Leverage Cycle over the Life Cycle: A Quantitative Model of Endogenous Leverage

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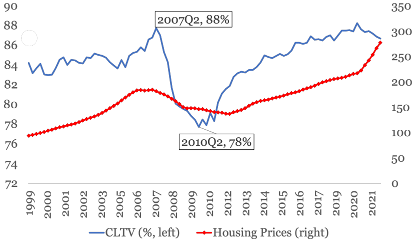
Introduction

This paper

- A quantitative model that rationalizes two facts on the US housing market
  - Leverage co-moves with housing prices
  - Mortgage spreads move in the opposite direction
- Three key elements
  - Overlapping-generation framework
  - Endogenous determination of leverage
  - Large aggregate shocks

Motivating Fact 1

- Leverage co-moves with housing prices (0.53)
- $\text{CLTV} \equiv \dfrac{\text{Total newly issued loan}}{\text{Appraised value of housing collateral}}$

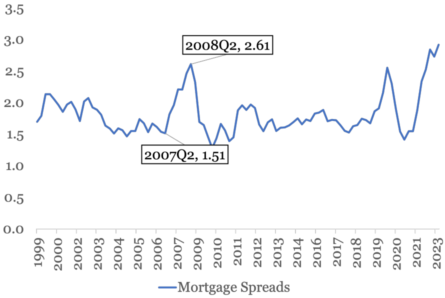


Data source: CLTV from Freddie Mac Single-Family Loan-Level Dataset, Case-Shiller housing price index from FRED, index Jan 2000=100.

Figure: Mean CLTV of First-time Homebuyers

Motivating Fact 2

- Mortgage spreads move in opposite direction (-0.29)



Data source: FRED MORTGAGE30US, DGS10.

Mortgage spread: average 30-year fixed-rate mortgage - 5-year treasury bill yield

Why do we care?

- Empirical importance: 94% of home purchases are financed through mortgages in the US
- Endogenous leverage theory (Fostel and Geanakoplos (2008) etc)
  - Leverage is a key driver in housing price volatility
  - A channel ignored by most macro-finance models
- Frictions in collateralized borrowing create room for policy interventions

Life Cycle Patterns

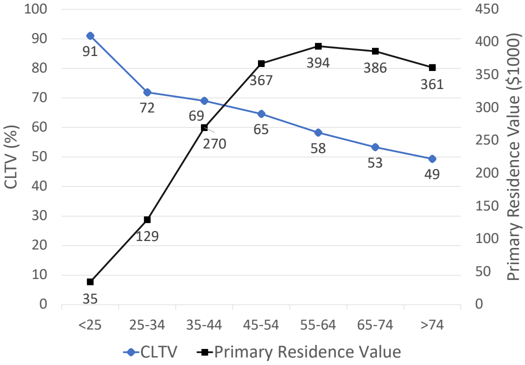


Figure: Housing Wealth and Leverage by Age

Data sources: CLTV: 2023 HDMA national loan-level dataset. Primary residence values: 2022 Survey of Consumer Finances dataset.

Model

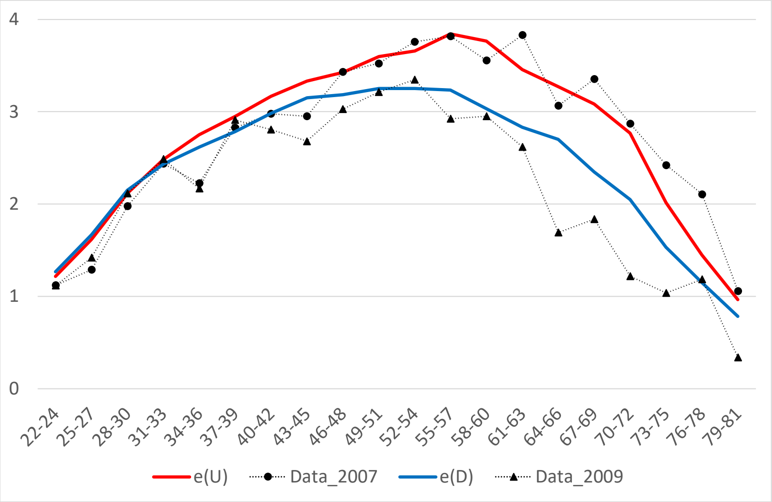
Model: Environment

- **Age**: $a \in \{1, \dots, A\}$
  - No uncertainty about lifetime
  - No heterogeneity within cohorts
- **Uncertainty**
  - $z_t \in \{U, D\}$, Markov process
- **Commodities**

