Projectivity Criterira Reading Seminars Fall 2024

Note 2 — 11, 09, 2024 (draft version 0)

Yi Li

1 Overview

Last time we proved the Seshadri projectivity criterion of Moishezon variety.

This time we first introduce the Chow-Barlet cycle space and prove a technical result which shows that we can approximate the Chow-Barlet cycle space by countable many projective morphism. Using this result, we can prove the main theorem of today's seminar that is some lower semi-continuity result on Seshadri constant.

Let us brief sketch the idea of the proof.

2 Questions from the previous time

3 Chow-Barlet cycle space

We can now state the result which allows to approximate the Chow-Barlet cycle space using countable many projective morphism

Theorem 3.1 (Approximation Chow-Barlet cycle space using countable many projective morphisms)**.** Let $g : X \to S$ be a proper morphism of complex analytic spaces that is bimeromorphic to a projective morphism. Fix $m \in \mathbb{N}$. Then there are countably many diagrams of complex analytic spaces over S,

$$\begin{array}{ccc} C_i & \hookrightarrow & W_i \times_S X \\ {\scriptstyle w_i}\downarrow\uparrow{\scriptstyle \sigma_i} & & \\ W_i & & \end{array}$$

indexed by $i \in I$, such that

(1) the $w_i : C_i \to W_i$ are proper, of pure relative dimension 1 and flat over a dense, Zariski open subset $W_i^\circ \subset W_i$,

(2) the fiber of w_i over any $p \in W_i^\circ$ has multiplicity m at $\sigma_i(p)$,

(3) the W_i are irreducible, the structure maps $\pi_i : W_i \to S$ are projective, and

(4) the fibers over all the W_i° give all irreducible curves that have multiplicity m at the marked point.

Proof. □

4 Lower semi-continuity of Seshadri constant (openness of projectivity)

The main result that will be proved in today's seminar is

Theorem 4.1 (Openness of projectivity over Euclidean open subset, see [Kol22], Proposition 14.)**.**

Proof. □

References

[Kol22] János Kollár. "Seshadri's criterion and openness of projectivity". In: *Proc. Indian Acad. Sci. Math. Sci.* 132.2 (2022), Paper No. 40, 12. ISSN: 0253-4142,0973-7685. DOI: 10.1007/s12044-022-00680-9. URL: https://doi.org/10.1007/s12044-022-00680-9.