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DualSPHysics+: An enhanced DualSPHysics with improvements in accuracy, energy conservation and resolution of the continuity equation

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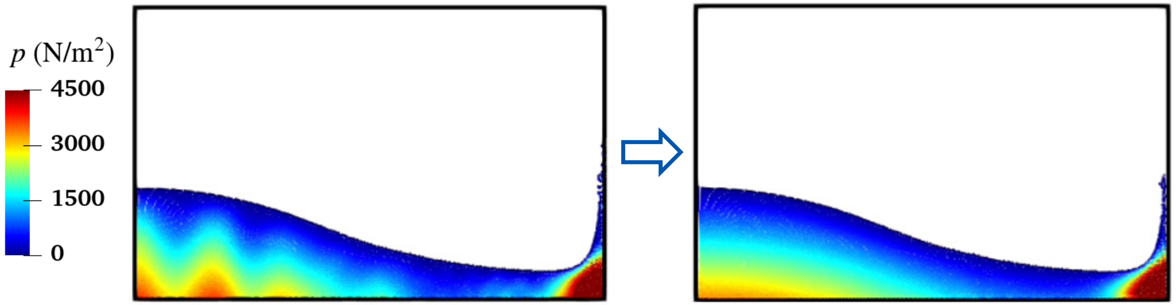
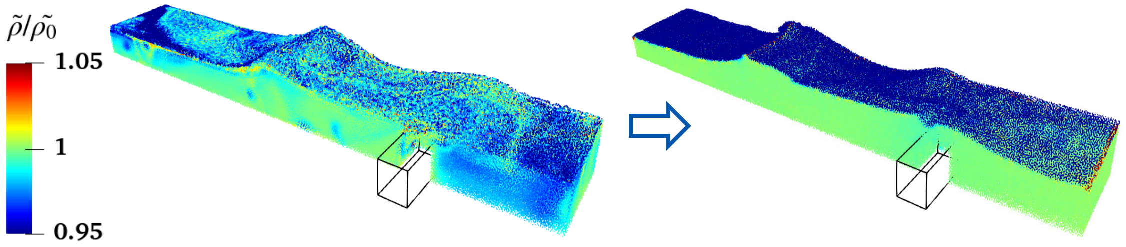
19 November 2024





1 Background

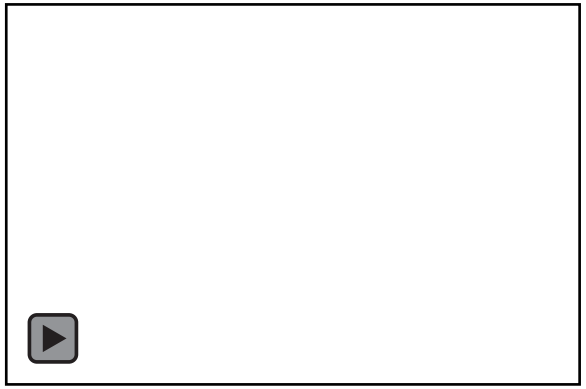
➢ Dam-break flow

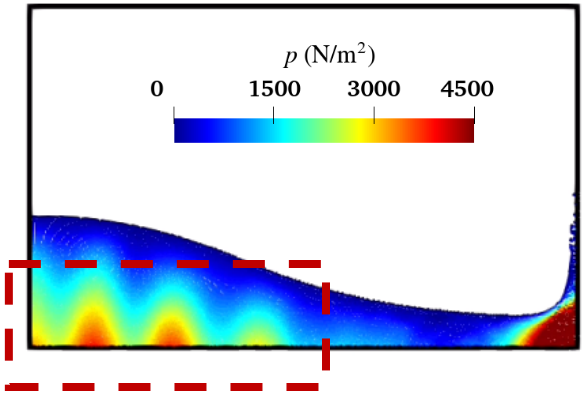
WCSPH

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \boldsymbol{u}$$

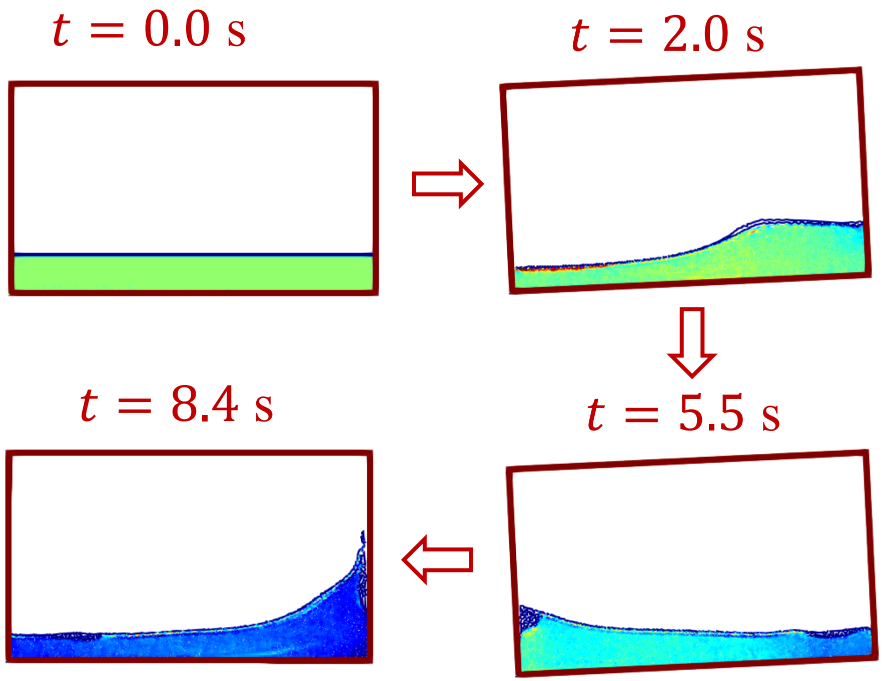
$$\frac{D\boldsymbol{u}}{Dt} = -\frac{\nabla p}{\rho} + \boldsymbol{g}$$

$$p = c_0^2(\rho - \rho_0)$$

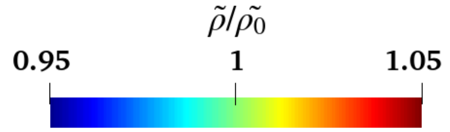




➢ Sloshing flow



$$\tilde{\rho}_a = \sum_b m_b W_{ab}$$



■ Oscillations of pressure
(acoustic wave + numerical discretization errors)

■ Volume non-conservation during simulation

2 Developments of DualSPHysics+: Overview

δ-SPH model in DualSPHysics

$$\frac{D\rho_a}{Dt} = -\rho_a \sum_b \boldsymbol{u}_{ab} \cdot \nabla_a W_{ab} V_b + D_a, \quad \frac{D\boldsymbol{u}_a}{Dt} = -\sum_b m_b \left(\frac{p_b + p_a}{\rho_b \rho_a} + \Pi_{ab} \right) \nabla_a W_{ab} + \boldsymbol{g}$$

DualSPHysics

Further enhancements corresponding to four aspects

i) Minimum energy dissipation ← Riemann stabilization term instead of artificial viscosity

ii) Consistent particle shifting at and in the vicinity of free surface ← OPS

iii) Enhanced resolution of the continuity equation ← VEM and VCS

iv) Effective cleaning of velocity-divergence errors ← Combination of VEM and HPDC

δR-SPH-OPS-VCS-VEM-HPDC in DualSPHysics+ [1]

$$\frac{D\rho_a}{Dt} = -\rho_a \sum_b \boldsymbol{u}_{ab} \cdot \nabla_a W_{ab} V_b + D_a, \quad \frac{D\boldsymbol{u}_a}{Dt} = -2\sum_b m_b \left(\frac{p^*}{\rho_a \rho_b} \right) \nabla_a W_{ab} + \boldsymbol{g} + \boldsymbol{a}_a^{\mathrm{VEM}} - \nabla \psi_a, \quad p^* = \frac{1}{2} F(p_L, p_R) + \frac{1}{2} \phi \bar{\rho} (u_L - u_R), \quad \boldsymbol{r}'_a = \boldsymbol{r}_a + \delta \boldsymbol{r}_a^{\mathrm{OPS}} + \delta \boldsymbol{r}_a^{\mathrm{VCS}}$$

[1] Zhan, Yi, Min Luo, and Abbas Khayyer. "DualSPHysics+: An enhanced DualSPHysics with improvements in accuracy, energy conservation and resolution of the continuity equation." *Computer Physics Communications* (2024): 109389.