		Spot Price	Agg. Supply
Consumption (c_t^a)	perishable	1	$\bar{e}(z_t)$
House (h_t^a)	perfectly durable, divisible	q_t	H

- **Endowments**
  - $\bar{e}(z_t) = \sum_{a=1}^{A} e^a(z_t) = \sum_{a=1}^{A} e_t^a$

Endowment age profiles



Model: Preferences

- Period Utility

$$u^a(c,h) = \begin{cases} \dfrac{(c^{1-\alpha}h^{\alpha})^{1-\rho}}{1-\rho} & a \in \{1,\ldots,A-1\} \\ \dfrac{c^{1-\rho}}{1-\rho} & a = A \end{cases}$$

- Lifetime Utility

$$U_t = E_t \sum_{a=1}^{A} \beta^{a-1} u^a(c_t^a, h_t^a)$$

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  - $\theta^a_{j,t} \in \mathbb{R}$
    - $\theta^a_{j,t} < 0$: short, borrow $|\pi_{j,t}\theta^a_{j,t}|$
    - $\theta^a_{j,t} > 0$: long, lend $|\pi_{j,t}\theta^a_{j,t}|$
  - Delivery: $\min\{j, q_{t+1}\}$
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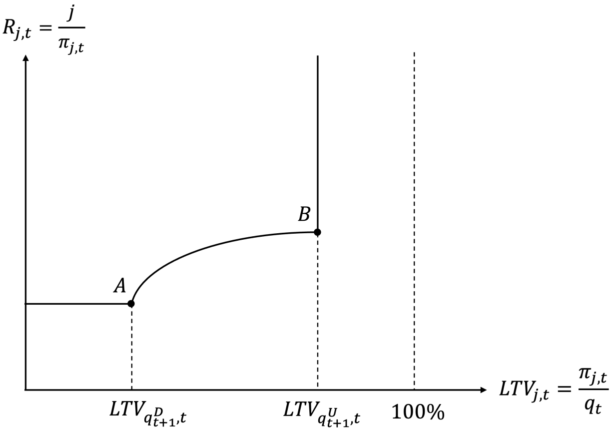
$$\left.\begin{array}{l} \text{▸ Leverage}: \; LTV_{j,t} = \dfrac{\pi_{j,t}}{q_t} \\[2ex] \text{▸ Gross interest rate}: \; R_{j,t} = \dfrac{j}{\pi_{j,t}} \end{array}\right\} \text{Credit Surface: } CS_t$$

Credit Surface (CS_t)

Following Fostel and Geanakoplos (2015)



Constraints

- **Budget Constraint**

$$c_t^a + q_t h_t^a + \int_{R_+} \theta_{j,t}^a \pi_{j,t} dj \leqslant e_t^a + q_t h_{t-1}^{a-1} + \underbrace{\int_{R_+} \theta_{j,t-1}^{a-1} \min\{j, q_t\} dj}_{\text{Total delivery of contracts}}$$

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$$\underbrace{\int_{j>q_t} q_t \, \theta_{j,t-1}^{a-1} dj}_{\text{Default: deliver } q_t} + \underbrace{\int_{j\leqslant q_t} j \, \theta_{j,t-1}^{a-1} dj}_{\text{Repay promised } j} =$$

Constraints

- **Collateral Constraint**

$$\int_{R_+} \max\{-\theta^a_{j,t}, 0\} dj \leqslant h^a_t$$

- **No short-selling Constraint**

$$h^a_t \geqslant 0$$

Definition: Collateral Equilibrium

- Allocations $\{c_t^a, h_t^a, (\theta_{j,t}^a)_{\forall j \in J_t}\}\ \forall t,\ a$
- Prices $\{q_t, (\pi_{j,t})_{\forall j \in J_t}\}\ \forall t$

s.t.

