

Leverage Cycle over the Life Cycle: A Quantitative Model of Endogenous Leverage

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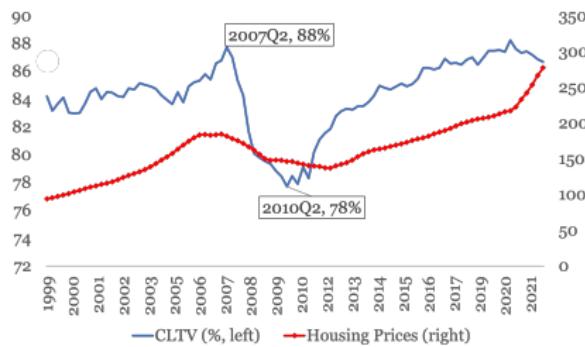
Introduction

This paper

- ▶ A quantitative model that rationalizes two facts on the US housing market
 - ▶ Leverage co-moves with housing prices
 - ▶ Mortgage spreads move in the opposite direction
- ▶ Three key elements
 - ▶ Overlapping-generation framework
 - ▶ Endogenous determination of leverage
 - ▶ Large aggregate shocks

Motivating Fact 1

- ▶ Leverage co-moves with housing prices (0.53)
- ▶ $\text{CLTV} \equiv \frac{\text{Total newly issued loan}}{\text{Appraised value of housing collateral}}$

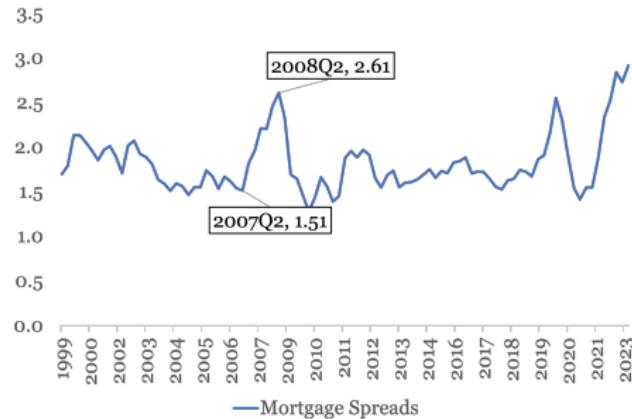


Data source: CLTV from Freddie Mac Single-Family Loan-Level Dataset, Case-Shiller housing price index from FRED, index Jan 2000=100.

Figure: Mean CLTV of First-time Homebuyers

Motivating Fact 2

- ▶ Mortgage spreads move in opposite direction (-0.29)



Data source: FRED MORTGAGE30US, DGS10.

Mortgage spread: average 30-year fixed-rate mortgage - 5-year treasury bill yield

Why do we care?

- ▶ Empirical importance: 94% of home purchases are financed through mortgages in the US
- ▶ Endogenous leverage theory (Fostel and Geanakoplos (2008) etc)
 - ▶ Leverage is a key driver in housing price volatility
 - ▶ A channel ignored by most macro-finance models
- ▶ Frictions in collateralized borrowing create room for policy interventions

Life Cycle Patterns

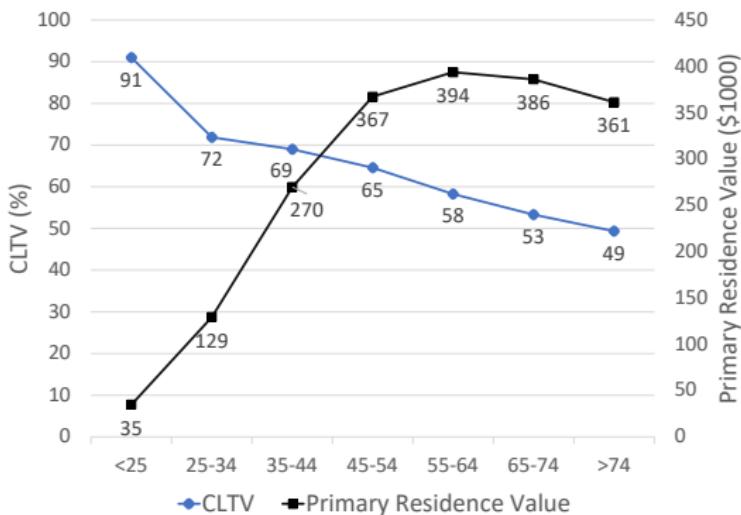


Figure: Housing Wealth and Leverage by Age

Data sources: CLTV: 2023 HDMA national loan-level dataset. Primary residence values: 2022 Survey of Consumer Finances dataset.

