EC 522 Homework 1

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Problem 1

i

$$\begin{aligned}\mathcal{F}(\cos(20\pi x)) &= \iint_{-\infty}^{\infty}\cos(20\pi x)\mathrm{d}x\mathrm{d}y = \iint_{-\infty}^{\infty}\frac{1}{2}(e^{-i20\pi x}+e^{i20\pi x})e^{-i2\pi(f_x x+f_y y)}\mathrm{d}x\mathrm{d}y \\ &= \int_{-\infty}^{\infty}e^{-i2\pi f_y y}\mathrm{d}y\int_{-\infty}^{\infty}\frac{1}{2}(e^{-i20\pi x}+e^{i20\pi x})e^{-i2\pi f_x x}\mathrm{d}x \\ &= \frac{\delta(y)}{2}\int_{-\infty}^{\infty}(e^{-i2\pi(10+f_x)x}+e^{-i2\pi(-10+f_x)x})e^{-i2\pi f_x x}\mathrm{d}x \\ &= [\delta(f_x+10)+\delta(f_x-10)]\delta(f_y) = \begin{cases}\infty & f_x=\pm10\ \&\ f_y=0 \\ 0 & \text{others}\end{cases}\end{aligned}$$

$$\begin{aligned}\mathcal{F}(\cos(40\pi y)) &= \iint_{-\infty}^{\infty}\cos(40\pi y)\mathrm{d}x\mathrm{d}y = \iint_{-\infty}^{\infty}\frac{1}{2}(e^{-i40\pi y}+e^{i20\pi x})e^{-i2\pi(f_x x+f_y y)}\mathrm{d}x\mathrm{d}y \\ &= \delta(f_x)[\delta(f_y+20)+\delta(f_y-20)] = \begin{cases}\infty & f_y=\pm20\ \&\ f_x=0 \\ 0 & \text{others}\end{cases}\end{aligned}$$

$$\begin{aligned}\mathcal{F}(\cos(20\pi x+40\pi y)) &= \iint_{-\infty}^{\infty}\cos(20\pi x+40\pi y)\mathrm{d}x\mathrm{d}y \\ &= \iint_{-\infty}^{\infty}\frac{1}{2}(e^{-i20\pi x-i40\pi y}+e^{i20\pi x+i40\pi y})e^{-i2\pi(f_x x+f_y y)}\mathrm{d}x\mathrm{d}y \\ &= [\delta(f_x-10)\delta(f_y-20)+\delta(f_x+10)\delta(f_y+20)] \\ &= \begin{cases}\infty & (f_x=10\ \&\ f_y=20)\ or\ (f_x=-10\ \&\ f_y=-20) \\ 0 & \text{others}\end{cases}\end{aligned}$$

$$\begin{aligned}\mathcal{F}(\cos(20\pi x)+\cos(40\pi y)) &= \iint_{-\infty}^{\infty}(\cos(20\pi x)+\cos(40\pi y))\mathrm{d}x\mathrm{d}y \\ &= \iint_{-\infty}^{\infty}\frac{1}{2}(e^{-i20\pi x}+e^{i20\pi x}+e^{-i40\pi y}+e^{i40\pi x})e^{-i2\pi(f_x x+f_y y)}\mathrm{d}x\mathrm{d}y \\ &= \{\delta(f_y)[\delta(f_x-10)+\delta(f_x+10)]+\delta(f_x)[\delta(f_y-20)+\delta(f_y+20)]\} \\ &= \begin{cases}\infty & (f_x=\pm10\ \&\ f_y=0)\ or\ (f_y=\pm20\ \&\ f_x=0) \\ 0 & \text{others}\end{cases}\end{aligned}$$