2.1 DualSPHysics+: Combination of VEM and HPDC

➢ Velocity-divergence Error Mitigating (VEM) [1]

$$\boxed{p_a^{\mathrm{VEM}}} = c_s^2 d\rho_a = c_s^2 \Delta t \left(\frac{D\rho}{Dt}\right)_a^{k-1} = -\rho_a c_s^2 \Delta t \boxed{\langle \nabla \cdot \boldsymbol{u} \rangle_a^{k-1}}$$

Pressure related to velocity divergence error

This error will be accumulated

$$\boldsymbol{a}_a^{\mathrm{VEM}} = -\frac{1}{\rho_a}\sum_b F\left(p_a^{\mathrm{VEM}}, p_b^{\mathrm{VEM}}\right)\nabla_a W_{ab} V_b$$

$$F\left(p_a, p_b\right) = \begin{cases} p_b + p_a & \left(p_a \geq 0 \cup a \notin \Omega_{IN}\right) \\ p_b - p_a & \left(p_a < 0 \cap a \in \Omega_{IN}\right) \end{cases}$$

$$\frac{D\boldsymbol{u}_a}{Dt} = -\sum_b m_b \left(\frac{F\left(p_a, p_b\right)}{\rho_a \rho_b} + \Pi_{ab}\right)\nabla_a W_{ab} + \boldsymbol{g}_a + \boxed{\boldsymbol{a}_a^{\mathrm{VEM}}}$$

➢ Hyperbolic/Parabolic Divergence Cleaning (HPDC) [2]

$$\begin{cases} \left(\dfrac{D\boldsymbol{u}}{Dt}\right)_\psi + \nabla \psi = 0 \\ \mathcal{D}(\psi) + \nabla \cdot \boldsymbol{u} = 0 \end{cases}$$

Hyperbolic term

Parabolic term

$$\frac{\partial^2 (\nabla \cdot \boldsymbol{u})}{\partial t} - \boxed{c_h^2 \nabla^2 (\nabla \cdot \boldsymbol{u})} + \boxed{\frac{c_h^2}{c_p^2}\frac{\partial (\nabla \cdot \boldsymbol{u})}{\partial t}} = 0$$

$$\frac{D\boldsymbol{u}_a}{Dt} = -2\sum_b m_b \left(\frac{p^*}{\rho_a \rho_b}\right)\nabla_a W_{ab} + \boldsymbol{g} + \boldsymbol{a}_a^{\mathrm{VEM}} \boxed{-\nabla \psi_a}$$

$\psi = 0$ for boundary particles

[1] Khayyer et al. "Enhanced resolution of the continuity equation in explicit weakly compressible SPH simulations of incompressible free-surface fluid flows." *Applied Mathematical Modelling* 116 (2023): 84-121.

[2] Fourtakas et al. "Divergence cleaning for weakly compressible smoothed particle hydrodynamics." *arXiv preprint arXiv*:2410.06038 (2024).

2.2 DualSPHysics+: Concept of VEM$^{\text{II}}$ and VEM$^{\text{III}}$

➤ VEM$^{\text{I}}$

$$\boxed{p_a^{\text{VEM}^{\text{I}}}} = c_s^2 \Delta t \left(\frac{D\rho}{Dt}\right)_a^{k-1} = -\rho_a c_s^2 \Delta t \langle \nabla \cdot \boldsymbol{u} \rangle_a^{k-1}$$

Based on instantaneous errors ⇩

$$\boldsymbol{a}_a^{\text{VEM}} = -\frac{1}{\rho_a} \sum_b F\left(p_a^{\text{VEM}}, p_b^{\text{VEM}}\right) \nabla_a W_{ab} V_b$$

$$F(p_a, p_b) = \begin{cases} p_b + p_a & (p_a \geq 0 \cup a \notin \Omega_{IN}) \\ p_b - p_a & (p_a < 0 \cap a \in \Omega_{IN}) \end{cases}$$

➤ VEM$^{\text{II}}$

$$\left\{ \nabla \cdot \left(\frac{\nabla \boxed{p^{\text{VEM}^{\text{II}}}}}{\rho} \right) \right\}_a^{k+1} = \frac{\langle \nabla \cdot \boldsymbol{u} \rangle_a^k}{\Delta t}$$

⇩ Based on the projection-based particle methods

$$\boldsymbol{a}_a^{\text{VEM}} = -\frac{1}{\rho_a} \sum_b F\left(p_a^{\text{VEM}}, p_b^{\text{VEM}}\right) \nabla_a W_{ab} V_b$$

$$F(p_a, p_b) = \begin{cases} p_b + p_a & (p_a \geq 0 \cup a \notin \Omega_{IN}) \\ p_b - p_a & (p_a < 0 \cap a \in \Omega_{IN}) \end{cases}$$

- VEM$^{\text{II}}$ → Projecting the instantaneous velocity field onto a velocity divergence space
- VEM$^{\text{III}}$ concurrently applies both VEM$^{\text{I}}$ and VEM$^{\text{II}}$

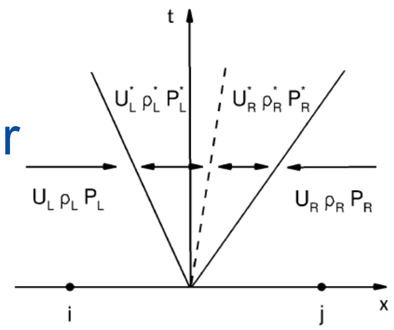
2.3 DualSPHysics+: δR-SPH

Artificial viscosity

$$\frac{D\boldsymbol{u}_a}{Dt} = -\sum_b m_b\left(\frac{p_b+p_a}{\rho_b\rho_a}+\boxed{\Pi_{ab}}\right)\nabla_a W_{ab}+\boldsymbol{g}$$

⇩ Riemann solver

$$\boxed{\frac{D\boldsymbol{u}_a}{Dt} = -2\sum_b m_b\left(\boxed{\frac{p^*}{\rho_a\rho_b}}\right)\nabla_a W_{ab}+\boldsymbol{g}}$$



Implicitly and properly introduce minimum dissipation via Riemannian solvers

- Traditional Riemann solver for continuity equation

$$\frac{D\rho_i}{Dt} = -\rho_i\sum_j \boldsymbol{u}_{ij}\cdot\nabla_i w_{ij}V_j + D_i^R, \boxed{D_i^R} = 2\rho_i c_0\sum_j \frac{\rho_j-\rho_i}{\rho_j+\rho_i}\frac{\boldsymbol{r}_{ij}}{|\boldsymbol{r}_{ij}|}\cdot\nabla_i w_{ij}V_j$$

$$\sum_i D_i^R V_i = \sum_i\sum_j 2\rho_i c_0 \frac{\rho_j-\rho_i}{\rho_j+\rho_i}\frac{\boldsymbol{r}_{ij}}{|\boldsymbol{r}_{ij}|}\cdot\nabla_i w_{ij}V_j V_i \boxed{\neq 0}$$

Diffusive term is not globally conserved [1]

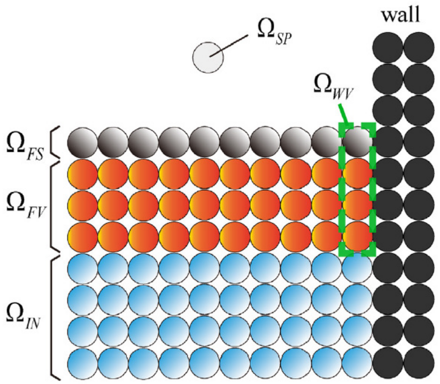
$$\boxed{D_a = \delta h c_s\sum_b \psi_{ab}\cdot\nabla_a W_{ab}V_b,\ \psi_{ab} = 2\left(\rho_{ba}^T-\rho_{ab}^H\right)\frac{r_{ab}}{|r_{ab}|^2}}$$

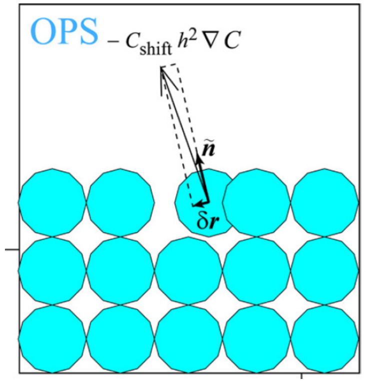
- The combination of the δ-SPH density diffusion term in the continuity equation and a Riemann stabilization term in the momentum equation is adopted

[1] Khayyer et al. "An improved Riemann SPH-Hamiltonian SPH coupled solver for hydroelastic fluid-structure interactions." *Engineering Analysis with Boundary Elements* 158 (2024): 332-355.

DualSPHysics+: Particle shifting

➢ Optimized Particle shifting (OPS) [1]





I. Conduct particle type detections
II. Compute shifting vector based on Fick's law of diffusion
III. Eliminate the normal shifting component for Ω_{FS}, Ω_{FV}

$$\boxed{C_a} = \sum_b W_{ab} V_b, \delta \boldsymbol{r}_a^{\text{OPS}} = -C_{\text{shift}} h^2 \left(\boldsymbol{I} - \tilde{\boldsymbol{n}}_a \otimes \tilde{\boldsymbol{n}}_a \right) \cdot \nabla C_a$$

Particle concentration

➢ Volume Conservation Shifting (VCS) [2]

$$\tilde{\rho}_a = \sum_b m_b W_{ab}$$

Volume conservation error ⇩

$$\left\{ \nabla \cdot \left(\frac{\nabla p^{\text{VCS}}}{\rho} \right) \right\}_a^{k+1} = \boxed{\frac{\tilde{\rho}_0 - \tilde{\rho}_a^k}{\tilde{\rho}_0 \Delta t^2}} + \frac{1}{\Delta t^2 \tilde{\rho}_0 c_s^2} p_a^{\text{VCS},k+1}$$

Explicitly solved ⇩

$$\delta \boldsymbol{r}_a^{\text{VCS}} = \Delta t^2 \left(-\frac{1}{\rho_a} \left\langle \nabla p^{\text{VCS}} \right\rangle_a^{\text{L}} \right)$$

$$\boldsymbol{r}_a' = \boldsymbol{r}_a + \boxed{\delta \boldsymbol{r}_a^{\text{OPS}}} + \boxed{\delta \boldsymbol{r}_a^{\text{VCS}}}$$

[1] Khayyer et al. "Comparative study on accuracy and conservation properties of two particle regularization schemes and proposal of an optimized particle shifting scheme in ISPH context." *Journal of Computational Physics* 332 (2017): 236-256.
[2] Khayyer et al. "Enhanced resolution of the continuity equation in explicit weakly compressible SPH simulations of incompressible free-surface fluid flows." *Applied Mathematical Modelling* 116 (2023): 84-121.