Model

Model: Environment

- ▶ **Age:** $a \in \{1, \dots, A\}$
 - ▶ No uncertainty about lifetime
 - ▶ No heterogeneity within cohorts
- ▶ **Uncertainty**
 - ▶ $z_t \in \{U, D\}$, Markov process

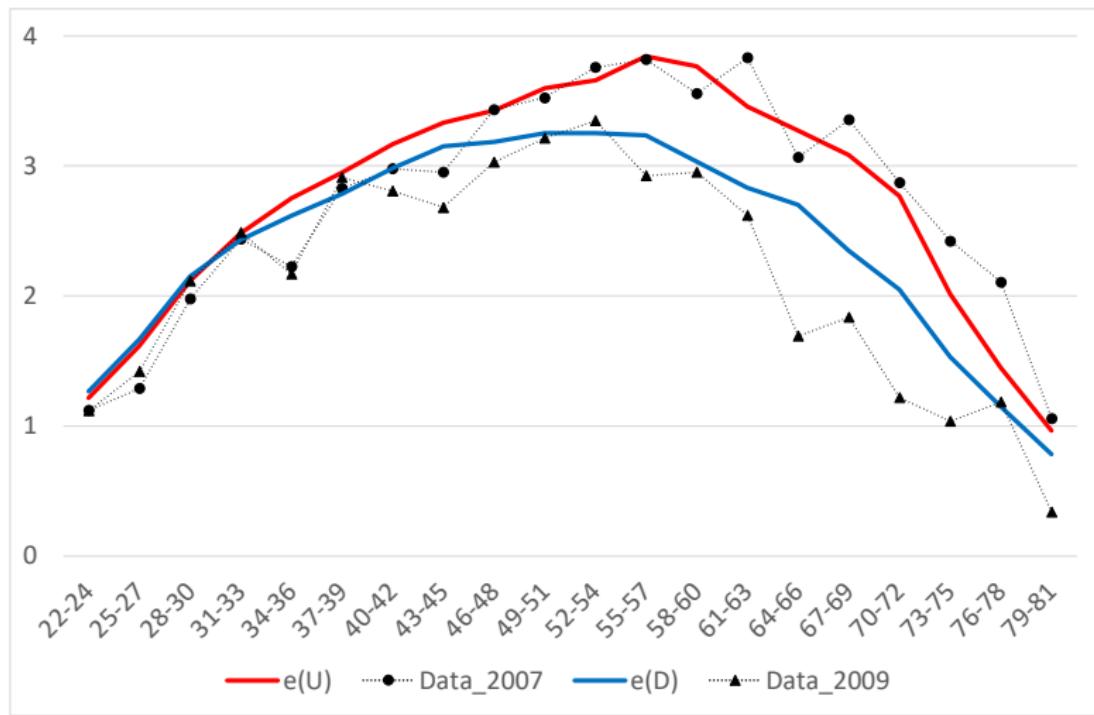
▶ Commodities

		Spot Price	Agg. Supply
Consumption (c_t^a)	perishable	1	$\bar{e}(z_t)$
House (h_t^a)	perfectly durable, divisible	q_t	H

▶ Endowments

- ▶ $\bar{e}(z_t) = \sum_{a=1}^A e^a(z_t) = \sum_{a=1}^A e_t^a$

Endowment age profiles



Model: Preferences

- ▶ Period Utility

$$u^a(c, h) = \begin{cases} \frac{(c^{1-\alpha} h^\alpha)^{1-\rho}}{1-\rho} & a \in \{1, \dots, A-1\} \\ \frac{c^{1-\rho}}{1-\rho} & a = A \end{cases}$$

- ▶ Lifetime Utility

$$U_t = E_t \sum_{a=1}^A \beta^{a-1} u^a(c_t^a, h_t^a)$$

Model: Debt Contracts

- ▶ Menu of one-period contracts: $(j, 1) \in J_t$

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 - ▶ $\theta_{j,t}^a \in \mathbb{R}$
 - ▶ $\theta_{j,t}^a < 0$: short, borrow $|\pi_{j,t}\theta_{j,t}^a|$
 - ▶ $\theta_{j,t}^a > 0$: long, lend $|\pi_{j,t}\theta_{j,t}^a|$
- ▶ Delivery: $\min\{j, q_{t+1}\}$
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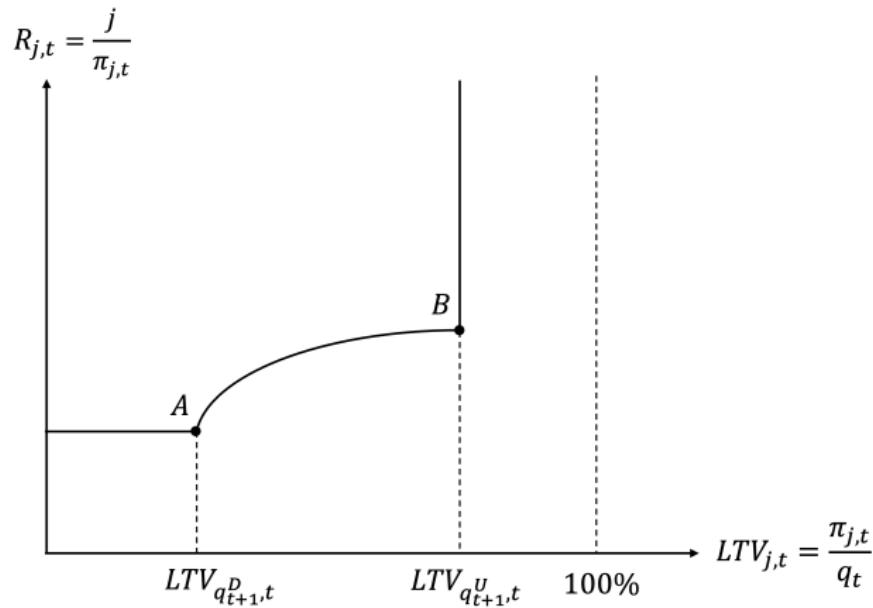
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- Credit Surface: CS_t

