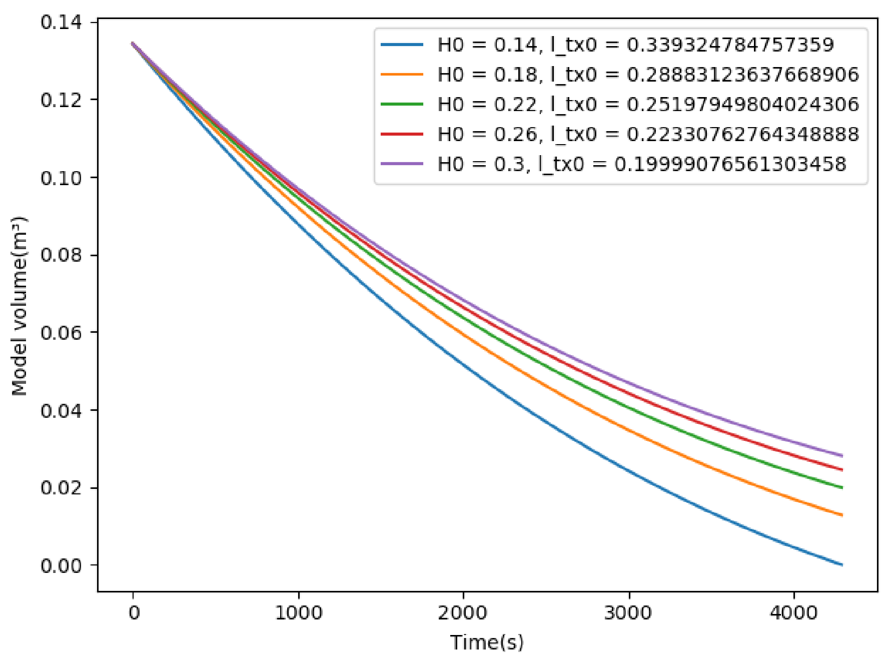


Figure 16: The process of a sand castle being hit by rain

From the function image, we can see that with the decrease of H_0, the rate of model volume decrease becomes larger. The model with $H_0 = 0.14\ m$ has been washed away by rain when the time reaches about 4290 seconds. So while keeping the volume and the area of the bottom ellipse constant, the higher the model is, the more resistant it is to the impact of rain.

6 The other methods to prolong sandcastles life

6.1 Compaction

The first way to increase the strength of a sandcastle is to hold the foundation firmly together, to reduce the gaps between the sand in it, to strengthen the intermolecular forces between the sand grains, and to integrate the foundation. An easier way is to fill the sand with a small bucket and press it down with a shovel or fist.

6.2 Build a wall and dig a moat

Adding a wall can protect the castle. This can not only reduce the impact of the sea on the sand castle to a certain extent, but also can increase the beauty of the sand castle more solemn

Digging a moat next to the wall. This ditch will retain most of the water when the waves arrive. And the slightly dry sand at the bottom of the ditch can absorb a lot of water.

6.3 Curing agent

We can also regard sandcastle as a building. In modern architecture, it is not enough to only use natural materials in nature, and most of them are mixed with additives. If there is a curing agent, the sand castle foundation can be the hardness

7 An Article to Fun in the sun

How to build a stable sandcastle scientifically

FROM: Team 2013484 , MCM B

To: Fun in the Sun

Date: March 08, 2020

No one could say no to playing sand on the beach. Is there any more interesting than creating a sandcastle step by step? A great day with sandcastle is never be dull.

But every time when your castle be crumbled by waves and tides,have you thought about how to strengthen your castle?

Today I will tell you the secrets of building a perfect long-lived castle using scientific methods!

1. The optimum sand water ratio

The first and most important thing you need to know about sandcastle is that its material is a mixture of sand and water.Only wet sand can be used to build a stable castle.

The principle of this is also well to explained by science.Inside the sand,there are hundreds of millions of sand particles.Water is connected between the sand particles as "bridge".The connection,which we called "Liquid Bridge" ensures structural stability.

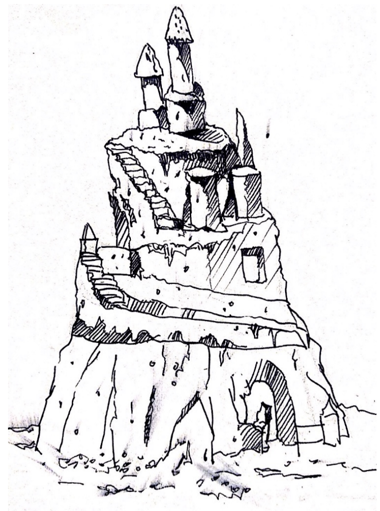
When the force between the sand particles reaches the maximum, the sand pile is the closest. In our model calculation, the overall ratio of water and sand is about 1:1.1

2. The shape of base

Playing sand is much more fun than pile a modular Lego.Let sand flow through your fingers and use your imagination to build your castle freely without being bound by the frame.