3 Numerical validations and applications

➢ Five cases

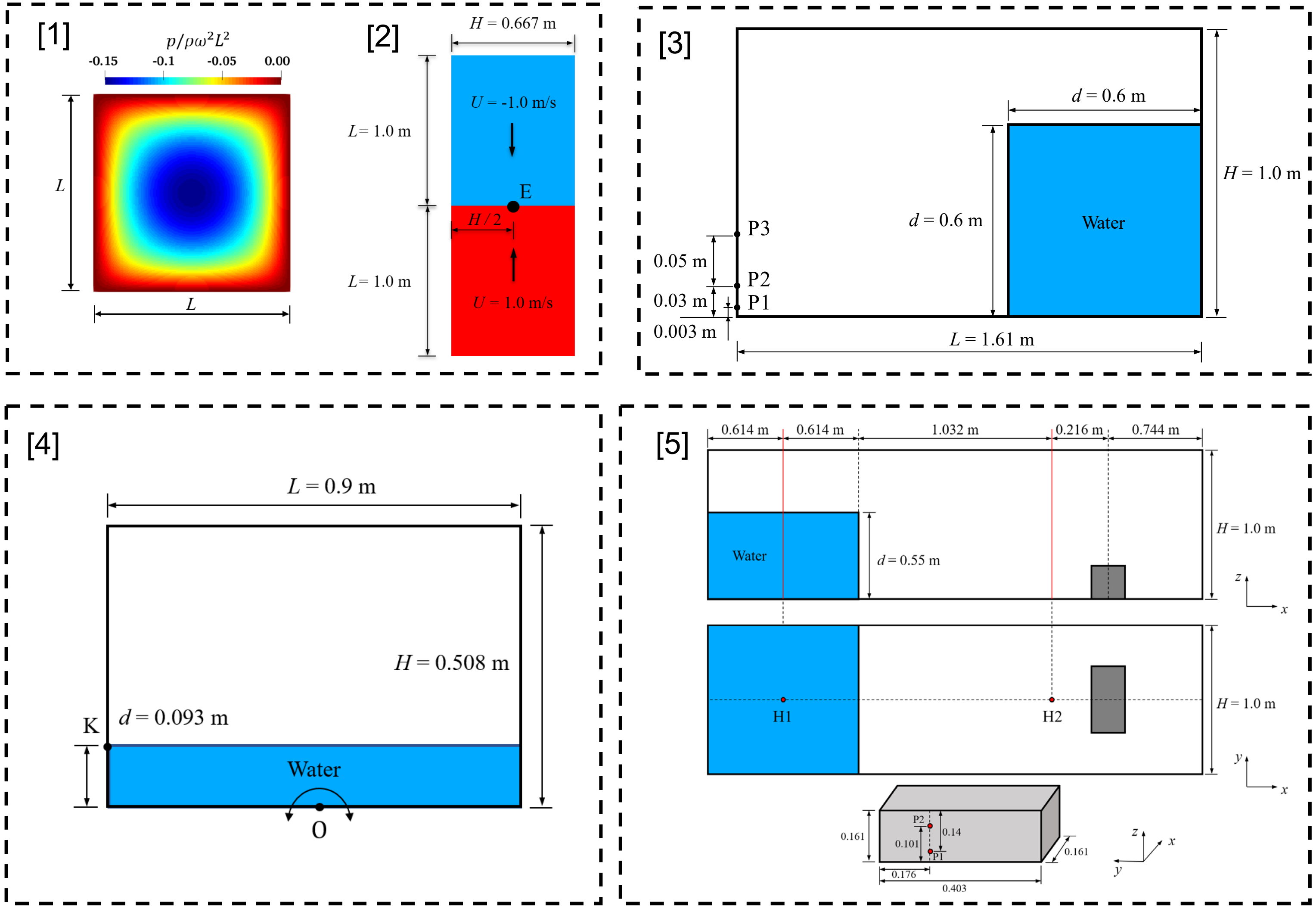
[1] A rotating fluid square

[2] Impact of two fluid patches

[3] Two-dimensional dam break

[4] Liquid sloshing

[5] Three-dimensional dam break



➢ References

[1] Le Touzé, D., et al. "A critical investigation of smoothed particle hydrodynamics applied to problems with free-surfaces." *IJNMF* (2013)

[2] Antuono et al. "Energy balance in the δ-SPH scheme." *CMAME* (2015)

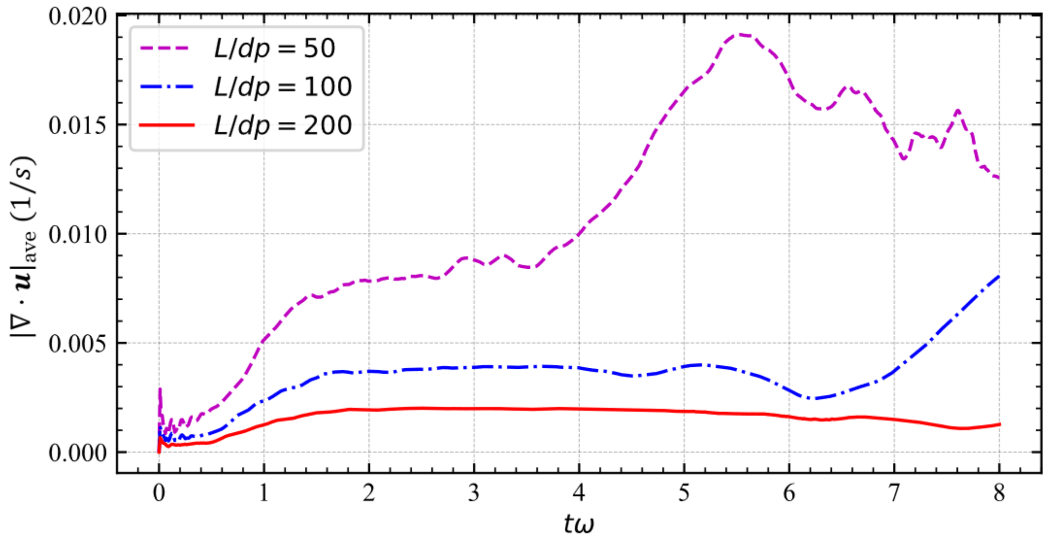
[3] Lobovský, Libor, et al. "Experimental investigation of dynamic pressure loads during dam break." *JFS* (2014)

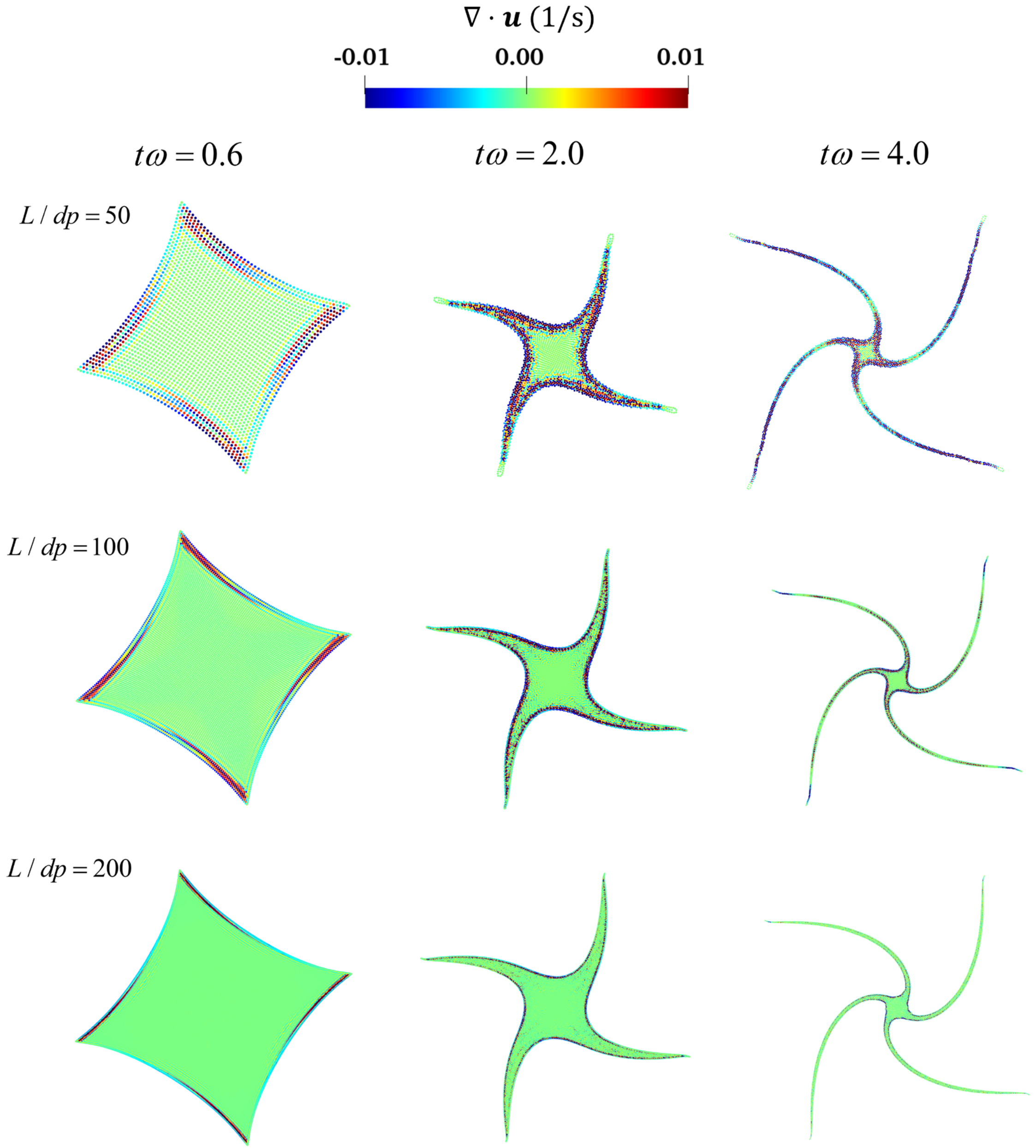
[4] Souto-Iglesias, A., et al. "A set of canonical problems in sloshing. Part 2: Influence of tank width on impact pressure statistics in regular forced angular motion." *OE* (2015)