Credit Surface (CS_t)

Following Fostel and Geanakoplos (2015)



Constraints

- **Budget Constraint**

$$c_t^a + q_t h_t^a + \underbrace{\int_{R_+} \theta_{j,t}^a \pi_{j,t} dj}_{\text{Total delivery of contracts}} \leq e_t^a + q_t h_{t-1}^{a-1} + \underbrace{\int_{R_+} \theta_{j,t-1}^{a-1} \min\{j, q_t\} dj}_{\text{Total delivery of contracts}}$$

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$$\underbrace{\int_{j > q_t} q_t \theta_{j,t-1}^{a-1} dj}_{\text{Default: deliver } q_t} + \underbrace{\int_{j \leq q_t} j \theta_{j,t-1}^{a-1} dj}_{\text{Repay promised } j} =$$

Constraints

- ▶ **Collateral Constraint**

$$\int_{R_+} \max\{-\theta_{j,t}^a, 0\} dj \leq h_t^a$$

- ▶ **No short-selling Constraint**

$$h_t^a \geq 0$$

Definition: Collateral Equilibrium

- ▶ Allocations $\{c_t^a, h_t^a, (\theta_{j,t}^a)_{\forall j \in J_t}\} \forall t, a$
- ▶ Prices $\{q_t, (\pi_{j,t})_{\forall j \in J_t}\} \forall t$

s.t.

- ▶ Households maximize life-time utility subject to the constraints, taking prices as given
- ▶ All markets clear

Solution Method

Two Challenges

- ▶ Conceptual Challenge
 - ▶ Infinitely many contracts, which ones are chosen in equilibrium?
- ▶ Computational Challenge
 - ▶ High degree of heterogeneity
 - ▶ Large aggregate shocks
 - ▶ Occasionally binding collateral constraint

Solutions

- ▶ Conceptual Challenge
 - ▶ Guess and verify
 - ▶ Guess only a finite number of contracts are traded, reducing KKT conditions to a finite number of equations
 - ▶ Verify if agents have incentives to deviate to trading contracts not included in the finite set
- ▶ Computational Challenge
 - ▶ Neural network

Parameters

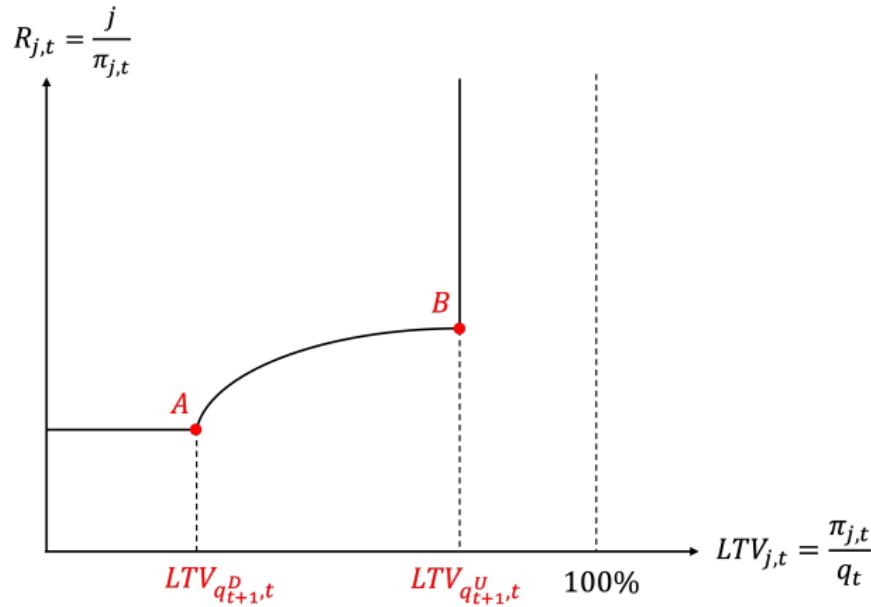
Parameter	Value	Interpretation
<i>Uncertainty</i>		
γ_{UD}	0.14	$\gamma_U = 85\%$
γ_{DU}	0.80	$d_U = 7, d_D = 1$
<i>Preferences</i>		
β	0.83	Annual discount rate 0.94
ρ	4.5	Risk-aversion coefficient
α	0.115	Housing share
<i>Endowments</i>		
$\{e^a(U)\}_{a=1}^{20}$		Income (2007)
$\{e^a(D)\}_{a=1}^{20}$		Income (2009)

Notes: γ_U represents the unconditional frequency of the U state. d_U and d_D represent the durations of the U and D states, respectively.