- Households maximize life-time utility subject to the constraints, taking prices as given
- All markets clear

Solution Method

Two Challenges

- Conceptual Challenge
  - Infinitely many contracts, which ones are chosen in equilibrium?
- Computational Challenge
  - High degree of heterogeneity
  - Large aggregate shocks
  - Occasionally binding collateral constraint

Solutions

- Conceptual Challenge
  - Guess and verify
    - Guess only a finite number of contracts are traded, reducing KKT conditions to a finite number of equations
    - Verify if agents have incentives to deviate to trading contracts not included in the finite set
- Computational Challenge
  - Neural network

Parameters

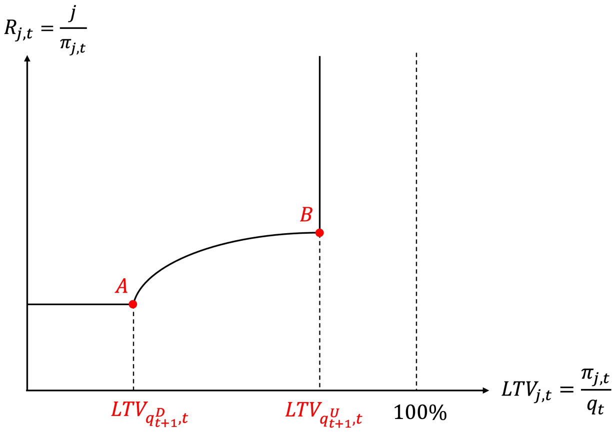
Parameter	Value	Interpretation
Uncertainty		
γ_{UD}	0.14	$\gamma_U = 85\%$
γ_{DU}	0.80	$d_U = 7, d_D = 1$
Preferences		
β	0.83	Annual discount rate 0.94
ρ	4.5	Risk-aversion coefficient
α	0.115	Housing share
Endowments		
$\{e^a(U)\}_{a=1}^{20}$		Income (2007)
$\{e^a(D)\}_{a=1}^{20}$		Income (2009)

Notes: γ_U represents the unconditional frequency of the U state. d_U and d_D represent the durations of the U and D states, respectively.

Results

Equilibrium

- 2 contracts traded in each period: Riskless: q^D_{t+1} and Risky: q^U_{t+1}



Equilibrium

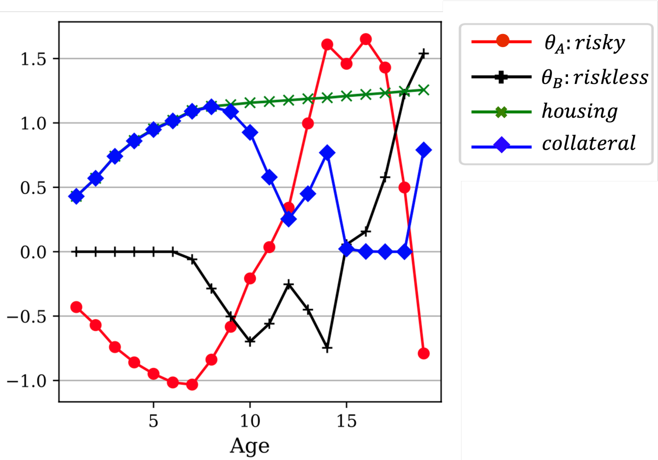
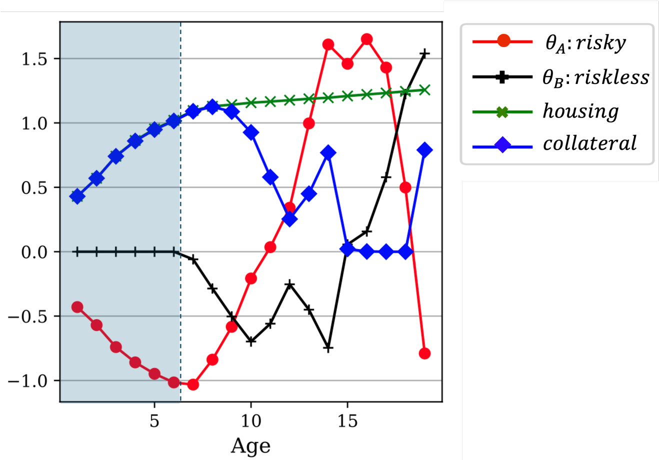


Figure: Portfolio Holdings (Mean)

Constrained borrowers



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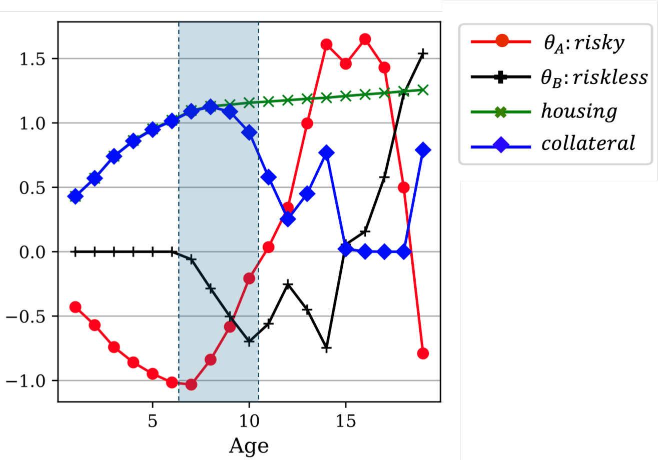