ii

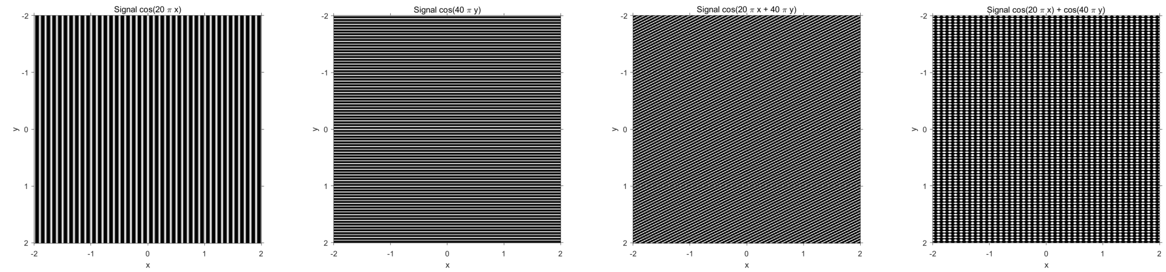

Fig 1.1 Raw signal image $f(x, y)$

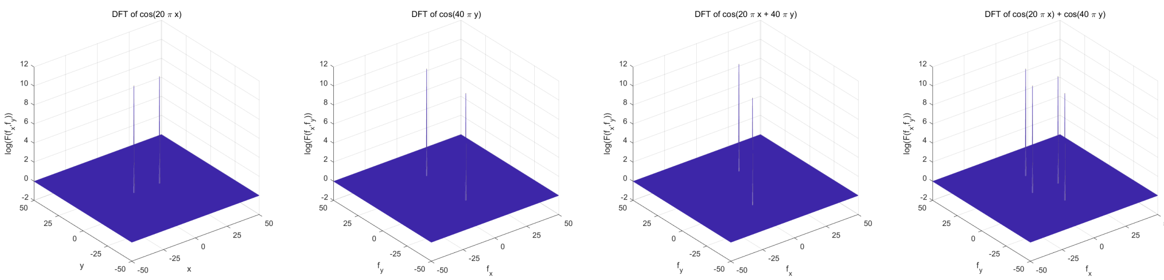

Fig 1.2 DFT of signal $F(f_x, f_y)$

Problem 2

a

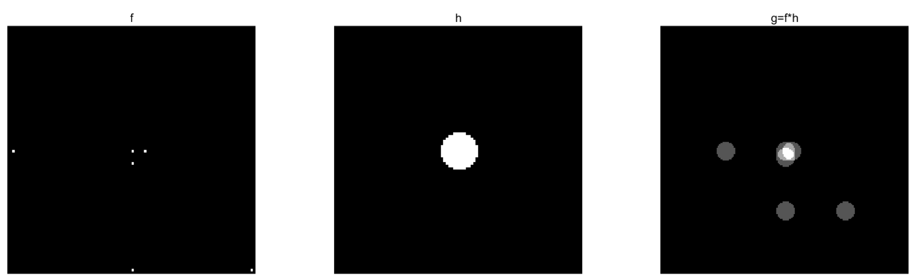

Fig 2.1 Direct method in the spatial domain

b

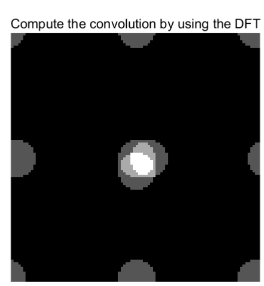

Fig 2.2 Compute the convolution by using the DFT

c

The DFT assumes the signal is periodic, which can introduce artifacts at the borders due to wrapping around. Convolution in the spatial domain attempts to handle edges by trimming the output to the size of the original image, but it doesn't inherently deal with the periodicity implied in the DFT-based convolution.
If the regular Fourier transform is used, it corresponds to a circular convolution. This can cause the signal to appear on the other side at the edge of the signal. In order to solve this problem, it is necessary to perform the Fourier transform by complementing both sides with 0. When the length of the zero-padding is the same as or greater than the length of the signal itself, there are no artifacts due to the periodicity of the signal.

