Problem 1: Kernel Trick (10%)

As noted in class, a (properly selected) kernel function can be expressed as an inner product between two data points which are projected onto a high-dimensional space, i.e., $K(\mathrm{x}, \mathrm{x}') = \Phi(\mathrm{x})^\top \Phi(\mathrm{x}')$, where K is the kernel function and $\Phi(\cdot)$ denotes feature mapping. Suppose that a mapping satisfies $\Phi(\mathrm{x}) : \mathbb{R}^2 \to \mathbb{R}^3$ with a corresponding kernel function $K(\mathrm{x}, \mathrm{x}') = (\mathrm{x}^\top \mathrm{x}')^2$. Given $\mathrm{x} = \begin{bmatrix} x_1 \ x_2 \end{bmatrix}^\top$, please derive the explicit form of $\Phi(\mathrm{x}) \in \mathbb{R}^3$ in terms of x_1, x_2.

Ans:

解

已知 $X = [X_1 \ X_2]^T$

令 $X' = Y = [Y_1 \ Y_2]^T$

則

$$
\begin{aligned}
K(X,X') = K(X,Y) &= (X^T Y)^2 \\
&= (X_1Y_1 + X_2Y_2)^2 \\
&= (X_1Y_1)^2 + (X_2Y_2)^2 + 2X_1X_2Y_1Y_2 \\
&= (X_1^2, X_2^2, \sqrt{2}X_1X_2)^T(Y_1^2, Y_2^2, \sqrt{2}Y_1Y_2) \\
&= \phi(X)^T \phi(Y) \\
&= \phi(X)^T \phi(X')
\end{aligned}
$$

得 $\forall X \in R^2$, $\phi(X) = \phi((X_1, X_2)^T)$

$$= (X_1^2, X_2^2, \sqrt{2}X_1X_2) \in R^3$$

Problem 2 : Color and Texture Segmentation (40%)

For this problem you will need to develop an unsupervised learning algorithm for image segmentation using either color or texture information.

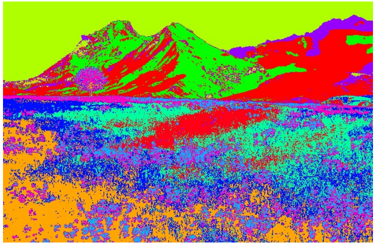
Image data and filter banks

- **filterBank.mat**: The given .mat file contains a set of 38 filters (also known as filter bank). This filter bank is stored as a 49 x 49 x 38 matrix (i.e., each filter is of size 49 x 49 pixels).
- **Images**: **zebra.jpg** and **mountain.jpg**

Problem sets

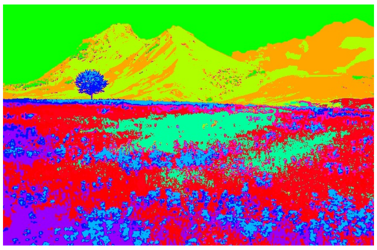
(a) (20%) **Color segmentation**: Given an *RGB color image* of size $n = w \times h$ pixels, each pixel can be viewed as a three dimensional feature, each dimension describes the value in each color channel. For the task of image segmentation, please run the k-means algorithm* to cluster these n pixels (in terms of their 3D features) into k groups.
(i) Plot the segmentation results for both images based on your clustering results. For visualization purposes, pixels in the same group should be represented by the same color, while those in different groups are shown in distinct colors, as shown in Fig. 1a.

Ans:



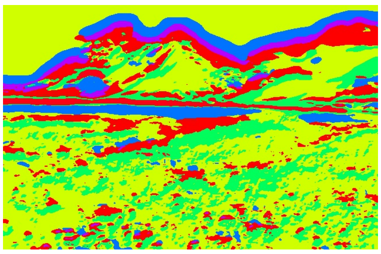
(ii) Convert both RGB images into Lab color space. Repeat the above clustering procedure and plot your segmentation results.
(*Please use $k = 10$ and maximum number of iterations $= 1000$ for both **zebra.jpg** and **mountain.jpg** when running the k-means algorithm.)

Ans:



(b) (20%) **Texture segmentation**: We now consider the use of texture information for image segmentation. For simplicity, please convert the color images into *grayscale* ones, before extracting image textural features via the provided filter bank. As a result, you would produce $n = w \times h$ 38-dimensional features, each dimension corresponds to a particular filter response. Similarly, please perform k-means clustering* to cluster these n features into k different groups.
(i) Please plot the texture segmentation results for both images, as depicted in Fig. 1b.

Ans:



(ii) Combine both color and texture features ($3 + 38 = 41$-dimensional features) for image segmentation. Repeat the above clustering procedure and plot your segmentation results.
(*Note that when calculating filter responses, please use symmetric padding to deal with pixels near image boundaries. Please use $k = 6$ and maximum number of iterations $= 1000$ for both **zebra.jpg** and **mountain.jpg**.)

Ans: