

# STATISTICS FOR BUSINESS AND ECONOMICS

## CHAPTER 9 HYPOTHESIS TESTING- PART2



# 本章重點

- 9.3. Population Mean:  $\sigma$  known
  - ▣ Test Statistics.
  - ▣ One-tailed Test.
    - Rejection Rule.
      - $p$ -Value.
      - Critical Value.
  - ▣ Two-tailed Test.
    - Rejection Rule.
      - $p$ -Value.
      - Critical Value.
  - ▣ Summary and Practical Advice.
  - ▣ Relationship between Interval Estimation and Hypothesis Testing.
- 9.4. Population Mean:  $\sigma$  unknown

# Population Mean: $\sigma$ known

- Applicable circumstances:
  - ▣ The sample is selected from a population that is normally distributed.
  - ▣ Also applicable if the sample size is large enough incases where it is not reasonable to assume the population is normally distributed.
- One-tailed tests
  - ▣ Left-tailed test:
    - $H_0: \mu \geq \mu_0$
    - $H_a: \mu < \mu_0$
  - ▣ Right-tailed test:
    - $H_0: \mu \leq \mu_0$
    - $H_a: \mu > \mu_0$

# Population Mean: $\sigma$ known-Test Statistic

- **Ex.**, The Federal Trade Commission (FTC) periodically conducts statistical studies designed to test the claims that manufacturers make about their product. For example, the label on a large can of Hilltop Coffee states that the can contains 1.36 kilograms of coffee. The FTC knows the Hilltop's production process cannot place exactly 1.36 kilograms of coffee in each can, even if the mean filling weight for the population of all cans filled is 1.36 kilograms per can. However, as long as the population mean filling weight is at least 1.36 kilograms per can, the rights of consumers will be protected. Thus, the FTC interprets the label information on a large can of coffee as a claim by Hilltop that the population mean filling weight is at least 1.36 kilograms per can.

# Population Mean: $\sigma$ known-Test Statistic

- **Step1**-Develop the null and alternative hypotheses for the test.:
  - Use the claim by Hilltop as null hypothesis.
  - FTC test the Hilltop's claim, therefore the alternative hypothesis is that Hilltop's claim is incorrect.
    - Null hypothesis  $H_0: \mu \geq 1.36$ .
    - Alternative hypothesis  $H_a: \mu < 1.36$ .
      - **The hypothesized value of the population mean is  $\mu_0 = 1.36$ .**
- If the sample data indicate that  $H_0$  cannot be rejected, the statistical evidence does not support the conclusion that a label violation has occurred. Hence, no action should be taken against Hilltop.
- If the sample data indicate that  $H_0$  can be rejected, we will conclude that the alternative hypothesis,  $H_a: \mu < 1.36$  is true. In this case a conclusion of underfilling and a charge of a label violation against Hilltop would be justified.

# Population Mean: $\sigma$ known-Test Statistic

- **Step2**-Determining level of significance  $\alpha$ :
  - Suppose a sample of 36 cans of coffee is selected and the sample mean  $\bar{x}$  is computed as an estimate of the population mean  $\mu$ .
    - If the value of the sample mean  $\bar{x}$  is less than 1.36 kilograms, the sample results will cast doubt on the null hypothesis.
      - How much less than 1.36 kilograms must  $\bar{x}$  be before we would be willing to declare the difference significant and risk making a Type I error by falsely accusing Hilltop of a label violation. 要小於3磅多少, 才可以宣稱拒絕 $H_0$ ?
    - **How much risk of making a Type I error by falsely accusing Hilltop of a label violation can we take?**
  - The level of significance  $\alpha$ , is the probability of making a Type I error by rejecting  $H_0$  when it is true.
    - The decision maker must specify the level of significance.
    - If the cost of making a Type I error is high, a small value should be chosen for the level of significance.
  - If we set the level of significance for the hypothesis test at  $\alpha = 0.01$

# Population Mean: $\sigma$ known-Test Statistic

## □ **Step3**-Collect the sample data and compute the value of a test statistic:

### ■ Test statistic:

- Assume that the population standard deviation is known with a value of  $\sigma = 0.08$ .

- Sample size is 36.

- The **standard error** (standard deviation of a sampling distribution) of  $\bar{x}$  is given by:

- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.08}{\sqrt{36}} = 0.013$

- Assume that the sampling distribution of  $\bar{x}$  is normally distributed:

- $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{\bar{x} - 1.36}{0.013}$

- Use the standard normal probability table to find the lower tail probability corresponding to any z value.