Results

Equilibrium

- 2 contracts traded in each period: Riskless: q_{t+1}^D and Risky: q_{t+1}^U



Equilibrium

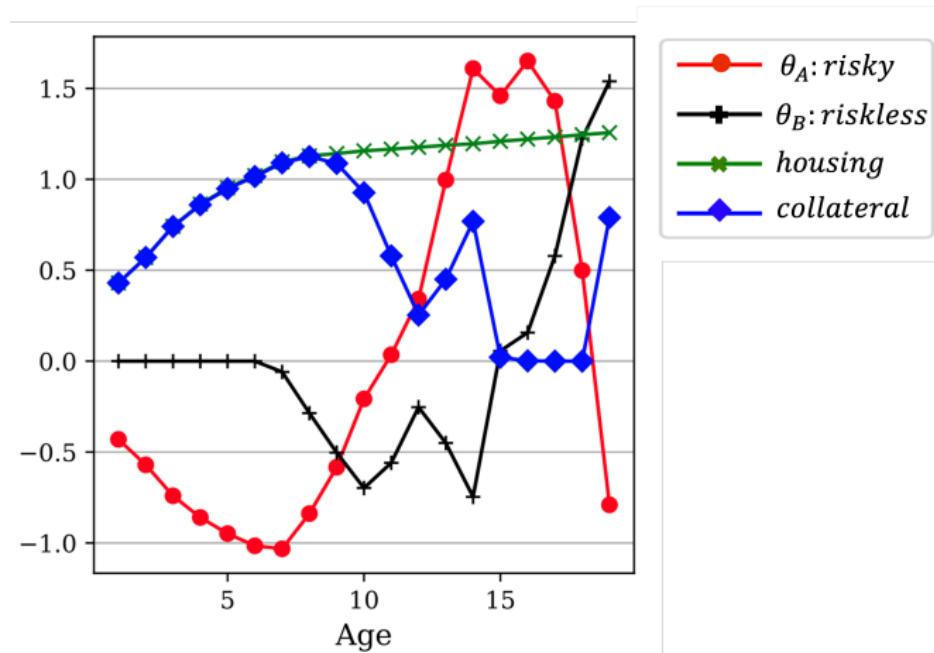


Figure: Portfolio Holdings (Mean)

Constrained borrowers

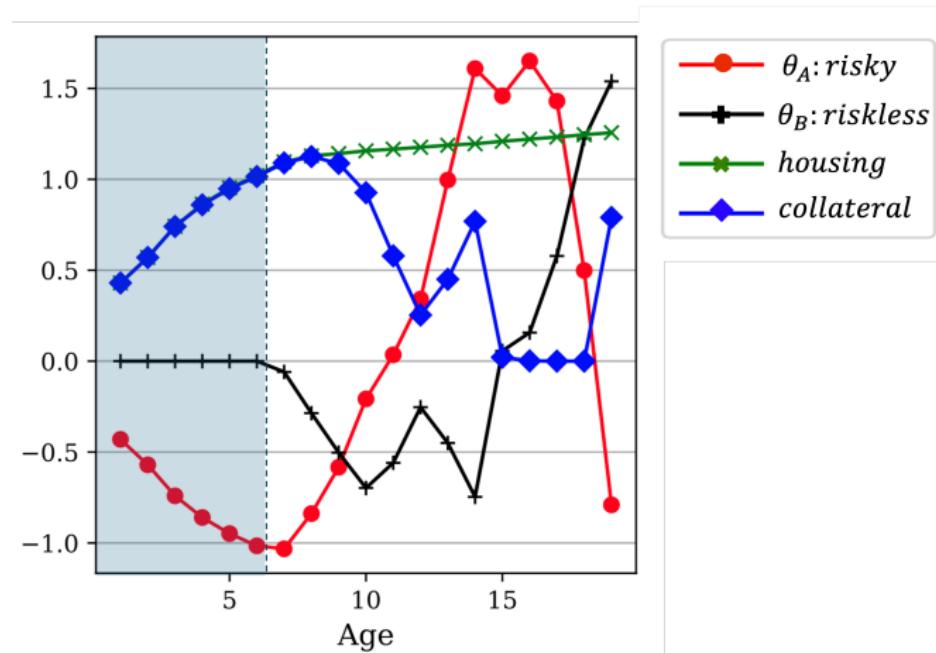


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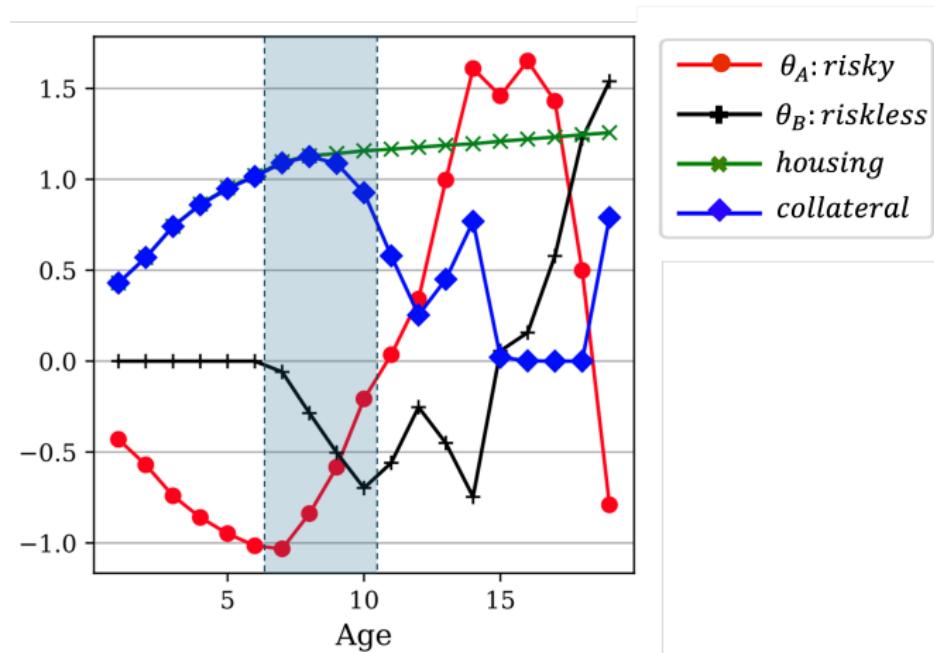