[5] Kleefsman, K. M. T., et al. "A volume-of-fluid based simulation method for wave impact problems.“ *JCP* (2005)

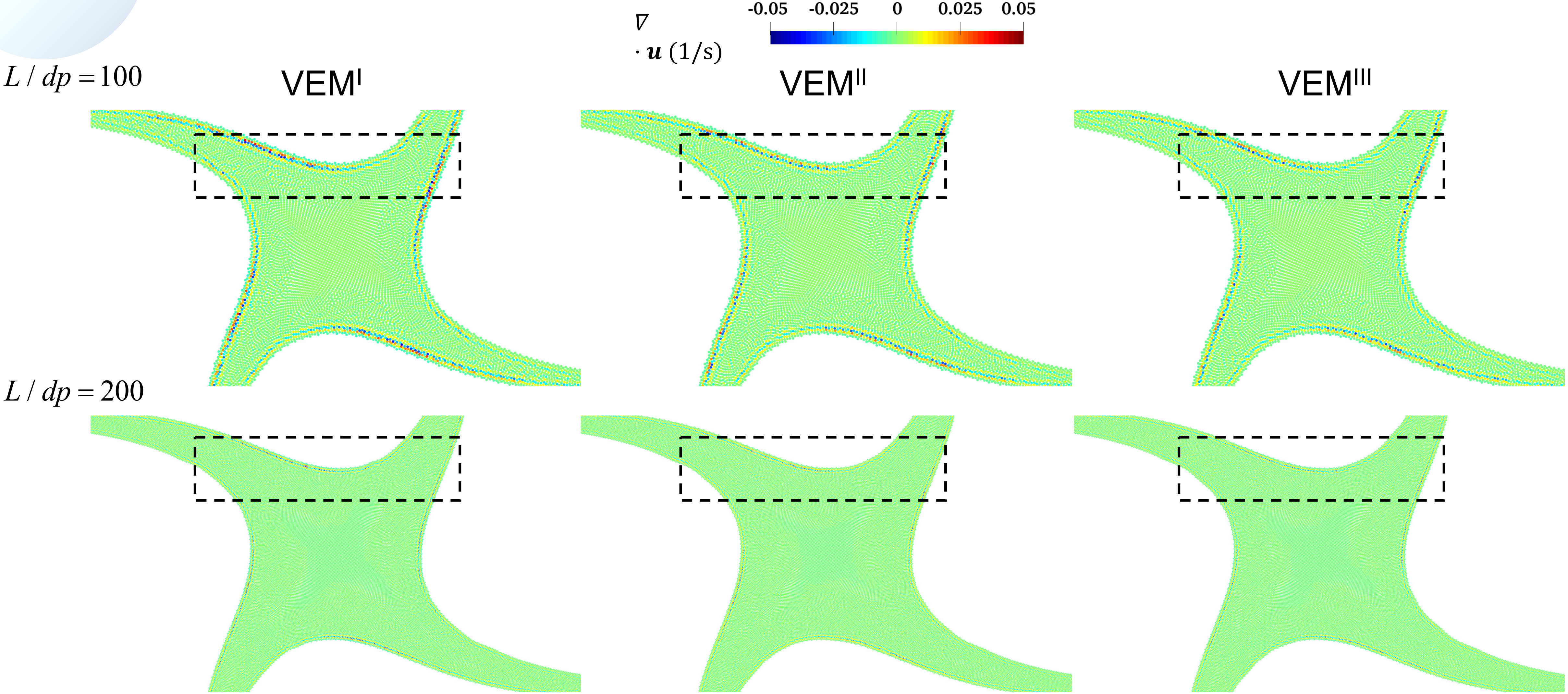
3.1 A rotating fluid square: Convergence analysis

- Pressure field smoother and highly-deformed tip shapes closer to the reference results
- Effectiveness and convergence of velocity divergence cleaning





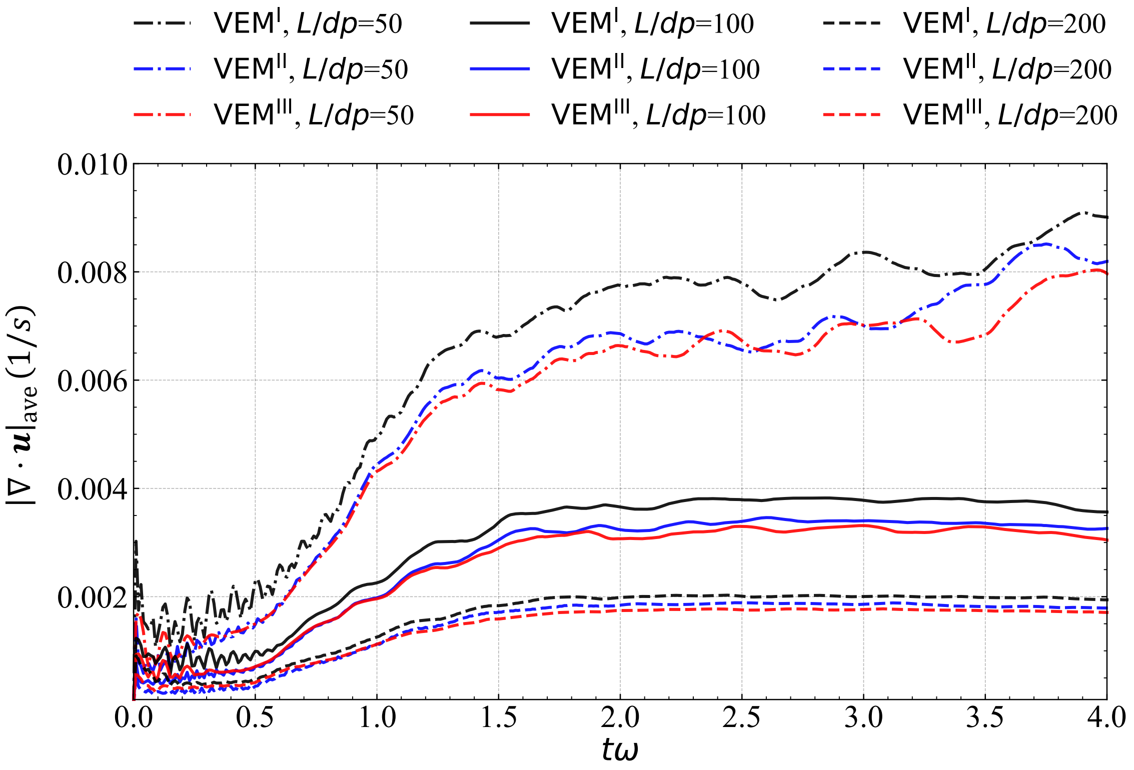
3.1 A rotating fluid square: Velocity divergence error



