

Hyperbolicity Course Notes

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Scribe: Yi Li

1 Overview

The topics in today's lecture are:

1. Proof of the Hodge structures in family satisfies the Griffiths transversality condition,
2. We will construct the Higgs metric (not necessarily positive definite) and Hodge metric (positive definite) on the Hodge bundle,
3. We will give a geometric description of the Higgs field (which is actually the Kodaira-Spencer map),
4. We will construct the compact dual, period domain, period mapping and we will introduce some basic properties about period mapping and period domain,
5. We will study the curvature property on the period domain. As an application, we will show the moduli space of Calabi-Yau manifold is hyperbolic.

- 2 The Griffiths transversality theorem
- 3 Construction of the Hodge metric and Higgs metric
- 4 Geometric interpretation of the Higgs field using Kodaira-Spencer map
- 5 Construction of period domain (as homogeneous space)
- 6 Holomorphicity of the period domain
- 7 Tangent space of the period domain
- 8 Tangent bundle of the period domain
- 9 Horizontal tangent bundle of the period domain
- 10 Curvature properties
- 11 Hyperbolicity on the moduli space of Calabi-Yau manifolds