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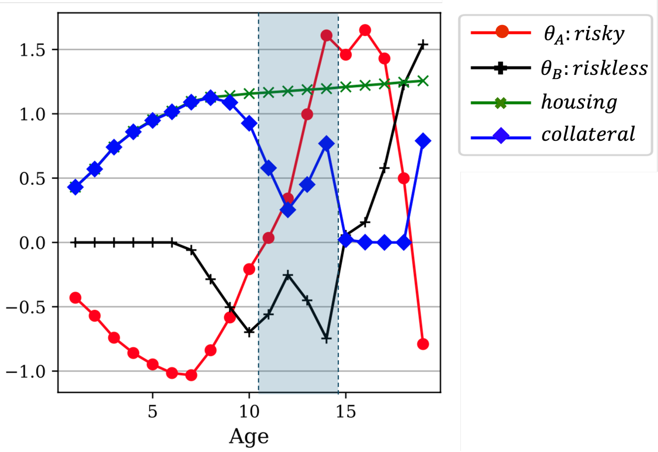
Long Risky, Short Riskless



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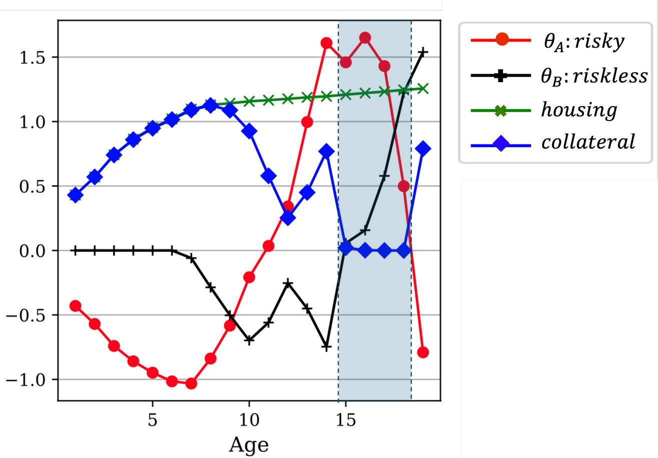
Lenders



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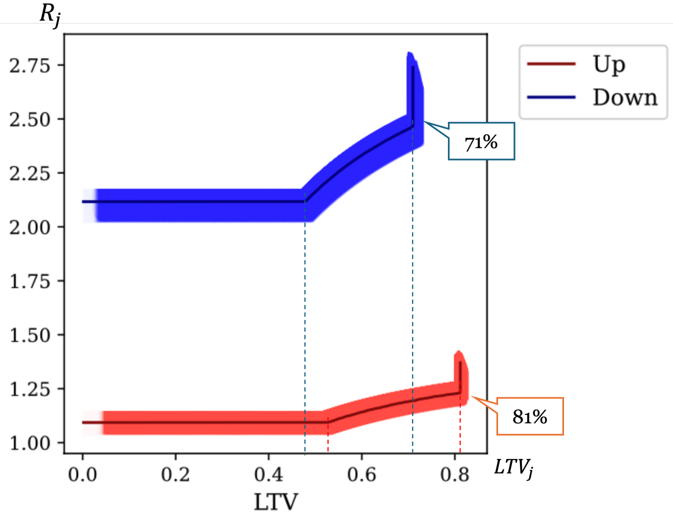
Credit Surface