□ For hypothesis tests about a population mean in the  $\sigma$  known case, we use the standard normal random variable z as a test statistic to determine whether  $\bar{x}$  deviates from **the hypothesized value** of  $\mu$  (which is  $\mu_0$ ) enough to justify rejecting the null hypothesis.

# Rejection Rule- *p*-value Approach

- Test statistic for hypothesis tests about a population mean:  $\sigma$  known
  - $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
- The key question for a **lower tail test** is: How small must the test statistic  $z$  be before we choose to reject the null hypothesis?
  - ***p*-Value approach:**
    - The *p*-value approach uses the value of the test statistic to compute a probability called a *p*-value..
    - A *p*-value is a probability that provides a measure of the evidence against the null hypothesis provided by the sample. Smaller *p*-values indicate more evidence against  $H_0$ .
    - Rejection rule (determine  $\alpha$ , calculate  $z$ , obtain *p*-value):
      - If  $p\text{-value} \leq \alpha$ , rejecting  $H_0$

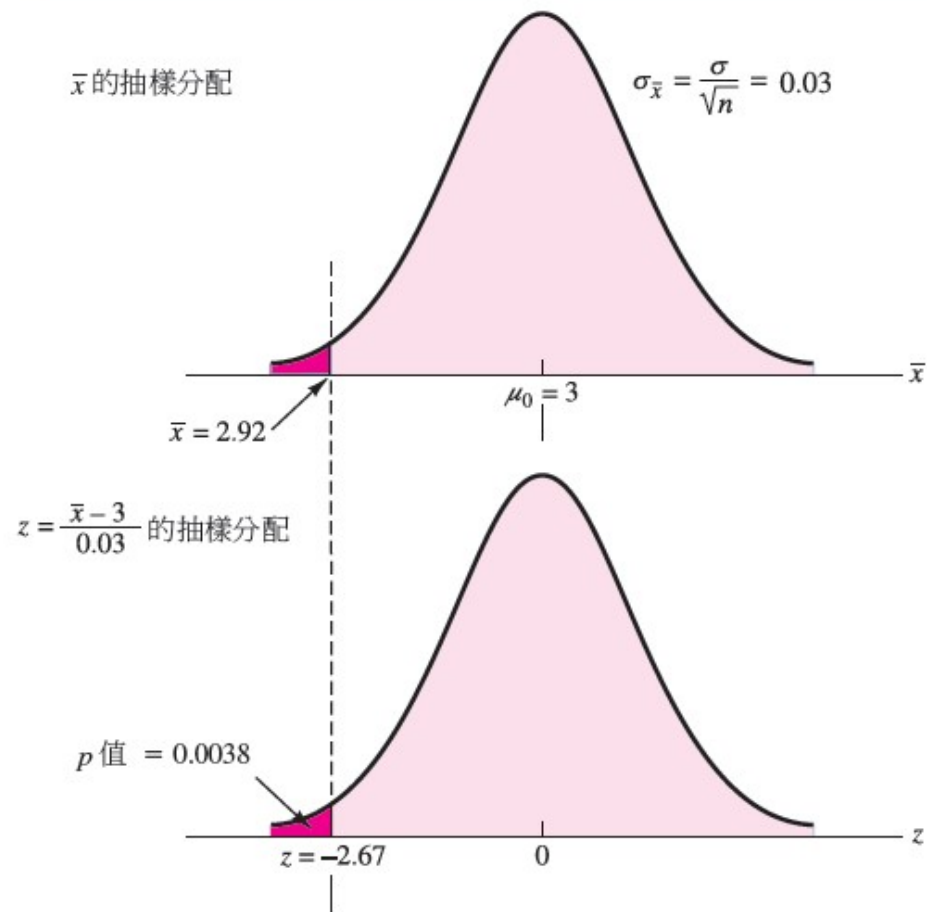


# Rejection Rule- *p*-value Approach

- **Ex.**, Suppose the sample of 36 Hilltop coffee cans provides a sample mean of  $\bar{x} = 1.325$  kilograms. Is this small enough to reject  $H_0$ ?
  - ▣  $\bar{x} = 1.325$ ,  $\sigma = 0.08$ ,  $n = 36$ , **test statistic** is the  $z$  value defined as:
    - $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{1.325 - 1.36}{0.08 / \sqrt{36}} = -2.63$
    - The  $p$ -value is the probability that  $z < -2.63$ , that is:  $P(z < -2.67) = 0.0043$ .
      - The lower tail area corresponding to the value of the test statistic.
    - The probability of obtaining a value of  $\bar{x} = 1.325$  or less when the null hypothesis is true as an equality ( $\mu_0 = 1.36$ ) is 0.43%.
  - ▣ If  $\alpha = 0.01$  is the **level of significance**.
    - It means that FTC is willing to tolerate a probability of 0.01 of rejecting the null hypothesis when it is true as an equality ( $\mu_0 = 1.36$ ).
