Nonvanishing conjecture reading seminars Spring 2024

Lecture 4 — 06, 06, 2024 (draft version)

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1 Overview

The aim of this note is to introduce the non-vanishing conjecture. We will prove the classical Shokurov non-vanishing theorem. We then try to prove the BCHM non-vanishing, which says that if the non-vanishing holds in dimension $(n-1)$ and special finiteness and existence of minimal conjecture hold in dimension n then the non-vanishing conjecture also true in dimension n. Note that by the spriling induction, this will implies the non-vanishing theorem under BCHM conditions. After that we will introduce some recent progress on non-vanishing conjecture.

2 Shokurov non vanishing theorem

3 BCHM version nonvanishing theorem

We will prove the following theorem in this section.

Theorem 1. Assume non-vanishing conjecture in dimension $n-1$, special finiteness in dimension n and existence of minimal model in dimension n. Then the non-vanishing conjecture is also true in dimension n.

Before proving the theorem, let us introduce several technical lemmas.

Lemma 2 (Deformation of effectiveness)**.**

Lemma 3 (Local uniqueness of canonical models)**.**

Lemma 4 (Improve that pair when positive part of Zariski decomposition is non-zero)**.**

Now we can prove the BCHM non-vanishing theorem.

Proof of Theorem 1. □

References