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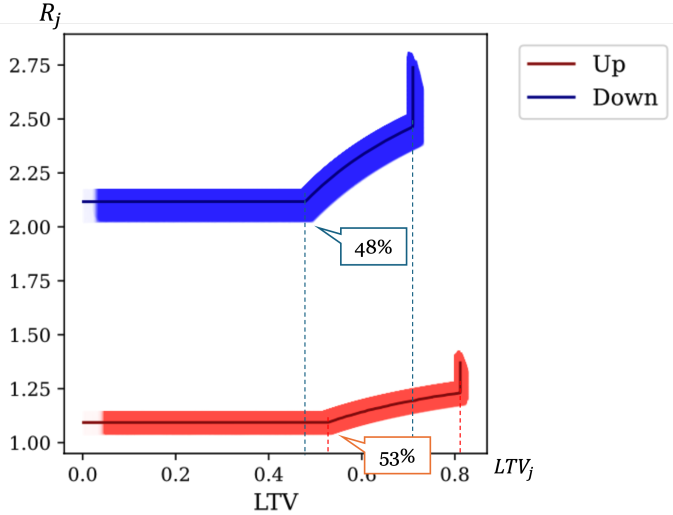

Aggregate Shocks Impact Housing Prices through Two Channels

$$q_t = \underbrace{\underbrace{\frac{Du_h^a(c_t^a, h_t^a)}{Du_c^a(c_t^a, h_t^a)}}_{\text{Rent}} + \underbrace{E_t\left[\beta \frac{Du_c^{a+1}(c_{t+1}^{a+1}, h_{t+1}^{a+1})}{Du_c^a(c_t^a, h_t^a)} q_{t+1}\right]}_{\text{PV}(q_{t+1})}}_{\text{Fundamental Value: Wealth Channel}} \qquad \underbrace{\frac{\mu_{h,t}^a}{Du_c^a(c_t^a, h_t^a)}}_{\text{Collateral Value: Leverage Channel}}$$

Wealth Channel: Unconstrained Agents

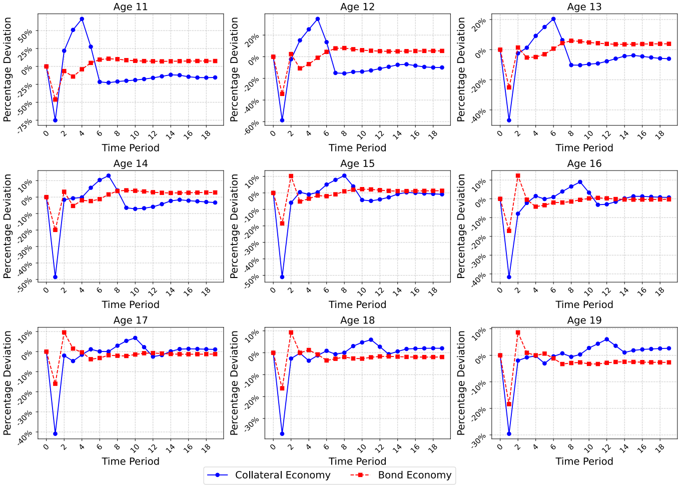

Figure: Percentage Deviations in Financial Wealth Under a D shock

Leverage Channel: Constrained Borrowers

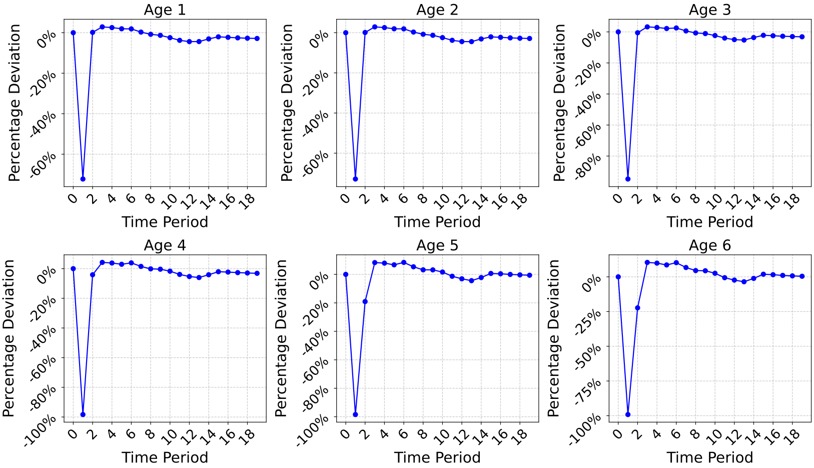

Figure: Percentage Deviations in Collateral Value for Constrained Borrowers Under a *D* shock