    - When we obtain a  $p$ -value of 0.43%, it means that when null hypothesis is correct, the probability of obtaining a sample mean  $\bar{x} = 1.325$  or less is 0.43%, which is smaller than  $\alpha = 0.01$ .

# Rejection Rule- $p$ -value Approach

圖 9.2  $\bar{x} = 2.92$  且  $z = -2.67$  時，Hilltop 咖啡研究中的  $p$  值



# Rejection Rule- $p$ -value Approach

- In the Hilltop Coffee test, the  $p$ -value of 0.0043 resulted in the rejection of the null hypothesis at the 1% significance level.
  - ▣ **The rejection rule depends on the level of significance  $\alpha$ .**
  - ▣ However, the observed  $p$ -value of 0.0043 means that we would reject  $H_0$  for any value of  $\alpha \geq 0.0043$ .
    - There fore, the  $p$ -value is also called **the observed level of significance**.
- Different decision makers may express different opinions concerning the cost of making a Type I error and may choose a different level of significance.
  - ▣ By providing the  $p$ -value as part of the hypothesis testing results, another decision maker can compare the reported  $p$ -value **to his or her own level of significance** and possibly **make a different decision** with respect to rejecting  $H_0$ .

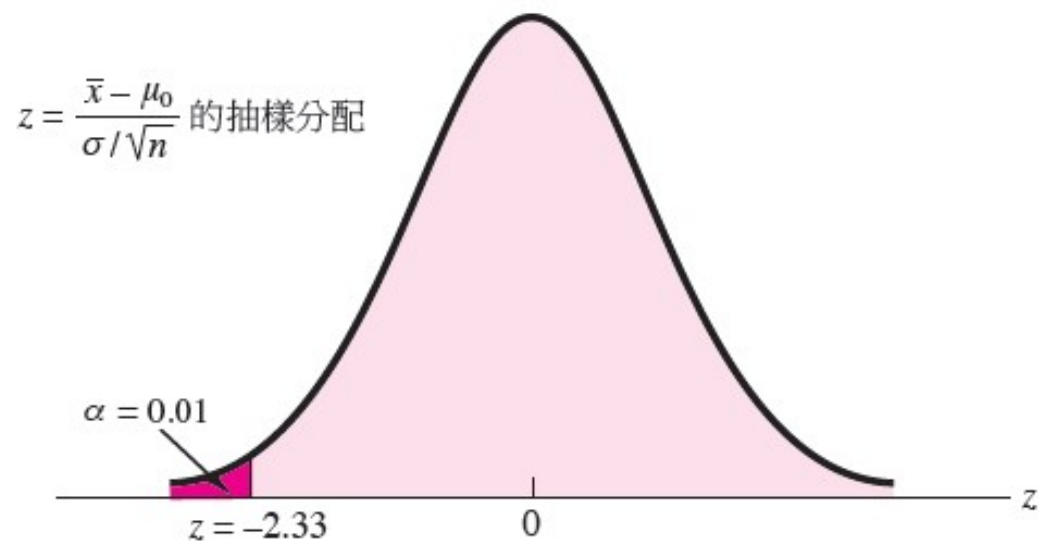
# Rejection Rule- Critical Value Approach

- Again, the key question for a **lower tail test** is: How small must the test statistic  $z$  be before we choose to reject the null hypothesis?
  - **Critical value approach:**
    - First determine a value for the test statistic called the critical value.
    - For a lower tail test, the critical value serves as a benchmark for determining whether the value of the test statistic is small enough to reject the null hypothesis.
      - It is the value of the test statistic that corresponds to an area of  $\alpha$  in the lower tail of the sampling distribution of the test statistic.
      - In other words, the critical value is **the largest value of the test statistic** that will result in the rejection of the null hypothesis.