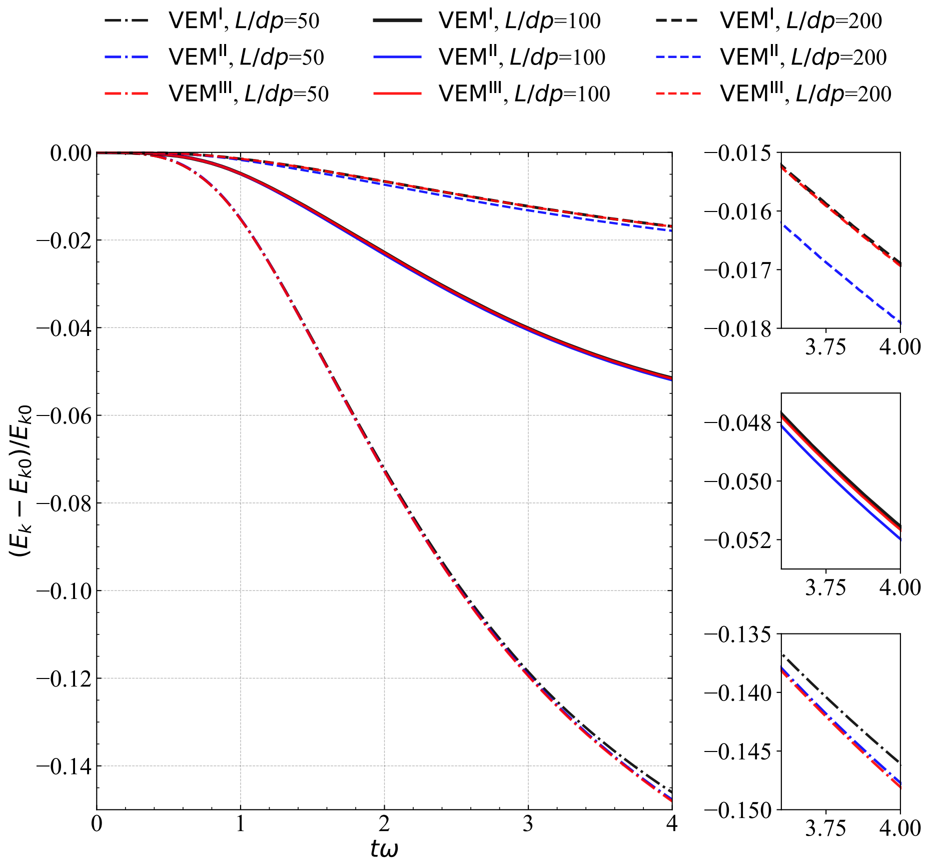
■ VEM$^{\mathrm{II}}$, VEM$^{\mathrm{III}}$ → Effective cleaning of velocity divergence error at and in the vicinity of free surfaces

3.1 A rotating fluid square: Velocity divergence and pressure

- Velocity divergence cleaning



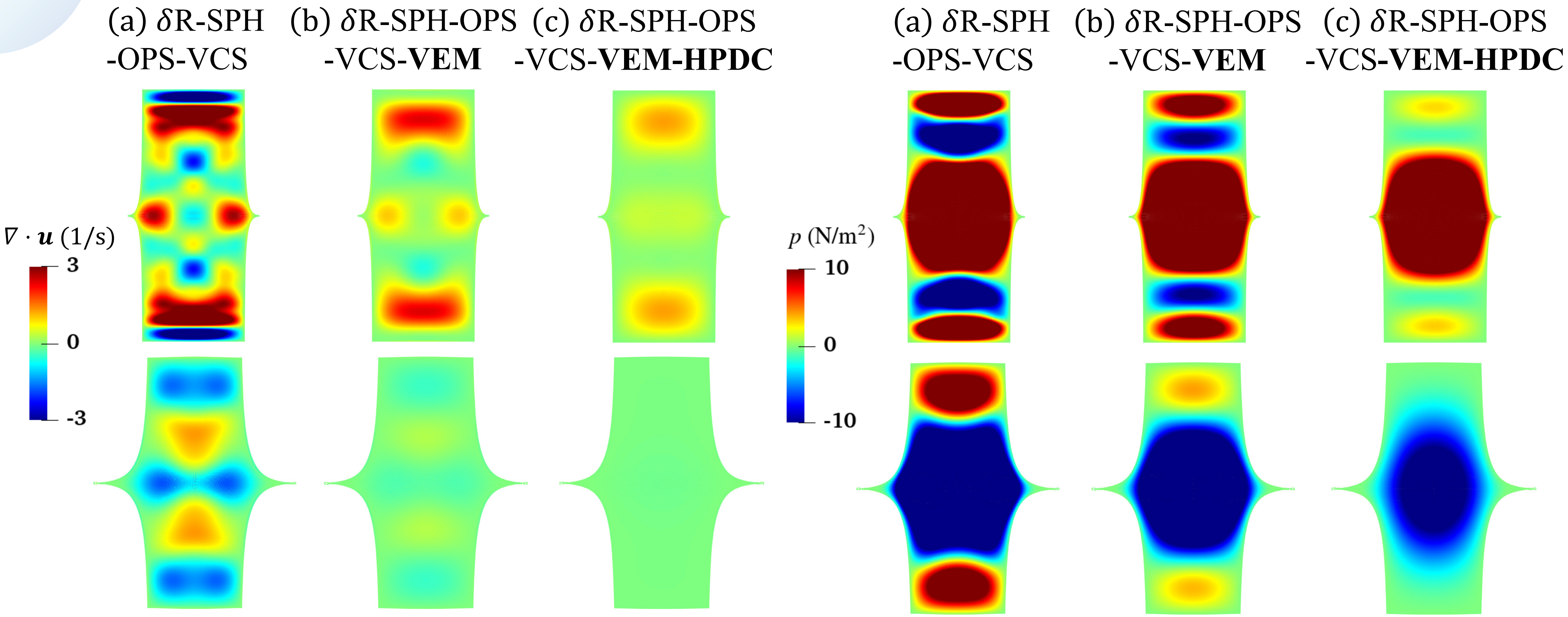
- Normalized mechanical energy



- VEM^{II} and VEM^{III} → More effective cleaning of velocity divergence error

- Without excessive dissipations in high resolutions

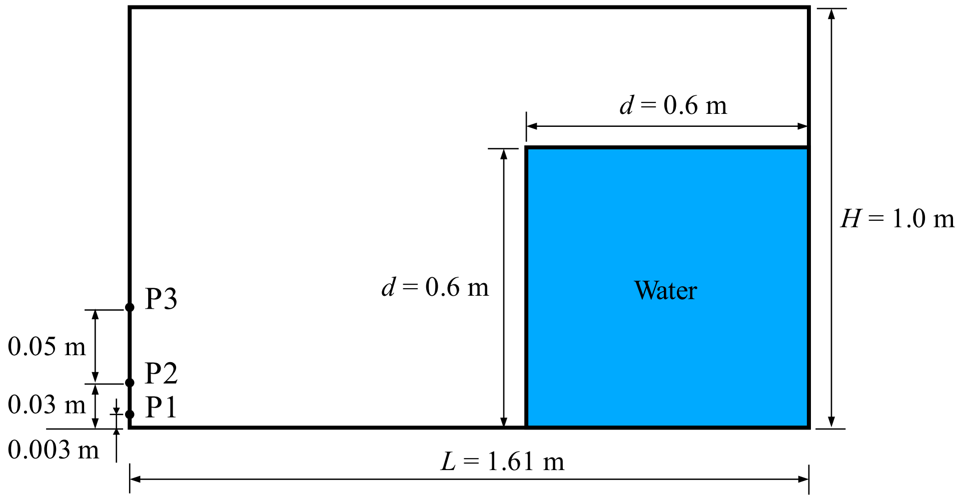
3.2 Impact of two fluid patches: Velocity divergence and pressure