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Long Risky, Short Riskless

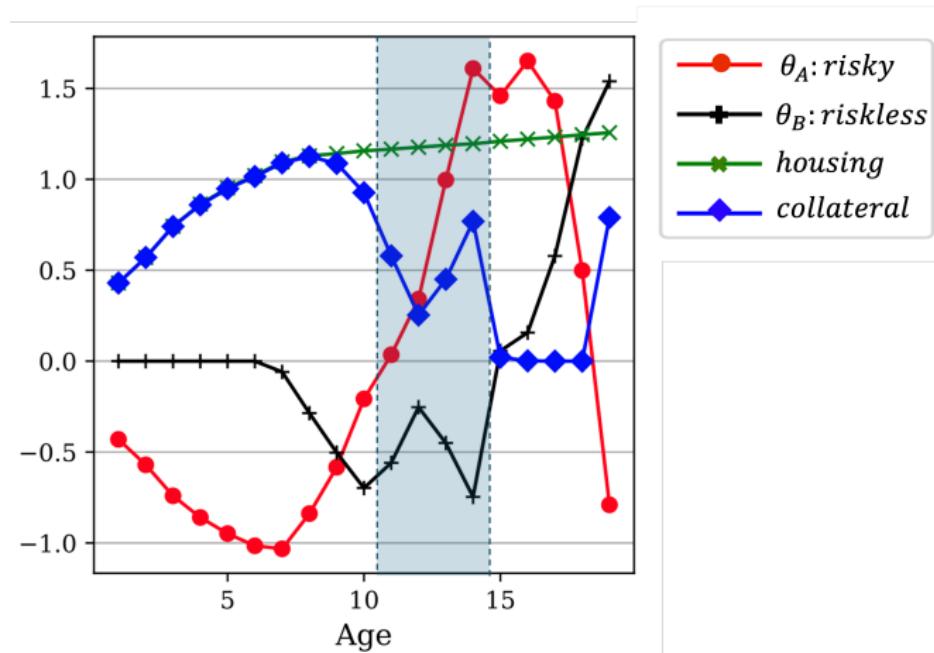


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Lenders

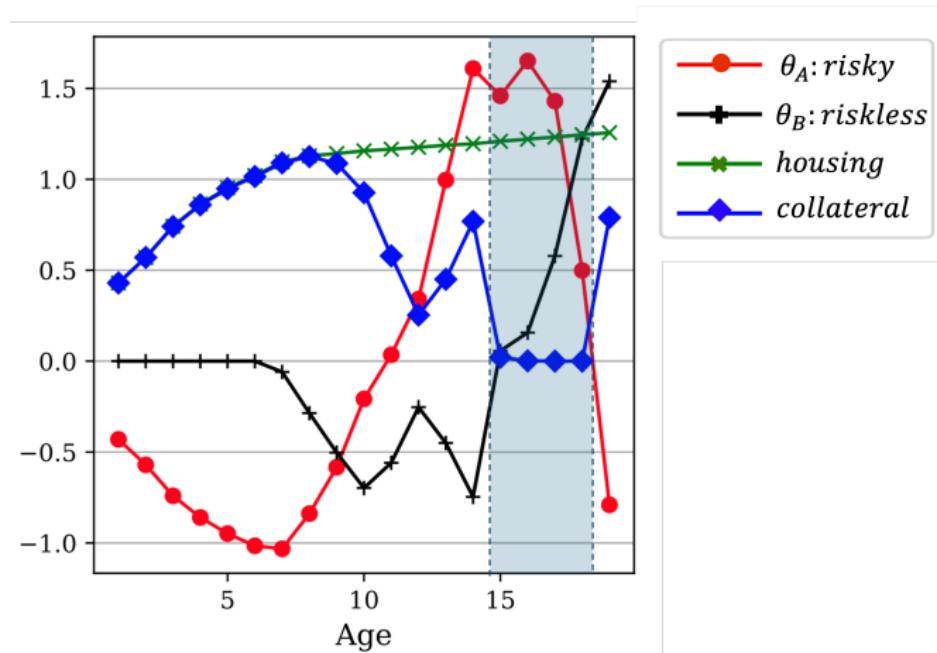
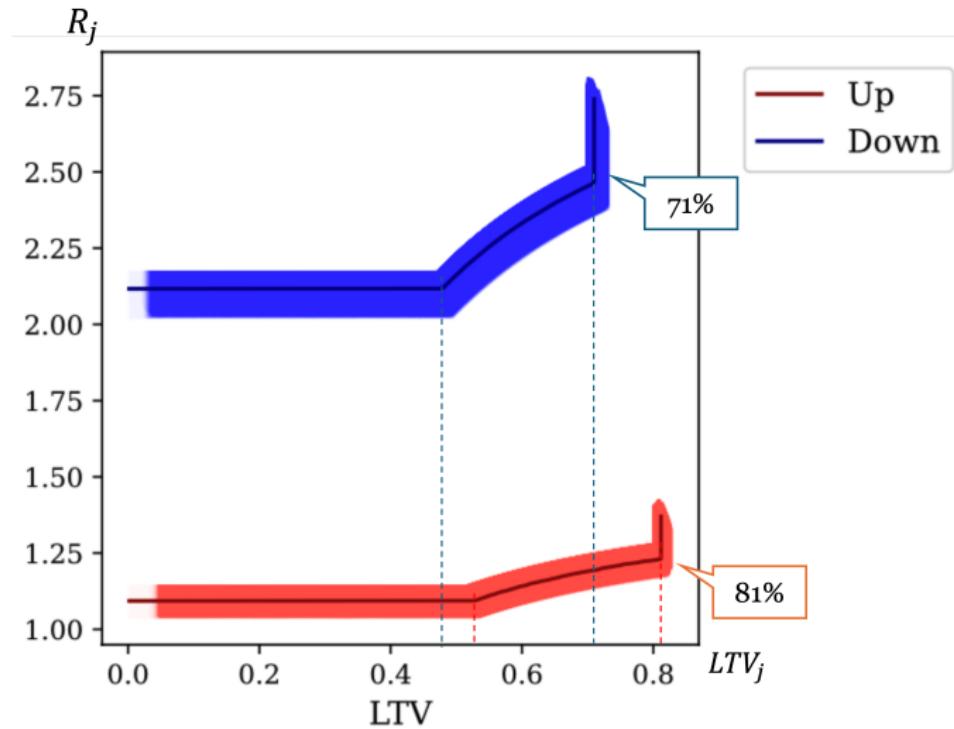
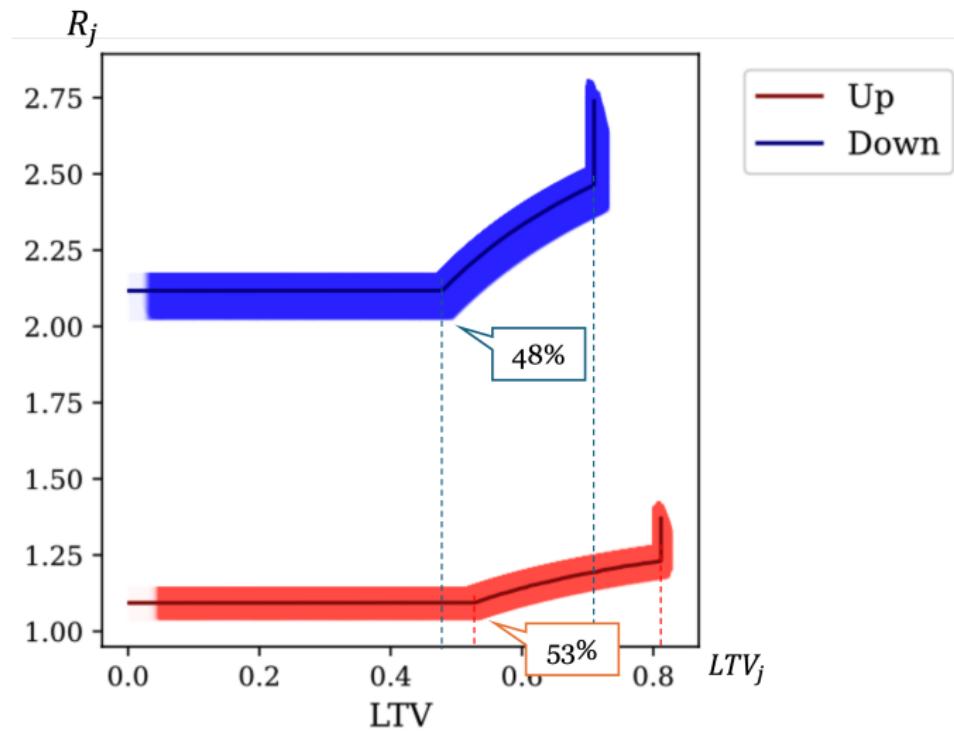


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Credit Surface



Credit Surface



Aggregate Shocks Impact Housing Prices through Two Channels

$$q_t = \underbrace{\frac{Du_h^a(c_t^a, h_t^a)}{Du_c^a(c_t^a, h_t^a)}}_{\text{Rent}} + \underbrace{\left[E_t \left[\beta \frac{Du_c^{a+1}(c_{t+1}^{a+1}, h_{t+1}^{a+1})}{Du_c^a(c_t^a, h_t^a)} q_{t+1} \right] \right]}_{\text{PV}(q_{t+1})} + \underbrace{\frac{\mu_{h,t}^a}{Du_c^a(c_t^a, h_t^a)}}_{\text{Collateral Value: Leverage Channel}}$$