# Rejection Rule- Critical Value Approach

圖 9.3

Hilltop 咖啡假設檢定的臨界值  $z = -2.33$



# Rejection Rule- Critical Value Approach

- **Ex.,** In the Hilltop Coffee example, assume that  $\alpha = 0.01$
- The critical value is the value of the test statistic that corresponds to an area of  $\alpha = 0.01$  in the lower tail of a standard normal distribution.
  - ▣ Using the standard normal distribution table, we find that  $z = -2.33$  provides an area of 0.01 in the lower tail.
  - ▣ Thus, if the sample results in a value of the test statistic that is less than or equal to -2.33, the corresponding  $p$ -value will be less than or equal to 0.01, reject  $H_0$ .
- The rejection rule at the 1% significance level is:
  - ▣ If  $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \leq -2.33$ , reject  $H_0$ .
- We have  $\bar{x} = 1.325$ , the standard error  $\sigma/\sqrt{n} = 0.08/\sqrt{36}$ , thus, the test statistic  $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = -2.63$ 
  - ▣ Because  $z = -2.63 < -2.33$ , we can reject  $H_0$ .
  - ▣ We can conclude that Hilltop Coffee is underfilling cans.

# Rejection Rule- Critical Value Approach

- Rejection rule for a lower tail test: Critical value approach:
  - **Reject  $H_0$  if  $z \leq -z_\alpha$ .**
  - $z_\alpha$  is the critical value, it is the  $z$  value that provides an area of  $\alpha$  in the lower tail of the standard normal distribution..

# Summary

- The  $p$ -value approach to hypothesis testing and the critical value approach will **always lead to the same rejection decision**. That is:
  - Whenever the  $p$ -value is less than or equal to  $\alpha$ .
    - The advantage of the  $p$ -value approach is that the  $p$ -value tells us **how significant the results are** (the observed level of significance).
  - The value of the test statistic will be less than or equal to the critical value.
    - If we use the critical value approach, we only know that the results are significant at the stated level of significance..



# Summary of One-tailed Tests

- Calculating test statistic:  $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$
- Lower tail test: Use the standard normal probability table, compute the probability that  $z$  is less than or equal to the value of the test statistic (area in the lower tail).
  - $H_0: \mu \geq \mu_0$
  - $H_a: \mu < \mu_0$
  - Rejection rule:
    - $p$ -value: the above-mentioned area is **smaller than or equal to** the significance level  $\alpha$ , reject  $H_0$ .
    - Critical value: if the above-mentioned  $z \leq -z_\alpha$ , reject  $H_0$ .
- Upper tail test: Use the standard normal probability table, compute the probability that  $z$  is greater than or equal to the value of the test statistic (area in the upper tail).
  - $H_0: \mu \leq \mu_0$
  - $H_a: \mu > \mu_0$
  - Rejection rule:
    - $p$ -value: the above-mentioned area is **smaller than or equal to** the significance level  $\alpha$ , reject  $H_0$ .
    - Critical value: if the above-mentioned  $z \geq z_\alpha$ , reject  $H_0$ .

# Two-Tailed Test

- Two-tailed test for population mean  $\mu$ :
- Calculating test statistic:  $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$
- Two-tailed test:
  - ▣  $H_0: \mu = \mu_0$
  - ▣  $H_a: \mu \neq \mu_0$
- (1) If the value of the test statistic is in the upper tail (檢定統計量為正), compute the probability that  $z$  is greater than or equal to the value of the test statistic (the upper tail area).
- (2) If the value of the test statistic is in the lower tail (檢定統計量為負), compute the probability that  $z$  is less than or equal to the value of the test statistic (the lower tail area).
- Double the probability (or tail area) from above-mentioned area to obtain  $p$ -value.

# Two-Tailed Test

- **Ex.**, The U.S. Golf Association (USGA) establishes rules that manufacturers of golf equipment must meet if their products are to be acceptable for use in USGA events. MaxFlight use a high-technology manufacturing process to produce golf balls with a mean driving distance of 295 yards. Sometimes, however, the process gets out of adjustment and produces golf balls with a mean driving distance different from 295 yards. When the mean distance falls below 295 yards, the company worries about losing sales because the golf balls do not provide as much distance as advertised. When the mean distance passes 295 yards, MaxFlight's golf balls may be rejected by the USGA for exceeding the overall distance standard concerning carry and roll.