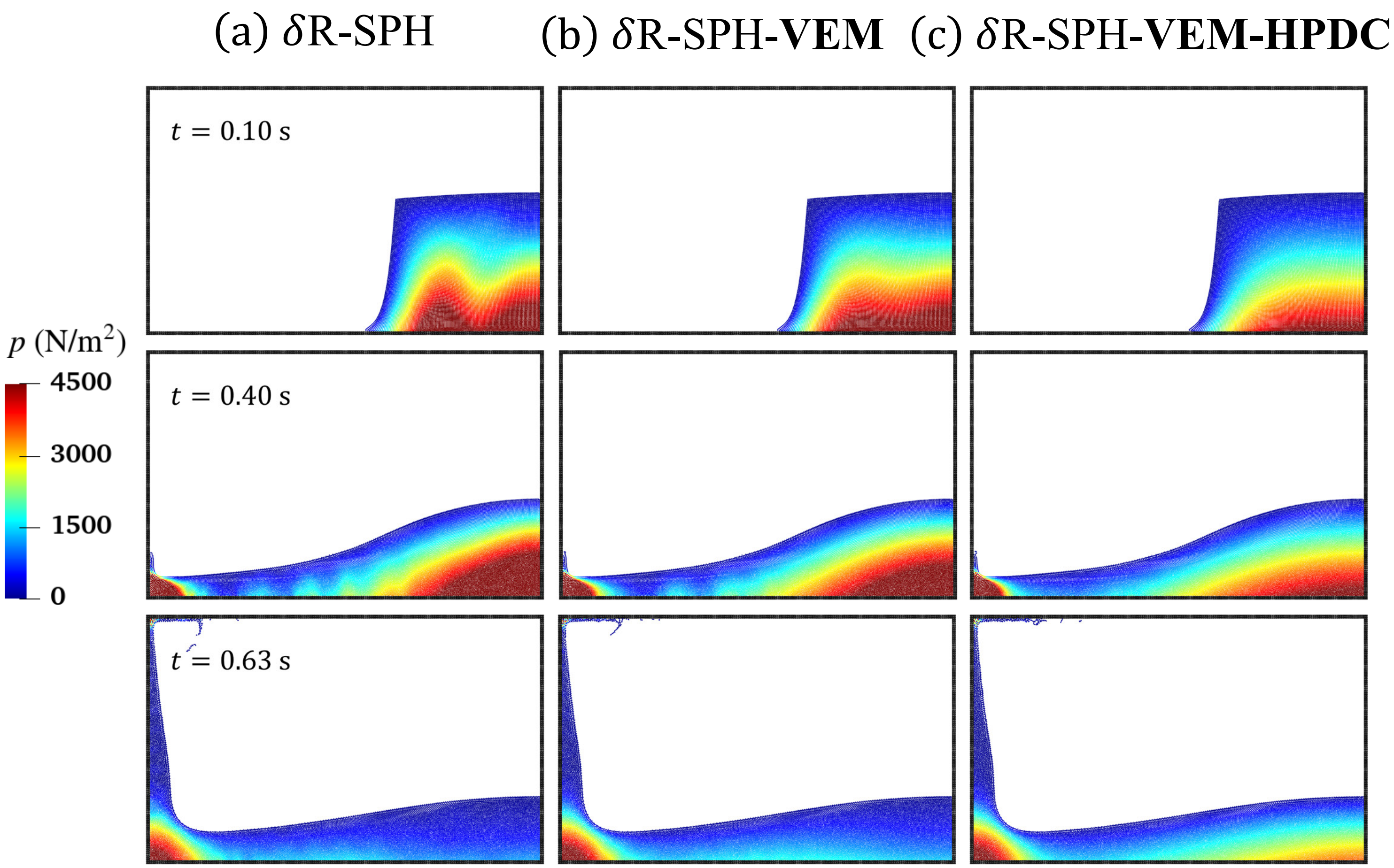


- Progressively adopting VEM and HPDC, velocity divergence errors reduced to nearly zero
- Compression/rarefaction waves partially reduced by VEM and further mitigated by the combination of VEM and HPDC

2D dam break: Pressure fields

- Numerical settings

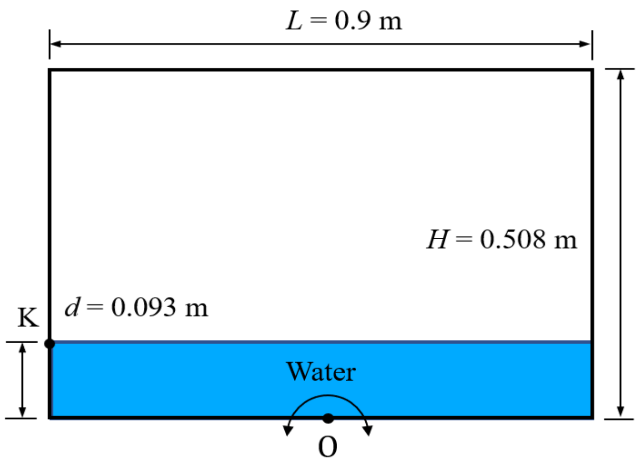


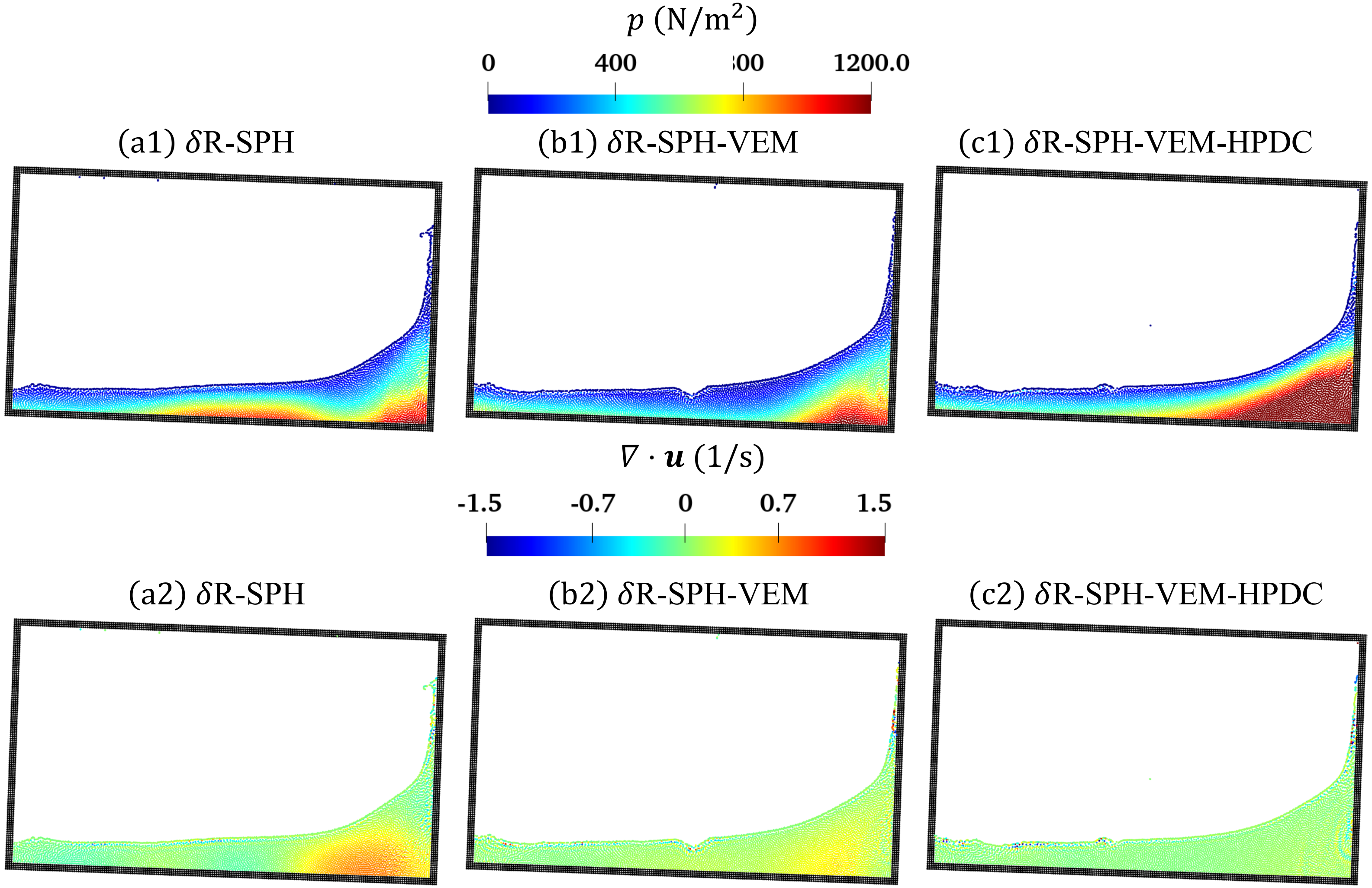


- Pressure noises partially removed by the VEM and almost eliminated by further incorporation of HPDC

