Simulation laboratory 1: Random number generation and Poisson process

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Goals

1. **Random number generation:**
   - Understand how to draw from a distribution
   - Apply the inverse transform method
2. **Poisson process:**
   - Understand how to generate events
   - Understand difference between homogeneous and nonhomogeneous Poisson processes
   - Apply the thinning algorithm

Overview

Implementation:

1. Exponential random numbers
2. Homogeneous Poisson process
3. Nonhomogeneous Poisson process

Steps:

1. Read the specifications (written in the distributed Python codes)
2. Implement the requested functions
3. Test the functions

TO DO:

- Use the inverse transform method given a uniform distributed random number
- Explore your programme: change number of draws and/or parameter, compare to theoretical distribution

Homogeneous Poisson process

TO DO:

- Use the function for exponential random number generation
- Let $\lambda = 4$ and $T = 1$.
- Plot empirical arrival time distribution.

Nonhomogeneous Poisson process

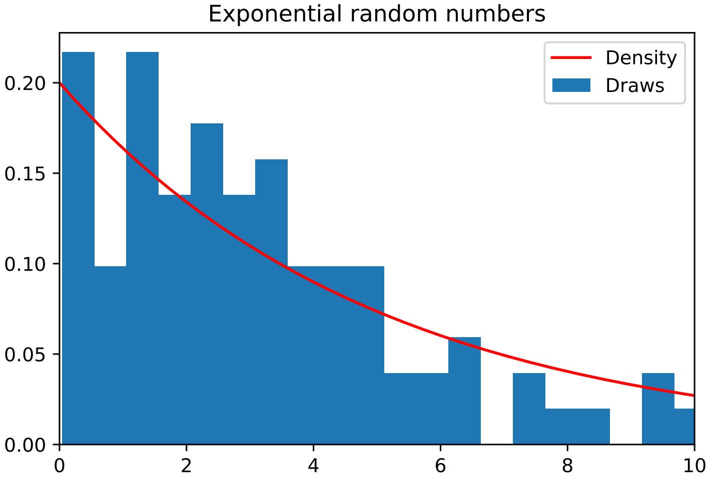
TO DO:

- Use the thinning algorithm: be aware of a function for $\lambda(t)$
- Let $\lambda(t) = \lambda \cdot \sin(t) + \lambda$ with $\lambda = 4$ and $T = 10$.
- Compare results to homogeneous Poisson process.

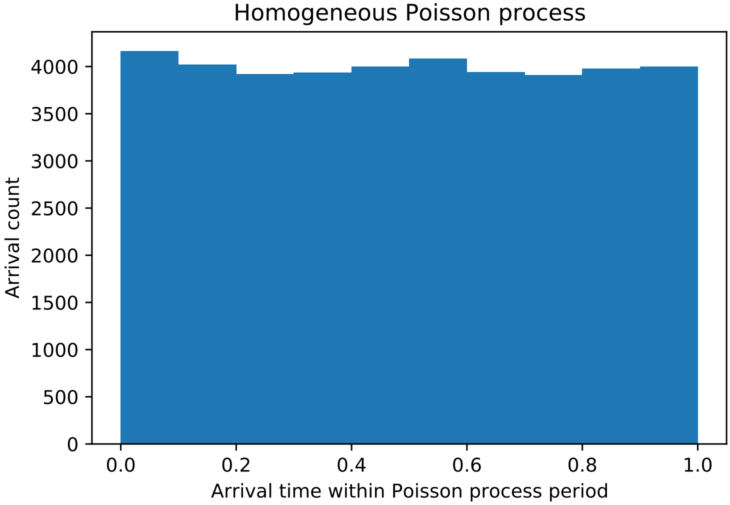
Extra questions

1. What is the efficiency, i.e., the number of accepted values over the total number of generated values, of this non homogeneous Poisson process with a unique λ where $\lambda(t) \leq \lambda$?
2. Can the efficiency be improved using several piecewise constant λ_i where $\lambda(t) \leq \lambda_i$, $t_{i-1} \leq t \leq t_i$?
3. Implement a non homogeneous Poisson process with multiple λ_is. What is the new efficiency? Attention at the transition between λ_i and λ_{i+1}.

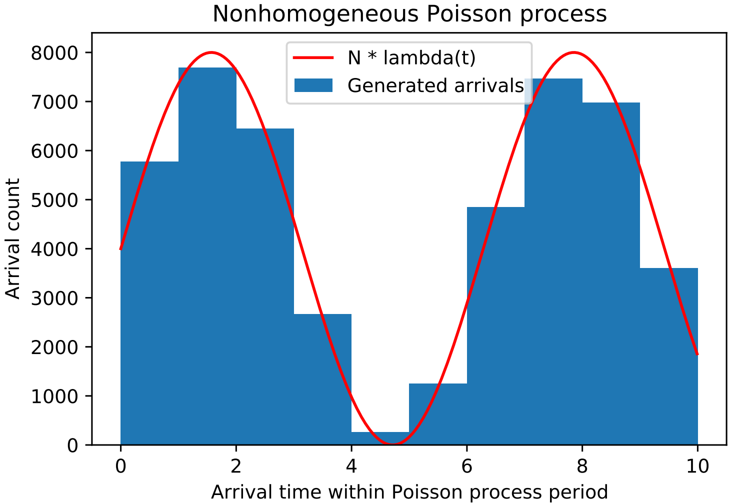
Exponential random numbers



Homogeneous Poisson process



Non-homogeneous Poisson process



Non-homogeneous Poisson process - Extra questions

1. **Q:** What is the efficiency, i.e., the number of accepted values over the total number of generated values, of this non homogeneous Poisson process with a unique λ where $\lambda(t) \leq \lambda$?
   A: 0.59
2. **Q:** Can the efficiency be improved using several piecewise constant λ_i where $\lambda(t) \leq \lambda_i$, $t_{i-1} \leq t \leq t_i$?
   A: Yes
3. **Q:** Implement a non homogeneous Poisson process with multiple λ_is. What is the new efficiency?
   A: $\lambda_i \in \{\lambda/2, \lambda\}$ and $t_i = i\pi$ so that $\lambda(t) \leq \lambda_i \leq \lambda$ is satisfied throughout the interval. New efficiency equals to **0.73**.


Any
Questions?