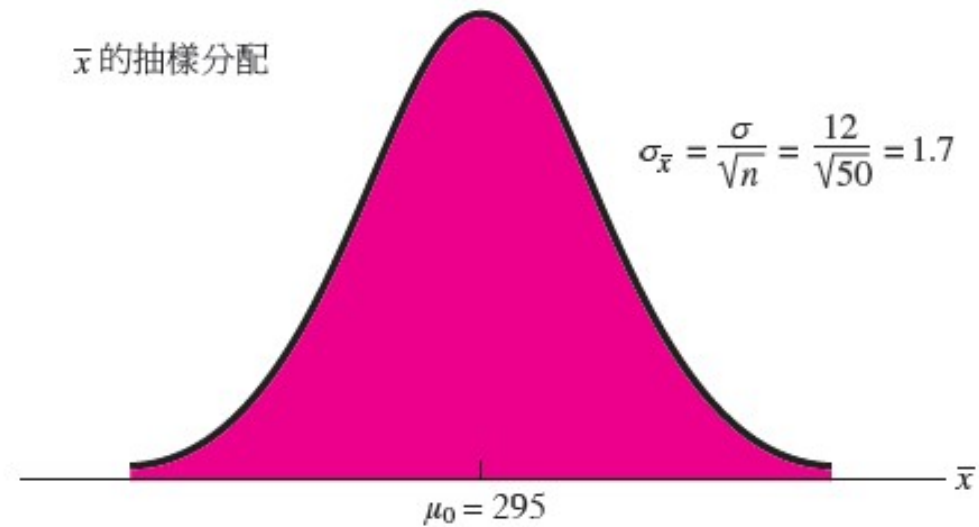
# Two-Tailed Test

- MaxFlight's quality control program involves taking periodic samples of 50 golf balls to monitor the manufacturing process. For each sample a hypothesis test is conducted to determine whether the process has fallen out of adjustment.
- We begin by assuming that the process is functioning correctly:
  - ▣  $H_0: \mu = 295$
  - ▣  $H_a: \mu \neq 295$
- The quality control team selected  $\alpha = 0.05$  as the level of significance for the test. From previous tests, the population standard deviation can be assumed known with a value of  $\sigma = 12$ , the standard error is  $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{50}} = 1.7$