Liquid sloshing : Velocity divergence and pressure

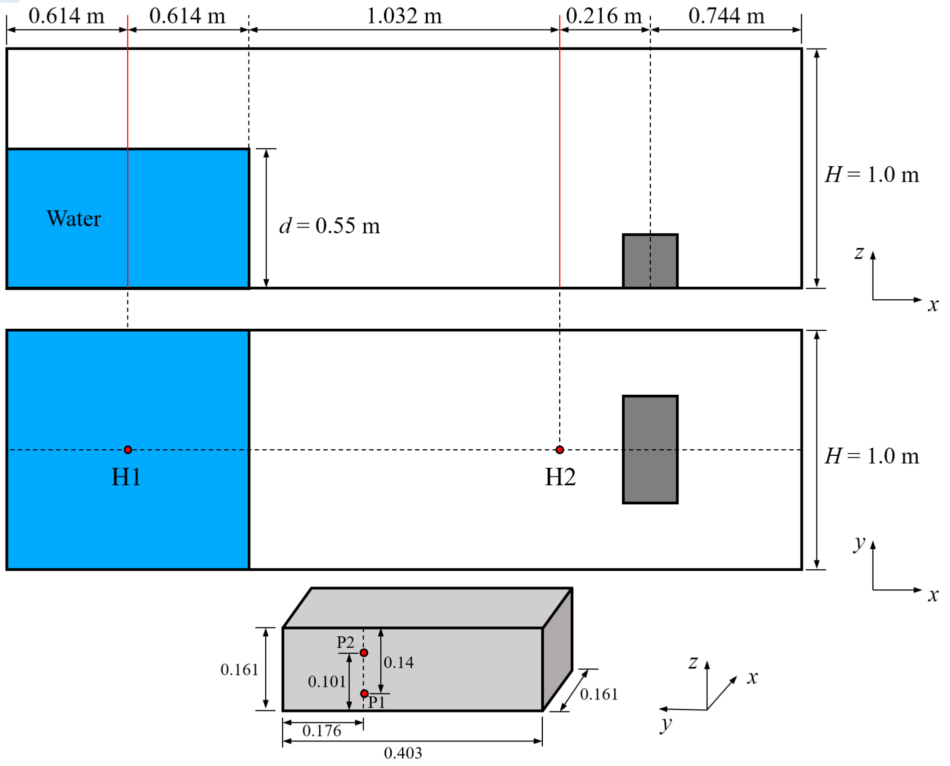
➢ Numerical settings



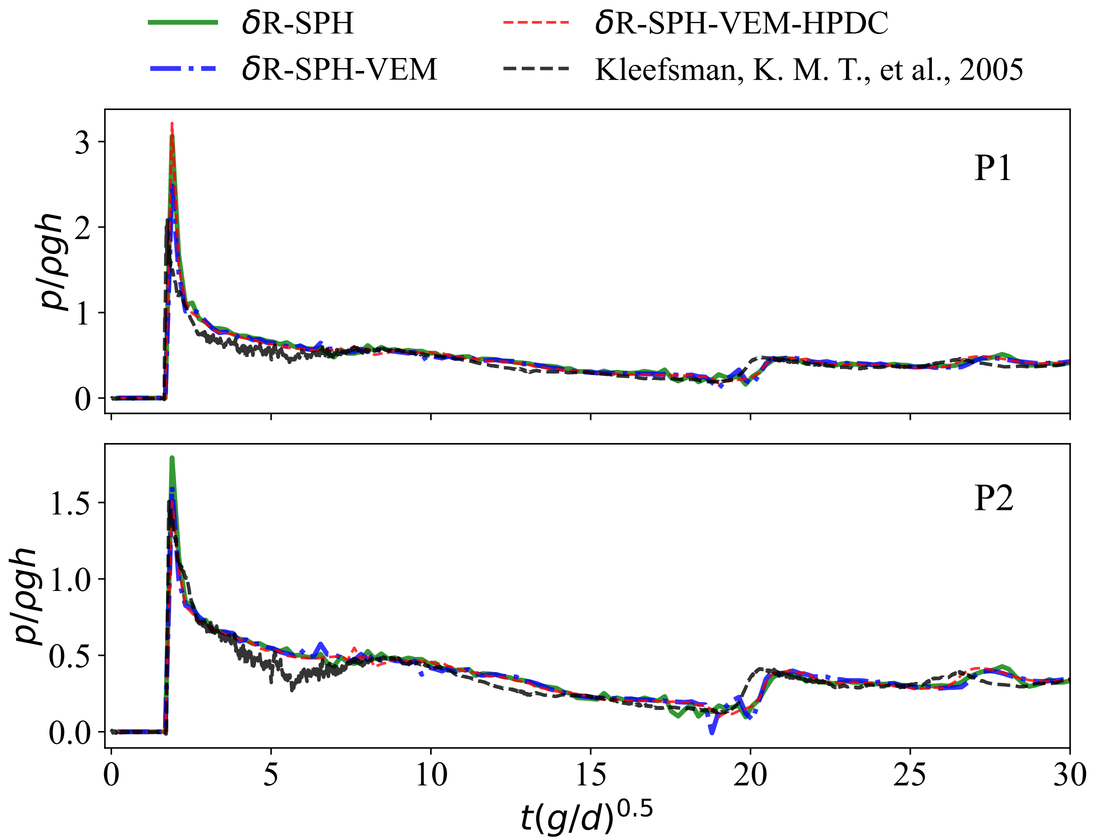


■ Pressure and velocity divergence fields are enhanced by VEM and HPDC

3.5 3D dam break: Pressure histories



dp = 0.01 m, N ≈ 1200000



- All schemes are implemented on both CPU and GPU → Efficient 3D simulations
- Pressure results are in satisfactory agreement with the experimental data

4 Summary: DualSPHyscis+

- Combination of VEM and HPDC → Cleans the velocity divergence errors effectively
- VEM^{II} → More effective cleaning of velocity divergence error at and in the vicinity of free surfaces
- δR-SPH → Retains numerical stability and mitigates excessive energy dissipation
- OPS → Implements particle shifting at and in the vicinity of free surfaces properly
- VEM, HPDC and VCS → Improve enforcement of velocity divergence-free and density invariance conditions, enhancing the resolution of the continuity equation

Parallelization on both the CPU (OpenMP) and GPU (CUDA)

CPC library

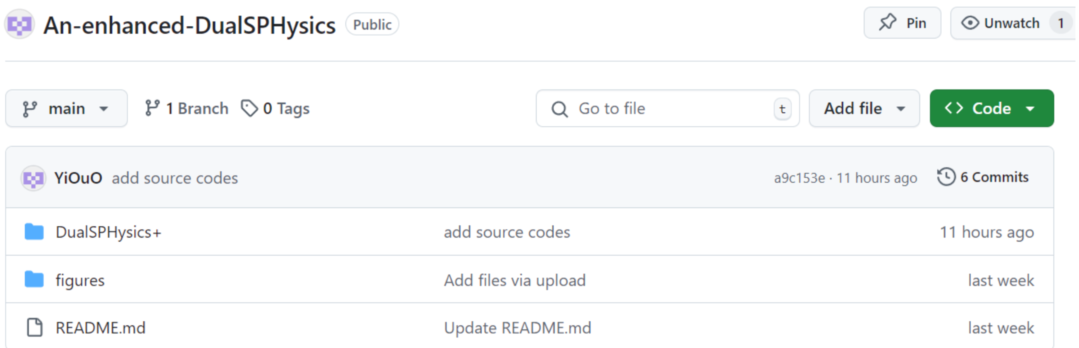
Mendeley Data

DualSPHysics+: An enhanced DualSPHysics with improvements in accuracy, energy conservation and resolution of the continuity equation

Published: 4 November 2024 | Version 1 | DOI: 10.17632/xnrfv9pgb5.1
Contributors: Yi Zhan, Min Luo, Abbas Khayyer

https://data.mendeley.com/datasets/xnrfv9pgb5/1

Long-term support



An-enhanced-DualSPHysics Public

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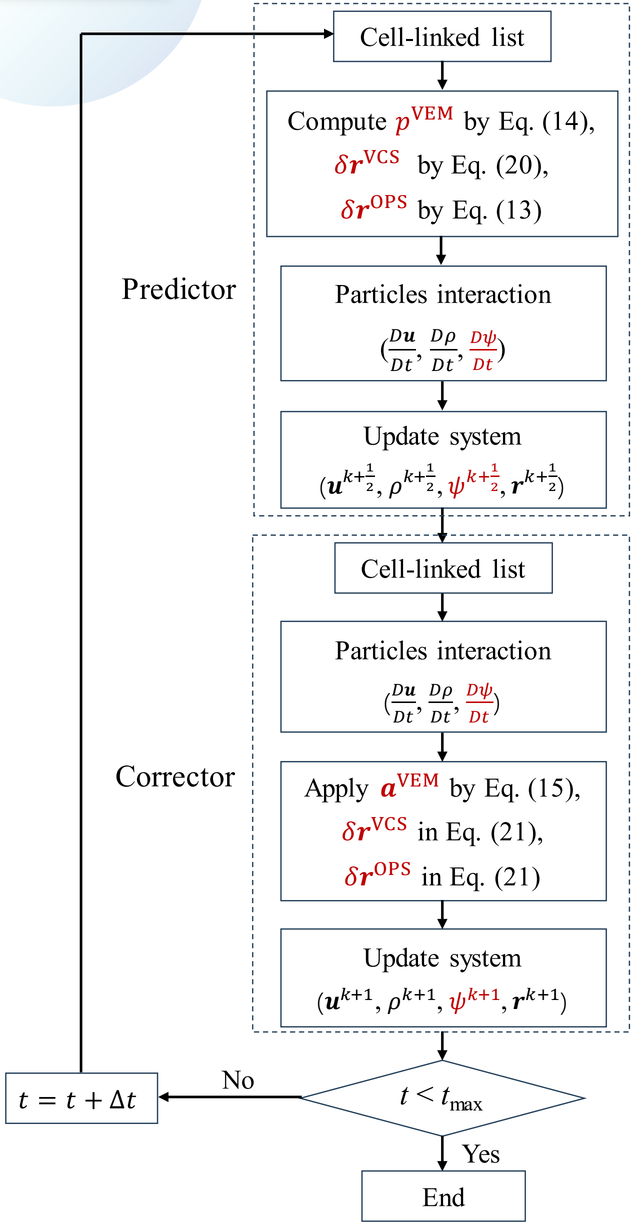
YiOuO add source codes	a9c153e · 11 hours ago	6 Commits
DualSPHysics+	add source codes	11 hours ago
figures	Add files via upload	last week
README.md	Update README.md	last week

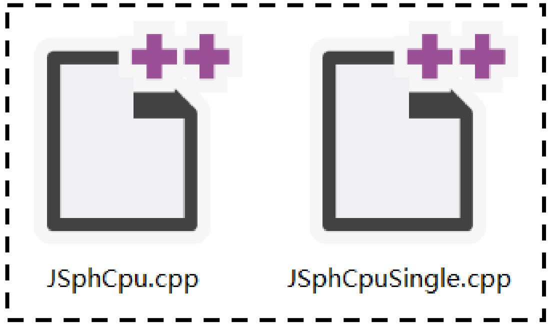
https://github.com/YiOuO/An-enhanced-DualSPHysics