Feedback Loop Between Leverage and Housing Prices

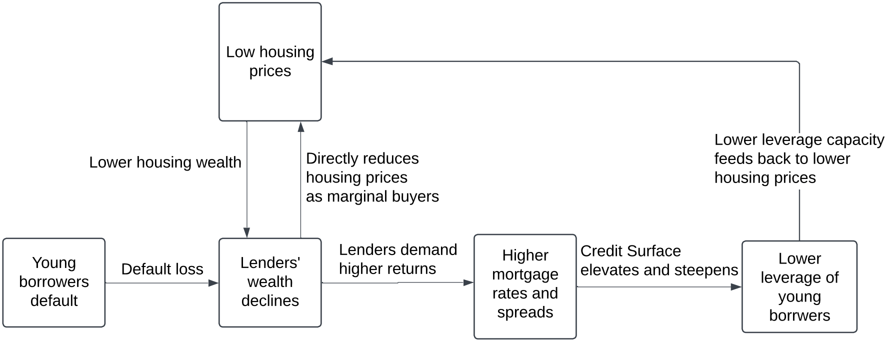

Figure: The Feedback Loop between Leverage and Housing Prices in Downturns

A bond economy

Consider a bond economy with exogenous LTV cap ϕ:

- $(b_t^a)_{a\in A}$: bond holdings
- p_t: the price of the bond

Agents maximize lifetime utility subject to the following constraints:

- Budget Constraint

$$c_t^a + q_t h_t^a + p_t b_t^a \leqslant e_t^a + q_t h_{t-1}^{a-1} + b_{t-1}^{a-1}$$

- Borrowing Constraint

$$-p_t b_t^a \leqslant \phi q_t h_t^a$$

Leverage Amplifies Volatility

Table: Comparison of Collateral Economy and Bond Economy

	Collateral Economy			Bond Economy		
	U	D	Δ	U	D	Δ
Housing Price	1.78	1.03	-42.1%	1.84	1.3	-29.3%
Aggregate debt	15.29	6.81	-55.5%	11.07	8.23	-25.7%
$DLTV^h$	43%	33.2%	-9.8 *pp*	30.1%	31.6%	1.5 *pp*
Volatility	16.2%			11.1%		

Notes:

- Volatility is measured by the coefficient of variation of housing prices over the simulation. 16.2% indicates that housing prices deviate by 16.2% from the mean on average.
- *DLTV* of housing is defined as the ratio of aggregate debt in the economy to the aggregate value of housing,

$$DLTV_t^h = \frac{p_t \sum_{a \in A} max\{-b_t^a, 0\}}{q_t H}.$$

Leverage Amplifies Volatility

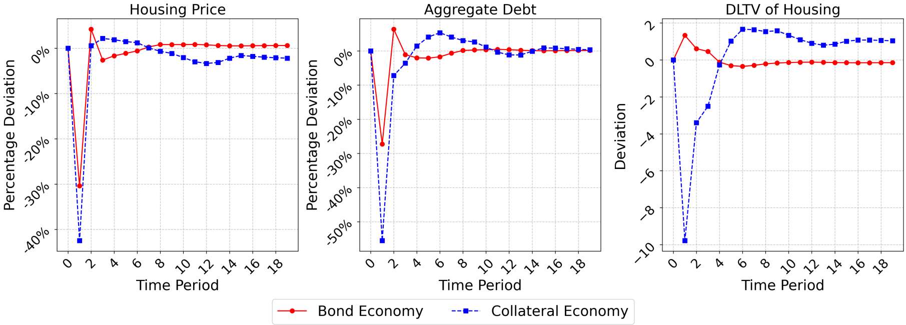

Figure: Deviations in Housing Price, Aggregate Debt and Leverage Under a *D* shock: Collateral vs Bond Economies

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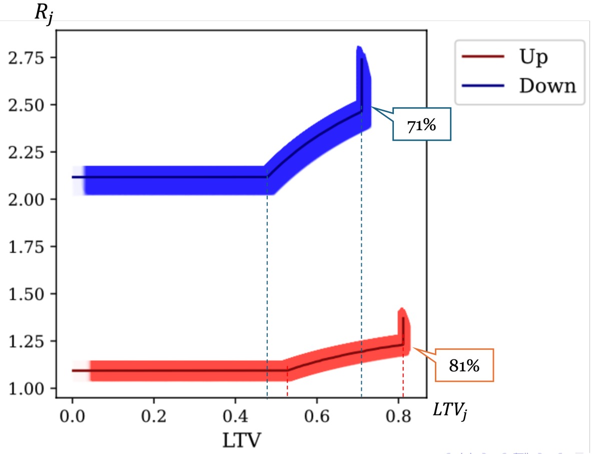

Thank You!