# Two-Tailed Test

圖 9.4

MaxFlight 假設檢定中， $\bar{x}$  的抽樣分配



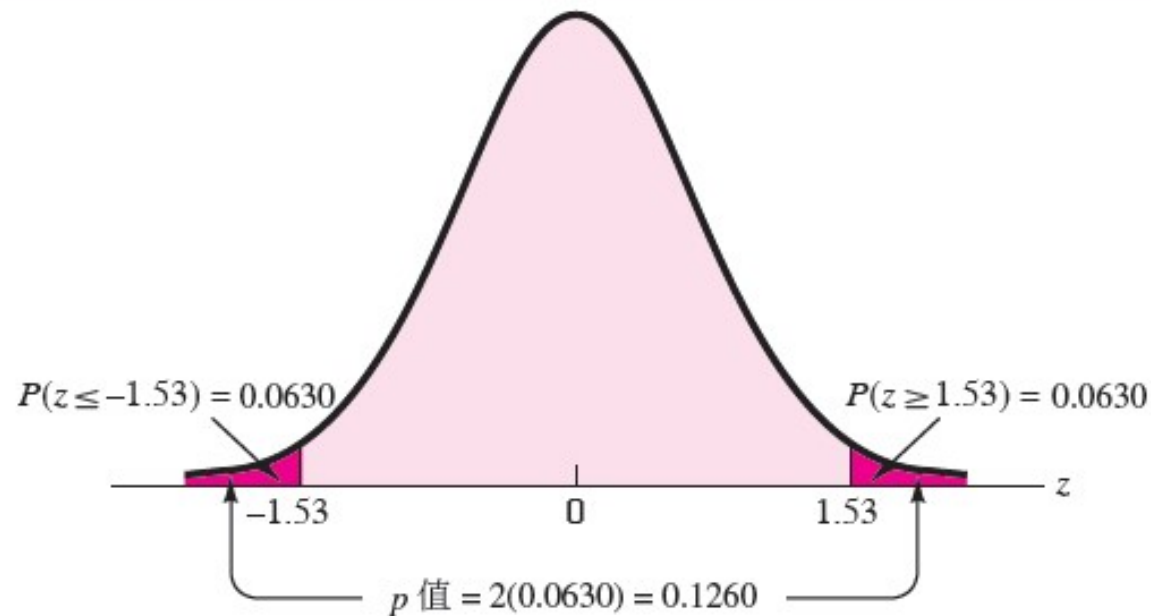
# Two-Tailed Test- $p$ -value Approach

- ▣ Suppose that a sample of 50 golf balls is selected and that the sample mean is  $\bar{x} = 297.6$  yards. Is this value of  $\bar{x}$  enough larger than 295 to cause us to reject  $H_0$ .
- ▣  **$p$ -value approach:**
  - Calculating the test statistic:  $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{297.6 - 295}{12 / \sqrt{50}} = 1.53$
  - The test statistic is positive, therefore we look up the standard normal probability table and find  $P(z \geq 1.53) = 1 - 0.9370 = 0.0630$
  - Double the probability (area).
  - $\mu$  is not likely equal to 295:  $z \geq 1.53$  and  $z \leq -1.53$ . The probability should be  $P(z \geq 1.53) + P(z \leq -1.53) = 0.0630 + 0.0630 = 0.1260$
  - Because the significance level  $\alpha = 0.05$ ,  $p$ -value is  $0.1260 > 0.05$ , we cannot reject  $H_0$

# Two-Tailed Test- $p$ -value Approach

圖 9.5

MaxFlight 假設檢定的  $p$  值



# Two-Tailed Test- Critical Value Approach

## ■ Critical value approach:

- Calculating the test statistic:  $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{297.6 - 295}{12 / \sqrt{50}} = 1.53$
- With a level of significance of  $\alpha = 0.05$ , the area **in each tail** corresponding to the critical values is  $\frac{\alpha}{2} = 0.025$ . Using the standard normal probability table, we find the critical values for the test statistic are  $-z_{0.025} = -1.96$  and  $z_{0.025} = 1.96$ .
- Rejection rule:
  - If  $z \leq -1.96$  or  $z \geq 1.96$ , reject  $H_0$ .
- Because the value of the test statistic for the MaxFlight study is  $z = 1.53$ , the statistical evidence will not permit us to reject the null hypothesis  $H_0$  at the 0.05 level of significance.



# Summary and Practical Advice

- Steps of hypothesis testing:
  - **Step1**: Develop the null and alternative hypotheses.
  - **Step2**: Specify the level of significance  $\alpha$ .
  - **Step3**: Collect the sample data and compute the value of the test statistic.
- Rejection rule:
  - **p-value**:
    - Use the value of the test statistic to compute the p-value (use the standard normal probability table).
      - **Note**: when it is a two-tailed test, double the probability.
    - Reject  $H_0$  if  $p\text{-value} \leq \alpha$
  - **Critical value**:
    - Use the level of significance to determine the critical value and the rejection rule.
      - **Note**: when it is a two-tailed test, find  $z_{\alpha/2}$ .
    - Use the value of the test statistic and the rejection rule to determine whether to reject  $H_0$ .

# Summary and Practical Advice

表 9.2  $\sigma$  已知時，母體平均數假設檢定彙整表

	左尾檢定	右尾檢定	雙尾檢定
假設	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
檢定統計量	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
拒絕法則： $p$ 值法	若 $p$ 值 $\leq \alpha$ ， 則拒絕 $H_0$	若 $p$ 值 $\leq \alpha$ ， 則拒絕 $H_0$	若 $p$ 值 $\leq \alpha$ ， 則拒絕 $H_0$
拒絕法則： 臨界值法	若 $z \leq -z_{\alpha}$ ， 則拒絕 $H_0$	若 $z \geq z_{\alpha}$ ， 則拒絕 $H_0$	若 $z \leq -z_{\alpha/2}$ ， 或 $z \geq z_{\alpha/2}$ ， 則拒絕 $H_0$

# Relationship between Interval Estimation and Hypothesis Testing

- In Chapter 8, if  $\sigma$  is known and the confidence coefficient is  $1 - \alpha$ , then the  $(1 - \alpha)\%$  confidence interval estimation of a population mean is:
  - ▣  $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
- In Chapter 9, a two-tailed hypothesis test about a population mean is:
  - ▣  $H_0: \mu = \mu_0$
  - ▣  $H_a: \mu \neq \mu_0$
- We know that  $100 * (1 - \alpha)\%$  of the **confidence intervals generated** will contain the population mean.
- and  $100\alpha\%$  of the **confidence intervals generated** will not contain the population mean..
  - ▣ Thus, if we reject  $H_0$  whenever the confidence interval does not contain  $\mu_0$ , we will be rejecting the null hypothesis when it is true ( $\mu = \mu_0$ ) with probability  $\alpha$ .