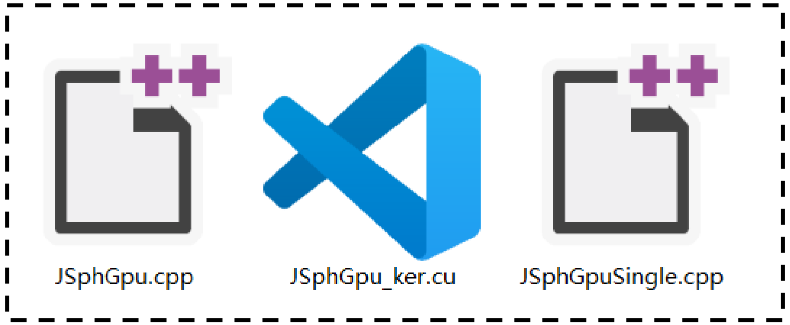
5 Program documentation: Overview



Program unit	Categories	Description
JSphCpu::InteractionForcesFluid KerInteractionForcesFluid	R	Riemann stabilization term based on Eq. (25).
JSphCpu::RunVEMVCSOPS_StepI KerRunVEMVCSOPS_StepI	VEM	Compute p_a^{VEM} by Eq. (14).
JSphCpu::RunVEMVCSOPS_StepII KerRunVEMVCSOPS_StepII	VEM	Compute a_a^{VEM} by Eq. (15) and add it into the momentum equation by Eq. (17).
JSphCpu::InteractionForcesFluid KerInteractionForcesFluid	HPDC	Compute the time derivative of ψ by Eq. (34) and add it to the momentum equation by Eq. (32).
JSphCpu::ComputeSymplecticPre JSphCpu::ComputeSymplecticCorr JSphGpu::ComputeSymplecticPre JSphGpu::ComputeSymplecticCorr	HPDC	Update ψ with predictor-corrector time integration scheme.
JSphCpu::InteractionForcesBoundDummy KerInteractionForcesBoundDummy	HPDC	Apply the Dirichlet boundary condition for ψ.
JSphCpu::ShiftFluid KerRunVEMVCSOPS_StepII JSphShifting::RunOPS	OPS	Compute the Particle Shifting vector by Eq. (13).
JSphCpu::ComputeSymplecticCorr JSphGpu::ComputeSymplecticCorr	OPS	Apply the Particle Shifting vectors based on Eq. (21).
JSphCpu::RunVEMVCSOPS_StepI KerRunVEMVCSOPS_StepI	VCS	Compute p^{VCS} by Eq. (19).
JSphCpu::RunVEMVCSOPS_StepII KerRunVEMVCSOPS_StepII	VCS	Compute δr_a^{VCS} by Eq. (20).
JSphCpuSingle::GetNormals KerGetNormals	Others	Compute normal vectors of particles based on Eq. (34) of Khayyer et al. [21].
JSphCpuSingle::FreeSurfaceDetction KerFreeSurfaceDetction	Others	Identify free-surface and free-surface-vicinity particles, inner fluid particles and wall vicinity particles.
JSphCpu::GetCorrectMatrix KerGetAuxarray	Others	Compute the renormalization matrix for gradient operators.

- Core files in CPU and GPU





5 Program documentation: Compile and run

Essential Software

C++ IDE

Visual studio 2022

GPU acceleration

CUDA C/C++

Post-processing

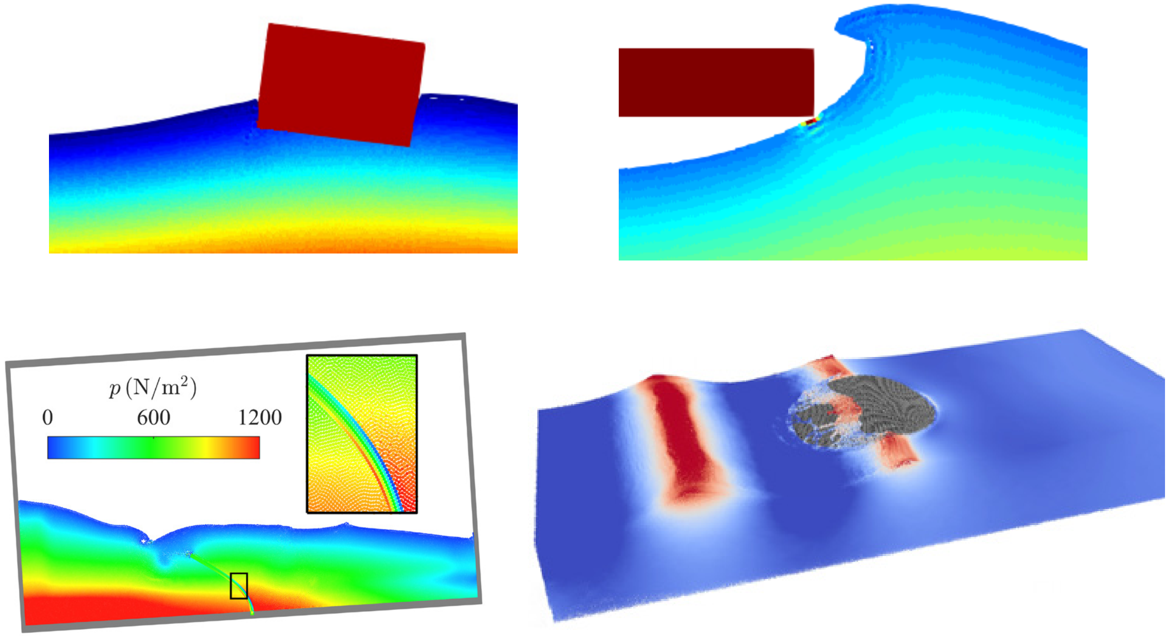
ParaView

➢ Steps to compile and run DualSPHysics+

- Step 1: Download DualSPHysics (V5.2 BETA for CPU and V5.2 for GPU) from its official homepage and replace the source files with files of DualSPHysics+
- Step 2: Recompile DualSPHysics+ via IDE or CMake
- Step 3: Create a new case folder and copy the case files from DualSPHysics+
- Step 4: Modify the relative path in batch file and case configuration in xml file to run the case

6 Ongoing/future work based on DualSPHysics+

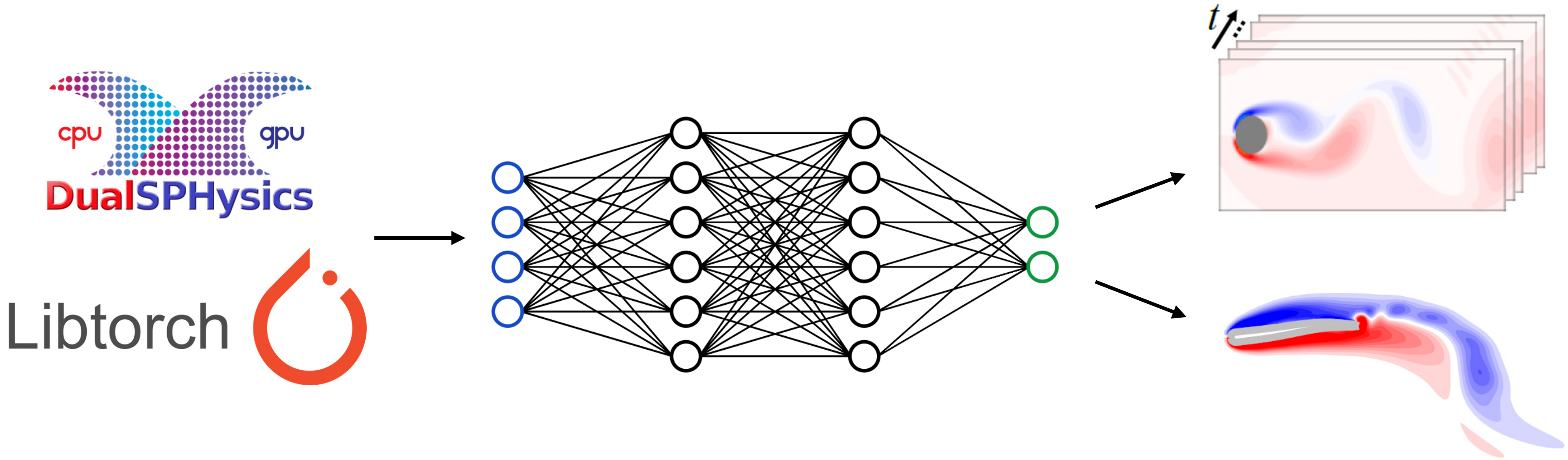
➢ Wave-structure interaction



- Wave interaction with flexible structures
- Wave slamming on structures
- Wave interaction with floating structures, e.g., floating breakwater, multi-connected floating photovoltaic
- 3D real-world WSI problems

6 Ongoing/future work based on DualSPHysics+

➢ AI-assisted DualSPHyscis+



- Coupling DualSPHysics+ with machine learning libraries
- More efficient simulations of fluid flows and WSIs
- Active control of ocean devices, e.g., underwater soft robot, marine renewable energy devices