Whether it's a skyscraper or a sandcastle,needs a sturdy base or bottom.If the bottom isn't strong enough to hold it up,it could crumble or tip over!

You must be thinking about what shape of foundation is the strongest



and the most resistant to the impact of seawater.Is it rectangle?Is it circle?Is it hexagon? According to our physical model ,the conclusion is that the best impact-resistant shape is oval.

No matter which part of the ellipse the sea's impact hits,the oval will spread outward with the impact point as the center to ensure every part is under the same pressure,it will not be crushed by the sudden impact of waves.

3. Other skills

In rainy weather, the amount of water added to the sand can be appropriately reduced because the humidity of the air is high. Similarly, in the clear sun, don't forget to sprinkle some water on the outside of the sand castle, otherwise the outer wall of the sand castle will crack, and the future of this powerful city is collapse.

There are other clever ways to ensure the longevity of the sand castle.The construction of city walls and moats can effectively reduce the impact of seawater, and sand castles will look more majestic and tall.

How interesting it is to create a beach castle!When you know these sand casting skills, you will appreciate the charm of science in your next play.

How about go to the beach next time?

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Appendices

Here is Code we used in our model, which python is the main development language.

Appendices A Relationship between the ratio of long and short axes and Cd

```
import numpy as np
from scipy import interpolate
import matplotlib.pyplot as plt

x = [1.0, 2.0, 4.0, 8.0]
laminar = [1.2, 0.60, 0.35, 0.25]
turbulent = [0.3, 0.2, 0.15, 0.1]
x_new = np.linspace(1, 8, 1000)
Interpolations = ['linear', 'zero', 'cubic', '
   quadratic', 'slinear']
# laminar as the y_data
plt.xlabel(u'Long axis / short axis')
plt.ylabel(u'Cd in laminar')
tck = interpolate.splrep(x, laminar)
plt.plot(x, laminar, "o",  label=u"origin data")
for mode in Interpolations:
    plt.plot(x_new, interpolate.interp1d(x, laminar,
       kind=mode)(x_new), label="{} interpolation".
       format(mode))
plt.plot(x_new, interpolate.splev(x_new, tck), label=u
   "B-spline interpolation")
plt.legend()
plt.show()
# turbulent as the y_data
plt.xlabel(u'Long axis / short axis')
plt.ylabel(u'Cd in turbulent')
tck = interpolate.splrep(x, turbulent)
plt.plot(x, turbulent, "o",  label=u"origin data")
for mode in Interpolations:
```

```
    plt.plot(x_new, interpolate.interp1d(x, turbulent,
         kind=mode)(x_new), label="{} interpolation".
        format(mode))
plt.plot(x_new, interpolate.splev(x_new, tck), label=u
   "B-spline interpolation")
plt.legend()
plt.show()
```

Appendices B The process of a sand castle being hit by rain(sensitivity analysis)

```
import math
import numpy as np
import scipy.optimize
import matplotlib.pyplot as plt


def get_b0(H):
    A = (2.5 * math.pi - 5 / 3) * H
    B = H / 3
    C = 2 * H / 15 - 0.13424
    return scipy.optimize.fsolve(lambda x: A * x ** 2
       + B * x + C, 0)[0]


def ellipse_platform(a, b0, H0, t_Data):
    V_Data = []
    for t in t_Data:
        b, H = get_params(H0, b0, delta_l, t)
        V_Data.append(2.5 * math.pi * H * b ** 2 + (5
           * a ** 2 + 5 * a * b - 10 * b ** 2) * H /
           6)
    return V_Data


def get_params(H0, b0, delta_l, t):
    H = H0 - delta_l * t
    b = b0 - delta_l * t
    return b, H
```

```
a = 0.4
H0_data = np.linspace(0.14, 0.3, 5)
e_b = math.pi * 2.5 * a ** 2
delta_l = 0.005 * 18.7 * e_b / 3600
t_data = np.linspace(0, 10000, 10000)
H_test = H0_data[0]
b_test = get_b0(H_test)
t_max = 0

# Determine the range of t
for t in t_data:
    b, H = get_params(H_test, b_test, delta_l, t)
    if 2.5 * math.pi * H * b ** 2 + (5 * a ** 2 + 5 *
       a * b - 10 * b ** 2) * H / 6 <= 0:
        t_max = int(t)
        break
t_data = np.linspace(0, t_max, 10 * t_max)

# draw the function image
plt.xlabel(u'Time(s)')
plt.ylabel(u'Model volume(m³)')
for H0 in H0_data:
    b0 = get_b0(H0)
    plt.plot(t_data, ellipse_platform(a, b0, H0,
       t_data), label='H0 = {}, l_tx0 = {}'.format(H0,
        b0))
plt.legend()
plt.show()
```