# Relationship between Interval Estimation and Hypothesis Testing

- A confidence interval approach to testing a hypothesis of the form:
  - ▣ **Step1**, Develop the null and alternative hypotheses.
    - $H_0: \mu = \mu_0$
    - $H_a: \mu \neq \mu_0$
  - ▣ **Step2**, Select a simple random sample from the population and use the value of the sample mean  $\bar{x}$  to develop the confidence interval for the population mean  $\mu$ .
    - $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$
  - ▣ **Step3**, If the confidence interval contains the hypothesized value  $\mu_0$ , do not reject  $H_0$ . Otherwise, reject  $H_0$

# Relationship between Interval Estimation and Hypothesis Testing

- **Ex.**, The MaxFlight hypothesis test takes the following form:
  - ▣  $H_0: \mu = 295$
  - ▣  $H_a: \mu \neq 295$
- We sampled 50 golf balls and found a sample mean distance of  $\bar{x} = 297.6$ . The population standard deviation is  $\sigma = 12$ . The standard error is  $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{50}} = 1.7$ . With a level of significance  $\alpha = 0.05$ . We construct a 95% confidence interval estimate:
  - ▣  $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \rightarrow 297.6 \pm z_{0.025} * 1.7 \rightarrow 297.6 \pm 1.96 * 1.7 \rightarrow (294.3, 300.9)$
  - ▣ The hypothesized value for the population mean,  $\mu_0 = 295$ , is in this interval.
  - ▣ The hypothesis testing conclusion is that the null hypothesis,  $H_0: \mu = 295$ , cannot be rejected..

# Population Mean: $\sigma$ unknown-Test Statistic

- Because the  $\sigma$  is unknown, the sample must be used to develop an estimate of both  $\mu$  and  $\sigma$ .
- In the  $\sigma$  unknown case, the sample standard deviation  $s$  is used as an estimate of  $\sigma$ . The sampling distribution of the test statistic follows the  $t$  distribution.
- Test statistic for hypothesis tests about a population mean:  $\sigma$  unknown:
  - ▣  $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
  - ▣ where  $s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$  is the sample standard deviation.

# Population Mean: $\sigma$ unknown- One tailed test

- **Ex.**, A business travel magazine wants to classify transatlantic gateway airports according to the mean rating for the population of business travelers. A rating scale with a low score of 0 and a high score of 10 will be used, and airports with a population mean rating greater than 7 will be designated as a superior service airports. The sample for London's Heathrow airport provided a sample mean rating of  $\bar{x} = 7.25$  and a sample standard deviation of  $s = 1.052$  with a sample size of 60 business travelers. Do the data indicate that Heathrow should be designated as a superior service airport?

# Population Mean: $\sigma$ unknown- One tailed test

- **Step1**- Develop null and alternative hypotheses:
  - ▣  $H_0: \mu \leq 7$
  - ▣  $H_a: \mu > 7$ 
    - This is a upper tail test.
- **Step2**- Determine the level of significance:
  - ▣ We will use  $\alpha = 0.05$  as the level of significance for the test.
- **Step3**- The value of test statistic:
  - ▣ Because  $\sigma$  is unknown,  $s$  is used to estimate  $\sigma$ , we must use  $t$  distribution.
  - ▣  $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{7.25 - 7}{1.052/\sqrt{60}} = 1.84$




# Population Mean: $\sigma$ unknown- One tailed test

## □ Rejection rule:

### □ **$p$ -value approach:**

- The sampling distribution of  $t$  has  $n - 1 = 60 - 1 = 59$  degrees of freedom.
- Because this is an upper tail test, the  $p$ -value is  $P(t \geq 1.84)$ , that is, the upper tail area corresponding to the value of the test statistic  $t = 1.84$ .
  - Unlike standard normal probability table, the  $t$  distribution table will not contain sufficient detail to determine the exact  $p$ -value.

右尾面積	0.20	0.10	0.05	0.025	0.01	0.005
$t$ 值 (自由度 59)	0.848	1.296	1.671	2.001	2.391	2.662

  $t = 1.84$

- We see that  $t = 1.84$  is between 1.671 and 2.001.
- The  $p$ -value must be less than 0.05 and greater than 0.025.
- Thus,  $p\text{-value} < \alpha$ , we reject the null hypothesis  $H_0$ .