d

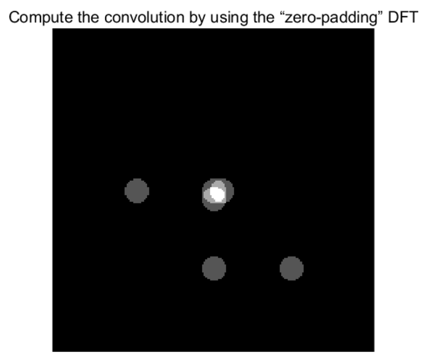

Fig 2.2 Compute the convolution by using the “zero-padding” DFT

Problem 3

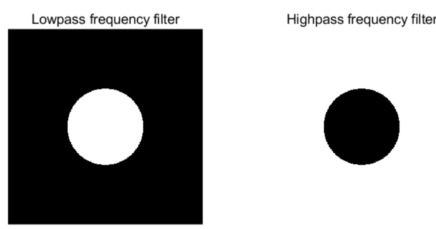

Fig 3.1 Lowpass and highpass filter

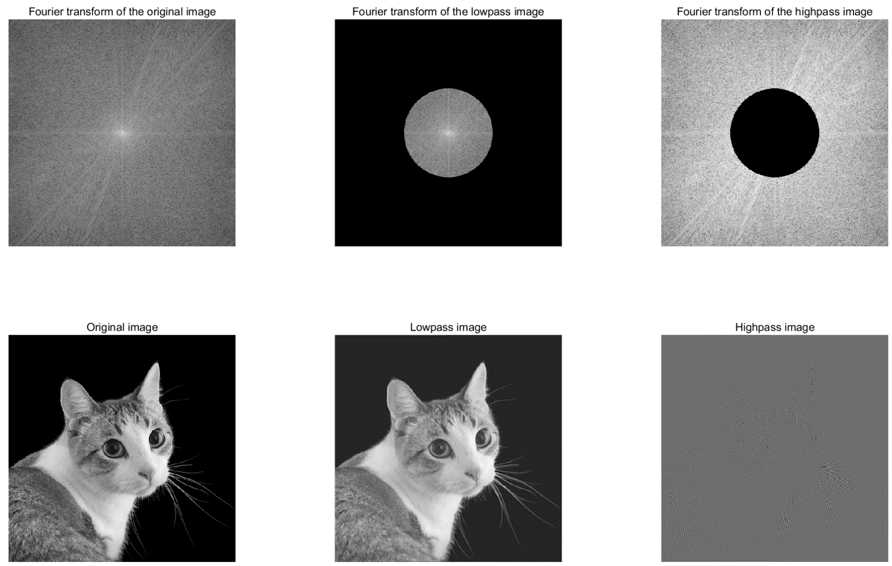

Fig 3.2 Original and filtered Fourier spectra and photographs

After the lowpass filter, the low-frequency information of the image is retained while the high-frequency signal is filtered. After the inverse Fourier transform, it's obviously that the subject outline of the image is still present. The details are missing and image becomes blurred.
After the highpass filter, conversely, high frequency signals are retained while low frequency signals are filtered out. Thus the image after the inverse Fourier transform has only the details of the contours.

Problem 4

a & b

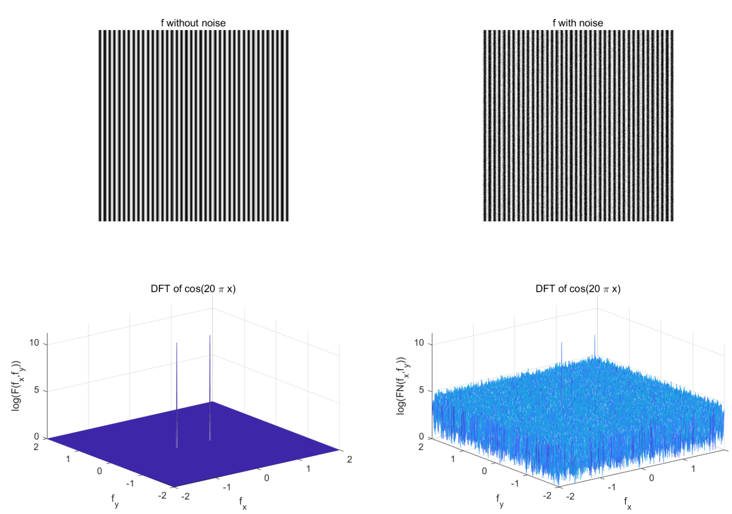

Fig 4.1 Signal and Fourier transform before and after adding noise

In the spatial domain, the addition of noise can blur a clear signal. As you can see in the image, the addition of Gaussian noise to the signal f results in a visually perceptible graininess or fuzziness overlaid on the

original signal.
In the Fourier domain, noise generally introduces additional frequency components that were not present in the original signal. Since Gaussian noise is broadband, covering a wide range of frequencies, the DFT of the noisy signal f_n will show increased magnitudes across a broader spectrum of frequencies

c

SNR is calculated as:

$$SNR = \frac{mean(f^2)}{mean((f_n - f)^2)}$$

After several attempts, the value of SNR was found to be around 12.5. This is because the average of cos^2 is 0.5, and the average energy of noise with std = 0.2 is 0.04. The SNR should be $0.5/0.04 = 12.5$.