G33D - Class Note 2

Value-at-Risk

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https://github.com/YiLiu6240/bham_ECONG33_Regulation-Supervision

February 10, 2017

Concept

(For detailed discussion of Value-at-Risk please refer to Hull (2012, Chap.9)).

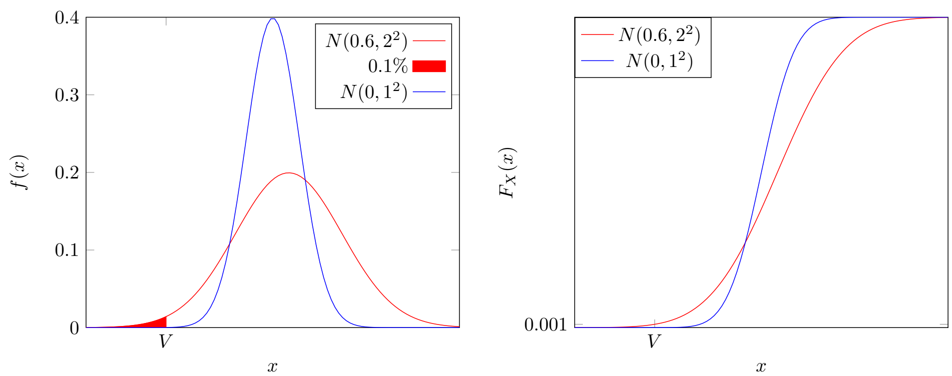
What we try to get from Value-at-Risk is to measure the maximum level of losses given a confidence level, or "how bad can the losses be for p% of all the possible scenarios".

Using the scenario we discussed in Class set 1, where the bank's profit follows a normal distribution of $X \sim N(\mu = 0.6, \sigma^2 = 2^2)$, we try to pinpoint a V which divides the area of probabilities (under the probability density function *pdf* curve) into two portions 0.1% and 99.9%. It's even easier for us to spot V that is associated with a cumulative probability of 0.001 on the cumulative distribution function *cdf* plot. The V is the VaR we are looking for (Figure 1).

Under such a normal distribution setting, we calculate VaR from the "transformation" formula[1]:

$$VaR_{1-\alpha} = \mu + Z_\alpha \sigma$$

Figure 1: VaR in a Normal Distribution



pdf *cdf*

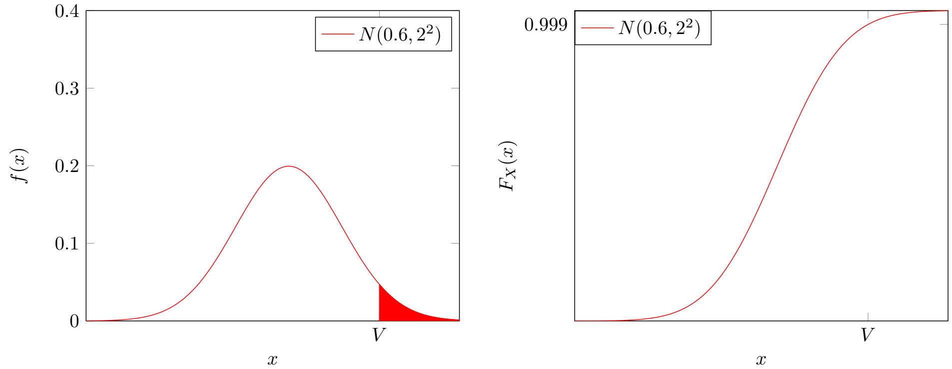
In addition, note that the VaR is represented differently if we denote x as losses (Figure 2).

We discussed in the class tutorial how VaR can be obtained for other kinds of continuous distributions. We also discussed how VaR can be applied when dealing with discrete outcomes. Refer to Hull (2012, p.186) for details.

[1] which is derived from the normal distribution quantile function (inverse function of *cdf*):

$$VaR_{1-\alpha} = F_X^{-1}(x) = \mu + \sigma\sqrt{2}erf^{-1}(2F - 1)$$

Figure 2: VaR measured as losses



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Portfolio VaR Hull (2012, p.192-194, p.195-197)

From portfolio theory we derive the returns and volatility of an investment portfolio from the elements (weight w_i, return R_i, volatility σ_i, correlation $\rho_{i,j}$) of individual investment:

$$\mu_p = E(R_p)$$

$$R_p = \sum_{i=1}^{n} w_i R_i$$

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2 \sum_{1 \leq i < j \leq n} \rho_{i,} w_i w_j \sigma_i \sigma_j$$

Suppose we still retain the normal distribution assumption for the returns of individual investment in a portfolio, then we can derive $VaR_{1-\alpha,p} = \mu_p + Z_\alpha \sigma$ by sustituting the individual elements into the equation.

The multi-period VaR can essentially be represented as a portfolio VaR where we should take autocorrelation into account.

If we further impose the condition that the individual returns are $i.i.d$ then we can derive simpler expression for the portfolio/multi-period VaR. However this strong condition is generally not held.

References

Hull, John (2012), *Risk Management and Financial Institutions,+ Web Site*, Vol. 733, John Wiley & Sons.