# Population Mean: $\sigma$ unknown- One tailed test

## ■ Critical value approach:

- The critical value corresponding to an area of  $\alpha = 0.05$  in the upper tail of a  $t$  distribution with 59 degrees of freedom is  $t_{0.05} = 1.671$ .
- Thus, the rejection rule using the critical value approach is to reject  $H_0$  if  $t \geq 1.671$ .
- Test statistic is  $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{7.25 - 7}{1.052/\sqrt{60}} = 1.84 \geq 1.671$ 
  - Reject  $H_0: \mu \leq 7$ .
  - Heathrow should be classified as a superior service airport..

# Population Mean: $\sigma$ unknown- Two-tailed test

- Ex., A company, Holiday Toys, manufactures and distributes its products through more than 1000 retail outlets. For this year's most important new toy, Holiday's marketing director is expecting demand to average 40 units per retail outlet. Prior to making the final production decision based upon this estimate, Holiday decided to survey a sample of 25 retailers in order to develop more information about the demand for the new product. With  $\mu$  denoting the population mean order quantity per retail outlet, the sample data will be used to conduct the following two-tailed hypothesis test:

# Population Mean: $\sigma$ unknown- Two-tailed test

- **Step1**-Develop null and alternative hypotheses:
  - ▣  $H_0: \mu = 40$
  - ▣  $H_a: \mu \neq 40$ 
    - It's a two-tailed test.
- **Step2**-Determine the level of significance:
  - ▣ Assume that  $\alpha = 0.05$ .
- **Step3**-Calculating test statistic:
  - ▣ The population standard deviation  $\sigma$  is unknown, and we use sample standard deviation  $s$  to estimate  $\sigma$ . Therefore, we should use  $t$  distribution. The sample of 25 retailers provided a mean of  $\bar{x} = 37.4$  and a standard deviation  $s = 11.79$ . The degrees of freedom is  $n - 1 = 25 - 1 = 24$ .
  - ▣  $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{37.4 - 40}{11.79/\sqrt{25}} = -1.10$


# Population Mean: $\sigma$ unknown- Two-tailed test

## □ Rejection rule:

### □ **$p$ -value approach:**

- The sampling distribution is a  $t$  distribution with 24 degrees of freedom.
- Because it is a two-tailed test and the test statistic  $t = -1.10$  is negative. Therefore, the  $p$ -value is two times the area under the curve of 24 degrees of freedom provides the following information.
  - The  $t$  distribution table only contains positive  $t$  value. Because the  $t$  distribution is symmetric, however, the upper tail area at  $t = 1.10$  is the same as the lower tail area at  $t = -1.10$ .

右尾面積	0.20	0.10	0.05	0.025	0.01	0.005
$t$ 值 (自由度 24)	0.857	1.318	1.711	2.064	2.492	2.797

  $t = 1.10$

- We see that  $t = 1.10$  is between 0.857 and 1.318
- The area in the upper tail at  $t = 1.10$  is between 0.20 and 0.10. We have to **double these amount**.
- $p$ -value must be between 0.40 and 0.20, and  $p\text{-value} > \alpha$ , cannot reject  $H_0$ .

# Population Mean: $\sigma$ unknown- Two-tailed test

## ▣ Critical value approach:

- With  $\alpha = 0.05$  and the  $t$  distribution with 24 degrees of freedom.
- $\alpha/2 = 0.025$ , the critical values are  $t_{0.025} = 2.064$  and  $-t_{0.025} = -2.064$ .
- Using the test statistic:  $t \leq -2.064$  or  $t \geq 2.064$ , reject  $H_0$ .
- The test statistic is  $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = -1.10$ , which is not smaller than -2.064.
  - $H_0: \mu = 40$  cannot be rejected.
  - This result indicates that Holiday should continue its production planning for the coming season based on the expectation that  $\mu = 40$ .

# Population Mean: $\sigma$ unknown- Two-tailed test

表 9.3  $\sigma$  未知時，母體平均數假設檢定的彙整表

	左尾檢定	右尾檢定	雙尾檢定
假設	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
檢定統計量	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
拒絕法則： $p$ 值法	若 $p$ 值 $\leq \alpha$ ， 則拒絕 $H_0$	若 $p$ 值 $\leq \alpha$ ， 則拒絕 $H_0$	若 $p$ 值 $\leq \alpha$ ， 則拒絕 $H_0$
拒絕法則： 臨界值法	若 $t \leq -t_\alpha$ ， 則拒絕 $H_0$	若 $t \geq t_\alpha$ ， 則拒絕 $H_0$	若 $t \leq -t_{\alpha/2}$ 或 $t \geq t_{\alpha/2}$ ， 則拒絕 $H_0$