Hyperbolicity Course Notes Spring 2024

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1 Overview

The topics in today's lecture are:

(1) Proof of the Hodge structures in family satisfies the Griffiths transversality condition, (2) We will construct the Higgs metric (not necessarily positive definition) and Hodge metric (positive definite) on the Hodge bundle, (3) We will give a geometric discription of the Higgs field (which is actually the Kodaira-Spencer map), (4) We will construct the compact dual, period domain, period mapping and we will introduce some basic properties about period mapping and period domain, (5) We will compute the tangent bundle of period domain, and derivative of period mapping.

2 The Griffiths transversality theorem

3 Construction of the Hodge metric and Higgs metric

4 Geometric interpretation of the Higgs field using Kodaira-Spencer map

5 Construction of period domain (as homogenuous space)

6 Holomorphicity of the period domain

7 Tangent space of the period domain

8 Tangent bundle of the period domain

9 Horizental tangent bundle of the period domain

10 Curvature properties

11 Hyperbolicity on the moduli space of Calabi-Yau manifolds