Fundamental Value: Wealth Channel

Wealth Channel: Unconstrained Agents

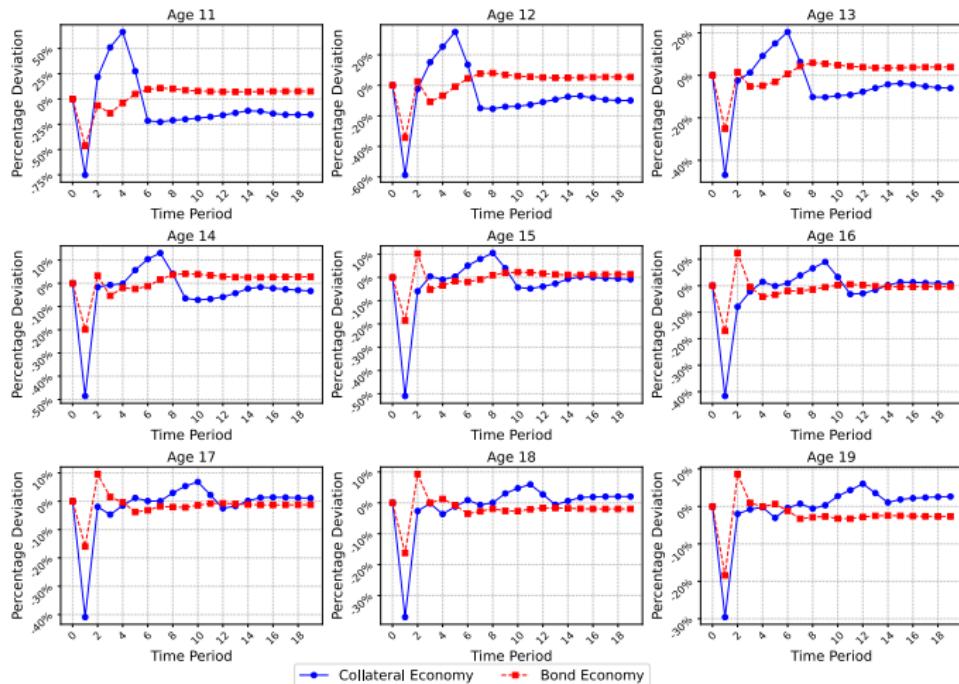


Figure: Percentage Deviations in Financial Wealth Under a D shock

Leverage Channel: Constrained Borrowers

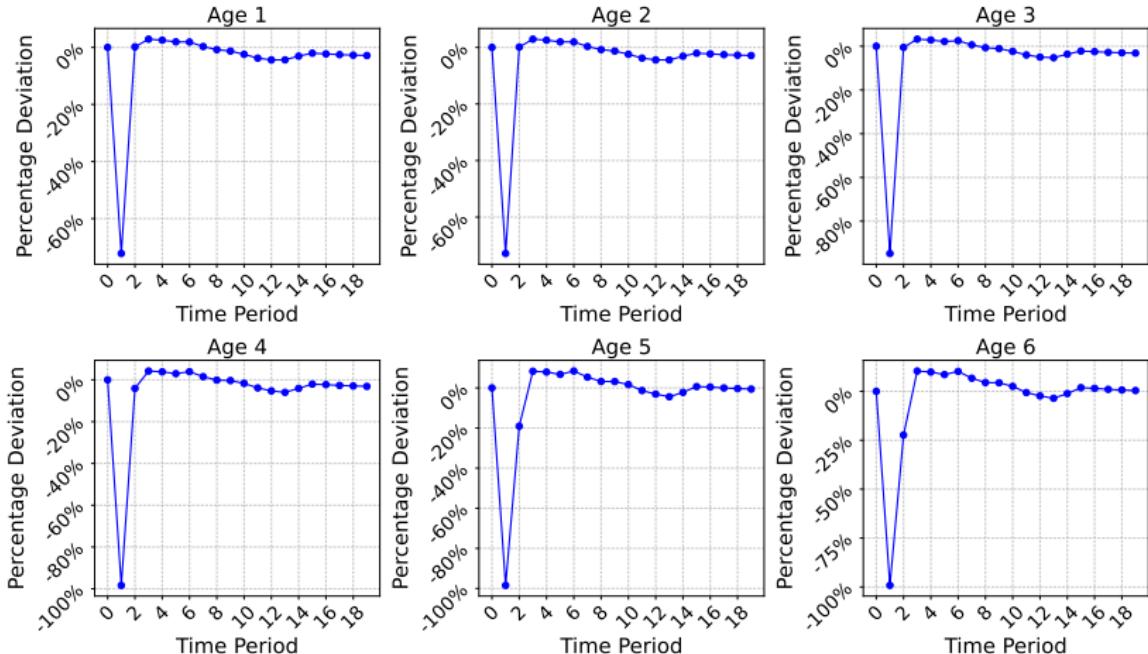
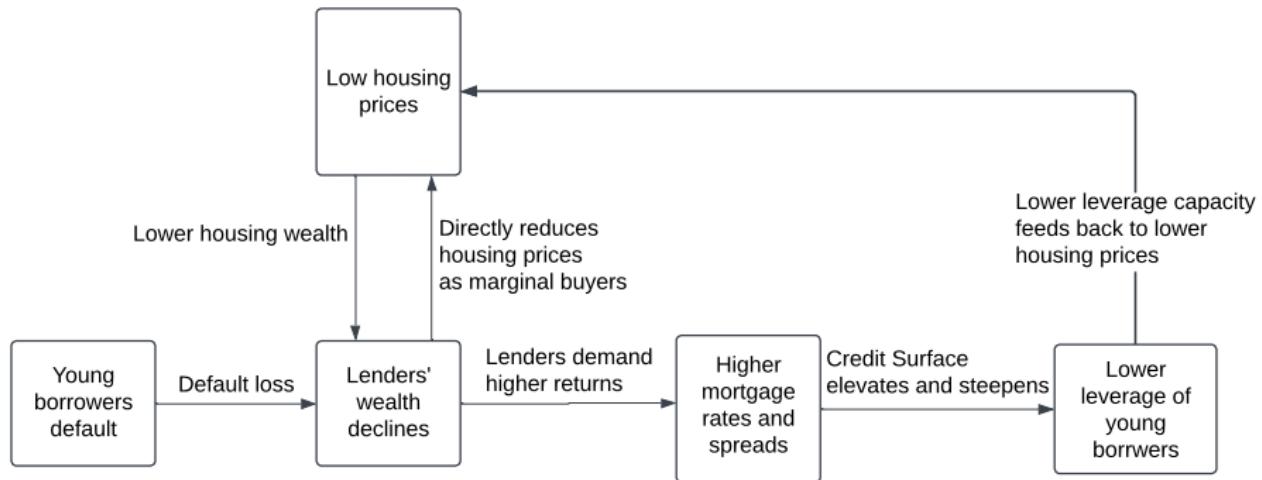


