1.a)

1. a) $y = \sum_j m_j w_j x_j$

$$E(y) = E\left(\sum_j m_j w_j x_j\right)$$
$$= \sum_j E(m_j w_j x_j)$$
$$= \sum_j E(m_j) E(w_j) E(x_j)$$

Since we know that $E(m_j) = \frac{1}{2}$

then

$$\boxed{= \frac{1}{2}\sum_j E(w_j) E(x_j)}$$

Since we are given X and W

then $E(w_j) = w_j$, $E(x_j) = x_j$

then we get

$$\boxed{= \frac{1}{2}\sum_j w_j x_j}$$

$$Var(y) = Var\left(\sum_j m_j w_j x_j\right)$$

$$= \sum_j var\left(\sum_j m_j w_j x_j\right)$$

Since X, W are given

$$= \sum_j (w_j x_j)^2 Var(m_j)$$

According to question, we know m_j are all iid Bernoulli random variable, then

$$Var(m_j) = \frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{1}{4}$$

$$\boxed{Var(y) = \sum_j \frac{1}{4}(w_j x_j)^2}$$

b)

b) According to a) $E(y) = \frac{1}{2}\sum_j w_j x_j$

then $\boxed{\hat{w}_j = \frac{1}{2} w_j}$

c)

c) $J = \frac{1}{2N}\sum_{i=1}^{N} E[(y^{(i)} - t^{(i)})^2]$

$= \frac{1}{2N}\sum_{i=1}^{N} E[(y^{(i)})^2 - 2(y^{(i)}t^{(i)}) + (t^{(i)})^2]$

$= \frac{1}{2N}\sum_{i=1}^{N} (E(y^{(i)2}) - 2E(y^{(i)}t^{(i)}) + E(t^{(i)2}))$

Here $E(y^{(i)2}) = Var(y^{(i)}) + E(y^{(i)})^2$

$2E(y^{(i)}t^{(i)}) = 2(\tilde{y}^{(i)}t^{(i)})$ since b) and $t^{(i)}$ is constant

$E(t^{(i)})^2 = t^{(i)2}$ Since $t^{(i)}$ is lable so it is constant

So $J = \frac{1}{2N}\sum_{i=1}^{N} (Var(y^{(i)}) + E(y^{(i)})^2 - 2\tilde{y}^{(i)}t^{(i)} + t^{(i)2})$

Since $Var(y^{(i)}) = \frac{1}{4}\sum_j w_j x_j^{(i)}$

$E(y^{(i)})^2 = \tilde{y}^{(i)2}$

$J = \frac{1}{2N}\sum_{i=1}^{N}((\frac{1}{4}\sum_j (w_j x_j^{(i)})^2) + \tilde{y}^{(i)2} - 2\tilde{y}^{(i)}t^{(i)} + t^{(i)2})$

$= \frac{1}{2N}\sum_{i=1}^{N}(\tilde{y}^{(i)} - t^{(i)})^2 + \underbrace{\frac{1}{8N}\sum_{i=1}^{N}\sum_j (w_j x_j^{(i)})^2}_{R}$

according to b), $w_j = 2\tilde{w}_j$

then $\boxed{R} = \frac{1}{8N}\sum_{i=1}^{N}\sum_j (2\tilde{w}_j x_j^{(i)})^2$

$\boxed{= \frac{1}{2N}\sum_{i=1}^{N}\sum_{j=1}^{D}(\tilde{w}_j x_j^{(i)})^2}$ Here according to question, j is from 1 to D.

Then

$J = \boxed{\frac{1}{2N}\sum_{i=1}^{N}(\tilde{y}^{(i)} - t^{(i)})^2 + R(\tilde{w}_1 \dots \tilde{w}_D)}$

2)

2. For $t \geq 1$,

we can get

$h^{(t)} = g(Ux^{(t)} + Wh^{(t-1)} + bh)$

$y^{(t)} = g(Vh^{(t)} + b_y)$ where g is active function.

we can let $\boxed{g(f) = \begin{cases} 1 & \text{if } f > 0 \\ 0 & \text{if } f \leq 0 \end{cases}}$

Here we can let $\boxed{U = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}}$

then $Ux^{(t)} = x_1^{(t)} + x_2^{(t)}$ at time t.

Set $\boxed{W = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}}$ we can get

$h_1^{(t)} = g(x_1^{(t)} + x_2^{(t)} + h_1^{(t-1)} + bh_1)$

$h_2^{(t)} = g(x_1^{(t)} + x_2^{(t)} + h_1^{(t-1)} + bh_2)$

$h_3^{(t)} = g(x_1^{(t)} + x_2^{(t)} + h_1^{(t-1)} + bh_3)$

here we can let $\boxed{b_h = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}}$

We can let $\boxed{V = (-1, 1, 1)}$ $\boxed{b_y = 0}$,

then $y^{(t)} = g(-h_1^{(t)} + h_2^{(t)} + h_3^{(t)})$

then if Sum $(x_1^{(t)}, x_2^{(t)}, h_1^{(t-1)}) = 3$, $y^{(t)} = 1$, $h^{(t)} = 1$

Sum $= 2$ $y^{(t)} = 0$ $h^{(t)} = 1$

Sum $= 1$ $y^{(t)} = 1$ $h^{(t)} = 0$

Sum $= 0$ $y^{(t)} = 0$ $h^{(t)} = 0$