Vaccination strategy project

Optimization and Simulation Course (MATH-600)

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Introduction

You are in charge of managing the vaccination process by age groups to stop the spreading of a pandemic.

The decisions that you have to make are:

- What age group should get vaccinated first to minimize the spreading.
- The minimum percentage of vaccination in each age group to ensure that the spreading of the virus is controlled.

The aim of the "Simulation Project" is to develop a discrete event simulation that represents the system and to evaluate the performance of the simulation outputs.

During the "Optimization Project", the discrete event simulation is expanded, and a good solution in terms of minimum number of deaths and the minimum number of economic losses is identified by an optimization algorithm.

Develop the discrete event simulation with a modular structure. It should be possible to modify the various components, such as the initial number of people that are classified as S (*Susceptible*), I (*Infected*), R (*Recovered*), and D (*Dead*) during the "Optimization Project" in order to test different policies.

Project description

We consider a population of size $N = 100,000$ individuals, we segment the population into three age groups: children, adults, and elders. The representation per age group is the following: 20% are children belonging to the group $g = 1$, 65% are adults belonging to $g = 2$, and 15% are elders belonging to $g = 3$. We define N_g as the number of individuals belonging to group g. The planning horizon for the vaccination process is 60 days. Time is discretized in days and indexed by t. A Markov chain process defines each transition between health states: S, I, R, D, $\forall g = 1,\ 2,\ 3$.

The transition from S to I depends on the probability of infection per contact, i.e., λ_g, the contact matrix, c, and the proportion of infected people among these contacts, i.e., $\frac{I_g}{N_g}$. We model the transition between S to I as a binary choice model. The sets of utilities are $[V_I^1, V_S^1], [V_I^2, V_S^2], [V_I^3, V_S^3]$.

$$V_I^1 = \lambda_1 c_{11} \frac{I_1}{N_1} +\ \lambda_2 c_{12} \frac{I_2}{N_2} +\ \lambda_3 c_{13} \frac{I_3}{N_3}$$

$$V_I^2 = \lambda_1 c_{21} \frac{I_1}{N_1} +\ \lambda_2 c_{22} \frac{I_2}{N_2} +\ \lambda_3 c_{23} \frac{I_3}{N_3}$$

$$V_I^3 = \lambda_1 c_{31} \frac{I_1}{N_1} +\ \lambda_2 c_{32} \frac{I_2}{N_2} +\ \lambda_3 c_{33} \frac{I_3}{N_3}$$

$$V_S^1, V_S^2, V_S^3 = 0$$

where the contact matrix c is equal to:

$$c = \begin{bmatrix} 7 & 1 & 1 \\ 1 & 5 & 0 \\ 1 & 0 & 6 \end{bmatrix},$$

and $\lambda = [0.5\ 0.55\ 0.65]$. Finally the probabilities of transition become ($\mu = 0.5$):

$$P\left(S_1^{t+1} \middle| S_1^t\right) = 1 - P(I_1^{t+1}|S_1^t)$$

$$P(I_1^{t+1}|S_1^t) = \mu \frac{V_I^1}{\lambda_1 c_{11} + \lambda_2 c_{12} + \lambda_3 c_{13}}$$

$$P\left(S_2^{t+1}\middle|S_2^t\right) = 1 - P(I_2^{t+1}|S_2^t)$$

$$P(I_2^{t+1}|S_2^t) = \mu\frac{V_I^2}{\lambda_1 c_{21} + \lambda_2 c_{22} + \lambda_3 c_{23}}$$

$$P\left(S_3^{t+1}\middle|S_3^t\right) = 1 - P(I_3^{t+1}|S_3^t)$$

$$P(I_3^{t+1}|S_3^t) = \mu\frac{V_I^3}{\lambda_1 c_{31} + \lambda_2 c_{32} + \lambda_3 c_{33}}$$

Table 1 describes the constant transition rates for the rest of the states within a segment of the population. For instance, the $P(R_1^{t+1}|I_1^t) = 0.13$. Note that $N = N_1 + N_2 + N_3$, We impose that there are 5 individuals infected in each group as initial conditions.

	$g = 1$				$g = 2$				$g = 3$			
t \ $t+1$	$\boldsymbol{S}$	$\boldsymbol{I}$	$\boldsymbol{R}$	$\boldsymbol{D}$	$\boldsymbol{S}$	$\boldsymbol{I}$	$\boldsymbol{R}$	$\boldsymbol{D}$	$\boldsymbol{S}$	$\boldsymbol{I}$	$\boldsymbol{R}$	$\boldsymbol{D}$
$\boldsymbol{S}$	$P\left(S_1^{t+1} \mid S_1^t\right)$	$P\left(I_1^{t+1} \mid S_1^t\right)$	0	0	$P\left(S_1^{t+1} \mid S_1^t\right)$	$P\left(I_2^{t+1} \mid S_2^t\right)$	0	0	$P\left(S_1^{t+1} \mid S_1^t\right)$	$P\left(I_3^{t+1} \mid S_3^t\right)$	0	0
$\boldsymbol{I}$	0	0.8	0.13	0.07	0	0.83	0.1	0.07	0	0.87	0.05	0.08
$\boldsymbol{R}$	0	0	1	0	0	0	1	0	0	0	1	0
$\boldsymbol{D}$	0	0	0	1	0	0	0	1	0	0	0	1

Table 1: Transition rates for all age groups

We assume that the number of susceptible individuals in a group g decreases to account for vaccination. For instance, if 15% of the children are vaccinated, probability for the susceptible group of children to become infected decreases by 15%.

$$P'\left(S_1^{t+1}\middle|S_1^t\right) = (1 + 0.15)P\left(S_1^{t+1}\middle|S_1^t\right)$$

$$P'\left(I_1^{t+1}\middle|S_1^t\right) = (1 - 0.15)P\left(I_1^{t+1}\middle|S_1^t\right)$$

The government controls the vaccination process by the limit of doses per day due to the personnel available, which is 500. For ethical purposes we will not set the cost of life, instead study the pareto frontier to analyze the trade-off between economic losses and deaths per day. For modeling the economic losses we assume that every time an adult gets infected, she does not work for a day, and therefore 100 CHF are lost.

Simulation

For the simulation project, you are requested to:

- Identify the control variables, the list of events, the state variables, and the indicators/output.
- Develop a discrete event simulation to represent the described project.
- Define the metrics used to quantify the quality of the service
  - Remember that extreme cases are important; evaluate other metrics in addition to the mean.
  - Report the mean squared error of your estimation using bootstrapping when necessary.
  - Use variance reduction techniques to reduce the computational time.
- Evaluate the number of deaths under the following scenarios:
  - We give priority to the elders for the vaccination process.
  - The government offers only twenty doses of vaccine at a time per age group. First, it offers the vaccine to the elders and as soon as it reaches its limit the government offers the vaccine to the adults, and finally to the children.
  - (or other scenarios which show your simulation performance well)
- Make any necessary assumptions.

Optimization

For the optimization project, you are requested to:

- Identify the decision variables of the vaccination management problem.
- Define the objective function.
- Design an optimization algorithm and apply it to solve the problem. The value of the objective function is evaluated by discrete event simulation.
- Like in the simulation project, the objective function can reflect various policies of the decision maker: whether they want to optimize over the average, best, worst, or certain percentile of the objective function

distribution. Decide what your position is and justify it, or present results for several alternatives.

- Use your creativity and define a new policy to offer vaccines to the population that can improve the strategy, i.e., reducing the total number of deaths and minimizing the economic loss. What if we prioritize the adults?