Figure: Percentage Deviations in Collateral Value for Constrained Borrowers Under a D shock

Feedback Loop Between Leverage and Housing Prices



A bond economy

Consider a bond economy with exogenous LTV cap ϕ :

- ▶ $(b_t^a)_{a \in A}$: bond holdings
- ▶ p_t : the price of the bond

Agents maximize lifetime utility subject to the following constraints:

- ▶ Budget Constraint

$$c_t^a + q_t h_t^a + p_t b_t^a \leq e_t^a + q_t h_{t-1}^{a-1} + b_{t-1}^{a-1}$$

- ▶ Borrowing Constraint

$$-p_t b_t^a \leq \phi q_t h_t^a$$

Leverage Amplifies Volatility

Table: Comparison of Collateral Economy and Bond Economy

	Collateral Economy			Bond Economy		
	U	D	Δ	U	D	Δ
Housing Price	1.78	1.03	-42.1%	1.84	1.3	-29.3%
Aggregate debt	15.29	6.81	-55.5%	11.07	8.23	-25.7%
$DLTV^h$	43%	33.2%	-9.8 pp	30.1%	31.6%	1.5 pp
Volatility	16.2%			11.1%		

Notes:

- Volatility is measured by the coefficient of variation of housing prices over the simulation. 16.2% indicates that housing prices deviate by 16.2% from the mean on average.
- $DLTV$ of housing is defined as the ratio of aggregate debt in the economy to the aggregate value of housing,

$$DLTV_t^h = \frac{p_t \sum_{a \in A} \max\{-b_t^a, 0\}}{q_t H}.$$

Leverage Amplifies Volatility

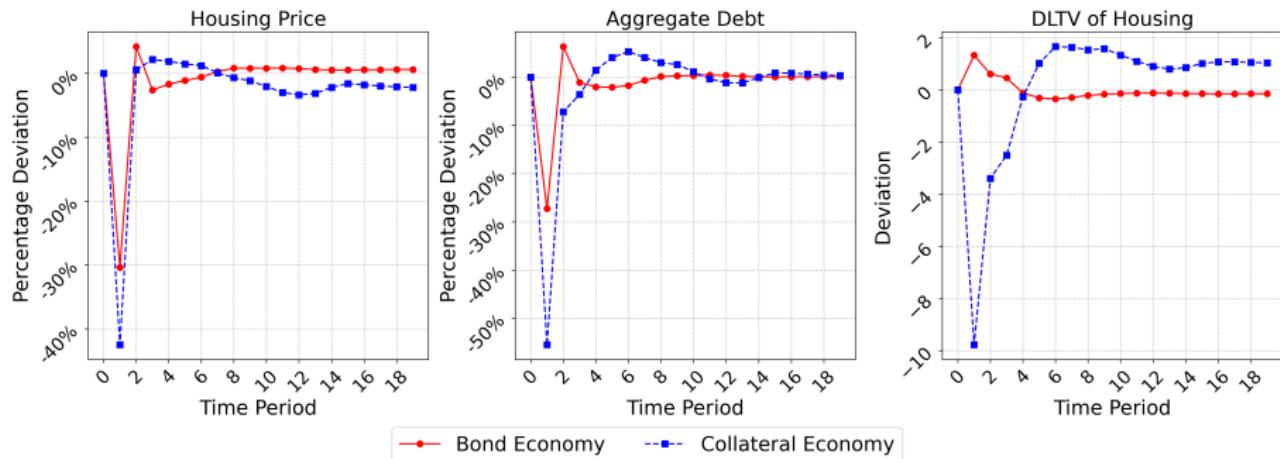


Figure: Deviations in Housing Price, Aggregate Debt and Leverage Under a D shock: Collateral vs Bond Economies

This paper

- ▶ Rationalizes 2 facts on the US housing market
 - ▶ Leverage co-moves with housing prices
 - ▶ Mortgage spread moves in opposite direction
- ▶ Demonstrates how leverage amplifies housing price volatility
 - ▶ Wealth channel: unconstrained agents
 - ▶ Leverage channel: constrained agents

Thank You!